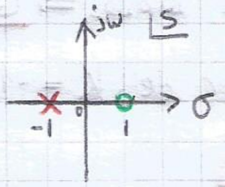
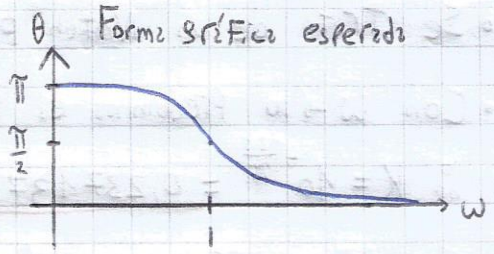
TS5

1) a) $T(s) = \dfrac{s-1}{s+1}$ (Pasa-Todo: rotador de Fase)



Forma gráfica esperada



$$\tau(\omega) = \frac{1}{1+\omega^2} - \frac{(-1)}{1+\omega^2} = \boxed{\frac{2}{1+\omega^2}}$$

Con Singularidades simples

$$\left(\tau(\omega) = \sum \frac{\sigma_{z_i}}{\sigma_{z_i}^2+\omega^2} - \sum \frac{\sigma_{p_i}}{\sigma_{p_i}^2+\omega^2}\right)$$

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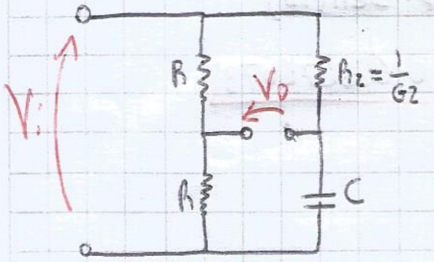
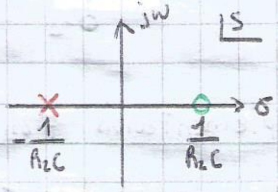
b) Topología Pasiva: Lattice



$$T(s) = \frac{V_o}{V_i} = \frac{1}{2} - \frac{G_2}{sC+G_2} = \frac{sC+G_2-2G_2}{2sC+2G_2} = \frac{1}{2}\,\frac{sC-G_2}{sC+G_2}$$

$$T(s) = \frac{1}{2}\,\frac{s-\frac{G_2}{C}}{s+\frac{G_2}{C}} = \boxed{\frac{1}{2}\,\frac{s-\frac{1}{R_2C}}{s+\frac{1}{R_2C}}}$$



$\Delta\theta = 15° = \theta(0) - \theta(1)$

Segundo Cuadrante $\to 1$ — Primer Cuadrante $\to 1$

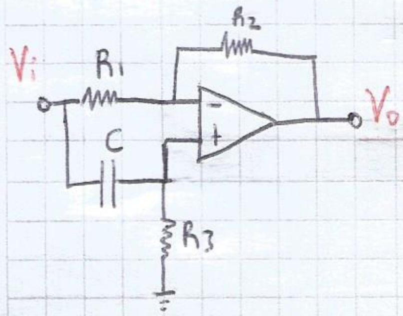
$$\theta_{(\omega=1)} = 180° - 15° = \frac{11\pi}{12} = \arctan\left(\frac{\omega}{\frac{-1}{R_2C}}\right) - \arctan\left(\frac{\omega}{\frac{1}{R_2C}}\right) = \pi - \arctan(R_2\cdot C) - \arctan(R_2\cdot C)$$

$$\therefore\ \pi - 2\arctan(R_2\cdot C) = \frac{11\pi}{12} \Longrightarrow \arctan(R_2\cdot C) = \frac{1}{24}\pi$$

$$R_2\cdot C = 0{,}130159643$$

Elijo: $\boxed{R_2 = 1} \Longrightarrow \boxed{C = 0{,}130159643}$ Elijo $\boxed{R = R_2 = 1}$

Topología Activa:



$\underline{R_1 = R_2}$

$$T(s) = \frac{s - \frac{1}{\frac{R_1}{R_2}\cdot CR_3}}{s+\frac{1}{CR_3}} \Longrightarrow T(s) = \frac{s-\frac{1}{CR_3}}{s+\frac{1}{CR_3}}$$

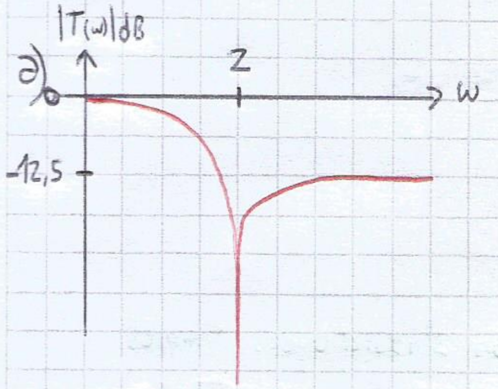
Nuevamente $R_3\cdot C = 0{,}130159643 \Longrightarrow \begin{cases} R_3 = 1 = R_1 = R_2 \\ C = 0{,}130159643 \end{cases}$

NOTA

2) $T(s) = K \cdot \dfrac{s^2 + s \cdot \frac{\omega_n}{Q_n} + \omega_n^2}{s^2 + s \cdot \frac{\omega_p}{Q_p} + \omega_p^2}$

Denominador Butterworth

$\therefore \boxed{Q_p = \frac{1}{\sqrt{2}}}$ Siempre

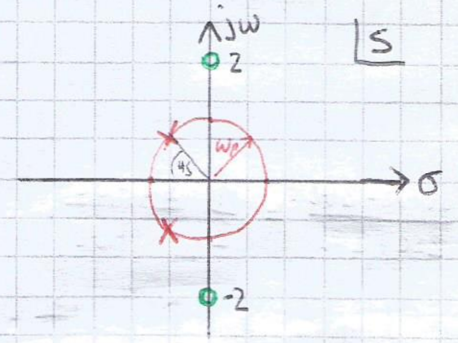


- Se trata de un Notch pasa bajos $\Rightarrow$ $Q_n \to \infty$
- Con $\omega \to \infty$ predomina el valor de $K = -12{,}5$dB

  $\therefore K = 10^{-\frac{12,5}{20}} = 0{,}23713737$

- Con $\omega = 0$ predomina $K \cdot \frac{\omega_n^2}{\omega_p^2} = 0\text{dB} \equiv 1$
- $\omega_n = 2$ (cero de transmisión) $\Rightarrow$ $\omega_p = \omega_n \cdot \sqrt{K}$

  $\omega_p = 0{,}97393505$

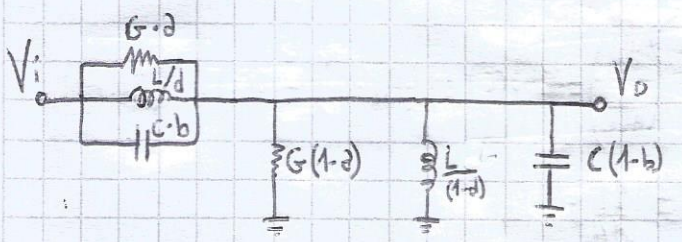


- Ceros en $\pm j2 = \pm j\omega_n$
- Polos: $-\omega_p \cdot \cos(45°) \pm j\,\omega_p \cdot \operatorname{Sen}(45°)$

  $\equiv -0{,}688676078 \pm j\,0{,}688676078$

Implementación Pasiva:

Empleando el levantamiento de admitancia:



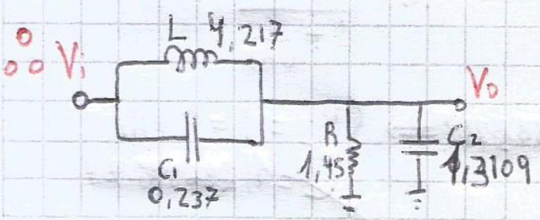
$$\frac{V_o}{V_i} = b\, \frac{s^2 + s\, \overbrace{\frac{G}{C}\frac{a}{b}}^{\emptyset} + \frac{1}{LC}\frac{d}{b}}{s^2 + s\frac{G}{C} + \frac{1}{LC}}$$

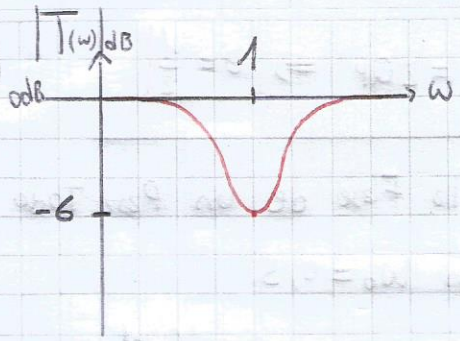
- Para ser notch: $a = 0$
- Elijo normalizar por ω_n $\Rightarrow$ $\frac{1}{LC}\frac{d}{b} = \omega_n'^2 = 1 \;\wedge\; \frac{1}{LC} = \frac{\omega_p^2}{\omega_n^2} = 0{,}23713737$

  $LC = 4{,}216965037$

- $b = K = 0{,}23713737 \Rightarrow d = LC \cdot b = 1$
- $\frac{1}{RC} = \frac{\omega_p}{Q_p} \Rightarrow R \cdot C = \frac{Q_p}{\frac{\omega_p}{\omega_n}} = 1{,}452061472$

Elijo $C = 1 \Rightarrow R = 1{,}452061472 \;\wedge\; L = 4{,}216965037$



b) 

$$T_{(s)} = \frac{s^2 + \frac{\omega_n}{Q_n} s + \omega_n^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2} \cdot K$$

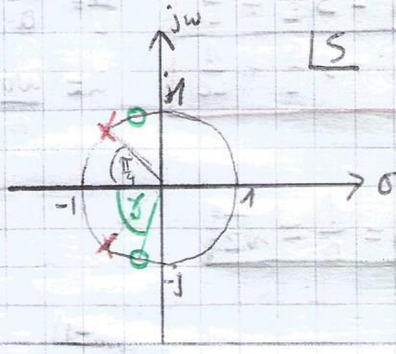
$$T_{(j\omega)} = \frac{-\omega^2 + j\omega \frac{\omega_n}{Q_n} + \omega_n^2}{-\omega^2 + j\omega \frac{\omega_p}{Q_p} + \omega_p^2}$$

$$|T_{(j\omega)}| = \frac{\sqrt{(\omega_n^2-\omega^2)^2 + \left(\omega \frac{\omega_n}{Q_n}\right)^2}}{\sqrt{(\omega_p^2-\omega^2)^2 + \left(\omega \frac{\omega_p}{Q_p}\right)^2}} K \Longrightarrow \begin{cases} \omega = 0 : |T_{(0)}| = 1 = K \frac{\omega_n}{\omega_p} \\ \omega \to \infty : |T_{(\infty)}| = 1 = \frac{\sqrt{\omega^4}}{\sqrt{\omega^4}} \cdot K = K \to \boxed{K=1} \end{cases}$$

$\therefore \frac{\omega_n}{\omega_p} = 1 \Longrightarrow \boxed{\omega_n = \omega_p \equiv \omega_0 = 1}$ de la gráfica (se sabe que $|T_{(\omega_0)}|$ es el mínimo valor)

$$|T_{(j\omega)}| = \frac{\sqrt{(1-\omega^2)^2 + \frac{\omega^2}{Q_n^2}}}{\sqrt{(1-\omega^2)^2 + \frac{\omega^2}{Q_p^2}}} \Longrightarrow |T_{(j1)}| = \frac{\frac{1}{Q_n}}{\frac{1}{Q_p}} = \boxed{\frac{Q_p}{Q_n}} = -6\,dB = \frac{1}{2}$$

$$Q_n = 2\,Q_p = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$$

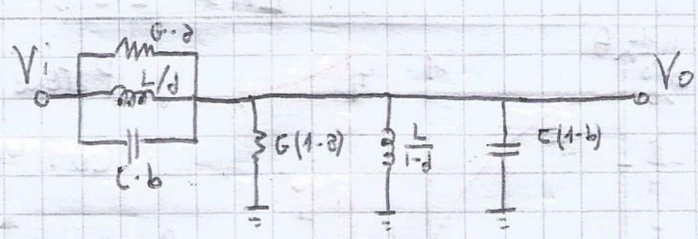


$$Q_n = \frac{1}{2 \cdot \cos\gamma} \Longrightarrow \gamma = \arccos\left(\frac{1}{2Q_n}\right) = 1{,}2094\ rad$$

$\therefore$ Ceros: $-\cos(\gamma) \pm j\,\mathrm{sen}(\gamma) = \underline{-0{,}35355339 \pm j0{,}9354143466}$

Polos: $-\cos\left(\frac{\pi}{4}\right) \pm j\,\mathrm{sen}\left(\frac{\pi}{4}\right) = \underline{-\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}}$

Implementación Pasiva: Nuevamente:



$$T_{(s)} = b\,\frac{s^2 + s\frac{G}{C}\frac{a}{b} + \frac{1}{LC}\frac{d}{b}}{s^2 + s\frac{G}{C} + \frac{1}{LC}}$$

- $\boxed{b = K = 1}$
- Normalizo por $\omega_0 \Longrightarrow \underline{\frac{1}{LC} = 1}$ (queda igual) $\Longrightarrow \frac{1}{LC} \cdot \frac{d}{b} = 1 \Longrightarrow \boxed{d = b = 1}$
- Elijo $\boxed{C = L = 1} \Longrightarrow \frac{G}{C} = \frac{1}{Q_p} = G \Longrightarrow \boxed{R = Q_p = \frac{1}{\sqrt{2}}}$

• $\frac{G}{C}\cdot\frac{a}{b} = \frac{W_0}{Q_n} = \frac{1}{2Q_p}$

$\frac{1}{R}\cdot\frac{a}{b} = \frac{1}{2Q_p} \Rightarrow R\,\frac{b}{a} = 2Q_p \Rightarrow \frac{Q_p}{a} = 2Q_p \Rightarrow \boxed{a = \frac{1}{2}}$

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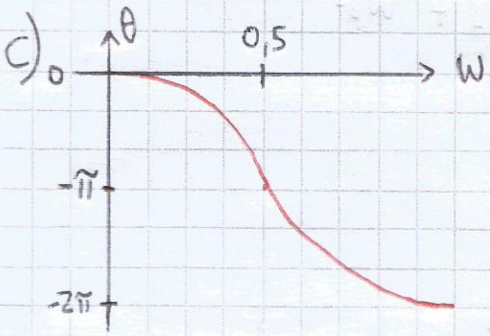
c)



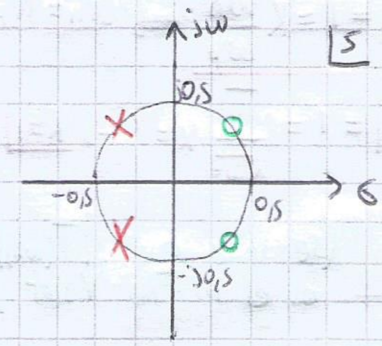
Corresponde a la Fase de un Pasa Todo de 2do orden con $W_0 = 0,5$

$$T(s) = \frac{s^2 - \frac{W_0}{Q}s + W_0^2}{s^2 + \frac{W_0}{Q}s + W_0^2}$$

$\therefore K = 1$

$Q_n = Q_p = \frac{1}{\sqrt{2}}$

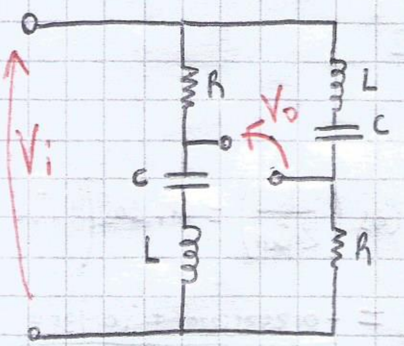
$W_n = W_p = 0,5$



Ceros: $0,5\cdot\cos\left(\frac{\pi}{4}\right) \pm j\,0,5\cdot\operatorname{sen}\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{4} \pm j\frac{\sqrt{2}}{4}}$

Polos: $\boxed{-\frac{\sqrt{2}}{4} \pm j\frac{\sqrt{2}}{4}}$

Implementación Pasiva:



$$V_o = V_i\left(\frac{\frac{1}{sC} + sL}{R + sL + \frac{1}{sC}} - \frac{R}{R + sL + \frac{1}{sC}}\right)$$

$$\frac{V_o}{V_i} = \frac{s^2LC - sCR + 1}{s^2LC + sCR + 1} = \boxed{\frac{s^2 - s\frac{R}{L} + \frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}}$$

Normalizando con W_0: $\left.\begin{array}{l}\frac{1}{LC} = 1 \\ \text{Elijo } L = C = 1\end{array}\right\} \Rightarrow \frac{R}{L} = \frac{1}{Q} \Rightarrow \boxed{R = \frac{1}{Q} = \sqrt{2}}$

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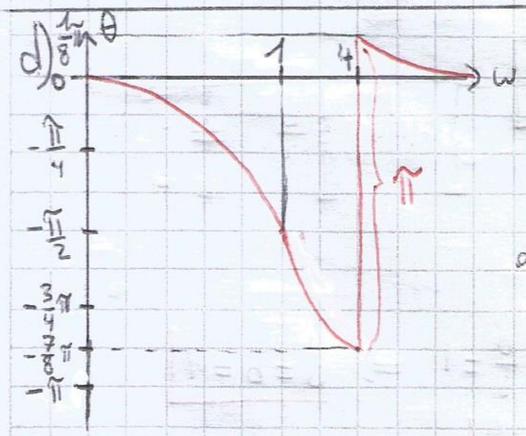
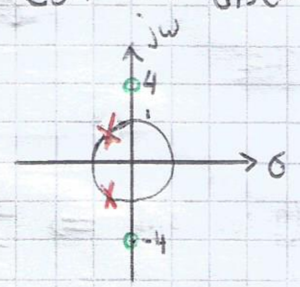
d)



Parece ser la Fase de un notch de $W_0 = 1$, pero con una discontinuidad finita de π en $w = 4$



Polos: $\boxed{-\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}}$

Ceros: $\boxed{\pm j4}$

NOTA

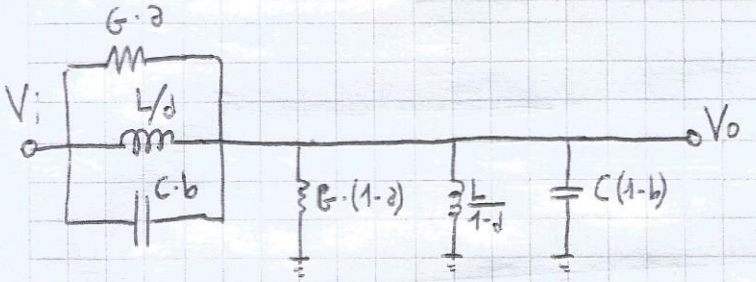
$$T(s)=K\cdot\frac{s^2+\omega_n^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

$\omega_n=4$ $Q_n\to\infty$

$\omega_p=1$ $\underline{K>0}$ (Indeterminable con solo la fase)

$Q_p=\frac{1}{\sqrt{2}}$

Implementación Pasiva:



$$\frac{V_o}{V_i}=b\,\frac{s^2+s\frac{G}{C}\cdot\frac{a}{b}+\frac{1}{LC}\frac{d}{b}}{s^2+s\frac{G}{C}+\frac{1}{LC}}$$

- $a=0$
- $\frac{1}{LC}=1 \Longrightarrow \frac{d}{b}=\omega_n^2=16$
- $b=K \Longrightarrow$ Para tener la menor atenuación posible $d=1$ (máximo)

$\therefore\ b=\frac{d}{16}=\boxed{0{,}0625}$ (máximo)

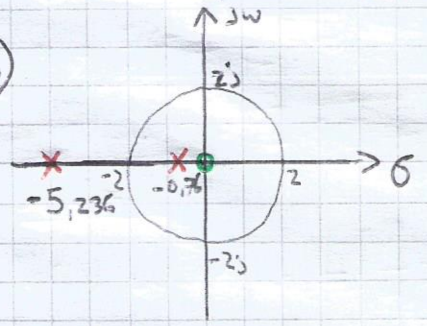
Hago $L=C=1 \Longrightarrow \frac{1}{R}=\frac{1}{Q_p} \Longrightarrow \boxed{R=\frac{\sqrt{2}}{2}}$

3) $\theta(\omega)=\frac{\pi}{2}-\operatorname{arctg}\left(\frac{6\omega}{-\omega^2+4}\right)$

- hay cero en el origen. (Único porque solo hay 2 polos)
- El arctan negativo es del denominador.

a) $\therefore\ F_{(j\omega)}=K\cdot\frac{j\omega\cdot\frac{\omega_0}{Q}}{j\underbrace{6}_{\frac{\omega_0}{Q}}\omega-\omega^2+\underbrace{4}_{\omega_0^2}} \Longrightarrow \boxed{F_{(s)}=K\cdot\frac{s\cdot 6}{s^2+6s+4}}=F_{(j\omega)}\Big|_{\omega=\frac{s}{j}}$

b)



Gráficamente:

$\theta(0)=\boxed{\frac{\pi}{2}}$

$\theta(\omega)\Big|_{\omega\to\infty}=\frac{\pi}{2}-\frac{\pi}{2}\cdot 2=\boxed{-\frac{\pi}{2}}$

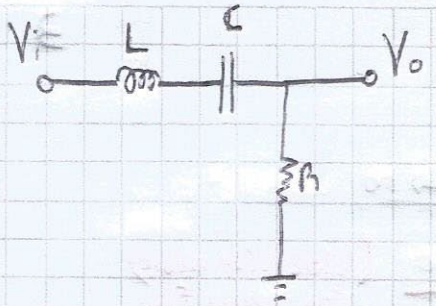
Expresión:

$\theta(0)=\frac{\pi}{2}-\operatorname{arctan}\left(\frac{6\cdot 0}{4}\right)=\boxed{\frac{\pi}{2}}$

$\theta(\omega)\Big|_{\omega\to\infty}=\frac{\pi}{2}-\operatorname{arctan}\left(\frac{\frac{\omega}{\omega}6}{-\omega+\frac{4}{\omega}}\right) \to \emptyset$

$\theta(\omega)\Big|_{\omega\to\infty}=\frac{\pi}{2}-\pi=\boxed{-\frac{\pi}{2}}$

NOTA

c) 

$$\frac{V_o}{V_i} = \frac{R}{R + \frac{1}{SC} + SL} = \frac{SCR}{SCR + 1 + S^2LC}$$

$$T(s) = \frac{s\frac{R}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$\therefore \frac{1}{LC} = 4 \Rightarrow$ Elijo $L = C = 0{,}5$

$\frac{R}{L} = 6 \Rightarrow R = 6 \cdot L = 6 \cdot 0{,}5 = 3$