

Symbol	Description	Value	Unit
	Module Dimensions	10 x 8 x 5	cm ³
L_1	Main arm length	29.8	cm
L_2	Load arm length	15.6	cm
	Distance between joint to middle of load arm		
d_{12}	Arm Anchor Point 1	21.0	cm
d_{12}	Arm Anchor Point 2	23.5	cm
d_{12}	Arm Anchor Point 3	26.0	cm
	Module body mass	0.3	kg
m_1	Main arm mass	0.064	kg
m_2	Load arm mass	0.03	kg
K_{enc}	Encoder resolution (in quadrature mode)	4096	Counts/Rev
K_1	Spring #1 stiffness	187	N/m
K_2	Spring #2 stiffness	313	N/m
K_3	Spring #3 stiffness	565	N/m

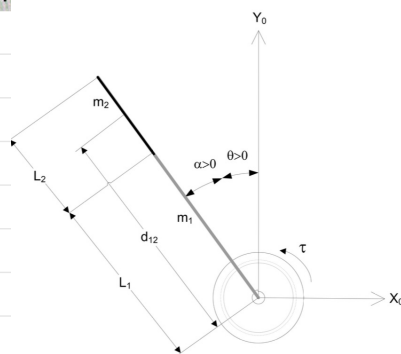
Symbol	Description	Value	Variation
V_{nom}	Motor nominal input voltage	6.0 V	
R_m	Motor armature resistance	2.6 Ω	$\pm 12\%$
L_m	Motor armature inductance	0.18 mH	
k_t	Motor current-torque constant	7.68×10^{-3} N-m/A	$\pm 12\%$
k_m	Motor back-emf constant	7.68×10^{-3} V/(rad/s)	$\pm 12\%$
K_g	High-gear total gear ratio	70	
	Low-gear total gear ratio	14	
η_m	Motor efficiency	0.69	$\pm 5\%$
η_g	Gearbox efficiency	0.90	$\pm 10\%$
$J_{m,rotor}$	Rotor moment of inertia	3.90×10^{-2} kg-m ²	$\pm 10\%$
J_{tach}	Tachometer moment of inertia	7.06×10^{-8} kg-m ²	$\pm 10\%$
J_{eq}	High-gear equivalent moment of inertia without external load	2.087×10^{-3} kg-m ²	
	Low-gear equivalent moment of inertia without external load	9.7585×10^{-5} kg-m ²	
B_{eq}	High-gear Equivalent viscous damping coefficient	0.015 N-m/(rad/s)	
	Low-gear Equivalent viscous damping coefficient	1.50×10^{-4} N-m/(rad/s)	
m_b	Mass of bar load	0.038 kg	
L_b	Length of bar load	0.1525 m	
m_d	Mass of disc load	0.04 kg	
r_d	Radius of disc load	0.05 m	
m_{max}	Maximum load mass	5 kg	
f_{max}	Maximum input voltage frequency	50 Hz	
I_{max}	Maximum input current	1 A	
ω_{max}	Maximum motor speed	628.3 rad/s	

Symbol	Description	Value
K_{gi}	Internal gearbox ratio	14
$K_{ge,low}$	Internal gearbox ratio (low-gear)	1
$K_{ge,high}$	Internal gearbox ratio (high-gear)	5
m_{24}	Mass of 24-tooth gear	0.005 kg
m_{72}	Mass of 72-tooth gear	0.030 kg
m_{120}	Mass of 120-tooth gear	0.083 kg
r_{24}	Radius of 24-tooth gear	6.35×10^{-3} m
r_{72}	Radius of 72-tooth gear	0.019 m
r_{120}	Radius of 120-tooth gear	0.032 m

Table 3.2: SRV02 Gearhead Specifications

Symbol	Description	Value	Variation
K_{pot}	Potentiometer sensitivity	35.2 deg/V	$\pm 2\%$
K_{enc}	Encoder sensitivity	4096 counts/rev	
K_{tach}	Tachometer sensitivity	1.50 V/kRPM	$\pm 2\%$

Table 3.3: SRV02 Encoder Specifications



$$J_L = J_1 + J_2 = \frac{m_1 L_1^2}{3} + \frac{m_2 L_2^2}{12} + m_2 d_{12}^2$$

$$\frac{\partial^2 L}{\partial t \partial q_i} - \frac{\partial L}{\partial q_i} = Q_i; \quad q_i = \begin{bmatrix} \theta \\ \alpha \end{bmatrix}$$

$$L = T - V$$

$$Q_1 = T - B_{eq} \dot{\theta}$$

$$Q_2 = -B_L \dot{\alpha}$$

$$T = \frac{\eta_g K_g \eta_m k_t}{R_m} (V_m - k_g k_m \dot{\theta})$$

$$\bullet L = T - V$$

$$T = \frac{1}{2} J \omega^2 = \frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} J_L (\dot{\theta} + \dot{\alpha})^2$$

$$V = \frac{1}{2} K_s \alpha^2$$

$$\bullet \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i$$

$$\triangleright (J_{eq} + J_L) \ddot{\theta} + J_L \ddot{\alpha} + B_{eq} \dot{\theta} = \tau$$

$$\triangleright J_L \ddot{\theta} + J_L \ddot{\alpha} + B_L \dot{\alpha} + K_s \alpha = 0$$

$$\left. \begin{array}{l} \text{Subs} \\ \begin{array}{l} (J_{eq} + J_L) \ddot{\theta} + J_L \ddot{\alpha} + B_{eq} \dot{\theta} = \tau \\ J_L \ddot{\theta} + J_L \ddot{\alpha} + B_L \dot{\alpha} + K_s \alpha = 0 \end{array} \end{array} \right\} \Rightarrow$$

$$\ddot{\theta} = \frac{(T - B_{eq} \dot{\theta} + B_L \dot{\alpha} + K_s \alpha)}{J_{eq}}$$

$$\ddot{\alpha} = \left[-\frac{1}{J_L} \tau - \left(\frac{J_{eq} + J_L}{J_L J_{eq}} \right) \dot{\alpha} \dot{\theta} + \frac{1}{J_{eq}} B_{eq} \dot{\theta} - \left(\frac{J_{eq} + J_L}{J_L J_{eq}} \right) K_s \alpha \right]$$

$$\begin{array}{l} x_1 = \theta \\ x_2 = \dot{\theta} \\ x_3 = \alpha \\ x_4 = \dot{\alpha} \end{array} \quad \frac{d}{dt} \quad \begin{array}{l} \dot{x}_1 = \dot{\theta} = x_2 \\ \dot{x}_2 = \ddot{\theta} \\ \dot{x}_3 = \dot{\alpha} = x_4 \\ \dot{x}_4 = \ddot{\alpha} \end{array}$$

To find the stiffness we need to find the natural frequency of the flexible joint. This is the frequency where the link attached to the flexible joint begins to oscillate the most. To find this frequency, we use a Sine Sweep signal. The Sine Sweep is a sine wave that goes through a range of frequencies, i.e., from low to high. We can then generate a power spectrum from the measured signal and identify the frequency with the largest amplitude - the natural frequency.

$$J(l) = \begin{cases} 3.28 \times 10^{-3} & \text{arm load 1, } d_{12} = 0.210 \\ 3.62 \times 10^{-3} & \text{arm load 2, } d_{12} = 0.235 \\ 3.99 \times 10^{-3} & \text{arm load 3, } d_{12} = 0.260 \end{cases}$$

To get the measured stiffness, substitute the calculated inertia and the measured frequency, which was found in Ans.2.13, into Equation 2.21

$$K_s = 0.00328(19.9)^2 = 1.3 \text{ N m/rad.} \quad (\text{Ans.2.14})$$

This result was found for arm load position 1. Results will vary depending on the load position.

$$K_s = J_L \omega_n^2 = 1.3 \text{ N } \frac{\text{m}}{\text{rad}}$$

$$\omega_n = 2\pi f$$

(3.846, 14.5)
↑
Frequency

$$\omega_n = 24.1651$$

$$J_L = 0.004$$

$$K_s = 2.3$$

Resultals
TFP
Natural frequency

data: data