FLEA

first order proving with equality
master project

Alexander Maringele
Supervisor: Georg Moser

Sep 2016 – Obergurgl

Project overview

Goals and requirements

- Goals
  - ATP for FOL with equality
  - Master thesis
- Requirements
  - Input: problems in clausal normal form
  - Data: clauses, literals, terms, indices, etc.
  - Algorithms: substitution, unification, etc.
  - **Proof search** (strategies)
- Non-Goals and Non-Requirements
  - CASC
  - FOF $\approx$ CNF

Clausal normal form

TPTP Syntax

```
...
cnf(same_hates, hypothesis, ( ~hates(agatha,X) | hates(butler,X) )).
...
```

$$\{\ \ldots,\ \neg \mathsf{hates}(\mathsf{agatha}, x) \vee \mathsf{hates}(\mathsf{butler}, x),\ \ldots\}$$

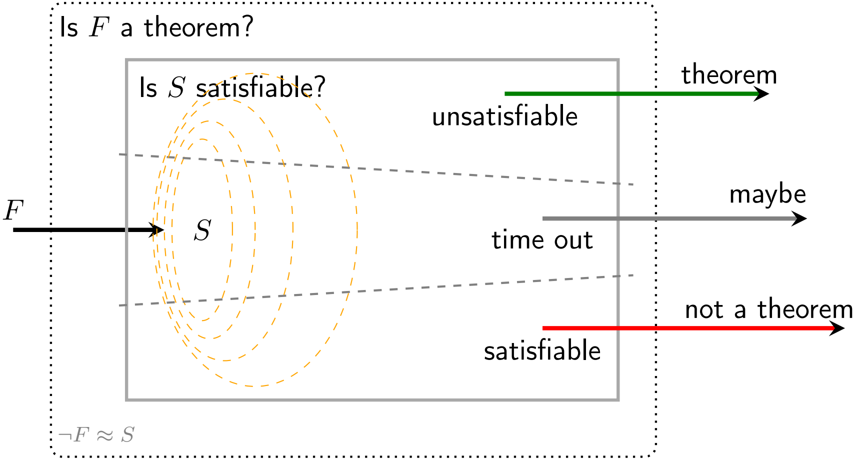
$$\equiv$$

$$\ldots\ \wedge\ \forall x\ (\neg \mathsf{hates}(\mathsf{agatha}, x) \vee \mathsf{hates}(\mathsf{butler}, x))\ \wedge\ \ldots$$

G. Sutcliffe, *The TPTP Problem Library and Associated Infrastructure: The FOF and CNF Parts, v3.5.0*, Journal of Automated Reasoning **43** (2009), no. 4, 337–362.

Refutation

Proof search



Ordered Resolution

$$\frac{L \vee C \quad \neg L' \vee D}{(C \vee D)\sigma}$$

where

$L\sigma$ strictly maximal in $C\sigma$, $\neg L'\sigma$ maximal in $D\sigma$, $\sigma = \mathrm{mgu}(L, L')$.

$$S = \{\mathsf{P}(x) \vee \neg\mathsf{P}(y), \neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b})\}$$

$$\cfrac{\cfrac{\mathsf{P}(x) \vee \neg\mathsf{P}(y) \quad \neg\mathsf{P}(\mathsf{a})}{\neg\mathsf{P}(y)}\; x \mapsto \mathsf{a} \qquad \mathsf{P}(\mathsf{b})}{\square}\; y \mapsto \mathsf{b}$$

Inst-Gen

$$\frac{L \lor C \quad \neg L' \lor D}{(L \lor C)\sigma \quad (\neg L' \lor D)\sigma}$$

where

$$\mathrm{sel}(L \lor C) = L \qquad \mathrm{sel}(\neg L' \lor D) = \neg L' \qquad \sigma = \mathrm{mgu}(L, L')$$

Selection

$S = \{\mathsf{P}(x) \lor \neg\mathsf{P}(y), \neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b})\}$ unsatisfiable

$P = (p_* \lor \neg p_*) \land \neg p_a \land p_b$ satisfiable

$S_0 = \{\mathsf{P}(x) \vee \neg\mathsf{P}(y), \neg\mathsf{P}(\mathsf{a}), \mathsf{P}(\mathsf{b})\}$ unsatisfiable

$P_0 = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b$ satisfiable

$$\frac{\mathsf{P}(x) \vee \neg\mathsf{P}(y)) \quad \neg\mathsf{P}(\mathsf{a})}{\mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(y)}\ x \mapsto \mathsf{a}$$

$P_1 = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b \wedge (p_a \vee \neg p_*)$ satisfiable

$$\frac{\mathsf{P}(\mathsf{b}) \quad \mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(y)}{\mathsf{P}(\mathsf{a}) \vee \neg\mathsf{P}(\mathsf{b})}\ y \mapsto \mathsf{b}$$

$P_2 = (p_* \vee \neg p_*) \wedge \neg p_a \wedge p_b \wedge (p_a \vee \neg p_*) \wedge (p_a \vee \neg p_b)$ unsatisfiable

Equality as predicate

$S = \{\mathsf{P}(\mathsf{a}), \neg\mathsf{P}(\mathsf{f}(x, \mathsf{b})), \mathsf{f}(x, \mathsf{b}) = x\}$ saturated

$P = p \wedge \neg q \wedge e$ satisfiable

$$S_0 = \{\mathsf{P}(\mathsf{a}), \neg\mathsf{P}(\mathsf{f}(\mathsf{a},\mathsf{b})), \mathsf{f}(x,\mathsf{b}) = x\}$$ unsatisfiable

$$P_0 = p_a \wedge \neg p_1 \wedge e_1$$ satisfiable

$$\mathsf{a} \neq y \vee \neg\mathsf{P}(\mathsf{a}) \vee \mathsf{P}(y)$$ $\mathsf{P}(\mathsf{a})$, congruence

$$P_1 = p_a \wedge \neg p_1 \wedge e_1 \wedge (\neg e_2 \vee \neg p_a \vee p_*)$$

Schemata

$x = x$	$s \neq s$	reflexivity
$x \neq y \vee y = x$	$s \neq t$	symmetry
$x \neq y \vee y \neq z \vee x = z$	$s \neq t$	transitivity
$x_1 \neq y_1 \vee x_2 \neq y_2 \vee \mathsf{f}(x_1, x_2) = \mathsf{f}(y_1, y_2)$	$\mathsf{f}(s_1, s_2) \neq \mathsf{f}(t_1, t_2)$	
$x \neq y \vee \neg\mathsf{P}(x) \vee \mathsf{P}(y)$	$\mathsf{P}(s)$	
$x \neq y \vee \neg\mathsf{P}(x) \vee \mathsf{P}(y)$	$\neg\mathsf{P}(s)$	congruence

Lemma

Symmetry and transitivity are consequences of reflexivity and congruence.

Symmetry.

$$\dfrac{\dfrac{}{x_1 = y_1 \wedge x_2 = y_2 \wedge x_1 = x_2 \to y_1 = y_2}\ \text{congruence}}{\dfrac{x = y \wedge x = x \wedge x = x \to y = x}{x = y \to y = x}\ \text{reflexivity}}\ x_1 \mapsto x, x_2 \mapsto x, y_1 \mapsto y, y_2 \mapsto x$$

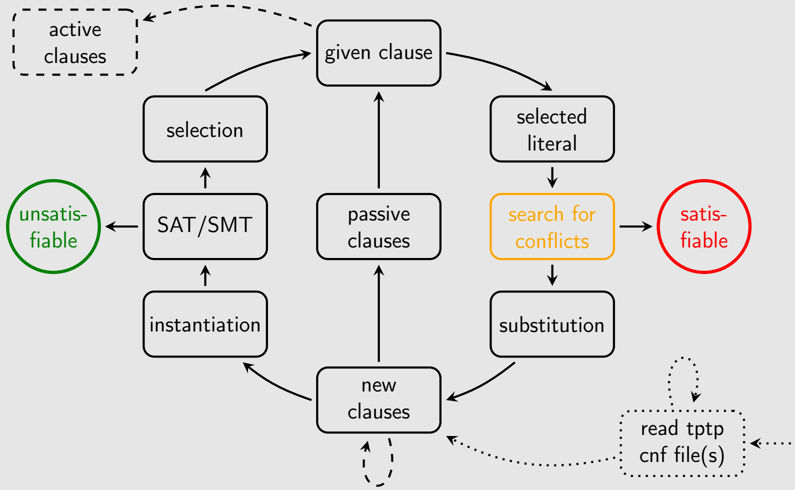
□

Transitivity.

$$\dfrac{\dfrac{}{x_1 \neq y_1 \vee x_2 \neq y_2 \vee x_1 \neq x_2 \vee y_1 = y_2}\ \text{congruence}}{\dfrac{x \neq x \vee y \neq z \vee x \neq y \vee x = z}{x \neq y \vee y \neq z \vee x = z}\ \text{reflexivity}}\ x_1 \mapsto x, x_2 \mapsto y, y_1 \mapsto x, y_2 \mapsto z$$

□

Run-loop


active clauses
given clause
selection
selected literal
unsatis-fiable
SAT/SMT
passive clauses
search for conflicts
satis-fiable
instantiation
substitution
new clauses
read tptp cnf file(s)