Advanced and Multivariable Control

28/6/2011

Exercise 1

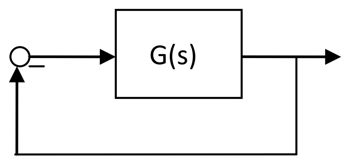
Consider the system

$$x_1(k+1) = 0.5x_1(k) + 3x_1^2(k)$$
$$x_2(k+1) = 0.5x_2(k) + x_1(k)x_2(k)$$

a. Compute the equilibrium states;
b. Study their stability with the linearized system;
c. With reference to the stable equilibrium, check its stability (with reference to the nonlinear system), with the Lyapunov function $V(x) = \alpha x_1^2 + bx_2^2$, *a*>0, *b*>0.

Exercise 2

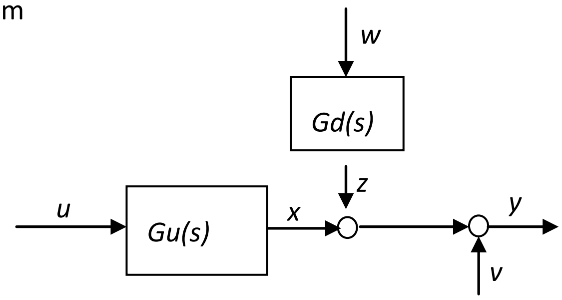
Consider the following feedback system, where $G(s) = \dfrac{\mu}{1+sT}$ is the loop transfer function, and assume that $\overline{G}(s) = \dfrac{\mu}{1+s\overline{T}}$ is the corresponding nominal model



a. Draw the scheme highlighting the nominal transfer function and the multiplicative uncertainty;
b. Compute the form of this multiplicative uncertainty;
c. By means of the Small Gain Theorem find the condition to be fulfilled in order to guarantee the stability of the perturbed closed-loop system provided that the nominal one is stable.

Exercise 3

Consider the following system



where $Gu(s) = \dfrac{1}{s+a}, a > 0, Gd(s) = \dfrac{1}{s}, w = WN(0,1), v = WN(0,1)$

a. Design a Kalman predictor from a state-space representation of the system;
b. Check if the matrix *P*, solution of the Riccati equation of the predictor, can converge to a semidefinite positive solution;
c. Can *P* converge to a definite positive solution?;

d. Verify that $P=\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ is a solution of the stationary Riccati equation of the predictor;
e. Compute the corresponding gain *L* and the eigenvalues of the predictor;
f. In the design of an LQG regulator for *Gu(s)*, consider *Gd(s)* as a weighting function in an *H2* problem and indicate the corresponding sensitivity function (sensitivity, complementary sensitivity, control sensitivity).

Stationary Riccati equation for LQ control

$$0 = A'P + PA + Q - PBR^{-1}B'P$$

Exercise 4

Given the discrete time system with transfer function $G(z)=\dfrac{b}{z-a}$, show how to design a regulator with integral action with the following methods:

a. Directly using the transfer function;
b. Using a state space representation.
c.

Exercise 5

Describe the Loop Transfer Recovery (LTR) procedure, its goal and the conditions required for its applicability.