

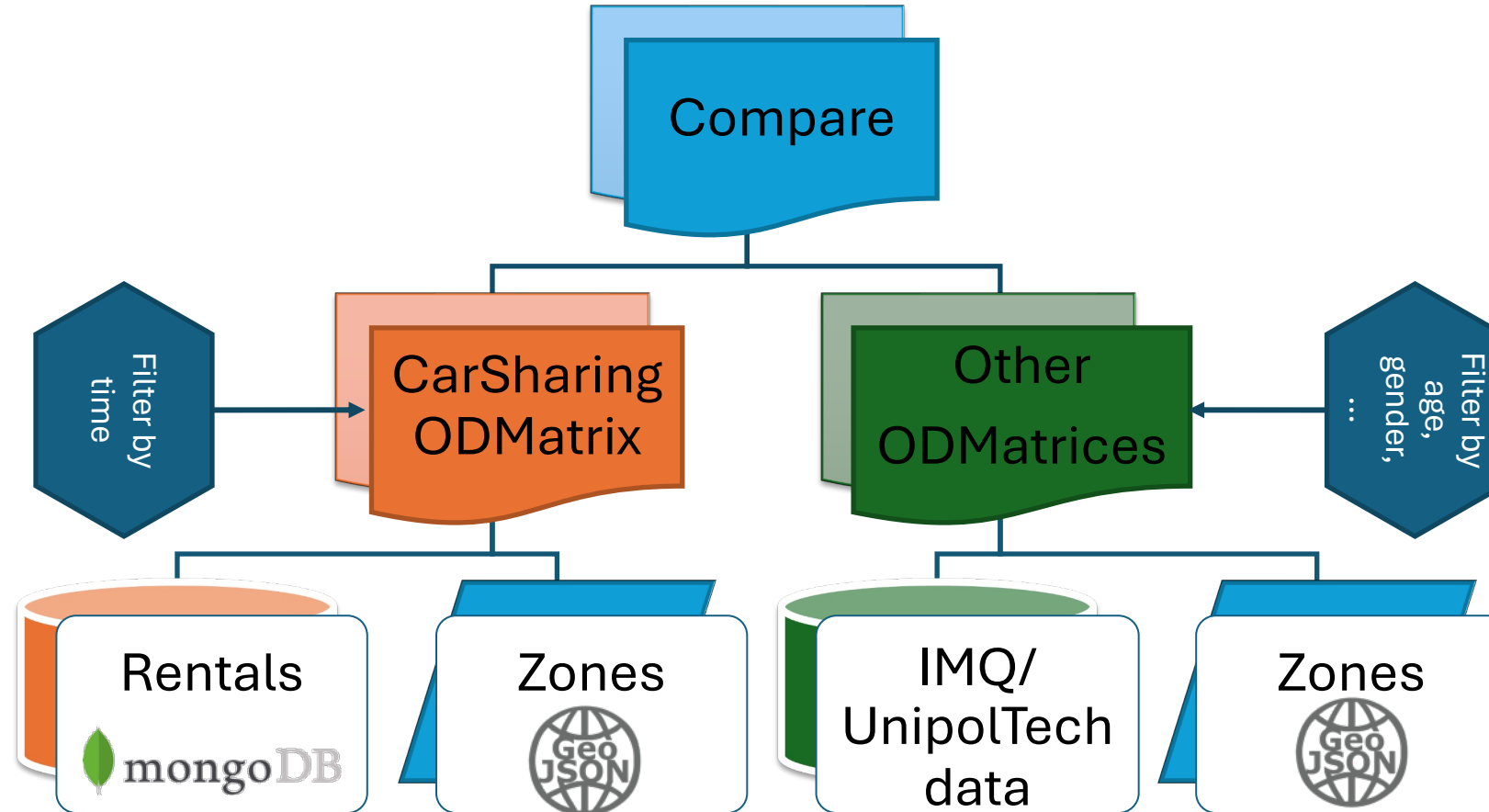
LAB 2

OD Matrices analytics

Goal

- This laboratory builds on the data analyzed during the first lab
- We want to find out which are the most likely class of users of car sharing system
- For this, we compare the OD matrices obtained from different sources
- Use metadata to filter
 - Based on gender, age, goals of trip, etc.
- And obtain the most similar OD matrix as the one from car sharing users

Overall view



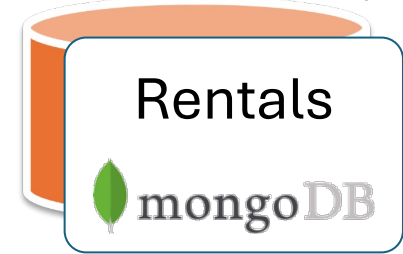
Datasets

3 Datasets

- Car Sharing rentals to obtain OD matrix of car sharing users
 - Car rentals for Enjoy and Car2Go
 - 2 months of data, with Origin/Destination (indexed for running geospatial queries)
 - Stored on MongoDB
- Open data from IMQ 2013 (Indagine Mobilità e Qualità)
 - <https://mtm.torino.it/it/dati-statistiche/indagini/matrici-od-imq-2013/>
- Data from UnipolTech collected in 2024
- **DO NOT SHARE carsharing and UnipolTech data**

Carsharing dataset

- Rentals are available on MongoDB
 - **ictts_PermanentBookings**
 - **ictts_enjoy_PermanentBookings**
- There are 3 indexes
 - `init_loc` and `final_loc` with 2D coordinates to support geospatial queries
 - `init_time` for filtering over time

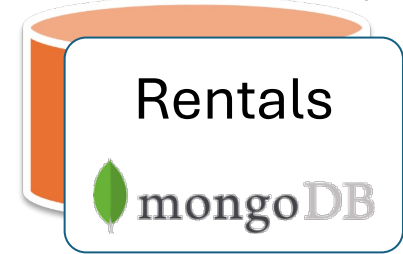


Extracting data from rentals

- We can filter
 - By time
 - By position
- Useful **time operators** in `$aggregate` pipeline
 - `$hours` – extract the hour in UTC from a `ISODate`
 - E.g., filter on rentals between midnight and midday
 - `$dayOfWeek` – Returns the day of the week for a date as a number
 - 1 (Sunday) and 7 (Saturday)
 - E.g., filter on rentals on Monday, Tuesday,..., Friday only

Example

```
db.ictts_PermanentBookings.aggregate([
  { $project: {
    hour: { $hour: "$init_date" },
    day: { $dayOfWeek: "$init_date" },
    init_loc: 1,
    final_loc: 1
  }
},
  { $match: {
    hour: { $gte: 0, $lt: 12 },
    day: { $gt: 1, $lt: 7 }
  }
},
  { $count: "tot"
}
])
```



*Transform the init_date
to get the hour and day*

*Filter on morning
and weekdays (1=Sunday, ...)*

Then count

Geospatial queries

- MongoDB's geospatial indexing allows you to efficiently execute spatial queries on a collection that contains geospatial shapes and points
- the geometry data in the location field must follow the **GeoJSON format**

```
<field>: { type: <GeoJSON type> , coordinates: <coordinates> }
```

- Type can be
 - Point: { type: "Point", coordinates: [40, 5] }
 - Linestring: { type: "LineString", coordinates: [[40, 5], [41, 6]] }
 - Polygon: { type: "Polygon", coordinates: [[[0 , 0] , [3 , 6] , [6 , 1] , [0 , 0]]] }
 - ...

Geospatial queries

- MongoDB support geospatial indexes that allow efficient queries
- A `2dsphere` index supports queries that calculate geometries on an earth-like sphere

Possible queries

- `$geoWithin`: query for location data found within a GeoJSON polygon
 - location data must be stored in GeoJSON format

```
db.<collection>.find( { <location field> :  
  { $geoWithin :  
    { $geometry :  
      { type : "Polygon",  
        coordinates : [ <coordinates> ]  
      }  
    }  
  }  
}
```

Geospatial queries

Other possible queries:

- `$geoIntersects` to select all indexed points and shapes that intersect with the polygon defined by the coordinates array
- `$geoNear` operator or `$geoNear` return the points closest to the defined point and sorts the results by distance

```
db.<collection>.find( { <location field> :  
  { $geoNear :  
    { $geometry :  
      { type : "Point" ,  
        coordinates : [ <longitude> , <latitude> ] },  
      $maxDistance : <distance in meters>  
    }  
  }  
} )
```

Example of geospatial query

- Find all cars available in a region of 1000m from Politecnico di Torino entrance

```
db.getCollection('ActiveParkings').find(  
  {loc:  
    { $geoNear:  
      { $geometry:  
        { type: "Point",  
          coordinates: [ 7.660, 45.0645 ]  
        },  
        $maxDistance: 1000  
      }  
    }  
  }  
)
```

Warning
Cost of geoqueries is not negligible

Extracting data from rentals

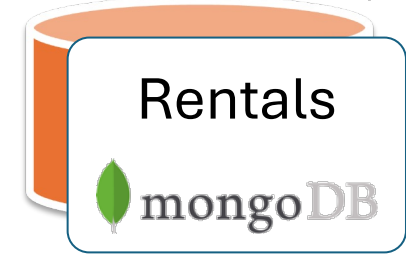


- Useful **geospatial operators** in \$aggregate pipeline
 - \$geoWithin: selects documents with geospatial data that exists entirely within a specified shape
 - The shape can be
 - GeoJSON Polygon
 - GeoJSON MultiPolygon (Polygon with “holes”)
 - The \$geoWithin operator uses the \$geometry operator to specify the GeoJSON object

```
{  
  <location field>: {  
    $geoWithin: {  
      $geometry: {  
        type: <"Polygon" or "MultiPolygon"> ,  
        coordinates: [ <coordinates> ]  
      }  
    }  
  }  
}
```

Extracting the O/D matrix

```
var orig_zone = ... // put here the arrays describing the zone
var dest_zone = ... // put here the arrays describing the zone
db.ictts_PermanentBookings.aggregate([
  { $match: {
    init_loc: { $geoWithin :
      { $geometry: {
        "type": "MultiPolygon",
        "coordinates": orig_zone
      }
    },
    final_loc: { $geoWithin :
      { $geometry: {
        "type": "MultiPolygon",
        "coordinates": dest_zone
      }
    }
  }
},
  { $count: "tot"
}
])
```



*Filter
on init_loc that falls
within the orig_zone*

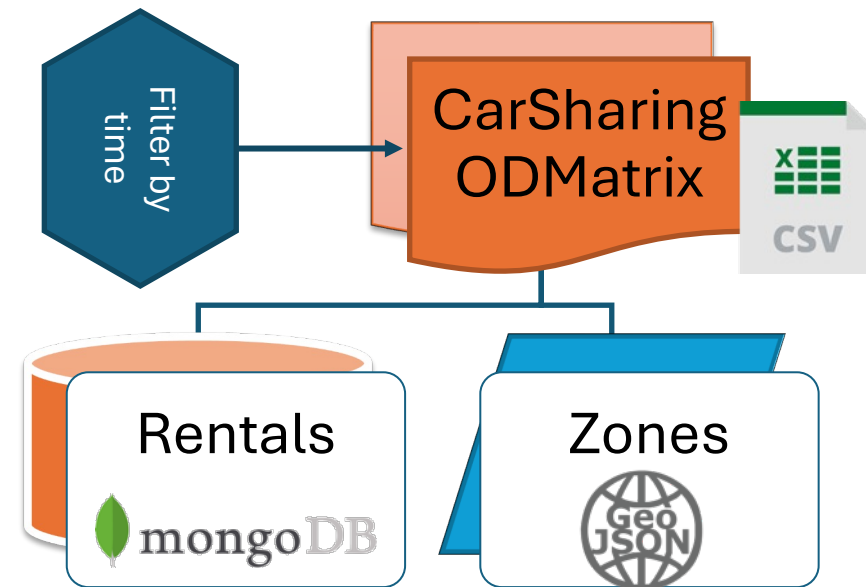
*and final_loc that falls
within the dest_zone*

Then count

Putting everything together

Prepare a javascript file with the program to run

- For each orig_zone
 - For each dest_zone
 - Count rentals in ODMatrix
 - From orig_zone
 - To dest_zone
 - In a given timeslot
- Derive different ODMatrixes
 - For Enjoy, Car2Go
 - Considering
 - All rentals
 - Rentals in weekends or weekdays only
 - Mornings and afternoons
 - ...
- Save in a CSV file



UnipolTech dataset



- Data collected by company – Anonymized
 - One week in September 2024 in Torino
- Data in 2 .csv files, link them by vehicle ID (`id_veicolo`)
- Information of users/vehicles (`Info_TO.csv`)
 - In total about 38k users
 - Gender, age, whether for commercial use
- Information of trips (`Trips_OD_TO.csv`)
 - Origin/Destination location (latitude and longitude)
 - Departure/arrival time
 - In total 545k trip records

Dataset – IMQ

- IMQ 2013 contains data collected with phone interviews
 - 52120 completed interviews
 - 105099 trips reported
 - Trips from Monday to Friday
 - Covers 185 zones of Piedmont
 - **23 zones in Torino, we will use them**
- Interviews are stratified by
 - Gender (male/female)
 - Age (11-19, 20-49, 50-64, 65+)
 - Motivation (go to work, got to school, visiting parents, ...)

Dataset - IMQ

- 183 zones
- Defined in a shape file
 - Resource: **Zone_IMQ_TO.rar**
- Available as geojson shape
 - Resource: **zoneBeauty.geojson**
- 23 zones in Torino
 - Resource: **TorinoZonescol.geojson**
 - Resource: **TorinoZonesArray.geojson**

Q001	TORINO - CENTRO
Q002	TORINO - S.SALVARIO
Q003	TORINO - CROCETTA
Q004	TORINO - S.PAOLO
Q005	TORINO - CENISIA
Q006	TORINO - S.DONATO
Q007	TORINO - AURORA
Q008	TORINO - VANCHIGLIA
Q009	TORINO - NIZZA-MILLEFONTI
Q010	TORINO - LINGOTTO
Q011	TORINO - S.RITA
Q012	TORINO - MIRAFIORI NORD
Q013	TORINO - POZZO STRADA
Q014	TORINO - PARELLA
Q015	TORINO - VALLETTE
Q016	TORINO - MADONNA CAMPAGNA
Q017	TORINO - B.TA VITTORIA
Q018	TORINO - B.RA MILANO
Q019	TORINO - FALCHERA
Q020	TORINO - REGIO PARCO
Q021	TORINO - MADONNA PILONE
Q022	TORINO - CAVORETTO
Q023	TORINO - MIRAFIORI SUD

IMQ data



- Phone interviews
 - From Tuesday to Saturday - 9.30 and 21.30
- Asked about movements done the day before
 - **Only obtained trips during weekdays**
 - Hour, departure place, arrival place, reason for the trip, modes,...
 - <http://mtm.torino.it/it/dati-statistiche/imq-alle-fasi-finali>
 - Sampling specifications:
 - 3% in the Turin metropolitana area
 - At least 60 interviews per zone
- Results are available as open data
 - <http://mtm.torino.it/it/dati-statistiche/indagine-imq-2013/base-dati-imq-2013>
[not available anymore]

IMQ data

- Raw data in Microsoft Access from http://mtm.torino.it/it/dati-statistiche/indagine-imq-2013/base-dati-imq-2013/IMQ2013_opendata.zip [not available anymore]
- There are several tables
 - Spostamenti: information about the movements done in the previous day
 - Interviste: information about the person
 - Tab_*: tables explaining the mapping for each category
- We have extracted the subset of movements done in the Torino area
 - From zone Q* to zone Q*
 - 16567 trips in total

IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR
8139335	1	2	1	2	C006	5	Q005	1	02/02/17 10:00	Q023	1	02/02/17 10:20
8139641	1	2	2	4	C009	4	Q002	1	02/02/17 16:30	Q023	1	02/02/17 16:50
8139830	1	2	1	2	C004	4	Q019	1	02/02/17 14:30	Q019	1	02/02/17 14:50
8140616	1	2	1	2	C003	9	Q003	1	02/02/17 20:00	Q005	1	02/02/17 20:30
8140678	1	2	2	3	C011	4	Q009	1	02/02/17 09:00	Q009	1	02/02/17 09:10
8140745	1	2	2	3	C004	4	Q015	1	02/02/17 18:00	Q021	1	02/02/17 18:15
8140799	1	2	2	3	C006	4	Q010	1	02/02/17 10:45	Q020	1	02/02/17 11:00
8141314	1	2	2	3	C009	4	Q022	1	02/02/17 18:15	Q022	1	02/02/17 18:30
8141976	2	2	1	4	C003	9	Q001	1	02/02/17 15:40	Q007	1	02/02/17 16:10
8142800	1	2	2	2	C003	7	Q012	1	02/02/17 19:30	Q019	1	02/02/17 19:45
8143087	1	2	2	3	C003	4	Q006	1	02/02/17 15:00	Q006	1	02/02/17 15:10
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35

IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR
8139335	1	2	1	2	C006	5	Q005	1	02/02/17 10:00	Q023	1	02/02/17 10:20
8139641	1	2	2	4	C009	4	Q002	1	02/02/17 16:30	Q023	1	02/02/17 16:50
8139830	1	2	1	2	Gender						1	02/02/17 14:50
8140616	1	2	1	2	1 Male						1	02/02/17 20:30
8140678	1	2	2	3	2 Female						1	02/02/17 09:10
8140745	1	2	2	3	C004	4	Q015	1	02/02/17 18:00	Q021	1	02/02/17 18:15
8140799	1	2	2	3	C006	4	Q010	1	02/02/17 10:45	Q020	1	02/02/17 11:00
8141314	1	2	2	3	C009	4	Q022	1	02/02/17 18:15	Q022	1	02/02/17 18:30
8141976	2	2	1	4	C003	9	Q001	1	02/02/17 15:40	Q007	1	02/02/17 16:10
8142800	1	2	2	2	C003	7	Q012	1	02/02/17 19:30	Q019	1	02/02/17 19:45
8143087	1	2	2	3	C003	4	Q006	1	02/02/17 15:00	Q006	1	02/02/17 15:10
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35

IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETÀ	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR
8139335	1	2	1	2	C006	5	Q005	1	02/02/17 10:00	Q023	1	02/02/17 10:20
8139641	1	2	2	4	C009	4	Q002	1	02/02/17 16:30	Q023	1	02/02/17 16:50
8139830	1	2	1	2	C006	Age					1	02/02/17 14:50
8140616	1	2	1	2	C006	1 From 11 to 19					1	02/02/17 20:30
8140678	1	2	2	3	C006	2 From 20 to 49					1	02/02/17 09:10
8140745	1	2	2	3	C006	3 From 50 to 64					1	02/02/17 18:15
8140799	1	2	2	3	C006	4 65+					1	02/02/17 11:00
8141314	1	2	2	3	C009	4	Q022	1	02/02/17 18:15	Q022	1	02/02/17 18:30
8141976	2	2	1	4	C003	9	Q001	1	02/02/17 15:40	Q007	1	02/02/17 16:10
8142800	1	2	2	2	C003	7	Q012	1	02/02/17 19:30	Q019	1	02/02/17 19:45
8143087	1	2	2	3	C003	4	Q006	1	02/02/17 15:00	Q006	1	02/02/17 15:10
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35

IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR
8139335	1	2	1	2	C006	5	G					
8139641	1	2	2	4	C009	4	G					
8139830	1	2	1	2	C004	4	G					
8140616	1	2	1	2	C003	9	G					
8140678	1	2	2	3	C011	4	G					
8140745	1	2	2	3	C004	4	G					
8140799	1	2	2	3	C006	4	G					
8141314	1	2	2	3	C009	4	G					
8141976	2	2	1	4	C003	9	G					
8142800	1	2	2	2	C003	7	G					
8143087	1	2	2	3	C003	4	G					
8143375	1	2	1	4	C004	9	G					
8146677	2	2	2	3	C023	7	G					
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35

Motivation

1 Go to work

2 Working reason

3 Study

4 Shopping

5 Bring someone

6 Cures or medical visits

7 Sport or leisure

8 Going back home

9 Visiting relatives or friends

10 other

11 Going back home on the day of the interview



IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_F		
										Q002	TORINO - S.SALVARIO
										Q003	TORINO - CROCETTA
8139335	1	2	1	2	C006	5	Q005	1	02/02/17	Q004	TORINO - S.PAOLO
										Q005	TORINO - CENISIA
8139641	1	2	2	4	C009	4	Q002	1	02/02/17	Q006	TORINO - S.DONATO
										Q007	TORINO - AURORA
8139830	1	2	1	2	C004	4	Q019	1	02/02/17	Q008	TORINO - VANCHIGLIA
										Q009	TORINO - NIZZA-MILLEFONTI
8140616	1	2	1	2	C003	9	Q003	1	02/02/17	Q010	TORINO - LINGOTTO
										Q011	TORINO - S.RITA
8140678	1	2	2	3	C011	4	Q009	1	02/02/17	Q012	TORINO - MIRAFIORI NORD
										Q013	TORINO - POZZO STRADA
8140745	1	2	2	3	C004	4	Q015	1	02/02/17	Q014	TORINO - PARELLA
										Q015	TORINO - VALLETTE
8140799	1	2	2	3	C006	4	Q010	1	02/02/17	Q016	TORINO - MADONNA CAMPAGNA
										Q017	TORINO - B.TA VITTORIA
8141314	1	2	2	3	C009	4	Q022	1	02/02/17	Q018	TORINO - B.RA MILANO
										Q019	TORINO - FALCHERA
8141976	2	2	1	4	C003	9	Q001	1	02/02/17	Q020	TORINO - REGIO PARCO
										Q021	TORINO - MADONNA PILONE
8142800	1	2	2	2	C003	7	Q012	1	02/02/17	Q022	TORINO - CAVORETTO
										Q023	TORINO - MIRAFIORI SUD
8143087	1	2	2	3	C003	4	Q006	1	02/02/17		
8143375	1	2	1	4	C004	9	Q001	1	02/02/17		
8146677	2	2	2	3	C023	7	Q008	1	02/02/17		
8147093	1	2	1	1	C016	2	Q001	1	02/02/17		



Zone Description

Zones



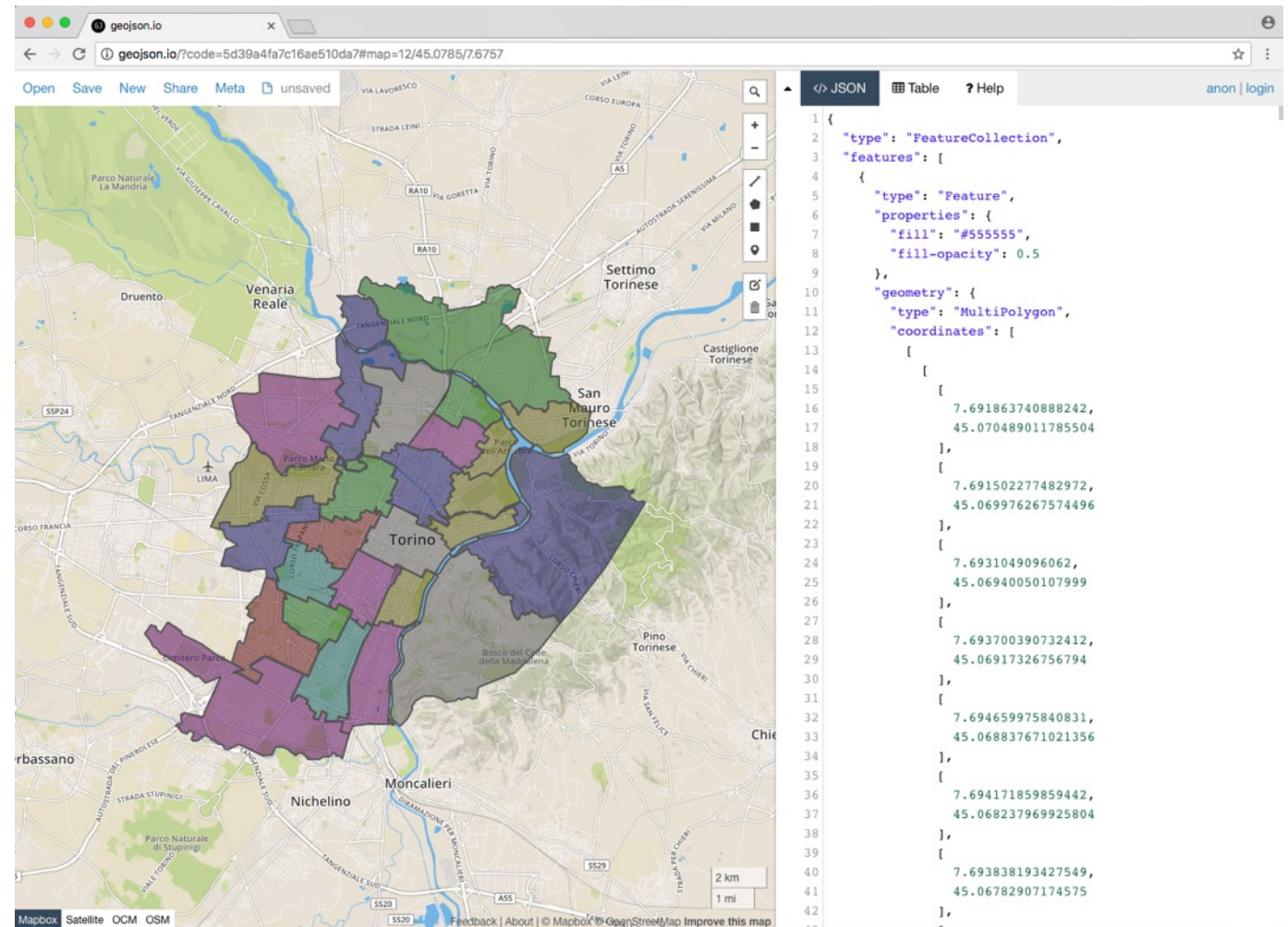
Extracting the zones

- There are 23 zones in Torino
 - Q001 – Q023
- These are defined in
 - **TorinoZonesArray.geojson**
 - A javascript array ready for use
 - **TorinoZonescol.geojson**
 - Original geojson shape
 - Play with it in <http://geojson.io/>

Q001	TORINO - CENTRO
Q002	TORINO - S.SALVARIO
Q003	TORINO - CROCETTA
Q004	TORINO - S.PAOLO
Q005	TORINO - CENISIA
Q006	TORINO - S.DONATO
Q007	TORINO - AURORA
Q008	TORINO - VANCHIGLIA
Q009	TORINO - NIZZA-MILLEFONTI
Q010	TORINO - LINGOTTO
Q011	TORINO - S.RITA
Q012	TORINO - MIRAFIORI NORD
Q013	TORINO - POZZO STRADA
Q014	TORINO - PARELLA
Q015	TORINO - VALLETTE
Q016	TORINO - MADONNA CAMPAGNA
Q017	TORINO - B.TA VITTORIA

Zones in Torino

- Check the map at <http://geojson.io/>



Analyzing OD matrix

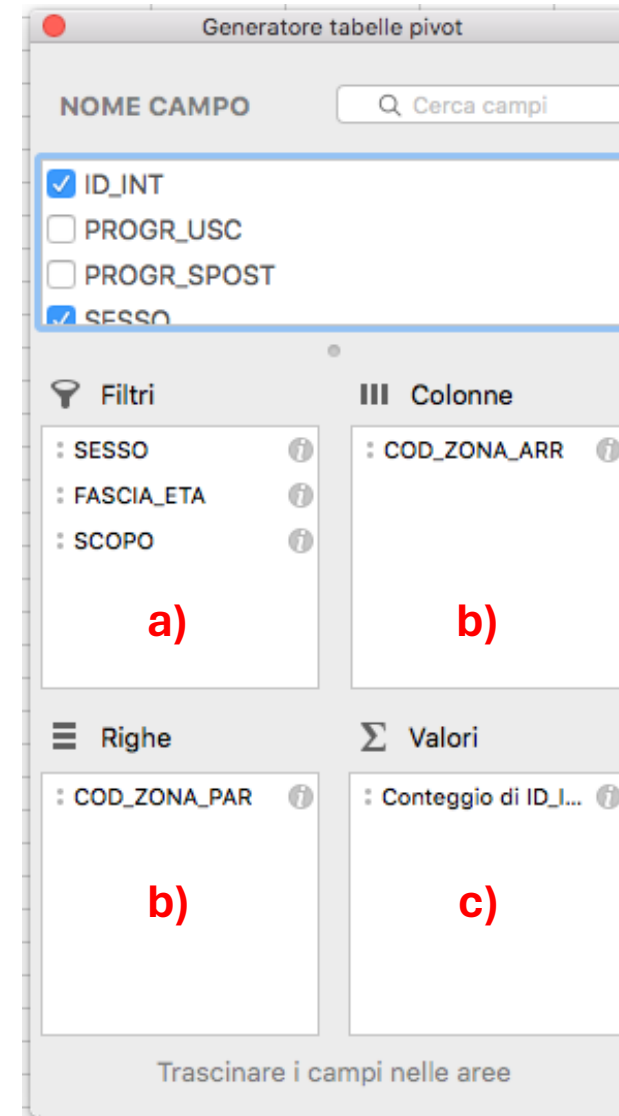
Goal: extract OD matrix

- OD matrix[i][j]: fraction of trips from zone i to zone j
- The simplest way to extract the OD matrix is using **pivot tables**
 - Pivot tables are simple ways to summarize data
 - They allow to process and transform data
 - And filter data
- You can do it with pandas dataframe
 - https://pandas.pydata.org/docs/reference/api/pandas.pivot_table.html

Create a pivot table with Excel

[Example in Excel – but any spreadsheet allow you to do it]

1. open a new spreadsheet and import the CVS file
2. Add a pivot table
[menu->data->add pivot table]
3. Drag elements
 - a) In filters: sesso, scopo, eta, ...
 - b) Rows and columns: zona_par, zona_arr
 - c) Value: any field
[we are going to "count" occurrences]



Result

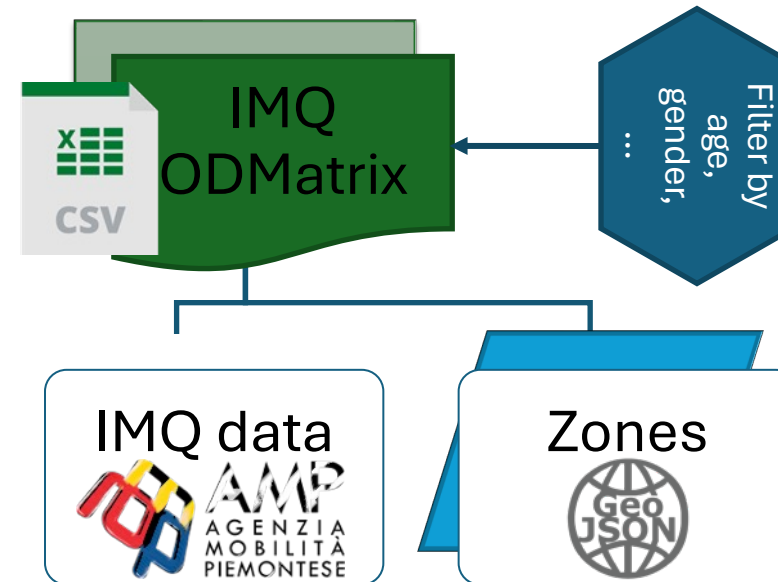
You can get different ODmatrixes by selecting different filters

FASCIA_ETA	(più elementi)	▼
SCOPO	(più elementi)	▼
SESSO	(Tutto)	▼
ORA_ARR	(Tutto)	▼

Conteggio di PR Etichette di col																								
Etichette di r	Q001	Q002	Q003	Q004	Q005	Q006	Q007	Q008	Q009	Q010	Q011	Q012	Q013	Q014	Q015	Q016	Q017	Q018	Q019	Q020	Q021	Q022	Q023	Totale com
Q001	102	11	25	1	18	15	10	19	11	11	3	6	9	12	3	3	4	3	1	1	9	8	8	293
Q002	37	27	12	1	5	1	6	6	16	2	1	4	3	3	1	2	1	1	1	3	2	4	5	144
Q003	35	7	41	7	11	8	5	4	9	9	6	2	2	1	2		3		2			2	1	157
Q004	18	8	7	11	16	8	4	3	6	2	8	8	7	3	1	1	2	1			1		5	120
Q005	28	7	17	6	33	14	4	4	5	2	5	6	21	8	3		5		1			5	5	179
Q006	50	7	14	3	14	70	12	15	13	8	2	4	6	13	6	7	14		3	2	2	4	5	274
Q007	44	5	11	2	10	18	36	15	5	3	3	7	4	8	2	9	8	7	17	5	2	3	2	226
Q008	44	6	1	5	3	3	12	32	3	2	3	2	1	2	1	4	3	4	3	5	4	7		150
Q009	27	14	6	2	5	2	10	4	46	14	2	1	1	4	1	2	4	2	1	1	1	2	2	154
Q010	21	5	11	2	3	3	4	6	14	29	7	7	7	2		3	5		1	3	1	1	15	150
Q011	15	8	8	7	6	4	7	2	4	8	27	14	2	1	3	4	1			1			4	126
Q012	19	4	8	3	10	11	4	2	15	14	13	27	9	2		3	2	1	2		2	2	16	169
Q013	34	4	15	10	9	11	3	8	8	4	4	7	46	28	5	4	3		4	2	1	1	6	217
Q014	28	6	6	4	6	23	5	3	4	4	5	6	15	41	1	2	6		3	1			2	171
Q015	17	3	3	2	1	11	1	6	6	3	1	2	2	7	20	9	4		5		1			104
Q016	30	2	9	4	9	15	6	7	6	7		2	4	5	12	22	17	5	10	4	1		8	185
Q017	35	6	7	2	5	18	11	6	3	8	4	2	3	11	8	12	57	3	7	3	1		5	217
Q018	25	6	5	2	3	10	25	7	2	2	2	1	1	2	3	8	9	35	25	16	2	3	5	199
Q019	19	4	5	1	1	4	6	2	1			1		1	2	2	6	10	25	5	1		1	97
Q020	13	2	4			2	3	5	3						1	2	2	2	5	14			2	60
Q021	17	1		3	5	4	5	15	1	4		1	1		1	1	2	3	1		11	6	2	84
Q022	16	12	1		1	4	2	6	3	1	1	1	1	3	1					1	2	16	4	76
Q023	26	4	6	2	5	3	1	7	12	22	2	6	1	1			2	1	1	2	2	3	27	136
Totale compless	700	159	222	80	179	262	182	184	196	159	99	117	146	158	77	100	160	78	118	69	46	67	130	3688

Results

- you can compute different OD matrixes
 - For different age
 - For different sex
 - For different trip reason
 - ...



Last step: compare similarity

- Given matrices $\mathbf{A}=[a_{ij}]$ and $\mathbf{B}=[b_{ij}]$ similarity can be defined as the sum of “distance” between each element

$$d_1(\mathbf{A}, \mathbf{B}) = \sum_{i=1}^n \sum_{j=1}^n |a_{ij} - b_{ij}|$$

$$d_2(\mathbf{A}, \mathbf{B}) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n (a_{ij} - b_{ij})^2}$$

$$d_\infty(\mathbf{A}, \mathbf{B}) = \max_{1 \leq i \leq n} \max_{1 \leq j \leq n} |a_{ij} - b_{ij}|$$

$$d_m(\mathbf{A}, \mathbf{B}) = \max \{ \|(\mathbf{A} - \mathbf{B})\mathbf{x}\| : \mathbf{x} \in \mathbb{R}^n, \|\mathbf{x}\| = 1 \}$$

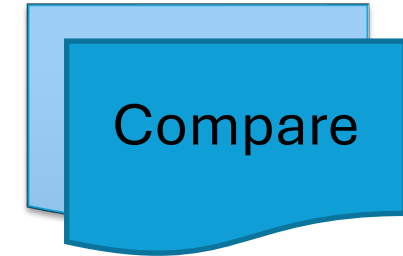


Compare

Last step: compare similarity

To better appreciate the distance computation, compare

- Two uniform random matrices
 - $a_{ij} = \text{rnd}(0,1)$ and $b_{ij} = \text{rnd}(0,1)$
- Two OD matrices extracted from the same dataset
 - From car sharing data: A = OD(first week), B = OD(second week)
 - From car sharing data: A = OD(enjoy), B = OD(Car2Go)
 - From IMQ: A = OD(all), B = OD(males), ...
- A uniform random matrix and a matrix from the data



Warning

1. Normalize elements, e.g., $\sum_{ij}(a_{ij})=1$
2. Compute distance of each element

$$d_{ij}=(a_{ij} - b_{ij})^n \text{ similarity}$$

1. Sum all distances

$$D_n(A,B)=\sum_{ij}(d_{ij})$$

