

FLEA

first order proving with equality
master project

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Project overview

Goals and requirements

- Goals
 - ATP for FOL with equality
 - Master thesis
- Requirements
 - Input: problems in clausal normal form
 - Data: clauses, literals, terms, indices, etc.
 - Algorithms: substitution, unification, etc.
 - **Proof search** (strategies)
- Non-Goals and Non-Requirements
 - CASC
 - $\text{FOF} \approx \text{CNF}$

Clausal normal form

PUZ001-1

...

`cnf(same_hates, hypothesis, (\neg hates(agatha,X) | hates(butler,X))).`

...

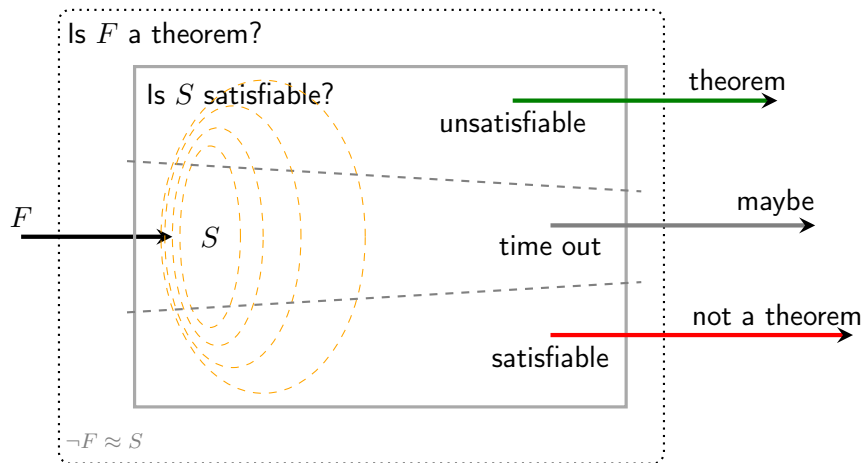
$$\{ \dots, \neg \text{hates}(\text{agatha}, x) \vee \text{hates}(\text{butler}, x), \dots \}$$

$$\equiv$$

$$\dots \wedge \forall x (\neg \text{hates}(\text{agatha}, x) \vee \text{hates}(\text{butler}, x)) \wedge \dots$$


G. Sutcliffe, *The TPTP Problem Library and Associated Infrastructure: The FOF and CNF Parts, v3.5.0*, Journal of Automated Reasoning **43** (2009), no. 4, 337–362.

Refutation



Ordered Resolution

$$\frac{L \vee C \quad \neg L' \vee D}{(C \vee D)\sigma}$$

where

$L\sigma$ strictly maximal in $C\sigma$, $\neg L'\sigma$ maximal in $D\sigma$, $\sigma = \text{mgu}(L, L')$.

$$S = \{P(x) \vee \neg P(y), \neg P(a), P(b)\}$$

$$\frac{\frac{P(x) \vee \neg P(y) \quad \neg P(a)}{\neg P(y)} \quad x \mapsto a \quad P(b)}{\square} \quad y \mapsto b$$

Inst-Gen

$$\frac{L \vee C \quad \neg L' \vee D}{(L \vee C)\sigma \quad (\neg L' \vee D)\sigma}$$

where

$$\text{sel}(L \vee C) = L \quad \text{sel}(\neg L' \vee D) = \neg L' \quad \sigma = \text{mgu}(L, L')$$

Selection

$$S = \{P(x) \vee \neg P(y), \neg P(a), P(b)\}$$

unsatisfiable

$$P = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b$$

satisfiable

$$S_0 = \{P(x) \vee \neg P(y), \neg P(a), P(b)\}$$

unsatisfiable

$$P_0 = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b$$

satisfiable

$$\frac{P(x) \vee \neg P(y) \quad \neg P(a)}{P(a) \vee \neg P(y)} \quad x \mapsto a$$

$$P_1 = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b \wedge (p_a \vee \neg p_*)$$

satisfiable

$$\frac{P(b) \quad P(a) \vee \neg P(y)}{P(a) \vee \neg P(b)} \quad y \mapsto b$$

$$P_2 = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b \wedge (p_a \vee \neg p_*) \wedge (p_a \vee \neg p_b)$$

unsatisfiable

Equality

$$S = \{^1: P(a), ^2: \neg P(f(x, b)), ^3: f(x, b) = x\}$$

saturated

$$P = p_1 \wedge \neg p_2 \wedge e_3$$

satisfiable

Equality Axioms

$$\frac{x = x \quad s \neq s \vee \mathcal{C}}{s = s} \quad x \mapsto s$$

reflexivity

$$\frac{x \neq y \vee y = x \quad s \neq t \vee \mathcal{C}}{t \neq s \vee s = t} \quad x \mapsto t, y \mapsto s$$

symmetry

$$\frac{x \neq y \vee y \neq z \vee x = z \quad s \neq t \vee \mathcal{C}}{s \neq y \vee y \neq t \vee s = t} \quad x \mapsto s, y \mapsto t$$

transitivity

$$\frac{x \neq y \vee f(x) = f(y) \quad f(s) \neq f(t)}{s \neq t \vee f(s) = f(t)} \quad x \mapsto s, y \mapsto t$$

f-congruence

$$\frac{x \neq y \vee L^c(x) \vee L(y) \quad L(s) \vee \mathcal{C}}{s \neq y \vee L^c(s) \vee L(y)} \quad x \mapsto s$$

P-congruence

$$S_0 = \{^1: P(a), ^2: \neg P(f(x, b)), ^3: f(x, b) = x\}$$

unsatisfiable

$$P_0 = p_1 \wedge \neg p_2 \wedge e_3$$

satisfiable

$$^4: x \neq f(x, b) \vee \neg P(x) \vee P(f(x, b))$$

 $^{2:1} P_{\neg}$ -congruence

$$P_1 = P_0 \wedge (\neg e_4 \vee \neg p_* \vee p_2)$$

$$^5: f(x, b) \neq x \vee x = f(x, b)$$

 $^{4:1}$ symmetry

$$P_2 = P_0 \wedge (\neg e_4 \vee \neg p_* \vee p_2) \wedge (\neg e_3 \vee e_4)$$

$$^6: a \neq f(a, b) \vee \neg P(a) \vee P(f(a, b))$$

 $^{4:1,1:1} x \mapsto a$

$$^7: \neg P(f(a, b))$$

 $^{2:1,6:1} x \mapsto a$

$$P_3 = P_2 \wedge (\neg e_6 \vee \neg p_1 \vee p_6) \wedge \neg p_6$$

$$^8: f(a, b) \neq a \vee a = f(a, b)$$

 $^{1:1}$ -symmetry

$$^9: f(a, b) = a$$

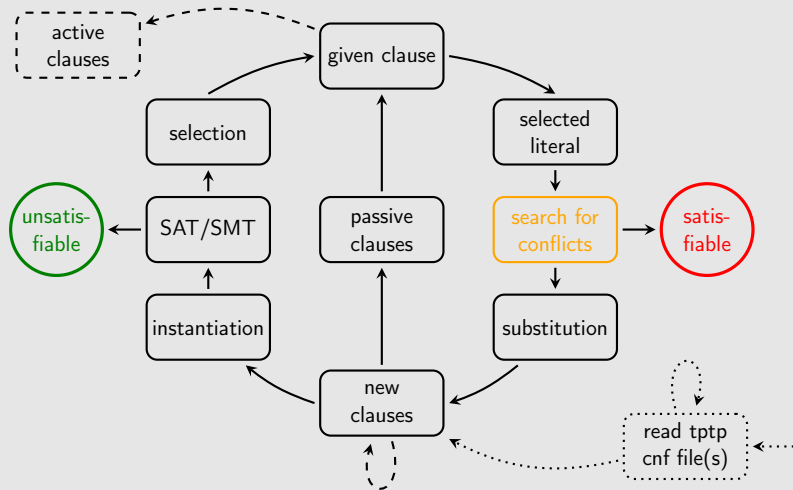
 $^{3:1,8:1} x \mapsto a$

$$P_4 = P_3 \wedge (\neg e_8 \vee e_6) \wedge e_8$$

unsatisfiable

- requirements
 - Linux, macOS
 - Flex, Bison, Clang, Swift 3
 - Yices
- structure
 - wrapper for yices, parsing, syslog, and some POSIX APIs
 - data and algorithms
 - strategy: sequential processing of growing list of clauses
- optimizations
 - indexing of selected literals
 - indexing of clauses
 - sharing of terms
- in progress = missing
 - alternative procedure for equality
 - experiments
 - written thesis

Run-loop



Lemma

Symmetry and transitivity are consequences of reflexivity and congruence.

Symmetry.

$$\begin{array}{c}
 \frac{x_1 = y_1 \wedge x_2 = y_2 \wedge x_1 = x_2 \rightarrow y_1 = y_2}{x = y \wedge x = x \wedge x = x \rightarrow y = x} \text{=-congruence} \\
 \frac{}{x = y \rightarrow y = x} \text{reflexivity}
 \end{array}$$



Transitivity.

$$\begin{array}{c}
 \frac{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}{x \neq x \vee y \neq z \vee x \neq y \vee x = z} \text{=-congruence} \\
 \frac{}{x \neq y \vee y \neq z \vee x = z} \text{reflexivity}
 \end{array}$$

