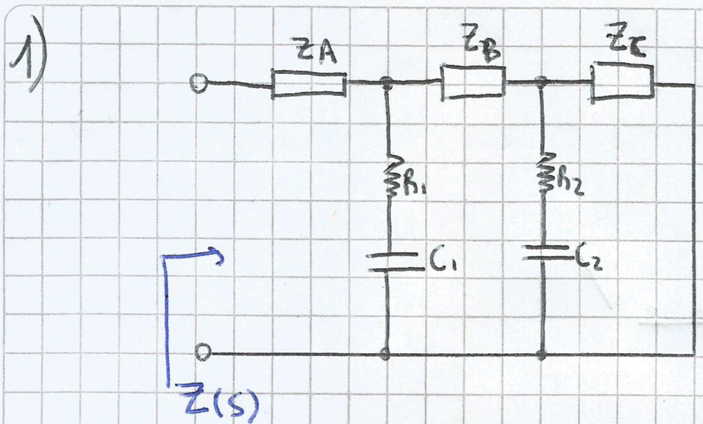
TS11

Hoja: 1

Fecha:

1)



- $R_1 \cdot C_1 = \frac{1}{6}$
- $R_2 \cdot C_2 = \frac{2}{7}$
- $Z(s) = \frac{s^2+6s+8}{s^2+4s+3}$

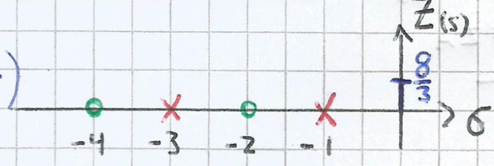
- Para cumplir con las constantes de tiempo:

$$Y_{RC_i} = \frac{1}{R_i + \frac{1}{sC_i}} = \frac{sC_i}{sC_iR_i+1} = \frac{s\frac{1}{R_i}}{s+\frac{1}{R_iC_i}} = \frac{sK_i}{s+\sigma_i}$$

Se debe retirar un polo en admitancia con $\sigma_i = (R_iC_i)^{-1}$

- Análisis Gráfico:

① $\frac{1}{T}$)

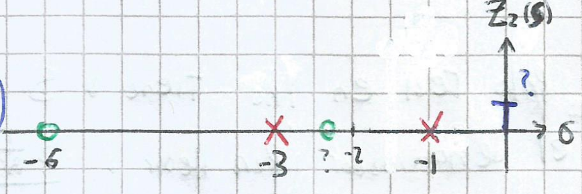


$$Z(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

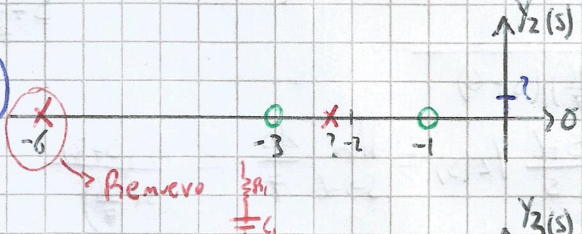
- Debo hacer una remoción parcial de impedancia tal que me quede un cero en $\sigma = -6$, y luego al pasar a admitancia tenga el polo en $-6 = \frac{-1}{R_1 C_1}$
- El único cero que puedo colocar en -6 es el de -4.

∴ Debo remover parcialmente la constante en infinito. K_∞ (Es posible y seguro dado que $Z(0) > Z(\infty)$)

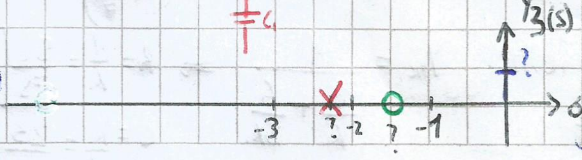
② $\frac{?}{T}$)



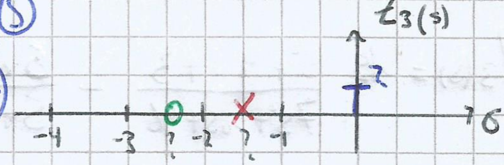
③ $\frac{?}{T}$)



④ $\frac{?}{T}$)

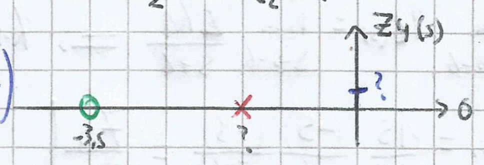


⑤ $\frac{?}{T}$)



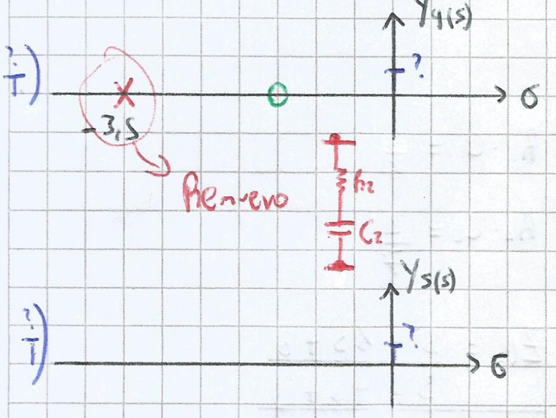
- Remuevo parcialmente K_∞'' para llevar el cero a $-\frac{7}{2} = \frac{-1}{R_2 \cdot C_2}$ (Es seguro porque $Z_{3(\infty)} < Z_{3(0)}$)

⑥ $\frac{?}{T}$)



Notas:

○ Paso a admitancia y remuevo el polo en $-\frac{7}{2}$:



Termino ✓

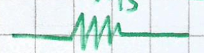
• Cálculo Algebraico

• Remoción Parcial para lograr cero en $\sigma=-6$:

$$Z_2(s) = Z(s) - K_\infty' \qquad \text{Tal que} \qquad \left[Z(s) - K_\infty'\right]\Big|_{s=-6} = Z_2(s)\Big|_{s=-6} = 0$$

$$\therefore K_\infty' = Z(-6) = \frac{36-36+8}{36-24+3} = \boxed{\frac{8}{15}}$$

$8/15 = Z_A$



○ Paso a admitancia, y luego remuevo polo en -6:

$$Z_2(s) = \frac{s^2+6s+8}{s^2+4s+3} - \frac{8}{15} = \frac{s^2+6s+8 - \frac{8}{15}s^2 - \frac{32}{15}s - \frac{8}{5}}{s^2+4s+3} = \frac{\frac{7}{15}s^2 + \frac{58}{15}s + \frac{32}{5}}{s^2+4s+3}$$

$$Y_2(s) = 15\cdot\frac{s^2+4s+3}{7s^2+58s+96} = \frac{15}{7}\,\frac{(s+1)(s+3)}{\left(s+\frac{16}{7}\right)(s+6)}$$

$$Y_3(s) = 15\,\frac{s^2+4s+3}{7s^2+58s+96} - \frac{s\cdot K_1}{s+6}$$

El polo real en Y_{RC} tiene a s en el denominador (para llegar a $\frac{s\frac{1}{R}}{s+\frac{1}{RC}}$)

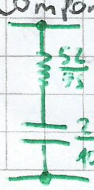
$$Y_2(s) = Y_3(s) + \frac{sK_1}{s+6}$$

$$\lim_{s\to-6} Y_2(s) = \lim_{s\to-6}\frac{sK_1}{s+6} \Longrightarrow K_1 = \lim_{s\to-6}\frac{s+6}{s}\,Y_2(s) = \lim_{s\to-6}\frac{15}{7}\cdot\frac{(s+1)(s+3)}{s\left(s+\frac{16}{7}\right)}$$

$$K_1 = \frac{15}{\cancel{7}}\,\frac{(-5)(-3)}{-6\left(-\frac{26}{\cancel{7}}\right)} = \boxed{\frac{75}{52}}$$

∴ Componentes: $Y_{RC_1} = \dfrac{1}{\frac{1}{K_1} + \frac{6}{sK_1}} \Rightarrow R = \frac{1}{K_1} = \boxed{\frac{52}{75}}$

$C = \frac{K_1}{6} = \boxed{\frac{25}{104}}$



- obtengo $Z_{3(s)}$ para luego hacer la remoción parcial forzando un cero en $-\frac{7}{2}$:

$$Y_{3(s)} = \frac{15}{7}\,\frac{s^2+4s+3}{(s+\frac{16}{7})(s+6)} - \frac{sK_1}{(s+6)}\frac{(s+\frac{16}{7})7}{(s+\frac{16}{7})7} = \frac{15s^2+60s+45-s^2K_1\cdot 7 - sK_1\cdot 16}{7(s+\frac{16}{7})(s+6)}$$

$$Y_{3(s)} = \frac{(15-K_1 7)s^2 + (60-K_1\cdot 16)s + 45}{7(s+\frac{16}{7})(s+6)} = \frac{\frac{255}{52}s^2 + \frac{480}{13}s + 45}{7(s+\frac{16}{7})(s+6)}$$

$$Y_{3(s)} = \frac{255}{364}\,\frac{(s+\frac{26}{17})\cancel{(s+6)}}{(s+\frac{16}{7})\cancel{(s+6)}}$$

$$\boxed{Z_{3(s)} = \frac{364}{255}\cdot\frac{s+\frac{16}{7}}{s+\frac{26}{17}}}$$

$$Z_{4(s)} = Z_{3(s)} - K_\infty'' \quad \text{Tal que} \quad \left[Z_{3(s)} - K_\infty''\right]\Big|_{s=-\frac{7}{2}} = Z_{4(s)}\Big|_{s=-\frac{7}{2}} = 0$$

$$\therefore K_\infty'' = Z_{3(-\frac{7}{2})} = \frac{364}{255}\cdot\frac{-\frac{7}{2}+\frac{16}{7}}{-\frac{7}{2}+\frac{26}{17}} = \boxed{\frac{884}{1005}}$$

$\frac{884}{1005} = Z_B$

- obtengo $Z_{4(s)}$, lo paso a admitancia y remuevo el polo en $-\frac{7}{2}$:

$$Z_{4(s)} = \frac{364s+832}{255\cdot s+390} - \frac{884}{1005} = \frac{365820\cdot s + 836160 - 225420\cdot s - 344760}{256275\cdot s + 391950}$$

$$Z_{4(s)} = \frac{140400\cdot s + 491400}{256275\cdot s + 391950} = \boxed{\frac{624}{1139}\;\frac{s+\frac{7}{2}}{s+\frac{26}{17}}}$$

$$Y_{4(s)} = \frac{1139}{624}\;\frac{s+\frac{26}{17}}{s+\frac{7}{2}}$$

$$Y_{5(s)} = Y_{4(s)} - \frac{s\cdot K_2}{s+\frac{7}{2}} \implies Y_{4(s)} = Y_{5(s)} + \frac{sK_2}{s+\frac{7}{2}}$$

$$\lim_{s\to -\frac{7}{2}} Y_{4(s)} = \lim_{s\to-\frac{7}{2}} \frac{sK_2}{s+\frac{7}{2}} \implies K_2 = \lim_{s\to-\frac{7}{2}} \frac{s+\frac{7}{2}}{s} Y_{4(s)} = \lim_{s\to-\frac{7}{2}} \frac{1139}{624}\;\frac{s+\frac{26}{17}}{s}$$

Notas:

$$K_2 = \frac{1139}{624}\left(1+\frac{26}{17\cdot\left(-\frac{7}{2}\right)}\right) = \boxed{\frac{4489}{4368}}$$

∴ Componentes: $Y_{RC_2} = \dfrac{1}{\dfrac{1}{K_2}+\dfrac{7/2}{s K_2}}$

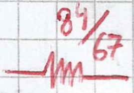
$$\begin{cases} R_2 = \dfrac{1}{K_2} = \boxed{\dfrac{4368}{4489}} \\[2ex] C_2 = \dfrac{K_2}{\frac{7}{2}} = \boxed{\dfrac{4489}{15288}} \end{cases}$$

• Obtengo $Y_{5(s)}$, y luego $Z_{5(s)} = Z_C$

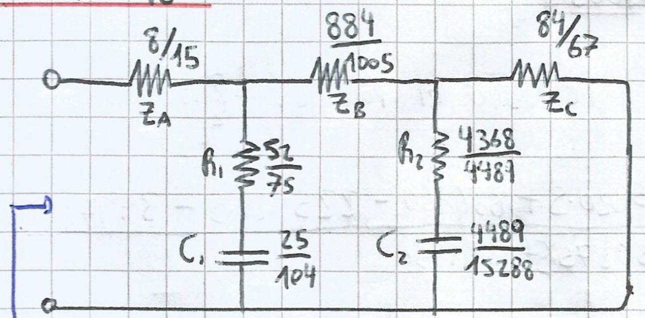
$$Y_{5(s)} = \frac{1139}{624}\,\frac{s+\frac{26}{17}}{s+\frac{7}{2}} - \frac{s K_2}{s+\frac{7}{2}} = \frac{\left(\frac{1139}{624}-K_2\right)s+\frac{67}{24}}{s+\frac{7}{2}} = \frac{\frac{67}{84}s+\frac{67}{24}}{s+\frac{7}{2}}$$

$$Y_{5(s)} = \frac{67}{84}\,\frac{\cancel{s+\frac{7}{2}}}{\cancel{s+\frac{7}{2}}} = \frac{67}{84}$$

$$\boxed{Z_{5(s)} = \frac{84}{67} = Z_C}$$



∴ Finalmente:



$$Z(s) = \frac{s^2+6s+8}{s^2+4s+3}$$

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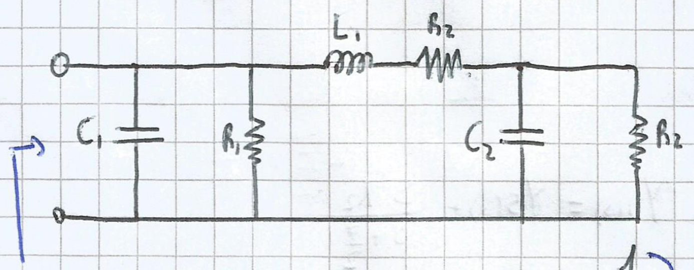
2)



$$Z(s) = \frac{s^2+s+1}{(s^2+2s+5)(s+1)} \begin{cases} Z(0) = \dfrac{1}{5} \\ Z(\infty) = 0 \end{cases}$$

Se trata de una impedancia capacitiva

- Para obtener C_1 debo trabajar con $Y(s)$ y restar $K_\infty \cdot S = C_1 \cdot S$:

$$Y(s) = \frac{(s^2+2s+5)(s+1)}{s^2+s+1} = \frac{s^3+2s^2+5s+s^2+2s+5}{s^2+s+1} = \frac{s^3+3s^2+7s+5}{s^2+s+1}$$

- Para restar $K_\infty \cdot S$ uso el método de división de polinomios:

$Y_2 = Y - SC_1$

$$Y_2 = \frac{Resto}{Q} = \frac{2s^2+6s+5}{s^2+s+1} \quad \begin{cases} Y_{2(0)} = 5 \\ Y_{2(\infty)} = 2 \end{cases}$$

$$\begin{array}{r|l} s^3+3s^2+7s+5 & s^2+s+1 \\ -\;s^3+s^2+s+0 & s \\ \hline Resto\;\; 2s^2+6s+5 & \end{array}$$

$C_1 = 1$

- Debo remover una constante (R_1) de una admitancia inductiva; donde $Y_{LC(0)} > Y_{LC(\infty)}$. Por lo que para que siga siendo FRP debo remover del infinito (mínimo valor):

- Para remover K_∞ uso Cauer por división de polinomios:

$$Y_3 = Y_2 - \frac{1}{R_1}$$

$$\begin{array}{r|l} 2s^2+6s+5 & s^2+s+1 \\ -\;2s^2+2s+2 & 2 \\ \hline Resto\;\; 4s+3 & \end{array}$$

$R_1 = \frac{1}{2}$

$$Y_{3(s)} = \frac{4s+3}{s^2+s+1}$$

$\therefore$ $$Z_{3(s)} = \frac{s^2+s+1}{4s+3} \quad \begin{cases} Z_{3(0)} = \frac{1}{3} \\ Z_{3(\infty)} = \infty \end{cases}$$ Impedancia Inductiva

- Para obtener L_1 debo remover el polo en infinito:

$$Z_4 = Z_3 - SL_1$$

$$\begin{array}{r|l} s^2+s+1 & 4s+3 \\ -\;s^2+\frac{3}{4}s+0 & \frac{1}{4}s \\ \hline Resto\;\; \frac{1}{4}s+1 & \end{array}$$

$L_1 = 1/4$

$$Z_4 = \frac{\frac{1}{4}s+1}{4s+3} \quad \begin{cases} Z_{4(0)} = \frac{1}{3} \\ Z_{4(\infty)} = \frac{1}{16} \end{cases}$$

Notas:

- Para obtener R_2 se debe remover la mínima parte real, en este caso en infinito:

$$\begin{array}{r|l} \frac{1}{4}s + 1 & 4s+3 \\ \hline -\left(\frac{1}{4}s + \frac{3}{16}\right) & \frac{1}{16} \\ \hline \text{Resto } \frac{13}{16} & \end{array}$$

$\frac{1}{16}$ R_2

$$Z_5 = Z_4 - R_2$$

$$Z_5 = \frac{\frac{13}{16}}{4s+3}$$

- Para remover el Capacitor C_2 Paso a admitancia:

$$Y_5 = \frac{4s+3}{\frac{13}{16}} \quad \nearrow Y_{5(0)} = \frac{48}{13} \quad \searrow Y_{5(\infty)} = \infty$$

El Capacitor es el Polo en infinito:

$$\begin{array}{r|l} 4s+3 & \frac{13}{16} \\ \hline -\left(4s+0\right) & \frac{64}{13}s \\ \hline \text{Resto } 3 & \end{array}$$

C_2 $\frac{64}{13}$

$$Y_6 = Y_5 - sC_2$$

$$Y_6 = \frac{3}{\frac{13}{16}} = \frac{48}{13} \Longrightarrow Z_6 = \frac{13}{48} = R_3$$

- Finalmente: