



flea
bit(e)s and pieces

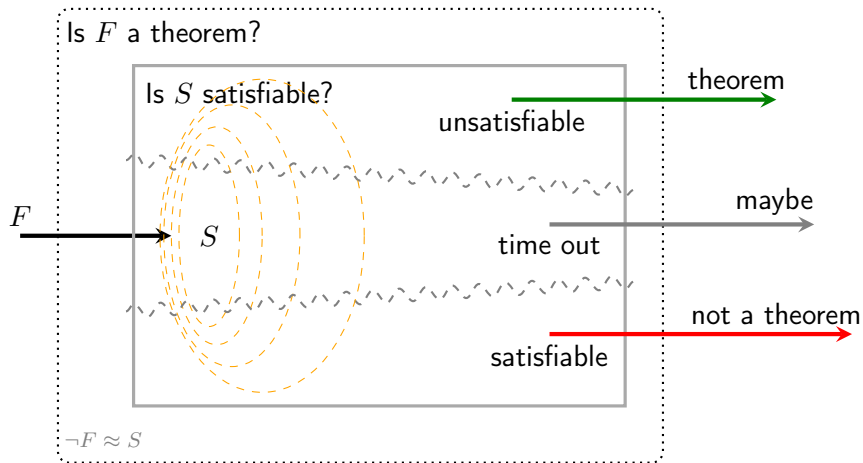
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June 15th, 2016

- 1 Previously
- 2 Procedure
- 3 Equality
- 4 Bernays–Schönfinkel–Ramsey
- 5 Summary and Outlook

References

-  Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, Inc., New York, NY, USA, 1979.
-  Robert Nieuwenhuis, Thomas Hillenbrand, Alexandre Riazanov, and Andrei Voronkov, *On the evaluation of indexing techniques for theorem proving*, Automated Reasoning (Rajeev Goré, Alexander Leitsch, and Tobias Nipkow, eds.), Lecture Notes in Computer Science, vol. 2083, Springer Berlin Heidelberg, 2001, pp. 257–271.



Definition (Ordered Resolution)

$$\frac{L \vee C \quad \neg L' \vee D}{(C \vee D)\sigma}$$

where

$L\sigma$ strictly maximal in $C\sigma$, $\neg L'\sigma$ maximal in $D\sigma$, $\sigma = \text{mgu}(L, L')$.

Definition (Inst-Gen)

$$\frac{L \vee C \quad \neg L' \vee D}{(L \vee C)\sigma \quad (\neg L' \vee D)\sigma}$$

where

$$\text{sel}(L \vee C) = L \quad \text{sel}(\neg L' \vee D) = \neg L' \quad \sigma = \text{mgu}(L, L')$$

Example (Resolution)

$$S = \{P(x) \vee \neg P(y), \neg P(a), P(b)\}$$

$$\frac{\frac{P(x) \quad \neg P(y)}{P(y)} \quad x \mapsto a \quad P(b)}{\square} \quad y \mapsto b$$

Example (Inst-Gen)

$$S_0 \perp = \{P(\perp) \vee \neg P(\perp), \neg P(a), P(b)\} \quad \text{satisfiable}$$

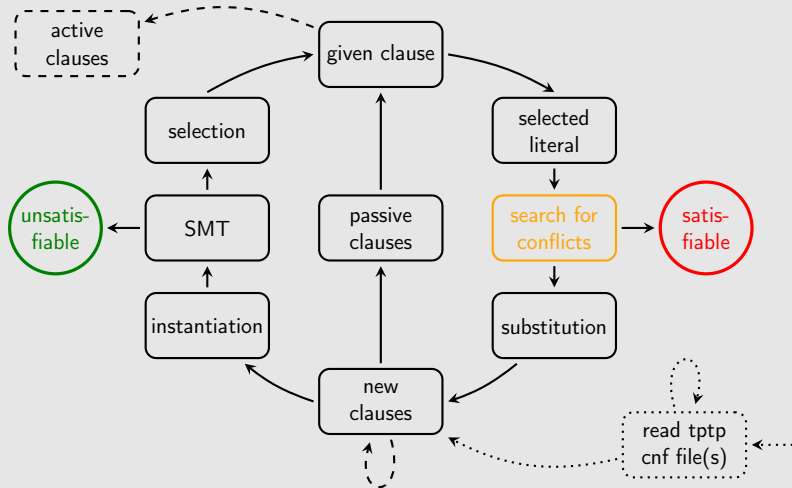
$$\frac{P(x) \vee \neg P(y) \quad \neg P(a)}{P(a) \vee \neg P(y)} \quad x \mapsto a$$

$$S_1 \perp \supsetneq \{\neg P(a), P(b), P(a) \vee \neg P(\perp)\} \quad \text{satisfiable}$$

$$\frac{P(b) \quad P(a) \vee \neg P(y)}{P(a) \vee P(b)} \quad y \mapsto b$$

$$S_2 \perp \supsetneq \{\neg P(a), P(b), P(a) \vee \neg P(b)\} \quad \text{unsatisfiable}$$

Run-Loop



Subsumption

$S = \{C, D, \dots\} \quad \exists \theta \ C\theta \subseteq D \quad C \text{ subsumes } D$

$S \text{ satisfiable} \xrightarrow{\checkmark} (S \setminus D) \text{ satisfiable}$

$\theta \text{ is proper, } S \perp \text{ satisfiable} \xrightarrow{\times} (S \setminus D) \perp \text{ satisfiable}$

$\theta \text{ is renaming, } S \perp \text{ satisfiable} \xrightarrow{\checkmark} (S \setminus D) \perp \text{ satisfiable}$

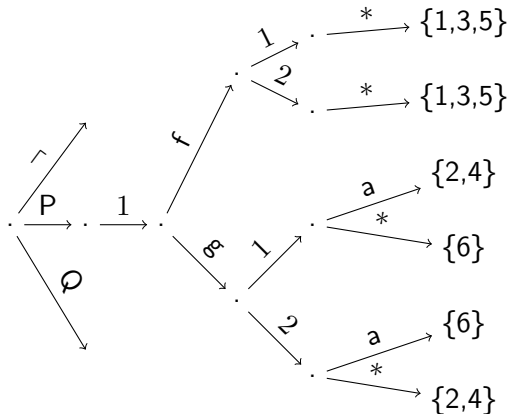
Example

$\{P(x, y), \neg P(a, z)\}$	$\{P(x, y), \neg P(a, z), P(a, z)\}$
$\{P(\perp, \perp), \neg P(a, \perp)\}$	$\{P(\perp, \perp), \neg P(a, \perp), P(a, \perp)\}$

data structures

- 1 trees to represent clauses, literals and terms
- 2 path indexing for fast retrieval of clashing selected literals
- 3 discrimination trees for fast retrieval of clause variants

$$\{^1: P(f(x, x)), ^2: P(g(a, x)), ^3: P(f(y, z)), ^4: P(g(a, y)), ^5: P(f(y, x)), ^6: P(g(y, a))\}$$



$$\neg P(g(b, z)) \mapsto \{P.1.g.1.b, P.1.g.2.*\} \mapsto \{6\} \cap \{2, 4, 6\}$$

$$\ell_3 \mapsto \text{P.f.h.a.a}$$


Implementation

$$\text{Int} \mapsto \text{Clause}$$

```
int32_t yices_assert_formula(context_t *ctx, term_t t);

term_t yices_or(uint32_t n, term_t arg[]);
term_t yices_not(term_t arg);
term_t yices_eq(term_t left, term_t right);

term_t yices_application(term_t fun, uint32_t n, const term_t arg[]);
term_t yices_new_uninterpreted_term(type_t tau);

type_t yices_function_type(uint32_t n, const type_t dom[], type_t range);
type_t yices_bool_type(void);
type_t yices_new_uninterpreted_type(void);

smt_status_t yices_check_context(context_t *ctx, const param_t *params);
model_t *yices_get_model(context_t *ctx, int32_t keep_subst);
int32_t yices_get_bool_value(model_t *mdl, term_t t, int32_t *val);
```

Example

$S = \{\mathbf{P(a)}, \neg P(f(a, b)), f(x, b) = x\}$	unsatisfiable
$S_{\perp} = \{\mathbf{P(a)}, \neg P(f(a, b)), f(\perp, b) = \perp\}$	satisfiable
$a \neq y \vee \neg P(a) \vee P(y)$	$P(a)$, congruence

Schemata

$x = x$	$s \neq s$	reflexivity
$x \neq y \vee y = x$	$s \neq t$	symmetry
$x \neq y \vee y \neq z \vee x = z$	$s \neq t$	transitivity
$x_1 \neq y_1 \vee x_2 \neq y_2 \vee \mathbf{f(x_1, x_2) = f(y_1, y_2)}$	$\mathbf{f(s_1, s_2) \neq f(t_1, t_2)}$	
$x \neq y \vee \neg \mathbf{P(x)} \vee P(y)$	$P(s)$	
$x \neq y \vee \neg P(x) \vee \mathbf{P(y)}$	$\neg P(s)$	congruence

Lemma

Symmetry and transitivity are consequences of reflexivity and congruence.

Symmetry.

$$\begin{array}{c}
 \frac{x_1 = y_1 \wedge x_2 = y_2 \wedge x_1 = x_2 \rightarrow y_1 = y_2}{x = y \wedge x = x \wedge x = x \rightarrow y = x} \text{congruence} \\
 \frac{}{x = y \rightarrow y = x} \text{reflexivity}
 \end{array}$$



Transitivity.

$$\begin{array}{c}
 \frac{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}{x \neq x \vee y \neq z \vee x \neq y \vee x = z} \text{congruence} \\
 \frac{}{x \neq y \vee y \neq z \vee x = z} \text{reflexivity}
 \end{array}$$



The Bernays–Schönfinkel–Ramsey class of first-order formulae is a decidable fragment of first-order logic. Each formula in this fragment is equivalent to a satisfiable formula

$$\exists a_1 \dots a_m \forall y_1 \dots y_n F$$

where F is quantifier free and does not contain function symbols.

Example

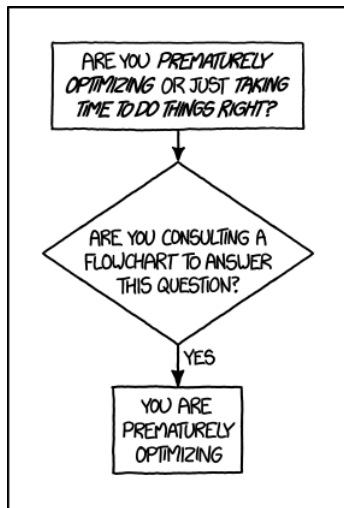
$$a \neq x \vee Q(a, x), Q(y, b)$$

Remark

In practice the addition of equality axioms reduces the success rates of (instantiation-based) automated theorem provers drastically.

- Done:
 - Scanning and parsing with `flex` and `bison`
 - Application logic with two simple strategies
 - SMT encoding to Yices 2
- Ongoing:
 - Integration of Unit Superposition calculus
 - Integration of Z3 for unsatisfiable core and maximal completion
 - Tests and experiments
- Missing:
 - Additional strategies
 - Combinations of strategies
 - Migration to Linux

Optimization



xkcd.com/1691/