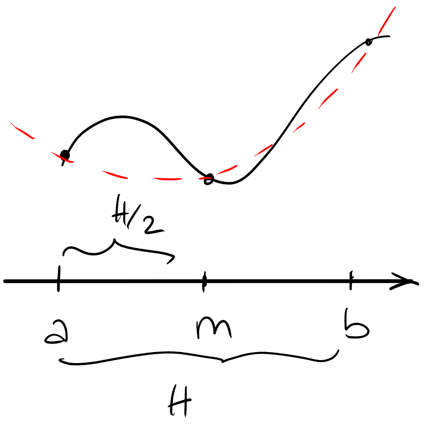
Dimostrazione formula di Simpson



--- parabola $y = \alpha_0 + \alpha_1 x + \alpha_2 x^2$

Deve passare per $(a, f(a))$
$(m, f(m))$
$(b, f(b))$

Cambio coordinate per comodità:
$\tilde{x} = x - a$

Cerco $y = \beta_0 + \beta_1 \tilde{x} + \beta_2 \tilde{x}^2$

passante per $(0, f(a))$
$(H/2, f(m))$
$(H, f(b))$

$\tilde{x} = 0 \qquad f(a) = \beta_0$

$$\left.\begin{aligned} \tilde{x} = H/2 \qquad & f(m) = \beta_1 H/2 + \beta_2 \frac{H^2}{4} + f(a) \\ \tilde{x} = H \qquad & f(b) = \beta_1 H + \beta_2 H^2 + f(a) \end{aligned}\right\} \quad f(b) - 2f(m) = \beta_2 \frac{H^2}{2} - f(a) \qquad \beta_2 = \frac{2f(a) + 2f(b) - 4f(m)}{H^2}$$

$$f(b) = \beta_1 H + 2f(b) - 4f(m) + 3f(a) \qquad \beta_1 = \frac{4f(m) - f(b) - 3f(a)}{H}$$

$$\int_0^H \beta_0 + \beta_1 \tilde{x} + \beta_2 \tilde{x}^2 = \beta_0 H + \frac{\beta_1 H^2}{2} + \frac{\beta_2 H^3}{3} = f(a)H + \frac{H}{2}\left(4f(m) - f(b) - 3f(a)\right)$$
$$+ \frac{H}{3}\left(2f(b) - 4f(m) + 2f(a)\right)$$

$$= \frac{H}{6}\left(6f(a) + 12f(m) - 3f(b) - 9f(a) + 4f(b) - 8f(m) + 4f(a)\right)$$

$$= \frac{H}{6}\left[f(a) + 4f(m) + f(b)\right] \; !$$