Term-Indexing

for Instantiation-Based First Order Theorem Proving

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January 27th, 2016

References

- Alexandre Riazanov and Andrei Voronkov, *Efficient instance retrieval with standard and relational path indexing*, Automated Deduction – CADE-19 (Franz Baader, ed.), Lecture Notes in Computer Science, vol. 2741, Springer Berlin Heidelberg, 2003, pp. 380–396 (English).
- R. Sekar, I. V. Ramakrishnan, and Andrei Voronkov, *Term indexing*, Handbook of Automated Reasoning (Alan Robinson and Andrei Voronkov, eds.), Elsevier Science Publishers B. V., Amsterdam, The Netherlands, The Netherlands, 2001, pp. 1853–1964.

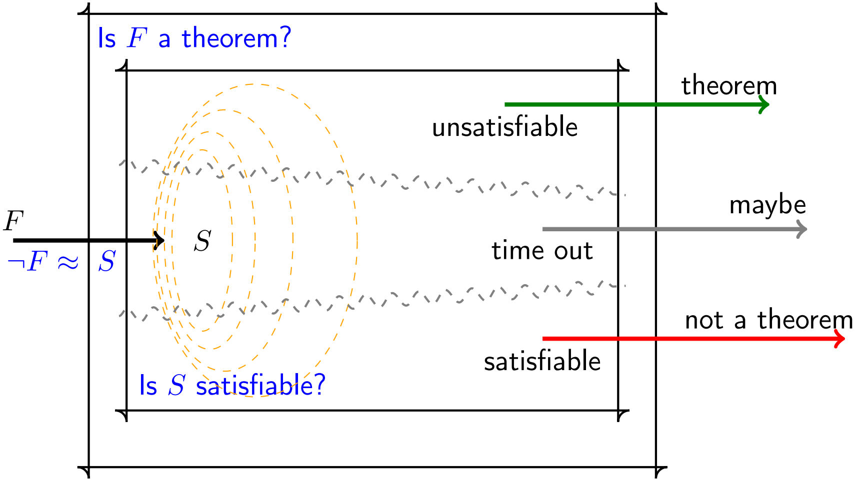
Outline

Notation (First Order Logic)

- $\Sigma = (\mathcal{V}, \mathcal{F}, \mathcal{P})$ signature
- $\mathcal{T} = \mathcal{V} \cup \{\, f(t_1, \ldots, t_n) \mid f \in \mathcal{F}_{(n)}, t_i \in \mathcal{T}\}$ terms
- $\mathcal{A} = \{P(t_1, \ldots, t_n) \mid P \in \mathcal{P}_{(n)}, t_i \in \mathcal{T}\} \cup \{s \approx t \mid s, t \in \mathcal{T}\}$ atoms
- $\mathcal{L} = \mathcal{A} \cup \{\neg A \mid A \in \mathcal{A}\}$ literals
- $\mathcal{C} = 2^{\mathcal{L}}$, e.g. $\mathsf{f}(x) \approx \mathsf{a} \vee \mathsf{a} \not\approx \mathsf{b}$ clauses
- $\mathcal{S} = 2^{\mathcal{C}}$, e.g. $\{\mathsf{f}(x) \approx \mathsf{a} \vee \mathsf{a} \not\approx \mathsf{b}, \mathsf{f}(x) \approx \mathsf{b}\}$ clause sets

$$\{C_1, \ldots, C_n\} \equiv \forall \mathcal{V}\mathsf{ar}(C_1) C_1 \wedge \ldots \wedge \forall \mathcal{V}\mathsf{ar}(C_1) C_n$$

Resolution


Is F a theorem?
theorem
unsatisfiable
maybe
F
S
$\neg F \approx S$
time out
not a theorem
satisfiable
Is S satisfiable?

Goal

A sound, refutational complete, and *effective* calculus.

- *Reduce* search space
  - Ordered Resolution
  - ...selection functions for clauses and literals
- *Reduce* redundancy
  - Ignore clause C, if C subsumes D, i.e. $C\tau \subseteq D$?
  - ...depends on the calculus

Example

$$S = \{{}^{t_1:}\mathsf{P}(x, y), {}^{t_2:}\neg\mathsf{P}(\mathsf{a}, z), {}^{t_3:}\mathsf{P}(\mathsf{a}, z')\}$$

t_1 subsumes t_3

$$S\bot = \{\mathsf{P}(\bot, \bot), \neg\mathsf{P}(\mathsf{a}, \bot), \mathsf{P}(\mathsf{a}, \bot)\}$$

InstGen/SMT

$$\frac{\mathsf{P}(x, y) \quad \neg\mathsf{P}(\mathsf{a}, z)}{\square}\ \{x \mapsto \mathsf{a}, y \mapsto z\}$$

Resolution

unit superposition inference rules

$$\frac{s \approx t \quad L[s']}{(L[t]) \cdot \sigma} \text{ unit paramodulation}$$

where $\sigma = \mathrm{mgu}(s, s')$, $s' \notin \mathcal{V}$, $t\sigma \not\succeq s\sigma$

$$\frac{s \approx t \quad u[s'] \not\approx v}{(u[t] \not\approx v) \cdot \sigma} \text{ unit superposition } \frac{s \approx t \quad u[s'] \approx v}{(u[t] \approx v) \cdot \sigma}$$

where $\sigma = \mathrm{mgu}(s, s')$, $s' \notin \mathcal{V}$, $t\sigma \not\succeq s\sigma$, $v\sigma \not\succeq u[s']\sigma$

$$\frac{s \not\approx t}{\square} \text{ unit equality resolution} \qquad \frac{A \quad \neg B}{\square} \text{ unit resolution}$$

where s and t (A and B respectively) are unifiable

Term retrieval problems

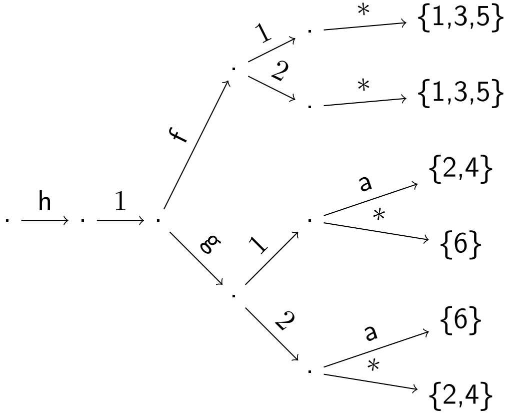
Definition

In a given set of terms

- find terms that are variants of a given term.
  $\mathrm{variant}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t$ and σ is renaming.
- find terms that are unifiable with a given term.
  $\mathrm{unifiable}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t\sigma$
- find terms that are instances of a given term.
  $\mathrm{instance}(\ell, t) \Leftrightarrow \exists\sigma\ \ell = t\sigma$
- find terms that are generalizations of a given term.
  $\mathrm{generalization}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t$

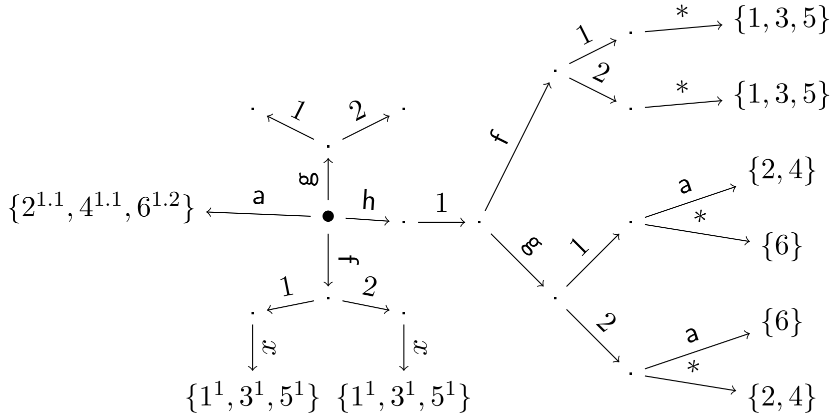
Definition

$$\{^{1:}\mathsf{h}(\mathsf{f}(x,x)), ^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), ^{3:}\mathsf{h}(\mathsf{f}(y,z)) ^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), ^{5:}\mathsf{h}(\mathsf{f}(y,x)), ^{6:}\mathsf{h}(\mathsf{g}(y,a))\}$$

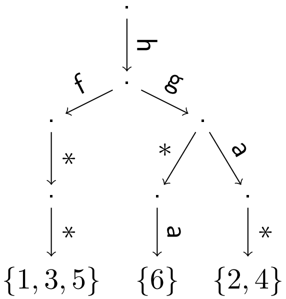


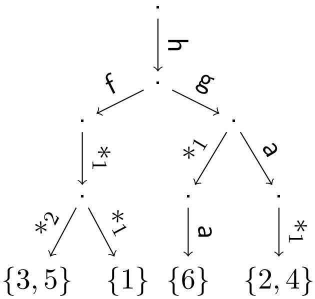
$\mathsf{h}(\mathsf{g}(y,x)) \mapsto \{\ \mathsf{h}.1.\mathsf{g}.1.*, \mathsf{h}.1.\mathsf{g}.2.*\}$

$$\{{}^{1:}\mathsf{h}(\mathsf{f}(x,x)), {}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), {}^{3:}\mathsf{h}(\mathsf{f}(y,z)) {}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), {}^{5:}\mathsf{h}(\mathsf{f}(y,x)), {}^{6:}\mathsf{h}(\mathsf{g}(y,a))\}$$



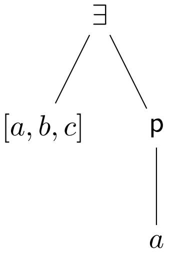
$$\left\{ \begin{array}{lll} {}^{1:}\mathsf{h}(\mathsf{f}(x,x)), & {}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), & {}^{3:}\mathsf{h}(\mathsf{f}(y,z)), \\ {}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), & {}^{5:}\mathsf{h}(\mathsf{f}(y,x)), & {}^{6:}\mathsf{h}(\mathsf{g}(y,a)) \end{array} \right\}$$





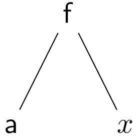
$\mathsf{h}(\mathsf{f}(y,x)) \mapsto \mathsf{h.f.}{*}.{*}$

$\mathsf{h}(\mathsf{f}(y,x)) \mapsto' \mathsf{h.f.}{*_1}.{*_2}$



$(\exists[a, b, c] : (\mathsf{p}(a)))$

$\mathsf{f}(\mathsf{a}, x)$



$\mathsf{f}(\mathsf{a}, x) \approx \mathsf{f}(x, \mathsf{a})$