

# First-Order Term-Indexing

Alexander Maringele

January 27th, 2016

# References

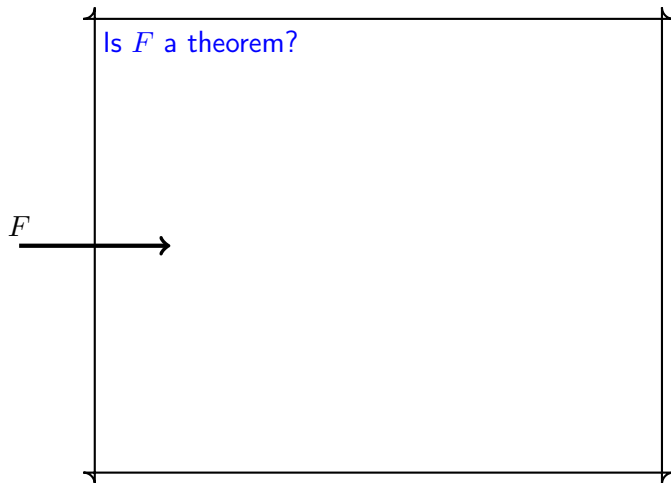


R. Sekar, I. V. Ramakrishnan, and Andrei Voronkov, *Term indexing*, Handbook of Automated Reasoning (Alan Robinson and Andrei Voronkov, eds.), Elsevier Science Publishers B. V., Amsterdam, The Netherlands, 2001, pp. 1853–1964.

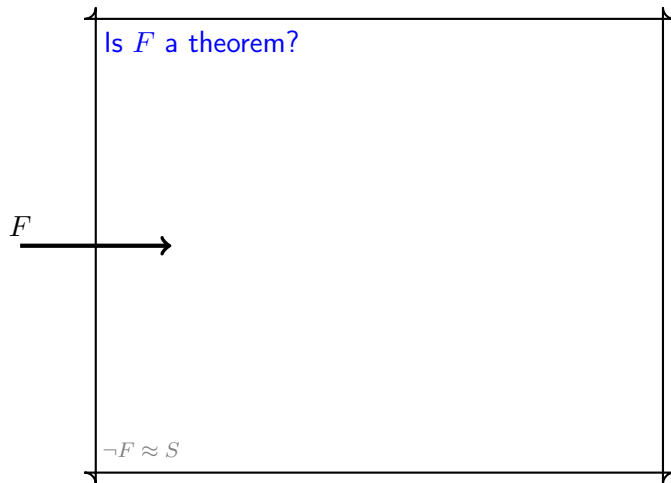
# Outline

- 1 Motivation
- 2 Term Structure
- 3 Path Indexing
- 4 Discrimination Trees
- 5 Substitution Trees
- 6 Search times

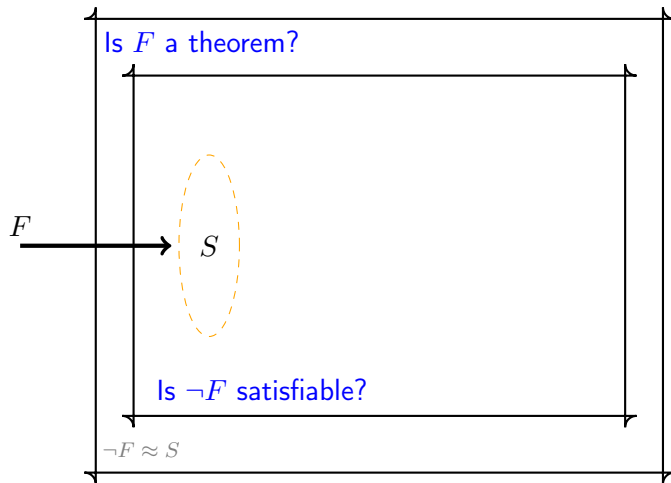
# Refutation



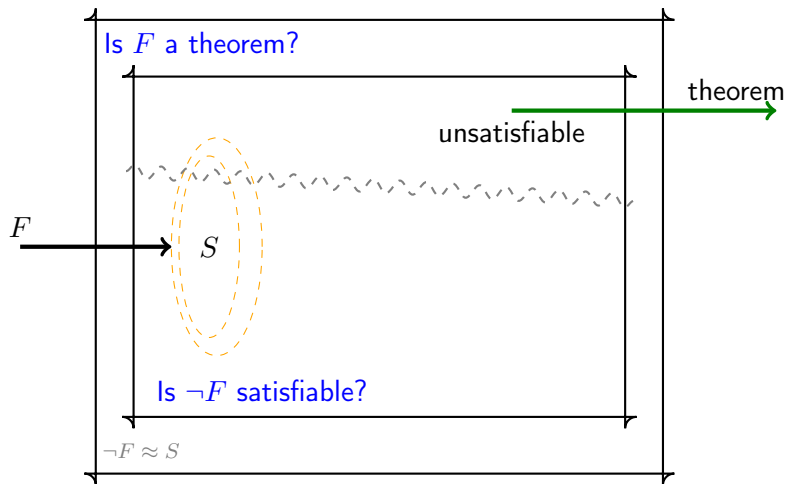
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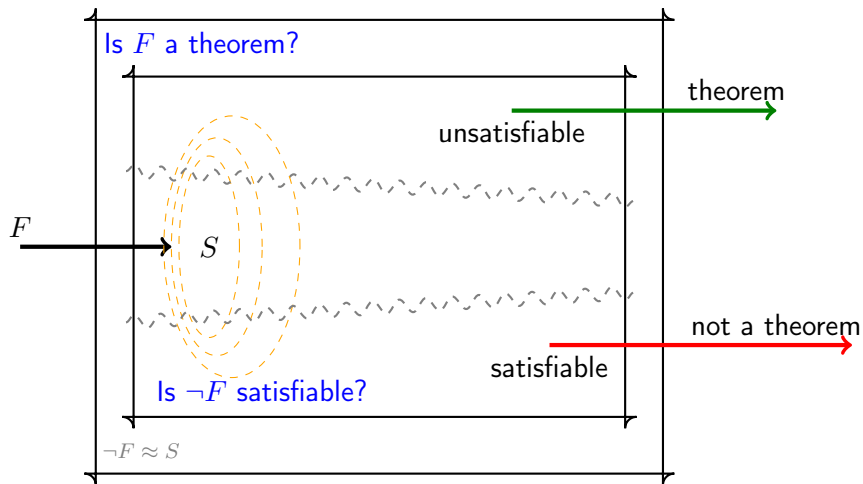
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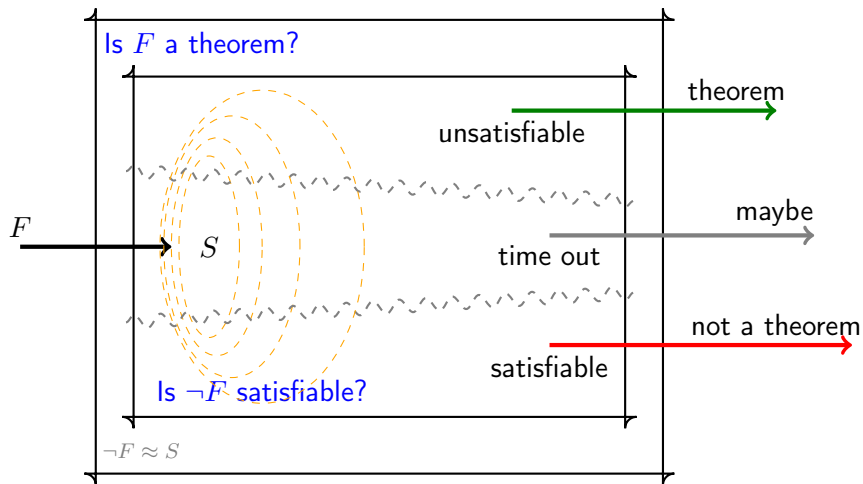


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## Clausal form

$$\{ P(f(x)) \vee f(x) \not\approx a, g(x, y) \approx a \vee \neg Q(x, y), \mathcal{C}_3 \}$$

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$$\equiv$$

$$\forall x (P(f(x)) \vee f(x) \not\approx a)$$

$$\wedge$$

$$\forall xy (g(x, y) \approx a \vee \neg Q(x, y))$$

$$\wedge$$

$$\forall \text{Var}(\mathcal{C}_3) (\mathcal{C}_3)$$

## Goal

A sound and refutation complete calculus.

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## Resolution (without equality)

Resolve and factor all clauses and literals in an unsatisfiable set

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$$\sigma = \text{mgu}(A, B)$$

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and the empty clause will be derived eventually.

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and the empty clause will be derived eventually.

## Observation

Usually the set grows too fast to obtain a result.



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  - e.g. discard clauses that are subsumed by other clauses

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- 1 *Reduce* search space
  - Ordered Resolution, Strategies, ...
  - ... with selection functions for clauses and literals
- 2 *Reduce* redundancy
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  - ... depending on the calculus

## Example (forward subsumption)

$$S = \{^1P(x, y), ^2\neg P(a, z)\} \cup \{^3P(a, z')\} \quad t_1 \text{ subsumes } t_3$$

$$\frac{P(x, y) \quad \neg P(a, z)}{\square} \{x \mapsto a, y \mapsto z\} \quad \text{Resolution}$$

$$S \perp = \{P(\perp, \perp), \neg P(a, \perp), P(a, \perp)\} \quad \text{InstGen / SMT}$$

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- *variants*

variant removal

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- *instances*

variant removal  
backward subsumption

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- *generalizations*

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of a query term in a given set of terms.

## Observation

Deduction rate drops quickly with sequential search.

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## Observation

Deduction rate drops quickly with sequential search.

## Term Indexing

Data structures and algorithms for fast retrieval of matching terms.

## Definition (Position Strings)

$$\mathcal{Pos}^{\Sigma}(t) = \left\{ \right.$$

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$$\mathcal{Pos}^{\Sigma}(h(f(a, y))) = \{\langle \epsilon, h \rangle, \langle 1, f \rangle, \langle 11, a \rangle, \quad \}$$

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## Term traversals

$$\mathcal{Pos}^{\Sigma}(h(f(a, y))) = \{\langle \epsilon, h \rangle, \langle 1, f \rangle, \langle 11, a \rangle, \langle 12, y \rangle\}$$

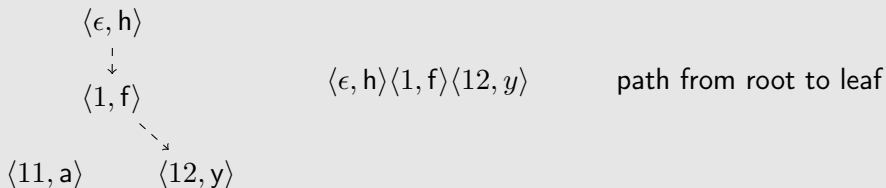
 $\langle \epsilon, h \rangle$ 
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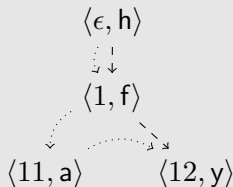


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$\langle \epsilon, h \rangle \langle 1, f \rangle \langle 12, y \rangle$  path from root to leaf  
 $\langle \epsilon, h \rangle \langle 1, f \rangle \langle 11, a \rangle \langle 12, y \rangle$  pre-order traversal

## Variables

Variants of terms generate the same position strings

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- if variable names are ignored

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In the first case even non-variants of terms generate the same strings.

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$h.1.f.2.*$

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- and traversal strings  $\langle \epsilon, h \rangle \langle 1, f \rangle \langle 11, * \rangle \langle 12, * \rangle$  h.f.a.\*

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- and traversal strings  $\langle \epsilon, h \rangle \langle 1, f \rangle \langle 11, * \rangle \langle 12, * \rangle$  h.f.a.\*  
when traversal order and arities of symbols are fixed.

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$t_1 \Rightarrow \{h.1.f.1.*, h.1.f.2.*\}$$

$$t_2 \Rightarrow \{h.1.f.1.*, h.1.f.2.a\}$$

$$t_3 \Rightarrow \{h.1.f.1.a, h.1.f.2a\}$$

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$$\begin{array}{c} \cdot \\ \vdots \\ h \\ \vdots \\ \cdot \end{array}$$

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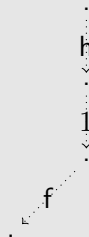
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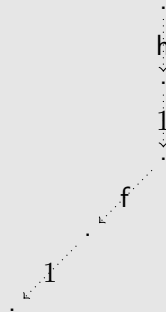
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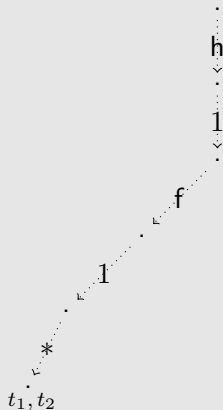
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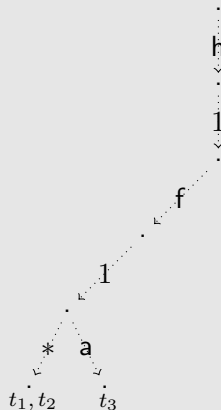
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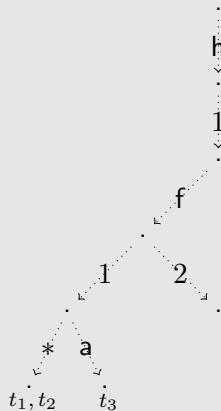
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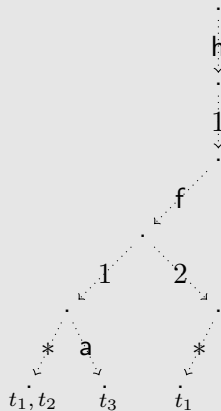
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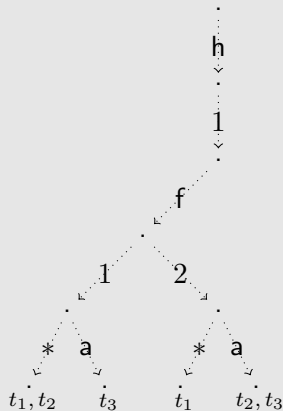
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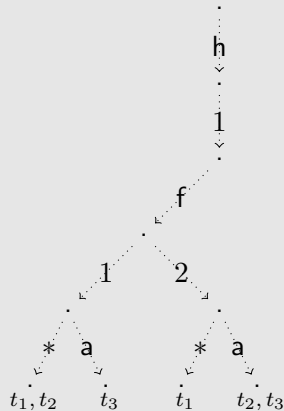


## Retrieve

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$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(\textcolor{red}{x}', \textcolor{green}{b})) \mapsto$$



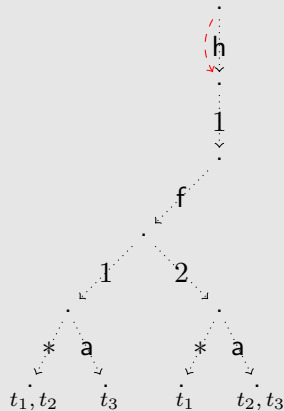


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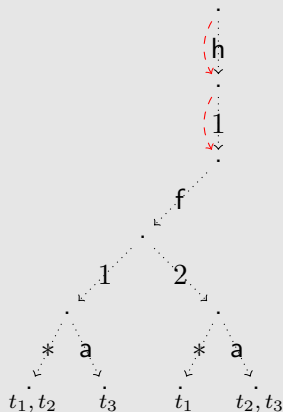


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$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(\textcolor{red}{x}', \textcolor{green}{b})) \mapsto$$

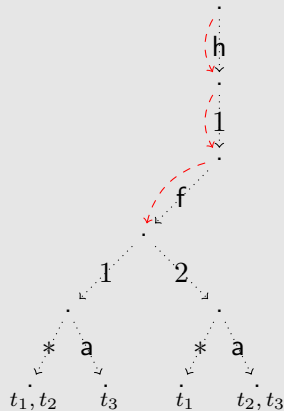


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto$$

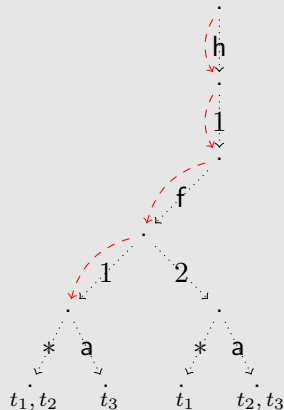


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b))) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto$$

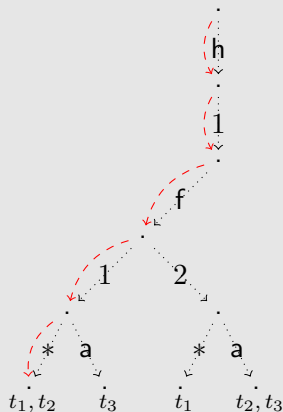


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, \}$$

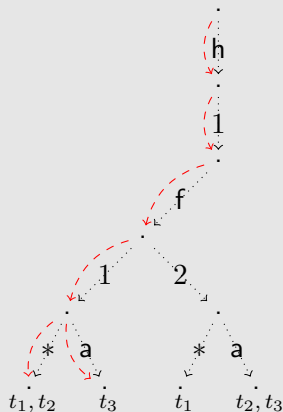


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\}$$

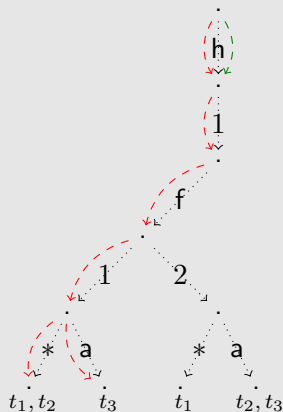


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\}$$

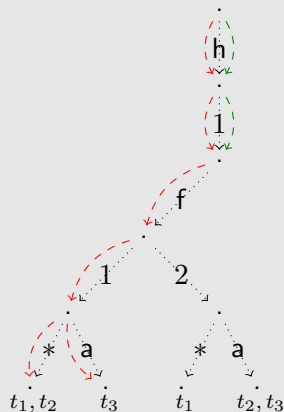


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\}$$



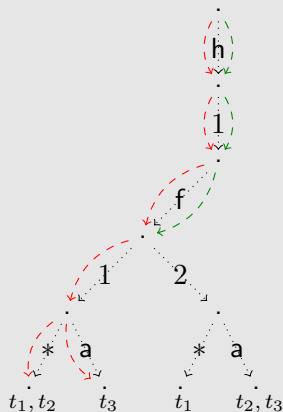


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\}$$

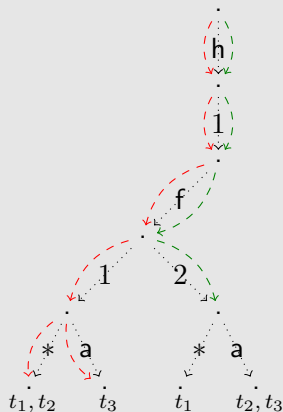


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\}$$

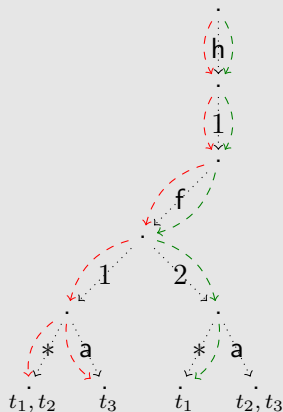


## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{t_1\}$$



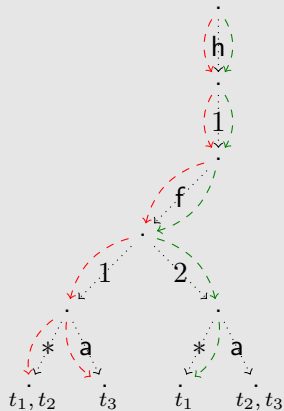
## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{t_1\}$$

$$i : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{\}$$



## Retrieve

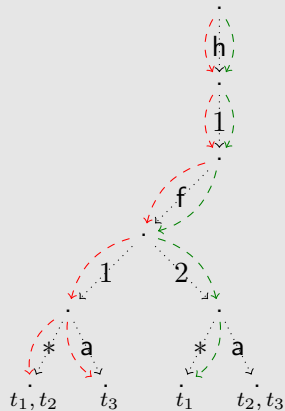
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{t_1\}$$

$$i : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{\}$$

$$g : h(f(x', b)) \mapsto \{t_1, t_2\} \cap \{t_1\}$$



## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

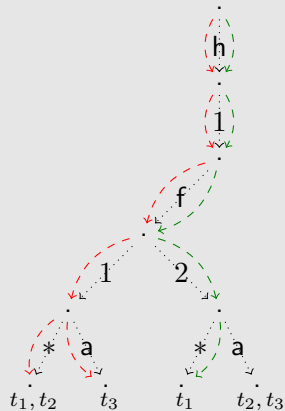
$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

$$u : h(f(\textcolor{red}{x}', \textcolor{green}{b})) \mapsto \{\textcolor{green}{t}_1, t_2, t_3\} \cap \{\textcolor{green}{t}_1\}$$

$$i : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{\}$$

$$g : h(f(x', b)) \mapsto \{\textcolor{green}{t}_1, t_2\} \cap \{\textcolor{green}{t}_1\}$$

$$v : h(f(x', b)) \mapsto \{t_1, t_2\} \cap \{\}$$



## Retrieve

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$

$$h(f(x, b)) \Rightarrow \{h.f.*, h.f.b\}$$

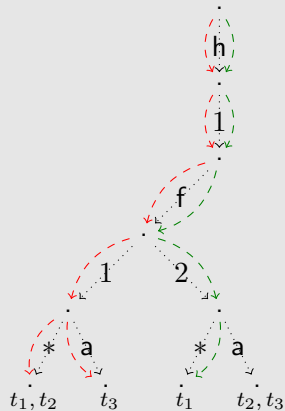
$$u : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{t_1\}$$

$$i : h(f(x', b)) \mapsto \{t_1, t_2, t_3\} \cap \{\}$$

$$g : h(f(x', b)) \mapsto \{t_1, t_2\} \cap \{t_1\}$$

$$v : h(f(x', b)) \mapsto \{t_1, t_2\} \cap \{\}$$

$$v : h(f(x', x')) \mapsto \{t_1, t_2\} \cap \{t_1\}$$



## Unit Superposition Inference Rules

$$\frac{s \approx t \quad L[s']}{(L[t]) \cdot \sigma} \quad \begin{array}{l} \text{unit} \\ \text{paramodulation} \end{array}$$

where  $\sigma = \text{mgu}(s, s')$ ,  $s' \notin \mathcal{V}$ ,  $t\sigma \neq s\sigma$

$$\frac{s \approx t \quad u[s'] \not\approx v}{(u[t] \not\approx v) \cdot \sigma} \quad \begin{array}{l} \text{unit} \\ \text{superposition} \end{array} \quad \frac{s \approx t \quad u[s'] \approx v}{(u[t] \approx v) \cdot \sigma}$$

where  $\sigma = \text{mgu}(s, s')$ ,  $s' \notin \mathcal{V}$ ,  $t\sigma \neq s\sigma$ ,  $v\sigma \neq u[s']\sigma$

$$\frac{s \not\approx t}{\square} \quad \begin{array}{l} \text{unit equality} \\ \text{resolution} \end{array}$$

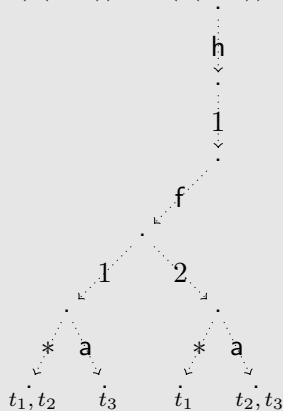
$$\frac{A \quad \neg B}{\square} \quad \begin{array}{l} \text{unit} \\ \text{resolution} \end{array}$$

where  $s$  and  $t$  ( $A$  and  $B$  respectively) are unifiable



$$^{t_4}f(x, a) \approx x$$

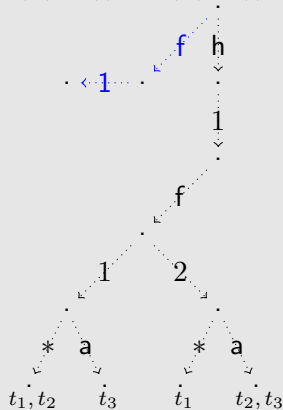
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$





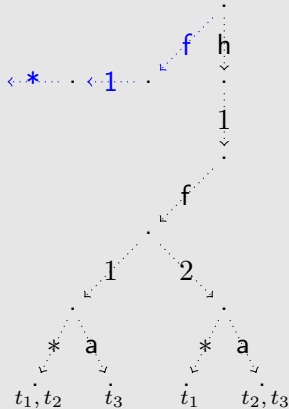
$$^{t_4}f(x, a) \approx x$$

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$



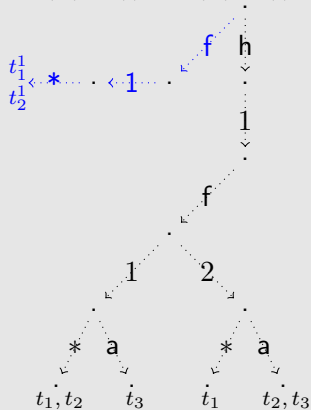
$$^{t_4}f(x, a) \approx x$$

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$



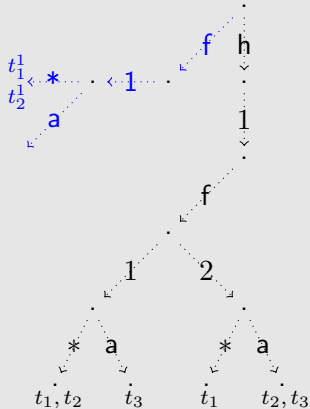
$${}^{t_4}f(x, a) \approx x$$

$${}^{t_1}h(f(x, y)), {}^{t_2}h(f(x, a)), {}^{t_3}h(f(a, a))$$



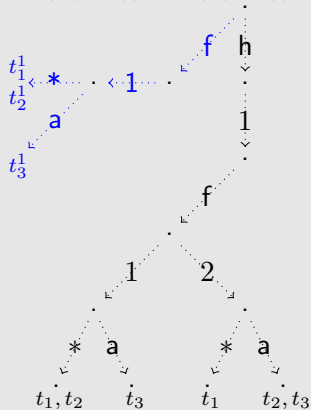
$${}^{t_4}f(x, a) \approx x$$

$${}^{t_1}h(f(x, y)), {}^{t_2}h(f(x, a)), {}^{t_3}h(f(a, a))$$



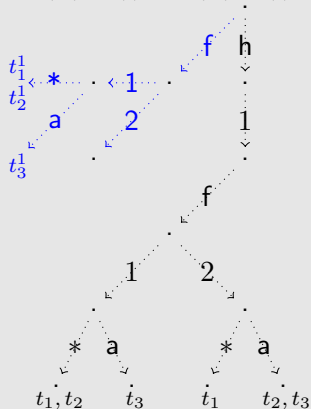
$$^{t_4}f(x, a) \approx x$$

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$



$$^{t_4}f(x, a) \approx x$$

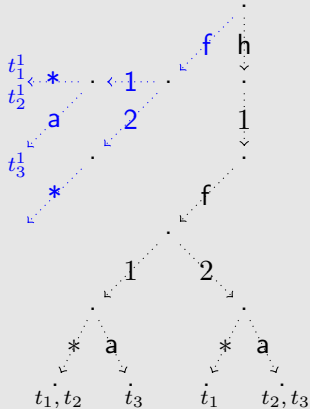
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$





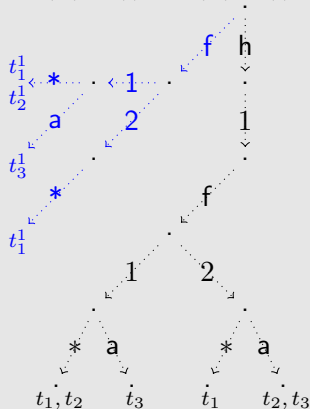
$$^{t_4}f(x, a) \approx x$$

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$



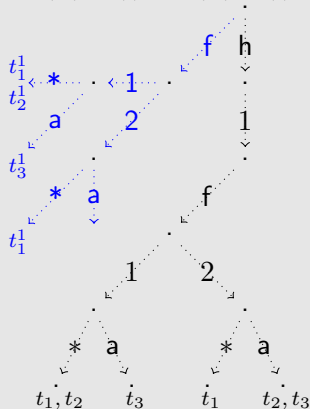
$$t_4: f(x, a) \approx x$$

$$t_1: h(f(x, y)), t_2: h(f(x, a)), t_3: h(f(a, a))$$



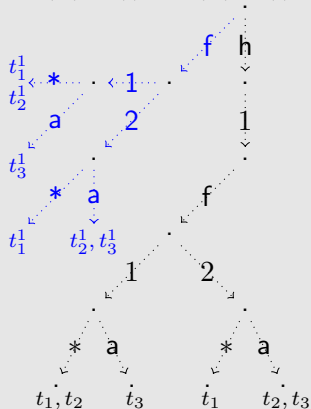
$$t_4: f(x, a) \approx x$$

$$t_1: h(f(x, y)), t_2: h(f(x, a)), t_3: h(f(a, a))$$



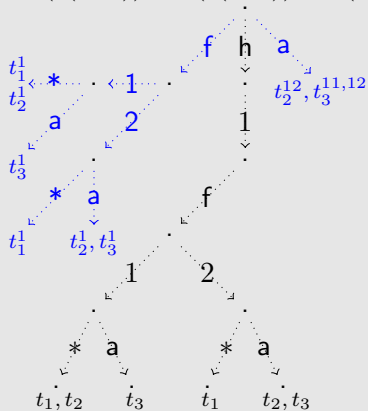
$${}^{t_4}f(x, a) \approx x$$

$${}^{t_1}h(f(x, y)), {}^{t_2}h(f(x, a)), {}^{t_3}h(f(a, a))$$



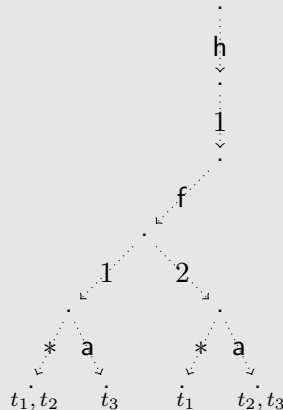
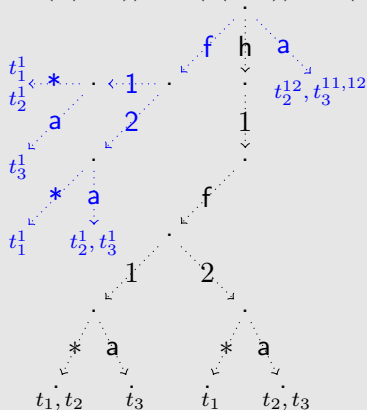
$${}^{t_4}f(x, a) \approx x$$

$${}^{t_1}h(f(x, y)), {}^{t_2}h(f(x, a)), {}^{t_3}h(f(a, a))$$



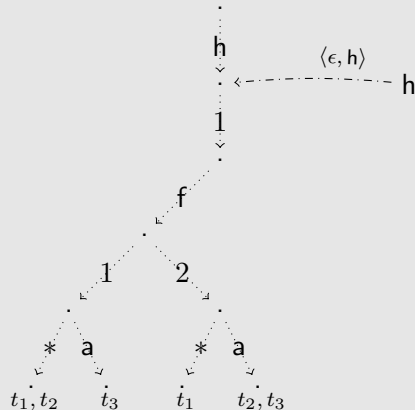
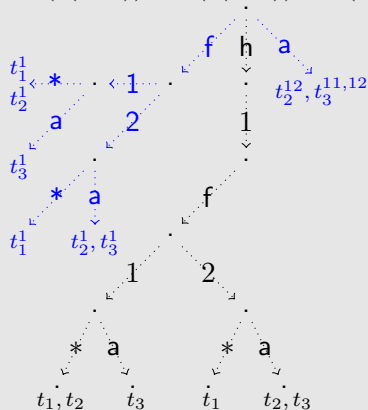
$$t_4: f(x, a) \approx x$$

$$t_1: h(f(x, y)), t_2: h(f(x, a)), t_3: h(f(a, a))$$

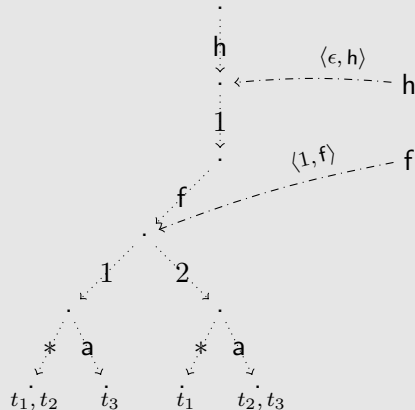


$$t_4: f(x, a) \approx x$$

$$t_1: h(f(x, y)), t_2: h(f(x, a)), t_3: h(f(a, a))$$



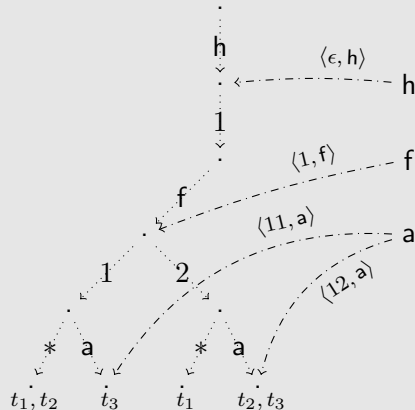
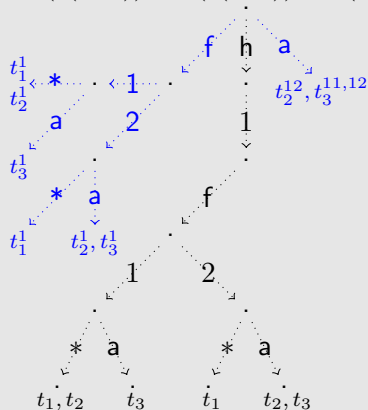
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, a)), ^{t_3}h(f(a, a))$$





$$t_4: f(x, a) \approx x$$

$$t_1: h(f(x, y)), t_2: h(f(x, a)), t_3: h(f(a, a))$$



## Insert

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$

$$t_1 \Rightarrow h.f.*.*$$

$$t_2 \Rightarrow h.f.*.h.a$$

$$t_3 \Rightarrow h.f.h.a.a$$

## Insert



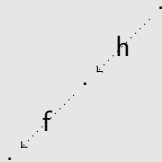
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$

$$t_1 \Rightarrow h.f.*.*$$

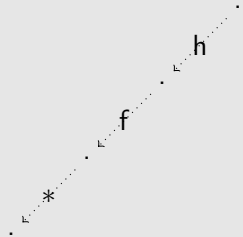
$$t_2 \Rightarrow h.f.*.h.a$$

$$t_3 \Rightarrow h.f.h.a.a$$

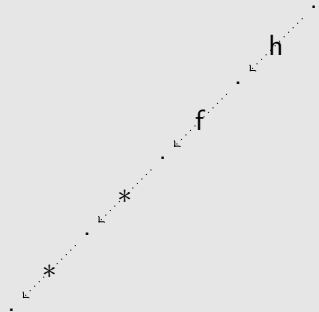
## Insert

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 


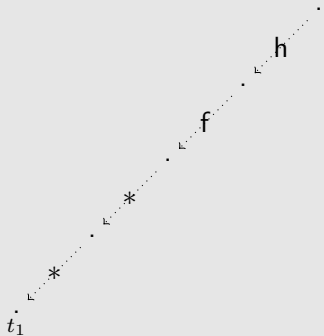
## Insert

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 


## Insert

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 


## Insert

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 


Insert

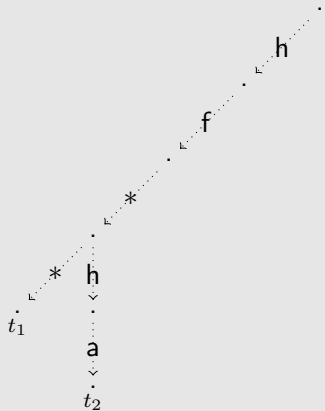
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$
$$t_1 \Rightarrow \text{h.f.*.*}$$
$$t_2 \Rightarrow \text{h.f.*.h.a}$$
$$t_3 \Rightarrow \text{h.f.h.a.a}$$



Insert

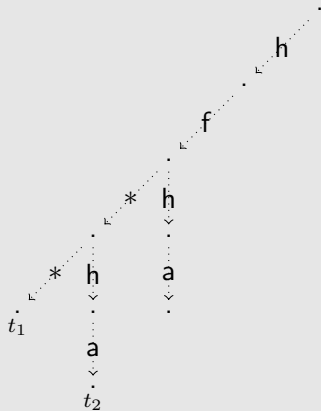
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$
$$t_1 \Rightarrow \text{h.f.*.*}$$
$$t_2 \Rightarrow \text{h.f.} * \text{h.a}$$
$$t_3 \Rightarrow \text{h.f.h.a.a}$$

## Insert

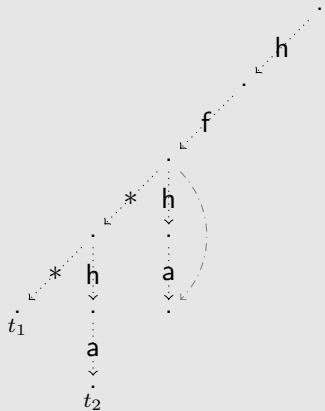
 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 




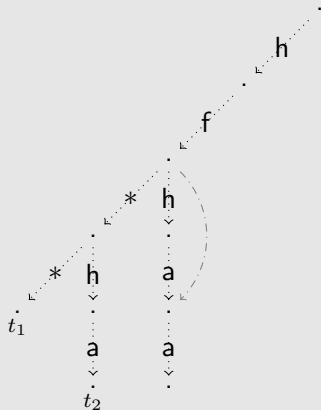
Insert

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$
$$t_1 \Rightarrow \text{h.f.*.*}$$
$$t_2 \Rightarrow \text{h.f.} * \text{h.a}$$
$$t_3 \Rightarrow \text{h.f.h.a.a}$$


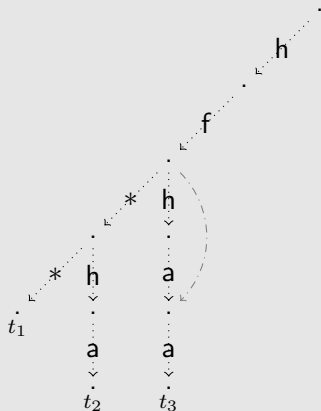
## Insert

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 


Insert

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$
$$t_1 \Rightarrow \text{h.f.*.*}$$
$$t_2 \Rightarrow \text{h.f.*.h.a}$$
$$t_3 \Rightarrow \text{h.f.h.a.a}$$


## Insert

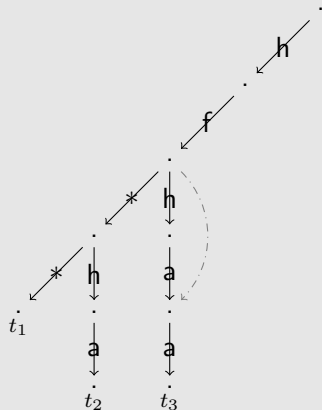
 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 
 $t_1 \Rightarrow h.f.*.*$ 
 $t_2 \Rightarrow h.f.*.h.a$ 
 $t_3 \Rightarrow h.f.h.a.a$ 


## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{ \quad \}$$



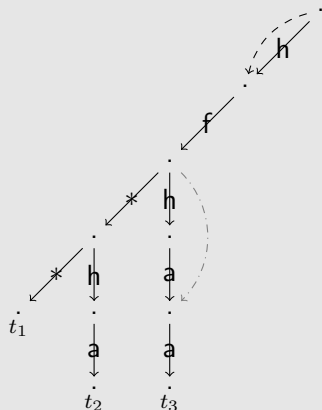


## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{ \quad \}$$

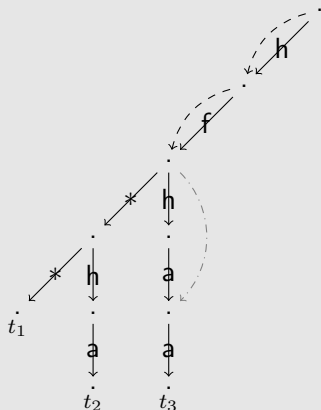


## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{ \quad \}$$

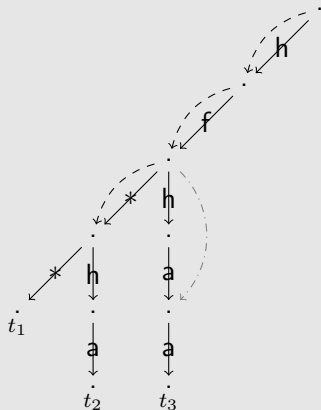


## Retrieve

$$^{t_1:}h(f(x, y)), ^{t_2:}h(f(x, h(a))), ^{t_3:}h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', \mathbf{a})) \mapsto \{ \quad \}$$

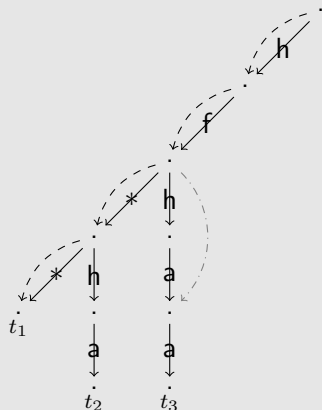


## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{t_1, \quad \}$$

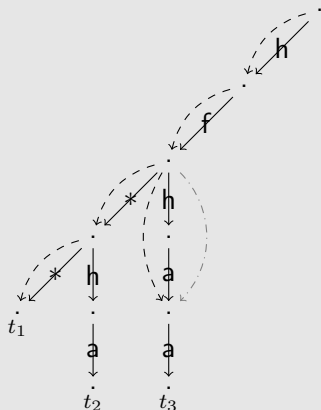


## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{t_1, \quad \}$$

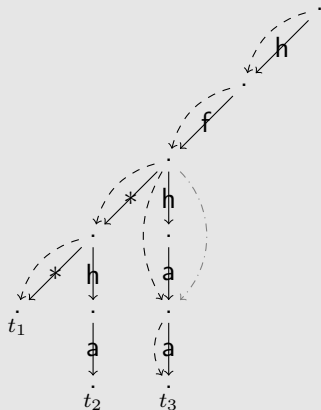


## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{t_1, t_3\}$$



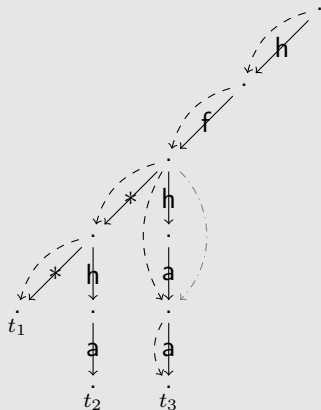
## Retrieve

$$t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', a)) \mapsto \{t_1, t_3\}$$

$$i : h(f(x', a)) \mapsto \{t_3\}$$



## Retrieve

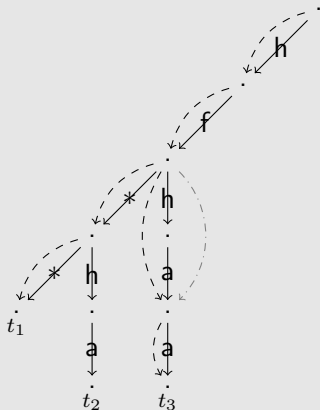
$$^{t_1:}h(f(x, y)), ^{t_2:}h(f(x, h(a))), ^{t_3:}h(f(h(a), a))$$

$$h(f(x', a)) \Rightarrow h.f.*.a$$

$$u : h(f(x', \mathbf{a})) \mapsto \{t_1, t_3\}$$

$$i : \mathbf{h}(\mathbf{f}(x', \mathbf{a})) \mapsto \{t_3\}$$

$$g : h(f(x', a)) \mapsto \{t_1\}$$





## Retrieve

$$^{t_1:}h(f(x, y)), ^{t_2:}h(f(x, h(a))), ^{t_3:}h(f(h(a), a))$$

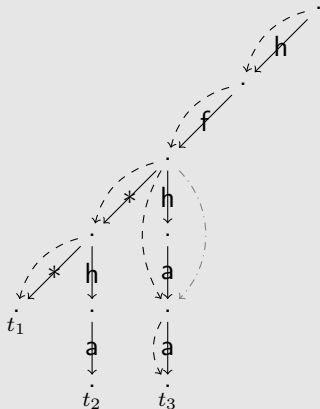
$$h(f(x', a)) \Rightarrow h.f.*.a$$

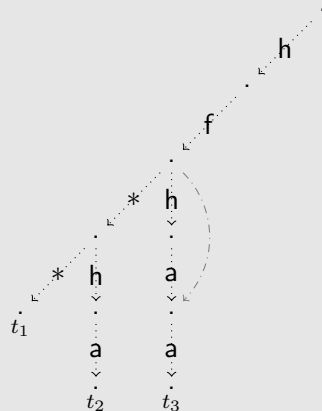
$$u : h(f(x', \mathbf{a})) \mapsto \{t_1, t_3\}$$

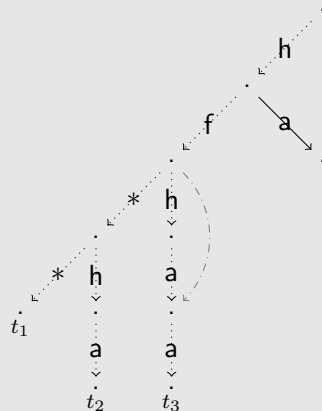
$$i : h(f(x', \mathbf{a})) \mapsto \{t_3\}$$

$$g : h(f(x', a)) \mapsto \{t_1\}$$

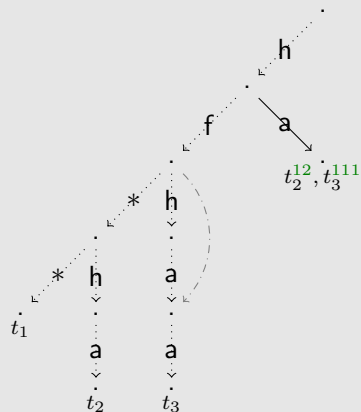
$$v : \mathbf{h}(\mathbf{f}(x', \mathbf{a})) \mapsto \{ \}$$



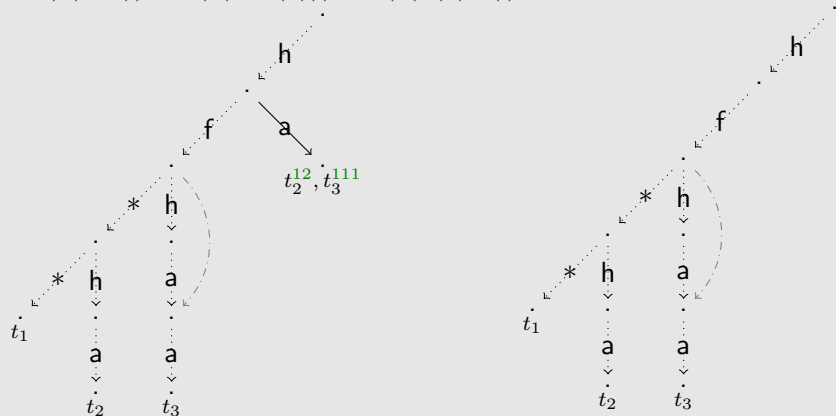
$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$


$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$


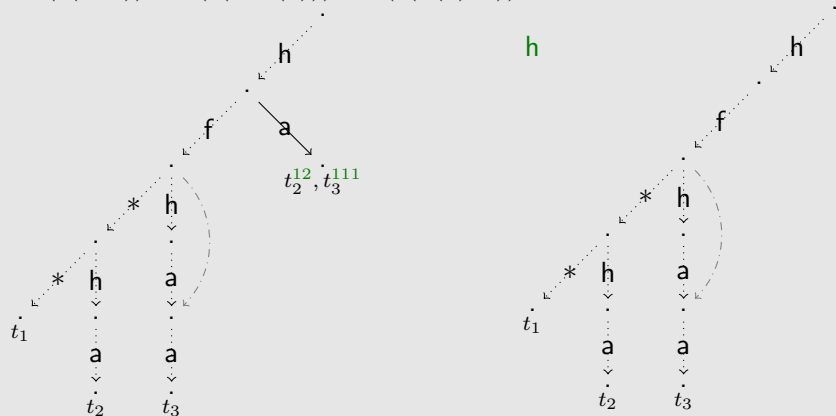
## Subterms

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 


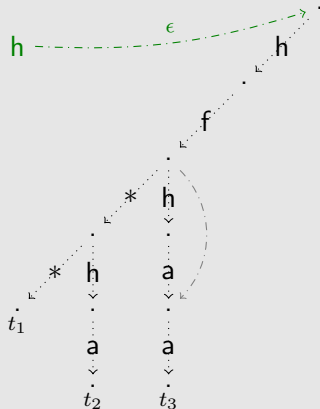
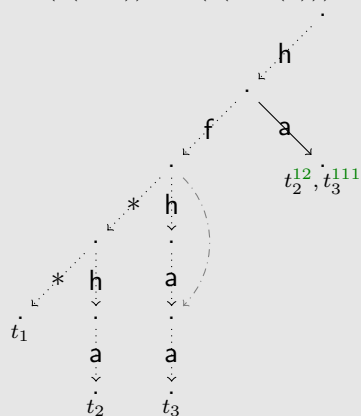
## Subterms

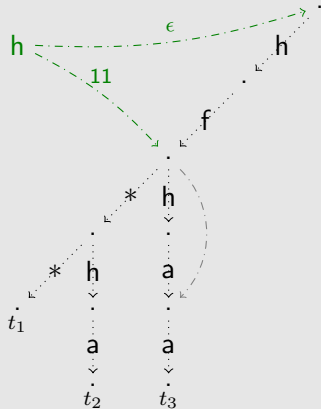
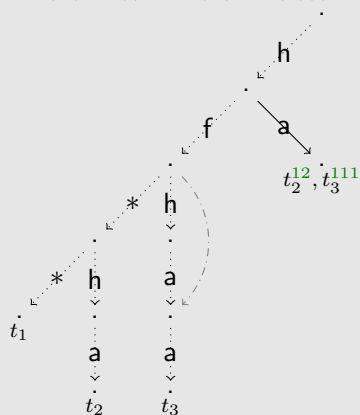
 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 


## Subterms

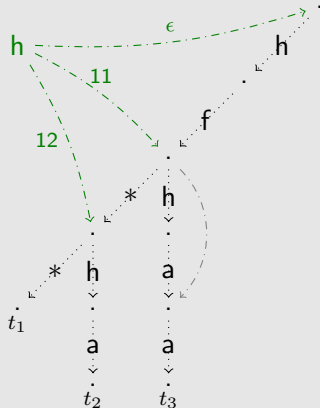
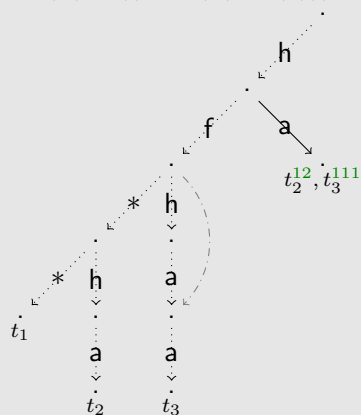
 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 


## Subterms

 $t_1: h(f(x, y)), t_2: h(f(x, h(a))), t_3: h(f(h(a), a))$ 


$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$




$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a))$$


## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\cdot$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

$$\swarrow$$

$$*_2 \mapsto x$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

$$\swarrow$$

$$*_2 \mapsto x$$

$$\swarrow$$

$$*_3 \mapsto y$$

$$t_1$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

$$\swarrow$$

$$*_2 \mapsto x$$

$$\swarrow$$

$$*_3 \mapsto y$$

$$t_1$$

$$\searrow$$

$$*_3 \mapsto h(a)$$

$$t_2$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

$$\swarrow \quad \searrow$$

$$*_2 \mapsto x \qquad *_3 \mapsto a$$

$$\swarrow \quad \searrow$$

$$*_3 \mapsto y \qquad *_3 \mapsto h(a)$$

$$t_1 \qquad t_2$$



## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

$$\swarrow \quad \searrow$$

$$*_2 \mapsto x \quad *_3 \mapsto a$$

$$\swarrow \quad \searrow \quad \swarrow$$

$$*_3 \mapsto y \quad *_3 \mapsto h(a) \quad *_2 \mapsto h(a)$$

$$t_1 \quad t_2 \quad t_3$$

## Build

$$^{t_1}h(f(x, y)), ^{t_2}h(f(x, h(a))), ^{t_3}h(f(h(a), a)), ^{t_4}h(f(a, a)))$$

$$\downarrow$$

$$*_0 \mapsto h(*_1)$$

$$\downarrow$$

$$*_1 \mapsto f(*_2, *_3)$$

$$\swarrow \quad \searrow$$

$$*_2 \mapsto x \quad *_3 \mapsto a$$

$$\begin{array}{cc} \swarrow & \searrow \\ *_3 \mapsto y & *_3 \mapsto h(a) \end{array} \quad \begin{array}{cc} \swarrow & \searrow \\ *_2 \mapsto h(a) & *_2 \mapsto a \end{array}$$

$$\begin{array}{cc} t_1 & t_2 \end{array} \quad \begin{array}{cc} t_3 & t_4 \end{array}$$

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals					sequential	index	speed
new	total	$(\ell_1, \ell_2)$	$A, \neg B$		search	search	up

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals		$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
new	total					
1 000	1 000	500 000	761	726ms	70ms	10

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals		$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
new	total					
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals		$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
new	total					
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals		$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
new	total					
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72



TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals		$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
new	total					
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals		$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
new	total					
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95
1 000	32 000	31 500 000	592	40s	489ms	82

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals new	total	$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95
1 000	32 000	31 500 000	592	40s	489ms	82
1 000	64 000	63 500 000	1167	80s	697ms	115

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals new	total	$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95
1 000	32 000	31 500 000	592	40s	489ms	82
1 000	64 000	63 500 000	1167	80s	697ms	115
1 000	128 000	127 500 000	1479	160s	13s	12

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals new	total	$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95
1 000	32 000	31 500 000	592	40s	489ms	82
1 000	64 000	63 500 000	1167	80s	697ms	115
1 000	128 000	127 500 000	1479	160s	13s	12
1 000	256 000	255 500 000	1097	320s	440s	1

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals new	total	$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95
1 000	32 000	31 500 000	592	40s	489ms	82
1 000	64 000	63 500 000	1167	80s	697ms	115
1 000	128 000	127 500 000	1479	160s	13s	12
1 000	256 000	255 500 000	1097	320s	440s	1
1 000	512 000	511 500 000	1440	640s	348s	2

TPTP/Problems/HWV/HWV134-1.p

2 332 428 formulae, 6 570 884 literals

literals new	total	$(\ell_1, \ell_2)$	$A, \neg B$	sequential search	index search	speed up
1 000	1 000	500 000	761	726ms	70ms	10
1 000	2 000	1 500 000	812	2s	69ms	29
1 000	4 000	3 500 000	723	4s	75ms	53
1 000	8 000	7 500 000	433	9s	125ms	72
1 000	16 000	15 500 000	742	21s	221ms	95
1 000	32 000	31 500 000	592	40s	489ms	82
1 000	64 000	63 500 000	1167	80s	697ms	115
1 000	128 000	127 500 000	1479	160s	13s	12
1 000	256 000	255 500 000	1097	320s	440s	1
1 000	512 000	511 500 000	1440	640s	348s	2
1 000	1 024 000	1023 500 000	1534	1280s	348s	4