Politecnico di Milano

AUTOMATION AND CONTROL ENGINEERING

Study and Control of the Mechanichal System: Rotary Flexible Joint

Course

Automation and Control Laboratory

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Contents

Problem Description

This report will describe the model of the system, our solution and some attempts to describe and control the system.

The system is composed of a DC motor that provides torque to a metal base, over which a metal arm is fixed with an hinge and two symmetrical springs.

The length of the arm, hence its inertia, and the the equilibiurm length of the springs can be modified in a variety of different configurations.

The system has several interfaces that could be connected to an acquisition system (DAC/ADC + Amplifier) to acquire measurements and provide input signal, namely:

- Actuators:

  Power Supply input of the motor's module (changing the voltage);

- Sensors:

  Incremental Encoder for the position of the base with respect to the global reference frame;

  Incremental Encoder for the relative position of the arm with respect to the base.

The acquisition system composed by ADC/DAC + Amplifier is already configured, it doesn't require our attention, for this reason it will not be discussed in this report.

The main task of this project is to provide a basic control strategy for such system and to develope a more advanced control strategy to accomodate all possible configurations of the system in question.

This goal is divided in sub-tasks to be achieved:

1. position control of the base, with a frequency based approach;
2. position control of the arm tip, with a frequency based or a state space approach;
3. manage uncertainties and control the position of the arm tip with the system in different configurations, with a state space approach or other advanced control techniques.

Model Identification

The system could be schematized as:

It is possible to recognize models of a DC motor powered by a voltage V_{dc} coupled with a gearbox (both modeled in the same box) that provides torque u to the flexible joint. Finally two encoders read the angular positions y and send the estimated values $\hat{y}$ to the ADC converter.

2.1 Mathematical Model

2.1.1 DC motor equations

Before starting to model the DC motor we can get the time constant of its dynamics from the values inthe datasheet:

$$\frac{R}{L} = \frac{2.6\,\Omega}{0.18\,mH} \approx 15\,KHz$$

As this is clearly above the frequency range of the mechanical system, we can neglect its dynamics and model only the static contribution.

The physical equations of the DC Motor then become:

$$\begin{cases} V_a = R_a I_a + E \\ E = k_m \dot{\theta} \\ \tau = k_t I_a \end{cases}$$

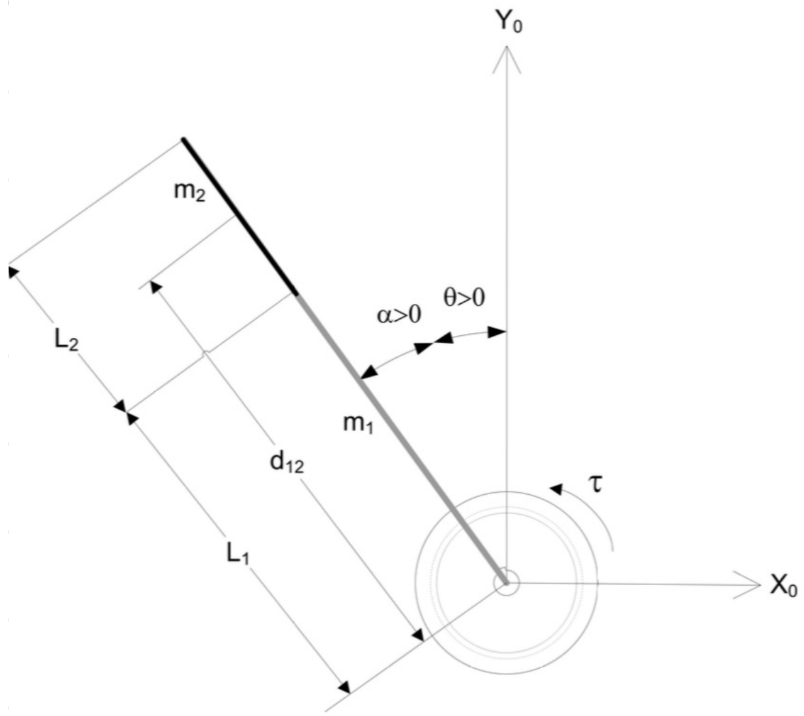
After several mathematical steps and considering the gearbox ratio K_g and the conversion efficiencies we get:

$$\tau = \frac{\eta_m \eta_g k_t K_g (V - K_g k_m \dot{\theta})}{R_m}$$

Where τ is the torque applied to the system and θ is the angular position of the base.

2.1.2 Flexible joint equations

We can represent our system as follows:



Here we have a 2-dofs mechanical system that can be modelled using a Lagrangian approach.

The 2 deegrees of freedom are:

- θ: the absolute angular position of the base;
- α: the relative angular position of the arm with respect to to the base.

The kinetic energy:

$$T = \frac{1}{2}J_{eq}\dot{\theta}^2 + \frac{1}{2}J_L(\dot{\theta} + \dot{\alpha})^2$$

where J_{eq} refers to the equivalent inertia of the motor + gearbox, and J_L refers to the inertia of the arm.

The Potential Energy:

$$V = \frac{1}{2}K_s\dot{\alpha}^2$$

where K_s refers to the linearized stiffness of the equivalent torsional spring.

The Dissipative Function:

$$D = \frac{1}{2}B_{eq}\dot{\theta}^2 + \frac{1}{2}B_L\dot{\alpha}^2$$

where B_{eq} and B_L refer respectively to the equivalent friction of the motor + gearbox and the equivalent friction of the arm.

The dynamics of the system can be found applying the Euler-Lagrange equations for each deegree of freedom :

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \left(\frac{\partial T}{\partial x}\right) + \left(\frac{\partial D}{\partial \dot{x}}\right) + \left(\frac{\partial V}{\partial x}\right) = \left(\frac{\delta W}{\delta x}\right)$$

finally we get the following system of equation:

$$\begin{cases} J_{eq}\ddot{\theta} + J_L(\ddot{\theta} + \ddot{\alpha}) + B_{eq}\dot{\theta} = \tau \\ J_L(\ddot{\theta} + \ddot{\alpha}) + B_L\dot{\alpha} + K_s\alpha = 0 \end{cases}$$

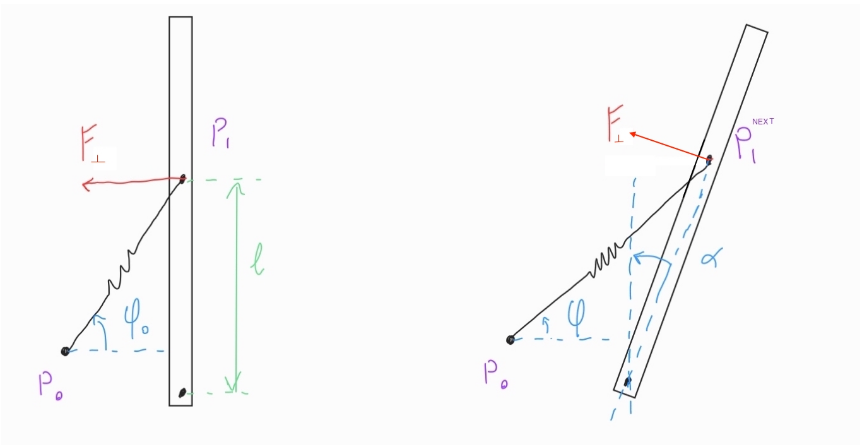
Non-Linear Model for The Spring

The reason of considering the couple of spring as an equivalent torsional one is to reduce the complexity of the system. The validity of this assumption comes from a study that we did on the error that the linear approximation provides with respect to to the real model.

Considering the General equation of a spring

$$F = K_s(x_k - x_0)$$

in the following situations (we consider in this treatment only one spring, but the discussion is valid due to the symmetry for the entire couple)



On the left the equilibrium position $x_k = x_0$, where:

$$\varphi_0 = atan\left(\frac{P_{1_y}}{P_{0_x}}\right)$$

$$F_\perp = F \cdot cos(\varphi_0)$$

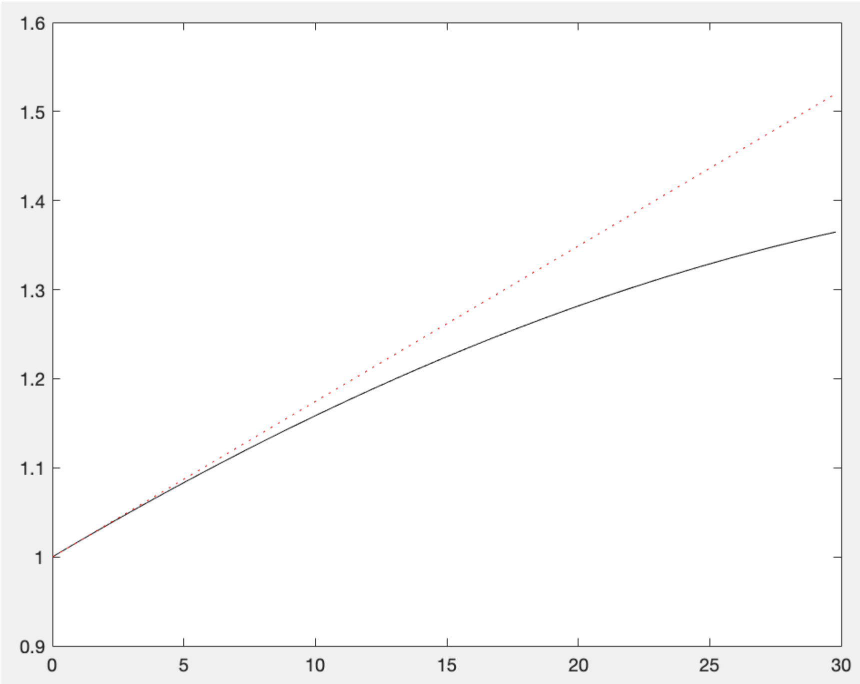
$$x_0 = \sqrt{(P_{1_x} - P_{0_x})^2 + (P_{1_y} - P_{0_y})^2}$$

On the right you are perturbed position, so:

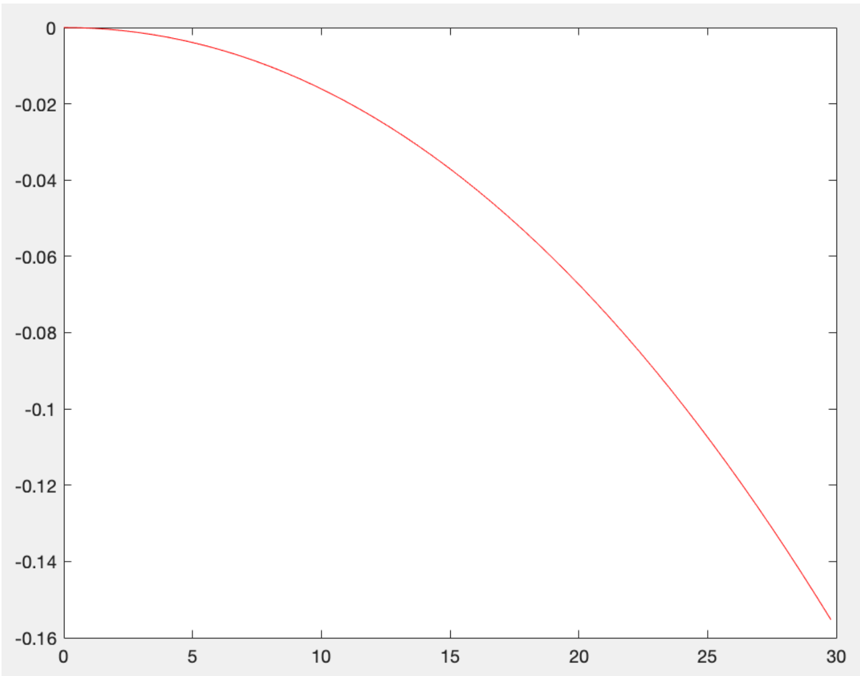
$$x_k = \sqrt{(P_{1_x} - P_{0_x})^2 + (P_{1_y} - P_{0_y})^2} \rightarrow \Delta x_k = x_k - x_0$$

$$P_1^{NEXT} = \begin{bmatrix} l \cdot sin(\alpha) \\ l \cdot (1 - cos(\alpha)) \end{bmatrix} \rightarrow F_{\perp} = F \cdot cos(\varphi + \alpha)$$

Plotting the value of the $F_{\perp}$ in function of the angle α compared with a linear increase we obtain:



the error between the two curves:



it is possible to see from the graph that the error starts to be non-negligible above 25 degrees, but from measurements the angle remains under 10 degrees. For this reason we can consider the Force with a linear behavior and the K_s constant.

2.1.3 Development of The State Space Model

Continuous Time

Starting from the equations:

$$\begin{cases} J_{eq}\ddot{\theta} + J_L(\ddot{\theta} + \ddot{\alpha}) + B_{eq}\dot{\theta} = \frac{\eta_m \eta_g k_t K_g (V - K_g k_m \dot{\theta})}{R_m} \\ J_L(\ddot{\theta} + \ddot{\alpha}) + B_L\dot{\alpha} + K_S\alpha = 0 \end{cases}$$

we develop the State Space system in continuous time, where the state are:

$$\begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \\ \dot{\alpha} \end{bmatrix}$$

the matrix A:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\text{eq}} R_m}{J_{\text{eq}} R_m} & \frac{K_s}{J_{\text{eq}}} & \frac{B_L}{J_{\text{eq}}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\text{eq}} R_m}{J_{\text{eq}} R_m} & -K_S\left(\frac{J_{\text{eq}} + J_L}{J_{\text{eq}} J_L}\right) & -B_L\left(\frac{J_{\text{eq}} + J_L}{J_{\text{eq}} J_L}\right) \end{bmatrix}$$

and the B matrix:

$$\begin{bmatrix} 0 \\ \frac{\eta_m \eta_g k_t K_g}{R_m J_{\text{eq}}} \\ 0 \\ -\frac{\eta_m \eta_g k_t K_g}{R_m J_{\text{eq}}} \end{bmatrix}$$

Considering as the outputs of the system the angular positions θ and α.

Discrete Time

One last step is to compute the model in discrete time, this is necessary for the last and definitive approach we used in the identification procedure.

Considering a sampling time Δ the A matrix:

$$\begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 - \Delta\frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\text{eq}} R_m}{J_{\text{eq}} R_m} & \Delta\frac{K_{\text{stiff}}}{J_{\text{eq}}} & \Delta\frac{B_L}{J_{\text{eq}}} \\ 0 & 0 & 1 & \Delta \\ 0 & \Delta\frac{\eta_m \eta_g k_t k_m K_g^2 + B_{\text{eq}} R_m}{J_{\text{eq}} R_m} & -\Delta K_S\left(\frac{J_{\text{eq}} + J_L}{J_{\text{eq}} J_L}\right) & 1 - \Delta B_L\left(\frac{J_{\text{eq}} + J_L}{J_{\text{eq}} J_L}\right) \end{bmatrix}$$

And the B matrix:

$$\begin{bmatrix} 0 \\ \Delta\frac{\eta_m \eta_g k_t K_g}{R_m J_{\text{eq}}} \\ 0 \\ -\Delta\frac{\eta_m \eta_g k_t K_g}{R_m J_{\text{eq}}} \end{bmatrix}$$

the outputs remain the same.

2.2 Identification Technique

We proceeded in 3 different way, increasing the complexity, trying to fit as possible all the dynamics of the system. The first two methods didn't provide us enough good results, but are reported because guide us in the choice of a good method for the identification and the model produced is reliable.

2.2.1 Deprecated Methods

Stiffness Identification

The first method sticks too much on the reliability of the parameters from the data sheet: we tried to identify just the value of the stiffness of the spring using a step signal and analyzing the frequency of the peak of resonance:

$$K_s = J_L \cdot \omega_n^2$$

As result our model didn't fit a lot the real system and the results was so bad that encourage us to proceed in a complete different direction.

Identification Toolbox

Due to high number of possible uncertainties we look for a different approach that could work around the small number of possible types of experiments and the direct inaccessibility of some parameters. An interesting example of this last consideration is the impossible measurement of the current inside the armature to get a measurement of the resistance R_m.

For these reasons we choose to look for an optimization method that can provide the values of the state space matrices. The first attempt consisted in the usage of the model identification toolbox that, given the order of the system, provides a transfer function representation of the system.

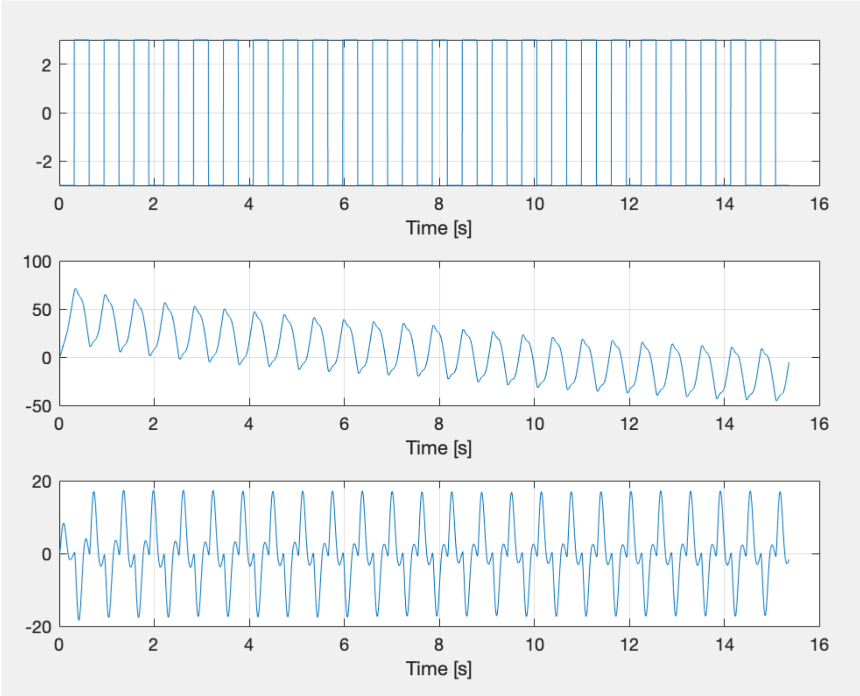
I will not go in deep with this method became as first step in that direction we didn't put too much effort. In fact, we let Matlab works on its own to get the model however the results weren't good enough and in this way we lost the physical meaning of the provided quantities.

2.2.2 Model Identification using CVX

This is the definitive method that we used. CVX is a package allows, giving constraints and objectives, to implement a convex optimization in Matlab in the form:

$$\begin{aligned} \text{minimize} \ & \|Ax - b\|_2 \\ \text{subject to} \ & Cx = d \\ & \|x\|_\infty \leq e \end{aligned}$$

As dataset we collect the values of the 2 outputs with the system subjected to a square wave of period of $T = 0,63s$



For the optimization we started from the nominal parameters, considering the idea that the real values should be not too far.

The nominal parameters:

- For the motor:

  Motor armature resistance: $R_m = 2.6\Omega$

  Motor current-torque constant: $K_t = 0.00768 Nm/A$

  Motor back-emf constant: $K_m = 0.00768 V/(rad/s)$

  High-gear total gear ratio: $K_g = 70$

  Motor efficiency: $\eta_m = 0.69$

  Gearbox efficiency: $\eta_g = 0.9$

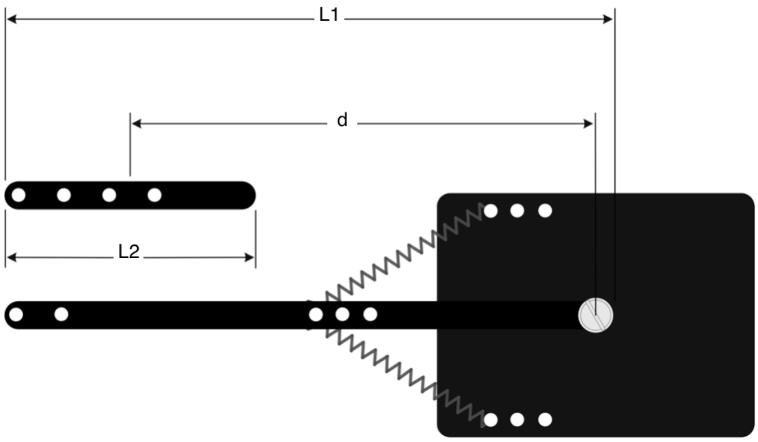
- The equivalent mechanical system of the motor + gearbox:

  Equivalent moment of inertia: $J_{eq} = 0.002087 Kgm^2$

  Equivalent viscous damping coefficient: $B_{eq} = 0.015 Nm/(rad/s)$

For the parameters of the rotating arm we compute the values of the inertia, following its geometry, as:

$$J_L = m_1 \cdot \frac{L_1^2}{3} + m_2 \cdot \frac{L_2}{12} + m_2 \cdot d^2 = 0.0032 Kgm^2$$

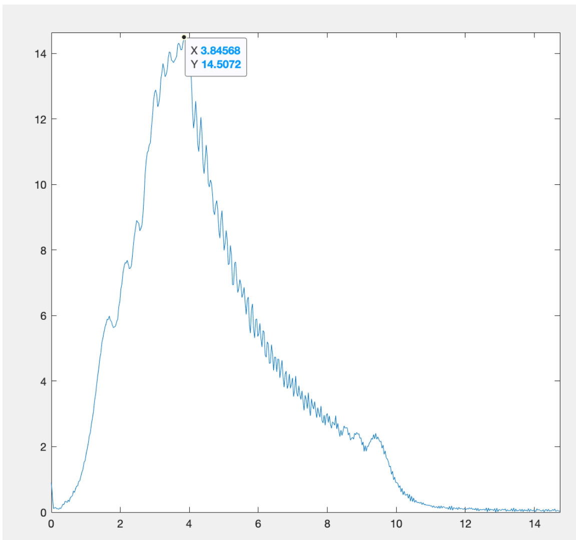


the value of the friction coefficient was supposed initially null:

$$B_L = 0$$

and the value of the K_s we use the value generated in the analysis stiffness identification:

the analysis provide a Fourier transfer of the second output (the relative position of the tip) as in the figure:



the peak is at:

$$f = 3.84568Hz$$

as result we assign the stiffness initial value as:

$$K_s = J_L \cdot \omega_n^2 = 1.8426N/m$$

Position Control of the Base

Position Control of The Tip

Position Control of the Tip with Uncertanties

Conclusioni