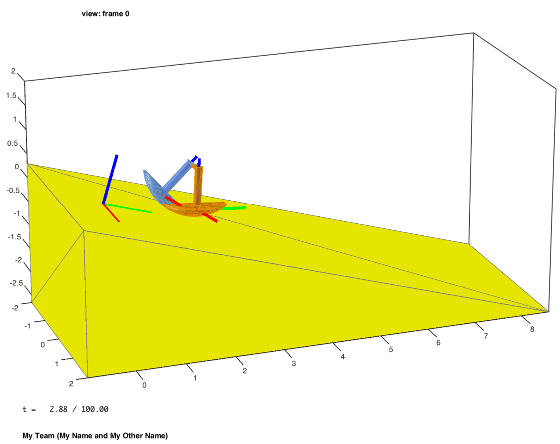
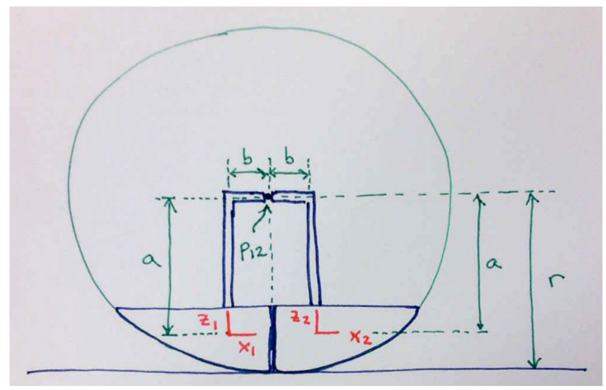
AE352 Homework #8:
Rolling and Impact: Passive Walker

(due at the beginning of class on Friday, November 20)





The goal this week is to simulate the motion of the two-legged robot shown above left. To do so, you will be adding code to the MATLAB script `hw8.m`, available on the course website. Groups of lines are labeled "must change" (for things like implementing coordinate transformations and finding rates of change), "can change" (for things like specifying initial conditions or making movies), and "can't change" (for things that happen behind the scenes). Much of the code will be familiar to you from HW1-HW7. Again this week, every function ends with an "`end`" statement. If you create one or more functions yourself, you'll need to end them with an "`end`" statement as well.

Each foot of the robot is half a spherical cap. Frame 0 is fixed to the ground. Frame 1 is fixed to the left foot, with its origin at the center of mass. Frame 2 is fixed to the right foot, with its origin at the center of mass. The two feet are connected by a revolute joint, which allows them to rotate with respect to each other about an axis parallel to x_1 (equivalently, x_2). The point at which the two feet are connected together is p_{12}. It should be clear from the picture shown above right that:

$$p_{12}^1 = \begin{bmatrix} b \\ 0 \\ a \end{bmatrix} \qquad\qquad p_{12}^2 = \begin{bmatrix} -b \\ 0 \\ a \end{bmatrix}$$

These parameters are all constant, and are given to you in the code. Other useful parameters are also given, for example the mass and moment of inertia of everything (m_L, J_L^1, etc.) and the gravity force vector written in the coordinates of frame 0:

$$g^0 = 9.81 \begin{bmatrix} 0 \\ \sin\phi \\ -\cos\phi \end{bmatrix},$$

where *phi* is the angle of the slope down which the robot is walking (you can change this angle!). As usual, see `GetGeometryOfRobot` for a list of these parameters.

The robot is subject to four sources of force and torque:

- The force of gravity. This force acts on both feet.
- The force applied by the left foot to the right foot through the revolute joint, denoted by f_{12}^1, acting at p_{12}. An equal and opposite force acts on the left foot.
- The torque applied by the left foot to the right foot through the revolute joint, denoted by τ_{12}^1. There is not motor this week—the torque is due only to the constraint and to viscous friction. I recommend you write

$$\tau_{12}^1 = t_{12}\left(-k\left(\text{“blah”}\right)\right) + S_{12}r_{12}$$

  for appropriate constants t_{12} and S_{12}, where “blah” is an expression for the angular velocity of the right foot with respect to the left foot about the joint axis (i.e., it is the projection of $w_{1,2}^2$ onto the x_2 axis). An equal and opposite torque acts on the left foot.
- The force applied by the ground on one of the two feet. Only one foot is in contact with the ground at any given time. Assume that friction is sufficiently high that this foot rolls without slipping. Denote the force by f_{01}^0 or f_{02}^0 depending on if the left foot (1) or right foot (2) is in contact with the ground. The z_0 component of this force is the “normal force” that keeps the foot from falling through the ground. The x_0 and y_0 components of this force make up the “tangential force” or “friction force” that keeps the foot from slipping.

You can take almost exactly the same approach to modeling this system as you took last week. There are two differences.

The first difference, which is *not* a big deal, is that you have to model the system twice—once assuming the left foot is in contact with the ground, and once assuming the right foot is in contact. We will describe the position and orientation of each foot separately—as (o_1^0, R_1^0) and (o_2^0, R_2^0), where each rotation matrix is parameterized by a separate ZYX Euler Angle sequence. There will be a total of 34 unknowns:

- the linear and angular acceleration of each frame (12 unknowns)

$$\dot{v}_{0,1}^0, \dot{w}_{0,1}^1, \dot{v}_{0,2}^0, \dot{w}_{0,2}^2$$

- the constraint force and torque associated with the revolute joint (5 unknowns)

$$f_{12}^1, r_{12}$$

- the constraint force associated with the rolling contact (3 unknowns)

$$f_{01}^0 \text{ or } f_{02}^0$$

You will derive a total of 20 equations:

- Newton's Equation and Euler's Equation for each body (12 equations)
- the constraint on the relative position of one foot with respect to the other due to the revolute joint—to derive this constraint, write an expression for p_{12}^1 in terms of p_{12}^2 and take the time derivative twice (3 equations)

- the constraint on the relative orientation of one foot with respect to the other due to the revolute joint—to derive this constraint, write an expression for $S_{12}^T w_{1,2}^2$ and take the time derivative (2 equations)
- the constraint that the contact foot rolls without slipping (3 equations)

It may sound like deriving two models is a lot of work, but it's not really, because the models for the left foot and the right foot are almost identical. You should find the paperwork even easier this week than last week.

The second difference, which *is* a big deal, is that you have to model the impact that occurs when switching from one foot to another. We will discuss how to do so in class. Happily, the equations you'll derive to model this impact are almost identical to the ones described above.

Please submit the following things. Like last week, you may—but are not required to—work in pairs. If you choose to work with a partner, please submit *one copy of your assignment* with both your and your partner's name on the front page.

1. (120 pts) You must derive the equations of motion and write them in matrix form as $F\gamma = h$, where γ is a column matrix of unknowns, so you can solve easily in MATLAB as $\gamma = F^{-1}h$.
2. (60 pts) You must implement everything marked "must change" in `hw8.m`. (You may, of course, also play around with anything marked "can change.") Submit your entire code online—details of the submission process will be forthcoming.
3. (60 pts) You must choose a task that is of interest to you and complete it. Examples of "a task" will be forthcoming. You must submit the following things:
   - A description of what you wanted to accomplish and why.
   - A movie showing the results you were able to achieve, submitted as in HW4-HW7.

   If you believe that you were not successful in doing what you wanted (e.g., if you encountered a coding error that you could not resolve), then please attempt to describe what went wrong in your video. If you do this then you can still receive full credit for this part of the assignment.

Like last week, it is important that you **start this assignment right away**, even though it is not due until the day before fall break. You should proceed carefully and methodically through each part of the derivation and implementation.

Remember how important it is to be clear and organized in your approach to the derivation, so that you can compare your work to others. If you are working with a partner, both of you should do the derivation, and check to see if you agree. If you are working alone, it will still benefit you to check your work with others.