EXERCISE 7
TRASVERSE-FIELD ISING MODEL

QUANTUM INFORMATION AND COMPUTING COURSE 2021/2022

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EXERCISE GOALS

Consider N spin-$^1/_2$ particles on a one-dimensional lattice, described by the Hamiltonian

$$H = \lambda \sum_i^N \sigma_z^i - \sum_i^{N-1} \sigma_X^i \, \sigma_X^{i+1}$$

where σ's are the Pauli matrices and λ is the interaction strength.

- Write a program that computes the $2^N \times 2^N$ matrix representation of the Hamiltonian H for different N
- Diagonalize H for different N = 1, … , N_{max} and λ ∈ [0 : 3]. How big is N_{max} you can reach?
- Plot the first k levels as a function of λ for different N. Comment the spectrum.

IN THEORY…

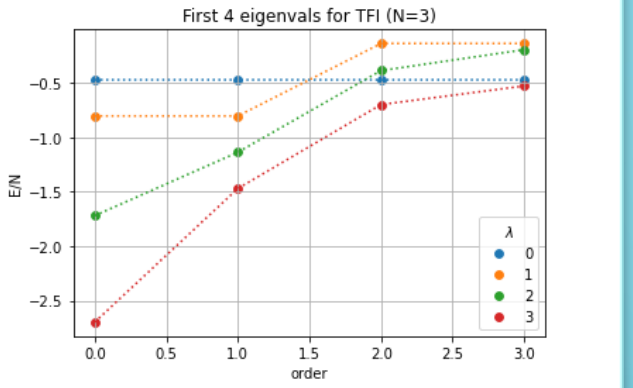
- System subject to quantum fluctuations: at critical point we face quantum phase transition (analytical solutions possible)
- For $\lambda = 0$: minimum energy if all spins aligned (in X), if $\lambda \gg 1$: first term dominates and minimal energy for all spins along Z
- To enhance programming, recall that $1_m \otimes 1_n = 1_{nm}$
- Enlarging system size is hard: H scales exponentially with N

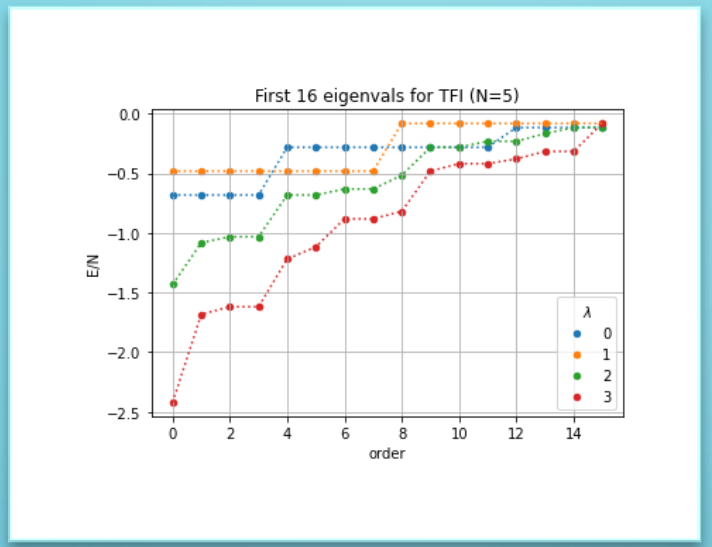
CODE DEVELOPMENT:

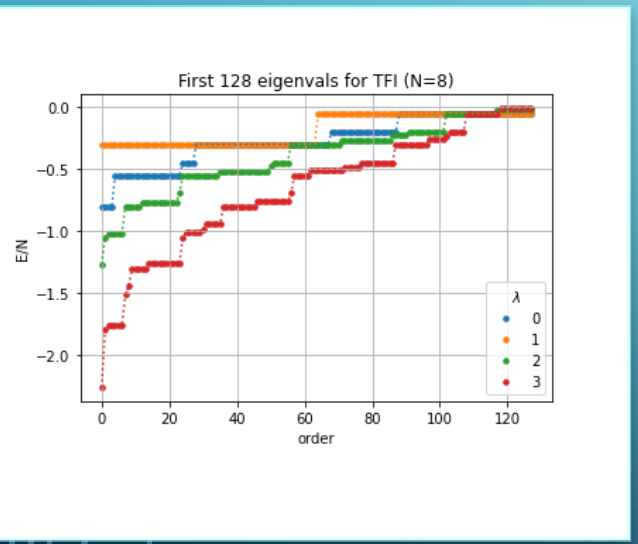
- Diagonalization via ZHEEV (LAPACK)
- Computational time bearable (O(s)) up to N = 12
- For N = 13, time = O(min); N = 16 implies memory error

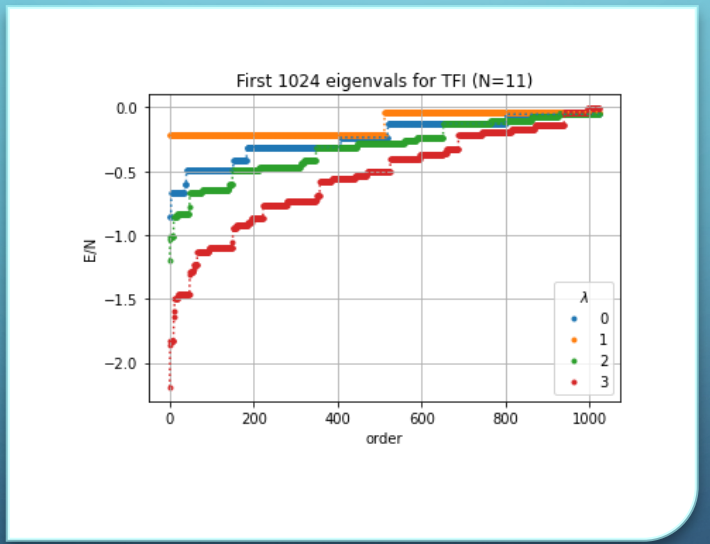
TFI generator

```
H = 0.d0

! Define once for all the tensor product of two sigma X
sx_X_sx = TensorProduct(sigma_X(),sigma_X())
! Define once for all the tensor product of sigma Z and ID(2)
sz_X_id = TensorProduct(sigma_Z(),Identity(2))

! First multi-site term
H = - TensorProduct(sx_X_sx,Identity(2**(System_size-2)))

! First interaction term
H = H + lambda * TensorProduct(sigma_Z(),Identity(2**(System_size-1)))

! Other multi-site terms
do ii=2,System_size-1

    H = H - TensorProduct(TensorProduct(Identity(2**(ii-1)),sz_X_id),Identity(2**(System_size-ii-1)))
    H = H + lambda * TensorProduct(TensorProduct(Identity(2**(ii-1)),sz_X_id),Identity(2**(System_size-ii-1)))

end do
```









SPECTRA FOR DIFFERENT N, λ:

- Large degeneracy for smaller λ values
- Spectrum is symmetric in energy
- Larger λ allows to reach lower energy per spin
- Neat jump in energy for $\lambda = 1$: first 2^{N-2} eigenvals are in degenerate ground state, then 2^{N-2} degenerate in an higher energy state (no similar trends for other values of λ).

NUMERICAL GROUND STATE EIGENVECTORS (N=3):

$\lambda = 0$ $\quad |\psi> = 0.354 \cdot \sum_{i=0}^{7} |i>$

$\lambda = 1$ $\quad |\psi> = 0.493 \cdot (|000> + |111>) + 0.085 \cdot (|001> + |110>) + 0.085 \cdot (|010> + |101>) + 0.493 \cdot (|011> + |100>)$

$\lambda = 2$ $\quad |\psi> = 0.577 \cdot (|000> + |111>) + 0.382 \cdot (|001> + |110>) + 0.130 \cdot (|010> + |101>) + 0.065 \cdot (|011> + |100>)$

$\lambda = 3$ $\quad |\psi> = 0.597 \cdot (|000> + |111>) + 0.325 \cdot (|001> + |110>) + 0.155 \cdot (|010> + |101>) + 0.117 \cdot (|011> + |100>)$

For $\lambda = 0$ the behaviour is greatly different from others, but a general symmetry is respected for all Runs (in particular, amplitude probabilities are not affected by global bit flips). For large λ changes in the absolut value of coeffs is minimal.