Homework SSSP

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1 Abstract

In this assignment, we developed a Wave Digital Filter (**WDF**) model of a piezoelectric **MEMS** loudspeaker, based on its linear lumped-element model (**LEM**) representation. Starting from the equivalent mechanical domain circuit, the corresponding **WDF** was derived according to the following schematic:

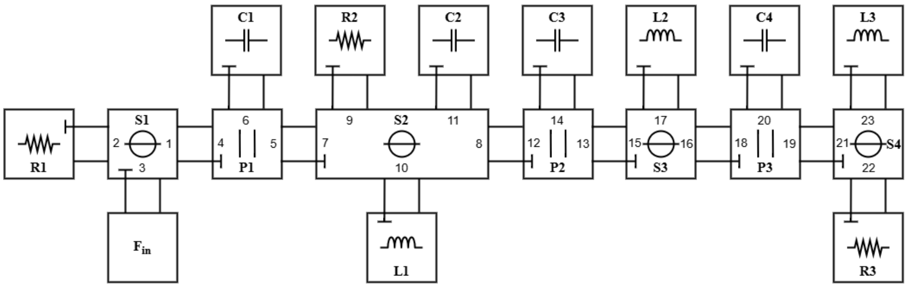

Figure 1: WDF Representation

2 Wave Digital Filter Architecture

- **Root Element**:
  The root of the WDF is the only non-adaptable element: the ideal voltage source F_{in} (corresponding to a force generator in the mechanical domain in impedance analogy).
- **Series Junctions**:
  Denoted as **S1**, **S2** (the only five-port series junction; its scattering matrix was obtained using the fundamental loop matrix **B** method in order to optimize computational efficiency and improve performances), **S3** and **S4** (connected to the resistor **R3**, from which the output signal is extracted for comparison).
- **Parallel Junctions**:
  Denoted as **P1**, **P2**, **P3**

3 Configuration of Free Parameters

The **adaptation conditions** for the individual ports are defined as:

$$Z_2 = R_1, \quad Z_6 = \frac{T_s}{2C_1}, \quad Z_9 = R_2, \quad Z_{10} = \frac{2L_1}{T_s}, \quad Z_{11} = \frac{T_s}{2C_2},$$
$$Z_{14} = \frac{T_s}{2C_3}, \quad Z_{17} = \frac{2L_2}{T_s}, \quad Z_{20} = \frac{T_s}{2C_4}, \quad Z_{22} = R_3, \quad Z_{23} = \frac{2L_3}{T_s}.$$

In order to achieve **reflection-free adaptors**, the following composite impedances were computed:

$$Z_{21} = Z_{22} + Z_{23}, \quad Z_{18} = \frac{Z_{19} \cdot Z_{20}}{Z_{19} + Z_{20}}, \quad Z_{15} = Z_{16} + Z_{17},$$
$$Z_{12} = \frac{Z_{13} \cdot Z_{14}}{Z_{13} + Z_{14}}, \quad Z_7 = \sum_{i=8}^{11} Z_i, \quad Z_4 = \frac{Z_5 \cdot Z_6}{Z_5 + Z_6}, \quad Z_3 = Z_1 + Z_2.$$

The expressions of the Scattering Matrices can be found in the code implementation.

4 Output Signal Analysis

The comparison between the output signal of the Wave Digital Filter (**WDF**) and the **ground-truth** signal is presented below, both in the time and frequency domains.

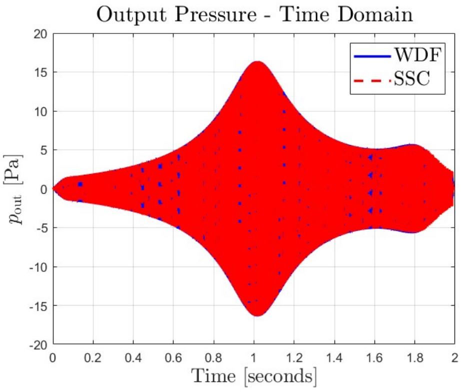

Figure 2: Time Domain analysis

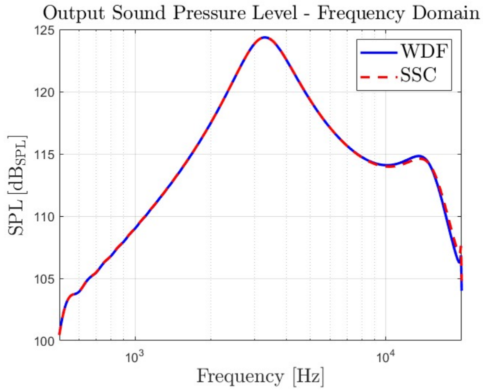

Figure 3: Frequency Domain Analysis

As shown in the plots, the **WDF** output closely matches the reference signal within the frequency range of interest, that is the **audio bandwidth** spanning roughly from 20 Hz to 20 kHz.

The Mean Squared Error (**MSE**) between the two signals is **0.0119**, and the corresponding error signal is also plotted in time for further inspection.

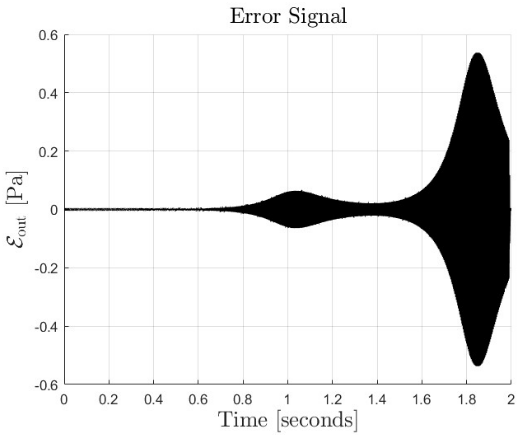

Figure 4: Error Signal

From the error analysis, the model shows good accuracy in the low- to mid-frequency range, with performance degrading at higher frequencies. This behavior is expected, given the input sweep from 500 Hz to 20 kHz and the frequency warping introduced by the bilinear transform. Although a high sampling rate helps reducing this effect, as applied here, some distortion remains above 10 kHz. However, even in this range, the error is still within acceptable limits (max peak $\approx \pm 0.5$ `Pa`).

5 Conclusion

In conclusion, the results confirm the effectiveness of the proposed **WDF** model in replicating the behavior of the piezoelectric **MEMS** loudspeaker across the target frequency range. The use of reflection-free ports ensured numerical stability and proper wave interaction. Furthermore, merging three series adaptors into a single five-port adaptor (**S2**) improved computational efficiency, reducing the simulation time from ≈ 11 to 9.5 seconds (of course it is processor-dependent).
Overall, the **WDF** approach proves to be well suited for modeling electromechanical systems with complex impedance, especially under performance or real-time constraints.