flea

bit(e)s and pieces

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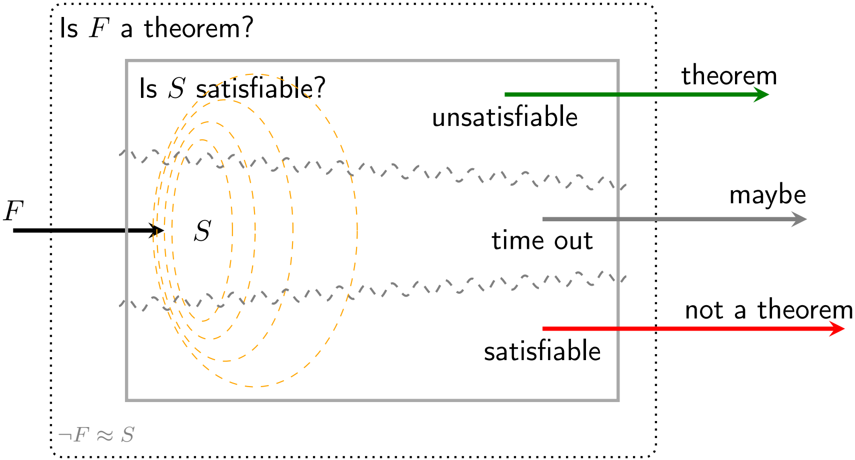
Hofstadter's Law: It always takes longer than you expect, even when you take into account Hofstadter's Law.

— Douglas Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid

References

- Bruno Dutertre, *Yices 2.2*, Computer-Aided Verification (CAV'2014) (Armin Biere and Roderick Bloem, eds.), Lecture Notes in Computer Science, vol. 8559, Springer, July 2014, pp. 737–744.
- G. Sutcliffe, *The TPTP Problem Library and Associated Infrastructure: The FOF and CNF Parts, v3.5.0*, Journal of Automated Reasoning **43** (2009), no. 4, 337–362.

Goal


Is F a theorem?
Is S satisfiable?
unsatisfiable
theorem
F
S
time out
maybe
satisfiable
not a theorem
$\neg F \approx S$

Definition (Ordered Resolution)

$$\frac{L \vee C \quad \neg L' \vee D}{(C \vee D)\sigma}$$

where

$L\sigma$ strictly maximal in $C\sigma$, $\neg L'\sigma$ maximal in $D\sigma$, $\sigma = \mathrm{mgu}(L, L')$.

Definition (Inst-Gen)

$$\frac{L \vee C \quad \neg L' \vee D}{(L \vee C)\sigma \quad (\neg L' \vee D)\sigma}$$

where

$$\mathrm{sel}(L \vee C) = L \qquad \mathrm{sel}(\neg L' \vee D) = \neg L' \qquad \sigma = \mathrm{mgu}(L, L')$$

Example (Resolution)

$$\dfrac{\dfrac{\mathsf{P}(x) \lor \neg\mathsf{P}(y) \quad \neg\mathsf{P}(\mathsf{a})}{\neg\mathsf{P}(y) \quad \mathsf{P}(\mathsf{b})}\ x \mapsto \mathsf{a}}{\square}\ y \mapsto \mathsf{b}$$

Example (Inst-Gen)

$$S_0\bot = \{\mathsf{P}(\bot) \lor \neg\mathsf{P}(\bot), \neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b})\} \qquad \text{satisfiable}$$

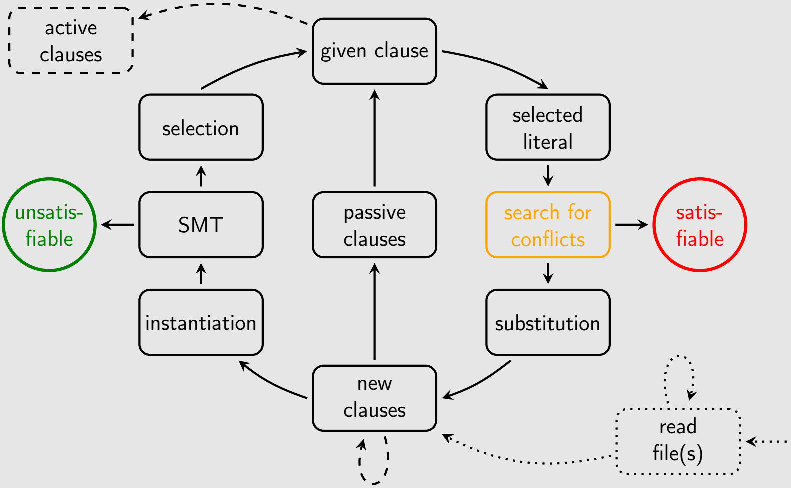
$$\dfrac{\mathsf{P}(x) \lor \neg\mathsf{P}(y)) \quad \neg\mathsf{P}(\mathsf{a})}{\mathsf{P}(\mathsf{a}) \lor \neg\mathsf{P}(y)}\ x \mapsto \mathsf{a}$$

$$S_1\bot \supsetneq \{\neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b}), \mathsf{P}(\mathsf{a}) \lor \neg\mathsf{P}(\bot)\} \qquad \text{satisfiable}$$

$$\dfrac{\mathsf{P}(\mathsf{b}) \quad \mathsf{P}(\mathsf{a}) \lor \neg\mathsf{P}(y)}{\mathsf{P}(\mathsf{a}) \lor \mathsf{P}(\mathsf{b})}\ y \mapsto \mathsf{b}$$

$$S_2\bot \supsetneq \{\neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b}), \mathsf{P}(\mathsf{a}) \lor \neg\mathsf{P}(\mathsf{b})\} \qquad \text{unsatisfiable}$$

Run-Loop


active clauses
given clause
selection
selected literal
unsatis-fiable
SMT
passive clauses
search for conflicts
satis-fiable
instantiation
substitution
new clauses
read file(s)

Subsumption

$$S = \{C, D, \ldots\} \qquad \exists\theta\ C\theta \subseteq D$$ C subsumes D

$$S \text{ satisfiable } \overset{\checkmark}{\iff} (S \setminus D) \text{ satisfiable}$$

$$\theta \text{ is proper, } S\bot \text{ satisfiable } \overset{\times}{\iff} (S \setminus D)\bot \text{ satisfiable}$$

$$\theta \text{ is renaming, } S\bot \text{ satisfiable } \overset{\checkmark}{\iff} (S \setminus D)\bot \text{ satisfiable}$$

Example

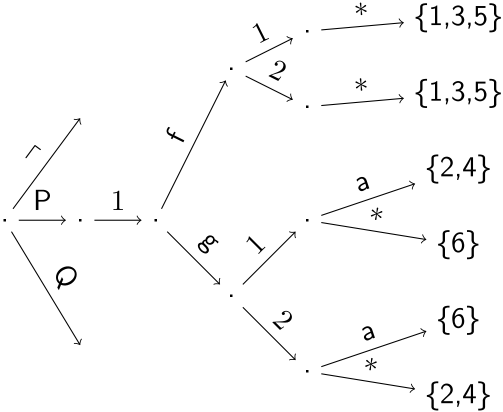
$$\{\mathsf{P}(x, y), \neg\mathsf{P}(\mathsf{a}, z)\} \qquad \{\mathsf{P}(x, y), \neg\mathsf{P}(\mathsf{a}, z), \mathsf{P}(\mathsf{a}, z)\}$$

$$\{\mathsf{P}(\bot, \bot), \neg\mathsf{P}(\mathsf{a}, \bot)\} \qquad \{\mathsf{P}(\bot, \bot), \neg\mathsf{P}(\mathsf{a}, \bot), \mathsf{P}(\mathsf{a}, \bot)\}$$

needed data structures

1. representation of clauses, literals and terms
2. fast retrieval of clauses with a selected literal that is unifiable with the negation selected literal of a given clause
3. fast retrieval of clauses with a set of literals that is a renamed subset of a given clause

$$\{{}^{1:}\mathsf{P}(\mathsf{f}(x,x)), {}^{2:}\mathsf{P}(\mathsf{g}(\mathsf{a},x)), {}^{3:}\mathsf{P}(\mathsf{f}(y,z)), {}^{4:}\mathsf{P}(\mathsf{g}(\mathsf{a},y)), {}^{5:}\mathsf{P}(\mathsf{f}(y,x)), {}^{6:}\mathsf{P}(\mathsf{g}(y,a))\}$$



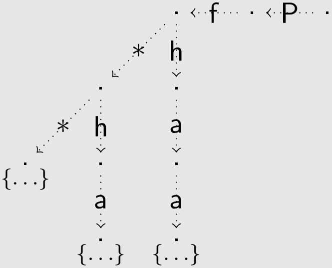
$$\neg\mathsf{P}(\mathsf{g}(\mathsf{b},z)) \mapsto \{\mathsf{P}.1.\mathsf{g}.1.\mathsf{b}, \mathsf{P}.1.\mathsf{g}.2.*\} \mapsto \{6\} \cap \{2,4,6\}$$

$$^{\ell_1:}\mathsf{P}(\mathsf{f}(x,y)), {}^{\ell_2:}\mathsf{P}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))), {}^{\ell_3:}\mathsf{P}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a}))$$

$$\ell_1 \mapsto \mathsf{P.f.}{*}.{*}$$

$$\ell_2 \mapsto \mathsf{P.f.}{*}.\mathsf{h.a}$$

$$\ell_3 \mapsto \mathsf{P.f.h.a.a}$$



Implementation

$$\texttt{Clause} \mapsto (\texttt{Int},\ \texttt{Set of term\_t})$$

$$\texttt{term\_t} \mapsto \texttt{Set of Int}$$

$$\texttt{Int} \mapsto \texttt{Clause}$$

Example

$$S = \{\mathsf{P}(\mathsf{a}), \neg\mathsf{P}(\mathsf{f}(\mathsf{a},\mathsf{b})), \mathsf{f}(x,\mathsf{b}) = x\} \qquad \text{unsatisfiable}$$

$$S\bot = \{\mathsf{P}(\mathsf{a}), \neg\mathsf{P}(\mathsf{f}(\mathsf{a},\mathsf{b})), \mathsf{f}(\bot,\mathsf{b}) = \bot\} \qquad \text{satisfiable}$$

$$\mathsf{a} \neq y \lor \neg\mathsf{P}(\mathsf{a}) \lor \mathsf{P}(y) \qquad \mathsf{P}(\mathsf{a}), \text{ congruence}$$

Schemata

$$\begin{array}{rl}
x = x & s \neq s \\
x \neq y \lor y = x & s \neq t \\
x \neq y \lor y \neq z \lor x = z & s \neq t \\
x_1 \neq y_1 \lor x_2 \neq y_2 \lor \mathsf{f}(x_1, x_2) = \mathsf{f}(y_1, y_2) & \mathsf{f}(s_1, s_2) \neq \mathsf{f}(t_1, t_2) \\
x \neq y \lor \neg\mathsf{P}(x) \lor \mathsf{P}(y) & \mathsf{P}(s) \\
x \neq y \lor \neg\mathsf{P}(x) \lor \mathsf{P}(y) & \neg\mathsf{P}(s)
\end{array}$$

Lemma

Symmetry and transitivity are consequences of reflexivity and congruence.

Symmetry.

$$\dfrac{\dfrac{}{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}\ \mathsf{C}}{\dfrac{x \neq y \vee x \neq x \vee x \neq x \vee y = x}{x \neq y \vee y = x}\ \mathsf{R}}\ x_1 \mapsto x, x_2 \mapsto x, y_1 \mapsto y, y_2 \mapsto x$$

□

Transitivity.

$$\dfrac{\dfrac{}{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}\ \mathsf{C}}{\dfrac{x \neq x \vee y \neq z \vee x \neq y \vee x = z}{x \neq y \vee y \neq z \vee x = z}\ \mathsf{R}}\ x_1 \mapsto x, x_2 \mapsto y, y_1 \mapsto x, y_2 \mapsto z$$

□

The Bernays–Schönfinkel-Ramsey class of first-order formulae is a decidable fragment of first-order logic. Each formula in this fragment is equivalent to a satisfiable formula

$$\exists a_1 \ldots a_m \forall y_1 \ldots\ y_n\, F$$

where F is quantifier free and does not contain function symbols.

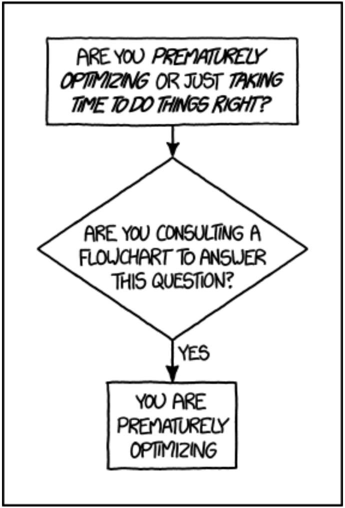
Example

$$\mathsf{a} \neq x \vee \mathsf{Q}(\mathsf{a}, x), \mathsf{Q}(y, \mathsf{b})$$

Remark

In practice the additon of equality axioms reduces the success rates of (instantiation-based) automated theorem provers drastically (even in this segment).

- migrate to Linux / Swift 3
- integrate unit superposition
- integrate ordered maximal completion
- experiments
- optimizations


ARE YOU *PREMATURELY OPTIMIZING* OR JUST *TAKING TIME TO DO THINGS RIGHT?*
ARE YOU CONSULTING A FLOWCHART TO ANSWER THIS QUESTION?
YES
YOU ARE PREMATURELY OPTIMIZING