MEG con PIVOTING

Data: Data $A^{(1)} = A$ e $P = I$;
for $k = 1, \dots, n-1$ **do**
  $f \leftarrow |a_{fk}^{(k)}| = \max_{r=k,\dots,n} |a_{rk}^{(k)}|$;
  scambio la riga k con la riga f in A, in P e in $\boldsymbol{b}$;
  for $i = k+1, \dots, n$ **do**
    $l_{ik} \leftarrow \dfrac{a_{ik}^{(k)}}{a_{kk}^{(k)}}$;
    for $j = k+1, \dots, n$ **do**
      $a_{ij}^{(k+1)} \leftarrow a_{ij}^{(k)} - l_{ik} a_{kj}^{(k)}$;
    end
    $b_i^{(k+1)} \leftarrow b_i^{(k)} - l_{ik} b_k^{(k)}$;
  end
end

// cerco la riga che massimizza il pivot
// scambio le righe
// non ho più divisione per 0!
(come il caso base)

ESEMPIO $A = A^{(1)} = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 2 & 2 \\ 3 & 6 & 4 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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  end
end

k = 1

$a_{r1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $A^{(1)} = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix}$ $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

i = 2

$\ell_{2,1} \leftarrow \dfrac{a_{21}}{a_{11}} = \dfrac{2}{3}$

j = 2

$a_{2,2}^{(2)} \leftarrow 2 - \tfrac{2}{3} \cdot 6 = -2$

j = 3

$a_{2,3}^{(2)} \leftarrow 2 - \tfrac{2}{3} \cdot 4 = \dfrac{-2}{3}$

$A^{(2)} = \begin{bmatrix} 3 & 6 & 4 \\ 0 & -2 & -2/3 \\ 0 & -1 & 5/3 \end{bmatrix}$

i = 3

$\ell_{3,1} \leftarrow \dfrac{a_{31}}{a_{11}} = 1/3$

j = 2

$a_{3,2}^{(2)} \leftarrow 1 - \tfrac{1}{3} \cdot 6 = -1$

j = 3

$a_{3,3}^{(2)} \leftarrow 3 - \tfrac{1}{3} \cdot 4 = 5/3$

k = 2

$a_{r2}^{(2)} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$ non faccio scambi

i = 3

$\ell_{3,2} \leftarrow \dfrac{a_{3,2}^{(2)}}{a_{2,2}} = 1/2$

j = 3

$a_{3,3}^{(3)} \leftarrow 5/3 - \tfrac{1}{2}(-2/3) = 2$

$A^{(3)} = U = \begin{bmatrix} 3 & 6 & 4 \\ 0 & -2 & -2/3 \\ 0 & 0 & 2 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & 1/2 & 1 \end{bmatrix}$

$LU = PA \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 6 & 4 \\ 0 & -2 & -2/3 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix}$