POLITECNICO
MILANO 1863

DC Motor Drive

Dynamical Models and Cascade Control Methods

Dynamics of Electrical Machines and Drives
(10CFU)

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Electrical Machines, Drives and Power Electronics Research Group

Suggested Book
Exercises and Embedded Implementation

- The exercises presented in this course are extensively detailed in the book:

[Introduction to Microcontroller Programming for Power Electronics Control Applications: Coding with MATLAB® and Simulink® (1st ed.)]

CRC Press, 2021
https://doi.org/10.1201/9781003196938

M. Rossi, N. Toscani, M. Mauri, and F. Castelli-Dezza

This book also cover the related *embedded implementation aspects on MCUs* which are not covered in this course.

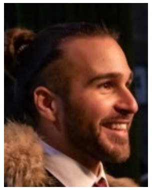
Available in the campus library at:
https://ebookcentral.proquest.com/lib/polimi/detail.action?docID=6710080

Practical Lectures Overview
Exercises

- ❑ In the next two lectures the first candidate as **Report Exercise** will be presented

- ❑ **Reminder:**
  the exercises relates to using computer simulations tools (MATLAB/Simulink) to:
  - analyze the behavior of a dynamic system
  - analyze the effect of the control loop on the performance of an electrical drive
  - the report has to show the definition of the parameters of the control system and the results/figures of the Simulink simulation
  - exercises can be done as groups… BUT each student MUST present its own report

Answering questions:



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What is an Electrical Drive

Industrial Terminology

- The standard from *"Comitato Elettrotecnico Italiano"* :

CEI 301-1

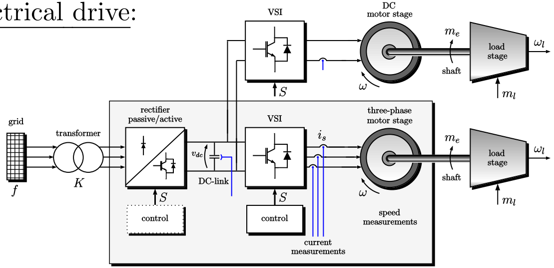
An electrical drive system is a system which converts electric energy into mechanical energy, by making use of power electronic apparatuses, following an input function (and according to established programme)

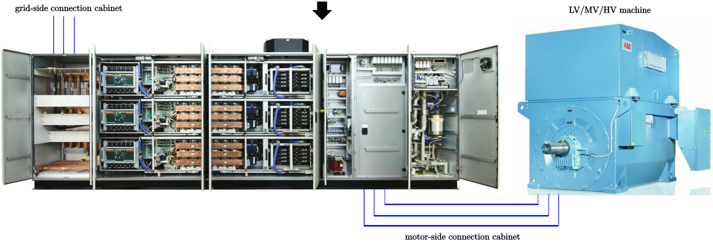
- However the industrial terminology refers to an electrical drive as the combination of **power converter stage** and **motor stage**
- Hence, it refers intrinsically to a motor oriented applications (rotating element based)
- We are mainly dealing with:
    - Traction (train, subway, trolley bus)
    - Industrial process (compressor, distillation tower, fluid handling)
    - Motorsport (formula-E, KERS)

Industrial Aspect

Example of MV-Drive

This is an electrical drive:

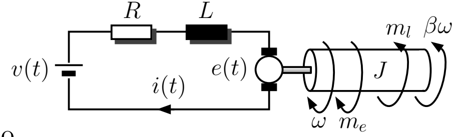




PM-DC Motor #1

Dynamical Model

Consider a permanent magnet DC motor:



1. **Electrical part:**

   $$L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + Ri(t) + e(t) = v(t)$$

   The back-emf $e(t)$ is proportional to the motor speed: $e(t) = K\omega(t)$

2. **Mechanical part:**

   $$J\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} + \beta\omega(t) = m_e(t) - m_l(t)$$

   The torque $m_e(t)$ is proportional to the armature current: $m_e(t) = Ki(t)$

PM-DC motor model

$$L\frac{\mathrm{d}i(t)}{\mathrm{d}t} = v(t) - Ri(t) - K\omega(t)$$

$$J\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = Ki(t) - m_l(t) - \beta\omega(t)$$

- On the other hand, they can be built different **state space model** according to which output would be analyzed (what we need to control)

PM-DC Motor #2

Choice of state variables

MIMO system: output $y = x$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \omega \\ i \end{bmatrix}$$

$$\boldsymbol{y} = [\ \omega\ \ i\ ]^T$$

$$\boldsymbol{u} = [\ m_l\ \ v\ ]^T$$

$$\dot{x}_1(t) = \frac{K}{J} x_2(t) - \frac{\beta}{J} x_1(t)$$

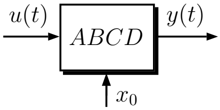
$$\dot{x}_2(t) = \frac{1}{L} u(t) - \frac{R}{L} x_2(t) - \frac{K}{L} x_1(t)$$

DC motor state space model

$$\dot{\boldsymbol{x}}(t) = \underbrace{\begin{bmatrix} -\frac{\beta}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix}}_{\boldsymbol{A}} \boldsymbol{x}(t) + \underbrace{\begin{bmatrix} -\frac{1}{J} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}}_{\boldsymbol{B}} \boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{C}} \boldsymbol{x}(t)$$

Represented by:



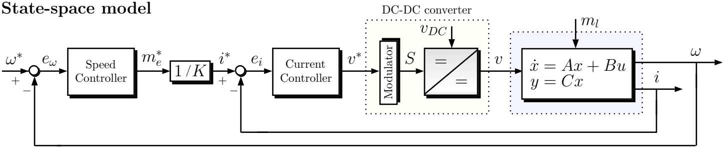
goto **Simulink**

Control Strategy

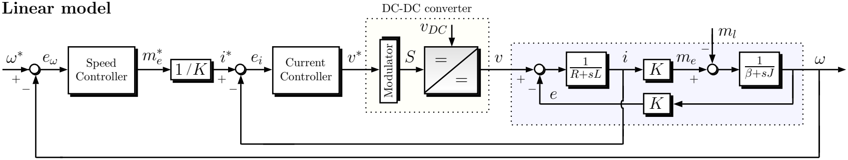
Transfer Functions

- Starting from the previous models, the block schemes are the following:

State-space model



Linear model



- **Remark**: state space models graphically hide the coupling effects (they consider them inside the matrices)

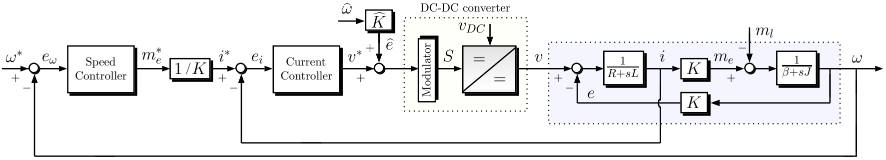
Decoupling Method

Back-emf compensation

- From the previous figure note that the electrical dynamic and the mechanical dynamic are **coupled** through the back-emf
- Assuming to have a good speed estimator or a speed sensor we can add a feedforward action providing a back-emf compensation $\widehat{e}(t)$

Feedforward term

$$\widehat{e}(t) = \widehat{K}\omega(t)$$



- **Remark**: back-emf compensation will take effect only at steady-state

Cascade Control Approach

Decoupling Method

- Now the two dynamics are effectively decoupled
- We can design the two PI controller $(k_p + k_i/s)$ independently:

Speed Control $R_\omega(s)$

$$G_\omega(s) = \frac{1}{\beta + sJ} \rightarrow L_\omega(s)$$

Current Control $R_i(s)$

$$G_i(s) = \frac{1}{R + sL} \rightarrow L_i(s)$$

- Remember the **cascade loop theory**: (brief remark)

Definition

the outer loop must be at least one decade slower then the inner loop in order to see a unitary closed loop transfer function for the inner one

- This imply at least $\omega_\omega \leqslant \frac{\omega_i}{10}$ to see $F_i = 1$ from the speed control loop

Exercise

Speed Control of a PM-DC Motor

Simulink

A DC motor equipped with a speed sensor has to follow a step-command speed reference from 0 to 300 rad/s.

- Assume a load torque as a step-command from 0 to $3Nm$ at time instant $0.05s$
- Select a proper bandwidth for the two control loop able to present
  - (phase margin $\varphi_m \geqslant 60°$) and cascade constraints

system parameters					
R	$0.6\ \Omega$	K	$0.04\ (Nm/A)(Vs/rad)$	β	$0.01\ Nms/rad$
L	$0.002\ H$	J	$6 \cdot 10^{-5}\ kgm^2$	f_{sw}	$10\ kHz$

- Write a MATLAB script to compute k_p and k_i given the motor parameter and/or use an automatic tool

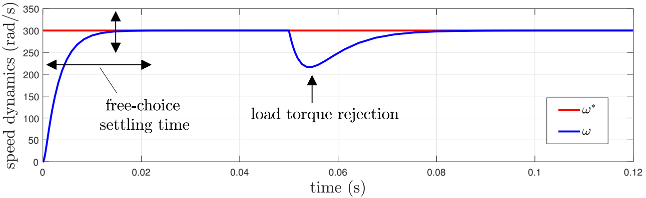
Exercise

Speed Control of a PM-DC Motor (2/2)

Which Loop has to be Designed First?

Note that we do not have any requirements on the settling time of the dynamical response (both for speed and current).
This means that we can start the design from inner loop as well as the outer one without drawbacks.

- ....in presence of a constraints on the settling time of the closed-loop speed response, from which loop should you start?



SeparatelyExcited DC Motor

Dynamical Model

Consider a separately excited DC motor: (*armature circuit* + *excitation circuit*)

1. **Electrical part:**

   $$L_a \frac{\mathrm{d}i_a(t)}{\mathrm{dt}} + R_a i_a(t) + e(t) = v_a(t)$$

   The back-emf is exploit as function of $i_e(t)$ as $e(t) = K_s i_e(t) \omega(t)$

   $$L_e \frac{\mathrm{d}i_e(t)}{\mathrm{dt}} + R i_e(t) = v_e(t)$$

   The excitation current controls $e(t)$ by changing the excitation flux $\psi_e(t)$

2. **Mechanical part:**

   $$J \frac{\mathrm{d}\omega(t)}{\mathrm{dt}} + \beta\omega(t) = m_e(t) - m_l(t)$$

   The torque is exploit as function of $i_e(t)$ as $m_e(t) = K_s i_e(t) i_a(t)$

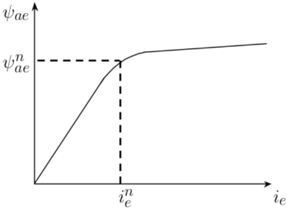
SE-DC motor model

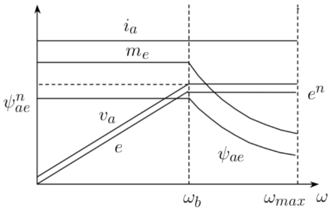
$$L_a \frac{\mathrm{d}i_a(t)}{\mathrm{dt}} = v_a(t) - R_a i_a(t) - K_s i_e(t)\omega(t)$$
$$L_e \frac{\mathrm{d}i_e(t)}{\mathrm{dt}} = v_e(t) - R_e i_e(t)$$
$$J \frac{\mathrm{d}\omega(t)}{\mathrm{dt}} = K_s i_e(t) i_a(t) - m_l(t) - \beta\omega(t)$$

SeparatelyExcited DC Motor

Excitation Circuit

- To properly use the ferromagnetic material, the motor should operate at the knee of the *magnetizing curve* → **rated linked flux** ψ_{ae}^n





- Above the base speed $\omega(t) > \omega_b$, the voltage $v_a(t)$ has reached the rated value (converter saturation) so $e(t)$ has to remain constant

$$e(t) = \text{cost} = \underbrace{K_s\, i_e(t)}_{\downarrow}\,\underset{\uparrow}{\omega(t)} = e^n$$

We need to control $i_e(t)$ in order to compensate the effect of increasing $\omega(t)$ and mantain constant $e(t)$

Control of Excitation Circuit

Excitation Current vs Operating region

- Let's exploit the operating region behaviour:
  - if $\omega < \omega_b$

$$i_e = i_e^n = \text{cost} \quad \rightarrow \quad e = \underbrace{K_s i_e^n}_{K} \omega$$

  similar to the coefficient K in the PM case

  .

  - if $\omega > \omega_b$

$$e = e^n = \text{cost} \quad \rightarrow \quad e^n = K_s i_e \omega$$

  then we can isolate the excitation current as

$$i_e = \frac{e^n}{K_s \omega}$$

- According to that if we properly control the current i_e we are able to run the motor over the ω_b → **field weakening**

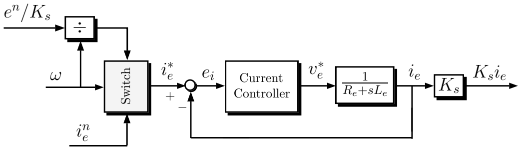
Control of Excitation Circuit

Field Weakening Control

- Explicit the current i_e as state variable we can summarize:

$$\begin{cases} i_e = i_e^n & \omega < \omega_b \\ i_e = \frac{e^n}{K_s \omega} & \omega > \omega_b \end{cases}$$

- The control loop is



- Note that the time constant $\tau_e = L_e/R_e$ is larger than $\tau_a = L_a/R_a$
- This imply to wait until the excitation circuit has reach the steady-state condition before to apply a speed reference command to the motor $\rightarrow$ **starting condition**

Exercise

Speed Control of a SF-DC Motor for Traction

A DC self excited motor is used to move an ATM tramway vehicle " *Carelli 1928* " with the following characteristics:

- Line voltage : $600\,V$
- Motor rated speed : $314 rad/s$
- Efficiency: 0.9 (neglecting excitation losses and iron losses)
- Armature circuit time constant :$10 ms$
- Excitation circuit rated voltage : $120\,V$
- Excitation circuit rated current : $1\,A$
- Excitation circuit time constant : $1s$

The tramway should accelerate from 0 to $60km/h$ in $25s$. The tramway mass is $10\,T$ and you should consider 200 people as the tramway trainload with a standard weight of $80kg$. The friction force is proportional to the speed and at rated speed ($60km/h$ or $314rad/s$) is 1/3 of traction force.

Exercise

Speed Control of a SF-DC Motor for Traction

ATM project (Milan)

- Find the design parameters of the DC motor according to the data
- Design and simulate speed and current control in order to cover a $10km$ track considering the Table I characteristics. The slope is $s\% = 100\tan(\theta)$

track	slope %	speed
$0 - 1\,km$	0	$35\ km/h$
$1 - 3\,km$	0	$60\ km/h$
$3 - 4\,km$	5%	$60\ km/h$
$4 - 6\,km$	0	$75\ km/h$
$6 - 8\,km$	0	$60\ km/h$
$8 - 9\,km$	-5%	$60\ km/h$
$9 - 10\ km$	0	$35\ km/h$