TS 12

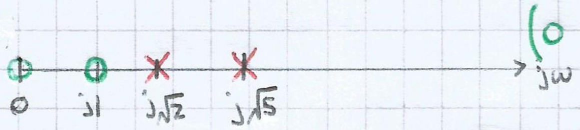
1) (Ej 6 TP7)

Sintetizar Cuadripolo que cumpla con

$$\begin{cases} Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0} = \dfrac{3s\left(s^2+\frac{7}{3}\right)}{(s^2+2)(s^2+5)} = \dfrac{\text{Impar}}{\text{Par}} \text{ (LC)} \\[2ex] Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0} = \dfrac{s(s^2+1)}{(s^2+2)(s^2+5)} \end{cases}$$

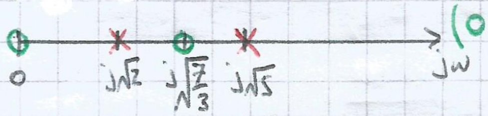
a) Análisis Gráfico:

- Y_{21}:



∴ [Al sintetizar la red a partir de Y_{11}, debo remover polos en origen, j1 e infinito.]

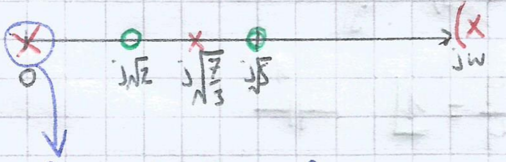
- Y_{11}: Conviene retirar admitancia, porque $\dfrac{-Y_{21}}{Y_{11}} = \left.\dfrac{-I_2}{I_1}\right|_{V_2=0}$, entonces si excito con corriente debo tener el primer componente en paralelo para que impacte en la transferencia.



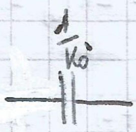
- Evidentemente, en este caso no se puede remover admitancia inicialmente.

∴ Solo voy a enfocarme en cumplir los parámetros.

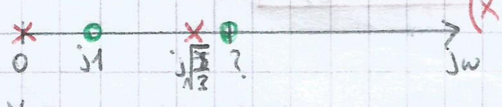
Z_{11}:



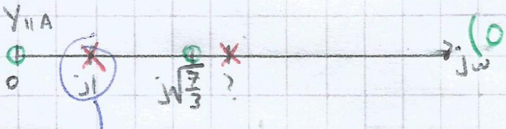
Remoción Parcial para mover cero de $j\sqrt{2}$ a $j1$: $\dfrac{K_0'}{s}$



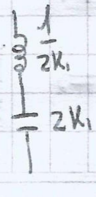
Z_{11A}



Y_{11A}



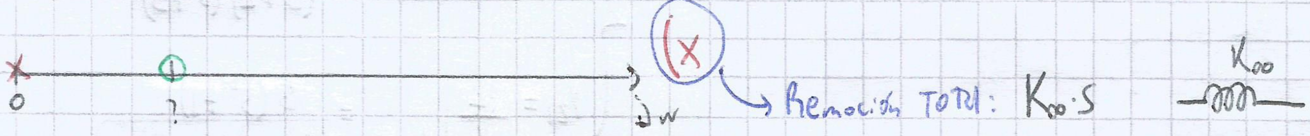
Remoción Total $\quad \dfrac{2K_1 s}{s^2+1} = \dfrac{1}{\dfrac{s}{2K_1} + \dfrac{1}{2K_1 s}}$



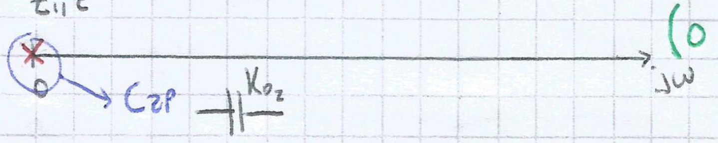
Y_{11B}

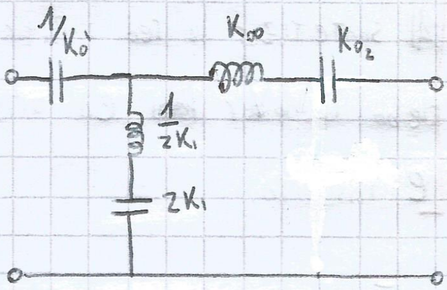
Remuevo polo en infinito:

Z_{11B}



Z_{11C}





b) Síntesis analítica

$$Z_{11} = \frac{(S^2+2)(S^2+5)}{3S\left(S^2+\frac{7}{3}\right)}$$

- Remoción parcial para forzar cero en j1:

$$Z_{11_A} = Z_{11} - \frac{K_0'}{S} \qquad \text{Tal que} \qquad Z_{11A(j)} = 0 = Z_{11(j)} - \frac{K_0'}{j}$$

$$K_0' = j \cdot Z_{11(j)} = \cancel{j}\,\frac{(-1+2)(-1+5)}{3\cancel{j}\left(-1+\frac{7}{3}\right)} = \frac{4\cdot 1}{7-3} = \frac{4}{4} = 1$$

$$Z_{11A} = \frac{(S^2+2)(S^2+5)}{3S\left(S^2+\frac{7}{3}\right)} - \frac{1}{S} = \frac{S^4+7S^2+10-3S^2-7}{3S\left(S^2+\frac{7}{3}\right)} = \frac{S^4+4S^2+3}{3S\left(S^2+\frac{7}{3}\right)}$$

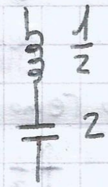
$$Y_{11A} = \frac{3S\left(S^2+\frac{7}{3}\right)}{(S^2+1)(S^2+3)}$$

- Remoción Total de j1:

$$Y_{11B} = Y_{11A} - \frac{2K_1 S}{S^2+1} \Rightarrow \lim_{S^2\to -1} Y_{11A} = \lim_{S^2\to -1} \frac{2K_1 S}{S^2+1} \Rightarrow 2K_1 = \lim_{S^2\to -1} \frac{(S^2+1)}{S}\, Y_{11A}$$

NOTA

$$Z K_1 = \lim_{s^2\to -1} \frac{3(s^2+\frac{7}{3})}{s^2+3} = \frac{-3+7}{2} = \boxed{2}$$



$$Y_{11B} = \frac{3s^3+7s}{(s^2+1)(s^2+3)} - \frac{2s}{s^2+1} = \frac{3s^3+7s-2s(s^2+3)}{(s^2+1)(s^2+3)} = \frac{3s^3+7s-2s^3-6s}{(s^2+1)(s^2+3)} = \frac{s^3+s}{(s^2+1)(s^2+3)}$$

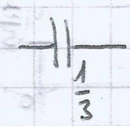
$$Y_{11B} = \frac{s}{s^2+3}$$

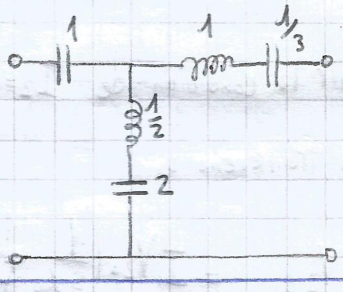
$$Z_{11B} = \frac{s^2+3}{s}$$

- Remuevo polo en infinito

$$Z_{11C} = Z_{11B} - K_\infty \cdot s \Rightarrow \lim_{s\to\infty} Z_{11B} = \lim_{s\to\infty} K_\infty \cdot s \Rightarrow K_\infty = \lim_{s\to\infty} \frac{Z_{11B}}{s} = \lim_{s\to\infty} \frac{s^2+3}{s^2} = \boxed{1}$$

$$Z_{11C} = \frac{s^2+3}{s} - s = \frac{s^2+3-s^2}{s} = \frac{3}{s} = \boxed{\frac{1}{s\cdot\frac{1}{3}}}$$





2) $T(s) = \frac{V_2}{V_1}\Big|_{I_2=0} = \frac{K(s+1)}{(s+2)(s+4)}$ Solo tiene ceros en -1 e ∞

a) Topología circuital que respete $T(s)$, usando parámetros Z e Y:

$$T(s) = \frac{P}{Q} = \frac{P/D}{Q/D} = \frac{\frac{K(s+1)}{D}}{\frac{(s+2)(s+4)}{D}} = \frac{Z_{21}}{Z_{11}}\Big|_{I_2=0} = -\frac{Y_{21}}{Y_{22}}\Big|_{I_2=0}$$

1) Empiezo con parámetros Z:

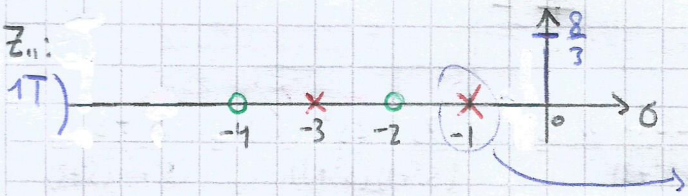
No es par ni impar $\frac{(s+2)(s+4)}{D} = Z_{11}$ (D disipativo) Tengo que:
- Generar alternancia entre polos y ceros
- Si quiero red RC, debo cumplir $Z_{RC}(0) > Z_{RC}(\infty)$

NOTA

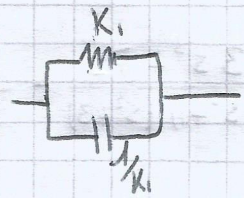
Elijo $D = (S+3)\cdot(S+1)$ / $\dfrac{Q}{D} = \dfrac{(S+2)(S+4)}{\underbrace{(S+3)}_{\text{genera alternancia}}\underbrace{(S+1)}}= Z_{11}$ $\left\langle \begin{array}{l} Z_{(0)} = \frac{8}{3} \\ Z_{(\infty)} = 1 \end{array}\right.$ $Z_{(0)} > Z_{(\infty)}$ (Verifica para Z_{RC})

Al sintetizar Z_{11} solo puedo remover Polo en -1 e ∞:

Z_{11}:

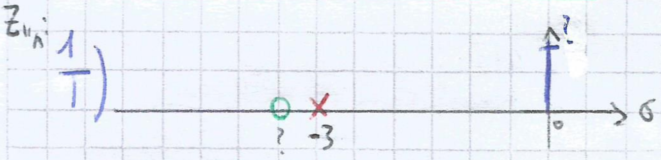


$$\frac{K_1}{s+1} = \frac{1}{\frac{s}{K_1}+\frac{1}{K_1}}$$

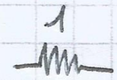


∘ Remuevo Polo en -1 completamente:

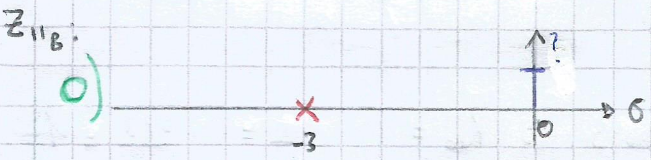
Z_{11_A}:



∘ Remuevo Resistencia en infinito: $K_\infty = 1$

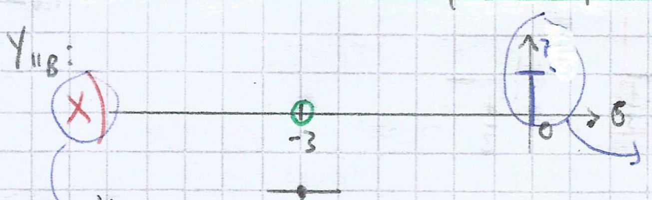


Z_{11_B}:

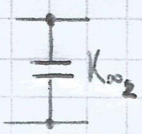


∘ Deseo Remover de ∞; por lo que antes remuevo la conductancia en el origen, manteniendo: $Y_{RC(0)} < Y_{RC(\infty)}$

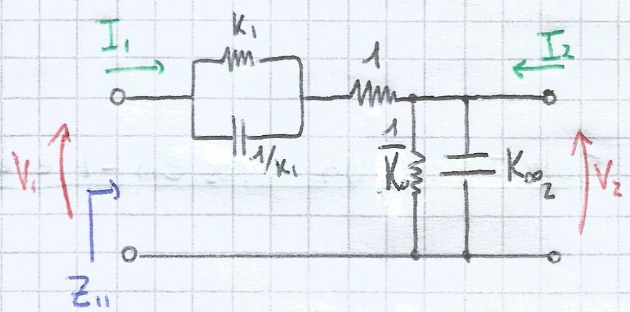
Y_{11_B}:



$K_{\infty_2}\cdot S$



esto es ideal para que al cumplir la condición de $I_2 = 0$, este último componente actúe en la transferencia.



Verifico $Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = \dfrac{I_1\cdot\left(\dfrac{1}{sK_{\infty_2}+K_0}\right)}{I_1}$

$$Z_{21} = \frac{1}{K_{\infty_2}\left(S+\frac{K_0}{K_{\infty_2}}\right)}$$

(Si $\dfrac{K_0}{K_{\infty_2}} = 3$ $\Rightarrow$ $Z_{21} = K\cdot\dfrac{1}{s+3} = \dfrac{P}{D} = \dfrac{K\,\cancel{(S+1)}}{\cancel{(S+1)}(S+3)}$)

(es prometedor)

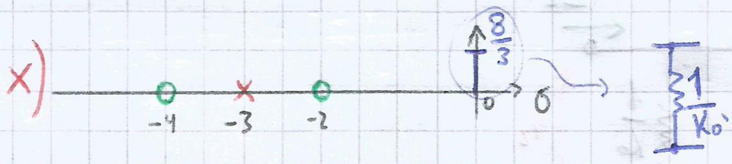
II) Sigo con parámetros Y:

$\dfrac{(S+2)(S+4)}{D} = Y_{22}$

- Alternancia entre polos y ceros
- Como quiero Y_{RC}: $Y_{RC(0)} < Y_{RC(\infty)}$

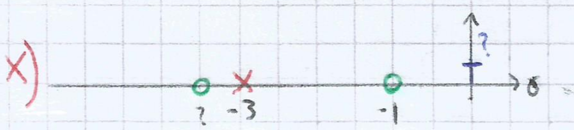
∘ Eligiendo $D = (S+3)$: $Y_{22} = \dfrac{(S+2)(S+4)}{S+3}$ $\left\langle \begin{array}{l} Y_{RC(0)} = \frac{8}{3} \ ✓ \\ Y_{RC(\infty)} = \infty \end{array}\right.$

Y_{22}:



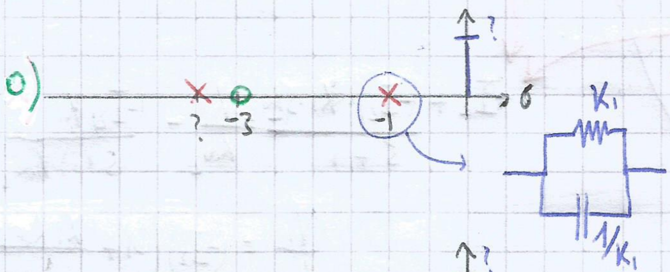
- Primero remuevo parcialmente una constante del origen para que quede un cero en -1 (y posteriormente remover de ahí), y asegurar a su vez un resistor en derivación para lograr que el último componente actúe en la transferencia incluso con $I_2 = 0$.

Y_{22_A}:

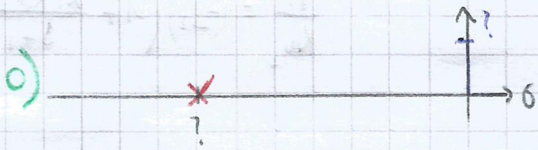


- Paso a impedancia y remuevo polo en -1.

Z_{22_A}:

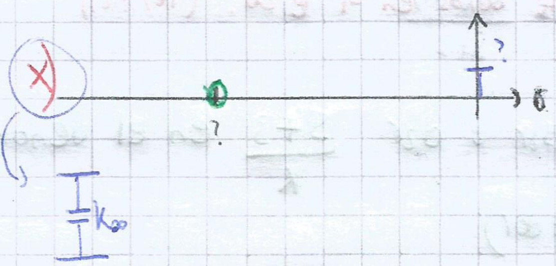


Z_{22_B}:



- Paso a admitancia y remuevo polo en infinito:

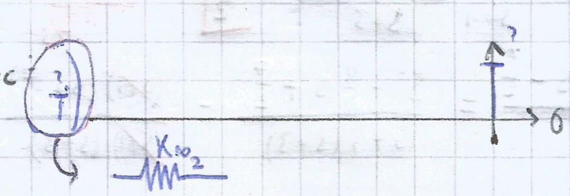
Y_{22_B}:



Y_{22_C}:

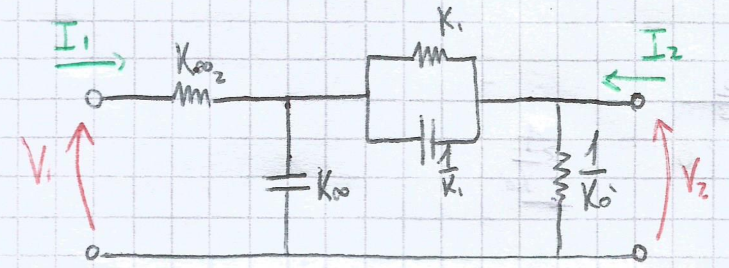
- Me debe quedar una impedancia en serie para que actúe en la transferencia al tener una excitación de tensión:

Z_{22_C}:

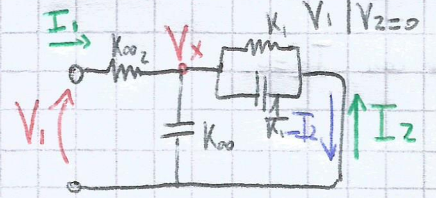


Como no quedan más polos ni ceros, asumo que Z_{22_C} es una CTE. (Último componente)

∴ Finalmente:



Verifico $-Y_{21} = \left.\frac{-I_2}{V_1}\right|_{V_2=0} =$



$$\begin{cases} I_1 = \frac{V_1 - V_x}{K_{\infty_2}} \\ V_x = (-I_2)\cdot \frac{1}{\frac{1}{K_1}s+\frac{1}{K_1}} \\ I_1 = (-I_2) + V_x \cdot s K_\infty \end{cases}$$

$$-I_2 + V_x \cdot sK_\infty = \frac{V_1 - V_x}{K_{\infty_2}}$$

$$-I_2 + \frac{(-I_2)\,K_1 \cdot sK_\infty}{s+1} = \frac{V_1}{K_{\infty_2}} - \frac{(-I_2)\,K_1}{(s+1)\,K_{\infty_2}}$$

$$\frac{V_1}{K_{\infty_2}} = (-I_2)\left(1 + \frac{K_1 K_\infty \cdot s}{s+1} + \frac{K_1}{(s+1)\,K_{\infty_2}}\right)$$

$$V_1 = -I_2\left(\frac{(s+1)K_{\infty_2} + sK_1K_\infty K_{\infty_2} + K_1}{(s+1)\,\cancel{K_{\infty_2}}}\cdot \cancel{K_{\infty_2}}\right)$$

$$\frac{-I_2}{V_1} = \frac{s+1}{(s+1)K_{\infty_2} + sK_1K_\infty K_{\infty_2} + K_1} = -Y_{21} = \frac{P}{D} = \frac{K(s+1)}{s+3}$$

$$-\frac{I_2}{V_1} = \frac{s+1}{s(K_{\infty_2} + K_1 K_\infty K_{\infty_2}) + K_1 + K_{\infty_2}}$$

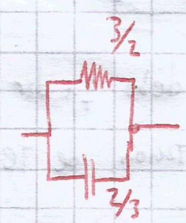
Puede llegar a dar $\frac{s+3}{K}$ en el denominador (Prometedor)

b) Cálculo de valor de componentes y K.

1) Parametros Z (sintetizando Z_{11}):

∘ Remuevo polo en -1 completamente:

$$Z_{11A} = Z_{11} - \frac{K_1}{s+1} \Rightarrow K_1 = \lim_{s\to -1}(s+1)Z_{11} = \lim_{s\to -1}\frac{(s+2)(s+4)}{s+3} = \boxed{\frac{3}{2}}$$



$$\therefore Z_{11A} = \frac{(s+2)(s+4)}{(s+1)(s+3)} - \frac{3/2}{s+1} = \frac{s^2+6s+8-\frac{3}{2}s-\frac{9}{2}}{(s+1)(s+3)} = \frac{s^2+\frac{9}{2}s+\frac{7}{2}}{(s+1)(s+3)} = \frac{\cancel{(s+1)}(s+\frac{7}{2})}{\cancel{(s+1)}(s+3)}$$

- Remuevo CTE en infinito:

$$Z_{11_B} = Z_{11_A} - K_\infty \qquad \text{donde} \quad K_\infty = \lim_{s\to\infty} Z_{11_A} = \lim_{s\to\infty} \frac{s+\frac{7}{2}}{s+3} = \boxed{1}$$

1

$$Z_{11B} = \frac{s+\frac{7}{2}}{s+3} - 1 = \frac{s+\frac{7}{2}-s-3}{s+3} = \boxed{\frac{\frac{1}{2}}{s+3}}$$

- Remuevo CTE en origen en admitancia:

$$Y_{11B} = 2(s+3)$$

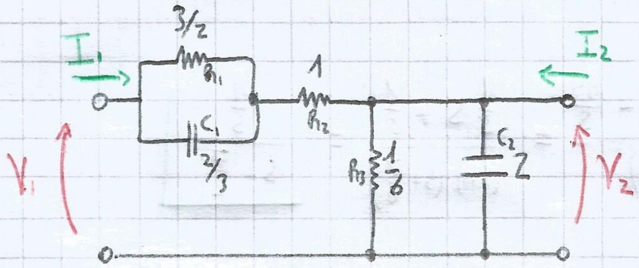
$$Y_{11C} = Y_{11B} - K_0 \qquad \text{donde} \quad K_0 = Y_{11B(0)} = \boxed{6}$$

$\frac{1}{6}$

$$Y_{11C} = 2s+6-6 = \boxed{2s} \qquad (K_{\infty_2} = 2)$$

$2 = K_{\infty_2}$

Finalmente:



$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{R_3 /\!/ \frac{1}{sC_2}}{R_3/\!/\frac{1}{sC_2} + R_2 + \frac{1}{sC_1}/\!/R_1} = \frac{\frac{1}{1/R_3 + sC_2}}{\frac{1}{R_3'+sC_2} + R_2 + \frac{1}{sC_1 + 1/R_1}} = \frac{\frac{1}{6+s2}}{\frac{1}{6+s\cdot 2} + 1 + \frac{1}{\frac{2}{3}+\frac{2}{3}s}}$$

$$\left.\frac{V_2}{V_1}\right|_{I_2} = \frac{\frac{1}{\cancel{2(s+3)}}}{\frac{\frac{2}{3}(s+1) + 2(s+3)\frac{2}{3}(s+1) + 2(s+3)}{\cancel{2(s+3)}(s+1)\frac{2}{3}}} = \frac{\frac{2}{3}(s+1)}{\frac{2}{3}s + \frac{2}{3} + 2s + 6 + \frac{4}{3}s^2 + \frac{4}{3}s + 4s + 4}$$

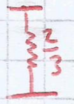
$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{\frac{2}{3}(s+1)}{\frac{4}{3}s^2 + 8s + \frac{32}{3}} = \frac{\frac{2}{3}(s+1)}{\frac{4}{3}(s+2)(s+4)} = \boxed{\frac{\frac{1}{2}(s+1)}{(s+2)(s+4)}} \qquad \left(K = \frac{1}{2}\right)$$

II) Parametros Y (sintetizando Y_{22}):

- Remoción parcial de cte en origen, generando cero en -1:

$$Y_{22_A} = Y_{22} - K_0' \qquad \text{Tal que} \qquad Y_{22_A(-1)} = Y_{22(-1)} - K_0' = 0$$

$$\therefore K_0' = Y_{22(-1)} = \frac{(-1+2)(-1+4)}{(-1+3)} = \boxed{\frac{3}{2}}$$



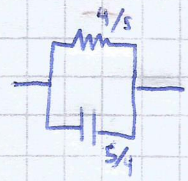
$$Y_{22_A} = \frac{s^2+6s+8-\frac{3}{2}s-\frac{9}{2}}{s+3} = \frac{s^2+\frac{9}{2}s+\frac{7}{2}}{(s+3)} = \frac{(s+1)(s+\frac{7}{2})}{s+3}$$

- Paso a impedancia y remuevo polo en -1:

$$Z_{22_A} = \frac{s+3}{(s+1)(s+\frac{7}{2})}$$

$$Z_{22_B} = Z_{22_A} - \frac{K_1}{s+1} \Rightarrow \lim_{s\to -1} Z_{22_A} = \lim_{s\to -1} \frac{K_1}{s+1} \Rightarrow K_1 = \lim_{s\to -1} (s+1) Z_{22_A}$$

$$K_1 = \lim_{s\to -1} \frac{s+3}{s+\frac{7}{2}} = \frac{2}{\frac{5}{2}} = \boxed{\frac{4}{5}}$$

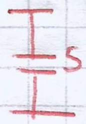


$$\therefore Z_{22_B} = \frac{s+3}{(s+1)(s+\frac{7}{2})} - \frac{\frac{4}{5}}{s+1} = \frac{s+3-\frac{4}{5}s-\frac{14}{5}}{(s+1)(s+\frac{7}{2})} = \frac{\frac{1}{5}s+\frac{1}{5}}{(s+1)(s+\frac{7}{2})} = \boxed{\frac{\frac{1}{5}}{s+\frac{7}{2}}}$$

- Paso a admitancia y remuevo polo en ∞:

$$Y_{22_B} = 5(s+\tfrac{7}{2})$$

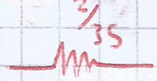
$$Y_{22C} = Y_{22B} - K_\infty \cdot s \Rightarrow K_\infty = \lim_{s\to\infty} \frac{Y_{22B}}{s} = \boxed{5}$$



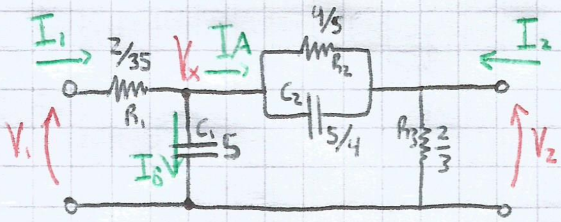
$$Y_{22C} = 5s + \frac{35}{2} - 5s = \boxed{\frac{35}{2}}$$

- Paso a impedancia:

$$Z_{22C} = \boxed{\frac{2}{35}}$$



∴ Finalmente:



$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{I_A \cdot R_3}{V_1}$$

$$I_A = \frac{V_x}{R_3 + \frac{1}{sC_2 + \frac{1}{R_2}}} = \frac{V_x}{\frac{2}{3} + \frac{1}{s\cdot\frac{5}{4}+\frac{5}{4}}} = \frac{V_x}{\frac{\frac{5}{6}(s+1)+1}{\frac{5}{4}(s+1)}} \Rightarrow V_x = I_A \cdot \frac{\frac{5}{6}(s+1)+1}{\frac{5}{4}(s+1)}$$

$$I_1 = I_A + I_B \Rightarrow \frac{V_1 - V_x}{R_1} = I_A + V_x \cdot sC_1 \Rightarrow \frac{V_1}{R_1} - \left(\frac{1}{R_1}+sC_1\right)V_x = I_A$$

$$I_A = \frac{V_1}{R_1} - \left(\frac{1}{R_1}+sC_1\right) I_A \cdot \frac{\left(\frac{5}{6}(s+1)+1\right)}{\frac{5}{4}(s+1)}$$

$$I_A = V_1 \cdot \frac{35}{2} - \left(\frac{35}{2}+s\cdot 5\right)\cdot \frac{\frac{2}{3}s + \overbrace{\frac{2}{3}+\frac{4}{5}}^{22/15}}{(s+1)}\, I_A$$

$$I_A = V_1 \frac{35}{2} - \frac{\frac{35}{3}s + \frac{77}{3} + \frac{10}{3}s^2 + \frac{22}{3}s}{s+1}\, I_A = V_1 \cdot \frac{35}{2} - \frac{\frac{10}{3}s^2 + 19s + \frac{77}{3}}{s+1}\, I_A$$

$$I_A\left(1 + \frac{\frac{10}{3}s^2+19s+\frac{77}{3}}{s+1}\right) = V_1\cdot\frac{35}{2}$$

$$I_A \frac{s+1+\frac{10}{3}s^2+19s+\frac{77}{3}}{s+1} = I_A \cdot \frac{\frac{10}{3}(s+2)(s+4)}{s+1} = V_1 \frac{35}{2} \Rightarrow I_A = V_1\cdot\frac{35}{2}\cdot\frac{s+1}{\frac{10}{3}(s+2)(s+4)}$$

$$I_A = V_1 \cdot \frac{21}{4}\cdot\frac{s+1}{(s+2)(s+4)} \Rightarrow \left.\frac{V_2}{V_1}\right|_{I_2=0} = \cancel{V_1}\cdot\frac{21}{4}\cdot\frac{s+1}{(s+2)(s+4)}\cdot\frac{2}{3\cdot\cancel{V_1}}$$

$$\boxed{\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{7}{2}\,\frac{s+1}{(s+1)(s+4)}} \quad \left(K=\frac{7}{2}\right)$$