Advanced and Multivariable Control
30 August 2021

with solutions

Surname ....Name ....

Personal ID ....Signature ....

Dear students, in this exam you will find two types of answers:

1. closed form (choose one over pre-defined answers), +3 for any correct answer, -0.33 for any wrong answer

2. open questions (questions 8 and 9) with answers to be written in the available sheets of paper.

During the exam, you will not be allowed to use books, notes, electronic devices. You cannot exchange any kind of information with anyone.

If something unusual happens, an oral exam will be required.

1

*

(3 punti)

Consider the system

$$\dot{x}_1(t) = x_1(t) - x_2(t) + u_1(t) + u_2(t)$$
$$\dot{x}_2(t) = u_2(t)$$

For this system it is possible to design a pole placement control law provided that:

- ○ in any case, it is not possible to arbitrarly assign the eigenvalues
- ○ no answer
- ○ only u1 can be used
- ○ only u2 can be used
- ⊗ both the inputs u1 and u2 are used

$\dot{x} = Ax + Bu$, $A = \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}$, $B = \begin{array}{cc} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ b_1 \quad b_2 \end{array}$

rank $B = 2$ → reachable with both the inputs

$W_{r1} = |b_1 \; Ab_1| = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$ not reachable with u_1

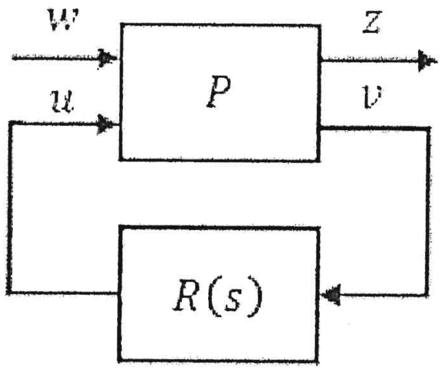
$W_{r2} = |b_2 \; Ab_2| = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}$ not reachable with u_2

2

With reference to infinite horizon LQ control, select the WRONG ANSWER *
(3 punti)

- [x] The only condition for the closed-loop stability is the reachability of the system under control
- [ ] Under the conditions for closed-loop stability of the nominal system, in continuous time a gain variation in the range [+0.5,inf] degrees does not affect the closed loop stability
- [ ] In discrete time, the guaranteed gain margin is smaller than in continuous time
- [ ] no answer
- [ ] it can be used to stabilize systems with integrators

no, the pair (A, Q) must be observable



Consider the two-by-two system reported in the figure. For this system select the couple of correct statements among the following ones:

A. In order to specify frequency characteristics of the corresponding sensitivity functions it is necessary to consider suitable shaping functions.

B. The system can consider only problems without reference signals.

C. Among the elements of the performance vector z it is possible to include also the control variable.

D. The design of the regulator with Hinf control always guarantees better performance than the one based on H2 control. *
(3 punti)

- ○ B - C
- ○ C - D
- ○ no answer
- ○ B - D
- ☒ A - C

4

*

(3 punti)

Given the system $\dot{x}(t) = Ax(t)$ and the Lyapunov equation $A'P + PA = -Q$, which is the wrong statement?

- [x] In order to verify the asymptotic stability of the system one should check that for all the possible Q>0 the corresponding solutions P are greater than zero
- [ ] Necessary condition for the asymptotic stability of the system is that for any Q>0 the solution P is greater than zero.
- [ ] no answer
- [ ] In order to verify the asymptotic stability of the system, it is sufficient to take a generic Q>0 and check that corresponding solution P is greater than zero
- [ ] Sufficient condition for the asymptotic stability of the system is that for any Q>0 the solution P is greater than zero.

5

The MPC method based on the step response of the system *
(3 punti)

- [ ] no answer
- [ ] Requires to derive the impulse response coefficients
- [ ] Can be used only in SISO problems
- [x] Can be applied to asymptotically stable processes
- [ ] Can be used only if the plant under control contains an integral action

6

With a suitable modification of the LQ control it is possible to force the closed-loop eigenvalues to have a real part smaller than -***a***, ***a***>0 (continuous time) or with absolute value smaller than ***a*** (***a***>0) (discrete time) *
(3 punti)

- ○ yes, but only for continuous time systems
- ○ no
- ○ yes, provided that the open-loop system has eigenvalues ***a*** or -***a***.
- ○ no answer
- ☒ yes

7

Concerning linear systems, the separation principle (select the proper answer): *
(3 punti)

- ○ it holds true in case of any linear stabilizing regulator
- ○ applies only to continuous time systems and regulators
- ○ it holds true only in the case of regulators designed with pole placement
- ○ no answer
- ☒ it holds true in case of any linear regulator

Design a reduced order observer for the following system
(5 punti)

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = ax_1(k) + bx_2(k) + u(k)$$
$$y(k) = x_1(k)$$

$$\begin{cases} x_2(h+1) = a\,y(h) + b_2\, x_2(h) + u(h) \\ y(h+1) = x_2(h) \end{cases}$$

<u>observer</u>

$$\hat{x}_2(h+1) = a\,y(h) + b_2\,\hat{x}_2(h) + u(h) + l\left[y(h+1) - \hat{x}_2(h)\right]$$

(7 punti)

Consider the system

$$\dot{x}_1(t) = x_2^3(t)$$
$$\dot{x}_2(t) = u(t)$$

Check with the Lyapunov theorem and the Kraswoski LaSalle result if:
A) the control law $u(t) = -x_2(t) - x_2^2(t)x_1(t)$ is such that the origin is a stable, asymptotically stable, or unstable equilibrium. Use the Lyapunov function $V(x) = 0.5(x_1^2 + x_2^2)$;
B) the control law $u(t) = -x_1(t) - x_2(t)$ is such that the origin is a stable, asymptotically stable, or unstable equilibrium. Use the Lyapunov function $V(x) = 0.5x_1^2 + 0.25x_2^4$

A) $\dot{V} = x_1 \dot{x}_1 + x_2 \dot{x}_2$

$$\begin{cases} \dot{x}_1 = x_2^3 \\ \dot{x}_2 = -x_2 - x_2^2 x_1 \end{cases} \Rightarrow \dot{V} = -x_2^2 \leq 0$$

origin stable equilibrium

KLS Theorem

$\dot{V} = 0 \rightarrow x_2 = 0 \rightarrow \dot{x}_1 = 0 \rightarrow x_1 = \text{constant}$

no asymptotic stability can be concluded

B) $\begin{cases} \dot{x}_1 = x_2^3 \\ \dot{x}_2 = -x_1 - x_2 \end{cases}$, $\dot{V} = -x_2^4 \leq 0$ stability of the origin

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KLS Theorem $\dot{V} = 0 \rightarrow x_2 = 0,\ \dot{x}_2 = 0 \rightarrow x_1 = 0$

origin asymptotically stable equilibrium