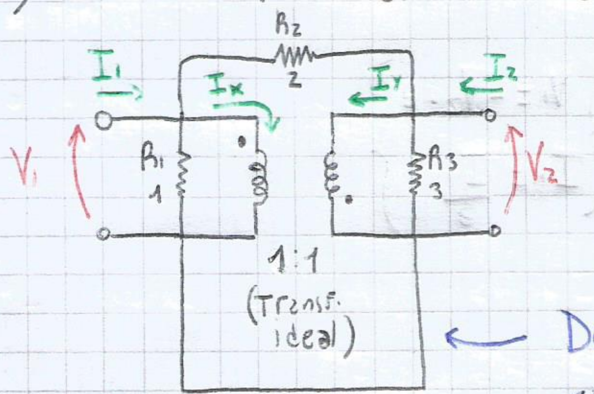
TS7

1) Calcular parámetros Z de:



Debe haber un cable para cerrar el circuito y que circule corriente por R_2.

$$\begin{cases} V_1 = -V_2 \\ I_x = I_Y \end{cases}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \; ; \; Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \; ; \; Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \; ; \; Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + I_x \qquad\qquad I_2 = \frac{V_2}{R_3} - \frac{V_1 - V_2}{R_2} + I_Y$$

• $I_2 = 0$:

$$I_Y = \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} = I_x \implies I_x = \frac{V_1}{R_2} - V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$I_x = \frac{V_1}{2} + V_1\left(\frac{1}{2} + \frac{1}{3}\right) = \frac{4}{3}V_1$$

$$\therefore I_1 = V_1 + \frac{V_1}{2} + \frac{V_1}{2} + \frac{4}{3}V_1 = \frac{10}{3}V_1 \implies I_1 = -\frac{10}{3}V_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{3}{10} = 0{,}3 \qquad\qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -\frac{3}{10} = -0{,}3$$

• $I_1 = 0$:

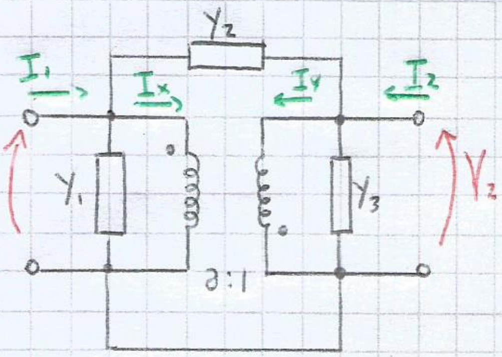
$$I_x = \frac{V_2 - V_1}{R_2} - \frac{V_1}{R_1} = I_Y \implies I_Y = \frac{V_2}{2} + \frac{V_2}{2} + \frac{V_2}{1} = 2V_2$$

$$\therefore I_2 = \frac{V_2}{3} + \frac{V_2}{2} + \frac{V_2}{2} + 2V_2 = \frac{10}{3}V_2 \implies Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 0{,}3 \;\wedge\; Z_{12} = \left.\frac{-V_2}{I_2}\right|_{I_1=0} = -0{,}3$$

NOTA

$$Z = \begin{pmatrix} 0{,}3 & -0{,}3 \\ -0{,}3 & 0{,}3 \end{pmatrix}$$

2)



$$\begin{cases} V_1 = -V_2 \cdot a \\ I_x = \dfrac{I_y}{a} \end{cases}$$

$$\begin{cases} I_1 = V_1 \cdot Y_1 + (V_1 - V_2) Y_2 + I_x \\ I_2 = V_2 \cdot Y_3 + (V_2 - V_1) Y_2 + I_y \end{cases}$$

$I_2 = 0$:

$$I_y = V_1 Y_2 - V_2 (Y_2 + Y_3) = a \cdot I_x$$

$$\therefore I_1 = V_1 Y_1 + (V_1 - V_2) Y_2 + V_1 \frac{Y_2}{a} - V_2 \frac{Y_2 + Y_3}{a}$$

$$I_1 = V_1 Y_1 + \left(V_1 + \frac{V_1}{a}\right) Y_2 + V_1 \frac{Y_2}{a} + V_1 \frac{Y_2 + Y_3}{a^2}$$

$$\frac{I_1}{V_1} = Y_1 + Y_2 + 2\frac{Y_2}{a} + \frac{Y_2 + Y_3}{a^2} = Z_{11}^{-1}$$

$$\frac{I_1}{V_1} = \frac{I_1}{-V_2 \cdot a} \Rightarrow \frac{I_1}{V_2} = -\left(Y_1 \cdot a + Y_2 \cdot a + 2 Y_2 + \frac{Y_2 + Y_3}{a}\right) = Z_{21}^{-1}$$

$I_1 = 0$:

$$I_x = V_2 \cdot Y_2 - V_1 (Y_1 + Y_2) = \frac{I_y}{a}$$

$$\therefore I_2 = V_2 Y_3 + (V_2 - V_1) Y_2 + V_2 \cdot Y_2 \cdot a - V_1 \cdot a (Y_1 + Y_2)$$

$$I_2 = V_2 Y_3 + (V_2 + V_2 a) Y_2 + V_2 Y_2 a + V_2 a^2 (Y_1 + Y_2)$$

$$\frac{I_2}{V_2} = Y_3 + Y_2 + 2 Y_2 \cdot a + a^2 (Y_1 + Y_2) = Z_{22}^{-1}$$

$$\frac{I_2}{V_2} = \frac{I_2}{-\frac{V_1}{a}} \Rightarrow \frac{I_2}{V_1} = -\left(a Y_1 + a Y_2 + 2 Y_2 + \frac{Y_3 + Y_2}{a}\right) = Z_{12}^{-1}$$

- Se observa que $Z_{21}^{-1} = Z_{12}^{-1} \Rightarrow Z_{12} = Z_{21} \Rightarrow$ la red es recíproca para todo Y_1, Y_2, Y_3, a. Esto concuerda con el hecho de que el circuito es pasivo.

- Para ser simétrico, $Z_{11} = Z_{22} \Rightarrow Z_{11}^{-1} = Z_{22}^{-1}$

$$\therefore \quad Y_1 + Y_2 + 2\frac{Y_2}{a} + \frac{Y_2+Y_3}{a^2} = a^2(Y_1+Y_2) + 2Y_2 \cdot a + Y_2 + Y_3$$

- Si $a=1$, la red es simétrica $\forall Y_1, Y_2, Y_3$.