

$$\begin{array}{c}
\frac{F \quad G}{F \wedge G} \wedge i \quad \frac{F \wedge G}{G} \wedge e_1 \quad \frac{F \wedge G}{F} \wedge e_2 \\
\frac{F}{\neg\neg F} \neg\neg i \quad \frac{\neg\neg F}{F} \neg\neg e \\
\frac{F \quad F \rightarrow G}{\neg G} \text{modus ponens} \quad \frac{F \rightarrow G \quad \neg G}{\neg F} \text{modus tollens} \\
\frac{\boxed{\begin{array}{c} F \\ \vdots \\ G \end{array}}}{F \rightarrow G} \rightarrow i \quad \frac{F \vee G \quad \boxed{\begin{array}{c} F \\ \vdots \\ H \end{array}} \quad \boxed{\begin{array}{c} G \\ \vdots \\ H \end{array}}}{H} \vee e \\
\frac{F}{F \vee G} \vee i_1 \quad \frac{G}{F \vee G} \vee i_2 \\
\frac{\perp}{F} \perp e \quad \frac{F \quad \neg F}{\perp} \neg e \quad \frac{\boxed{\begin{array}{c} F \\ \vdots \\ \perp \end{array}}}{\neg F} \neg i \\
\frac{\boxed{\begin{array}{c} \neg F \\ \vdots \\ \perp \end{array}}}{F} PBC \quad \frac{}{F \vee \neg F} LEM \\
\frac{}{t = t} = i \quad \frac{s = t \quad F\{x \mapsto s\}}{F\{x \mapsto t\}} = e \\
\frac{\forall x F}{F\{x \mapsto t\}} \forall e \quad \frac{\boxed{\begin{array}{c} x_0 \\ \vdots \\ F\{x \mapsto x_0\} \end{array}}}{\forall F} \forall i \\
\frac{F\{x \mapsto x_0\}}{\exists x F} \exists i \quad \frac{\exists x F \quad \boxed{\begin{array}{c} x_0 \quad F\{x \mapsto x_0\} \\ \vdots \\ H \end{array}}}{H} \exists e
\end{array}$$

Table 1: Natural Deduction Rules