Term-Indexing

for Instantiation-Based First Order Theorem Proving

Alexander Maringele

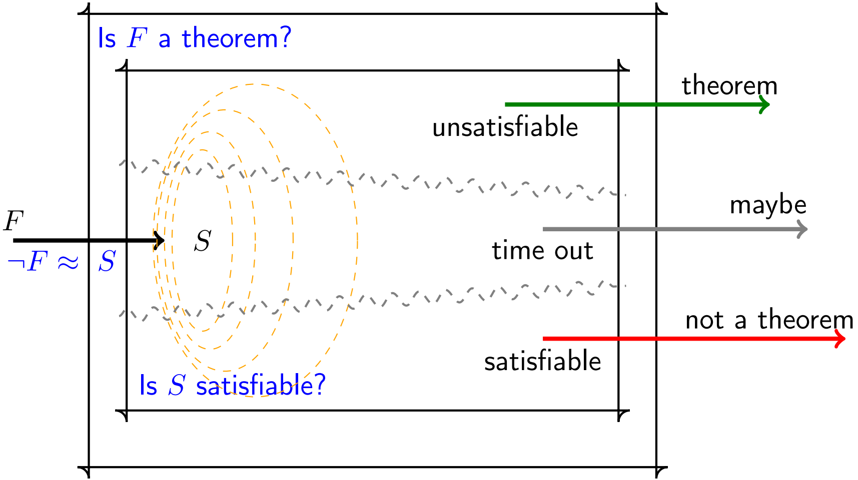
January 27th, 2016

References

- Alexandre Riazanov and Andrei Voronkov, *Efficient instance retrieval with standard and relational path indexing*, Automated Deduction – CADE-19 (Franz Baader, ed.), Lecture Notes in Computer Science, vol. 2741, Springer Berlin Heidelberg, 2003, pp. 380–396 (English).
- R. Sekar, I. V. Ramakrishnan, and Andrei Voronkov, *Term indexing*, Handbook of Automated Reasoning (Alan Robinson and Andrei Voronkov, eds.), Elsevier Science Publishers B. V., Amsterdam, The Netherlands, The Netherlands, 2001, pp. 1853–1964.

Outline

Resolution



Is F a theorem?
theorem
unsatisfiable
maybe
F
S
$\neg F \approx S$
time out
not a theorem
satisfiable
Is S satisfiable?

Goal

A sound, refutational complete, and effective procedure.

-
- Avoid redundancy

superposition inference rules

$$\frac{A \vee \mathcal{C} \quad \neg B \vee \mathcal{D}}{(\mathcal{C} \vee \mathcal{D})\,\mathrm{mgu}(A,B)}\ \textit{ordered resolution} \qquad \frac{A \vee B' \vee \mathcal{C}}{(A \vee \mathcal{C})\,\mathrm{mgu}(A,B)}\ \textit{ordered factoring}$$

$$\frac{s \approx t \vee \mathcal{C} \quad \neg A[s'] \vee \mathcal{D}}{(\mathcal{C} \vee \neg A[t] \vee \mathcal{D})\ \mathrm{mgu}(s,s')}\ \textit{ordered paramodulation}\ \frac{s \approx t \vee \mathcal{C} \quad A[s'] \vee \mathcal{D}}{(\mathcal{C} \vee A[t] \vee \mathcal{D})\ \mathrm{mgu}(s,s')}$$

$$\frac{s \approx t \vee \mathcal{C} \quad u[s'] \not\approx v \vee \mathcal{D}}{(\mathcal{C} \vee u[t] \not\approx v \vee \mathcal{D})\ \mathrm{mgu}(s,s')}\ \textit{superposition}\ \frac{s \approx t \vee \mathcal{C} \quad u[s'] \approx v \vee \mathcal{D}}{(\mathcal{C} \vee u[t] \approx v \vee \mathcal{D})\ \mathrm{mgu}(s,s')}$$

$$\frac{s \not\approx s' \vee \mathcal{C}}{\mathcal{C}\,\mathrm{mgu}(s,s')}\ \textit{equality resolution} \qquad \frac{s \approx s' \vee u \approx v \vee \mathcal{C}}{(v \not\approx s' \vee u \approx s' \vee \mathcal{C})\,\mathrm{mgu}(s,s')}\ \textit{equality factoring}$$

Definition

The *u*nit superposition calculus includes

- unit paramodulation (UP)

$$\frac{s \approx t \quad L[s']}{(L[t])\,\sigma}\ (UP)$$

  where $\sigma = \mathrm{mgu}(s, s')$ is defined, $s' \notin \mathcal{V}$, $t\sigma \not\succeq s\sigma$;

- unit superposition (US_-, US_+)

$$\frac{s \approx t \quad u[s'] \not\approx v}{(u[t] \not\approx v)\,\sigma}\ (US_-) \qquad \frac{s \approx t \quad u[s'] \approx v}{(u[t] \approx v)\,\sigma}\ (US_+)$$

  where $\sigma = \mathrm{mgu}(s, s')$ is defined, $s' \notin \mathcal{V}$, $t\sigma \not\succeq s\sigma$, $v\sigma \not\succeq u[s']\sigma$;

- unit equality resolution $(UR_\approx)$, and unit resolution (UR)

$$\frac{s \not\approx t}{\square}\ (UR_\approx) \qquad \frac{A \quad \neg B}{\square}\ (UR)$$

Term retrieval problems

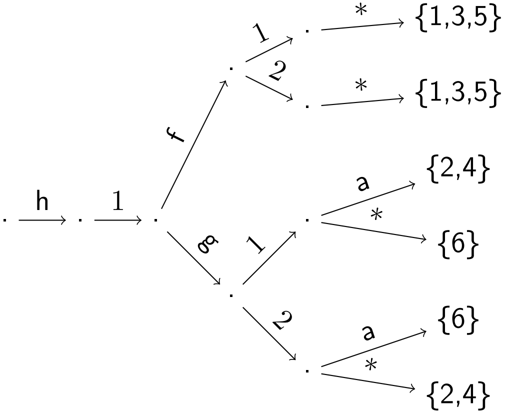
Definition

In a given set of terms

- find terms that are variants of a given term.
  $\mathrm{variant}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t$ and σ is renaming.
- find terms that are unifiable with a given term.
  $\mathrm{unifiable}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t\sigma$
- find terms that are instances of a given term.
  $\mathrm{instance}(\ell, t) \Leftrightarrow \exists\sigma\ \ell = t\sigma$
- find terms that are generalizations of a given term.
  $\mathrm{generalization}(\ell, t) \Leftrightarrow \exists\sigma\ \ell\sigma = t$

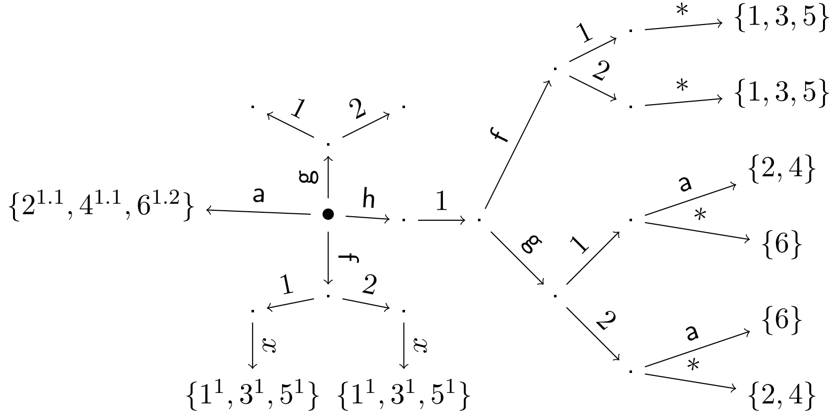
Definition

$$\{{}^{1:}\mathsf{h}(\mathsf{f}(x,x)),{}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)),{}^{3:}\mathsf{h}(\mathsf{f}(y,z)){}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)),{}^{5:}\mathsf{h}(\mathsf{f}(y,x)),{}^{6:}\mathsf{h}(\mathsf{g}(y,a))\}$$

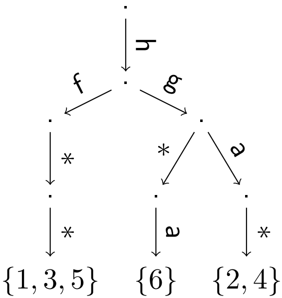


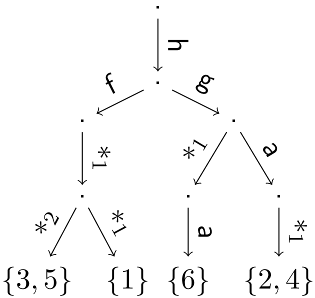
$$\mathsf{h}(\mathsf{g}(y,x)) \mapsto \{\ \mathsf{h}.1.\mathsf{g}.1.*, \mathsf{h}.1.\mathsf{g}.2.*\}$$

$$\{{}^{1:}\mathsf{h}(\mathsf{f}(x,x)), {}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), {}^{3:}\mathsf{h}(\mathsf{f}(y,z)) \; {}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), {}^{5:}\mathsf{h}(\mathsf{f}(y,x)), {}^{6:}\mathsf{h}(\mathsf{g}(y,a))\}$$



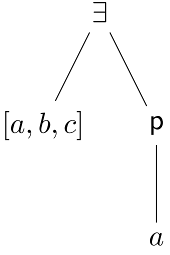
$$\left\{ \begin{array}{lll} {}^{1:}\mathsf{h}(\mathsf{f}(x,x)), & {}^{2:}\mathsf{h}(\mathsf{g}(\mathsf{a},x)), & {}^{3:}\mathsf{h}(\mathsf{f}(y,z)), \\ {}^{4:}\mathsf{h}(\mathsf{g}(\mathsf{a},y)), & {}^{5:}\mathsf{h}(\mathsf{f}(y,x)), & {}^{6:}\mathsf{h}(\mathsf{g}(y,a)) \end{array} \right\}$$





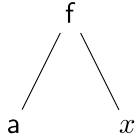
$\mathsf{h}(\mathsf{f}(y,x)) \mapsto \mathsf{h.f.}{*}.{*}$

$\mathsf{h}(\mathsf{f}(y,x)) \mapsto' \mathsf{h.f.}{*_1}.{*_2}$



$(\exists[a,b,c] : (\mathsf{p}(a)))$

$\mathsf{f}(\mathsf{a}, x)$



$\mathsf{f}(\mathsf{a}, x) \approx \mathsf{f}(x, \mathsf{a})$