bits and pieces
status quo

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June 15th, 2016

1 Previously
Resolution and InstGen
Examples
Subsumption

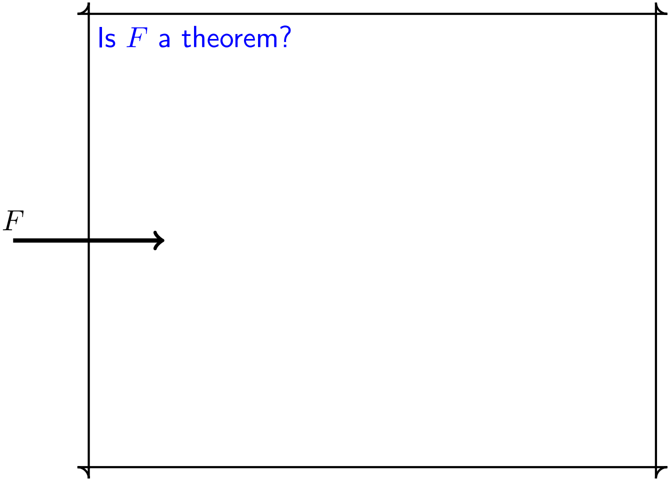
References

Christoph Sticksel, *Efficient equational reasoning for the Inst-Gen framework*, Ph.D. thesis, School of Computer Science, University of Manchester, 2011.

Hofstadter's Law: It always takes longer than you expect,
even when you take into account Hofstadter's Law.

— Douglas Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid

Goal



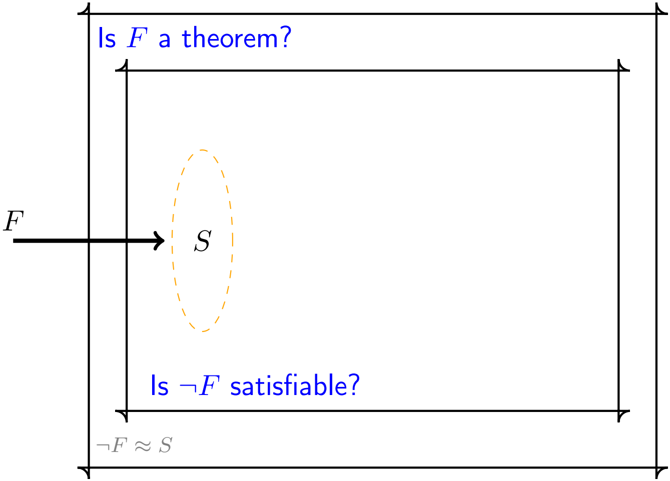
Goal

Is F a theorem?

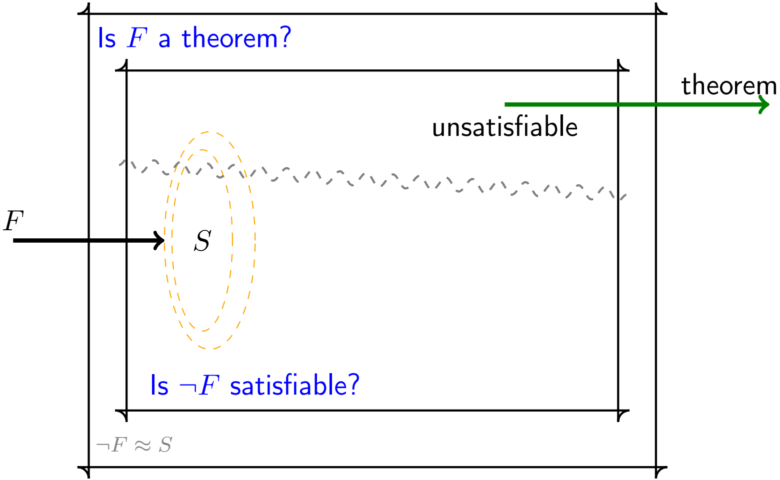
F

$\neg F \approx S$

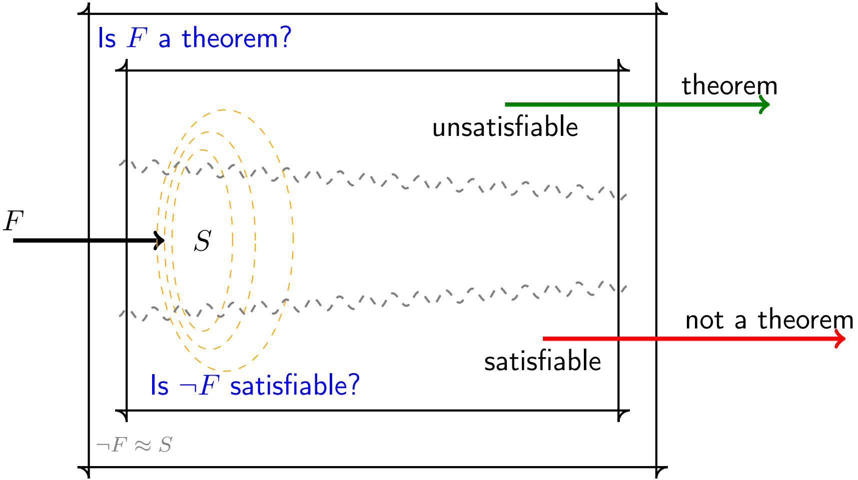
Goal



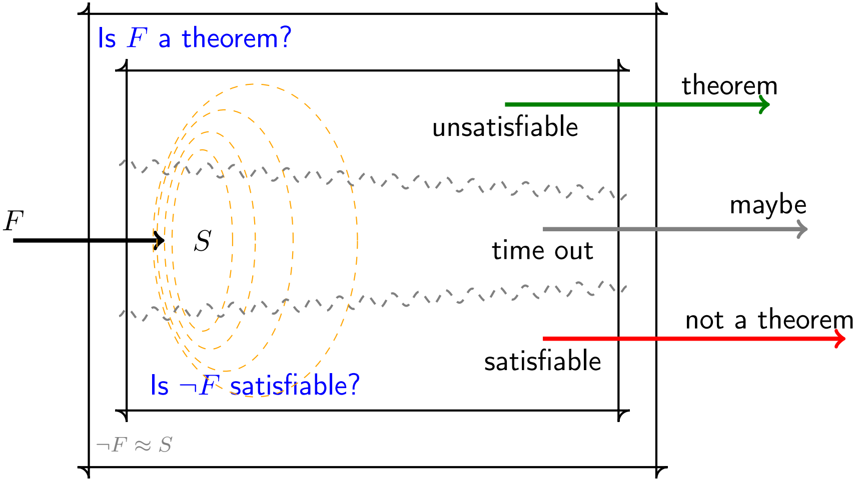
Goal


Is F a theorem?
theorem
unsatisfiable
F
S
Is $\neg F$ satisfiable?
$\neg F \approx S$

Goal


Is F a theorem?
theorem
unsatisfiable
F
S
not a theorem
satisfiable
Is $\neg F$ satisfiable?
$\neg F \approx S$

Goal


Is F a theorem?
theorem
unsatisfiable
F
S
maybe
time out
not a theorem
satisfiable
Is $\neg F$ satisfiable?
$\neg F \approx S$

Definition (Ordered Resolution)

$$\frac{L \vee C \quad \neg L' \vee D}{(C \vee D)\sigma}$$

where

$L\sigma$ strictly maximal in $C\sigma$, $\neg L'\sigma$ maximal in $D\sigma$, $\sigma = \mathrm{mgu}(L, L')$.

Definition (Inst-Gen)

$$\frac{L \vee C \quad \neg L' \vee D}{(L \vee C)\sigma \quad (\neg L' \vee D)\sigma}$$

where

$$\mathrm{sel}(L \vee C) = L \qquad \mathrm{sel}(\neg L' \vee D) = \neg L' \qquad \sigma = \mathrm{mgu}(L, L')$$

Example (Resolution)

$$\cfrac{\cfrac{\mathsf{P}(x) \vee \neg\mathsf{P}(y) \quad \neg\mathsf{P}(\mathsf{a})}{\neg\mathsf{P}(y) \quad \mathsf{P}(\mathsf{b})}\; x \mapsto \mathsf{a}}{\square}\; y \mapsto \mathsf{b}$$

Example (Inst-Gen)

$$S_0\bot = \{\mathsf{P}(\bot) \vee \neg\mathsf{P}(\bot), \neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b})\} \qquad \text{satisfiable}$$

$$\frac{\mathsf{P}(x) \vee \neg\mathsf{P}(y)) \quad \neg\mathsf{P}(\mathsf{a})}{\mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(y)}\; x \mapsto \mathsf{a}$$

$$S_1\bot \supsetneq \{\neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b}), \mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(\bot)\} \qquad \text{satisfiable}$$

$$\frac{\mathsf{P}(\mathsf{b}) \quad \mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(y)}{\mathsf{P}(\mathsf{a}) \vee \mathsf{P}(\mathsf{b})}\; y \mapsto \mathsf{b}$$

$$S_2\bot \supsetneq \{\neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b}), \mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(\mathsf{b})\} \qquad \text{unsatisfiable}$$

Subsumption

$$S = \{C, D, \dots\} \qquad \exists\theta\ C\theta \subseteq D$$ C subsumes D

$$S \text{ satisfiable } \overset{\checkmark}{\Longleftrightarrow} (S \setminus D) \text{ satisfiable}$$

$$\theta \text{ is proper, } S\bot \text{ satisfiable } \overset{\times}{\Longleftrightarrow} (S \setminus D)\bot \text{ satisfiable}$$

$$\theta \text{ is renaming, } S\bot \text{ satisfiable } \overset{\checkmark}{\Longleftrightarrow} (S \setminus D)\bot \text{ satisfiable}$$

Example

$$\{\mathsf{P}(x, y), \neg\mathsf{P}(\mathsf{a}, z)\} \qquad \{\mathsf{P}(x, y), \neg\mathsf{P}(\mathsf{a}, z), \mathsf{P}(\mathsf{a}, z)\}$$

$$\{\mathsf{P}(\bot, \bot), \neg\mathsf{P}(\mathsf{a}, \bot)\} \qquad \{\mathsf{P}(\bot, \bot), \neg\mathsf{P}(\mathsf{a}, \bot), \mathsf{P}(\mathsf{a}, \bot)\}$$