

# Asymmetric neural networks and the process of learning

To cite this article: G Parisi 1986 *J. Phys. A: Math. Gen.* **19** L675

View the [article online](#) for updates and enhancements.

## You may also like

- [Efficient temporal processing with biologically realistic dynamic synapses](#)  
Thomas Natschläger, Wolfgang Maass and Anthony Zador
- [Gate-tunable plasticity in artificial synaptic devices based on four-terminal amorphous gallium oxide memristors](#)  
Taishi Ikeuchi, Yusuke Hayashi, Tetsuya Tohei et al.
- [Ionically gated transistors based on two-dimensional materials for neuromorphic computing](#)  
Ke Xu and Susan K Fullerton-Shirey

## LETTER TO THE EDITOR

# Asymmetric neural networks and the process of learning

Giorgio Parisi

Dipartimento di Fisica, Il Università' di Roma 'Tor Vergata', Via Orazio Raimondo, Roma 00173, Italy and INFN, Sezione di Roma, Italy

Received 15 April 1986

**Abstract.** In this letter we study the influence of a strong asymmetry of the synaptic strengths on the behaviour of a neural network which works as an associative memory. We find that the asymmetry in the synaptic strengths may be crucial for the process of learning.

In order to understand how neural networks are able to learn and to work as associative memories, at the present moment it is convenient to consider very simplified models.

A very interesting and widely studied model is the following: the neurons can be firing or quiescent and are represented by variables  $S_i$  which may only take the values  $\pm 1$  and the synaptic strengths (the influence of the neuron  $i$  on the neuron  $k$ ) are real numbers  $J_{i,k}$ . The dynamics is very simple: it can be fully deterministic (the so-called zero temperature limit) or a finite amount of randomness may be present (finite temperature).

In the Hopfield model (Hopfield 1982) the  $J_{i,k}$  are symmetric and the process of learning is the following: each time that a pattern  $\mu_i$  is learnt (the  $\mu_i$  are also  $\pm 1$ ) the  $J$  are changed according to the following formula:

$$J_{i,k}^{\text{new}} = J_{i,k}^{\text{old}} + \mu_i \mu_k \quad (1)$$

which is a generalisation of the original Hebb rule.

The assumption of a symmetric distribution of the  $J$  has been criticised for not being realistic (if the neuron  $i$  influences the neuron  $k$  there is no reason why the neuron  $k$  should influence the neuron  $i$ ). If the Hopfield model were a model for a properly working memory, this criticism would not be very relevant. Indeed, it seems that a not too large amount of asymmetry in the synaptic strengths does not qualitatively change the behaviour of the system (Amit 1985).

The aim of this letter is to show that the performance of the Hopfield model is not that which is required from a good memory; in particular, the learning procedure is quite problematic. In contrast, if the model is strongly modified in such a way that the asymmetry of the synaptic strength plays a crucial role, the memory performs much better and the learning can be done in a simple way. It may be possible for the qualitative differences between the two models to be experimentally observed.

Before presenting the criticism of the Hopfield model we first summarise what is known about this model from both analytic and numerical computations (Amit *et al* 1985, Kinzel 1985, Mezard *et al* 1986a). Let us work under the simplifying hypothesis that  $N$  is large (e.g. 100-1000) and that all the inputs are orthogonal and the number

of learnt patterns ( $p$ ) is proportional to  $N$ :

$$p = \alpha N. \quad (2)$$

It is normally assumed that an input pattern is presented to the network by forcing the state of the network to coincide with the input pattern. After this moment, the system evolves with its own dynamics and the output of the system is the state of the network after some time. If the network had a perfect memory, each time that the input pattern had some resemblance to one of the memorised patterns, the output should be the memorised pattern.

If  $\alpha$  is smaller than a critical value ( $\approx 0.14$ ) and the input pattern is sufficiently near to one of the memorised patterns, the retrieval procedure is successful with probability 1 when  $N \rightarrow \infty$  (apart from about a 1-2% discrepancy between the memorised pattern and the output pattern). On the other hand, if the input is not sufficiently near to one of the memorised patterns, the network is confused and it goes into some state which is very far from every memorised pattern. The nearer  $\alpha$  is to the critical value 0.14, the nearer the input state must be to one of the learnt patterns in order to avoid confusion. When  $\alpha$  becomes greater than the critical value the network is always confused; also, if the input pattern is one of the memorised ones, it becomes confused and goes into a state which is very far from every memorised pattern.

These results hold exactly at zero temperature; if a non-zero amount of noise is allowed and  $\alpha$  is not too small (greater than 0.05), the state in which the system stays near one of the learnt patterns is metastable. If the input pattern is near one of the memorised patterns the system goes into a state very similar to the learnt pattern but after a very long time the network becomes confused and it goes into some state which is very far from every memorised pattern.

It has recently been recognised that the state of total confusion may be avoided by a simple modification of the generalised Hebb rule. For example, we can write (Changeaux *et al* 1986, Mezard and Nadal 1986)

$$J_{i,k}^{\text{new}} = (1 - \lambda) J_{i,k}^{\text{old}} + \mu_i \mu_k \quad (3)$$

or more generally (Parisi 1986)

$$J_{i,k}^{\text{new}} = g(J_{i,k}^{\text{old}} + \mu_i \mu_k) \quad (4)$$

where  $g$  is an appropriate non-linear function such that the  $J$  cannot become arbitrarily large (or small).

If the parameter  $\lambda$  is well tuned, only the last  $\alpha N$  patterns ( $\alpha \approx 0.03-0.05$ ) which are learnt are correctly memorised, while the others are forgotten. In other words, the memory has a finite capacity and the learning of a new pattern forces the forgetting of an old pattern. In this way the memory never reaches the state of total confusion. This modification of the generalised Hebb rule is crucial if we want to understand how the learning process works in detail.

The main disadvantage of a memory which works according to the Hopfield model is that, in both cases ((a) retrieval of one of the input patterns and (b) confusion), after a short transient time the network goes to a time-independent state and it is not possible to discriminate between the two cases. In other words, the outside world which examines the output is unable to discriminate between a valid output (a) and an unreliable output (b). It is clear that a capability to discriminate between case (a) and case (b) would be very useful: an error can be tolerated, if it is identified as such,

but the possibility of having non-recognised errors may affect the reliability of the whole neural system of which our memory is supposed to be one component.

The incapacity to discriminate between case (a) and case (b) becomes really dramatic if we consider the learning mechanism in detail. The crucial question is the following: does the modification of the synaptic strength (according to a generalised Hebb rule) happen only when the network is requested to do so or can the synaptic strengths always be modified? In other words, does the network learn only when it is instructed to learn or is it the network which decides when to learn?

The first option (learning under request) is clearly possible. For example, some chemical modifications of the environment of the network could trigger the application of the Hebb rule. In this letter we explore the feasibility of the second possibility, i.e. that the Hebb rule is always applied, with the exception of times when the network is confused. The new form of the Hebb rule should be

$$dJ_{i,k}(t)/dt = -\lambda J_{i,k}(t) + S_i(t)S_k(t) \quad (5a)$$

if the network is not in a confused state or

$$dJ_{i,k}(t)/dt = 0 \quad (5b)$$

if the network is in a confused state, where  $t$  is the time. The synaptic strengths are modified only when the network is not in a confused state.

In this way the behaviour of our ideal memory should be the following: the network remembers the states in which it has been for a sufficiently long time. In this way the memory learns a pattern if it is forced to stay in the corresponding state a sufficiently long time and, on the other hand, the process of retrieval of a pattern starts when the memory is set in the corresponding state for a short time and after the memory is left free to evolve according to its own laws.

In other words, if a pattern is presented to the memory for a long time it is memorised; if it presented to a memory for a short time, the pattern is searched for in the memory. If the pattern is not found, the memory becomes confused and nothing happens. However, if the retrieval procedure ends with success the state of the neural network coincides with the found pattern for a certain time (during this time the pattern found becomes better memorised); at later times the memory jumps again in the confused states and the content of the memory is not changed until a new pattern is presented.

If we disregard the case of short term memory and we consider only long term memory, this proposal makes sense only if the application of the Hebb rule is inhibited when the network is in a confused state. Indeed, in the absence of input the memory goes into a confused random state and after some time only this state would be memorised if the Hebb rule (equation (5a)) were always operating. This problem is solved by the introduction of equation (5b).

The Hopfield model satisfies all the necessary requirements. Unfortunately, in this model (as we have already remarked) it is impossible to discriminate between the two cases ((a) retrieval of one of the input patterns and (b) confusion). Our proposal is that the neural network should work as the Hopfield network with the main difference being that, when the input state is such that it does not lead to the retrieval of one of the learnt states and the network goes into a state of confusion, the state of the network becomes time dependent in a chaotic way.

In this way, after the input has been presented to the system, if the  $S_i$  are time independent, the retrieval of one of the input states has been completed; on the other

hand, if the  $S_i$  are time dependent the network stays in a confused state. In other words, only the outputs that are time independent can be considered as valid outputs and the outputs which depend on time should be disregarded.

If this happens, the Hebb rule can be modified in the following way:

$$dJ_{i,k}(t)/dt = [-\lambda J_{i,k}(t) + \bar{S}_i(t)\bar{S}_k(t)]f(\bar{S}_i(t)\bar{S}_k(t)) \quad (6)$$

$$\bar{S}_i(t) = 1/T \int_0^T d\tau S_i(t-\tau)$$

when  $f(x)$  is a function which is practically zero below a threshold ( $x_c$ ) and is very near to one for  $x$  greater than the threshold and  $\bar{S}_i(t)$  is the average over a time  $T$  of the status of the  $i$ th neuron (although  $\bar{S}$  depends on  $T$ , we have not indicated this dependence in order to simplify the notation).

In other words, the updating of the synaptic strengths (which is at the basis of the learning process) is sensitive only to the average value of the neurons in the most recent past and it happens only if the neurons do not flip from one state to another too quickly.

Having established how our ideal neural network should work, we should discuss how one can realise a network which works in the way we have described. The claim of this letter is that strong asymmetry of the synaptic strengths is needed to reach this goal. We can arrive at this conclusion by studying the Hopfield model in detail.

The simplest form of the dynamic of the neural network is the following; the time is discretised (in a realistic model the time steps are of the order of a millisecond) and in the so-called zero temperature limit the variables  $S_i$  are updated by applying the following equation to all the neurons (sequentially or in random order):

$$S_i = \text{sgn}(h_i) \quad h_i \equiv \sum_{k=1,N} J_{i,k} S_k. \quad (7)$$

At finite temperature some noise is present, equation (6) holds only in an approximate way and we have

$$S_i = \text{sgn}(h_i) \quad \text{with probability } p_i \equiv 1/[1 + \exp(-\beta h_i)] \quad (8)$$

$$S_i = -\text{sgn}(h_i) \quad \text{with probability } 1 - p_i.$$

Obviously in the limit  $\beta \rightarrow \infty$  we recover equation (7). The great advantage of the Hopfield model is that it can be studied in great detail analytically because its properties coincide with those of a common statistical mechanical system. Due to the symmetry of the synaptic strengths we can define an energy

$$E[S] = \sum_{i,k=1,N} J_{i,k} S_i S_k \quad (9)$$

and the probability distribution of the  $S$  at large time is given by the usual Gibbs formula†:

$$P[S] \propto \exp[-\beta E[S]]. \quad (10)$$

† This is not true for parallel updating in which equation (8) is applied to all the neurons simultaneously. It is only true if equation (8) is applied to each neuron at different times as stated in the text (independent updating). Also, parallel updating has a description in statistical mechanics and the differences are not very strong (generally speaking, the result on the large time behaviour is true only if the temperature ( $1/\beta$ ) is not strictly equal to zero; one should carefully note that the two limits, time going to infinity and temperature going to zero, do not commute).

The zero temperature dynamics corresponds to searching for the minimum of  $E[S]$  using the fastest descent algorithm: each neuron flips its state (from firing to quiescent or vice versa) if the energy decreases by doing so. Each time that the neuron is flipped the energy decreases so that we must reach a stable state after a not too large number of steps.

On the other hand, if the neuron strengths are asymmetric ( $J_{i,k} \neq J_{k,i}$ ) the energy function does not exist anymore and the large time behaviour may be much more complicated at zero temperature. For example, we could have that the  $S_i$  become a periodic function of the time (limit cycle). It is also possible that the length of this cycle is very large (proportional to  $\exp(N)$ ); in this case we say that the system behaves in a chaotic way.

We can be more quantitative by introducing the order parameter  $q(t)$  (Edwards and Anderson 1975, Mezard *et al* 1986b) as

$$q(t) = (1/N) \sum_{i=1, N} [\bar{S}_i(t)]^2. \quad (11)$$

It is well known that at low temperature  $q$  is different from zero (also for large average time  $T$ ) both when the network is near to one of the input states and when the network is confused. However, we would like it that when the network is a confused state  $q$  is zero (or small) and consequently the function  $f$  in equation (6) is very near to zero and the synaptic strengths are time independent.

We have checked numerically that if the synaptic strengths are random and asymmetric ( $J_{i,k}$  is independent from  $J_{k,i}$ ),  $q$  remains zero also in the low temperature limit. In other words, in the equivalent of the spin glass phase for an asymmetric network, the system does not order itself in a random direction but has a chaotic behaviour.

There are many ways in which the asymmetry may be introduced in the Hebb rule. For example, a synapse going from neuron  $i$  to neuron  $k$  may exist with probability  $p$ : not all the neurons are connected one with the other (diluted network) and the dilution is done in an asymmetric way. In other words, there are  $N \times N$  random variables  $C_{i,k}$  ( $C_{i,k} = 0$  or  $1$ ) such that  $\bar{C} = p$ : only if  $C_{i,k} = 1$  does the synaptic connection exist. If we apply one of the various generalised Hebb rules only to the connections which exist (i.e.  $C_{i,k} = 1$ ), we generate an asymmetric set of  $J$ , whose asymmetry is stronger for smaller values of  $p$ .

It is clear that the various parameters of the model must be correctly tuned in order to avoid spurious states in which half of the network is near to one of the input states while the other half of the network is near to another input state (in the Hopfield model this is true when  $\alpha > 0.03$ ).

The condition that the network should automatically go into the confused state (also after a successful retrieval operation) is the most delicate point (which we have not investigated numerically). It is very reasonable that (as in the Hopfield model) the ordered state is metastable and after some time it decays into the confused state only if  $\alpha$  is sufficiently large. If the memory is blank when we start to use the memory and we learn the first patterns,  $\alpha$  is by definition zero and some problems may be present in initialising the memory. It is rather likely that the suggestion (Changeaux *et al* 1985) that the memory is not blank at the initial time will play a crucial role here, although with some imagination one could start to build a theory for the imprinting.

We could also consider more complex models. The synapses are divided in two groups: the Hebb rule applies only to the synapses of the first group while the others are not modified by the learning process. The synapses of the second kind are needed

in order to guarantee that, no matter what happens to the synapses of the first kind, the ordered state is always metastable and the network jumps automatically to the chaotic state. For example, it may be possible that only the excitatory synapses participate in the process of learning while the inhibitory synapses are not modified during the learning and are responsible for bringing the system into the chaotic phase. It seems to me that a modification of this kind will strongly increase the robustness of the network.

Summarising, we propose that a neural network learns automatically any input pattern which is presented to it for a long enough time without the need for any explicit chemical order and that a neural network of a long term memory is in a chaotic time-dependent state when it is not active. The correctness of these two proposals (especially of the second) can clearly be experimentally verified. This behaviour of the network is possible only if the synaptic strengths are sufficiently asymmetric. It is difficult at this stage to discriminate between the many possible models for the asymmetry because models (like the one proposed in this letter) in which we assume that the synapses are randomly distributed are probably not realistic because the process of connecting the neurons is not a pure random process but contains a strong deterministic part.

It is a pleasure for me to thank D Amit, M Mezard, J P Nadal and M Virasoro for useful discussions on the subject of this letter.

## References

- Amit D J 1985 *Jerusalem preprint*
- Amit D J, Gutfreund H and Sompolinsky H 1985 *Phys. Rev. Lett.* **55** 1930
- Changeaux J P, Dehaene S, Nadal J P and Toulouse G 1986 *Europhys. Lett.* to be published
- Changeaux J P, Nadal J P and Toulouse G 1985 *ENS, Paris Preprint*
- Edwards S F and Anderson P W 1975 *J. Phys. F: Met. Phys.* **5** 965
- Hopfield J J 1982 *Proc. Natl Acad. Sci. USA* **79** 2554
- Kinzel W Z 1985 *Z. Phys. B* **60** 205
- Mezard M and Nadal J P 1986 Private communication
- Mezard M, Parisi G and Virasoro M 1986a *Rome preprint*
- 1986b *The Spin Glass Theory and Beyond* (Singapore: World Scientific) to be published
- Parisi G 1984 *Mathematical Physics VII* ed W E Brittin, K E Gustafson and W Wyss (Amsterdam: North-Holland)
- 1986 *J. Phys. A: Math. Gen.* **19** L617