Politecnico di Milano
AUTOMATION AND CONTROL ENGINEERING

Study and Control of the Mechanichal System: Rotary Flexible Joint

Course
Automation and Control Laboratory

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Problem Description

This report will describe the model of the system, our solution and some attempts to describe and control the system.

The system is composed by a motor's module that provide torque to a turret, above the turret there's a beam which is attached at one of the two edges through a screw to the turret. The beam will follow the movement of the base due to two springs attached between the turret and the beam.

The system has several interfaces that could be connected to an acquisition system (DAC/ADC + Amplifier) to acquire measurements and provide input signal, the interfaces are:

- Actuators:

  Power Supply input of the motor's module (changing the voltage);

- Sensors:

  Incremental Encoder for the position of the turret with respect to to the motor's module;

  Incremental Encoder for the relative position of the arm with respect to the turret.

The acquisition system composed by ADC/DAC + Amplifier is already configured, it doesn't require our attention, for this reason it will not treat in this report.

The main task is to control a low damped system with variable parameters, this goal is divided in sub-tasks to be achieved:

1. position control of the top base, with a frequency based approach;
2. position control of the arm tip, with a frequency based or a state space approach;
3. position control of the arm tip with uncertainty in the spring stiffness and arm moment of inertia, with a state space approach or other advanced control techniques.

Model Identification

The system could be schematized:

It is possible to recognize models of a DC electric motor powered by a voltage V_{dc} coupled with a gearbox (both modeled in the same box) that provide torque u to a flexible (due the springs) joint, at last an encoder to model all the conversion's dynamics between the angular positions y and the red ones $\hat{y}$.

2.1 Mathematical Model Creation

2.1.1 DC Motor Equations

The first task is to decide the shape of the model in terms of which dynamics consider or neglect.

Starting from the DC motor we assumed a static model due to the fact that from the data sheet of the motor, it should have a dynamic given by the inductance at a frequency:

$$\frac{R}{L} = \frac{2.6\Omega}{0.18mH} \approx 15KHz$$

this is clearly above the frequency range of the mechanical system, that for its nature should have a frequency in the order at last of $100Hz$ (deeper treatment in its section).

The physical equations of the Static DC Motor:

$$\begin{cases} V_a = R_a I_a + E \\ E = k_m \Omega \\ \tau = k_t I_a \end{cases}$$

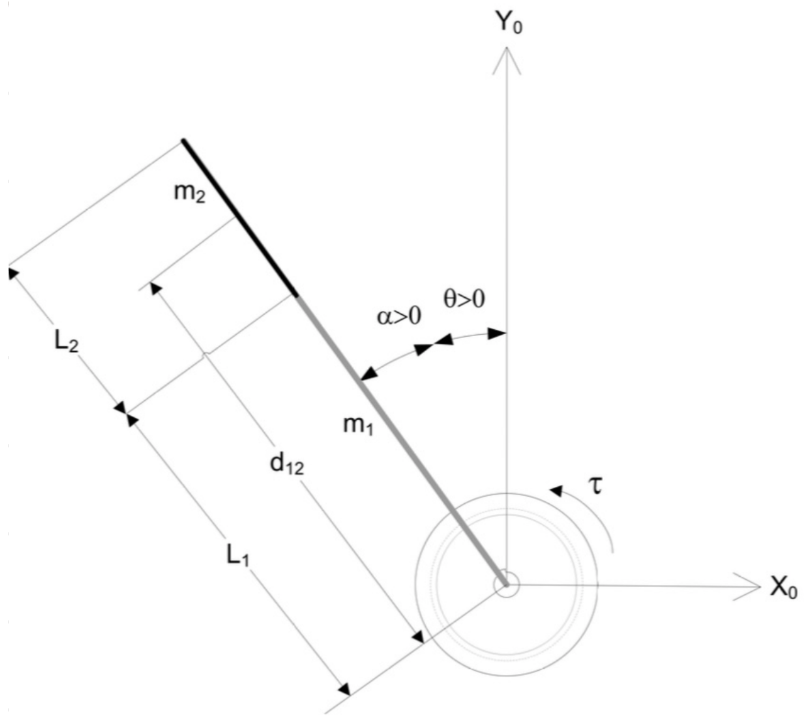
After several mathematician steps and considering the gearbox effect:

$$\tau = \frac{\eta_m \eta_g k_t K_g (V - K_g k_m \dot{\theta})}{R_m}$$

this is the output of the DC Motor's Model, where has been added the gear ratio K_g to provide the angular position at the attaching point of the turret.

2.1.2 Flexible Joint and The Gearbox Equations

The model of the beam consider the scheme of the beam as:



in this model the system is a 2-dofs mechanical system and can be modeled following a Lagrangian's Approach.

The 2 deegrees of freedom are:

- θ: the absolute angular position of the base of the turret;
- α: the relative angular position of the tip with respect to to the base of the turret.

The Kinetic Energy:

$$V = \frac{1}{2}J_{eq}\dot{\theta}^2 + \frac{1}{2}J_L(\dot{\theta} + \dot{\alpha})^2$$

where J_{eq} refers to the equivalent inertia of the motor + gearbox, instead J_L refers to the inertia of the beam.

The Potential Energy:

$$V = \frac{1}{2}K_s\dot{\alpha}^2$$

where K_s refers to the linearized stiffness of the equivalent torsional spring. This assumption will be clarified later, but for readability reasons not here.

The Dissipative Function:

$$D = \frac{1}{2}B_{eq}\dot{\theta}^2 + \frac{1}{2}B_L\dot{\alpha}^2$$

where B_{eq} refers to the equivalent friction of the motor + gearbox, instead B_L refers to the equivalent friction that the single beam is subjected.

Applying the Lagrange treatment for each deegree of freedom :

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \left(\frac{\partial T}{\partial x}\right) + \left(\frac{\partial D}{\partial \dot{x}}\right) + \left(\frac{\partial V}{\partial x}\right) = \left(\frac{\delta W}{\delta x}\right)$$

and after several mathematical steps the system of equation becomes:

$$\begin{cases} J_{eq}\ddot{\theta} + J_L(\ddot{\theta} + \ddot{\alpha}) + B_{eq}\dot{\theta} = \tau \\ J_L(\ddot{\theta} + \ddot{\alpha}) + B_L\dot{\alpha} + K_{stiff}\alpha = 0 \end{cases}$$

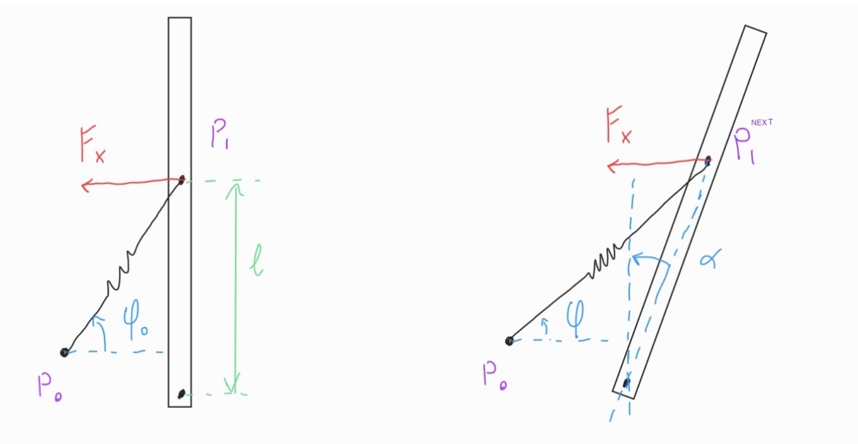
Non-Linear Model for The Spring

The reason of considering the couple of spring as an equivalent torsional one is to reduce the complexity of the system. The validity of this assumption comes from a study that we did on the error that the linear approximation provides with respect to to the real model.

Considering the General equation of a spring

$$F = K_s(x_k - x_0)$$

in the following situations (we consider in this treatment only one spring, but the discussion is valid due to the symmetry for the entire couple)



On the left the equilibrium position $x_k = x_0$, where:

$$\varphi_0 = atan\left(\frac{P_{1_y}}{P_{0_x}}\right)$$

$$F_x = F \cdot cos(\varphi_0)$$

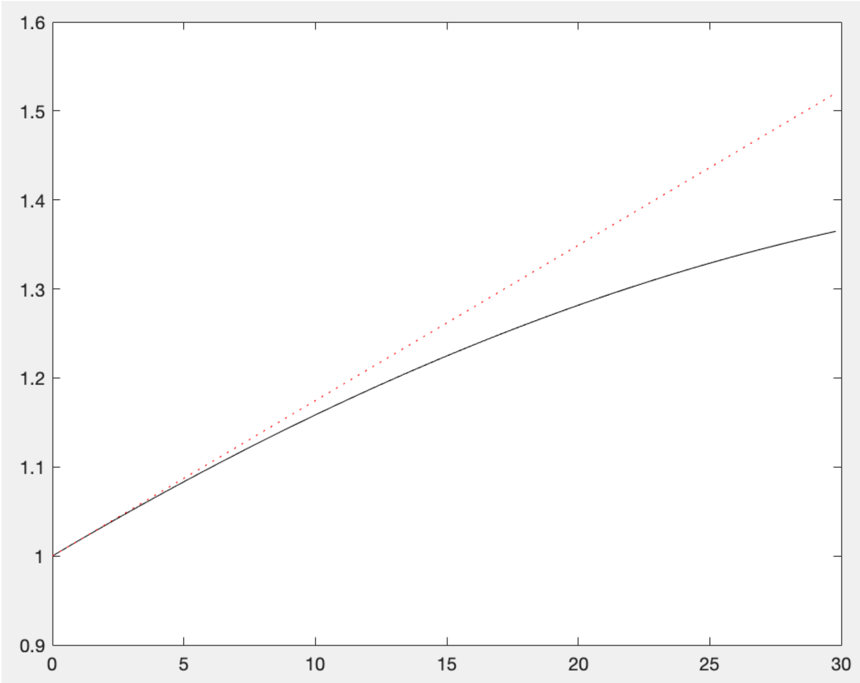
$$x_0 = \sqrt{(P_{1_x} - P_{0_x})^2 + (P_{1_y} - P_{0_y})^2}$$

On the right you are perturbed position, so:

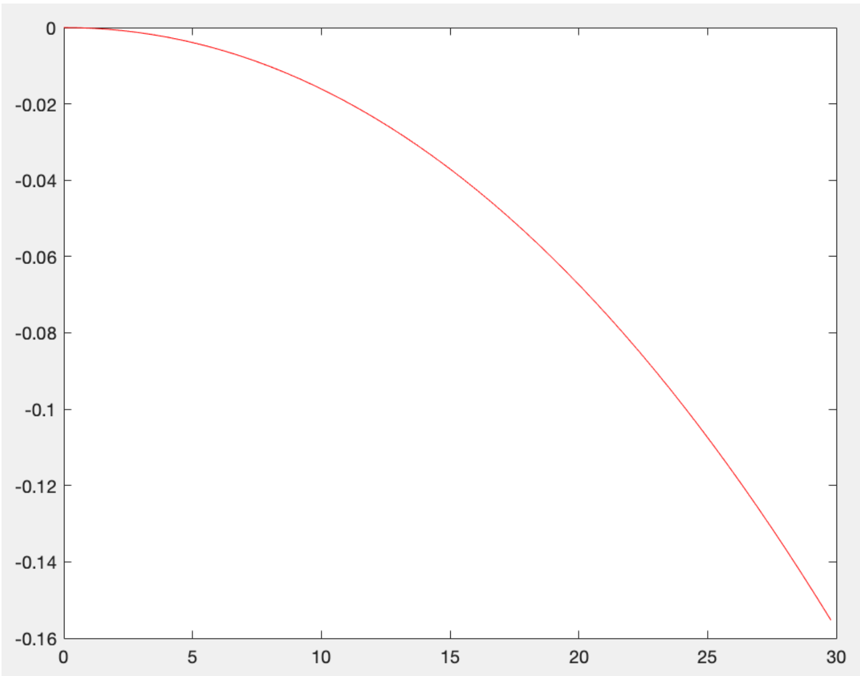
$$x_k = \sqrt{(P_{1_x} - P_{0_x})^2 + (P_{1_y} - P_{0_y})^2} \rightarrow \Delta x_k = x_k - x_0$$

$$P_1^{NEXT} = [l \cdot cos(\alpha), l \cdot (1 - sin(\alpha))] \rightarrow F_x = F \cdot cos(\varphi + \alpha)$$

Plotting the value of the F_x in function of the angle α compared with a linear increase we obtain:



the error between the two curves:



it is possible to see from the graph that the error starts to be non-negligible above 25 degrees, but from measurements the angle remains under 10 degrees. For this reason we can consider the Force with a linear behavior and the K_s constant.

2.1.3 Development of The State Space Model

Continuous Time

Starting from the equations:

$$\begin{cases} J_{eq}\ddot{\theta} + J_L(\ddot{\theta}+\ddot{\alpha}) + B_{eq}\dot{\theta} = \frac{\eta_m \eta_g k_t K_g (V - K_g k_m \dot{\theta})}{R_m} \\ J_L(\ddot{\theta}+\ddot{\alpha}) + B_L\dot{\alpha} + K_S\alpha = 0 \end{cases}$$

we develop the State Space system in continuous time, where the state are:

$$\begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \\ \dot{\alpha} \end{bmatrix}$$

the matrix A:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\mathrm{eq}} R_m}{J_{\mathrm{eq}} R_m} & \frac{K_s}{J_{\mathrm{eq}}} & \frac{B_L}{J_{\mathrm{eq}}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\mathrm{eq}} R_m}{J_{\mathrm{eq}} R_m} & -K_S\left(\frac{J_{\mathrm{eq}}+J_L}{J_{\mathrm{eq}} J_L}\right) & -B_L\left(\frac{J_{\mathrm{eq}}+J_L}{J_{\mathrm{eq}} J_L}\right) \end{bmatrix}$$

and the B matrix:

$$\begin{bmatrix} 0 \\ \frac{\eta_m \eta_g k_t K_g}{R_m J_{\mathrm{eq}}} \\ 0 \\ -\frac{\eta_m \eta_g k_t K_g}{R_m J_{\mathrm{eq}}} \end{bmatrix}$$

Considering as the outputs of the system the angular positions θ and α.

Discrete Time

One last step is to compute the model in discrete time, this is necessary for the last and definitive approach we used in the identification procedure.

Considering a sampling time Δ the A matrix:

$$\begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1-\Delta\frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\mathrm{eq}} R_m}{J_{\mathrm{eq}} R_m} & \Delta\frac{K_{\mathrm{stiff}}}{J_{\mathrm{eq}}} & \Delta\frac{B_L}{J_{\mathrm{eq}}} \\ 0 & 0 & 1 & \Delta \\ 0 & \Delta\frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\mathrm{eq}} R_m}{J_{\mathrm{eq}} R_m} & -\Delta K_S\left(\frac{J_{\mathrm{eq}}+J_L}{J_{\mathrm{eq}} J_L}\right) & 1-\Delta B_L\left(\frac{J_{\mathrm{eq}}+J_L}{J_{\mathrm{eq}} J_L}\right) \end{bmatrix}$$

And the B matrix:

$$\begin{bmatrix} 0 \\ \Delta\frac{\eta_m \eta_g k_t K_g}{R_m J_{\mathrm{eq}}} \\ 0 \\ -\Delta\frac{\eta_m \eta_g k_t K_g}{R_m J_{\mathrm{eq}}} \end{bmatrix}$$

the outputs remain the same.

2.2 Identification Technique

We proceeded in 3 different way, increasing the complexity, trying to fit as possible all the dynamics of the system. The first two methods didn't provide us enough good results, but are reported because guide us in the choice of a good method for the identification and the model produced is reliable.

2.2.1 Deprecated Methods

Stiffness Identification

The first method sticks too much on the reliability of the parameters from the data sheet: we tried to identify just the value of the stiffness of the spring using a step signal and analyzing the frequency of the peak of resonance:

$$K_s = J_L \cdot \omega_n^2$$

As result our model didn't fit a lot the real system and the results was so bad that encourage us to proceed in a complete different direction.

Identification Toolbox

Due to high number of possible uncertainties we look for a different approach that could work around the small number of possible types of experiments and the direct inaccessibility of some parameters. An interesting example of this last consideration is the impossible measurement of the current inside the armature to get a measurement of the resistance R_m.

For these reasons we choose to look for an optimization method that can provide the values of the state space matrices. The first attempt consisted in the usage of the model identification toolbox that, given the order of the system, provides a transfer function representation of the system.

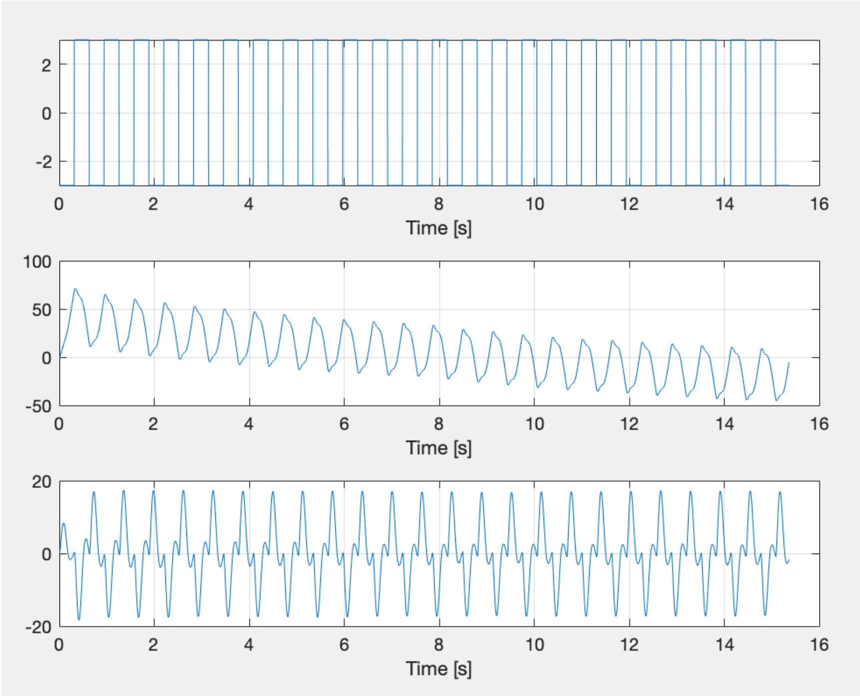
I will not go in deep with this method became as first step in that direction we didn't put too much effort. In fact, we let Matlab works on its own to get the model however the results weren't good enough and in this way we lost the physical meaning of the provided quantities.

2.2.2 Model Identification using CVX

This is the definitive method that we used. CVX is a package allows, giving constraints and objectives, to implement a convex optimization in Matlab in the form:

$$\begin{aligned} \text{minimize } & \|Ax - b\|_2 \\ \text{subject to } & Cx = d \\ & \|x\|_\infty \leq e \end{aligned}$$

As dataset we collect the values of the 2 outputs with the system subjected to a square wave of period of $T = 0,63s$



For the optimization we started from the nominal parameters, considering the idea that the real values should be not too far.

The nominal parameters:

- For the motor:

  Motor armature resistance: $R_m = 2.6\Omega$

  Motor current-torque constant: $K_t = 0.00768 Nm/A$

  Motor back-emf constant: $K_m = 0.00768 V/(rad/s)$

  High-gear total gear ratio: $K_g = 70$

  Motor efficiency: $\eta_m = 0.69$

  Gearbox efficiency: $\eta_g = 0.9$

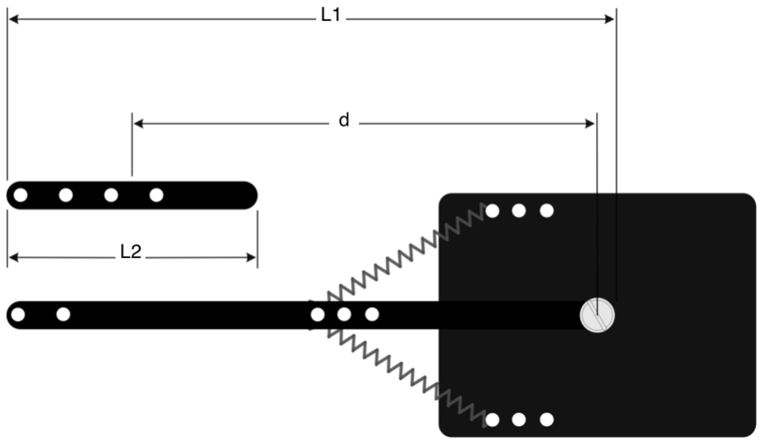
- The equivalent mechanical system of the motor + gearbox:

  Equivalent moment of inertia: $J_{eq} = 0.002087 Kgm^2$

  Equivalent viscous damping coefficient: $B_{eq} = 0.015 Nm/(rad/s)$

For the parameters of the rotating arm we compute the values of the inertia, following its geometry, as:

$$J_L = m_1 \cdot \frac{L_1^2}{3} + m_2 \cdot \frac{L_2}{12} + m_2 \cdot d^2 = 0.0032 Kgm^2$$

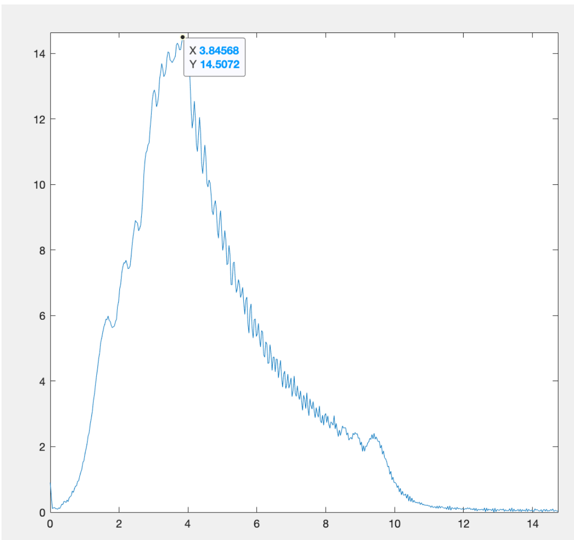


the value of the friction coefficient was supposed initially null:

$$B_L = 0$$

and the value of the K_s we use the value generated in the analysis stiffness identification:

the analysis provide a Fourier transfer of the second output (the relative position of the tip) as in the figure:



the peak is at:

$$f = 3.84568Hz$$

as result we assign the stiffness initial value as:

$$K_s = J_L \cdot \omega_n^2 = 1.8426N/m$$

Position Control of the Base

Position Control of The Tip

Position Control of the Tip with Uncertanties

Conclusioni