

Data: Data $A^{(1)} = A$;

for $k = 1, \dots, n-1$ do

for $i = k+1, \dots, n$ do

$$l_{ik} \leftarrow \frac{a_{ik}^{(k)}}{a_{kk}^{(k)}};$$

for $j = k+1, \dots, n$ do

$$a_{ij}^{(k+1)} \leftarrow a_{ij}^{(k)} - l_{ik} a_{kj}^{(k)};$$

end

$$b_i^{(k+1)} \leftarrow b_i^{(k)} - l_{ik} b_k^{(k)};$$

end

end

// costruisce $A^{(2)}, A^{(3)}$ eliminando x_2, x_3
// ciclo sulle righe da $k+1$ all'ultima

// trova il moltiplicatore

// ciclo sulle colonne: le prime k sono 0

// le altre si trovano sottraendo la riga k

// uguale sul termine noto

ESEMP: $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 5 & 1 \\ 6 & 1 & 10 \end{bmatrix} \cdot A^{(1)} = A$

($k=1$)

$i=2$

$$l_{21} \leftarrow \frac{a_{21}^{(1)}}{a_{11}^{(1)}} = \frac{4}{2} = 2$$

$j=2$

$$a_{2,2}^{(2)} \leftarrow 5 - 2 \cdot 1 = 3$$

$j=3$

$$a_{2,3}^{(2)} \leftarrow 1 - 2 \cdot 1 = -1$$

$i=3$

$$l_{31} \leftarrow \frac{a_{31}^{(1)}}{a_{11}^{(1)}} = 3$$

$j=2$

$$a_{3,2}^{(2)} \leftarrow 1 - 3 \cdot 1 = -2$$

$j=3$

$$a_{3,3}^{(2)} \leftarrow 10 - 3 \cdot 1 = 7$$

Data: Data $A^{(1)} = A$;

for $k = 1, \dots, n-1$ do

for $i = k+1, \dots, n$ do

$$l_{ik} \leftarrow \frac{a_{ik}^{(k)}}{a_{kk}^{(k)}};$$

for $j = k+1, \dots, n$ do

$$a_{ij}^{(k+1)} \leftarrow a_{ij}^{(k)} - l_{ik} a_{kj}^{(k)};$$

end

$$b_i^{(k+1)} \leftarrow b_i^{(k)} - l_{ik} b_k^{(k)};$$

end

end

$$\Rightarrow A^{(2)} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & -2 & 7 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & ? & 1 \end{bmatrix}$$

$$A^{(3)} = U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 19/3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -2/3 & 1 \end{bmatrix}$$

$k=2$

$i=3$

$$l_{32} \leftarrow \frac{a_{32}^{(2)}}{a_{22}^{(2)}} = -\frac{2}{3}$$

$j=3$

$$a_{3,3}^{(3)} = 7 - \left(-\frac{2}{3}\right)(-1) = \frac{19}{3} \Rightarrow$$

Verifica: $L \cdot U = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -2/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 19/3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 5 & 1 \\ 6 & 1 & 10 \end{bmatrix} \quad \checkmark$$