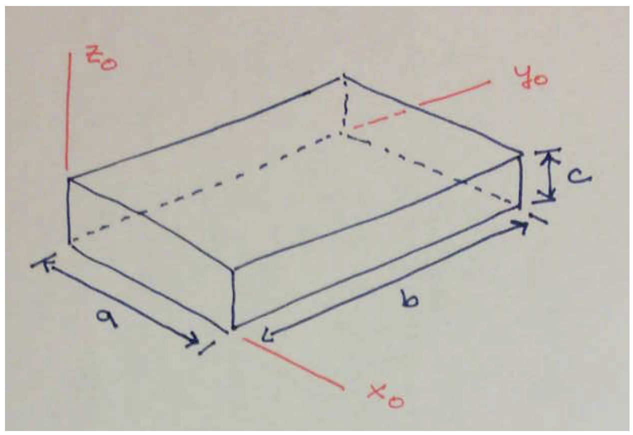
AE352 Homework #4: Mass, Inertia, Design

(due at the beginning of class on Wednesday, September 30)

1. Suppose that frame k is glued to a rigid body and that o_k is at the center of mass. Also, suppose that m is the mass of the rigid body and that J^k is the moment of inertia matrix in the coordinates of frame k. Prove that:

$$J^j = R_k^j J^k {R_k^j}^T + m \widehat{o_k^j}^T \widehat{o_k^j}$$

This formula is often called the "parallel axis theorem."



2. Consider the box shown above. Assume it has uniform, unit density. Note that, as implied by the picture, o_0 is at one corner of the box.

(a) Compute the mass of the box by evaluating the integral

$$\int dm$$

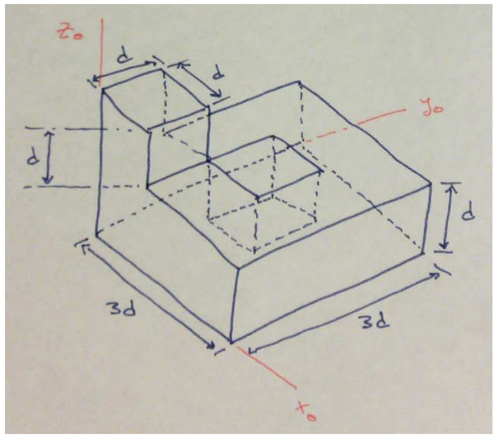
(b) Compute the center of mass of the box—a point, expressed in the coordinates of frame 0—by evaluating the integral

$$\frac{\int p^0 dm}{\int dm}$$

(c) Define another frame 1 with its coordinate axes aligned with frame 0 (i.e., with $R_1^0 = I$) and with its origin at the center of mass of the box. Compute the moment of inertia matrix of the box in the coordinates of frame 1 by evaluating the integral

$$J^1 = \int \widehat{p^1}^T \widehat{p^1} dm$$

Yes, you can look up formulas for all of these things online and in most textbooks. The point here is to derive them yourself.



3. Shown above is a box with a square base, with a cube cut out of its center and stacked on one corner. We will call this object "the body." Assume it has uniform, unit density. Note that, as implied by the picture, o_0 is at one corner of the body.

   (a) Compute the mass of the body.

   (b) Compute the center of mass of the body and express it in the coordinates of frame 0.

   (c) Define another frame 1 with its coordinate axes aligned with frame 0 (i.e., with $R_1^0 = I$) and with its origin at the center of mass of the body. Compute the moment of inertia matrix of the body in the coordinates of frame 1.

   It is possible to do all three of these things by evaluating the same integrals as in P2. However, it is easier to divide the body into three parts (the box, the "hole," and the cube), to compute the mass, center of mass, and moment of inertia of each part using the formulas you derived in P2, and then to combine your results (e.g., using the formula from P1).

4. The point of last week's homework was to visualize the motion of a *given* rigid body (a spacecraft) when it is subject to *given* applied forces (from thrusters). The point of this week's homework is to *design* a spacecraft and to *choose* a set of thrusters that allow you to achieve a given task. You have complete freedom in your design, subject to these rules:

   - The spacecraft must have mass 1 kg and a uniform density of 1 kg/m^3. It must be one continuous shape (i.e., everything must be "attached together") and it must have volume everywhere. It must have at most 10 thrusters. Each thruster must exert a force of magnitude at most 0.25 N. The point of application of this force must be somewhere on the surface of the spacecraft, and the direction must be inward normal to the surface at this point. No two thrusters can be placed at exactly the same point.

   The goal is to fly the spacecraft so that frame 3 is aligned with frame 0—as fast as possible, without running into the wall (!?!). You will be adding code to the MATLAB script `hw4.m`, available on the course website. You may work in pairs. The code allows you to record and make a movie of your flight. **Yes, this facilitates a friendly competition—stay tuned!** Please submit (1) your code, (2) a movie of your fastest flight, (3) a description of your final design—its geometry, mass, and inertial properties—clearly indicating that you followed the rules, and (4) a description of your design process, with whatever you think is necessary to justify your choice of final design (for example, how does your ability to turn quickly change as you change the geometry of the spacecraft, and can you explain this by looking at your moment of inertia matrix?). **Details will be discussed in class.**