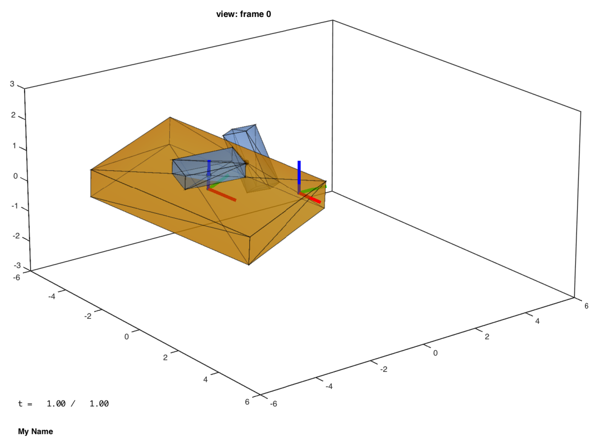
AE352 Homework #5:
Systems of Rigid Bodies: Spacecraft and Reaction Wheels

(due at the beginning of class on Wednesday, October 14)



The goal this week is to simulate the motion of the box-shaped spacecraft shown above (orange). Two box-shaped reaction wheels (blue) are attached to this spacecraft. A "reaction wheel" is simply another rigid body that is mounted on the shaft of a motor that is fixed to the spacecraft. When the spacecraft applies a torque to the reaction wheel with the motor, the reaction wheel applies an equal and opposite torque back to the spacecraft. Taken together, the spacecraft and the two reaction wheels are a *system of rigid bodies* that are subject to constraints. It may surprise you that—with some care—you can apply what you already know to model this system. The challenge is handling the constraints. We will learn how to handle them together in lecture.

You will be adding code to the MATLAB script `hw5.m`, available on the course website. Everything you need to know about both the spacecraft and the reaction wheels is in the script. Groups of lines are labeled "must change" (for things like implementing coordinate transformations and finding rates of change), "can change" (for things like specifying initial conditions or making movies), and "can't change" (for things that happen behind the scenes). Much of the code will be familiar to you from HW1-HW4. We will discuss what is not familiar in lecture.

If you like, you may work in pairs. You must submit the following:

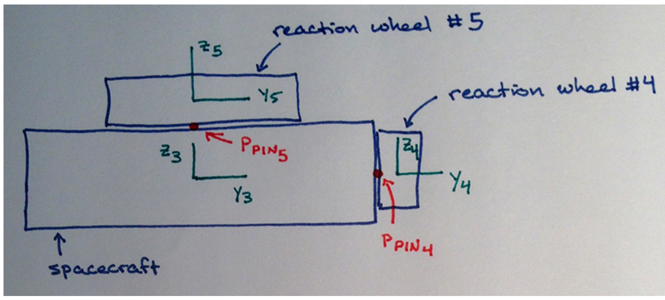
- A pencil-and-paper derivation of everything you implement.
- The lines of code in `hw5.m` that you add or change.
- A movie of results that are of interest to you—details of submission TBA. (Things that might be "of interest" include using the reaction wheels to either reorient or de-spin the spacecraft.)

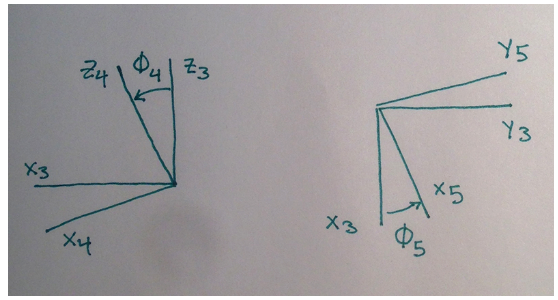
This assignment is obviously different from what you have seen in prior weeks. There is no busywork and far less structure. I want you to come away from this assignment with confidence that you can model a fairly complicated dynamic system from start to finish.

It is important that you start this assignment right away. If you do a little bit every day, you will be happy and at peace. If you wait until the night before, you will be in a world of hurt.

More about the geometry of the spacecraft

Frame 3 is fixed to the spacecraft, with o_3 at its center of mass. There are two reaction wheels, which we will call RW4 and RW5. Frame 4 is fixed to RW4, with o_4 at the center of mass of RW4. Frame 5 is fixed to RW5, with o_5 at the center of mass of RW5. Each reaction wheel rotates about a single axis that is fixed in the spacecraft. In particular, RW4 rotates about y_3 and RW5 rotates about z_3. Assuming that frames 3, 4, and 5 are all initially aligned, we call ϕ_4 the angle frame 4 has rotated about y_3, and call ϕ_5 the angle frame 5 has rotated about z_3. The pictures below should make these relationships clear.

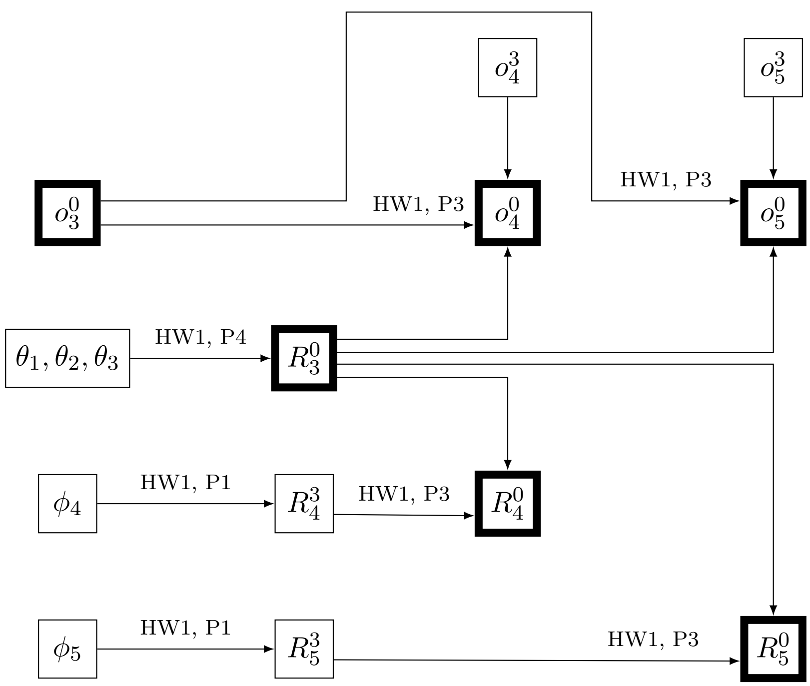




In the code, you'll also see the variables `robot.rw4.ppin_in3` and `robot.rw5.ppin_in3`. These variables describe (in the coordinates of frame 3) the points p_{pin_4} and p_{pin_5} at which RW4 and RW5, respectively, are attached to the spacecraft. These points are also indicated in the pictures.

More about how to compute position and orientation of everything

This diagram shows the "flow of information" to compute o and R for frames 3, 4, and 5. On the left are state variables (given to you). On the top are parameters (given to you). Everything else you compute. I've indicated the problems on HW1 that you should review, to remind you how.



Deliverables

Please submit the following things. If you choose to work with a partner, please submit *one copy of your assignment* with both your and your partner's name on the front page. You are also welcome to work by yourself.

1. (120 pts) You must derive expressions for the following things by hand, including any diagrams necessary to complete these derivations:

   - position and orientation of both reaction wheels

   $$o_4^0, R_4^0 \text{ (also requires } R_4^3), o_5^0, R_5^0 \text{ (also requires } R_5^3)$$

   - linear and angular velocity of both reaction wheels

   $$v_{0,4}^0, w_{0,4}^4 \text{ (also requires } w_{3,4}^4), v_{0,5}^0, w_{0,5}^5 \text{ (also requires } w_{3,5}^5)$$

   - linear and angular acceleration of both reaction wheels

   $$\dot{v}_{0,4}^0, \dot{w}_{0,4}^4, \dot{v}_{0,5}^0, \dot{w}_{0,5}^5$$

   - mass and moment of inertia of everything

   $$m, J^3, m_4, J_4^4, m_5, J_5^5$$

   - constraint forces and torques

   $$f_4^3, \tau_4^3, f_5^3, \tau_5^3$$

   - equations of motion

     (a) write both Newton's Equation and Euler's Equation for the spacecraft and for each reaction wheel separately

     (b) plug in for $\dot{v}_{0,4}^0$, $\dot{w}_{0,4}^4$, τ_4, $\dot{v}_{0,5}^0$, $\dot{w}_{0,5}^5$, and τ_5

     (c) put unknowns on left-hand-side and knowns on right-hand-side

     (d) write in matrix form as $Fg = h$ where g is a column matrix of unknowns, so you can solve easily in MATLAB as $g = F^{-1}h$

2. (60 pts) You must implement everything marked "must change" in `hw5.m`. (You may, of course, also play around with anything marked "can change.") Submit a print-out *only* of the lines of code that are changed, in the order in which these lines appear in `hw5.m`.

3. (60 pts) You must choose a task that is of interest to you and complete it. Examples of "a task" appear on the following page. You must submit the following things:

   - A description of what you wanted to accomplish and why.
   - A movie showing the results you were able to achieve, submitted as you did in HW4.

   If you believe that you were not successful in doing what you wanted (e.g., if you encountered a coding error that you could not resolve), then please attempt to describe what went wrong in your video. If you do this then you can still receive full credit for this part of the assignment.

Examples of "a task"

Here are four examples of "a task" that you might choose to accomplish.

- *Reorient the spacecraft.* Specify an initial orientation of the spacecraft in which frames 0 and 3 are not aligned. Show how to "fly" the spacecraft so that, in its final orientation, frames 0 and 3 are aligned (or nearly so). You might consider whether or not this is always possible, for any initial orientation. You might also consider—and describe in your video—how this maneuver could be used in a real spacecraft mission.

- *De-spin the spacecraft.* Specify an initial angular velocity of the spacecraft (you'll have to do this very carefully, if you don't want the spacecraft to drift away...). Show how to "fly" the spacecraft so that its final angular velocity is aligned with the x_0 axis (or nearly so). You might consider whether or not this is always possible, for any initial angular velocity. You might also consider—and describe in your video—how this maneuver could be used in a real spacecraft mission.

- *Analysis of momentum.* Show results (i.e., empirical data from simulation) verifying that the linear and angular momentum of the entire system—spacecraft and two reaction wheels, taken together—are conserved (i.e., remains constant) no matter what torques are applied to the reaction wheels by the spacecraft. Show results verifying that the linear and angular momentum of the spacecraft and each reaction wheel, taken separately, are not conserved. You might discuss—and describe in your video—what these results mean.

- *Analysis of performance as a function of reaction wheel shape and size.* Show results (i.e., empirical data from simulation) that demonstrate the effects of changing the shape and size (i.e., the mass and moment of inertia) of each reaction wheel. You might consider the implications of these results for spacecraft design. You might discuss—and describe in your video—when these results mean.

You may, of course, choose any other task of similar scope that is of more interest to you instead.