Term-Indexing

for Instantiation-Based First Order Theorem Proving

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References

Alexandre Riazanov and Andrei Voronkov, *Efficient instance retrieval with standard and relational path indexing*, Automated Deduction – CADE-19 (Franz Baader, ed.), Lecture Notes in Computer Science, vol. 2741, Springer Berlin Heidelberg, 2003, pp. 380–396 (English).

R. Sekar, I. V. Ramakrishnan, and Andrei Voronkov, *Term indexing*, Handbook of Automated Reasoning (Alan Robinson and Andrei Voronkov, eds.), Elsevier Science Publishers B. V., Amsterdam, The Netherlands, The Netherlands, 2001, pp. 1853–1964.

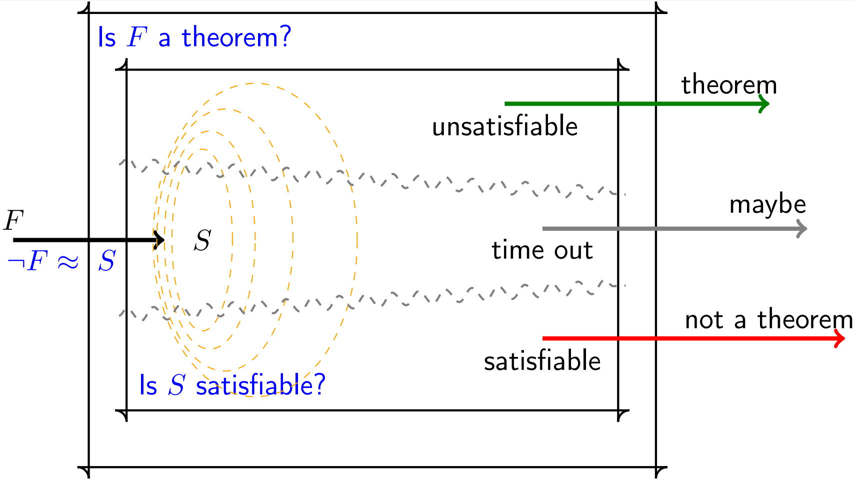
Outline

1. Motivation

2. path indexing

3. discrimination trees

Resolution



Goal

A sound$^{\circ}$, refutational complete$^{\diamond}$, and effective* procedure.

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Term retrieval problems

- Find terms that are variants of a given term.
  $\mathrm{variant}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t$ and σ is renaming.
- Find terms that are unifiable with a given term.
  $\mathrm{unifiable}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t\sigma$
- Find terms that are instances of a given term.
  $\mathrm{instance}(\ell, t) \Leftrightarrow \exists\sigma\ \ell = t\sigma$
- Find terms that are generalizations of a given term.
  $\mathrm{generalization}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t$

Definition

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A position is a sequence of positive integers. The empty sequence ε denotes the root position, pq denotes the concatenation of positions. $\mathcal{P}\mathsf{os}(t)$ denotes the set of positions in term t, and $t|_p$ denotes the subterm of t at positon $p \in \mathcal{P}\mathsf{os}(t)$.

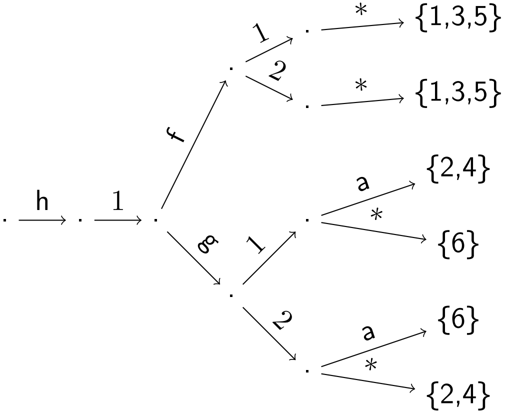
Definition

A postion string is a nonempty string of the form $\langle p_1, s_1\rangle \ldots \langle p_n, s_n\rangle$ where p_i are positions and s_i are function or variable symbols and

1. if p_i is a proper prefix of p_j then $i < j$
2.

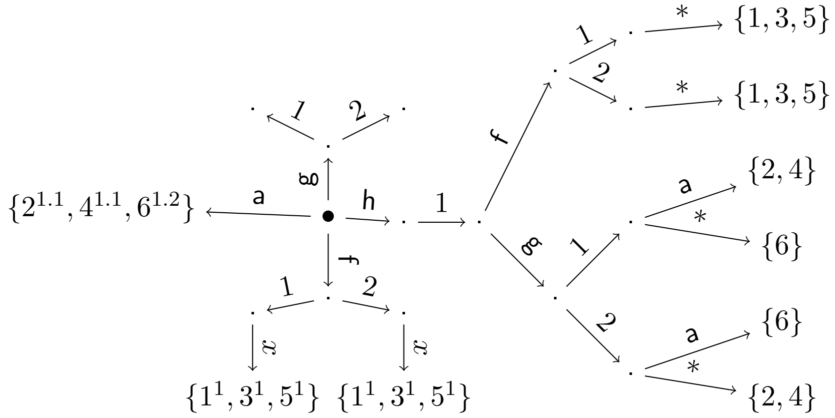
Definition

$$\{{}^{1:}\mathsf{h}(\mathsf{f}(x,x)), {}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), {}^{3:}\mathsf{h}(\mathsf{f}(y,z)) {}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), {}^{5:}\mathsf{h}(\mathsf{f}(y,x)), {}^{6:}\mathsf{h}(\mathsf{g}(y,a))\}$$

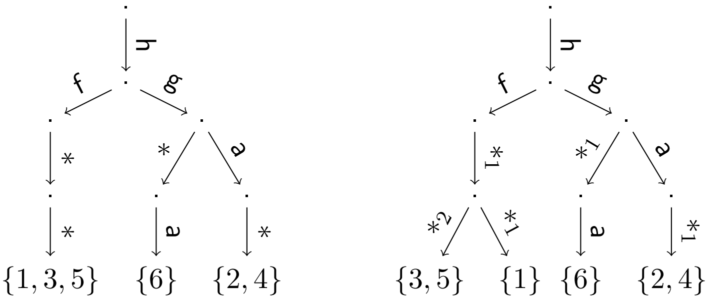


$\mathsf{h}(\mathsf{g}(y,x)) \mapsto \{\ \mathsf{h}.1.\mathsf{g}.1.*, \mathsf{h}.1.\mathsf{g}.2.*\}$

$$\{^{1:}\mathsf{h}(\mathsf{f}(x,x)),^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)),^{3:}\mathsf{h}(\mathsf{f}(y,z))^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)),^{5:}\mathsf{h}(\mathsf{f}(y,x)),^{6:}\mathsf{h}(\mathsf{g}(y,a))\}$$

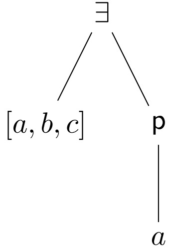


$$\left\{ \begin{array}{lll} {}^{1:}\mathsf{h}(\mathsf{f}(x,x)), & {}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), & {}^{3:}\mathsf{h}(\mathsf{f}(y,z)), \\ {}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), & {}^{5:}\mathsf{h}(\mathsf{f}(y,x)), & {}^{6:}\mathsf{h}(\mathsf{g}(y,a)) \end{array} \right\}$$



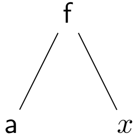
$\mathsf{h}(\mathsf{f}(y,x)) \mapsto \mathsf{h.f.}{*}.{*}$

$\mathsf{h}(\mathsf{f}(y,x)) \mapsto' \mathsf{h.f.}{*_1}.{*_2}$



$$(\exists[a, b, c] : (\mathsf{p}(a)))$$

$\mathsf{f}(\mathsf{a}, x)$



$\mathsf{f}(\mathsf{a}, x) \approx \mathsf{f}(x, \mathsf{a})$