SUPPLEMENTARY MATERIAL
A Distributed Approach for Fault Detection in Swarms of Robots

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We collect here the results of the statistical tests described in Section 5 of the main paper.

1 Single Fault Experiments

The permutation tests used on the results of the single fault experiments compare the means of two populations. The two hypotheses of the tests are:

$$\mathcal{H}_0 : \mu_0 = \mu_1 \text{ vs. } \mathcal{H}_1 : \mu_0 \neq \mu_1$$

where μ_0 is the mean of the first population, μ_1 is the mean of the second population, $\mathcal{H}_0$ is the null hypothesis, and $\mathcal{H}_1$ is the alternative hypothesis. In this case, given a behavior and a fault, the populations are the runs performed with different fault detection methods.

In Tables 1–4, we report the results: in each cell, we report the p-value of the permutation test. If, after the Bonferroni-Holm correction, the p-value remains lower than α, it is reported in bold.

		bact	*lact*	*ract*	*pmax*	*pmin*	*prnd*	*rofs*
CRM-B	**ML-B**	0.7742	0.9930	0.1089	0.6722	**0.0002**	0.7696	**0.0002**
CRM-B	**ML-N**	**0.0002**	**0.0002**	**0.0002**	0.2965	**0.0002**	0.0821	**0.0002**
ML-B	**ML-N**	**0.0002**	**0.0002**	**0.0002**	0.1750	**0.0002**	0.1652	**0.0002**

Table 1: Results of the permutation tests on single fault experiments: aggregation behavior.

2 Multiple Faults Experiments

The permutation tests of set I, used to assess the difference in the means varying the value of p, are the same as the ones used with the results of the experiments

		bact	*lact*	*ract*	*pmax*	*pmin*	*prnd*	*rofs*
CRM-B	**ML-B**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	0.8294	**0.0002**	**0.0002**
CRM-B	**ML-N**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0074**	**0.0002**	0.2917
ML-B	**ML-N**	**0.0002**	**0.0004**	0.0256	**0.0014**	**0.0077**	**0.0014**	**0.0002**

Table 2: Results of the permutation tests on single fault experiments: dispersion behavior.

		bact	*lact*	*ract*	*pmax*	*pmin*	*prnd*	*rofs*
CRM-B	**ML-B**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0030**	**0.0002**
CRM-B	**ML-N**	**0.0102**	**0.0004**	**0.0002**	**0.0014**	**0.0002**	**0.0058**	**0.0002**
ML-B	**ML-N**	**0.0004**	0.0554	**0.0042**	**0.0002**	0.5761	0.2264	**0.0038**

Table 3: Results of the permutation tests on single fault experiments: flocking behavior.

		bact	*lact*	*ract*	*pmax*	*pmin*	*prnd*	*rofs*
CRM-B	**ML-B**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	0.2955	**0.0002**	**0.0002**
CRM-B	**ML-N**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	0.2240	**0.0002**	**0.0002**
ML-B	**ML-N**	0.5289	0.1278	0.2753	0.4917	0.0216	0.4919	1.0000

Table 4: Results of the permutation tests on single fault experiments: homing behavior.

with a single fault. In this case, given a behavior, the populations are the runs performed with a fault detection method (ML-B or ML-N) and with different values of p. We report the results of these statistical tests in Tables 5–12. The values on the y-axes and the x-axes are the values of p in the first and second population, respectively.

The permutation tests of set II, used to assess the difference in the variance varying the value of p, have the following hypotheses:

$$\mathcal{H}_0 : \frac{\sigma_0^2}{\sigma_1^2} = 1 \text{ vs. } \mathcal{H}_1 : \frac{\sigma_0^2}{\sigma_1^2} > 1$$

where σ_0^2 is the variance of the first population, σ_1^2 is the variance of the second population, $\mathcal{H}_0$ is the null hypothesis, and $\mathcal{H}_1$ is the alternative hypothesis. Also in this case, given a behavior, the populations are the runs performed with a fault detection method (ML-B or ML-N) and with different values of p. We report the results of these statistical tests in Tables 13–20.

The one-sided permutation tests of set III, used to assess the difference in the means varying the type of features, have the following hypotheses:

$$\mathcal{H}_0 : \mu_b \geq \mu_n \text{ vs. } \mathcal{H}_1 : \mu_b < \mu_n$$

where μ_b is the mean of the results with binary features, μ_n is the mean of the results with numerical features, $\mathcal{H}_0$ is the null hypothesis, and $\mathcal{H}_1$ is the alternative hypothesis. We report the results of these statistical tests in Table 21.

In Tables 5–21 we use the following criterion to fill the cells: in each cell, we report the p-value of the permutation test. If, after Bonferroni-Holm correction, the p-value remains lower than α, it is reported in bold.

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1								
0.2	0.0754							
0.3	0.0358	0.6522						
0.4	0.5152	0.2242	0.1172					
0.5	0.5188	0.1866	0.0826	0.9696				
0.6	0.6622	0.0832	0.0370	0.7408	0.7658			
0.7	0.5762	0.0026	**0.0010**	0.1366	0.1164	0.1204		
0.8	0.6650	0.0052	0.0022	0.1880	0.1656	0.2064	0.8560	
0.9	0.4354	0.0026	**0.0014**	0.0788	0.0590	0.0728	0.6176	0.5422

Table 5: Results of the permutation tests of set I: aggregation behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1								
0.2	0.2180							
0.3	0.0450	0.4520						
0.4	0.2498	0.9576	0.4582					
0.5	0.0022	0.0308	0.1096	0.0398				
0.6	**0.0006**	0.0054	0.0354	0.0060	0.6588			
0.7	0.0016	0.0672	0.2498	0.0720	0.4582	0.1808		
0.8	0.0036	0.0678	0.2570	0.0772	0.5300	0.2616	0.9126	
0.9	0.0082	0.1498	0.4626	0.1616	0.3308	0.1350	0.7268	0.6714

Table 6: Results of the permutation tests of set I: aggregation behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1								
0.2	0.2876							
0.3	**0.0014**	0.0890						
0.4	0.0026	0.1152	0.9454					
0.5	0.0018	0.0362	0.5284	0.6228				
0.6	**0.0004**	0.0530	0.8138	0.9128	0.6540			
0.7	0.0146	0.4794	0.1382	0.2116	0.0542	0.0734		
0.8	**0.0008**	0.1214	0.7094	0.7362	0.3162	0.5232	0.1952	
0.9	**0.0004**	0.0370	0.6726	0.7912	0.7772	0.8474	0.0430	0.4018

Table 7: Results of the permutation tests of set I: dispersion behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1								
0.2	0.2482							
0.3	**0.0004**	0.0360						
0.4	**0.0004**	0.0370	0.8308					
0.5	**0.0000**	0.0104	0.4310	0.6160				
0.6	**0.0000**	0.0154	0.8546	0.9468	0.5062			
0.7	**0.0014**	0.1570	0.2522	0.2396	0.0624	0.1408		
0.8	**0.0000**	0.0192	0.9388	0.8712	0.4344	0.8894	0.1614	
0.9	**0.0000**	0.0052	0.4618	0.6978	0.8684	0.5698	0.0366	0.4710

Table 8: Results of the permutation tests of set I: dispersion behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1								
0.2	0.3074							
0.3	0.7754	0.4082						
0.4	0.1936	0.0122	0.0566					
0.5	0.7672	0.3260	0.9592	0.0102				
0.6	0.6432	0.0840	0.3356	0.1790	0.2438			
0.7	0.3732	0.0278	0.1338	0.3824	0.0486	0.5192		
0.8	0.0778	0.0018	0.0096	0.5212	**0.0012**	0.0310	0.0594	
0.9	0.0634	**0.0006**	0.0062	0.3774	**0.0004**	0.0154	0.0356	0.7162

Table 9: Results of the permutation tests of set I: flocking behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1									
0.2	0.4002								
0.3	0.8968	0.4250							
0.4	0.1284	0.0152	0.0590						
0.5	0.7608	0.4766	0.8528	0.0096					
0.6	0.7046	0.1738	0.5628	0.0910	0.3362				
0.7	0.5100	0.0922	0.3456	0.1550	0.1490	0.6824			
0.8	0.0926	0.0066	0.0340	0.8804	0.0024	0.0420	0.0820		
0.9	0.0760	0.0058	0.0262	0.7598	**0.0012**	0.0352	0.0620	0.8540	

Table 10: Results of the permutation tests of set I: flocking behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1									
0.2	0.6602								
0.3	0.0892	0.0094							
0.4	0.5006	0.1406	0.2086						
0.5	0.0324	0.0016	0.6174	0.0642					
0.6	0.7892	0.7440	0.0110	0.1564	**0.0008**				
0.7	0.1770	0.0156	0.5482	0.4232	0.2196	0.0182			
0.8	0.4304	0.0982	0.2676	0.8650	0.0856	0.1150	0.5374		
0.9	0.1956	0.0146	0.4750	0.4462	0.1860	0.0082	0.9278	0.5744	

Table 11: Results of the permutation tests of set I: homing behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1									
0.2	0.6428								
0.3	0.1470	0.0342							
0.4	0.7060	0.2914	0.2042						
0.5	0.0482	0.0040	0.6294	0.0520					
0.6	0.7822	0.7340	0.0322	0.3352	0.0020				
0.7	0.3410	0.0580	0.4538	0.4626	0.1626	0.0526			
0.8	0.9976	0.4882	0.0676	0.5798	0.0082	0.6150	0.1348		
0.9	0.5998	0.1606	0.2042	0.8690	0.0476	0.1452	0.4926	0.3650	

Table 12: Results of the permutation tests of set I: homing behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1		0.1946	0.5340	0.2400	0.0566	0.0024	**0.0000**	**0.0002**	**0.0000**
0.2	0.7924		0.8186	0.4654	0.1264	0.0058	**0.0000**	**0.0000**	**0.0000**
0.3	0.4912	0.1844		0.1656	0.0208	**0.0004**	**0.0000**	**0.0000**	**0.0000**
0.4	0.7480	0.5214	0.8264		0.2058	0.0226	**0.0000**	0.0052	**0.0002**
0.5	0.9500	0.8762	0.9754	0.7930		0.1094	**0.0008**	0.0192	**0.0008**
0.6	0.9978	0.9966	0.9998	0.9734	0.8904		0.0020	0.1198	0.0172
0.7	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982		0.9022	0.8534
0.8	1.0000	0.9998	1.0000	0.9972	0.9866	0.8790	0.1018		0.2840
0.9	1.0000	1.0000	1.0000	0.9996	0.9968	0.9836	0.1470	0.7198	

Table 13: Results of the permutation tests of set II: aggregation behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1		0.4610	0.2938	0.4474	0.4860	0.2944	0.0228	0.2162	0.2350
0.2	0.5480		0.2720	0.7104	0.8610	0.5378	0.0090	0.2428	0.1796
0.3	0.7070	0.7280		0.8390	0.9372	0.7278	0.0882	0.4586	0.4510
0.4	0.5532	0.3006	0.1636		0.6190	0.2768	0.0088	0.1258	0.1072
0.5	0.5304	0.1416	0.0740	0.3764		0.1636	0.0020	0.0568	0.0400
0.6	0.6942	0.4642	0.2660	0.7414	0.8356		0.0162	0.2256	0.1962
0.7	0.9772	0.9918	0.9140	0.9918	0.9972	0.9816		0.8962	0.9308
0.8	0.7852	0.7440	0.5266	0.8716	0.9390	0.7738	0.0984		0.4868
0.9	0.7656	0.8122	0.5480	0.8932	0.9594	0.7930	0.0604	0.5152	

Table 14: Results of the permutation tests of set II: aggregation behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1		0.8090	0.6536	0.6692	0.8876	0.6232	0.0198	0.1786	0.6490
0.2	0.1840		0.1834	0.5034	0.5472	0.1686	**0.0000**	0.0106	0.2154
0.3	0.3478	0.8190		0.5768	0.8342	0.4620	0.0070	0.0682	0.5348
0.4	0.3312	0.4944	0.4144		0.4856	0.3520	0.0156	0.1348	0.3750
0.5	0.1026	0.4264	0.1612	0.5330		0.1558	0.0016	0.0124	0.2014
0.6	0.3802	0.8268	0.5510	0.6510	0.8532		0.0092	0.1194	0.5634
0.7	0.9798	0.9990	0.9908	0.9832	0.9984	0.9918		0.9160	0.9726
0.8	0.8154	0.9874	0.9312	0.8768	0.9824	0.8702	0.0858		0.8628
0.9	0.3428	0.7826	0.4770	0.6416	0.8022	0.4324	0.0250	0.1338	

Table 15: Results of the permutation tests of set II: dispersion behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1		0.7584	0.8450	0.7526	0.9600	0.6878	0.0558	0.4838	0.7668
0.2	0.2388		0.3734	0.5352	0.7790	0.2262	0.0024	0.1058	0.3090
0.3	0.1564	0.6196		0.5476	0.8722	0.2620	0.0038	0.1090	0.4326
0.4	0.2432	0.4712	0.4548		0.5124	0.3654	0.0312	0.2540	0.3594
0.5	0.0320	0.2332	0.1362	0.4948		0.0636	0.0010	0.0216	0.1064
0.6	0.3094	0.7772	0.7368	0.6574	0.9416		0.0172	0.2820	0.6286
0.7	0.9392	0.9992	0.9944	0.9650	0.9988	0.9800		0.9404	0.9786
0.8	0.5178	0.8896	0.8950	0.7402	0.9722	0.7292	0.0528		0.7798
0.9	0.2332	0.7014	0.5666	0.6424	0.8890	0.3732	0.0206	0.2160	

Table 16: Results of the permutation tests of set II: dispersion behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1		0.4768	0.0408	**0.0008**	**0.0000**	**0.0004**	**0.0000**	**0.0000**	**0.0000**
0.2	0.4996		0.0436	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
0.3	0.9542	0.9542		0.0024	0.0038	0.0092	**0.0000**	**0.0000**	**0.0000**
0.4	0.9998	1.0000	0.9972		0.6318	0.6186	0.0520	0.0012	0.0318
0.5	0.9998	1.0000	0.9966	0.3628		0.5226	0.0394	0.0028	0.0130
0.6	0.9990	0.9998	0.9898	0.3836	0.4896		0.0980	0.0384	0.0624
0.7	1.0000	1.0000	1.0000	0.9498	0.9618	0.9018		0.0672	0.2272
0.8	1.0000	1.0000	1.0000	0.9984	0.9968	0.9610	0.9326		0.7038
0.9	1.0000	1.0000	1.0000	0.9714	0.9880	0.9392	0.7812	0.3084	

Table 17: Results of the permutation tests of set II: flocking behavior with binary features (ML-B).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1		0.6544	0.1530	0.0030	0.0010	0.0040	**0.0002**	**0.0000**	0.0012
0.2	0.3380		0.1172	0.0022	0.0030	0.0048	**0.0000**	**0.0002**	**0.0002**
0.3	0.8556	0.8704		0.0054	0.0166	0.0162	**0.0006**	**0.0000**	**0.0004**
0.4	0.9978	0.9988	0.9944		0.7754	0.6862	0.3200	0.0676	0.1344
0.5	0.9992	0.9964	0.9878	0.2130		0.4166	0.1194	0.0250	0.0356
0.6	0.9974	0.9942	0.9832	0.3140	0.5732		0.1940	0.0690	0.0896
0.7	0.9998	0.9998	0.9994	0.6828	0.8772	0.8150		0.0728	0.1556
0.8	1.0000	1.0000	1.0000	0.9342	0.9766	0.9328	0.9254		0.6124
0.9	1.0000	1.0000	1.0000	0.8662	0.9582	0.9056	0.8584	0.3816	

Table 18: Results of the permutation tests of set II: flocking behavior with numerical features (ML-N).

0.1		0.0874	0.4476	0.1192	0.3156	0.0188	0.1104	0.1642	0.0648
0.2	0.9076		0.9940	0.9306	0.9840	0.0158	0.8290	0.7594	0.5790
0.3	0.5636	0.0044		0.0472	0.3502	**0.0000**	0.0578	0.1192	0.0234
0.4	0.8910	0.0598	0.9552		0.8292	0.0014	0.5070	0.5282	0.3238
0.5	0.6824	0.0144	0.6398	0.1664		**0.0004**	0.1680	0.2162	0.0906
0.6	0.9828	0.9822	1.0000	0.9988	1.0000		0.9966	0.9188	0.9672
0.7	0.8898	0.1702	0.9366	0.5082	0.8282	0.0052		0.5274	0.3266
0.8	0.8344	0.2470	0.8886	0.4574	0.7780	0.0790	0.4690		0.3328
0.9	0.9402	0.4366	0.9704	0.6824	0.9086	0.0310	0.6716	0.6842	
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

Table 19: Results of the permutation tests of set II: homing behavior with binary features (ML-B).

0.1		0.1404	0.4562	0.1568	0.2606	0.0342	0.0936	0.0560	0.0560
0.2	0.8544		0.9650	0.6190	0.8840	0.0232	0.3996	0.1346	0.1504
0.3	0.5456	0.0326		0.0500	0.2490	**0.0000**	0.0146	**0.0006**	0.0016
0.4	0.8354	0.3766	0.9510		0.8006	0.0030	0.2602	0.0548	0.0658
0.5	0.7222	0.1206	0.7312	0.2002		**0.0002**	0.0586	0.0034	0.0058
0.6	0.9676	0.9744	1.0000	0.9966	1.0000		0.9628	0.6514	0.8452
0.7	0.9062	0.5988	0.9864	0.7416	0.9322	0.0354		0.1544	0.1916
0.8	0.9372	0.8656	0.9994	0.9480	0.9962	0.3560	0.8570		0.6294
0.9	0.9418	0.8492	0.9996	0.9372	0.9938	0.1498	0.8202	0.3836	
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

Table 20: Results of the permutation tests of set II: homing behavior with numerical features (ML-N).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
aggregation	**0.0012**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**
dispersion	0.5521	0.3741	0.6091	0.7974	0.8272	0.4959	0.9292	0.9994	0.9824
flocking	**0.0114**	**0.0006**	**0.0004**	**0.0002**	**0.0002**	**0.0002**	**0.0004**	**0.0010**	0.0966
homing	0.8720	0.9832	0.6480	0.6560	0.5735	0.9996	0.5265	0.1072	0.0680

Table 21: Results of the permutation tests of set III.