Trabajo Práctico Nº 5

5- $H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-0{,}8} \Rightarrow Y(z)\,(z-0{,}8) = X(z)\cdot z$

$Y(z)\cdot z - 0{,}8\cdot Y(z) = X(z)\cdot z$

$\Rightarrow \quad Y(z) - 0{,}8\cdot z^{-1}\cdot Y(z) = X(z)$

$\Rightarrow \quad Y[n] - 0{,}8\; Y[n-1] = X[n]$

$Y[n] = X[n] + 0{,}8\; Y[n-1]$

Si $\quad X[n] = 20\cdot \cos\left(\frac{\pi . n}{2} + 30°\right) = 20\cdot \cos\left[\frac{\pi}{2}\left(n + \frac{\pi}{6}\right)\right]$

De tabla:

$$F[n+a] \xrightarrow{z^{-1}} z^{a}\cdot F(z)$$

$$\cos(\Omega_0 . n)\cdot u[n] \xrightarrow{z^{-1}} \frac{z\,(z-\cos(\Omega_0))}{z^2 - 2 . z\cos(\Omega_0) + 1}$$

$\Rightarrow \bullet\; X(z) = 20\cdot z^{\pi/6}\cdot \frac{z\,[z-\cos(\pi/2)]}{z^2 - 2 . z\cos(\pi/2) + 1}$

$X(z) = 20\cdot z^{\pi/6}\; \frac{z^2}{z^2+1}$

$\Rightarrow \bullet\; Y(z) = X(z)\;\frac{z}{z-0{,}8} = 20\cdot z^{\pi/6}\; \overbrace{\frac{z^3}{(z^2+1)(z-0{,}8)}}^{A(z)}$

NOTA

HOJA N°

FECHA

$$A(z) = \frac{z^3}{(z^2+1)(z-0{,}8)} = \frac{A}{(z-0{,}8)} + \frac{B\cdot z + C}{(z^2+1)}$$

$$z^3 = A\,(z^2+1) + z\,(z-0{,}8)\cdot B + (z-0{,}8)\,C$$

Si $A = 0{,}8$ $\quad 0{,}512 = 1{,}64\,A \Rightarrow A = 64/205$

Si $z = 0$: $\quad 0 = 64/205 - 0{,}8\,C \Rightarrow C = 16/41$

Si $z = 1$: $\quad 1 = 128/205 + 0{,}2\,B + 16/205 \Rightarrow B = 61/41$

$$\Rightarrow Y(z) = 20 \cdot z^{\pi/6} \cdot \Bigg[\underbrace{\frac{64/205}{z-0{,}8}}_{B(z)} + \underbrace{\frac{61/41 \cdot z}{z^2+1}}_{C(z)} + \underbrace{\frac{16/41}{z^2+1}}_{D(z)} \Bigg]$$

Transformando por tabla:

- $C[n] = \frac{61}{41} \cdot \text{sn}\left(\frac{\pi}{2}\cdot n\right) \cdot u[n]$
- $B(z) = z^{-1}\,\frac{z}{z-0{,}8}\,\frac{64}{205} \longrightarrow B[n] = 0{,}8^{\,n-1} \cdot \frac{64}{205} \cdot u[n-1]$
- $D(z) = \frac{16}{41}\,z^{-1}\,\frac{z}{z^2+1} \longrightarrow D[n] = \frac{16}{41} \cdot \text{sn}\left[\frac{\pi}{2}(n-1)\right] \cdot u[n-1]$

$$\boxed{Y[n] = 20 \cdot \frac{61}{41} \cdot \text{sn}\left[\frac{\pi}{2}\left(n+\frac{\pi}{6}\right)\right] u\left[n+\frac{\pi}{6}\right] + 20 \cdot \left\{ 0{,}8^{\,n+\pi/6-1}\,\frac{64}{205} + \frac{16}{41}\,\text{sn}\left[\frac{\pi}{2}\left(n+\frac{\pi}{6}-1\right)\right]\right\} \cdot u\left[n+\frac{\pi}{6}-1\right]}$$

⑥ $y[n] = 2\,y[n-1] - 1{,}81\,y[n-2] + 0{,}68\,y[n-3] + x[n] + 3\,x[n-1] +$

$3\,x[n-2] + x[n-3]$

a) $Y(z) = 2\,Y(z)\,z^{-1} - 1{,}81\,Y(z)\cdot z^{-2} + 0{,}68\,Y(z)\,z^{-3} + X(z) + 3\,X(z)\,z^{-1} +$

$3\,X(z)\,z^{-2} + X(z)\,z^{-3}$

$$Y(z)\left(1 - 2z^{-1} + 1{,}81\,z^{-2} - 0{,}68\,z^{-3}\right) = X(z)\left(1 + 3z^{-1} + 3z^{-2} + z^{-3}\right)$$

$$H(z) = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1-2z^{-1}+1{,}81\,z^{-2}-0{,}68\,z^{-3}} = \boxed{\frac{z^3+3z^2+3z+1}{z^3-2z^2+1{,}81z-0{,}68}}$$

b) $$H(\Omega) = H(z)\Big|_{z=e^{j\Omega}} = \frac{e^{j3\Omega}+3\cdot e^{j2\Omega}+3\cdot e^{j\Omega}+e^{j0}}{e^{j3\Omega}-2\,e^{j2\Omega}+1{,}81\cdot e^{j\Omega}-e^{j0}\cdot 0{,}68}$$

$$= \frac{\left(e^{j\Omega}+e^{j0}\right)^3}{\cos(3\Omega)+j\operatorname{sen}(3\Omega)-2\cos(2\Omega)-2j\operatorname{sen}(2\Omega)+1{,}81\cos(\Omega)+j\operatorname{sen}(\Omega)\,1{,}81+0{,}68}$$

$$= \frac{\left[e^{j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}}+e^{-j\frac{\Omega}{2}}\right)\right]^3}{\cos(3\Omega)-2\cos(2\Omega)+1{,}81\cos(\Omega)-0{,}68+j\left[\operatorname{sen}(3\Omega)-2\operatorname{sen}(2\Omega)+1{,}81\cdot\operatorname{sen}(\Omega)\right]}$$

$$= \frac{\left[e^{j\frac{\Omega}{2}}\cdot 2\cos\left(\frac{\Omega}{2}\right)\right]^3}{\text{denominador}}$$

$$= \frac{8\cdot e^{j\frac{3}{2}\Omega}\cdot\cos^3\left(\frac{\Omega}{2}\right)}{\text{denominador}}$$

- $$|H(\Omega)| = \frac{8\cdot\cos^3\left(\frac{\Omega}{2}\right)}{\sqrt{\left[\cos(3\Omega)-2\cos(2\Omega)+1{,}81\cos(\Omega)-0{,}68\right]^2+\left[\operatorname{sen}(3\Omega)-2\operatorname{sen}(2\Omega)+1{,}81\operatorname{sen}(\Omega)\right]^2}}$$

- $$\varphi(\Omega) = \frac{3}{2}\Omega - \operatorname{arctg}\left(\frac{\operatorname{sen}(3\Omega)-2\operatorname{sen}(2\Omega)+1{,}81\operatorname{sen}(\Omega)}{\cos(3\Omega)-2\cos(2\Omega)+1{,}81\cos(\Omega)-0{,}68}\right)$$

NOTA

c) $X(t) = \underbrace{10}_{a(t)} + \underbrace{5\cos(2\pi \cdot 2000\, t - \overset{\pi/3}{60^\circ})}_{b(t)} + \underbrace{20 \cdot \operatorname{sen}(2\pi\, 8000 \cdot t + \overset{\pi/6}{30^\circ})}_{c(t)}$

- $a(t) = 10 \longrightarrow a(s) = \frac{10}{s}$

- $b(t) = 5 \cdot \cos\left[2\pi \cdot 2000 \left(t - \frac{1}{12000}\right)\right] \longrightarrow b(s) = e^{-\frac{1}{12000}} \quad \frac{s}{s^2 + (2\pi 2000)^2} \cdot 5$

- $c(t) = 20 \cdot \operatorname{sen}\left[2\pi\, 8000 \left(t - \frac{1}{96000}\right)\right] \longrightarrow c(s) = 20 \cdot e^{-\frac{1}{96000}} \quad \frac{2\pi\, 8000}{s^2 + (2\pi 8000)^2}$

- $X(s) = \frac{10}{s} + 5 \frac{s}{s^2 + (4000\pi)^2} + 20 \frac{\pi\, 16000}{s^2 + (16000\pi)^2}$

- $X(z) = \frac{z+1}{z-1} \cdot 10 + 5 \frac{\left(\frac{z-1}{z+1}\right)^2}{\left(\frac{z-1}{z+1}\right)^2 + (4000\pi)^2} + 20 \frac{\pi \cdot 16000}{\left(\frac{z-1}{z+1}\right)^2 + (16000\pi)^2}$

$= \frac{z+1}{z-1} \cdot 10 + 5 \frac{(z-1)^2}{(z-1)^2 + \left[4000\pi (z+1)\right]^2} + 20 \frac{(z+1)^2\, \pi\, 16000}{(z-1)^2 + \left[16000\pi (z+1)\right]^2}$