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$$\begin{tikzcd}
& \langle \epsilon, \mathsf{h}\rangle \arrow[d, dashed] \arrow[d, dotted, shift right] & \\
& \langle 1, \mathsf{f}\rangle \arrow[dl, dotted] \arrow[dr, dashed] & \\
\langle 11, \mathsf{a}\rangle \arrow[rr, dotted, bend left] & & \langle 12, \mathsf{y}\rangle
\end{tikzcd}$$

$\langle \epsilon, \mathsf{h}\rangle\langle 1, \mathsf{f}\rangle\langle 12, y\rangle$	path from root to leaf
$\langle \epsilon, \mathsf{h}\rangle\langle 1, \mathsf{f}\rangle\langle 11, \mathsf{a}\rangle\langle 12, y\rangle$	pre-order traversal

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  when traversal order and arity of symbols are fixed.