$$\frac{F \quad G}{F \wedge G}\ \wedge i \qquad \frac{F \wedge G}{G}\ \wedge e_1 \qquad \frac{F \wedge G}{F}\ \wedge e_2$$

$$\frac{F}{\neg\neg F}\ \neg\neg i \qquad \frac{\neg\neg F}{F}\ \neg\neg e$$

$$\frac{F \quad F \to G}{\neg G}\ \text{modus ponens} \qquad \frac{F \to G \quad \neg G}{\neg F}\ \text{modus tollens}$$

$$\frac{\boxed{\begin{array}{c} F \\ \vdots \\ G \end{array}}}{F \to G}\ \to i \qquad \frac{F \vee G \quad \boxed{\begin{array}{c} F \\ \vdots \\ H \end{array}} \quad \boxed{\begin{array}{c} G \\ \vdots \\ H \end{array}}}{H}\ \vee e$$

$$\frac{F}{F \vee G}\ \vee i_1 \qquad \frac{G}{F \vee G}\ \vee i_2$$

$$\frac{\bot}{F}\ \bot e \qquad \frac{F \quad \neg F}{\bot}\ \neg e \qquad \frac{\boxed{\begin{array}{c} F \\ \vdots \\ \bot \end{array}}}{\neg F}\ \neg i$$

$$\frac{\boxed{\begin{array}{c} \neg F \\ \vdots \\ \bot \end{array}}}{F}\ PBC \qquad \frac{}{F \vee \neg F}\ LEM$$

$$\frac{}{t = t}\ = i \qquad \frac{s = t \quad F\{x \mapsto s\}}{F\{x \mapsto t\}}\ = e$$

$$\frac{\forall x F}{F\{x \to t\}}\ \forall e \qquad \frac{\boxed{\begin{array}{ll} x_0 & \\ & \vdots \\ & F\{x \mapsto x_0\} \end{array}}}{\forall F}\ \forall i$$

$$\frac{F\{x \mapsto x_0\}}{\exists x F}\ \exists i \qquad \frac{\exists x F \quad \boxed{\begin{array}{ll} x_0 & F\{x \mapsto x_0\} \\ & \vdots \\ & H \end{array}}}{H}\ \exists e$$

Table 1: Natural Deduction Rules