Homework 5 Solution

Key:	Black text	= Original problem statement
	Blue text	= Solution and remarks
	Final solutions appear in $\boxed{\text{boxes}}$	

Overview and Motivation for Approach Used

In this homework assignment we wish to simulate a spacecraft with 2 reaction wheels. To do this we need to write the equations of motion for the spacecraft and each reaction wheel. Since we have 3 bodies, this will give us 6 equations. The differential equations we are trying to find (for a body with frame i) are

1) Newton's equation for translation motion

$$m\dot{v}_{0,i}^{i} = \sum_{j=1}^{n} f_j^{\,i}$$ where f_j are the external forces acting on the body

2) Euler's equation of rotation motion

$$J^i\dot{\omega}_{0,i}^{i} + \widehat{\omega_{0,i}^{i}}J^i\omega_{0,i}^{i} = \sum_{j=1}^{n}\tau_j^i$$ where τ_j are the external torques acting on the body

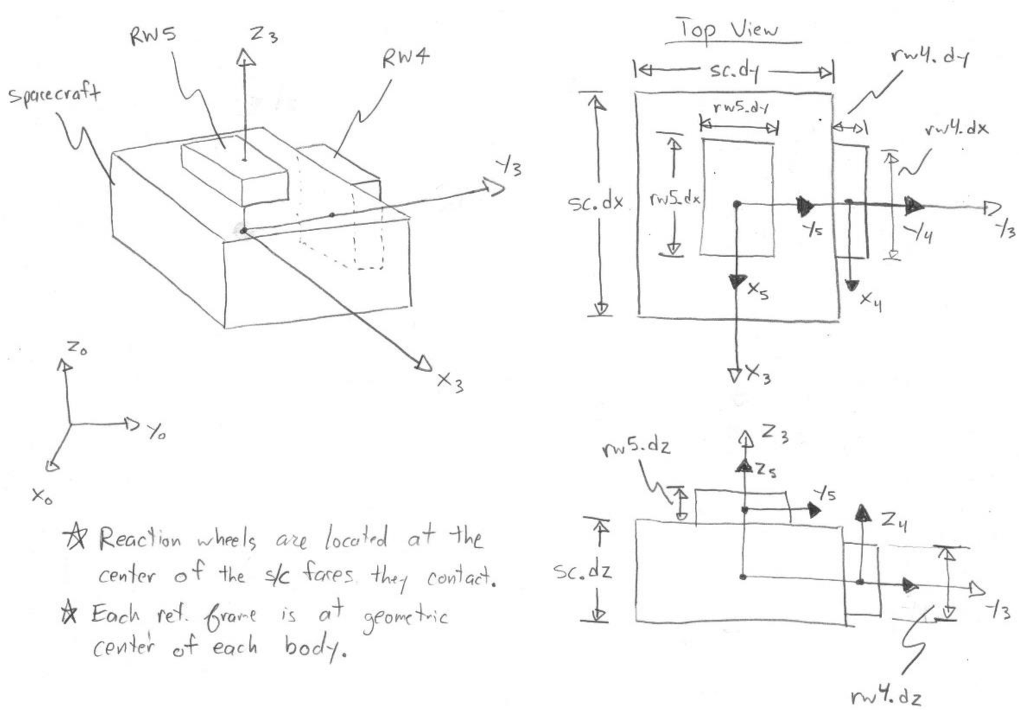
To write these equations for each body we need to find expressions for each variable in the equations.

1. (120 pts) You must derive expressions for the following things by hand, including any diagrams necessary to complete these derivations:
   - position and orientation of both reaction wheels

$$o_4^0, R_4^0 \text{ (also requires } R_4^3), o_5^0, R_5^0 \text{ (also requires } R_5^3)$$

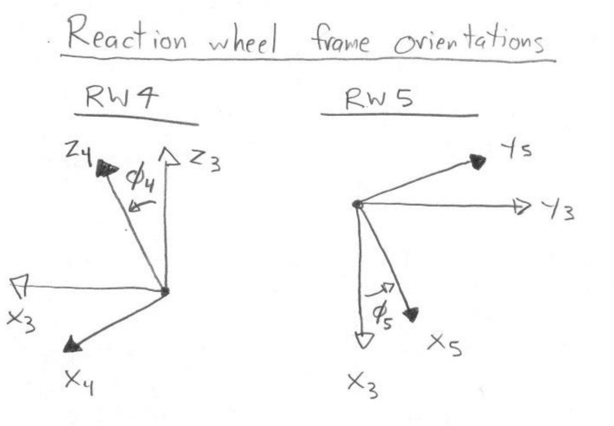
Let us start by sketching the spacecraft, reaction wheels, and their associated references frames.

Figure 1:



We can find the rotation matrices R_4^3 and R_5^3 with the next sketch.

Figure 2:



From this we write

$$\boxed{R_4^3 = \begin{bmatrix} c_4 & 0 & s_4 \\ 0 & 1 & 0 \\ -s_4 & 0 & c_4 \end{bmatrix}} \qquad \boxed{R_5^3 = \begin{bmatrix} c_5 & -s_5 & 0 \\ s_5 & c_5 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

From previous homework assignments, we also know we can write R_3^0 in terms of the XYZ Euler angles.

$$\boxed{R_3^0 = \begin{bmatrix} c_2c_3 & -c_2s_3 & s_2 \\ s_1s_2c_3 + c_1s_3 & -s_1s_2s_3 + c_1c_3 & -s_1c_2 \\ -c_1s_2c_3 + s_1s_3 & c_1s_2s_3 + s_1c_3 & c_1c_2 \end{bmatrix}}$$

We don't have to multiply out $R_3^0R_5^3$ or $R_3^0R_4^3$ by hand. We'll let the code take care of that.

Since o_4^0 and o_5^0 are effectively points (and in particular points that describe the origin of a reference frame) we can use our equation for point transformations.

We can now write

$$\boxed{\begin{aligned} o_4^0 &= o_3^0 + R_3^0 o_4^3 \\ R_4^0 &= R_3^0 R_4^3 \\ o_5^0 &= o_3^0 + R_3^0 o_5^3 \\ R_5^0 &= R_3^0 R_5^3 \end{aligned}}$$

- linear and angular velocity of both reaction wheels

$$v^0_{0,4},\, w^4_{0,4} \text{ (also requires } w^4_{3,4}),\, v^0_{0,5},\, w^5_{0,5} \text{ (also requires } w^5_{3,5})$$

$$v^0_{0,4} = \frac{d}{dt}[o^0_4]$$

$$= \frac{d}{dt}[o^0_3 + R^0_3 o^3_4]$$

$$= \dot{o}^0_3 + R^0_3 \widehat{\omega^3_{0,3}} o^3_4 + R^0_3 \dot{o}^3_4$$

, but $\dot{o}^3_4 = 0$ because the origin of RW4 is fixed w.r.t. frame 3.

$$\boxed{v^0_{0,4} = v^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} o^3_4}$$

Similarly, we can show that

$$\boxed{v^0_{0,5} = v^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} o^3_5}$$

We can calculate the requested ω by remembering that angular velocities are just vectors, and vectors written in the same frame can be added.

$$\omega^4_{0,4} = \omega^4_{0,3} + \omega^4_{3,4}$$

$$\omega^5_{0,5} = \omega^5_{0,3} + \omega^5_{3,5}$$

We will need to find $\omega^4_{3,4}$ and $\omega^5_{3,5}$. From **figure 2** we can calculate these by inspection

$$\omega^4_{3,4} = \begin{bmatrix} 0 \\ \dot{\phi}_4 \\ 0 \end{bmatrix} \qquad \omega^5_{3,5} = \begin{bmatrix} 0 \\ 0 \\ \dot{\phi}_5 \end{bmatrix}$$

We therefore have

$$\boxed{\omega^4_{0,4} = (R^3_4)^T \omega^3_{0,3} + \omega^4_{3,4} \quad \text{where } R^3_4 \text{ is known and } \omega^4_{3,4} = \begin{bmatrix} 0 \\ \dot{\phi}_4 \\ 0 \end{bmatrix}}$$

$$\boxed{\omega^5_{0,5} = \left(R^3_5\right)^T \omega^3_{0,3} + \omega^5_{3,5} \quad \text{where } R^3_5 \text{ is known and } \omega^5_{3,5} = \begin{bmatrix} 0 \\ 0 \\ \dot{\phi}_5 \end{bmatrix}}$$

- linear and angular acceleration of both reaction wheels

$$\dot{v}^0_{0,4}, \dot{w}^4_{0,4}, \dot{v}^0_{0,5}, \dot{w}^5_{0,5}$$

$$= \frac{d}{dt}\left[v^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} o^3_4\right]$$

$$= \dot{v}^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} \widehat{\omega^3_{0,3}} o^3_4 + R^0_3 \left(\widehat{\dot{\omega}^3_{0,3}} o^3_4 + \widehat{\omega^3_{0,3}} \dot{o}^3_4\right) \qquad \text{(Product rule twice)}$$

$$= \dot{v}^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} \widehat{\omega^3_{0,3}} o^3_4 + R^0_3 \widehat{\dot{\omega}^3_{0,3}} o^3_4 \qquad (\dot{o}^3_4 = 0)$$

$$\boxed{\dot{v}^0_{0,4} = \dot{v}^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} \widehat{\omega^3_{0,3}} o^3_4 - R^0_3 \widehat{o^3_4} \dot{\omega}^3_{0,3}}$$

I've simply reversed the order of the cross product in the last term in the last step. Later this will be helpful.

Similarly $\boxed{\dot{v}^0_{0,5} = \dot{v}^0_{0,3} + R^0_3 \widehat{\omega^3_{0,3}} \widehat{\omega^3_{0,3}} o^3_5 - R^0_3 \widehat{o^3_5} \dot{\omega}^3_{0,3}}$

$$\dot{\omega}^4_{0,4} = \frac{d}{dt}\left[(R^3_4)^T \omega^3_{0,3} + \omega^4_{3,4}\right]$$

$$= R^4_3 \widehat{\omega^3_{4,3}} \omega^3_{0,3} + (R^3_4)^T \dot{\omega}^3_{0,3} + \dot{\omega}^4_{3,4}$$

$$= R^4_3 (\widehat{R^3_4 \omega^4_{4,3}}) \omega^3_{0,3} + (R^3_4)^T \dot{\omega}^3_{0,3} + \dot{\omega}^4_{3,4} \qquad \text{use } \omega^4_{4,3} = -\omega^4_{3,4}$$

$$= -(R^3_4)^T R^3_4 \widehat{\omega^4_{3,4}} (R^3_4)^T \omega^3_{0,3} + (R^3_4)^T \dot{\omega}^3_{0,3} + \dot{\omega}^4_{3,4}$$

$$= \left(R^3_4 \widehat{\omega^4_{3,4}}\right)^T \omega^3_{0,3} + (R^3_4)^T \dot{\omega}^3_{0,3} + \dot{\omega}^4_{3,4} \qquad \text{(- sign from transpose of skew sym. matrix)}$$

$$\boxed{\dot{\omega}^4_{0,4} = \left(R^3_4 \widehat{\omega^4_{3,4}}\right)^T \omega^3_{0,3} + (R^3_4)^T \dot{\omega}^3_{0,3} + \dot{\omega}^4_{3,4} \quad , \omega^4_{3,4} = t_4 \dot{\phi}_4 = \begin{bmatrix}0\\1\\0\end{bmatrix} \dot{\phi}_4,\ \dot{\omega}^4_{3,4} = t_4 \ddot{\phi}_4 = \begin{bmatrix}0\\1\\0\end{bmatrix} \ddot{\phi}_4}$$

Similarly

$$\boxed{\dot{\omega}^5_{0,5} = \left(R^3_5 \widehat{\omega^5_{3,5}}\right)^T \omega^3_{0,3} + \left(R^3_5\right)^T \dot{\omega}^3_{0,3} + \dot{\omega}^5_{3,5} \quad , \omega^5_{3,5} = t_5 \dot{\phi}_5 = \begin{bmatrix}0\\0\\1\end{bmatrix} \dot{\phi}_5, \quad \dot{\omega}^5_{3,5} = t_5 \ddot{\phi}_5 = \begin{bmatrix}0\\0\\1\end{bmatrix} \ddot{\phi}_5}$$

- mass and moment of inertia of everything

$$m, J^3, m_4, J^4_4, m_5, J^5_5$$

Since the spacecraft and reaction wheels are all rectangular prisms, their masses and moments of inertia are easy to calculate. For the moment of inertia, use the formula calculated in homework 4.

Spacecraft: $\boxed{m = \rho_{sc} dx_{sc} dy_{sc} dz_{sc}}$

RW4: $\boxed{m_4 = \rho_4 dx_4 dy_4 dz_4}$

RW5: $\boxed{m_5 = \rho_5 dx_5 dy_5 dz_5}$

From homework 4 we have J. This equation can be applied to the spacecraft and reaction wheels.

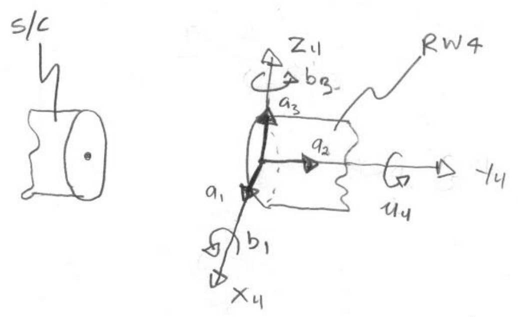
$$\boxed{J_i^i = \frac{m}{12}\begin{bmatrix} dy^2 + dz^2 & 0 & 0 \\ 0 & dx^2 + dz^2 & 0 \\ 0 & 0 & dx^2 + dy^2 \end{bmatrix}}$$

- constraint forces and torques

$$f_4^3, \tau_4^3, f_5^3, \tau_5^3$$

For reaction wheel 4, we draw the following diagram of the pin joint between the wheel and the spacecraft.

Figure 3 (RW4):

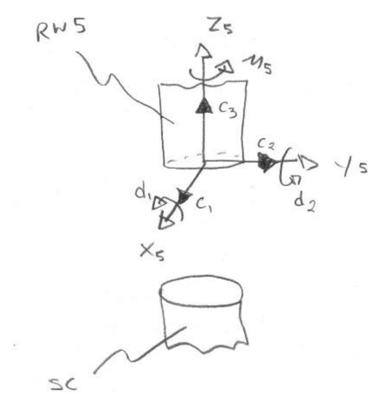


$$\boxed{f_4^3 = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}}$$

$$\tau_4^3 = t_4 u_4 + S_4 r_4$$

$$\boxed{\tau_4^3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_4 + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_3 \end{bmatrix}}$$

Figure 4 (RW5):



$$\boxed{f_5^3 = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}$$

$$\tau_5^3 = t_5 u_5 + S_5 r_5$$

$$\boxed{\tau_5^3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_5 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}}$$

We note that the forces acting on the spacecraft due to RW4 and RW5 are $-f_4^3$ and $-f_5^3$, respectively.

- equations of motion
  (a) write both Newton's Equation and Euler's Equation for the spacecraft and for each reaction wheel separately

Spacecraft

NE: $m\dot{v}^0_{0,3} = \sum_{i=1}^{n} f_i$ Each RW provides a force via the pin joint.

$m\dot{v}^0_{0,3} = -f^0_4 - f^0_5$

$$\boxed{m\dot{v}^0_{0,3} = -R^0_3 f^3_4 - R^0_3 f^3_5}$$

EE: $J^3\dot{\omega}^3_{0,3} + \widehat{\omega^3_{0,3}} J^3\omega^3_{0,3} = \sum_{i=1}^{n} \tau_i$

The torques acting on the spacecraft originate from 4 sources: 2 from the pure moments passed through the pin joint at each RW, and 2 from the reaction forces that act on the spacecraft's pin joints at a distance from the spacecraft's CM.

$$\boxed{J^3\dot{\omega}^3_{0,3} + \widehat{\omega^3_{0,3}} J^3\omega^3_{0,3} = (-\widehat{p^3_{pin4}} f^3_4 - \tau^3_4) + \left(-\widehat{p^3_{pin5}} f^3_5 - \tau^3_5\right)}$$

RW4

NE: $m_4\dot{v}^0_{0,4} = \sum_{i=1}^{n} f^0_i$

$m_4\dot{v}^0_{0,4} = f^0_4$ The only force acting on RW4 is the constraint force in the pin joint.

$$\boxed{m_4\dot{v}^0_{0,4} = R^0_3 f^3_4}$$

EE: $J^4_4\dot{\omega}^4_{0,4} + \widehat{\omega^4_{0,4}} J^4\omega^4_{0,4} = \sum_{i=1}^{n} \tau^4_i$

The torques acting on the RW originate from 2 sources: the pure moments carried by the pin joint, and the constraint forces that act at the pin joint, which is located at a distance from RW4's CM.

$$J^4_4\dot{\omega}^4_{0,4} + \widehat{\omega^4_{0,4}} J^4_4\omega^4_{0,4} = \widehat{p^4_{pin4}} f^4_4 + \tau^4_4$$

$$J^4_4\dot{\omega}^4_{0,4} + \widehat{\omega^4_{0,4}} J^4_4\omega^4_{0,4} = \widehat{p^4_{pin4}} (R^3_4)^T f^3_4 + (R^3_4)^T \tau^3_4$$

$$\boxed{J^4_4\dot{\omega}^4_{0,4} + \widehat{\omega^4_{0,4}} J^4_4\omega^4_{0,4} = \widehat{p^4_{pin4}} (R^3_4)^T f^3_4 + (R^3_4)^T \tau^3_4 \qquad where\ \tau^3_4 = t_4 u_4 + S_4 r_4}$$

RW5

NE: $\boxed{m_5\dot{v}^0_{0,5} = R^0_3 f^3_5}$

EE: $\boxed{J^5_5\dot{\omega}^5_{0,5} + \widehat{\omega^5_{0,5}} J^5_5\omega^5_{0,5} = \widehat{p^5_{pin5}} \left(R^3_5\right)^T f^3_5 + \left(R^3_5\right)^T \tau^3_5 \qquad where\ \tau^3_5 = t_5 u_5 + S_5 r_5}$

(b) plug in for $\dot{v}_{0,4}^{0}$, $\dot{w}_{0,4}^{4}$, τ_4, $\dot{v}_{0,5}^{0}$, $\dot{w}_{0,5}^{5}$, and τ_5

Spacecraft

NE: $\boxed{m\dot{v}_{0,3}^{0} = -R_3^0 f_4^3 - R_3^0 f_5^3}$ This was already in the form we wanted!

EE: $J^3\dot{\omega}_{0,3}^3 + \widehat{\omega_{0,3}^3} J^3 \omega_{0,3}^3 = (-\widehat{p_{pin4}^3} f_4^3 - \tau_4^3) + \left(-\widehat{p_{pin5}^3} f_5^3 - \tau_5^3\right)$

$$\boxed{J^3\dot{\omega}_{0,3}^3 + \widehat{\omega_{0,3}^3} J^3 \omega_{0,3}^3 = \left(-\widehat{p_{pin4}^3} f_4^3 - t_4 u_4 - S_4 r_4\right) + \left(-\widehat{p_{pin5}^3} f_5^3 - t_5 u_5 - S_5 r_5\right)}$$

RW4

NE: $m_4 \dot{v}_{0,4}^0 = R_3^0 f_4^3$

$$\boxed{m_4\left(\dot{v}_{0,3}^0 + R_3^0 \widehat{\omega_{0,3}^3}\widehat{\omega_{0,3}^3} o_4^3 - R_3^0 \widehat{o_4^3} \dot{\omega}_{0,3}^3\right) = R_3^0 f_4^3}$$

EE: $J_4^4 \dot{\omega}_{0,4}^4 + \widehat{\omega_{0,4}^4} J_4^4 \omega_{0,4}^4 = \widehat{p_{pin4}^4} (R_4^3)^T f_4^3 + (R_4^3)^T \tau_4^3$

$$\boxed{J_4^4\left(\left(R_4^3 \widehat{\omega_{3,4}^4}\right)^T \omega_{0,3}^3 + (R_4^3)^T \dot{\omega}_{0,3}^3 + t_4 \ddot{\phi}_4\right) + \widehat{\omega_{0,4}^4} J_4^4 \omega_{0,4}^4 = \widehat{p_{pin4}^4} (R_4^3)^T f_4^3 + (R_4^3)^T (t_4 u_4 + S_4 r_4)}$$

RW5

NE: $\boxed{m_5\left(\dot{v}_{0,3}^0 + R_3^0 \widehat{\omega_{0,3}^3}\widehat{\omega_{0,3}^3} o_5^3 - R_3^0 \widehat{o_5^3} \dot{\omega}_{0,3}^3\right) = R_3^0 f_5^3}$

EE:

$$\boxed{J_5^5\left(\left(R_5^3 \widehat{\omega_{3,5}^5}\right)^T \omega_{0,3}^3 + \left(R_5^3\right)^T \dot{\omega}_{0,3}^3 + t_5 \ddot{\phi}_5\right) + \widehat{\omega_{0,5}^5} J_5^5 \omega_{0,5}^5 = \widehat{p_{pin5}^5} \left(R_5^3\right)^T f_5^3 + \left(R_5^3\right)^T (t_5 u_5 + S_5 r_5)}$$

(c) put unknowns on left-hand-side and knowns on right-hand-side

Spacecraft

NE: $m\dot{v}_{0,3}^0 - R_3^0 f_4^3 - R_3^0 f_5^3$

$$\boxed{m\dot{v}_{0,3}^0 + R_3^0 f_4^3 + R_3^0 f_5^3 = 0}$$

EE: $J^3\dot{\omega}_{0,3}^3 + \widehat{\omega_{0,3}^3} J^3 \omega_{0,3}^3 = \left(-\widehat{p_{pin4}^3} f_4^3 - t_4 u_4 - S_4 r_4\right) + \left(-\widehat{p_{pin5}^3} f_5^3 - t_5 u_5 - S_5 r_5\right)$

$$\boxed{J^3\dot{\omega}_{0,3}^3 + \widehat{p_{pin4}^3} f_4^3 + \widehat{p_{pin5}^3} f_5^3 + S_4 r_4 + S_5 r_5 = -\widehat{\omega_{0,3}^3} J^3 \omega_{0,3}^3 - t_4 u_4 - t_5 u_5}$$

RW4

NE: $m_4\left(\dot{v}_{0,3}^0 + R_3^0\widehat{\omega_{0,3}^3}\widehat{\omega_{0,3}^3}o_4^3 - R_3^0\widehat{o_4^3}\dot{\omega}_{0,3}^3\right) = R_3^0 f_4^3$

$$\boxed{m_4\dot{v}_{0,3}^0 - m_4R_3^0\widehat{o_4^3}\dot{\omega}_{0,3}^3 - R_3^0 f_4^3 = -m_4R_3^0\widehat{\omega_{0,3}^3}\widehat{\omega_{0,3}^3}o_4^3}$$

EE: $J_4^4\left(R_4^3\widehat{\omega_{3,4}^4}\right)^T\omega_{0,3}^3 + J_4^4(R_4^3)^T\dot{\omega}_{0,3}^3 + J_4^4t_4\ddot{\phi}_4 + J_4^4\widehat{\omega_{0,4}^4}J_4^4\omega_{0,4}^4 = \widehat{p_{pin4}^4}(R_4^3)^Tf_4^3 + (R_4^3)^T(t_4u_4 + S_4r_4)$

$$\boxed{J_4^4(R_4^3)^T\dot{\omega}_{0,3}^3 + J_4^4t_4\ddot{\phi}_4 - \widehat{p_{pin4}^4}(R_4^3)^Tf_4^3 - (R_4^3)^TS_4r_4 = -J_4^4\left(R_4^3\widehat{\omega_{3,4}^4}\right)^T\omega_{0,3}^3 - \widehat{\omega_{0,4}^4}J_4^4\omega_{0,4}^4 + (R_4^3)^Tt_4u_4}$$

RW5

NE: $$\boxed{m_5\dot{v}_{0,3}^0 - m_5R_3^0\widehat{o_5^3}\dot{\omega}_{0,3}^3 - R_3^0 f_5^3 = -m_5R_3^0\widehat{\omega_{0,3}^3}\widehat{\omega_{0,3}^3}o_5^3}$$

EE:

$$\boxed{J_5^5\left(R_5^3\right)^T\dot{\omega}_{0,3}^3 + J_5^5t_5\ddot{\phi}_5 - \widehat{p_{pin5}^5}\left(R_5^3\right)^Tf_5^3 - \left(R_5^3\right)^TS_5r_5 = -J_5^5\left(R_5^3\widehat{\omega_{3,5}^5}\right)^T\omega_{0,3}^3 - \widehat{\omega_{0,5}^5}J_5^5\omega_{0,5}^5 + \left(R_5^3\right)^Tt_5u_5}$$

(d) write in matrix form as $Fg = h$ where g is a column matrix of unknowns, so you can solve easily in MATLAB as $g = F^{-1}h$

Let $g^T = [\dot{v}_{0,3}^0 \quad \dot{\omega}_{0,3}^3 \quad f_4^3 \quad f_5^3 \quad \ddot{\phi}_4 \quad \ddot{\phi}_5 \quad r_4 \quad r_5]$

Note that g is a $(18x1)$ vector of unknowns.

$$Fg = \overbrace{\begin{bmatrix} mI_{3\times3} & O_{3\times3} & R_3^0 & R_3^0 & O_{3\times1} & O_{3\times1} & O_{3\times2} & O_{3\times2} \\ O_{3\times3} & J_3 & \widehat{p_{pin4}^3} & \widehat{p_{pin5}^3} & O_{3\times1} & O_{3\times1} & S_4 & S_5 \\ m_4I_3 & \left(-m_4R_3^0\widehat{o_4^3}\right) & -R_3^0 & O_{3\times3} & O_{3\times1} & O_{3\times1} & O_{3\times2} & O_{3\times2} \\ O_{3\times3} & J_4^4(R_4^3)^T & -\widehat{p_{pin4}^4}R_4^{3^T} & O_{3\times3} & J_4^4t_4 & O_{3\times1} & -R_4^{3^T}S_4 & O_{3\times2} \\ m_5I_{3\times3} & \left(-m_5R_3^0\widehat{o_5^3}\right) & O_{3\times3} & -R_3^0 & O_{3\times1} & O_{3\times1} & O_{3\times2} & O_{3\times2} \\ O_{3\times3} & J_5^5R_5^{3^T} & O_{3\times3} & -\widehat{p_{pin5}^5}R_5^{3^T} & O_{3\times1} & J_5^5t_5 & O_{3\times1} & -R_5^{3^T}S_5 \end{bmatrix}}^{18} \begin{bmatrix} \dot{v}_{0,3}^0 \\ \dot{\omega}_{0,3}^3 \\ f_4^3 \\ f_5^3 \\ \ddot{\phi}_4 \\ \ddot{\phi}_5 \\ r_4 \\ r_5 \end{bmatrix} \; 18$$

$$h = \begin{bmatrix} 0 \\ -\widehat{w_{0,3}^{3}} J^{3} w_{0,3}^{3} - t_4 u_4 - t_5 u_5 \\ -m_4 R_3^{0} \widehat{w_{0,3}^{3}} \widehat{w_{0,3}^{3}} o_4^{3} \\ -J_4^{4} \left(R_4^{3} \widehat{w_{3,4}^{4}}\right)^{T} w_{0,3}^{3} - \widehat{w_{0,4}^{4}} J_4^{4} w_{0,4}^{4} + R_4^{3^T} t_4 u_4 \\ -m_5 R_3^{0} \widehat{w_{0,3}^{3}} \widehat{w_{0,3}^{3}} o_5^{3} \\ -J_5^{5} \left(R_5^{3} \widehat{w_{3,5}^{5}}\right)^{T} w_{0,3}^{3} - \widehat{w_{0,5}^{5}} J_5^{5} w_{0,5}^{5} + R_5^{3^T} t_5 u_5 \end{bmatrix}$$

2. (60 pts) You must implement everything marked "must change" in `hw5.m`. (You may, of course, also play around with anything marked "can change.") Submit a print-out *only* of the lines of code that are changed, in the order in which these lines appear in `hw5.m`.

To calculate the moment of inertia matrices, I coded a function called getJofBox. This function uses the equation you derived in homework 4. I also coded function to return rotation matrix and wedge matrix.

```
function J = getJofBox(m, dx, dy, dz)
% Get the moment of inertia of a box of mass m and side lengths dx, dy, dz
% (moment of inertia given w.r.t. center of mass of box)
J = (m/12) * diag([dy^2 + dz^2; dx^2 + dz^2; dx^2 + dy^2]);

function R = getR_xyz(theta)
s1 = sin(theta(1));
s2 = sin(theta(2));
s3 = sin(theta(3));
c1 = cos(theta(1));
c2 = cos(theta(2));
c3 = cos(theta(3));

R_3in0 = [c2*c3              -c2*s3                  s2;
    s1*s2*c3 + c1*s3      -s1*s2*s3 + c1*c3       -s1*c2;
    -c1*s2*c3 + s1*s3     c1*s2*s3 + s1*c3        c1*c2   ]

function w = wedge(a)
w = [0      -a(3)    a(2);
     a(3)    0        -a(1);
     -a(2)   a(1)     0 ];


% - Mass and moment of inertia of SC
robot.sc.m = robot.sc.rho * robot.sc.dx * robot.sc.dy * robot.sc.dz;
robot.sc.J_in3 = getJofBox(robot.sc.m, robot.sc.dx, robot.sc.dy,
robot.sc.dz);
```

```
% - Mass and moment of inertia of RW#4
robot.rw4.m = robot.rw4.rho * robot.rw4.dx * robot.rw4.dy * robot.rw4.dz;
robot.rw4.J_in4 = getJofBox(robot.rw4.m, robot.rw4.dx, robot.rw4.dy,
robot.rw4.dz);

% - Mass and moment of inertia of RW#5
robot.rw5.m = robot.rw5.rho * robot.rw5.dx * robot.rw5.dy * robot.rw5.dz;
robot.rw5.J_in5 = getJofBox(robot.rw5.m, robot.rw5.dx, robot.rw5.dy,
robot.rw5.dz);
```

% Part 2

```
    R_3in0 = getR_xyz(theta);
    o_4in0 = o_3in0 + R_3in0 * robot.o_4in3;
    o_5in0 = o_3in0 + R_3in0 * robot.o_5in3;

    c4     = cos(phi4);
    s4     = sin(phi4);
    R_4in3 = [ c4  0   s4;
               0   1   0;
               -s4 0  c4 ];
    R_4in0 = R_3in0 * R_4in3;

    c5     = cos(phi5);
    s5     = sin(phi5);
    R_5in3 = [ c5 -s5  0;
               s5  c5  0;
               0   0   1 ];
    R_5in0 = R_3in0 * R_5in3;

    for i=1:size(robot.sc.p_in3,2)
        robot.sc.p_in0(:,i) = o_3in0 + R_3in0*robot.sc.p_in3(:,i);
    end
    for i=1:size(robot.rw4.p_in4,2)
        robot.rw4.p_in0(:,i) = o_4in0 + R_4in0*robot.rw4.p_in4(:,i);
    end
    for i=1:size(robot.rw5.p_in5,2)
        robot.rw5.p_in0(:,i) = o_5in0 + R_5in0*robot.rw5.p_in5(:,i);
    end
```

GetRates function

```
function [o_3in0dot,thetadot,phi4dot,phi5dot,...                 % <- velocity
          v_03in0dot,w_03in3dot,phi4dotdot,phi5dotdot] = ...   % <- acceleration
             GetRates(o_3in0,theta,phi4,phi5,...                % <- configuration
                      v_03in0,w_03in3,phi4dot,phi5dot,...       % <- velocity
                      u4,u5,...                                 % <- input torques
                      robot)                                    % <- parameters that
describe the SC and RWs

% Define some variables to make F and h easier to read
R_3in0 = getR_xyz(theta);
s4 = sin(phi4);      c4 = cos(phi4);
s5 = sin(phi5);      c5 = cos(phi5);
```

```
R_4in3 = [c4 0 s4; 0 1 0; -s4 0 c4];
R_5in3 = [c5 -s5 0; s5 c5 0; 0 0 1];
S4 = [1 0; 0 0; 0 1];
S5 = [1 0; 0 1; 0 0];
t4 = [0; 1; 0];
t5 = [0; 0 ; 1];
m  = robot.sc.m;
m4 = robot.rw4.m;
m5 = robot.rw5.m;
J3 = robot.sc.J_in3;
J4 = robot.rw4.J_in4;
J5 = robot.rw5.J_in5;
w_04in4 = R_4in3'*w_03in3 + t4*phi4dot;
w_05in5 = R_5in3'*w_03in3 + t5*phi5dot;

F = [ m*eye(3), zeros(3,3),      R_3in0,      R_3in0,      zeros(3,6) ;
      zeros(3,3),  J3, wedge(robot.rw4.ppin_in3), wedge(robot.rw4.ppin_in3),
zeros(3,2), S4, S5;
      m4*eye(3,3),  -m4*R_3in0*wedge(robot.o_4in3), -R_3in0, zeros(3,9);
      zeros(3,3), J4*R_4in3', -wedge(robot.rw4.ppin_in4)*R_4in3', zeros(3,3), J4*t4,
zeros(3,1), -R_4in3'*S4, zeros(3,2);
      m5*eye(3,3), -m5*R_3in0*wedge(robot.o_5in3), zeros(3,3), -R_3in0, zeros(3,6);
      zeros(3,3), J5*R_5in3', zeros(3,3), -wedge(robot.rw5.ppin_in5)*R_5in3',
zeros(3,1), J5*t5, zeros(3,2), -R_5in3'*S5];

h = [ zeros(3,1);
      -wedge(w_03in3)*J3*w_03in3 - t4*u4 - t5*u5;
      -m4*R_3in0*wedge(w_03in3)*wedge(w_03in3)*robot.o_4in3;
      -J4*(R_4in3*wedge(t4*phi4dot))'*w_03in3 - wedge(w_04in4)*J4*w_04in4 +
R_4in3'*t4*u4;
      -m5*R_3in0*wedge(w_03in3)*wedge(w_03in3)*robot.o_5in3;
      -J5*(R_5in3*wedge(t5*phi5dot))'*w_03in3 - wedge(w_05in5)*J5*w_05in5 +
R_5in3'*t5*u5];

g = F\h;


s2 = sin(theta(2));
s3 = sin(theta(3));
c2 = cos(theta(2));
c3 = cos(theta(3));
B  = [ c3/c2        -s3/c2      0;
       s3           c3          0;
       -s2*c3/c2    s2*s3/c2    1];

thetadot    = B * w_03in3;
o_3in0dot   = v_03in0;
phi4dot     = phi4dot;
phi5dot     = phi5dot;
v_03in0dot = g(1:3);
w_03in3dot = g(4:6);
%f_4in3      = g(7:9);
%f_5in3      = g(10:12);
phi4dotdot = g(13);
phi5dotdot = g(14);
%r4          = g(15:16);
%r5          = g(17:18);
```

3. (60 pts) You must choose a task that is of interest to you and complete it. Examples of "a task" appear on the following page. You must submit the following things:

- A description of what you wanted to accomplish and why.
- A movie showing the results you were able to achieve, submitted as you did in HW4.

I decided to simulate detumbling (de-spinning) a spacecraft. This procedure is an important one become many satellites enter their orbits spinning at non-zero rates that must be reduced to 0 before mission operations can begin.

My first step was to alter the initial angular rate in the code.

```
w_03in3 = 0.1*randn(3,1);
```

Now each time the code starts my spacecraft will be spinning at a different rate and about a different axis.

When I tried to reduce my spacecraft's angular rate to 0, I had great difficult operating the controls. Part of the reason for this is that I am attempting 3-axis control using only 2 reaction wheels. For full 3-axis attitude control we need a minimum of 3 reaction wheel whose axes are not aligned. Since we only have 2 reaction wheels our maneuvering ability is diminished.