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*Exploiting Non-Slip Wall Contacts to Position Two Particles Using The Same Control Input*

Dear IEEE Transactions on Robotics Editorial Office,

Please find attached the revised paper, *Exploiting Non-Slip Wall Contacts to Position Two Particles Using The Same Control Input*, along with the document containing a response to the reviewers. We are grateful to the reviewers for helping us improve our manuscript through their comments and questions. Please let us know if further information is required. This work is extending the preliminary conference paper, "Algorithms for shaping a particle swarm with a shared input by exploiting non-slip wall contacts", presented at the 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). The conference paper considered only square workspaces. This work extends the analysis to convex workspaces and 3D positioning. This paper also implements the algorithms using a hardware setup inspired by the anatomy of the gastrointestinal tract.

The new revision of our paper includes multi-media so that others can build on our results. These include illustrative videos for the simulations and experiments: ??? SHIVA, WHERE ARE THE VIDEOS??? (1) *TorqueSoftware.mp4*, (2) *TorqueHardware.mp4*, (3) *video name*. We also include Mathematica Notebooks (.nb files) containing the simulation code.

Sincerely,

A handwritten signature in black ink that reads "Aaron+ Becker". The signature is written in a cursive, flowing style.

Aaron T. Becker (on behalf of all the authors)

## RESPONSE TO REVIEWERS

In the following document, we have provided detailed responses to the comments and questions of the reviewers. Comments and questions by reviewers are in blue, our responses are in black.

*Comments by Associate Editor*

[R 0.1] “The Associate Editor (AE) was able to obtain two high-quality reviews from researchers working directly in this field. After considering their reviews, it is the opinion of the AE that this paper is not currently suitable for publication in T-RO. However, the authors are encouraged to prepare a revised draft, addressing the concerns of the reviewers. Please also address how the technique could be practically applied to navigate inside an actual 3D intestine as opposed to a cross section in which the particles move on a planar surface. Would magnetic forces be sufficient to overcome gravity? Would the particles need to be made neutrally buoyant?”

We are grateful for the efforts of our reviewers. We have thoroughly revised this paper by (1) restructuring the paper as suggested by the reviewers (2) constructing reachability sets for all initial conditions, proving controllability as long as the initial positions are distinct and the final positions are not antipodes, and (3) updating almost all the figures and captions.

We made the model for both 2D and 3D more clear – the particles are assumed to be neutrally buoyant so that the same force is applied to each particle. Please see the detailed responses below.

*Comments by Reviewer #1*

[R 1.1] “This paper explains an underactuated control strategy for small teams of magnetic microparticles which move under the influence of a single broadcast signal. This underactuated control problem is relevant to microrobotic swarms moving under magnetic field input. In previous work, the authors have explored interesting control methods relying on obstacles in the environment, but the case studied here is more application-realistic. The control method here is clever and successful. The work is interesting to read and I think that anyone with a basic background in controls will find this work interesting. The main concept used here to achieve multi-particle control is that the particles experience a no-slip condition when pushed against the environment wall. The particles can be removed from the wall by pulling them. The paper is an extension of an IROS2017 paper which introduced the idea for square workspace only. This TRO submission generalizes to a non-square workspace, which is a notable improvement. This version of the work will thus be a helpful addition to the literature. The paper gives a very nice review of previous work in using broadcast signals for multi-particle control. ”

Thank you! We have completed a major revision, as explained below.

[R 1.2] “Is it expected that the no-slip condition will be a strongly satisfied assumption for in-vivo biomedical applications? In cases where there were some slip, how could that be handled?”

To maintain clarity, this paper focused on systems with non-slip boundaries. While several environments with non-slip boundaries exist (e.g. tripe, and intestines with villi), extending to lower friction values is a intriguing avenue that unfortunately does not fit within the 12 page limit for this journal, but is something we are investigating. The reachable sets with non-infinite friction are smaller. The conclusion now explains “ This paper assumed friction was sufficient to completely stop particles in contact with the boundary. The algorithms would require retooling to handle small friction coefficients.”

[R 1.3] “This control method requires advance knowledge of the workspace boundary geometry, in addition to micro-particle state feedback. It would be interesting to hear about how accurate the boundary geometry must be known for this control to work well, and perhaps compare this will medical imaging techniques such as CT or MRI which could potentially be used to generate such maps. For motion through 3D lumens, the cross-sectional geometry will change throughout the lumen.”

These are excellent points. This paper assumes perfect knowledge of the boundary and initial conditions of the particles. We added text explaining that systems with large numbers of protrusions or concave workspaces are best handled by motion planners such as RRT.

Section IV-F: “Workspaces that are tortuous or with many obstacles are better handled by other path planners, such as RRT [29], or [5], which used collisions with protrusions of the workspace to rearrange particles.”.

Also, during the hardware experiments, the particle that was pushed into the com stomach boundary disappeared from the camera, but control still was successful, indicating that perfect sensing is not necessary: “Our algorithm successfully delivered the particles to goal positions in 5 out of 5 trials.”

[R 1.4] “Abstract: It is stated that “given 3 orthogonal magnetic fields”. This statement should be made more precise, because the magnetic pulling here uses field gradients, not fields. In addition, these gradients are not orthogonal. An accurate statement would need to be more complex.”

Yes. To make this work as broadly applicable as possible (magnetic inputs are just one example) we changed the abstract to say “This paper investigates particle control with uniform forces (the same force is applied everywhere in the workspace). Given a controllable field that can generate forces in three orthogonal directions, steering one particle in 3D is trivial. ”

[R 1.5] “Fig 2: “first contact point” is not clear. There is no way given to tell where these line up with actual positions on the workspace. ”

Fig 2 is now Fig. 11. We rearranged the order because the optimization results can be applied to Alg. 1, but are not necessary to implement or understand the algorithm. We updated this figure to clarify what the contact points are by adding and labelling tick marks from 0 to 4 along the boundary of the square workspace and the distance plot. We also added angle markings and green points showing the optimal solution in both the workspace and the distance plots.

[R 1.6] “Fig 3: the meaning of grey areas should be mentioned in the fig or caption.”

This is now Figure 2. We added a legend and caption text stating: “Gray areas denote regions inaccessible by our motion planner. The particle start positions must be distinct ( $\|s_2 - s_1\| \geq \epsilon$ ), and at least one goal position must be farther than  $\epsilon$  from the boundary, where  $\epsilon$  is a small but nonzero user-specified constant.”

[R 1.7] “Figures are hard to read because there is a lot going on, the font is often too small and text sometimes is obscured.”

There were a number of problems with the initial figures. We have added legends to Figs. 2, 3, 5, 15, and added captions with visual elements to 2, 14, 16. We standardized our variable names and icons, and added Figs 8 and 9.

[R 1.8] “III.C. Its not clear why this section is relevant. Why is the shortest path which intersects the wall needed? Some writing structure could help the reader understand what the goal is here.”

To better present the motion planner, we moved this section, which covers an optimization result, to the end, and altered the framing text: “Algorithm 1 provided a technique to bring two particles to goal positions using global inputs, but did not optimize path length. Changing the relative positions of particles in any workspace requires making one particle contact the boundary. In this section we present two results that can be incorporated into Algorithms 2 and 3 to generate shorter motion paths.”

*Comments by Reviewer #6*

[R 2.1] “In this manuscript algorithms to position two particles in arbitrary locations under uniform actuation are proposed. The algorithms rely on differentiating between particles by contacting them with walls or boundaries of the domain, at which they are assumed to not move unless actuated in a direction away from the wall. The paper presents some results on shortest paths that contact surfaces (but see below on some presentation issues), then describes the algorithm, which basically entails moving both particles so that one particle contacts a boundary, adjusting the relative displacement between particles while the one particle remains at the boundary, then translating both particles to the desired location. Simulations of the algorithm are presented in square and circular domains, and experiments are described in circular cross sections inspired by intestinal and stomach scenarios. Although similar ideas were presented earlier by the authors using either an obstacle in the workspace or square workspaces, this paper extends that work to circular and convex polygonal workspaces. The overall idea is a novel contribution as other swarm control techniques focus on either heterogeneity of microrobots or of actuation (such as fields) rather than distinguishing particles by their proximity to boundaries. However, in my opinion the manuscript requires major revisions before being suitable for publication, mainly to address the theoretical effectiveness of the algorithm (as detailed below), and less so to address clarity of presentation. ”

Thank you. We were pleased to get this review, because it led us to think more deeply on how to present the results and led to the major revisions detailed below.

[R 2.2] “The main weakness of the paper in my opinion is that the authors never really prove that their algorithm works, or alternatively how general their algorithm is, i.e. what are its limits of applicability. To be more specific, in their Algorithm 1, if the desired configuration is not within the 2-move reachable set, then the algorithm targets instead the closest point in the 2-move reachable set, and then “iterates until we reach the goal.” When does such iteration actually achieve the goal? What are the attainable final goals that can be reached after iteration? Does the space of attainable goals depend on the initial positions of the particles? While there are simulations, the ones presented do not explore the entire possible space (which is understandable since they it is quite large). I think an analysis of the attainable space of their algorithm is needed.”

An excellent suggestion! This was a major oversight in the first version. It is possible to explore the possible space, and we can be quite clear about the locations that are attainable. We rearranged the paper and added Section IV-F, which analyzes the reachable set:

“The  $\Delta$  configuration enables an iterative method to compute the accessible workspace. Due to symmetry of the workspace, the fraction of the  $\Delta$  configuration space reachable in  $2k$  moves is a function of only the initial separation distance  $d_{12}$ . The angle  $\theta$  between the initial particle positions simply rotates the reachable  $\Delta$  configuration space. As long as the initial configurations are distinct ( $s_1 \neq s_2$ ), the reachable set grows quickly. This relationship is shown in Fig. 15. Only antipodal locations are unreachable ( $\|g_2 - g_1\| = 1$ ), but can be asymptotically approached. Indeed, even with a tiny initial separation of  $d_{12} = 0.001$ ,

after 14 moves 90% of the  $\Delta$  configuration space is reachable. In two moves, the maximum reachable fraction of 0.373 is achieved with  $d_{12} \approx 0.81$ .”

We added Figs. 8 and 9 which illustrate the effects of 2-move reachable sets, as well as 4-move, 6, 8, 10, 12, and 14-moves. Fig. 15 analyzes the worst case: when the goal locations are at the antipodes. Even in this situation we can approach arbitrarily close to the desired configuration, but can never achieve it. This figure shows that the approximation error decrease can be fit by a  $1/(\text{distance travelled}^3)$ .

[R 2.3] “There are issues with the presentation that can be easily improved. At the beginning of the paper many terms are not defined which makes figures and discussion hard to follow. Definitions do come later, but they should be moved up. For example, in Fig 2 which symbols are targets and which are initial conditions are not defined. In Fig 3 epsilon is shown but not defined nor is its significance explained.  $s$  and  $g$  and  $\Delta s$  and  $\Delta g$  are not defined. In Fig 12, where is the goal on the boundary? Can that be indicated on the figures?”

Thank you for detailed points. We performed a large restructuring. Fig 2 is now 11. We updated this figure to clarify what the contact points are by adding and labelling tick marks from 0 to 4 along the boundary of the square workspace and the distance plot. We also added angle markings and green points showing the optimal solution in both the workspace and the distance plots. We also added visual icons to the caption: “from starting positions ( $\square$ ,  $\square$ ) to goal positions ( $\circ$ ,  $\circ$ )”

Figure 3 is now 2, we added both a legend to the figure and a descriptive caption: “Particles move from start positions ( $\square$ ,  $\square$ ), to goal positions ( $\circ$ ,  $\circ$ ). Dashed lines show the shortest route if particles could be controlled independently. Solid arrows show path given by Alg. 1. Gray areas denote regions inaccessible by our motion planner. The particle start positions must be distinct ( $\|s_2 - s_1\| \geq \epsilon$ ), and at least one goal position must be farther than  $\epsilon$  from the boundary, where  $\epsilon$  is a small but nonzero user-specified constant.”

Figure 12 had the wrong caption. It now is Fig. 14, and has the caption: “Contour plots showing the number of moves and distance commanded if red particle’s goal position is varied in  $x$  and  $y$ . Starting positions of red and blue particles ( $\square$ ,  $\square$ ) and goal position of blue particle  $\circ$  are fixed. The top row has the blue particle’s goal position at the origin, generating symmetric contour plots. Moving the blue particles’ goal position to  $(-0.2, 0)$ , generates non-symmetric contour plots.”

We also added captions for start and end (colored squares and circles) to each drawing.

[R 2.4] “(more presentation) At the beginning of section IV it wasn’t clear to me whether Algorithm 1 had been described yet when it was first mentioned, and whether that sentence was a description of what was to come, or was as assertion that logically followed from the previous parts of the paper. ”

We rewrote this section. It now begins with: “This section presents an algorithm, Alg. 1, that uses non-slip contacts with walls to arbitrarily position two particles in a convex workspace. Workspaces are 2D convex polygons with no internal obstacles. ”

[R 2.5] “A minor comment, sections IIIB and C are a little confusing since it is not motivated why shortest path is being considered in the overall argument of the paper. Furthermore, it seems that shortest path in IIIB is used for the situation moving two particles, while in IIIC it is used for the situation of moving only one particle. Consistency in the presentation would help..”

We rearranged the order because the optimization results can be applied to Alg. 1, but are not necessary to implement or understand the algorithm. The results were combined into a new Section V, which opens by stating:

“Algorithm 1 provided a technique to bring two particles to goal positions using global inputs, but did not optimize path length. Changing the relative positions of particles in any workspace requires making one particle contact the boundary. In this section we present two results that can be incorporated into Algorithms 2 and 3 to generate shorter motion paths.”