Term-Indexing

Alexander Maringele

January 27th, 2016

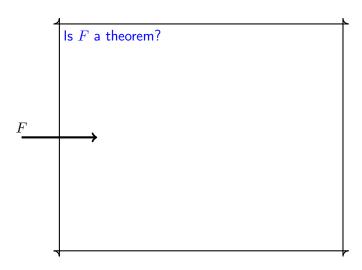
References

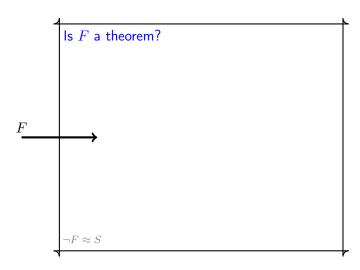


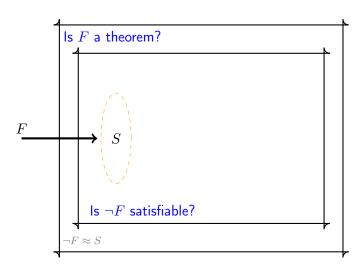
R. Sekar, I. V. Ramakrishnan, and Andrei Voronkov, *Term indexing*, Handbook of Automated Reasoning (Alan Robinson and Andrei Voronkov, eds.), Elsevier Science Publishers B. V., Amsterdam, The Netherlands, 2001, pp. 1853–1964.

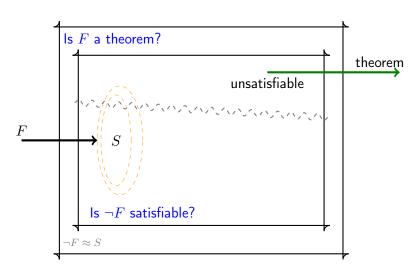
Outline

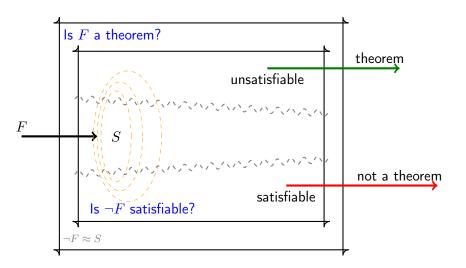
- Motivation
- 2 Term Structure
- 3 Path Indexing
- 4 Discrimination Trees
- 5 Substitution Trees
- 6 Experiment

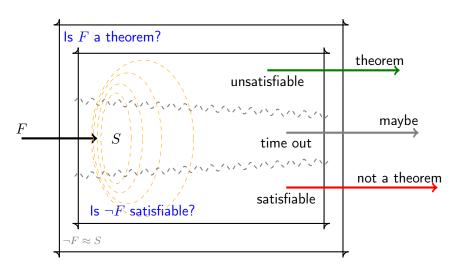












 $\{ \mathsf{P}(\mathsf{f}(x)) \lor \mathsf{f}(x) \not\approx \mathsf{a}, \; \mathsf{g}(x,y) \approx \mathsf{a} \lor \neg \mathsf{Q}(x,y), \; \mathcal{C}_3 \; \}$

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Clausal form

$$\left\{ \begin{array}{l} \mathsf{P}(\mathsf{f}(x)) \vee \mathsf{f}(x) \not\approx \mathsf{a}, \ \mathsf{g}(x,y) \approx \mathsf{a} \vee \neg \mathsf{Q}(x,y), \ \mathcal{C}_3 \end{array} \right\} \\ \equiv \\ \forall x \left(\mathsf{P}(\mathsf{f}(x)) \vee \mathsf{f}(x) \not\approx \mathsf{a} \right) \\ \wedge \\ \forall xy \left(\mathsf{g}(x,y) \approx \mathsf{a} \vee \neg \mathsf{Q}(x,y) \right) \\ \wedge \\ \forall \mathcal{V}\mathsf{ar}(\mathcal{C}_3) \left(\mathcal{C}_3 \right) \end{aligned}$$

A sound and refutation complete calculus.

A sound and refutation complete calculus.

Resolution (without equality)

Resolve and factor all clauses and literals in an unsatisfiable set

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A sound and refutation complete calculus.

Resolution (without equality)

Resolve and factor all clauses and literals in an unsatisfiable set

$$\frac{A \vee \mathcal{C} \quad \neg B \vee \mathcal{D}}{(\mathcal{C} \vee \mathcal{D})\sigma} \ (\sigma) \ \text{resolution} \qquad \frac{A \vee B \vee \mathcal{C}}{(A \vee \mathcal{C})\sigma} \ (\sigma) \ \text{factoring}$$

$$\sigma = \mathrm{mgu}(A, B)$$

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Resolution (without equality)

Resolve and factor all clauses and literals in an unsatisfiable set

$$\frac{A \vee \mathcal{C} \quad \neg B \vee \mathcal{D}}{(\mathcal{C} \vee \mathcal{D}) \sigma} \ (\sigma) \ \text{resolution} \qquad \frac{A \vee B \vee \mathcal{C}}{(A \vee \mathcal{C}) \sigma} \ (\sigma) \ \text{factoring}$$

$$\sigma = \mathrm{mgu}(A, B)$$

and the empty clause will be derived eventually.

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Resolution (without equality)

Resolve and factor all clauses and literals in an unsatisfiable set

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$$\sigma = \mathrm{mgu}(A, B)$$

and the empty clause will be derived eventually.

Observation

Usually the set grows too fast to obtain a result.

A sound, refutation complete, and

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A sound, refutation complete, and effective calculus.

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1 Reduce search space

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A sound, refutation complete, and effective calculus.

- 1 Reduce search space
 - Ordered Resolution, Strategies, ...

A sound, refutation complete, and effective calculus.

- 1 Reduce search space
 - Ordered Resolution, Strategies, ...
 - with selection functions for clauses and literals

A sound, refutation complete, and effective calculus.

- 1 Reduce search space
 - Ordered Resolution, Strategies, ...
 - with selection functions for clauses and literals
- 2 *Reduce* redundancy

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A sound, refutation complete, and *effective* calculus.

- 1 Reduce search space
 - Ordered Resolution, Strategies, ...
 - with selection functions for clauses and literals
- 2 *Reduce* redundancy
 - e.g. discard clauses that are subsumed by other clauses

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A sound, refutation complete, and effective calculus.

- 1 Reduce search space
 - Ordered Resolution, Strategies, ...
 - with selection functions for clauses and literals
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 - e.g. discard clauses that are subsumed by other clauses
 - ... depending on the calculus

A sound, refutation complete, and *effective* calculus.

- 1 Reduce search space
 - Ordered Resolution, Strategies, . . .
 - ... with selection functions for clauses and literals
- 2 Reduce redundancy
 - e.g. discard clauses that are subsumed by other clauses
 - ...depending on the calculus

Example (forward subsumption)

$$t_1$$
 subsumes t_3

$$\frac{\mathsf{P}(x,y) \quad \neg \mathsf{P}(\mathsf{a},z)}{\Box} \ \{x \mapsto \mathsf{a}, y \mapsto z\}$$

Resolution

$$S \perp = \{ \mathsf{P}(\perp, \perp), \neg \mathsf{P}(\mathsf{a}, \perp), \mathsf{P}(\mathsf{a}, \perp) \}$$

InstGen / SMT

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3 Quickly find

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- 3 Quickly find
 - variants

variant removal

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- 3 Quickly find
 - variants
 - instances

variant removal backward subsumption

A sound, refutation complete, and effective calculus.

- 3 Quickly find
 - variants
 - instances
 - generalizations

variant removal backward subsumption forward subsumption

A sound, refutation complete, and effective calculus.

- 3 Quickly find
 - variants
 - instances
 - generalizations
 - unifiable terms

variant removal backward subsumption forward subsumption resolution, demodulation

A sound, refutation complete, and effective calculus.

- 3 Quickly find
 - variants
 - instances
 - generalizations
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of a query term in a given set of terms.

variant removal backward subsumption forward subsumption resolution, demodulation

A sound, refutation complete, and effective calculus.

- 3 Quickly find
 - variants
 - instances
 - generalizations
 - unifiable terms

of a query term in a given set of terms.

variant removal backward subsumption forward subsumption resolution, demodulation

Observation

Deduction rate drops quickly with sequential search.

A sound, refutation complete, and effective calculus.

- 3 Quickly find
 - variants
 - instances
 - generalizations
 - unifiable terms

of a query term in a given set of terms.

variant removal backward subsumption forward subsumption resolution, demodulation

Observation

Deduction rate drops quickly with sequential search.

Term Indexing

Data structures and algorithms for fast retrieval of matching terms.

$$\mathcal{P}\mathsf{os}^\Sigma(t) = \bigg\{$$

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$$\mathcal{P} \mathsf{os}^\Sigma(t) = \left\{ \left\{ \left\langle \epsilon, x \right\rangle \right\} \right.$$

if $t = x \in \mathcal{V}$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

$$\mathcal{P}os^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{$$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

$$\mathcal{P}os^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{\langle \epsilon,\mathsf{h} \rangle, \}$$

 $\langle \epsilon, \mathsf{h} \rangle$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

$$\mathcal{P}os^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{\langle \epsilon,\mathsf{h} \rangle, \langle 1,\mathsf{f} \rangle, \}$$

 $\langle 1, \mathsf{f} \rangle$

 $\langle \epsilon, \mathsf{h} \rangle$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

 $\langle \epsilon, \mathsf{h} \rangle$

$$\mathcal{P}os^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{\langle \epsilon,\mathsf{h} \rangle, \langle 1,\mathsf{f} \rangle, \langle 11,\mathsf{a} \rangle, \}$$

 $\langle 1, f \rangle$

 $\langle 11, \mathsf{a} \rangle$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

$$\mathcal{P}$$
os $^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{\langle \epsilon,\mathsf{h} \rangle, \langle 1,\mathsf{f} \rangle, \langle 11,\mathsf{a} \rangle, \langle 12,y \rangle \}$

 $\langle 11, \mathsf{a} \rangle \qquad \langle 12, y \rangle$

 $\langle \epsilon, \mathsf{h} \rangle$

 $\langle 1, \mathsf{f} \rangle$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle\} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle\} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i)\} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

$$\mathcal{P} \mathsf{os}^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{\langle \epsilon, \mathsf{h} \rangle, \langle 1, \mathsf{f} \rangle, \langle 11, \mathsf{a} \rangle, \langle 12, y \rangle \}$$

$$\langle \epsilon, \mathsf{h} \rangle$$

$$\langle 1, \mathsf{f} \rangle$$

$$\langle \epsilon, \mathsf{h} \rangle \langle 1, \mathsf{f} \rangle \langle 12, y \rangle$$
 path from root to leaf
$$\langle 11, \mathsf{a} \rangle$$

$$\langle 12, y \rangle$$

$$\mathcal{P} \mathsf{os}^\Sigma(t) = \begin{cases} \{\langle \epsilon, x \rangle \} & \text{if } t = x \in \mathcal{V} \\ \{\langle \epsilon, f \rangle \} \cup \{\langle ip, s \rangle \mid \langle p, s \rangle \in \mathcal{P} \mathsf{os}^\Sigma(t_i) \} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

Term traversals

$$\mathcal{P} os^{\Sigma}(\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{y}))) = \{\langle \epsilon,\mathsf{h} \rangle, \langle 1,\mathsf{f} \rangle, \langle 11,\mathsf{a} \rangle, \langle 12,y \rangle\}$$

$$\langle \epsilon,\mathsf{h} \rangle$$

$$\langle 1,\mathsf{f} \rangle$$

$$\langle \epsilon,\mathsf{h} \rangle \langle 1,\mathsf{f} \rangle \langle 12,y \rangle$$
 path from root to leaf
$$\langle \epsilon,\mathsf{h} \rangle \langle 1,\mathsf{f} \rangle \langle 11,\mathsf{a} \rangle \langle 12,y \rangle$$
 pre-order traversal
$$\langle 11,\mathsf{a} \rangle$$

$$\langle 12,y \rangle$$

Variants of terms generate the same position strings

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Variants of terms generate the same position strings

• if variable names are ignored

$$\mathsf{f}(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, * \rangle \langle 2, * \rangle$$

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Variables

Variants of terms generate the same position strings

• if variable names are ignored

 $f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, * \rangle \langle 2, * \rangle$

or normalized

 $f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle$

Variables

Variants of terms generate the same position strings

- if variable names are ignored
- or normalized

$$\mathsf{f}(y,z) \Rightarrow \langle \epsilon,\mathsf{f} \rangle \langle 1,* \rangle \langle 2,* \rangle$$

$$\begin{array}{l} \mathsf{f}(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle \\ \mathsf{f}(y,y) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_1 \rangle \end{array}$$

Variants of terms generate the same position strings

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$$f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, * \rangle \langle 2, * \rangle$$

or normalized

$$\begin{array}{l} \mathsf{f}(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle \\ \mathsf{f}(y,y) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_1 \rangle \end{array}$$

In the first case even non-variants of terms generate the same strings.

Variables

Variants of terms generate the same position strings

if variable names are ignored

$$f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, * \rangle \langle 2, * \rangle$$

or normalized

$$f(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle$$

$$f(y,y) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_1 \rangle$$

In the first case even non-variants of terms generate the same strings.

Notation

We abbreviate

Variants of terms generate the same position strings

• if variable names are ignored

$$f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, * \rangle \langle 2, * \rangle$$

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$$f(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle$$

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In the first case even non-variants of terms generate the same strings.

Notation

We abbreviate

• path strings $\langle \epsilon, \mathsf{h} \rangle \langle 1, \mathsf{f} \rangle \langle 12, * \rangle$

h.1.f.2.*

Variables

Variants of terms generate the same position strings

• if variable names are ignored

$$f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, * \rangle \langle 2, * \rangle$$

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$$f(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle$$

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Notation

We abbreviate

• path strings $\langle \epsilon, \mathsf{h} \rangle \langle 1, \mathsf{f} \rangle \langle 12, * \rangle$

h.1.f.2.*

• and traversal strings $\langle \epsilon, h \rangle \langle 1, f \rangle \langle 11, * \rangle \langle 12, * \rangle$

h.f.a.*

Variants of terms generate the same position strings

• if variable names are ignored

$$f(y,z) \Rightarrow \langle \epsilon, f \rangle \langle 1, * \rangle \langle 2, * \rangle$$

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$$f(y,z) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_2 \rangle$$

$$f(y,y) \Rightarrow \langle \epsilon, \mathsf{f} \rangle \langle 1, x_1 \rangle \langle 2, x_1 \rangle$$

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Notation

We abbreviate

• path strings $\langle \epsilon, \mathsf{h} \rangle \langle 1, \mathsf{f} \rangle \langle 12, * \rangle$

h.1.f.2.*

• and traversal strings $\langle \epsilon, \mathsf{h} \rangle \langle 1, \mathsf{f} \rangle \langle 11, * \rangle \langle 12, * \rangle$ when traversal order and arities of symbols are fixed.

h.f.a.*

$$\begin{split} t_1 & \mapsto \mathsf{h}(\mathsf{f}(x,y)), {}^{t_2 :} \mathsf{h}(\mathsf{f}(x,\mathsf{a})), {}^{t_3 :} \mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ & t_1 \Rightarrow \mathsf{h}.1.\mathsf{f}.1.*, \mathsf{h}.1.\mathsf{f}.2.* \} \\ & t_2 \Rightarrow \mathsf{h}.1.\mathsf{f}.1.*, \mathsf{h}.1.\mathsf{f}.2.\mathsf{a} \} \\ & t_3 \Rightarrow \mathsf{h}.1.\mathsf{f}.1.\mathsf{a}, \mathsf{h}.1.\mathsf{f}.2\mathsf{a} \} \end{split}$$

h

$$\begin{split} t_1 : & \mathsf{h}(\mathsf{f}(x,y)), ^{t_2 :} \mathsf{h}(\mathsf{f}(x,\mathsf{a})), ^{t_3 :} \mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ & t_1 \Rightarrow \{\mathsf{h}.1.\mathsf{f}.1.*, \mathsf{h}.1.\mathsf{f}.2.*\} \\ & t_2 \Rightarrow \{\mathsf{h}.1.\mathsf{f}.1.*, \mathsf{h}.1.\mathsf{f}.2.\mathsf{a}\} \\ & t_3 \Rightarrow \{\mathsf{h}.1.\mathsf{f}.1.\mathsf{a}, \mathsf{h}.1.\mathsf{f}.2\mathsf{a}\} \end{split}$$

$$^{t_1:}\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$t_1 \Rightarrow \{\text{h.1.f.1.*}, \text{h.1.f.2.*}\}$$

 $t_2 \Rightarrow \{\text{h.1.f.1.*}, \text{h.1.f.2.a}\}$

$$t_3 \Rightarrow \{\text{h.1.f.1.a}, \text{h.1.f.2a}\}$$

$$t_1 \Rightarrow \{\text{h.1.f.1.*}, \text{h.1.f.2.*}\}$$

$$t_2 \Rightarrow \{\text{h.1.f.1.*}, \text{h.1.f.2.a}\}$$

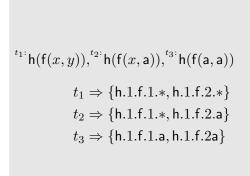
 $t_3 \Rightarrow \{\text{h.1.f.1.a}, \text{h.1.f.2a}\}$

h 1 .

 $t_1 = h(f(x,y)), t_2 = h(f(x,a)), t_3 = h(f(a,a))$ $t_1 \Rightarrow \{h.1.f.1.*, h.1.f.2.*\}$ $t_2 \Rightarrow \{h.1.f.1.*, h.1.f.2.a\}$ $t_3 \Rightarrow \{h.1.f.1.a, h.1.f.2a\}$

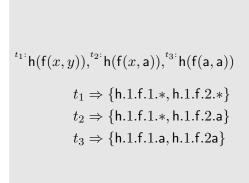






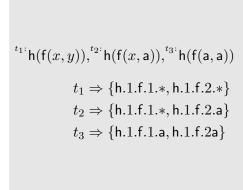






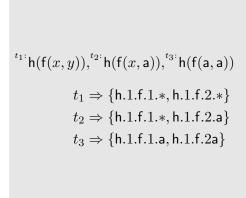






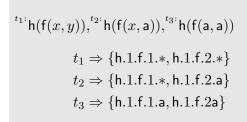


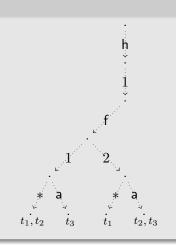








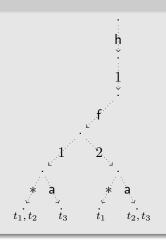




Retrieve

$$\begin{split} {}^{t_1:}\!\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{t_3:}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ \\ \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h.f.*},\mathsf{h.f.b}\} \end{split}$$

$$u:\mathsf{h}(\mathsf{f}(\pmb{z},\mathsf{b}))\mapsto$$

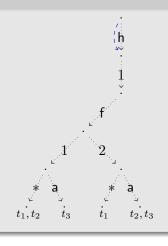


Retrieve

$$^{^{t_1}\!:}\!\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\!:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\!:}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$\mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h.f.*},\mathsf{h.f.b}\}$$

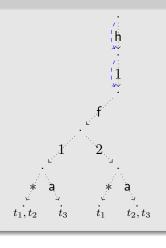
$$u: \mathsf{h}(\mathsf{f}(\boldsymbol{z},\mathsf{b})) \mapsto$$



$$^{t_1:}\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$$

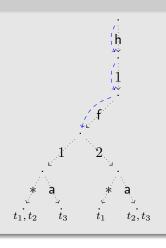
$$u: \mathsf{h}(\mathsf{f}(\boldsymbol{z},\mathsf{b})) \mapsto$$



$$^{^{t_1}\!:}\!\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\!:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\!:}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

 $h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$

$$u: h(f(z,b)) \mapsto$$

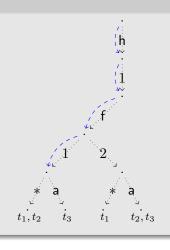


Retrieve

$$^{^{t_1}\!:}\!\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\!:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\!:}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

 $h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$

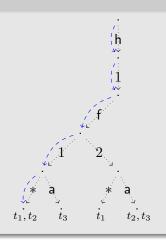
$$u: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto$$



Retrieve

$$\begin{split} {}^{t_1:}\!\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{t_3:}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ \\ \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h.f.*},\mathsf{h.f.b}\} \end{split}$$

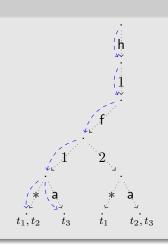
$$u: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2, \}$$



$$^{^{t_1}\cdot}\!\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\cdot}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\cdot}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$$

$$u:\mathsf{h}(\mathsf{f}(\pmb{z},\mathsf{b}))\mapsto\{t_1,t_2,t_3\}$$



$$^{^{t_1}\!\cdot\!}\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\!\cdot\!}\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\!\cdot\!}\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$$

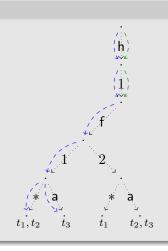
$$u:\mathsf{h}(\mathsf{f}(\pmb{z},\mathsf{b}))\mapsto\{t_1,t_2,t_3\}$$



$$^{^{t_1}\cdot}\!\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\cdot}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\cdot}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$\mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h.f.*},\mathsf{h.f.b}\}$$

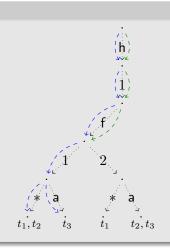
$$u:\mathsf{h}(\mathsf{f}(\pmb{z},\mathsf{b}))\mapsto\{t_1,t_2,t_3\}$$



$$^{^{t_1}\cdot}\!\mathsf{h}(\mathsf{f}(x,y)),^{^{t_2}\cdot}\!\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{^{t_3}\cdot}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$$

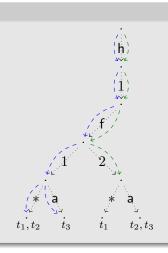
$$u:\mathsf{h}(\mathsf{f}(\pmb{z},\mathsf{b}))\mapsto\{t_1,t_2,t_3\}$$



$$^{^{t_1} \cdot} \mathsf{h}(\mathsf{f}(x,y)), ^{^{t_2} \cdot} \mathsf{h}(\mathsf{f}(x,\mathsf{a})), ^{^{t_3} \cdot} \mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))$$

$$h(f(z,b))) \Rightarrow \{h.f.*, h.f.b\}$$

$$u:\mathsf{h}(\mathsf{f}(\pmb{z},\mathsf{b}))\mapsto\{t_1,t_2,t_3\}$$



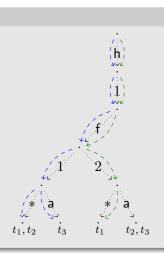
Retrieve

$$\begin{array}{c} ^{t_1} \mathsf{h}(\mathsf{f}(x,y)), ^{t_2} \mathsf{h}(\mathsf{f}(x,\mathsf{a})), ^{t_3} \mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ \\ \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h}.\mathsf{f}.*,\mathsf{h}.\mathsf{f}.\mathsf{b}\} \\ \\ u: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{t_1\} \end{array}$$

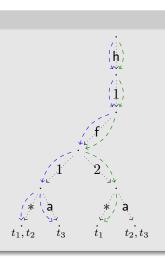
$$\begin{array}{c} {}^{t_1} \mathsf{h}(\mathsf{f}(x,y)), {}^{t_2} \mathsf{h}(\mathsf{f}(x,\mathsf{a})), {}^{t_3} \mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ \\ \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h}.\mathsf{f}.*,\mathsf{h}.\mathsf{f}.\mathsf{b}\} \\ \\ u: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{t_1\} \\ \\ i: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{\} \end{array}$$

Retrieve

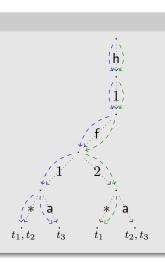
$$\begin{split} & \quad \quad \text{t_1'} \mathsf{h}(\mathsf{f}(x,y)), \\ & \quad \quad \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h}.\mathsf{f}.*,\mathsf{h}.\mathsf{f}.\mathsf{b}\} \\ & \quad \quad u : \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{t_1\} \\ & \quad \quad i : \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{\} \\ & \quad \quad g : \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2\} \cap \{t_1\} \end{split}$$



$$\begin{split} ^{t_1:}\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{a})),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ & \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h}.\mathsf{f}.*,\mathsf{h}.\mathsf{f}.\mathsf{b}\} \\ & u:\mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{t_1\} \\ & i:\mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{\} \\ & g:\mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2\} \cap \{t_1\} \\ & v:\mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2\} \cap \{\} \end{split}$$



$$\begin{split} ^{t_1:} \mathsf{h}(\mathsf{f}(x,y)), ^{t_2:} \mathsf{h}(\mathsf{f}(x,\mathsf{a})), ^{t_3:} \mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})) \\ & \mathsf{h}(\mathsf{f}(z,\mathsf{b}))) \Rightarrow \{\mathsf{h}.\mathsf{f}.*,\mathsf{h}.\mathsf{f}.\mathsf{b}\} \\ & u: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{t_1\} \\ & i: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2,t_3\} \cap \{\} \\ & g: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2\} \cap \{t_1\} \\ & v: \mathsf{h}(\mathsf{f}(z,\mathsf{b})) \mapsto \{t_1,t_2\} \cap \{\} \\ & v: \mathsf{h}(\mathsf{f}(z,z)) \mapsto \{t_1,t_2\} \cap \{t_1\} \end{split}$$



Unit Superposition Inference Rules

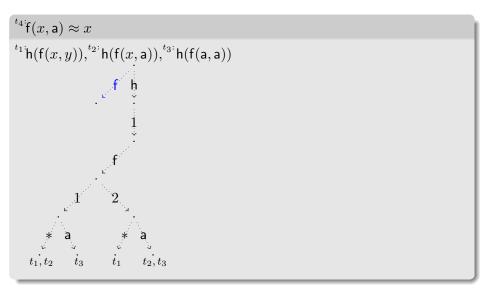
$$\frac{s \approx t \quad L[s']}{(L[t]) \cdot \sigma} \quad \underset{\text{paramodulation}}{\text{unit}}$$

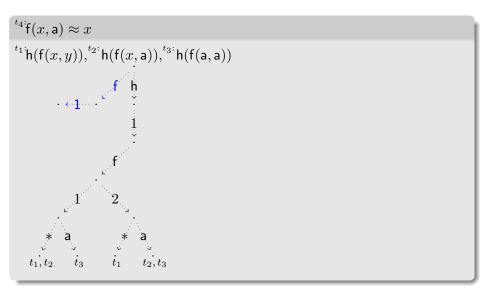
where $\sigma = \text{mgu}(s, s'), s' \notin \mathcal{V}, t\sigma \not\succeq s\sigma$

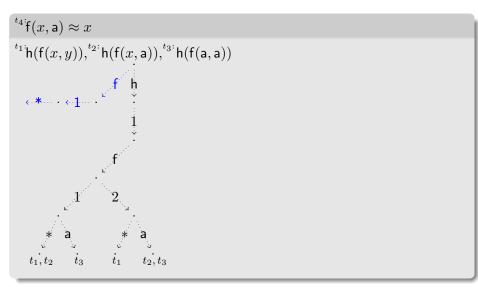
$$\frac{s \approx t \quad u[s'] \not\approx v}{(u[t] \not\approx v) \cdot \sigma} \text{ } \underset{\text{superposition}}{\text{unit}} \quad \frac{s \approx t \quad u[s'] \approx v}{(u[t] \approx v) \cdot \sigma}$$

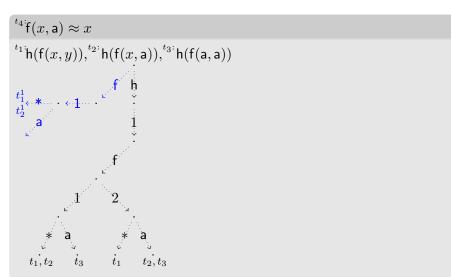
where $\sigma = \text{mgu}(s, s'), s' \notin \mathcal{V}, t\sigma \not\succeq s\sigma, v\sigma \not\succeq u[s']\sigma$

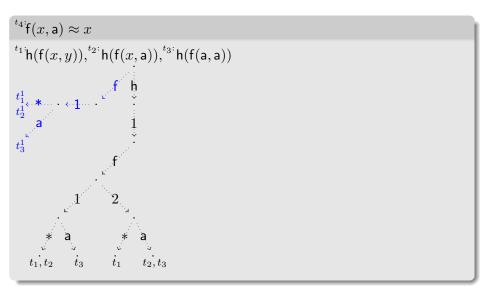
where s and t (A and B respectively) are unifiable

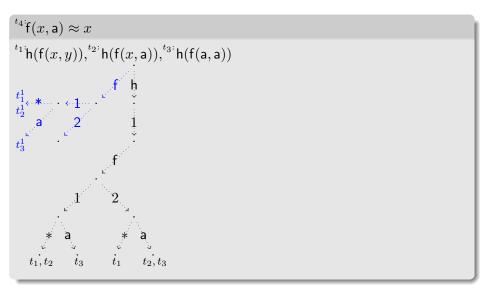


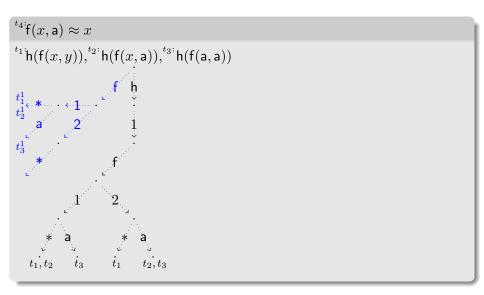


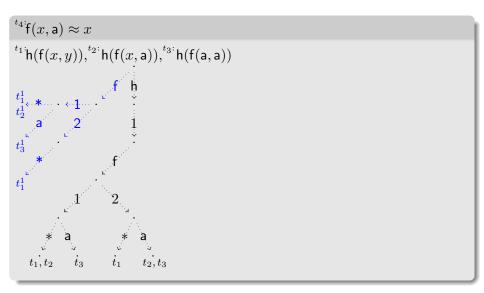


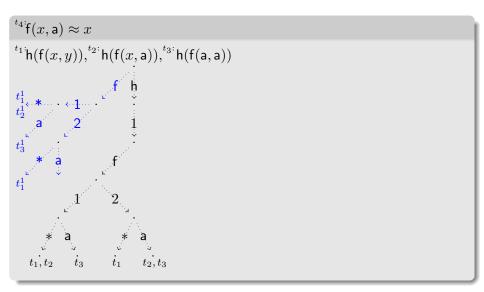


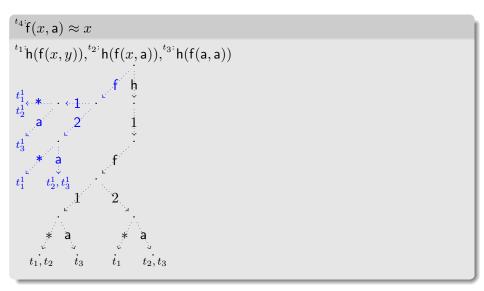


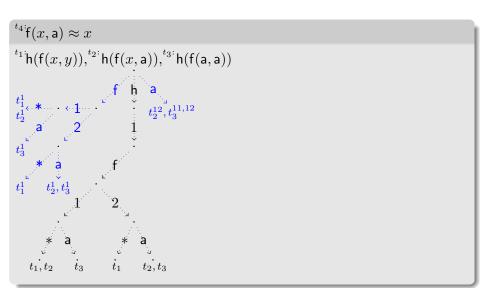


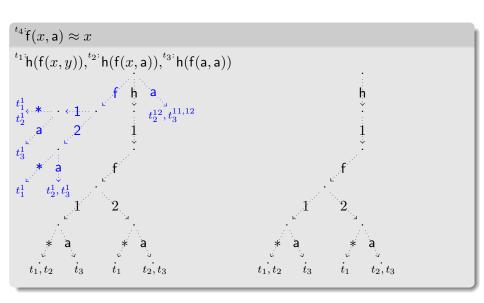


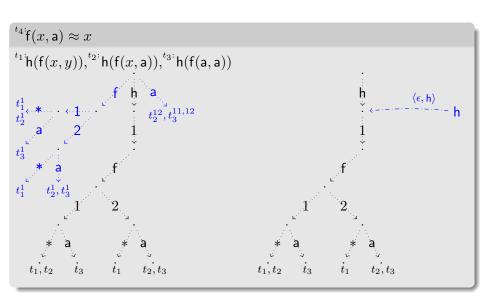


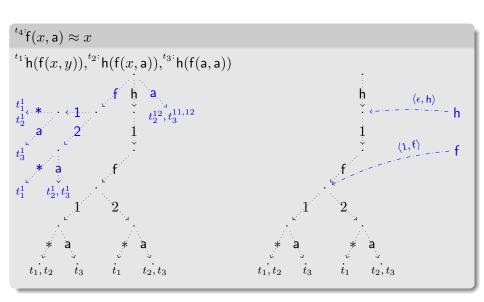












```
{}^{t_1}:h(f(x,y)), {}^{t_2}:h(f(x,h(a))), {}^{t_3}:h(f(h(a),a))
                          t_1 \Rightarrow \mathsf{h.f.}*.*
```

 $t_2 \Rightarrow \mathsf{h.f.*.h.a}$

 t_1 : $h(f(x,y)), {}^{t_2}$: $h(f(x,h(a))), {}^{t_3}$:h(f(h(a),a))

 $t_1 \Rightarrow \mathsf{h.f.}*.*$

 $t_2 \Rightarrow \mathsf{h.f.*.h.a}$

 t_1 h(f(x,y)), t_2 h(f(x,h(a))), t_3 h(f(h(a),a))

 $t_1 \Rightarrow \mathsf{h.f.}*.*$

 $t_2 \Rightarrow \mathsf{h.f.*.h.a}$

$$t_1$$
: $\mathsf{h}(\mathsf{f}(x,y)), t_2$: $\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))), t_3$: $\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a}))$ $t_1 \Rightarrow \mathsf{h.f.}*.*$ $t_2 \Rightarrow \mathsf{h.f.}*.\mathsf{h.a}$ $t_3 \Rightarrow \mathsf{h.f.}*.\mathsf{h.a}$



 t_1 h(f(x,y)), t_2 h(f(x,h(a))), t_3 h(f(h(a),a)) $t_1 \Rightarrow \mathsf{h.f.}*.*$ $t_2 \Rightarrow \mathsf{h.f.*.h.a}$

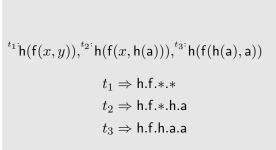
 t_1 h(f(x,y)), t_2 h(f(x,h(a))), t_3 h(f(h(a),a)) $t_1 \Rightarrow \mathsf{h.f.}*.*$ $t_2 \Rightarrow \mathsf{h.f.*.h.a}$ $t_3 \Rightarrow h.f.h.a.a$

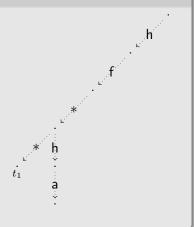


$$^{t_1:}$$
h(f(x,y)), $^{t_2:}$ h(f($x,$ h(a))), $^{t_3:}$ h(f(h(a), a))
 $t_1 \Rightarrow \text{h.f.}*.*$
 $t_2 \Rightarrow \text{h.f.}*.\text{h.a}$
 $t_3 \Rightarrow \text{h.f.}*.\text{h.a}$

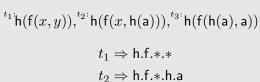






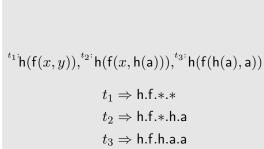


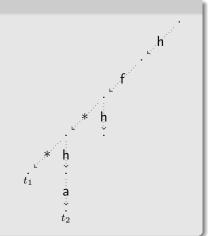




Build



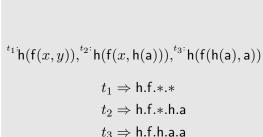




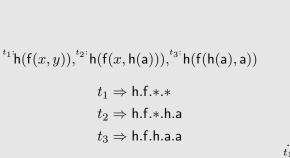
Build

ā



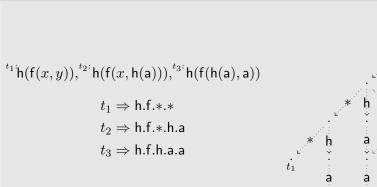






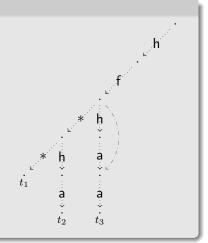






Insert

 t_1 : $h(f(x,y)), {}^{t_2}$: $h(f(x,h(a))), {}^{t_3}$:h(f(h(a),a)) $t_1 \Rightarrow \text{h.f.}*.*$ $t_2 \Rightarrow h.f.*.h.a$ $t_3 \Rightarrow h.f.h.a.a$



$$\begin{array}{c} ^{t_1:}\mathsf{h}(\mathsf{f}(x,y)), ^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))), ^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})) \\ \\ \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h.f.*.a} \\ \\ u: \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{ \qquad \} \end{array}$$



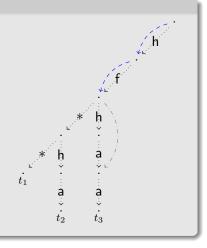
$$\mathsf{h}(\mathsf{f}(x,y)),^{t_2} \mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_3} \mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a}))$$

$$\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h}.\mathsf{f}.*.\mathsf{a}$$

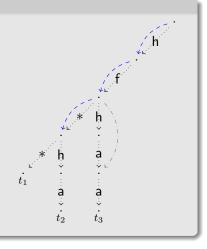
$$u : \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{\qquad \}$$



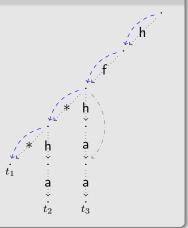
$$\begin{split} ^{t_1:}\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})) \\ & \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h.f.*.a} \\ & u:\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{ \qquad \} \end{split}$$



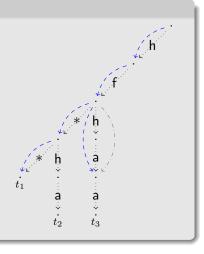
$$\begin{split} ^{t_1:}\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})) \\ & \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h.f.*.a} \\ & u:\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{ \qquad \} \end{split}$$



t_1
h $(f(x,y))$, t_2 h $(f(x,h(a)))$, t_3 h $(f(h(a),a))$
 $h(f(x',a)) \Rightarrow h.f.*.a$
 $u: h(f(x',a)) \mapsto \{t_1, \dots\}$



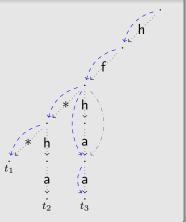
 t_1 : $h(f(x,y)), {}^{t_2}$: $h(f(x,h(a))), {}^{t_3}$: h(f(h(a),a)) $h(f(x',a)) \Rightarrow h.f.*.a$ $u: \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_1, \}$



$$\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a}))$$

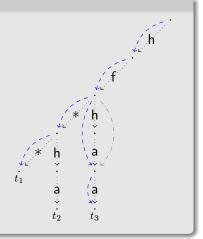
$$\mathsf{h}(\mathsf{f}(x',\mathsf{a}))\Rightarrow \mathsf{h}.\mathsf{f}.*.\mathsf{a}$$

$$u:\mathsf{h}(\mathsf{f}(x',\mathsf{a}))\mapsto \{t_1,t_3\}$$



$$\begin{split} {}^{t_1:}\!\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_3:}\!\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})) \\ & \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h.f.*.a} \\ & u: \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_1,t_3\} \end{split}$$

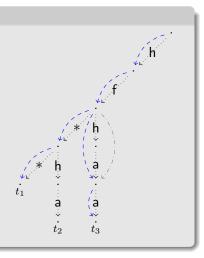
 $i: h(f(x', a)) \mapsto \{t_3\}$



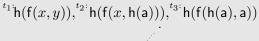
$$\begin{array}{c} {}^{t_1} \mathsf{h}(\mathsf{f}(x,y)), {}^{t_2:} \mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))), {}^{t_3:} \mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})) \\ \\ \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h.f.*.a} \\ \\ u: \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_1,t_3\} \\ \\ i: \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_3\} \\ \\ g: \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_1\} \end{array}$$

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$$\begin{split} ^{t_1:}\mathsf{h}(\mathsf{f}(x,y)),^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})) \\ & \mathsf{h}(\mathsf{f}(x',\mathsf{a})) \Rightarrow \mathsf{h.f.*.a} \\ \\ u:\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_1,t_3\} \\ \\ i:\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_3\} \\ \\ g:\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{t_1\} \\ \\ v:\mathsf{h}(\mathsf{f}(x',\mathsf{a})) \mapsto \{\ \} \end{split}$$



Subterms





Alexander Maringele

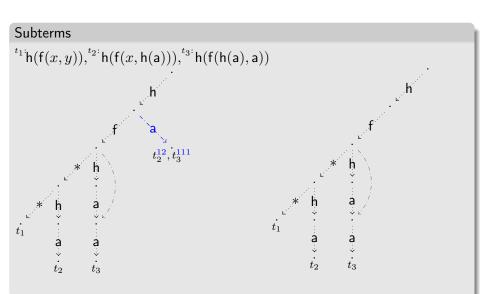
Subterms

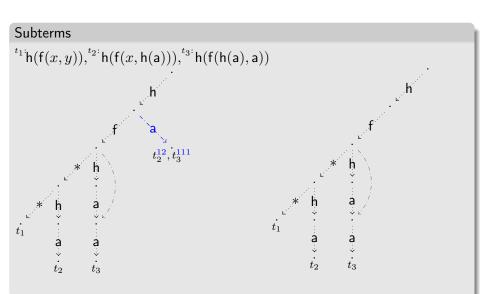
 ${}^{t_1}\dot{h}(f(x,y)), {}^{t_2}\dot{h}(f(x,h(a))), {}^{t_3}\dot{h}(f(h(a),a))$

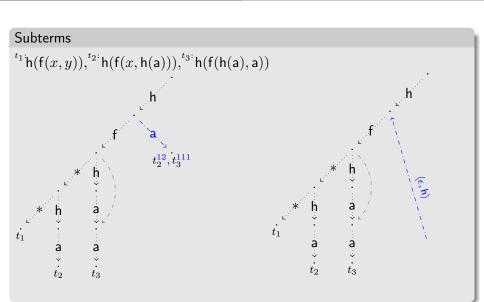


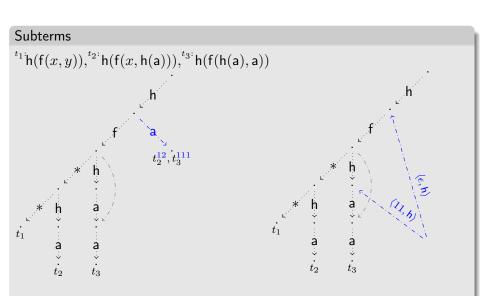


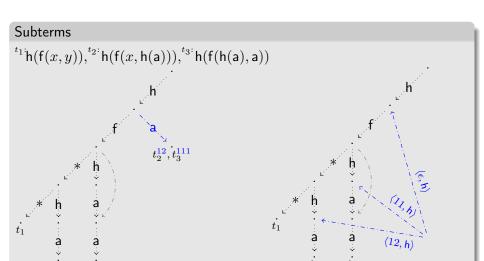
 ${}^{t_1}\dot{h}(f(x,y)), {}^{t_2}\dot{h}(f(x,h(a))), {}^{t_3}\dot{h}(f(h(a),a))$ t_2^{12}, t_2^{111}











Build

 t_1 : $h(f(x,y)), {}^{t_2}$: $h(f(x,h(a))), {}^{t_3}$: $h(f(h(a),a)), {}^{t_4}$:h(f(a,a)))

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Build

$$\overset{^{t_1}:}{\mathsf{h}}(\mathsf{f}(x,y)),\overset{^{t_2}:}{\overset{\mathsf{h}}{\mathsf{h}}}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),\overset{^{t_3}:}{\overset{\mathsf{h}}{\mathsf{h}}}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})),\overset{^{t_4}:}{\overset{\mathsf{h}}{\mathsf{h}}}(\mathsf{f}(\mathsf{a},\mathsf{a}))) \\
\overset{\cdot}{\downarrow} \\
*_0 \mapsto \mathsf{h}(*_1)$$

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Build

$$\begin{array}{c} {}^{t_1} \dot{\mathsf{h}}(\mathsf{f}(x,y)), {}^{t_2} \dot{\mathsf{h}}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))), {}^{t_3} \dot{\mathsf{h}}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})), {}^{t_4} \dot{\mathsf{h}}(\mathsf{f}(\mathsf{a},\mathsf{a}))) \\ \downarrow \\ *_0 \mapsto \mathsf{h}(*_1) \\ \downarrow \\ *_1 \mapsto \mathsf{f}(*_2,*_3) \end{array}$$

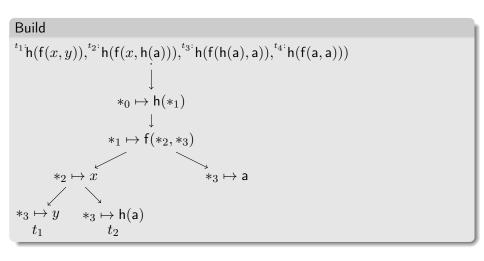
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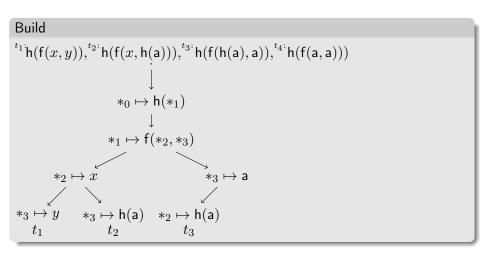
Build

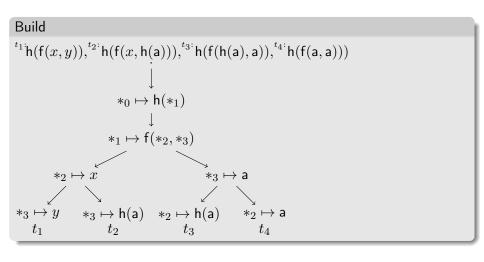
$$\begin{array}{c} ^{t_1:}\mathsf{h}(\mathsf{f}(x,y)), ^{t_2:}\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))), ^{t_3:}\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})), ^{t_4:}\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a}))) \\ \downarrow \\ *_0 \mapsto \mathsf{h}(*_1) \\ \downarrow \\ *_1 \mapsto \mathsf{f}(*_2,*_3) \\ & \\ *_2 \mapsto x \end{array}$$

Build t_1 : $h(f(x,y)), {}^{t_2}$: $h(f(x,h(a))), {}^{t_3}$: $h(f(h(a),a)), {}^{t_4}$:h(f(a,a))) $*_0 \mapsto \mathsf{h}(*_1)$ $*_1 \mapsto f(*_2, *_3)$ $*_2 \mapsto x$

Build $^{t_{1}:}\!\mathsf{h}(\mathsf{f}(x,y)),^{t_{2}:}\!\mathsf{h}(\mathsf{f}(x,\mathsf{h}(\mathsf{a}))),^{t_{3}:}\!\mathsf{h}(\mathsf{f}(\mathsf{h}(\mathsf{a}),\mathsf{a})),^{t_{4}:}\!\mathsf{h}(\mathsf{f}(\mathsf{a},\mathsf{a})))$ $*_0 \mapsto \mathsf{h}(*_1)$ $*_1 \mapsto f(*_2, *_3)$ $*_2 \mapsto x$ $*_3 \mapsto y \qquad *_3 \mapsto \mathsf{h}(\mathsf{a})$







checking 1000 new literals sequential path speed afterwards (ℓ_1,ℓ_2) $A, \neg B$ search index up

checking 1000 new literals			sequential	path	speed
afterwards	(ℓ_1,ℓ_2)	$A, \neg B$	search	index	up
1 000	500 000	761	726ms	70ms	10

checking afterwards	1000 new liter (ℓ_1, ℓ_2)		sequential search	path index	speed up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29

checking 1000 new literals			sequential	path	speed	
	afterwards	(ℓ_1,ℓ_2)	$A, \neg B$	search	index	up
	1 000	500 000	761	726ms	70ms	10
	2 000	1 500 000	812	2s	69ms	29
	4 000	3 500 000	723	4s	75ms	53

checking afterwards	1000 new liter (ℓ_1, ℓ_2)	rals $A, \neg B$	sequential search	path index	speed up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	720111s 2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

checking afterwards	1000 new lite (ℓ_1, ℓ_2)	rals $A, \neg B$	sequential search	path index	speed up
arterwards	(ϵ_1,ϵ_2)	A, D	Scarcii	IIIUEA	uр
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72
16 000	15 500 000	742	21s	221ms	95

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

	1000 new lite		sequential	path	speed
afterwards	(ℓ_1,ℓ_2)	$A, \neg B$	search	index	up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72
16 000	15 500 000	742	21s	221ms	95
32 000	31 500 000	592	40s	489ms	82

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

checking afterwards	1000 new lite (ℓ_1,ℓ_2)	rals $A, \neg B$	sequential search	path index	speed up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72
16 000	15 500 000	742	21s	221ms	95
32 000	31 500 000	592	40s	489ms	82
64 000	63 500 000	1 167	80s	697ms	115

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

g 1000 new lite	rals	sequential	path	speed
(ℓ_1,ℓ_2)	$A, \neg B$	search	index	up
500 000	761	726ms	70ms	10
1 500 000	812	2s	69ms	29
3 500 000	723	4s	75ms	53
7 500 000	433	9s	125ms	72
15 500 000	742	21s	221ms	95
31 500 000	592	40s	489ms	82
63 500 000	1 167	80s	697ms	115
127 500 000	1 479	160s	13s	12
	$\begin{array}{c} (\ell_1,\ell_2) \\ 500\ 000 \\ 1\ 500\ 000 \\ 3\ 500\ 000 \\ 7\ 500\ 000 \\ 15\ 500\ 000 \\ 31\ 500\ 000 \\ 63\ 500\ 000 \end{array}$	500 000 761 1 500 000 812 3 500 000 723 7 500 000 433 15 500 000 742 31 500 000 592 63 500 000 1 167	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

	1000 new lite	erals	sequential	path	speed
afterwards	(ℓ_1,ℓ_2)	$A, \neg B$	search	index	up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72
16 000	15 500 000	742	21s	221ms	95
32 000	31 500 000	592	40s	489ms	82
64 000	63 500 000	1 167	80s	697ms	115
128 000	127 500 000	1 479	160s	13s	12
256 000	255 500 000	1 097	320s	440s	1

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

checking afterwards	1000 new lite (ℓ_1,ℓ_2)	$^{rals}_{A,\neg B}$	sequential search	path index	speed up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72
16 000	15 500 000	742	21s	221ms	95
32 000	31 500 000	592	40s	489ms	82
64 000	63 500 000	1 167	80s	697ms	115
128 000	127 500 000	1 479	160s	13s	12
256 000	255 500 000	1 097	320s	440s	1
512 000	511 500 000	1 440	640s	348s	2

TPTP/Problems/HWV/HWV134-1.p 2 332 428 formulae, 6 570 884 literals

checkin afterwards	ig 1000 new lite (ℓ_1,ℓ_2)	$\begin{array}{c} rals \\ A, \neg B \end{array}$	sequential search	path index	speed up
1 000	500 000	761	726ms	70ms	10
2 000	1 500 000	812	2s	69ms	29
4 000	3 500 000	723	4s	75ms	53
8 000	7 500 000	433	9s	125ms	72
16 000	15 500 000	742	21s	221ms	95
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64 000	63 500 000	1 167	80s	697ms	115
128 000	127 500 000	1 479	160s	13s	12
256 000	255 500 000	1 097	320s	440s	1
512 000	511 500 000	1 440	640s	348s	2
1 024 000	1023 500 000	1 534	1280s	330s	4