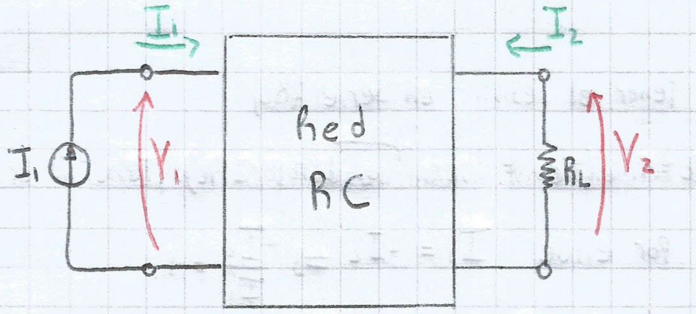
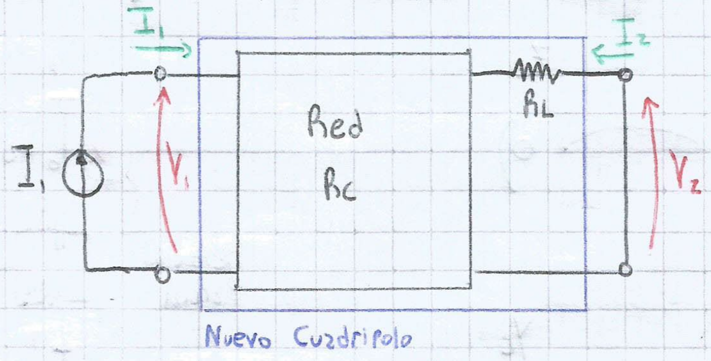
TS13

1) (Ej 5 TP7)



$$\begin{cases} -\dfrac{I_2}{I_1} = H \cdot \dfrac{s^2+5s+4}{s^2+8s+12} \\ Z_{21} = 6H \end{cases}$$

- Redefino el cuadripolo cargado en un nuevo cuadripolo descargado:

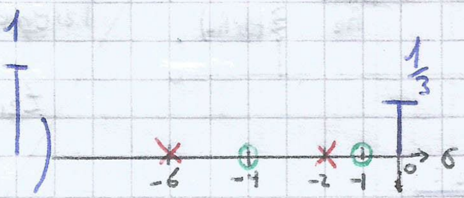


$$\therefore \ -\frac{I_2}{I_1}^{\text{(Antes) cargado}} = -\frac{I_2}{I_1}\bigg|_{V_2=0}^{\text{(Ahora) (descargado)}}$$

- Al sintetizar me debo cuidar de tener R_L en serie al final del circuito.

a) Síntesis Gráfica:

$$\frac{-I_2}{I_1}\bigg|_{V_2=0} = \frac{(s+1)(s+4)}{(s+2)(s+6)} \cdot H$$



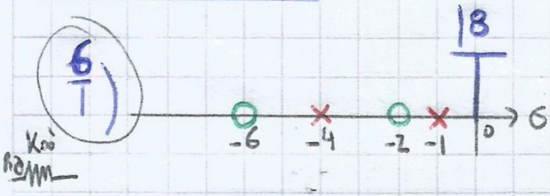
- Ceros de Transmisión en -4 y -1

- $$-\frac{I_2}{I_1} = \frac{P}{Q} = \frac{P/D}{Q/D} \Rightarrow \frac{Q}{D} = \frac{(s+2)(s+6)}{D}$$

- $$-\frac{I_2}{I_1}\bigg|_{V_2=0} = -\frac{Y_{21}}{Y_{11}} = \frac{Z_{21}}{Z_{22}} \longrightarrow$$ Como tengo Z_{21} de dato: $$\cancel{H} \cdot \frac{s^2+5s+4}{s^2+8s+12} = \frac{6\cancel{H}}{Z_{22}}$$

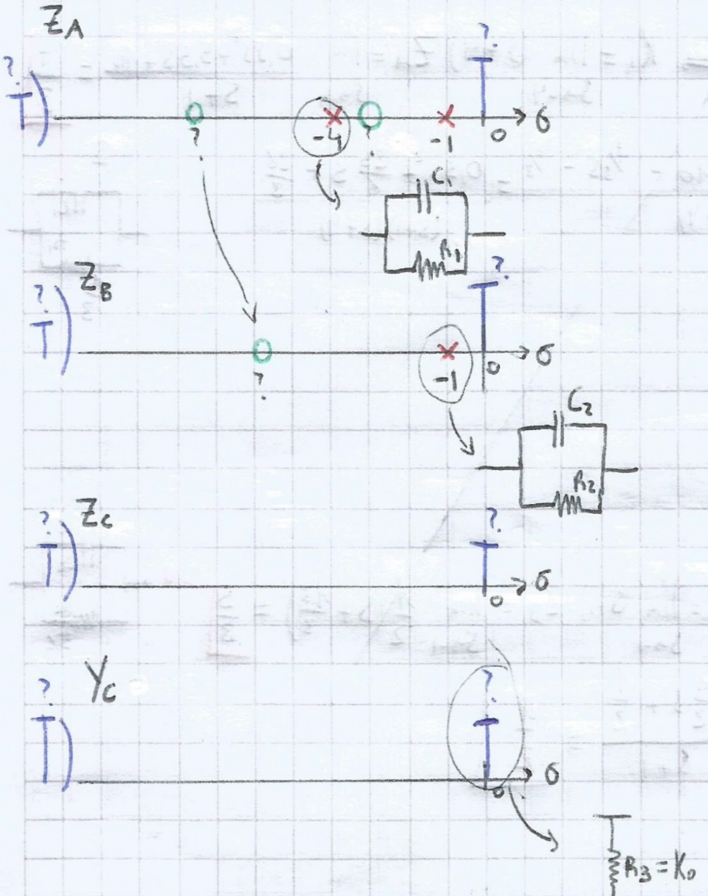
Sintetizo Z_{22}:

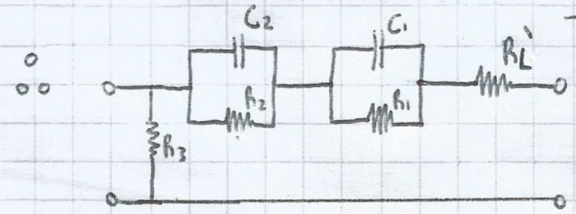
$$Z_{22} = 6 \cdot \frac{(s+2)(s+6)}{(s+1)(s+4)}$$



- Primero debo remover una constante de inf para tener el resistor en serie al final (R_L)
- Si removiera totalmente K_∞ y luego los polos en -4 y -1 no tendría un componente en derivación al comienzo de la red, lo cual es totalmente necesario dado que excito con una fuente de corriente ideal.

Hago una remoción parcial de K_∞, luego remuevo los polos en -4 y -1, paso a admitancia y termino de remover la constante restante (que queda en derivación al principio del circuito)



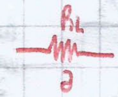


b) Síntesis analítica

∘ Remuevo Parcialmente de infinito:

¿Cuánto? • Por el momento no lo sé, así que lo dejo Parametrizado: $K_\infty' = a = R_L$

• Como $Z_{22(\infty)} = 6$, $0 < a < 6$



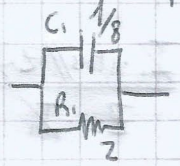
$$Z_A = Z_{22} - a = 6\,\frac{s^2+8s+12}{(s+1)(s+4)} - a = \frac{6s^2+48s+72 - a(s^2+5s+4)}{(s+1)(s+4)}$$

$$Z_A = \frac{(6-a)s^2 + (48-5a)s + 72 - 4a}{(s+1)(s+4)}$$

∘ Remuevo Polo en -4:

$$Z_B = Z_A - \frac{K_1}{s+4} \implies K_1 = \lim_{s\to -4} (s+4)\,Z_A = \frac{(6-a)16 + (48-5a)(-4) + 72 - 4a}{1-4}$$

$$K_1 = \frac{96 - 16a - 192 + 20a + 72 - 4a}{(-3)} = \boxed{8}$$



$$Z_B = Z_A - \frac{8}{s+4} = \frac{(6-a)s^2 + (48-5a)s + 72 - 4a - 8(s+1)}{(s+1)(s+4)} = \frac{(6-a)s^2 + (40-5a)s + 64 - 4a}{(s+1)(s+4)}$$

Auxiliarmente:

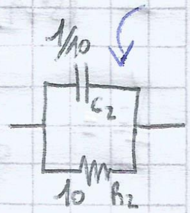
$$\begin{array}{rl|l}
 & (6-a)s^2 + (40-5a)s + 64 - 4a & s+4 \\ \cline{3-3}
- & (6-a)s^2 + (24-4a)s & (6-a)s + 16 - a \\ \cline{2-2}
 & (16-a)s + 64 - 4a & \\
- & (16-a)s + 64 - 4a & \\ \cline{2-2}
 & 0 &
\end{array}
\Bigg\}\; Z_B = \frac{\left[(6-a)s + 16 - a\right]\cancel{(s+4)}}{(s+1)\cancel{(s+4)}}$$

$$\therefore\; Z_B = \boxed{\frac{(6-a)s + 16 - a}{s+1}}$$

NOTA

- Remuevo Polo en -1:

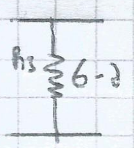
$$Z_C = Z_B - \frac{K_2}{s+1} \Longrightarrow K_1 = \lim_{s\to -1} (s+1)\, Z_B = (6-a)(-1) + 16 - a = \cancel{a} - 6 + 16 - \cancel{a} = \boxed{10}$$



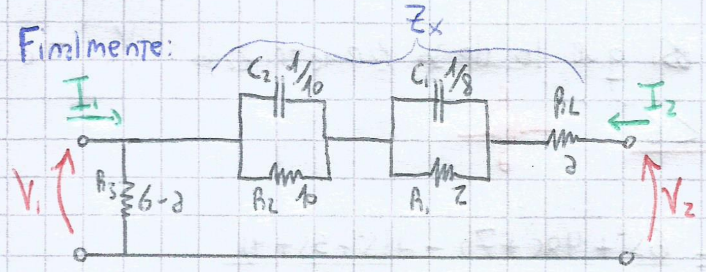
$$Z_C = \frac{(6-a)s + 16 - a - 10}{s+1} = \frac{(6-a)\cancel{(s+1)}}{\cancel{s+1}} = \boxed{6-a}$$

- Remuevo Constante en el origen (admitancia):

$$Y_C = \frac{1}{6-a} = \frac{1}{R_3}$$



- Finalmente:



- Solo falta definir a, para lo cual se calcula Z_{21} del cuadripolo original (sin R_L) y $\frac{-I_2}{I_1}\Big|_{V_2=0}$ del cuadripolo contemplando R_L, que es equivalente a $-\frac{I_2}{I_1}$ del cuadripolo original cargado con R_L.

- $$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = R_3 = 6-a = 6H \Longrightarrow H = \frac{6-a}{6}$$

- $$-\frac{I_2}{I_1}\Big|_{V_2=0} = \frac{V_1/Z_x}{I_1} = \frac{\cancel{I_1}\cdot(R_3 // Z_x)}{Z_x \cdot \cancel{I_1}} = \frac{R_3 \cdot \cancel{Z_x}}{\cancel{Z_x}(R_3+Z_x)} = \frac{R_3}{R_3+Z_x}$$

$$-\frac{I_2}{I_1}\Big|_{V_2=0} = \frac{6-a}{6-\cancel{a} + \frac{8}{s+4} + \frac{10}{s+1} + \cancel{a}} = \frac{6-a}{\frac{6(s+1)(s+4) + 8(s+1) + 10(s+4)}{(s+1)(s+4)}}$$

$$-\frac{I_2}{I_1}\Big|_{V_2=0} = \frac{(6-a)(s^2+5s+4)}{6s^2+30s+24+8s+8+10s+40} = \underbrace{\frac{6-a}{6}}_{H}\ \frac{s^2+5s+4}{s^2+8s+12}$$

Verifica $\forall a$

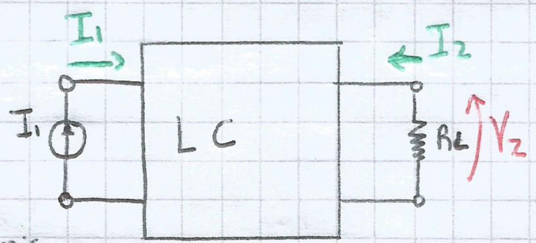
- Elijo $a = 1 \Longrightarrow R_L = 1 \wedge R_3 = 5$

NOTA

HOJA Nº

FECHA

2)



$$T(s) = \frac{V_2}{I_1} = \frac{s^2+9}{s^3+2s^2+2s+1}$$

1° Parte: Síntesis

- Condición de carga: $V_2 = -I_2 \cdot R_L \Rightarrow I_2 = \frac{-V_2}{R_L}$

- Evalúo Parámetros Z e Y:

$$\begin{cases} V_1 = Z_{11} \cdot I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases} \qquad \begin{cases} I_1 = Y_{11} \cdot V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{cases}$$

$$V_2 = Z_{21} I_1 + Z_{22} \cdot \left(\frac{-V_2}{R_L}\right)$$

$$V_2 \left(1 + \frac{Z_{22}}{R_L}\right) = Z_{21} I_1$$

$$\frac{V_2}{I_1} = \frac{Z_{21}}{1 + \frac{Z_{22}}{R_L}} \qquad (R_L = 1)$$

Con $\frac{V_2}{I_1} = \frac{P}{Q} = \frac{P}{m+n} = \begin{cases} \frac{P/m}{1 + n/m} \\ \frac{P/n}{1 + m/n} \end{cases}$

- P es par, Z_{21} debe ser impar $\Rightarrow \boxed{\frac{V_2}{I_1} = \frac{P/n}{1 + m/n} = \frac{Z_{21}}{1 + Z_{22}}}$ $(R_L = 1)$

$$\frac{V_2}{I_1} = \frac{s^2+9}{\underbrace{(s^3+2s)}_{n} + \underbrace{(2s^2+1)}_{m}} = \frac{\frac{s^2+9}{s^3+2s}}{1 + \frac{2s^2+1}{s^3+2s}}$$

a) Síntesis Gráfica

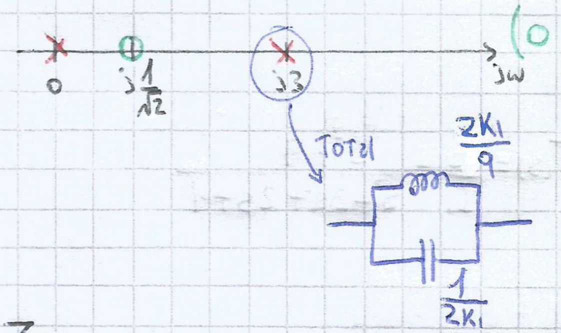
$$Z_{22} = \frac{2\left(s^2+\frac{1}{2}\right)}{s\left(s^2+2\right)}$$

Z_{22}:

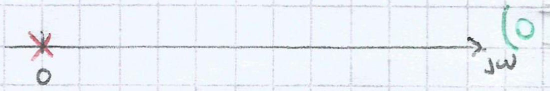
Se debe sintetizar removiendo polos en j3 e infinito (Ceros de transmisión)

$\frac{1}{Z_{22}}$:

Z_A

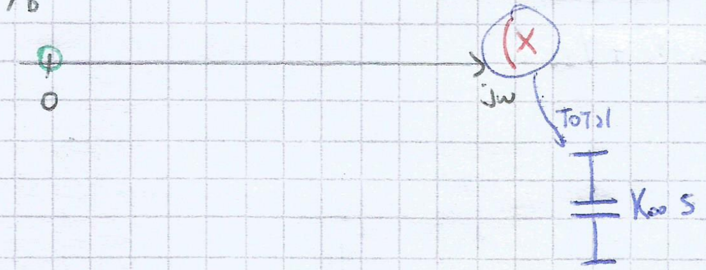


Z_B



∘ Fuerzo a tener un componente en derivación a la fuente de corriente:

Y_B



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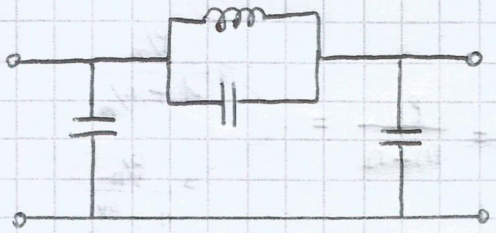


b) Síntesis Analítica:

Remoción Parcial en ∞:

$$\frac{1}{Z_{22}} = \frac{1}{2}\,\frac{s(s^2+2)}{s^2+1/2} \Longrightarrow Y_A = \frac{1}{Z_{22}} - K'_\infty \cdot s \quad \text{Tal que} \quad Y_{A(j3)} = 0$$

$$\therefore K'_\infty = \left.\frac{1}{Z_{22}\cdot s}\right|_{s=j3} = \frac{1}{2}\,\frac{2-9}{\frac{1}{2}-9} = \boxed{\frac{7}{17}}$$

$$Y_A = \frac{1}{2}\,\frac{s(s^2+2)}{s^2+1/2} - \frac{7}{17}s = \frac{s^3+2s-\frac{7}{17}s(s^2+1/2)\cdot 2}{2(s^2+1/2)} = \frac{s^3+2s-\frac{14}{17}s^3-\frac{7}{17}s}{2(s^2+1/2)} = \frac{\frac{3}{17}s^3+\frac{27}{17}s}{2(s^2+1/2)}$$

$$\boxed{Y_A = \frac{3}{34}\,\frac{s^3+9s}{s^2+1/2}}$$

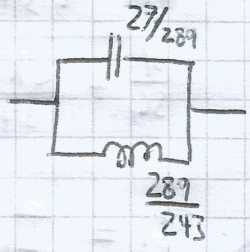
∘ Remuevo Polo en $j3$:

$$Z_A = \frac{34}{3}\,\frac{s^2+1/2}{s(s^2+9)} \Longrightarrow Z_B = Z_A - \frac{2K_1\cdot s}{s^2+9}$$

NOTA

$$\lim_{s^2\to -9} Z_A = \lim_{s^2\to -9} \frac{2K_1 s}{s^2+9} \Longrightarrow 2K_1 = \lim_{s^2\to -9} \frac{(s^2+9)}{s}\cdot Z_A$$

$$2K_1 = \lim_{s^2\to -9} \frac{34}{3}\,\frac{s^2+1/2}{s^2} = \frac{34}{3}\,\frac{1/2-9}{-9} = \boxed{\frac{289}{27}}$$

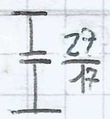


$$\therefore Z_B = \frac{34}{3}\,\frac{s^2+1/2}{s(s^2+9)} - \frac{289}{27}\cdot\frac{s}{s^2+9} = \frac{306(s^2+1/2)-289s^2}{27(s^2+9)s} = \frac{17s^2+153}{27(s^2+9)s}$$

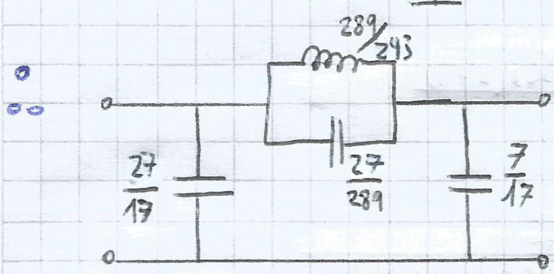
$$Z_B = \frac{17}{27}\,\frac{s^2+9}{s(s^2+9)} = \boxed{\frac{17}{27}\cdot\frac{1}{s}}$$

∘ Remoción total de polo en infinito:

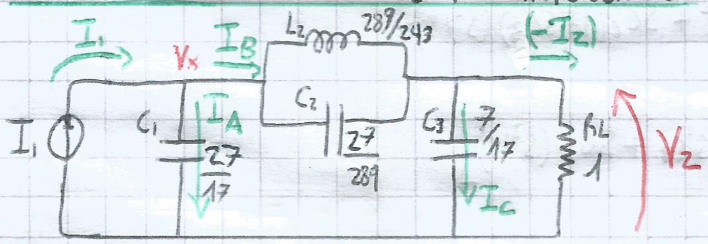
$$Y_B = \boxed{s\cdot\frac{27}{17}}$$



$\therefore$



2^da Parte: Verificación de transimpedancia



1. $\dfrac{V_2}{I_1}$

3. $I_1 = I_A + I_B \Rightarrow I_B = I_1 - I_A$

2. $V_2 = I_B\cdot\left(R_L /\!/ \dfrac{1}{sC_3}\right)$

4. $I_A = V_x\cdot sC_1$

5. $V_x = I_1\cdot\left[\dfrac{1}{sC_1} /\!/ \left(\dfrac{2K_1 s}{s^2+9} + \dfrac{1}{sC_3} /\!/ R_L\right)\right]$

$$(2)\quad V_2 = I_B \cdot \frac{1}{\frac{1}{R_L}+sC_3}$$

$$(5)\quad V_x = I_1 \cdot \frac{\frac{1}{sC_1}\cdot\left(\frac{2K_1 s}{s^2+9}+\frac{1}{\frac{1}{R_L}+sC_3}\right)}{\frac{1}{sC_1}+\frac{2K_1 s}{s^2+9}+\frac{1}{\frac{1}{R_L}+sC_3}}$$

$$(4)\quad I_A = V_x \cdot sC_1 = I_1 \cdot \frac{\frac{2K_1 s}{s^2+9}+\frac{1}{\frac{1}{R_L}+sC_3}}{\frac{1}{sC_1}+\frac{2K_1 s}{s^2+9}+\frac{1}{\frac{1}{R_L}+sC_3}}$$

$$(3)\quad I_B = I_1 - I_A = I_1\left(1-\frac{\frac{2K_1 s}{s^2+9}+\frac{1}{1/R_L+sC_3}}{\frac{1}{sC_1}+\frac{2K_1 s}{s^2+9}+\frac{1}{1/R_L+sC_3}}\right)$$

$$\therefore (2)\quad V_2 = I_1\cdot\left(\frac{1}{1/R_L+sC_3}-\frac{\frac{2K_1 s}{s^2+9}+\frac{1}{1/R_L+sC_3}}{\frac{1/R_L+sC_3}{sC_1}+\frac{2K_1 s(1/R_L+sC_3)}{s^2+9}+1}\right) \qquad (R_L=1)$$

$$\frac{V_2}{I_1} = \frac{1}{1+sC_3} - \frac{\frac{2K_1 s(1+sC_3)+s^2+9}{\cancel{(s^2+9)}(1+sC_3)}}{\frac{(1+sC_3)(s^2+9)+2K_1 s^2 C_1(1+sC_3)+sC_1(s^2+9)}{sC_1\cancel{(s^2+9)}}}$$

$$\frac{V_2}{I_1} = \frac{1}{1+sC_3} - \frac{2K_1 s\,(sC_1+s^2C_3C_1)+(s^2+9)sC_1}{(1+sC_3)^2(s^2+9)+2K_1 s^2 C_1(1+sC_3)^2+sC_1(1+sC_3)(s^2+9)}$$

$$\frac{V_2}{I_1} = \frac{(1+sC_3)(s^2+9)+\cancel{2K_1 s^2 C_1(1+sC_3)}+sC_1(s^2+9)-\cancel{2K_1 s^2 C_1(1+sC_3)}-(s^2+9)sC_1}{(1+sC_3)^2(s^2+9)+2K_1 s^2 C_1(1+sC_3)^2+sC_1(1+sC_3)(s^2+9)}$$

$$\frac{V_2}{I_1} = \frac{(s^2+9)\left[1+sC_3+\cancel{sC_1}-\cancel{sC_1}\right]}{(1+sC_3)\left[(1+sC_3)(s^2+9)+(1+sC_3)2K_1 s^2 C_1+sC_1(s^2+9)\right]}$$

$$\frac{V_2}{I_1} = \frac{\cancel{(1+sC_3)}(s^2+9)}{\cancel{(1+sC_3)}\left[(s^2+9)(1+s(C_1+C_3))+2K_1 s^2 C_1+2K_1 s^3 C_1 C_3\right]}$$

$$\frac{V_2}{I_1} = \frac{s^2+9}{(s^2+9)(1+2s)+17s^2+7s^3} = \frac{s^2+9}{s^2+2s^3+9+18s+17s^2+7s^3}$$

$$\frac{V_2}{I_1} = \frac{s^2+9}{9s^3+18s^2+18s+9}$$

$$\frac{V_2}{I_1} = \frac{1}{9}\,\frac{s^2+9}{s^3+2s^2+2s+1}$$