Chapter 23
Pricing Analytics

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1 Introduction

One of the most important decisions a firm has to take is the pricing of its products. At its simplest, this amounts to stating a number (the price) for a single product. But it is often a lot more complicated than that. Various pricing mechanisms such as dynamic pricing, promotions, bundling, volume discounts, segmentation, bidding, and name-your-own-price are usually deployed to increase revenues, and this chapter is devoted to the study of such mechanisms. Pricing and revenue optimization is known by different names in different domains, such as revenue management (RM), yield management, and pricing analytics. One formal definition of revenue management is the study of **how a firm should set and update pricing and product availability decisions across its various selling channels in order to maximize its profitability**. There are several key phrases in this definition: Firms should not only set but also update prices; thus, price setting should be dynamic and depend on many factors such as competition, availability of inventory, and updated demand forecasts. Firms not only set prices but also make product availability decisions; in other words, firms can stop offering certain products at a given price

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(such as the closing of low-fare seats on airlines) or offer only certain assortments in certain channels. Firms might offer different products at different prices across selling channels—the online price for certain products might be lower than the retail price!

The application of pricing and revenue management analytics in business management began in the 1970s. Airline operators like British Airways (then British Overseas Airways Corp.) and American Airlines began to offer differentiated fares for essentially the same tickets. The pioneer of this technique, called yield management, was Bob Crandall. Crandall, who eventually became chief executive of American Airlines, spearheaded a revolution in airline ticket pricing, but its impact would be felt across industries. Hotel chains, such as Marriott International, and parcelling services, like United Parcel Service, have used it to great effect.

These techniques have only become more refined in the decades since. The advent of big data has revolutionized the degree to which analytics can predict patterns of customer demand, helping companies adapt to trends more quickly than ever. Retail chains such as Walmart collect petabytes of data daily, while mobile applications like Uber rely on big data to provide the framework for their business model.

Yet even in its simplest form (a simple posted-price mechanism), pricing is tricky. If you set it too low or too high, you are losing out on revenue. On the other hand, determining the right price, either before or after the sale, may be impossible. Analytics helps; indeed, there are few other areas where data and analytics come together as nicely to help out the manager. That is because pricing is inherently about data and numbers and optimization. There are many unobservable factors such as a customer's willingness to pay and needs, so modeling plays a critical role. Here too, we restrict ourselves by and large to monopoly models, folding in, whenever possible, competitive prices and product features, but do not explicitly model strategic reactions and equilibria. We cover modeling of pricing optimization which by necessity involves modeling customer behavior and constrained optimization.

Moreover, the application of big data techniques to pricing methods raises concerns of privacy. As models become better at understanding customers, companies may find themselves rapidly entering an uncanny valley-like effect, where their clients find themselves disoriented and put off by the amount of precision with which they can be targeted. The European Union's General Data Protection Regulation is explicitly aimed at limiting the use and storage of personal data, necessitating a wide set of reforms by companies across sectors and industries.

The two building blocks of revenue management are developing quantitative models of customer behavior, that is, price-response curves, demand forecasts, market segmentation, etc., and tools of constrained optimization. The first building block is all about capturing details about the consumers at a micro-market level.For example, one might consider which customers shop at what times for which products at a given store of a food retailer. Then, one might model their sensitivity to price, product assortments, and product bundles. This data can be combined with inventory planning system information to set prices. The second building block reflects the fact that price should depend on availability. Therefore, capacity constraints play an important role in price optimization. In addition, there could be other simple constraints, such as inventory availability, route structure of an airline, network constraints that equate inflow and inventory to outflows, and consumption and closing inventory. More esoteric constraints are used to model customer switching behavior when presented with a choice of products or even the strategic behavior of customers in anticipation of a discount or price increase.

What sorts of questions does RM help answer? We have provided a partial list as follows:

- A hotel chain wants guidelines on how to design products for different customer segments. Price is not the only distinguishing feature. For example, hotels sell the same room as different products and at different prices, such as no refund, advance payment required, full refund, breakfast included, access to executive lounge included, etc!
- The owner of a health club wants to know whether the profits will increase if he sets different prices at different times and for different customers.
- A car manufacturer bidding on supply of a fleet of cars would like to know how to bid for a contract based on past bid information, current competition, and other factors to maximize expected profitability.
- A retail chain needs to decide when and how much to discount prices for a fashion good during a selling season to maximize expected revenue.
- In a downtown hotel, business travelers book closer to the date of stay than leisure travelers. Leisure travelers are more price sensitive than business travelers. The hotel manager has to decide how many rooms to save for business travelers.
- A hotel manager has to determine how to price a single-day stay vs. a multiple-day stay.
- A car rental agency has to decide whether it is profitable to transport cars from one location to another in anticipation of demand surge.
- A basketball franchise wants to explore differential pricing. It wants to evaluate whether charging different prices for different days, different teams, and different times of the day will increase revenue.
- How does the freedom to name your own price (invented by Priceline) work?

The analytics professional will recognize the opportunity to employ almost every tool in the analytics toolkit to solve these problems. First, data is necessary at the right granularity and from different sources including points of sales and reservation systems, surveys, and social media chatter. Information is also required on competitive offerings and prices. Data has to be gathered not only about sales but also no-shows and cancellations. Many a times, bookings are done in groups. These bookings have their own characteristics to record. Second, these data have to be organized in a form that reveals patterns and trends, such that revenue managers, product managers, and operations managers can coordinate their actions to change in demand and supply. Third, demand has to be forecast well into the future and at every market level. Some recent systems claim to even predict demand at a granularity of a single customer. The optimal RM solutions of prices and product availability have to be made available in an acceptable format to sales persons, agents, auction houses, etc. Thus, RM requires information, systems, technologies, and training, as well as disciplined action to succeed.

In the rest of the chapter, we provide a glimpse into the more commonly used RM techniques. These include capacity control, overbooking, dynamic pricing, forecasting for RM, processes used in RM, and network RM. We conclude with suggestions for further reading.

2 Theory

The factors that affect pricing are as follows:

1. The nature of the product or service (features, delivery, conditions of sale, channel) and the competing alternatives (both direct and indirect)
2. Customers' valuation of the product, needs, and purchasing behavior

The reader might have noticed that we did not include costs. That is because in this chapter, we do not discuss simple cost-based pricing such as markup rules (e.g., 10% margin), not because it is not practiced—indeed, it perhaps is the most popular methodology due to its simplicity—but because there is not much to say about such simple rules. Rather, we concentrate on market-based pricing that sets the price based on products and competition and what the consumer is willing to pay. Cost does play a role as a lower bound on the price, but the real decision is in setting the margin above the cost as a function of the market, product features, and customer preferences.

So we need a model of a market and the customer purchasing behavior. Both are somewhat abstract and imprecise concepts and only partially observable, but we do our best in modeling them so as to extract insight from observed data.

2.1 Basic Price Optimization Model

Let p represent price (of a single product) and $D(p)$ the demand at that price (assuming all other features are held the same). Revenue optimization is to find the price p that maximizes $R(p) = pD(p)$, and profit optimization is to maximize $(p - c)D(p)$ when c is the cost of producing one unit.

$D(p)$ is called the demand function, and it is natural to assume that it decreases as we increase price. It is also customary to assume it has some functional form, say

$D(p) = a - bp$ or $D(p) = ap^b$ where a and b are the *parameters* of the model that we estimate based on observed data.

> Example: Say, based on data, we estimate that demand for a certain product is $D(p) = 35.12 - 0.02p$ (i.e., demand is assumed to have a linear form $D(p) = a - bp$, where we calibrated $a = 35.12$ and $b = 0.02$). The revenue optimization problem is to maximize $p \times (35.12 - 0.02p)$. From calculus (take the derivative of the revenue function and set it to 0, so $35.12 - 2 \times 0.02p = 0$, and solve it for p), we obtain the optimal price to be $p^* = \frac{35.12}{2\times 0.02} = 878$.

Capacity restrictions introduce some complications, but, at least for the single product case, are still easy to handle. For instance, in the above example, if price is \$878, the demand estimate is $35.12 - 0.02 \times 878 = 17.56$. If, however, we have only ten units, it is natural to raise the price so that demand is exactly equal to 10, which can be found by solving $10 = 35.12 - 0.02p$ or $p = 1256$.

2.2 *Capacity Controls*

In this section, we look at the control of the sale of inventory when customers belong to different types or, using marketing terminology, segments. The segments are assumed to have different willingness to pay and also different preferences as to when and how they purchase. For example, a business customer for an airline may prefer to purchase close to the departure date, while a leisure customer plans well ahead and would like a guaranteed flight reservation. The original motivation of revenue management was an attempt to make sure that we set aside enough inventory for the late-coming, higher-paying business customer, yet continue selling at a cheaper price to the price-sensitive leisure segment.

We assume that we created products with sale restrictions (such as advance purchase required or no cancellations or weekend stay), and we label each one of these products as *booking classes*, or simply *classes*. All the products share the same physical inventory (such as the rooms of the hotel or seats on a flight). In practice, multiple RM products may be grouped into classes for operational convenience or control system limitations. If such is the case, the price attached to a class is some approximation or average of the products in that class.

From now on, we assume that each booking request is for a single unit of inventory.

2.2.1 Independent Class Models

We begin with the simplest customer behavior assumption, the *independent class* assumption: Each segment is identified with a single product (that has a fixed price), and customers purchase only that product. And if that product is not available for sale, then they do not purchase anything. Since segments are identified one-to-one with classes, we can label them as class 1 customers, class 2 customers, etc.

The goal of the optimization model is to find booking limits—the maximum number of units of the shared inventory we are willing to sell to that product—that maximize revenue.

Let's first consider the two-class model, where class 1 has a higher price than class 2, that is, $f_1 > f_2$, and class 2 bookings come first. The problem would be trivial if the higher-paying customers come first, so the heart of the problem is to decide a "protection level" for the later higher-paying ones and, alternately, a "booking limit" on when to stop sales to the lower-paying class 2 customers.

Say we have an inventory of r_0. We first make forecasts of the demand for each class, say based on historic demand, and represent the demand forecasts by D_j, $j = 1, 2$.

How many units of inventory should the firm protect for the later-arriving, but higher-value, class 1 customers? The firm has only a probabilistic idea of the class 1 demand (the problem would once more be trivial if it knew this demand with certainty).

The firm has to decide if it needs to protect r units for the late-arriving class 1 customers. It will sell the rth unit to a class 1 customer if and only if $D_1 \geq r$, so the *expected marginal revenue* from the rth unit is $f_1 P(D_1 \geq r)$. Intuitively, the firm ought to accept a class 2 request if and only if f_2 exceeds this marginal value or, equivalently, if and only if

$$f_2 \geq f_1 P(D_1 \geq r). \tag{23.1}$$

The right-hand side of (23.1) is decreasing in r. Therefore, there will be an optimal protection level for class 1, denoted r_1^*, such that we accept class 2 if the remaining capacity exceeds r_1^* and reject it if the remaining capacity is r_1^* or less. Formally, r_1^* satisfies

$$f_2 < f_1 P(D_1 \geq r_1^*) \text{ and } f_2 \geq f_1 P(D_1 \geq r_1^* + 1). \tag{23.2}$$

In practice, there are usually many products and segments, so consider $n > 2$ classes. We continue with the independent class assumption and that demand for the n classes arrives in n stages, one for each class in order of revenue with the highest-paying segment, class 1, arriving closest to the inventory usage time.

Let the classes be indexed so that $f_1 > f_2 > \cdots > f_n$. Hence, class n (the lowest price) demand arrives in the first stage (stage n), followed by class $n - 1$ demand in stage $n - 1$, and so on, with the highest-price class (class 1) arriving in the last stage (stage 1). Since, there is a one-to-one correspondence between stages and classes, we index both by j.

We describe now a heuristic method called the expected marginal seat revenue (EMSR) method. This heuristic method is used because solving the n class problem optimally is complicated. The heuristic method works as follows:

Consider stage $j+1$ in which the firm wants to determine protection level r_j for class j. Define the aggregated future demand for classes $j, j-1, \ldots, 1$ by

$$S_j = \sum_{k=1}^{j} D_k,$$

and let the weighted-average revenue (this is the heuristic part) from classes $1, \ldots, j$, denoted $\bar{f}_j$, be defined by

$$\bar{f}_j = \frac{\sum_{k=1}^{j} f_k E[D_k]}{\sum_{k=1}^{j} E[D_k]}, \tag{23.3}$$

where $E[D_j]$ denotes the expected class j demand.

Then, the EMSR protection level for class j and higher, r_j, is chosen by (23.2), to satisfy

$$P(S_j > r_j) = \frac{f_{j+1}}{\bar{f}_j}. \tag{23.4}$$

It is convenient to assume demand for each class j is normally distributed with mean μ_j and variance σ_j^2, in which case

$$r_j = \mu + z\sigma,$$

where $\mu = \sum_{k=1}^{j} \mu_k$ is the mean and $\sigma^2 = \sum_{k=1}^{j} \sigma_k^2$ is the variance of the aggregated demand to come at stage $j+1$ and $z = \Phi^{-1}(1 - f_{j+1}/\bar{f}_j)$ and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal c.d.f. One repeats this calculation for each j.

The EMSR heuristic method is very popular in practice as it is very simple to program and is robust with acceptable performance (Belobaba 1989). One can do the calculation easily enough using Excel, as it has built-in functions for the normal distribution and its inverse.

2.3 Overbooking

There are many industries where customers first reserve the service and then use it later. Some examples are hotels, airlines, restaurants, and rental cars. Now, when a customer reserves something for future use, their plans might change in the meantime. A *cancellation* is when the customer explicitly cancels the reservation, and a *no-show* is when they do not notify the firm but just do not show up at the scheduled time for the service. What the customer does depends on the reservation

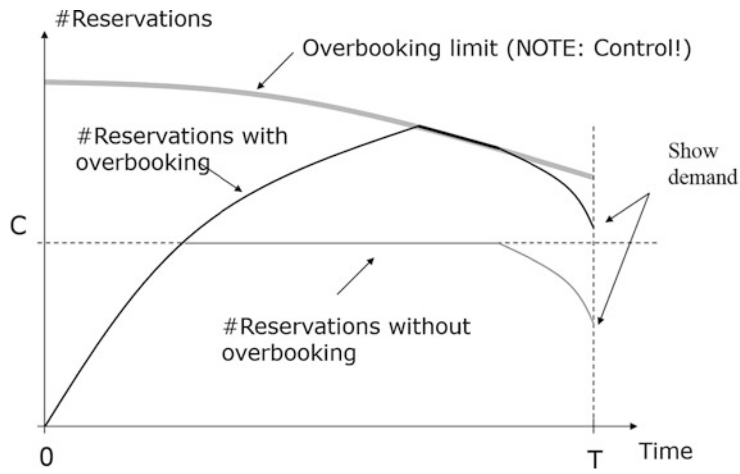

Fig. 23.1 Evolution of the overbooking limit

policies. If there is an incentive like partial or full refund of the amount, customers tend to cancel. If there is no refund, they will just opt for no-show.

Overbooking is a practice of taking more bookings than capacity, anticipating that a certain fraction of the customers would cancel or opt for no-show. This leads to better capacity utilization, especially on high-demand days when the marginal value of each unit of inventory is very high. The firm, however, has to be prepared to handle a certain number of customers who are denied service even though they have paid for the product and have a reservation contract. In many industries, the benefits of better capacity utilization dominate the risk of denying service, and overbooking has become a common practice.

Firms control overbooking by setting an upper limit on how much they overbook (called the overbooking limit). Typically as they come closer to the inventory usage time (say a flight departing), they have a better picture of demand, and they reduce the risk of overbooking if there appears to be high demand with few cancellations. Figure 23.1 shows the dynamics of a typical evolution of the overbooking limit. As the usage date nears and the firm fears that it might end up denying service to some customers, it brings down the overbooking limit toward physical capacity faster (can even be less than the current number of reservations on-hand also, to prevent new bookings).

Overbooking represents a trade-off: If the firm sells too many reservations above its capacity, it risks a scenario where more customers show up than there is inventory and the resulting costs in customer goodwill and compensation. If it does not overbook enough, it risks unsold inventory and an opportunity cost. Overbooking models are used to find the optimal balance between these two factors. We describe one such calculation below that, while not completely taking all factors into consideration, highlights this trade-off mathematically. It is reminiscent of the classical newsvendor model from operations management.

Let C_{DB} denote the *cost of a denied boarding*, that is, the estimated cost of denying service to a customer who has a reservation (which, as we mentioned earlier, includes compensation, loss of goodwill, etc.). Let C_u denote the opportunity cost of underused capacity, typically taken as the expected revenue for a unit of inventory. The overbooking limit we have to decide then is $\theta > C$, where C is the physical capacity.

For simplicity, we assume the worst and that demand will exceed overbooking limit, that is, we will be conservative in setting our limit. Let N be the number of no-shows/cancellations. Since we are not sure of the number of cancellations or no-shows, we model it as a random variable, say as a binomial random variable with parameters θ, p where p is the probability of a cancellation or no-show. Then, the number of customers who actually show up is given by $\theta - N$ (recall demand is conservatively assumed to be always up to θ).

Next, we pose the problem as the following marginal decision: Should we stay at the current limit θ or increase the limit to $\theta + 1$, continuing the assumption that demand is high and will also exceed $\theta + 1$? Two mutually exclusive events can happen: (1) $\theta - N < C$. In this case by moving the limit up by 1, we would increase our profit, or in terms of cost by $-C_u$. (2) $\theta - N \geq C$, and we incur a cost of C_{DB}.

So the expected cost per unit increase of θ is

$$-C_u Pr(\theta - N < C) + C_{DB} Pr(\theta - N \geq C).$$

Note that this quantity starts off negative (when $\theta = C$) as $Pr(\theta - N \geq C) = 0$ at that point, but as we keep increasing θ, it decreases, and the C_{DB} risk increases. So that tells us that we can increase profit as long as this is negative but incur a cost if it is positive, and the best decision is to stop when this quantity is 0. This results in a nice equation to determine the optimal θ,

$$\begin{aligned} -C_u Pr(\theta - N < C) + C_{DB} Pr(\theta - N \geq C) &= -C_u(1 - Pr(\theta - N \geq C)) \\ &\quad + C_{DB} Pr(\theta - N \geq C)0, \end{aligned}$$

or set θ, such that

$$Pr(\theta - N \geq C) = \frac{C_u}{C_u + C_{DB}}.$$

If we let $S(\theta)$ be the number of people who show up, an alternate view is that we need to set θ such that $Pr(S(\theta) \leq C) = \frac{C_u}{C_u + C_{DB}}$. If no-shows happen with probability p, shows also follow a binomial distribution with probability $1 - p$. So set θ such that (writing in terms of $\leq$ to suit Excel calculations)

$$Pr(S(\theta) \leq C) = 1 - \frac{C_u}{C_u + C_{DB}}.$$

Fig. 23.2 Computations for the overbooking example

	A	B	C	D	E	F
1		Critical ratio	0.3	p=	0.1	
2	theta	X such that PR(Show (*Theta-No-shows*) >= X) = 0.3				
3	100	92				
4	101	93				
5	102	93				
6	103	94				
7	104	95				
8	105	96				
9	106	97				
10	107	98				
11	108	99				
12	109	100	C			
13	110	101				
14	111	102				
15	112	103				
16	113	103				
17	114	104				
18	115	105				
19	116	106				
20	117	107				
21	118	108				
22	119	109				
23	120	110				
24	121	111				
25	122	112				
26	123	113				
27	124	113				

BINOM.INV(A3, 1-E1, 1-C1)

BINOM.INV function
Returns the smallest value for which the cumulative binomial distribution is greater than or equal to a criterion value

So we take Pr(Shows <= C) = 1 – 0.3.
If no-shows occur with probability, shows occur with probability 0.9.
So we use BINOM.INV(theta, 1-p, 1- Critical Ratio). We want to find the theta such that this value is egual to our capacity C

Example (Fig. 23.2): Say $C_u = \$150$ and $C_{DB} = \$350$. To calculate the overbooking limit, we first calculate the critical ratio:

$$\frac{C_u}{C_u + C_{DB}} = \frac{150}{500} = 0.3.$$

If we assume the distribution of N is approximately normal (quite accurate when $\theta p \geq 5$) with mean= θp and standard deviation = $\sqrt{\theta p}$, we can in fact use the InverseNormal (NORMINV in Excel) to do the calculations. Suppose $C = 100$ and $p = 0.1$ (that is 10% probability that a reservation will eventually cancel). The optimal solution is to overbook nine seats.

2.4 Dynamic Pricing

Dynamic pricing is a version of revenue management, simpler in some sense, but also requiring some close monitoring. There are usually no explicit virtual products aimed at different segments.[1] Rather, the firm changes the price of the product overtime as it observes changes in the many factors that would affect demand: such as time itself (e.g., because higher-valuation customers come after lower-valuation customers), weather, the customer mix, and competition.

[1] Although there could be, but let us not complicate unecessarily.

Over the last few years, dynamic pricing has taken on three distinct flavors: surge pricing, as practiced by Uber, Lyft, and utility companies; repricing, or competition-based pricing, as practiced by sellers on Amazon marketplace; and, finally, markdown or markup pricing, where prices are gradually decreased (as in fashion retail) or increased (as in low-cost airlines) as a deadline approaches.

2.4.1 Surge Pricing: Matching Demand and Supply

This is the newest and perhaps most controversial of dynamic pricing practices. The underlying economic reason is sound and reasonable. When there is more demand than supply, the price has to be increased to clear the market—essentially the good or service is allocated to those who value it the most. When asked to judge the fairness of this, most consumers do not have a problem with this principle, for instance, few consider auctions to be unfair.

However, when applied to common daily items or services, many consumers turn indignant. This is due to many reasons: (1) There is no transparency on how the prices move to balance demand and supply. (2) As the prices rise when a large number of people are in great need of it, they are left with a feeling of being price-gouged when they need the service most. (3) The item or service is essential or life-saving, such as pharmaceutical or ambulance service. Uber was a pioneer in introducing surge pricing into an industry used to a regulated fixed-price system (precisely to bring transparency and prevent price-gouging and also to avoid the hassle of bargaining). While initial reactions[2] have been predictable, it has, in a space of a few years, become a fact of life. This shows the importance of a firm believing in the economic rationale of dynamic pricing and sticking to the practice despite public resistance. Of course, consumers should find value in the service itself—as the prices are lower than alternatives (such as regular taxi service) during off-peak times, eventually consumers realize the importance and necessity of dynamic pricing.

2.4.2 Repricing: Competition-Driven Dynamic Pricing

A second phenomenon that has recently taken hold of is called "repricing" used in e-commerce marketplaces such as Amazon.com. It is essentially dynamic pricing driven by competition.

Many e-commerce platforms sell branded goods that are identical to what other sellers are selling. The seller's role in the entire supply chain is little more than stocking and shipping as warranties are handled by the manufacturer. Service does play a role, but many of the sellers have similar reviews and ratings, and often price is the main motivation of the customer for choosing one seller over the other, as the e-commerce platform removes all search costs.

[2] https://nyti.ms/2tybWiV, https://nyti.ms/2uwkLXR. Accessed on May 21, 2018.

Prices however fluctuate, the reasons often being mysterious. Some possible explanations are the firms' beliefs about their own attractiveness (in terms of ratings, reviews, and trust) compared to others and their inventory positions—a firm with low inventories may want to slow down sales by pricing higher. Another possible reason is that firms assess the profiles of customers who shop at different times of day and days of the week. A person shopping late at night is definitely not comparison-shopping from within a physical store, so the set of competitors is reduced.

Note that an e-commerce site would normally have more variables to price on, such as location and the past customer profile, but a seller in a marketplace such as Amazon or eBay has only limited information and has to put together competitor information from scraping the website or from external sources.

Repricing refers to automated tools, often just rules based, that change the price because of competitor moves or inventory. Examples of such rules are given in the exercise at the end of this chapter.

2.4.3 Markdown and Markup Pricing: Changing Valuations

Markdown pricing is a common tactic in grocery stores and fashion retailing. Here, the product value itself deteriorates, either physically because of limited shelf-life or as in the fashion and consumer electronics industry, as fresh collections or products are introduced. Markdowns in fashion have a somewhat different motivation from the markdowns of fresh produce. In fashion retail, the products cannot be replenished within the season because of long sales cycles, while for fresh groceries, the sell-by date reduces the value of the product because fresher items are introduced alongside.

Markdown pricing, as the name indicates, starts off with an initial price and then, gradually at various points during the product life cycle, reduces the price. The price reductions are often in the form of 10% off, 20% off, etc., and sometimes coordinated with advertised sales. At the end of the season or at a prescribed date, the product is taken off the shelf and sold through secondary channels at a steeply discounted price, sometimes even below cost. This final price is called the salvage value.

The operational decisions are how much to discount and when. There are various restrictions and business rules one has to respect in marking down, the most common one being once discounted, we cannot go back up in price (this is what distinguishes markdown pricing from promotions). Others limit the amount and quantities of discounting.

The trade-off involved is similar to what is faced by a newsvendor: Discounting too late would lead to excess inventory that has to be disposed, and discounting too soon will mean we sell off inventory and potentially face a stock-out.

In contrast to markdown pricing where prices go down, there are some industries that practice dynamic pricing with prices going up as a deadline approaches. Here, the value of the product does go down for the firm, but the customer mix may be changing when customers with higher valuations arrive closer to the deadline (either the type and mix of customers might be changing or even for the same customer, their uncertainty about the product may be resolved).

2.5 Forecasting and Estimation

For reasons of practicality, we try to keep models of demand simple. After all, elaborate behavioral models of demand would be useless if we cannot calibrate them from data and to optimize based on them. In any case, more complicated models do not necessarily mean they predict the demand better, and they often are harder to manage and control.

In this section, we concentrate on three simple models of how demand is explained as a function of price. All three are based on the idea of a potential population that is considering purchase of our product. The size of this population is M. Note that M can vary by day or day of week or time of day. Out of this potential market, a certain fraction purchase the product. We model the fraction as a function of price, and possibly other attributes as well.

Let $D(p)$ represent demand as a function of price p.

- In the *additive model of demand*,

  $$D(p) = M(a + bp)$$

  where a and b are parameters that we estimate from data. If there are multiple products, demand for one can affect the other. We can model demand in the presence of multiple products as

  $$D(p_i) = M(a_i + b_i p_i + \sum_{j \neq i} b_{ij} p_j).$$

  That is, demand for product i is a function of not just the price of i but also the prices of the other products, p_j, $j \neq i$. The parameters a_i, b_i, and b_{ij} are to be estimated from data.

  This model can lead to problems at the extremes as there is no guarantee that the fraction is between 0 and 1.

- In the *multiplicative model of demand*,

  $$D(p) = M(ap^b).$$

  When there are multiple products, the model is

  $$D(p_i) = M(a_i \prod p_j^{b_{ij}}).$$

This is usually estimated by taking the natural logarithms on both sides, so it becomes linear in the parameters. However, this model has to be handled with care in optimization as it can give funny results—essentially when we are optimizing both revenue and profits, the problem can become unbounded.

- *Choice model of demand*: In the choice model, each customer is assumed to make a *choice* among the available products. The following is sometimes called the multinomial-logit (MNL) model,

$$D(p_i) = M \frac{e^{a_i + b_i p_i}}{1 + \sum e^{a_j + b_j p_j}},$$

where e stands for the base of the natural logarithm. Note that this model has far fewer parameters than either the additive or multiplicative model and naturally limits the fraction to always lie between 0 and 1! This is the great advantage of this model.

We show in the exercises how these models can be used for price optimization. The case study on airline choice modeling (see Chap. 26), has a detailed exercise on estimation and use of choice models for price optimization and product design.

2.6 Processes for Capacity Control

Fixing prices for each product aimed at a segment, as outlined in Sect. 2.2.1, and controlling how much is sold at each price requires that we monitor how many bookings have been taken for each product and closing sales at that price whenever we sold enough.

So the sequence is (1) forecasting the demand for each RM product for a specific day and then (2) optimizing the controls given the forecasts and (3) controlling real-time sales for each product so they do not exceed the booking limits for that product.

We list below the main control forms used in RM industries. Because of the limitations of distribution systems that were designed many years ago, the output of our optimization step has to conform to these forms.

- *Nested allocations* or *booking limits*: All the RM products that share inventory are first ranked in some order, usually by their price.[3] Then, the remaining capacity is allocated to these classes, but the allocations are "nested," so the higher class has access to all the inventory allocated to a lower class. For example, if there are 100 seats left for sale and there are two classes, Y and B, with Y considered

[3] As we mentioned earlier, it is common to group different products under one "class" and take an average price for the class. In the airline industry, for instance, the products, each with a fare basis code (such as BXP21), are grouped into fare classes (represented by an alphabet, Y, B, etc.)

a higher class, then an example of a nested allocation would be Y100 B54. For example, if 100 Y customers were to arrive, the controls would allow sale to all of them. If 60 B customers were to show up, only 54 would be able to purchase. B is said to have an *allocation* or a *booking limit* of 54. Another terminology that is used is (nested) *protections*: Y is said to have (in this example) a protection of 46 seats.

The allocations are posted on a central reservation system and updated periodically (usually overnight). After each booking, the reservation system updates the limits. In the above example, if a B booking comes in, then (as the firm can sell up to 54 seats to B) it is accepted, so the remaining capacity is 99, and the new booking limits are Y99 B53. Suppose a Y booking comes in and is accepted, there are a couple of ways the firm can update the limits: Y99 B54 or Y99 B53. The former is called *standard* nesting and the latter *theft* nesting.

- *Bid prices*: For historic reasons, most airline and hotel RM systems work with nested allocations, as many global distribution systems (such as Amadeus or Sabre) were structured this way. Many of these systems allow for a small number of limits (10–26), so when the number of RM products exceeds this number, they somehow have to be grouped to conform to the number allowed by the system.

  The booking limit control is perfectly adequate when controlling a single resource (such as a single flight leg) independently (independent of other connecting flights, for instance), but we encounter its limitations when the number of products using that resource increases, say to more than the size of inventory. Consider network RM, where the products are itineraries, and there could be many itineraries that use a resource (a flight leg)—the grouping of the products a priori gets complicated and messy (although it has been tried, sometimes called *virtual nesting*).

  A more natural and appropriate form of control, especially for network RM, is a threshold-price form of control called *bid price* control. Every resource has a non-negative number called a bid price associated with it. A product that uses a combination of resources is sold if the price of the product exceeds the sum of the bid prices of the resources that the product uses. The bid prices are frequently updated as new information comes in or as the inventory is sold off. The next section illustrates the computation of bid prices.

2.7 Network Revenue Management

The Need for Network Revenue Management: In many situations, the firm has to compute the impact of a pricing, product offering, or capacity management decision on an entire network of resources. Consider the case of an airline that offers flights from many origins and to many destinations. In this case, passengers who are flying to different origin-destination (OD) pairs might use the same flight leg.

Say an airline uses Chicago as a hub. It offers itineraries from the East Coast of the USA, such as New York and Boston, to cities in the West, such as LA and San Francisco. Passengers who fly from New York to Chicago include those who travel directly to Chicago and also those traveling via Chicago to LA, San Francisco, etc. The firm cannot treat the flight booking on the New York to Chicago flight independently but has to consider the impact on the rest of the network as it reduces the capacity for future customers wishing to travel to LA and San Francisco.

Similarly, there are inter-temporal interactions when we consider multi-night-stay problems, for example, a car or a hotel room when rented over multiple days. Hence, the Monday car rental problem impacts the Tuesday car rental problem and so forth. Other examples of network revenue management include cargo bookings that consume capacity on more than one OD pair or television advertising campaigns that use advertisement slots over multiple shows and days.

Suboptimality of Managing a Network One Resource at a Time: It is easy to demonstrate that it is suboptimal to manage each resource separately. Consider a situation in which the decision maker knows that the flight from city A to B, with posted fare of 100, will be relatively empty, whereas the connecting flight from B to C, fare of 200, will be rather full. Some passengers want to just travel from A to B, and there are others who want to fly from A to C. Both use the AB leg. In this case, what will be the order of preference for booking a passenger from A to B vis-a-vis one who wants to travel from A to C and pays 275? Intuitively, we would remove the 200 from 275 and value the worth of this passenger to the airline on the AB leg as only 75. Therefore, total revenue might not be a good indicator of value. In this example, allocation of the 275 according to the distance between A–B and B–C might also be incorrect if, for example, the distances are equal. Allocations based on utilization or the price of some median-fare class would also be inappropriate. Therefore, any formula that allocates the value to different legs of the itinerary has to consider both the profitability of each leg and the probability of filling the seat.

An Example to Illustrate an Inductive Approach: Consider a simple example in which, as above, there is a flight from city A to city B and a connecting flight from B to C. The single-leg fares are 200 and 200, whereas the through fare from A to C is 360. There is exactly one seat left on each flight leg. Assume, as is done typically in setting up the optimization problem, time is discrete. It is numbered backward so that time n indicates that there are n time periods left before the first flight takes place. Also, the probability of more than one customer arrival in a time period is assumed to be negligible. Thus, either no customer arrives or one customer arrives. We are given there are three time periods left to go. In each period, the probability of a customer who wants to travel from A to B is 0.2, from B to C is 0.2, and from A to C is 0.45; thus, there is a probability of zero arrivals equal to 0.15. In this example, the arrival probabilities are the same in each period. It is easy to change the percentages overtime. What should be the airline's booking policy with one seat left on each flight?

This problem is best solved through backward induction. Define the state of the system as (n, i, j) where n is the period and i and j the numbers of unsold seats on legs AB and BC.

Consider the state (1, 1, 1). In this state, in the last period, the optimal decision is to sell to whichever customer who arrives. The expected payoff is $0.4 \times 200 + 0.45 \times 360 = 242$. We write the value in this state as $V(1, 1, 1) = 242$. The expected payoff in either state (1, 0, 1) or (1, 1, 0) is $0.2 \times 200 = 40$. We write $V(1, 0, 1) = V(1, 1, 0) = 40$. For completeness, we can write $V(n, 0, 0) = 0$.

When there are two periods to go, the decision is whether to sell to a customer or wait. Consider the state (2, 1, 1) and the value of being in this state, $V(2, 1, 1)$. Obviously, it is optimal to sell to an AC customer. Some calculations are necessary for whether we should sell to an AB or BC customer:

If an AB customer arrives: If we sell, we get $200 + V(1, 0, 1)$ (from selling the seat to a BC customer if they arrive in the last period) $= 240$. Waiting fetches $V(1, 1, 1) = 242$. Therefore, it is best to not sell.

If a BC customer arrives: Similar to the case above, it is better to wait.

If an AC customer arrives: Sell. We get 360.

Thus, $V(2, 1, 1) = 0.4 \times 242 + 0.45 \times 360 + 0.15 \times V(1, 1, 1) = 295.1$.

We can compute $V(2, 1, 0)(= V(2, 0, 1)) = 0.2 \times 200 + 0.8 \times V(1, 1, 0) = 72$.

In period 3, in the state (3, 1, 1), it is optimal to sell if an AC customer arrives.

If an AB (or BC) customer arrives, by selling we get $200 + V(2, 0, 1)(or\ V(2, 1, 0)) = 272$. This is smaller than $V(2, 1, 1)$. Therefore, it is better to wait.

This completes the analysis.

The reader can easily generalize to the case when there are different combinations of unsold seats. For example, having solved entirely for the case when a maximum of (k, m) seats are left in the last period, one can use backward induction to solve for the same when there are two periods to go, etc.

The backward induction method is called dynamic programming and can become quite cumbersome when the network is large and the number of periods left is large. It is stochastic in nature because of the probabilities. The astute reader might have noticed that these probabilities can be generated by using an appropriate model of customer choice that yields the probability of choosing an itinerary when presented with a set of options.

Bid Price Approach: Recall the bid price control of Sect. 2.6. The operating rule is to accept an itinerary if its fare exceeds the sum of bid prices on each leg used by the itinerary, if there is sufficient capacity left. The bid prices can be thought of as representing the marginal value of the units of capacity remaining.

But how do we calculate these bid prices? Many different heuristic approaches have been proposed and analyzed, both in the academic and in the practitioner literature (see, e.g., the references at the end of this chapter). These range from solving optimization models such as a deterministic linear program (DLP), a stochastic linear program (SLP), and approximate versions of the dynamic program (DP) illustrated above to a variety of heuristics. (The usual caveat is that the use of bid prices in this manner need not result in the optimal expected revenue. Take, for example, the decision rule that we derived using the dynamic program with three periods to go and one seat that is available on each flight leg. We need two bid prices (one per leg) such that each is greater than the fare on the single leg but their sum is less than the fare on the combined legs. Thus, we need prices b_1 and b_2 such that $b_1 > 200$, $b_2 > 200$, $b_1 + b_2 \leq 360$. Such values do not exist.)

In this chapter, we illustrate the DLP approach as it is practical and is used by hotels and airlines to solve the problem. In order to illustrate the approach, we shall first use a general notation and then provide a specific example. We are given a set of products, indexed by $i = 1\ to\ I$. The set of resources is labeled $j = 1\ to\ J$. If product i uses resource j, let $a_{ij} = 1\ else\ 0$. Let the revenue obtained from selling one unit of product i be R_i. We are given that the demand for product i is D_i and the capacity of resource j is C_j. Here, the demand and revenue are deterministic.

In the context of an airline, the products would be the itineraries, the resources are the flight legs, the coefficient $a_{ij} = 1$ if itinerary i uses flight leg j else 0, the capacity would be the unsold number of seats of resource j, and the revenue would be the fare of product i.

In a hotel that is planning its allocations of rooms for the week ahead, the product could be a stay that begins on a day and ends on another day, such as check-in on Monday and checkout on Wednesday. The resource will be a room night. The capacity will be the number of unsold rooms for each day of the week. The coefficient $a_{ij} = 1$ if product i requires stay on day j (e.g., a person who stays on Monday and Tuesday uses one unit of capacity on each room night). The revenue will be the price charged for the complete stay of product i. For a car rental problem, replace room night with car rental for each day of the week. Note that it is possible that two products use the same set of resources but are priced differently. Examples of these include some combination of room sold with/without breakfast, allowing or not allowing cancellation, taking payment ahead or at end of stay, etc.

The problem is to decide how many reservations X_i to accept of each product i. The general optimization problem can be stated as follows (DLP):

$$\max_{X} \sum_{i = 1\ to\ I} R_i X_i$$

s.t

$$\sum_{i} a_{ij} X_i \leq C_j, \quad j = 1\ to\ J, \tag{23.5}$$

$$X_i \leq D_i, \quad i = 1\ to\ I, \tag{23.6}$$

$$X_i = 0, 1, 2, \ldots, I$$

Here, constraints (23.5) make sure we don't sell more reservations than the capacity on each flight (on average); constraints (23.6) ensure that the number of reservations for an itinerary is less than the demand for that itinerary (mean of the demand—

remember this is just an approximation). The value of this optimization problem can in fact be shown to be an upper bound on the maximum expected revenue.

The following data is necessary to solve this problem: The demands have to be forecast. The capacity that is available will depend on the number of reservations that have already been accepted and has to be computed. The prices might be somewhat unknown because they fluctuate depending on the market conditions and the deals that are to be negotiated. The DLP will require estimates of the expected price. Moreover, it is assumed that there are no cancellations. It is also assumed there are no no-shows and that overbooking is not allowed. Some variations of the basic DLP do account for these factors.

Despite its limitations and simplicity, DLP is often used in practice because it is fast, convenient, and it uses readily available data. Frequent re-optimization and use of the most recent solution can yield good results. A concrete example will help illustrate the approach.

2.7.1 Case Study

A small hotel is planning its allocation of rooms for the week after the next week. For the purpose of planning, it assumes that the customers who stay on weekends belong to a different segment and do not stay over to Monday or check in before Saturday. It sells several products. Here, we consider the three most popular ones that are priced on the average at \$125, \$150, and \$200. These rates are applicable, respectively, if the customer (1) pays up front, (2) provides a credit card and agrees to a cancellation charge that applies only if the room reservation is cancelled with less than 1 day to go, and (3) is similar to (2) but also provides for free Internet and breakfast (that are virtually costless to the hotel). Customers stay for 1, 2, or 3 nights. The demand forecasts and rooms already booked are shown in Table 23.1. The hotel has a block of 65 rooms to allocate to these products.

In this example, there are 45 products and five resources. Each demand forecast pertains to a product. The available rooms on each of the 5 days constitute the five different resources. The Monday 1-night-stay product uses one unit of Monday capacity. The Monday 2-night-stay product uses one unit each of Monday and Tuesday room capacity, etc. The rooms available are 65 minus the rooms sold.

Table 23.1 Demand forecasts and rooms already booked

	125			150			200			Rooms sold
Monday	7	12	17	17	6	4	9	5	1	12
Tuesday	17	3	10	2	7	4	8	1	1	22
Wednesday	2	19	15	3	3	4	2	4	2	31
Thursday	15	11	0	20	9	0	6	0	0	24
Friday	20	0	0	9	0	0	4	0	0	15

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2				125	125	125	150	150	150	200	200	200	Rooms Sold		
3			Nights Stay	1	2	3	1	2	3	1	2	3			
4			Monday	7	12	17	17	6	4	9	5	1	12		
5			Tuesday	17	3	10	2	7	4	8	1	1	22		
6			Wednesday	2	19	15	3	3	4	2	4	2	31		
7			Thursday	15	11	7	20	9	6	6	0	0	24		
8			Friday	20	0	7	9	5	7	4	0	0	15		
9															
10			Decision Variables										Rooms Sold	Rooms Available	Revenue
11			Monday	1	1	1	1	1	1	1	1	1	9	53	2850
12			Tuesday	1	1	1	1	1	1	1	1	1	15	43	2850
13			Wednesday	1	1	1	1	1	1	1	1	1	18	34	2850
14			Thursday	1	1	1	1	1	1	1	1	1	18	41	2850
15			Friday	1	1	1	1	1	1	1	1	1	18	50	2850
16															
17				Total Revenue		14250									

Fig. 23.3 Decisions variables

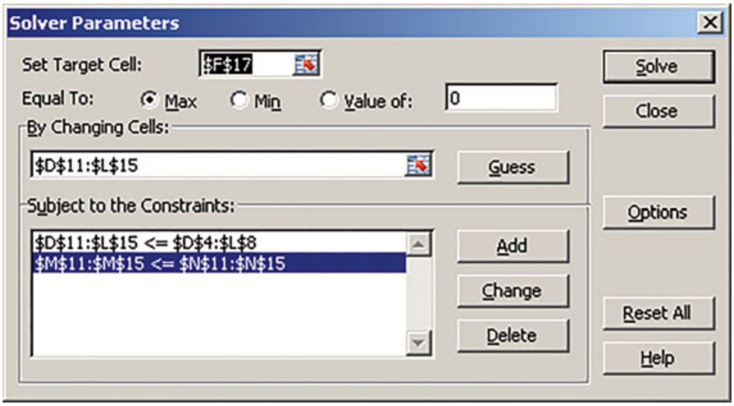

Fig. 23.4 Excel Solver setup

	125	125	125	150	150	150	200	200	200	Rooms Sold		
Nights Stay	1	2	3	1	2	3	1	2	3			
Monday	7	12	17	17	6	4	9	5	1	12		
Tuesday	17	3	10	2	7	4	8	1	1	22		
Wednesday	2	19	15	3	3	4	2	4	2	31		
Thursday	15	11	0	20	9	0	6	0	0	24		
Friday	20	0	0	9	0	0	4	0	0	15		
Decision Variables										Rooms Sold	Rooms Available	Revenue
Monday	7	4	0	17	6	4	9	5	1	53	53	12425
Tuesday	0	0	0	2	7	4	8	1	1	43	43	6800
Wednesday	0	0	0	3	1	4	2	4	2	34	34	5750
Thursday	0	2	0	8	9	0	6	0	0	41	41	5600
Friday	20	0	0	9	0	0	4	0	0	50	50	4650
	Total Revenue		35225									

Fig. 23.5 DLP solution

There are 45 decision variables in this problem. The screenshots of the data, decision variables (yellow), and Excel Solver setup are shown in Figs. 23.3 and 23.4.

Solving this problem as a linear program or LP (choose linear and non-negative in Solver), we obtain the solution shown in Fig. 23.5.

Table 23.2 Shadow price of constraints

Cell	Name	Final value	Shadow price	Constraint RH side	Allowable increase	Allowable decrease
$M11$	Monday rooms sold	53	100	53	2	0
$M12$	Tuesday rooms sold	43	150	43	0	2
$M13$	Wednesday rooms sold	34	150	34	2	1
$M14$	Thursday rooms sold	41	150	41	12	7.999999999
$M15$	Friday rooms sold	50	100	50	7.999999999	2.000000001

Note: Shadow prices were rounded to the nearest integer value

Table 23.3 Is rate class open or closed?

	125			150			200		
Night stay	1	2	3	1	2	3	1	2	3
Monday	OPEN	OPEN	CLOSED	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN
Tuesday	CLOSED	CLOSED	CLOSED	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN
Wednesday	CLOSED	CLOSED	CLOSED	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN
Thursday	CLOSED	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN
Friday	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN	OPEN

The optimal solution is to not accept many bookings in the \$125 rate class, except on Monday and Tuesday. Even some of the demand in the \$150 rate class is turned away on Wednesday and Thursday. One might simply use this solution as guideline for the next few days and then re-optimize based on the accepted bookings and the revised demand forecasts. Two potential opportunities for improvement are as follows: (1) The solution does not consider the sequence of arrivals, for example, whether the \$125 rate class customer arrives prior to the \$150. (2) The solution does not take into account the stochastic aspect of total demand. These can be partially remedied by use of the dual prices provided by the sensitivity analysis of the solution. The sensitivity analysis of the solution to the LP is obtained from any traditional solver including Excel. The sensitivity analysis of the room capacities is given in Table 23.2.

There is one shadow price per resource and day of stay. This can be used as the bid price for a room for that day. For example, if a customer were willing to pay \$225 for a 2-night stay beginning Monday, we would reject that offer because the price is less than the sum of the bid prices for Monday and Tuesday ($100 + 150$), whereas the hotel should accept any customer who is willing to pay for a 1-night stay on Monday or Friday if the rate exceeds \$100. One might publish what rate classes are open based on this logic as shown in Table 23.3.

We can also compute the minimum price for accepting a booking (or a group): In order to create the minimum price (see Table 23.4), we have rounded the shadow price manually to integer value. We emphasize that the bid price is an internal control mechanism that helps decisions makers in deciding whether to accept a customer. The bid price need not bear resemblance to the actual price. Also, note

Table 23.4 Minimum price based on shadow prices

Night stay	1	2	3
Monday	100	250	400
Tuesday	150	300	450
Wednesday	150	300	400
Thursday	150	250	250
Friday	100	100	100

Table 23.5 Tuesday night single-resource analysis

Product	Total revenue	Revenue for Tuesday
Monday 125 2 nights	250	150
Monday 125 3 nights	375	125
Monday 150 2 nights	300	200
Monday 150 3 nights	450	200
Monday 200 2 nights	400	300
Monday 200 3 nights	600	350
Tuesday 125 1 night	125	125
Tuesday 125 2 nights	250	100
Tuesday 125 3 nights	375	75
Tuesday 150 1 night	150	150
Tuesday 150 2 nights	300	150
Tuesday 150 3 nights	450	150
Tuesday 200 1 night	200	200
Tuesday 200 2 nights	400	250
Tuesday 200 3 nights	600	300

that even though the \$150 rate class for 1-night stay is open on Thursday, the LP solution does not accept all demand. Thus, the bid price is valid only for small change in the available capacity. Moreover, we may need to connect back to the single-resource problem to determine the booking limits for different rate classes. To see this, consider just the resource called Tuesday. Several different products use the Tuesday resource. Subtracting the bid price for the other days from the total revenue, we arrive at the revenue for Tuesday shown in Table 23.5.

Based on this table, we can infer that the DLP can also provide relative value of different products. This can be used in the single-resource problem to obtain the booking limits. We can also group products into different buckets prior to using the booking limit algorithm. Products with Tuesday revenue greater than or equal to 300 can be the highest bucket; the next bucket can be those with revenue between 200 and 250; the rest are into the lowest bucket.

Uses and Limitations of Bid Prices for Network Revenue Management: There are many practical uses of the bid prices. First and foremost, the approach shifts the focus of forecasting to the product level and away from the single-resource level. Thus, the decision maker generates demand forecasts for 1-night and 2-night stays separately instead of forecast for Tuesday night stay. The bid prices can help in route planning, shifting capacity if some flexibility is available, running promotions/shifting demand, identifying bid price trends, etc. For example, the management might decide not to offer some products on certain days, thereby shifting demand to other products. If there is some flexibility, a rental car company might use the bid price as guideline to move cars from one location with a low price to another with a high price. The product values might reveal systematic patterns of under- and over-valuation that can help decide whether to run a promotion for a special weekend rate or to a particular destination. Bid price trends that show a sustained increase over several weeks can indicate slackening of competitive pressure or advance bookings in anticipation of an event.

Several limitations of the approach have been mentioned in the chapter itself. More advanced material explaining the development of the network revenue management can be found in the references given in the chapter.

3 Further Reading

There are several texts devoted to revenue optimization. Robert Cross' book (2011) is one of the earliest ones devoted to the art and science of revenue management in a popular style. Many ideas discussed in this chapter and many more find a place in the book. Robert Phillips' book (2005) and Talluri and Van Ryzin's book (2006) contain a graduate level introduction to the subject. In addition, we have borrowed ideas from the papers listed at the end of the chapter (Bratu 1998; Lapp and Weatherford 2014; Talluri and van Ryzin 1998; Williamson 1992). The INFORMS Revenue Management and Pricing Section website[4] contains several useful references. Finally, there is a *Journal of Revenue and Pricing Management*[5] that is devoted to the topic.

Electronic Supplementary Material

All the datasets, code, and other material referred in this section are available in www.allaboutanalytics.net.

- Data 23.1: Opera.xls

Exercises

Ex. 23.1 (Protection Level) An airline offers two fare classes for economy class seats on its Monday morning flight: one class is sold at $400/ticket and another at

[4] http://www.informs.org/Community/revenue-mgt. Accessed on May 22, 2018.

[5] http://www.palgrave-journals.com/rpm/index.html. Accessed on May 22, 2018.

\$160/ticket. There are 225 economy seats on the aircraft. The demand for the \$400 fare (also called full-fare) seats has a mean of 46, a standard deviation of 16. Assume it follows a normal distribution. The demand for cheaper seats has an exponential distribution with mean of 177. A seat can be sold to either class. Further, the demand for the two fare classes can be assumed to be independent of one another. The main restriction is that the cheaper tickets must be purchased 3 weeks in advance.

(a) How many seats would you protect for the \$400 class customers?
(b) The forecast for *cheaper* class passengers has changed. It is now assumed to be less than 190 with probability 1. How many seats would you protect for full-fare customers given this information?
(c) Go back to the original problem. Suppose that unsold seats may sometimes be sold at the last minute at \$105. What effect will this have on the protection level (will you protect more or less seats or the same number of seats)? Why?
(d) Will your original answer change if the demands for the two classes are not independent of one another. Explain your answer if possible using an example.

Ex. 23.2 (Bid Price) Please see the data in the Excel sheet *Opera.xls* (available on website). The question is also given in the spreadsheet. It is reproduced below. All data is available in the spreadsheet.

Please carry out the following analysis based on the opera data. You are provided the cumulative booking for 1 year for two ticket classes. Assume that the opera house sells two types of tickets for their floor seats. The first is sold at \$145, and the ticket is nonrefundable. The second is for \$215 but refundable. The opera house has 245 floor seats. This data is given in two sheets in the spreadsheet.

You may verify (or assume) that the booking pattern is the same for most days. This is because we have normalized the data somewhat and got rid of peaks and valleys. The booking pattern is given 14 days prior to the concert onward. The final entry shows how many persons actually showed up for the concert on each day. Here is a sample of the data for \$145 seats:

	−1	0	1	2	3					
11/30/2011	143	143	133	124	116					

For example, today is November 30, 2011. For this date, 116 persons had booked seats with 3 days to go, 124 with 2 days to go, 133 with 1 day to go, and 143 the evening before the concert. Finally, 143 persons showed up on November 30 which was the day of the concert.

We have created a **forecast** for the demand for the two types of seats for the next ten days, December 1 through December 10. We have used the additive method to estimate the pickup (PU).

(In this method, we computed the difference between the average seats sold and seats booked with 1, 2, 3, ... days to go. That is the PU with 1, 2, 3, ... days to go). **See rows 40–44 in the first sheet of the spreadsheet** for the forecast.

Answer questions (a)–(d):

(a) Remember the opera has already sold some seats for the next 10 days. Compute the **available capacity** for the next 10 days (December 1 through December 10).
(b) Determine **how many seats to sell** at each price for the next 10 days. You have to set up a linear program for doing this.
(c) Comment on your solution. (How to use the shadow prices? What do the shadow prices reveal? What is necessary for implementing the solution?)
(d) Based on the data, can you provide advice on how to determine the overbooking level? Provide if possible an example using the data and any necessary assumption of the overbooking level and how it will be used by you in the optimization.

Ex. 23.3 (Choice Model) Daisy runs a small store in rural Indiana. Customers who come have to shop in her store or drive miles to go elsewhere. She has heard about revenue optimization! She always wondered at the rate at which customers gobbled her candy bars and always wondered whether she was pricing them right. The three best sellers are Milky Way, Starburst, and Coconut Bar. By gently asking some of her varied but trusted customers, she estimates their willingness to pay is around \$2.20, \$2.60, and \$2.00 for the three types of candy bars. The variance seems to be around 0.10 for each of these willingness-to-pay values. Currently, she charges \$2.00 for any of the candy bars. Typically, 100 customers visit her store every day.

(a) Estimate Daisy's current average sales and revenue.
(b) Daisy wants to run a promotion in her store by giving 10% off on one type of bar to customers. Which bar should she discount?
(c) What should be Daisy's optimal uniform price for the three types of candy bars? Would you recommend the price change?

Hint: Use the MNL model of choice. In this model, customers are assumed to be homogenous. They have an underlying utility U_i for product i. Each product is priced at p_i, i=1, 2, ..., n. The probability they will purchase product i is given by the following calculations:

$$\mu = \frac{\sqrt{(\text{variance} * 6)}}{\pi}$$

$$U_i = \textit{gross utility of product i} \ \ (\textit{assume equal to willingness to pay})$$

$$v_i = e^{((U_i - p_i)/\mu)} \qquad \textit{Prob}\,(\textit{Purchase}\,i) = \ v_i/(1 + v_1 + \ v_2 + \ldots + v_n)$$

Ex. 23.4 (Dynamic Pricing) Mike is the revenue management manager at Marriott Hotel on 49th St., Manhattan, New York. He is contemplating how to respond to last-minute "buy it now" requests from customers. In this sales channel, customers can bid a price for a room, and Mark can either take it or wait for the next bid. Customers are nonstrategic (in the sense, they don't play games with waiting to bid). Mark has observed that typically he gets at most one request every hour. Analysis indicates that he gets a request in an hour with probability 0.2. He is looking at the last 3 h of the decision before the booking for the next day closes. For example, if booking closes at midnight, then he is looking at requests between 9 and 10 PM, 10 and 11 PM, and 11 and midnight. Customers either ask for a low rate or a high rate. Typically, half of them ask for a room for $100 and the rest for $235 (which is the posted rate).

Help Mark structure his thoughts and come up with a decision rule for accepting or turning down bids. It may help to think that with 3 h to go he can at most sell three rooms, with 2 h to go he can sell at most two rooms, and with an hour to go he can sell at most one room. (Thus, he can give away excess rooms at any price beyond these numbers, etc.) Use the dynamic programming example.

Ex. 23.5 (Overbooking) Ms. Dorothy Parker is the admissions director at Winchester College that is a small liberal arts college. The college has a capacity of admitting 200 students a year. The classrooms are small and the college wants to maintain a strict limit. Demand is robust with over 800 applications the previous year, out of which 340 students were offered a place on a rolling basis and the target of 200 admissions was met.

However, 17 students who accepted the offer of admission did not show up. Subsequent enquiries revealed that four of them had a last-minute change of heart about their college choice, three decided to take a gap year, and there was no reply from the rest. They paid the deposit and forfeited the amounts by college rules. Admissions contacted those on the waiting list, but it was too late as most already joined other institutions. As a result, the cohort comprised only 183 students stressing the budgets.

Ms. Parker decided that a change of policy was needed, and for the next year, the college will overbook, that is, admit a cohort larger than the capacity of 200. The question is how many. The tuition fee for 1 year of study is $34,500.

(a) What data should Ms. Parker be collecting to make a decision on how many students to admit beyond the limit of 200?
(b) Can we assume that the cost of falling short by a student is the 4 years' worth of tuition revenue? Argue why or why not.
(c) What is the cost of taking on a student over the 200 limit? Explain how you came up with your number.
(d) Ms. Parker decided after some analysis that the lost revenue from a student was $100,000, and the cost of having more students than capacity is as follows:

Students	Cost
201	$10,000
202	$22,000
203	$40,000
204	$70,000
205	$100,000
206	$140,000

Beyond that, it is \$50,000 per student.

Is this data enough to set a target number of admissions? What other data would be useful? Based only on this data, how many students would you admit?

(e) Analyzing the previous 5 years of data, Ms. Parker observed that with the policy of admitting exactly 200 each year, the final number of students who showed up was as given below:

Admitted	Showed up
200	200
200	195
200	197
200	190
200	192
200	183

If Ms. Parker was to naively admit 217 students based on this year's observation of no-shows, what would be the expected cost? Based on the data, what is the optimal number to overbook?

Ex. 23.6 (Markdown Optimization) Xara is a speciality fashion clothing retail store focusing on the big-and-tall segment of the market. This year, it is selling approximately 12,000 SKUs, with each SKU further classified by sizes. The initial prices for each item are usually set by the headquarters, but once the shipment reaches the stores, the store managers have the freedom to mark down the items depending on sales. Store managers are evaluated based on the total revenue they generate, so the understanding is that they will try to maximize revenue.

The demand for the new line of jeans was estimated based on historical purchases as follows:

$$D(p) = 10{,}000(1 - 0.0105p)$$

Here, 10,000 stands for the potential market, and the interpretation of $(1 - 0.0105p)$ is the probability of purchase of each member of the market. That is, demand at price p is given by the preceding formula, where p is in the range of 0–\$95 (i.e., beyond \$95, the demand is estimated to be 0).

The season lasts 3 months, and leftover items have a salvage value of 25% of their initial price. The headquarters sets the following guidelines: Items once marked down cannot have higher prices later. Prices can only be marked down by 10, 20, 30, or 40%. It is assumed demand comes more or less uniformly over the 3 month season.

(a) Based on the demand forecast, what should be the initial price of the jeans, and how many should be produced?

(b) The manager of the store on Portal de l'Angel in Barcelona obtained an initial consignment of 300 jeans, calculated to be the expected demand at that store. After a while, he noticed that the jeans were selling particularly slowly. He had a stock of 200 items still, and it was already 2 months into the season, so it is likely the potential market for the store area was miscalculated. Should he mark down? If so, by how much? (Hint: Based on the expected demand that was initially calculated for the store, you need to derive the demand curve for the store.)

Ex. 23.7 (Repricing) Meanrepricer.com offers a complex rule option where you can set prices according to the following criteria:

- My Item Condition: the condition of your item
- Competitor Item Condition: the condition of your competitors' product
- Action: the action that needs to be taken when applying a rule
- Value: the difference in prices which needs to be applied when using a certain rule

Here are some sample rules. Discuss their rationale (if any) and how effective they are.

(a) If our price for Product A is 100 and our competitors' price for Product A is \$100, then the repricer will go ahead and reduce our price by 20% (i.e., from \$100 to \$80).
(b) In case your competitors' average feedback is lower than 3, chosen condition will instruct the repricer to increase your price by two units.
(c) Sequential rules, where the first applicable rule is implemented:
    (i) Reduce our price by two units if our competitors' product price is within a range of 300–800 units.
    (ii) Increase our price by two units if our competitors' product price is within a range of 500–600 units.

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