Completeness of Inst-Gen-Eq

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Harald Ganzinger and Konstantin Korovin.
Integrating Equational Reasoning into Instantiation-Based Theorem Proving.
In *18th CSL 2004. Proceedings*, volume 3210 of *LNCS*, pages 71–84, 2004.

$\succ_{gr}$	total simplification ordering on grounund terms
$\succ_{\ell}$	()
$\succ_{cl}$	()

Unit Paramodulation

Definition

$$\frac{(\ell \approx r) \cdot \sigma \quad L[\ell'] \cdot \sigma'}{L[r]\theta \cdot \rho}\ \theta \qquad\qquad \frac{(s \not\approx t) \cdot \tau}{\square}\ \mu$$

where

- $\ell\sigma \succ_{gr} r\sigma$, $\theta = \mathrm{mgu}(\ell, s)$, $\ell\sigma = \ell'\sigma' = \ell'\theta\rho$, $\ell' \notin \mathcal{V}$
- $s\tau = t\tau$, $\mu = \mathrm{mgu}(s, t)$

saturation process

$\mathcal{R}_{Inst}(S)$
selection function,
S-relevant

Lemma

Let $\mathcal{I}$ be an arbitrary total consistant extension of $\mathcal{I}_S$. Then for every $C \in S$ and $\sigma : \mathcal{V} \to \mathcal{T}(\mathcal{F}_\mathsf{f}, \emptyset)$.

$$\mathcal{I} \models C \cdot \sigma$$

Proof.

Assume minimal w.r.t. $\succ_{cl}$ ground instance $D' \cdot \sigma'$ of S with $\mathcal{I} \not\models D' \cdot \sigma'$. $\square$

Lemma

Let $M \cdot \tau = \min_{\succ_\ell} \{M' \cdot \tau' \mid M' \cdot \tau' \in \mathit{irred}_{R_S}(\mathcal{L}_S^{sat}), \mathcal{I} \not\models M'\tau'\}$
Then, $M \cdot \tau$ *is irreducible by* R_S*.*

Proof.

Assume $M \cdot \tau$ is reducible by $(\ell \to r) \in R_S$ and $(\ell \to r)$ is produced by $(\ell' \approx r') \cdot \rho \in \mathcal{L}_S^{sat}$. Bei construction τ is irreducible by R_S. Hence UP-inference is applicable:

$$\frac{(\ell' \approx r) \cdot \rho \quad M[\ell''] \cdot \tau}{M[r']\theta \cdot \mu}\ UP$$

$$\ell'\rho = \ell''\tau = \ell''\theta\mu,\ \theta = \mathrm{mgu}(\ell', \ell''),\ \mathcal{I} \not\models M[r']\theta\mu$$

- Assume $M[r']\theta \cdot \mu$ is UP-redundant in $\mathcal{L}_S^{sat}$.
  α is irreducible (lemma ..) by R_S. From definiton:
- Assume $M[r']\theta \cdot \mu$ is not UP-redundant in $\mathcal{L}_S^{sat}$.

$\square$