Advanced and Multivariable Control

7/2014

Exercise 1

Given the system

$$\dot{x}_1(t) = -x_1(t)$$
$$\dot{x}_2(t) = (x_1(t)-1)x_2(t)$$

Compute a Lyapunov function for the analysis of the equilibrium of the nonlinear system by means of the Lyapunov equation of the linearized model with Q=2I.
In view of the previous result, is it possible to draw any conclusion on the global stability of the origin of the nonlinear system?

Exercise 2

Consider the system

$$\dot{x}(t) = -x(t)+u(t)+v_x(t)$$
$$y(t) = x(t)+v_y(t)$$

where v_x is a WGN with null expected value and variance ρ^2, while v_y is a WGN with null expected value and variance equal to 1.

1. assuming that the state is measurable, consider an infinite LQ problem with cost function

$$J = \int_0^\infty (10x^2(\tau)+u^2(\tau))d\tau$$

compute the corresponding control law, the closed-loop eigenvalue, and the loop transfer function.

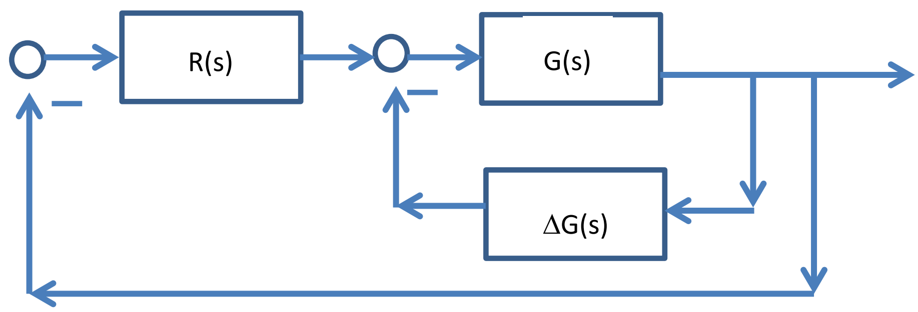
2. compute the Kalman filter (with ρ parametric), the regulator transfer function and the loop transfer function. Show that for increasing values of ρ the loop transfer function tends to the one with the LQ control.

LQ control Riccati equation

$$\dot{P}(t) = A'P(t)+P(t)A+Q-P(t)BR^{-1}B'P(t)$$

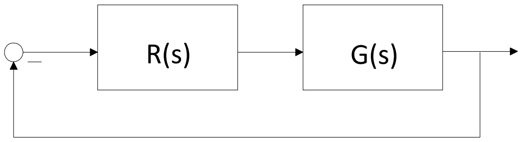
Exercise 3

Consider the feedback system shown in the following figure, where *ΔG(s)* is a feedback uncertainty, and apply the small gain theorem to obtain stability conditions for the perturbed closed-loop system. Assume that all the transfer function, including the one of the nominal feedback system, are asymptotically stable.



Exercise 4

Consider the feedback system



where

$$G(s)=\begin{bmatrix} \dfrac{s-1}{s+2} & 0 & \dfrac{1}{s+2} \\ \dfrac{s-0.5}{s+2} & \dfrac{s-1.5}{s-1} & \dfrac{2}{s+2} \end{bmatrix}, \quad R(s)=\begin{bmatrix} \dfrac{1}{s} & 0 \\ 0 & \dfrac{1}{s} \\ \dfrac{s+2}{s-1.5} & \dfrac{1}{s+2} \end{bmatrix}$$

Compute the poles and the zeros of G(s) and R(s) and check if it is possible to analyze the stability of the closed-loop system by considering only the sensitivity function (note that it is not necessary to develop all the computations).

Exercise 5

Briefly describe the relations between the gain and the singular values of linear, continuous time MIMO systems.