AMC - June 2021 *Solutions*

Dear students, in this exam you will find two types of answers:

1. closed form (choose one over pre-defined answers), +3 for any correct answer, -0.33 for any wrong answer

2. open questions (questions 8 and 9) with answers to be written in sheets of paper, each one reporting your Surname, your Name, your ID Polimi, your signature, and the indication "answer to question xx". Please use the sheets of paper that have been made available to you in advance (Beep portal). At the end of the exam, you will have to prepare a unique pdf file containing all these answers and upload it in another form which will be made available to you.

Answers to questions 8 and 9 will not be corrected if the overall grade for closed form answers is smaller that 8.2

During the exam, you will not be allowed to use books, notes, electronic devices (save for what you need at the end of the exam to prepare the pdf file and upload it). You cannot exchange any kind of information with anyone.
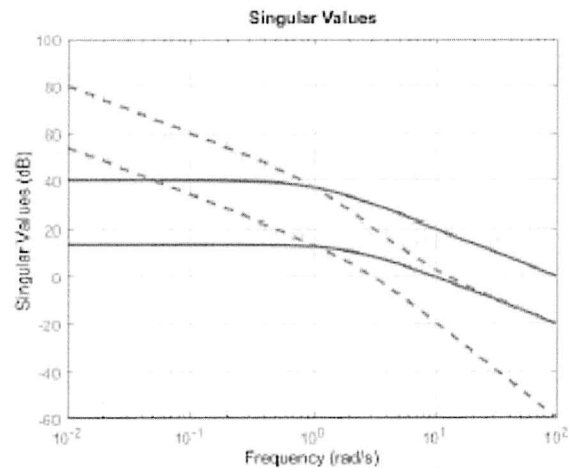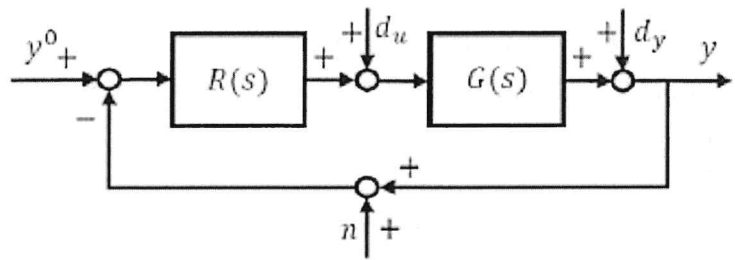
If something unusual happens, an oral exam will be required.

Good work!

(3 punti)

Consider a second order dynamic system with state $x = [\,x_1 \; x_2\,]'$, described by $\dot{x} = f(x)$ and $f(0) = 0$. For this system, take a Lyapunov function $V(x)$ globally positive definite and radially unbounded. Assume also that $\dot{V}(x) = (x_2^2 - 1)x_1^2$. The the origin is an equilibrium:

○ unstable

○ locally asympotically stable

○ globally asymptotically stable

⊗ stable

Singular Values

Consider the feedback system reported in the block diagram and two possible loop transfer functions L1(s)=R1(s)G1(s) and L2(s)=R2(s)G2(s) with the principal gains reported in the figure. Let L1(s) be associated to the dashed lines and L2(s) to the continuous ones. Select the true answer among the following ones:
(3 punti)

○ Assuming that at low and high frequency all the singular values diagrams have the same slope shown in the figure, it is likely that the two loop transfer functions do not have poles at the origin.

○ L1(s) and L2(s) have roughly the same crossover frequency.

○ At high frequency (ω > 20) the guaranteed attenuation of the noise disturbance n provided by L2(s) is always greater than the one guaranteed by L1(s).

✗ At low frequency (ω < 1) the guaranteed attenuation of the disturbance dy provided by L1(s) is always greater than the one guaranteed by L2(s).

3

Assume to have a linear system with n states and n-1 outputs. Then, it is possible to design with pole placement a regulator of order 1 such that all the eigenvalues of the closed-loop system are in prescribed positions.
(3 punti)

○ No, the regulator must be of order n

○ yes provided that the system is stabilizable and detectable

○ yes, provided that an estimator of unknown disturbances is also used

☒ Yes, provided that the system is reachable and observable

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(3 punti)

Consider the system described by

$$G(s) = \begin{bmatrix} \dfrac{(s+1)(s+\alpha)}{(s+5)^3} \\ \dfrac{2}{(s+1)(s+5)} \end{bmatrix}$$

and select the *wrong* statement

☒ the zeros are s=-1, s=-alfa

○ for a generic value of alfa, it is not possible to jointly bring the two outputs to arbitrary constant values

○ the poles are s=-1, s=-5 (multiplicity 3)

○ for alfa=0, only the second output can be asymptotically regulated to an arbitrary constant value

(3 punti)

Consider the system
$$\dot{x}(t) = 2x(t) + u(t)$$
For this system compute the infinite horizon LQ control law with $Q = 2.25$, $R = 1$.
The Riccati equation for LQ control is
$$-\dot{P}(t) = A'P(t) + P(t)A + Q - P(t)BR^{-1}B'P(t)$$
The corresponding closed-loop system is characterized by:

☒ pole in -2.5, gain margin (0.445,inf)

◯ pole in -2.5, gain margin (4.5,inf)

◯ pole in -4.5, gain margin (2,inf)

◯ pole in -2.5, gain margin undefined since the open loop system is unstable

6

In model order reduction methods:
(3 punti)

☒ It is possible to guarantee a prescribed approximation error expressed in terms of the Hinf norm of the difference between the real and the approximate transfer function

◯ with balanced residualization the approximation error is smaller than in balanced truncation (for a fixed order of the approximant)

◯ the procedure can be applied only to transfer functions of a regulator computed with H2 or Hinf control

◯ both balanced truncation and balanced reasidualization guarantee that the steady state gain is preserved

In Model Predictive Control stability of the origin can be obtained by including in the problem formulation
(3 punti)

☒ a zero terminal constraint and no weight on the final state

○ a quadratic terminal constraint and a quadratic weight on the final state ← y also accepted this one

○ a zero terminal constraint and a control horizon smaller than the prediction horizon

○ a quadratic terminal constraint and a linear weight on the final state

8

Domanda
(5 punti)

Consider the system

$$x(k+1) = x(k) + ax(k)u(k) + v_x(k)$$
$$y(k) = x(k) + v_y(k)$$

where a is an unknown, but constant parameter, and v_x, v_y satisfy the standard assumptions of Kalman filtering. Show how to implement an Extended Kalman Predictor to estimate x and a, including the computation of the matrices $A(k)$, $C(k)$ (Riccati equation not required).

See next sheets

(7 punti)

Consider the discrete time system

$$x(k + 1) = x(k) - 2u(k) + d(k)$$
$$y(k) = x(k)$$

for this system

(1) design with pole placement a stabilizing regulator guaranteeing closed-loop poles in $z = 0.5$;
(2) design with LQ_∞ a stabilizing controller. Consider a generic Q and $R = 8Q$.
(3) Show how to estimate the constant, but unknown disturbance d.

steady state Riccati equation:
$$P = A'PA + Q - A'PB(R + B'PB)^{-1}B'PA$$

See next sheets

Exercise 8

$$
\begin{cases}
x(h+1) = x(h) + a(h) x(h) u(h) + v_x(h) \\
a(h+1) = a(h) \\
y(h) = x(h) + v_y(h)
\end{cases}
$$

EKF

Let $\quad L(h) = \begin{vmatrix} \ell_1(h) \\ \ell_2(h) \end{vmatrix}$

$$
\begin{cases}
\hat{x}(h+1|h) = \hat{x}(h|h-1) + \hat{a}(h|h-1)\hat{x}(h|h-1) u(h) + \ell_2(h) \left[y(h) - \hat{x}(h|h-1) \right] \\
\hat{a}(h+1|h) = \hat{a}(h|h-1) + \ell_2(h) \left[y(h) - \hat{x}(h|h-1) \right]
\end{cases}
$$

$L(h)$ is obtained iteratively the discrete Riccati eq.
with matrices

$$
\hat{A}(h) = \begin{vmatrix} 1 + \hat{a}u & \hat{x}u \\ 0 & 1 \end{vmatrix} \quad \begin{array}{l} \hat{x} = \hat{x}(h|h-1) \\ \hat{a} = \hat{a}(h|h-1) \\ u = u(h) \end{array}
$$

$$
\hat{C}(h) = \begin{bmatrix} 1 & 0 \end{bmatrix}
$$

Exercise 9

$$\begin{cases} x(h_{+1}) = x(h) - 2u(h) + d(k) \\ y(h) = x(h) \end{cases}$$

1) $u(h) = -Kx(h)$ (state feedback) , $d = 0$

$x(h_{+1}) = (1 + 2K) x(h)$, $1 + 2K = 0.5 \Rightarrow K = -0.25$

2) $P = 2Q$, $K = \dfrac{B'PA}{R + B'PB} = \dfrac{-4Q}{8Q + 8Q} = -\dfrac{1}{4} = -0.25$

3)
$$\begin{cases} x(h_{+1}) = x(h) - 2u(h) + d(h) \\ d(h_{+1}) = d(h) \end{cases}$$

$$\begin{cases} \begin{vmatrix} x(h_{+1}) \\ d(h_{+1}) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} x(h) \\ d(h) \end{vmatrix} + \begin{vmatrix} -2 \\ 0 \end{vmatrix} u(h) \\ \\ y(h) = \begin{vmatrix} 1 & 0 \end{vmatrix} \begin{vmatrix} x(h) \\ d(h) \end{vmatrix} \end{cases}$$

This system is observable, so you can use a standard observer. The smartest of you have used a reduced order observer. Obviously it is correct.