

flea
bit(e)s and pieces

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*Hofstadter's Law: It always takes longer than you expect,
even when you take into account Hofstadter's Law.*

— Douglas Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid



- 1 Previously

- 2 Procedure

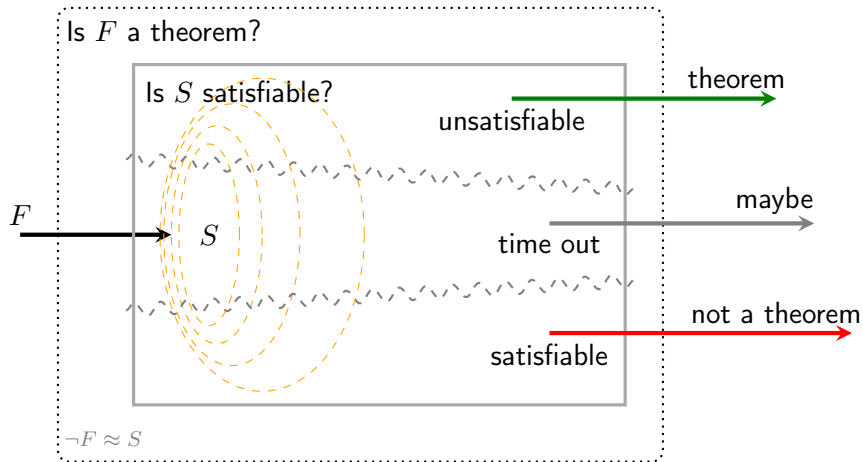
- 3 Equality

- 4 work to do

References

-  Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, Inc., New York, NY, USA, 1979.
-  Robert Nieuwenhuis, Thomas Hillenbrand, Alexandre Riazanov, and Andrei Voronkov, *On the evaluation of indexing techniques for theorem proving*, Automated Reasoning (Rajeev Goré, Alexander Leitsch, and Tobias Nipkow, eds.), Lecture Notes in Computer Science, vol. 2083, Springer Berlin Heidelberg, 2001, pp. 257–271.

Goal



Definition (Ordered Resolution)

$$\frac{L \vee C \quad \neg L' \vee D}{(C \vee D)\sigma}$$

where

$L\sigma$ strictly maximal in $C\sigma$, $\neg L'\sigma$ maximal in $D\sigma$, $\sigma = \text{mgu}(L, L')$.

Definition (Inst-Gen)

$$\frac{L \vee C \quad \neg L' \vee D}{(L \vee C)\sigma \quad (\neg L' \vee D)\sigma}$$

where

$$\text{sel}(L \vee C) = L \quad \text{sel}(\neg L' \vee D) = \neg L' \quad \sigma = \text{mgu}(L, L')$$

Example (Resolution)

$$\frac{\frac{P(x) \vee \neg P(y) \quad \neg P(a)}{\neg P(y) \quad P(b)} \quad x \mapsto a}{\square} \quad y \mapsto b$$

Example (Inst-Gen)

$$S_0 \perp = \{P(\perp) \vee \neg P(\perp), \neg P(a), P(b)\} \quad \text{satisfiable}$$

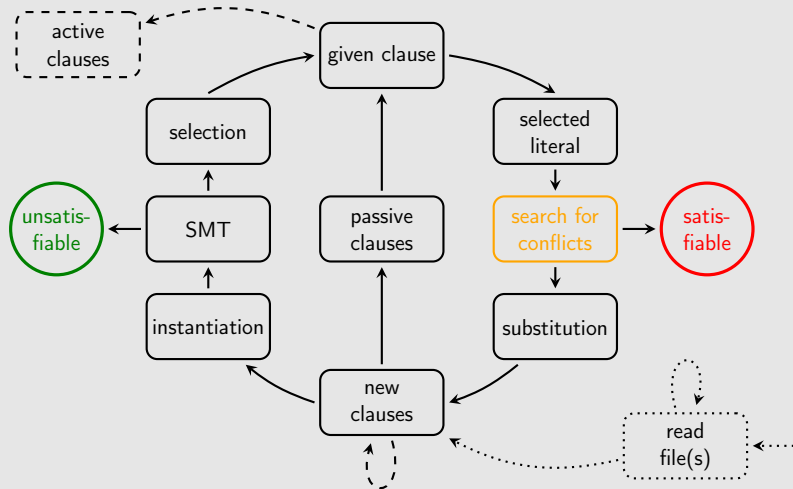
$$\frac{P(x) \vee \neg P(y) \quad \neg P(a)}{P(a) \vee \neg P(y)} \quad x \mapsto a$$

$$S_1 \perp \supsetneq \{\neg P(a), P(b), P(a) \vee \neg P(\perp)\} \quad \text{satisfiable}$$

$$\frac{P(b) \quad P(a) \vee \neg P(y)}{P(a) \vee P(b)} \quad y \mapsto b$$

$$S_2 \perp \supsetneq \{\neg P(a), P(b), P(a) \vee \neg P(b)\} \quad \text{unsatisfiable}$$

Run-Loop



Subsumption

$$S = \{C, D, \dots\} \quad \exists \theta \ C\theta \subseteq D \quad \text{C subsumes D}$$

$$S \text{ satisfiable} \xrightarrow{\checkmark} (S \setminus D) \text{ satisfiable}$$

$$\theta \text{ is proper, } S \perp \text{ satisfiable} \xrightarrow{\times} (S \setminus D) \perp \text{ satisfiable}$$

$$\theta \text{ is renaming, } S \perp \text{ satisfiable} \xrightarrow{\checkmark} (S \setminus D) \perp \text{ satisfiable}$$

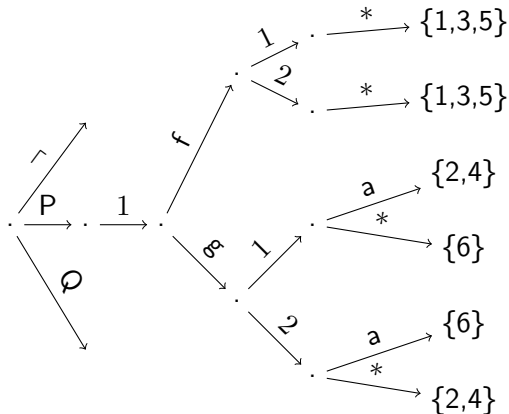
Example

$$\begin{array}{ll} \{P(x, y), \neg P(a, z)\} & \{P(x, y), \neg P(a, z), P(a, z)\} \\ \{P(\perp, \perp), \neg P(a, \perp)\} & \{P(\perp, \perp), \neg P(a, \perp), P(a, \perp)\} \end{array}$$

needed data structures

- 1 representation of clauses, literals and terms
- 2 fast retrieval of clauses with a selected literal that is unifiable with the negation selected literal of a given clause
- 3 fast retrieval of clauses with a set of literals that is a renamed subset of a given clause

$$\{^1: P(f(x, x)), ^2: P(g(a, x)), ^3: P(f(y, z)), ^4: P(g(a, y)), ^5: P(f(y, x)), ^6: P(g(y, a))\}$$



$$\neg P(g(b, z)) \mapsto \{P.1.g.1.b, P.1.g.2.*\} \mapsto \{6\} \cap \{2, 4, 6\}$$

Example

$S = \{\mathbf{P(a)}, \neg P(f(a, b)), f(x, b) = x\}$	unsatisfiable
$S \perp = \{\mathbf{P(a)}, \neg P(f(a, b)), f(\perp, b) = \perp\}$	satisfiable
$a \neq y \vee \neg P(a) \vee P(y)$	$P(a)$, congruence

Schemata

$x = x$	$s \neq s$
$x \neq y \vee y = x$	$s \neq t$
$x \neq y \vee y \neq z \vee x = z$	$s \neq t$
$x_1 \neq y_1 \vee x_2 \neq y_2 \vee \mathbf{f(x_1, x_2) = f(y_1, y_2)}$	$f(s_1, s_2) \neq f(t_1, t_2)$
$x \neq y \vee \neg \mathbf{P(x)} \vee P(y)$	$P(s)$
$x \neq y \vee \neg P(x) \vee \mathbf{P(y)}$	$\neg P(s)$

Lemma

Symmetry and transitivity are consequences of reflexivity and congruence.

Symmetry.

$$\begin{array}{c}
 \frac{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}{x \neq y \vee x \neq x \vee x \neq x \vee y = x} \text{C} \\
 \frac{x \neq y \vee x \neq x \vee x \neq x \vee y = x}{x \neq y \vee y = x} \text{R}
 \end{array}
 \quad
 \begin{array}{l}
 x_1 \mapsto x, x_2 \mapsto x, y_1 \mapsto y, y_2 \mapsto x
 \end{array}$$



Transitivity.

$$\begin{array}{c}
 \frac{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}{x \neq x \vee y \neq z \vee x \neq y \vee x = z} \text{C} \\
 \frac{x \neq x \vee y \neq z \vee x \neq y \vee x = z}{x \neq y \vee y \neq z \vee x = z} \text{R}
 \end{array}
 \quad
 \begin{array}{l}
 x_1 \mapsto x, x_2 \mapsto y, y_1 \mapsto x, y_2 \mapsto z
 \end{array}$$



- migrate to Linux / Swift 3
- integrate unit superposition
- integrate ordered maximal completion
- experiments
- optimizations

