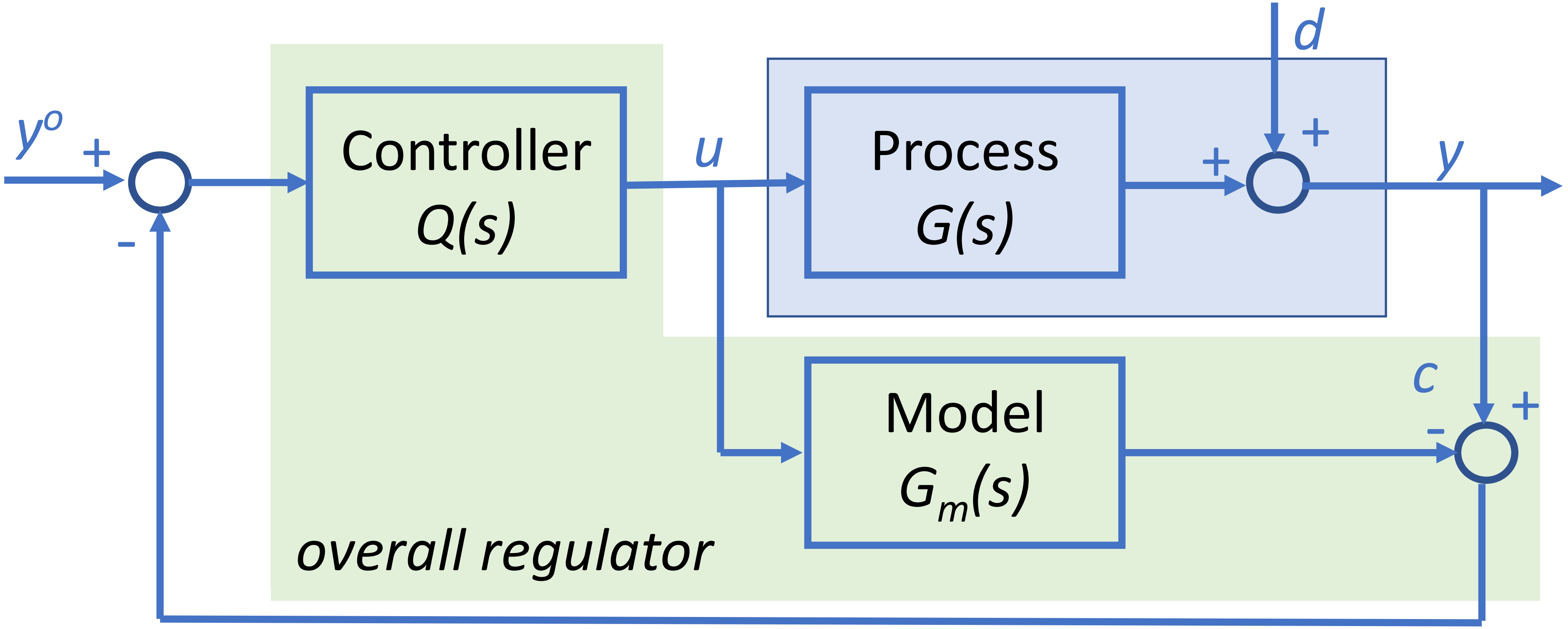
Advanced and Multivariable Control

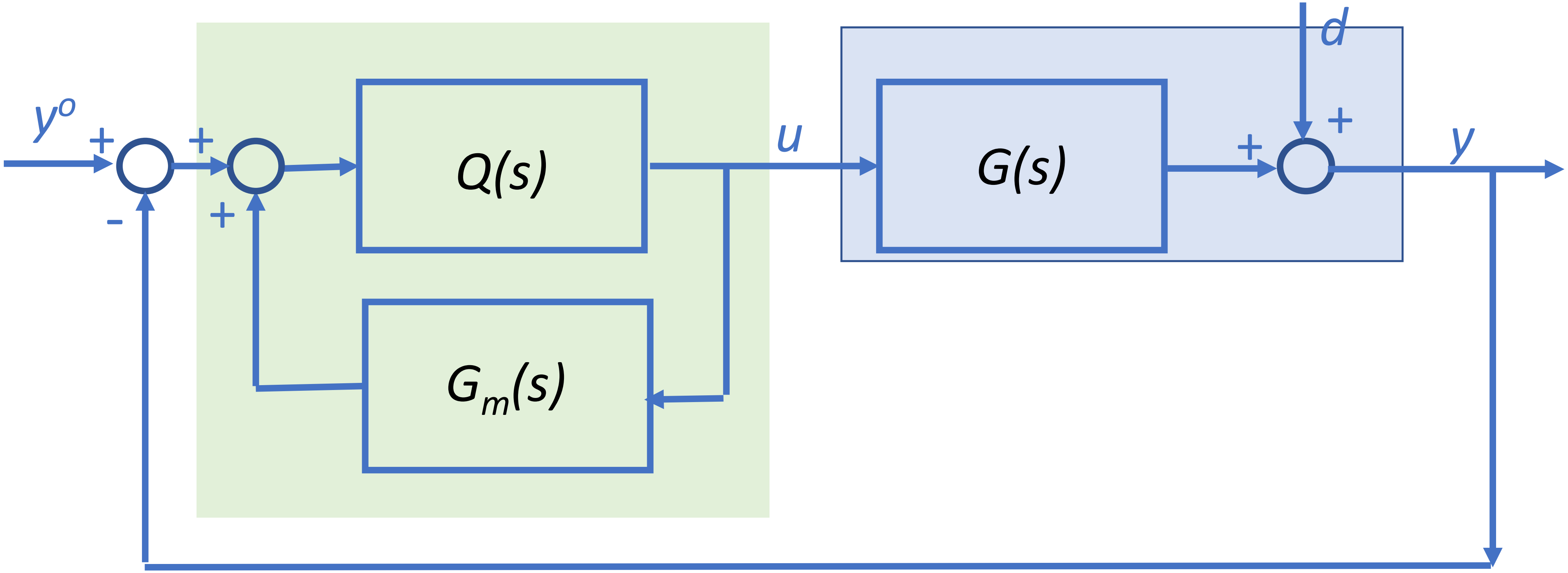
Internal Model Control

Riccardo Scattolini

Internal Model Control scheme

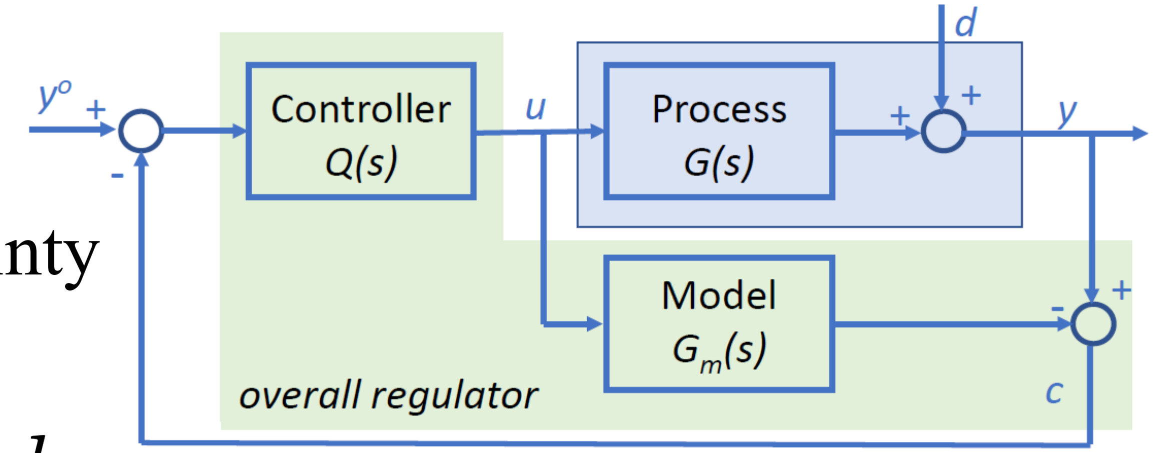


The process is *SISO, asymptotically stable* and described by the transfer function *G(s)*



Equivalent "standard " feedback regulator $R(s) = \frac{Q(s)}{1-Q(s)G_m(s)}$

Internal Model Control scheme



$C = (G\text{-}G_m)u+d$ null if there is no uncertainty

$$y = \frac{GQ}{1+Q(G-G_m)} y^o + \frac{1-QG_m}{1+Q(G-G_m)} d$$

If $\boxed{Q = G_m^{-1}}$, then $y = y^o$ *perfect tracking and disturbance rejection*

These properties hold also when $G \neq G_m$:

$$y = \frac{GQ}{\cancel{1}+Q(G-\cancel{G_m})} y^o + \frac{\cancel{1}-Q\cancel{G}_m}{1+Q(G-G_m)} d$$

Problems

The condition $Q = G_m^{-1}$ can be critical due to

- physical realizability of G_m^{-1} (more zeros than poles)
- "unstable" zeros or delays ($e^{-\tau s}$) of G_m → cancellations with unstable zeros/poles of G and/or anticipative terms must be avoided

Solution

Write $G_m = G_{mn} G_{mp}$ where G_{mp} contains all the unstable zeros and the delay of G_m (*nonminimum phase terms*, **not invertible**) and has *unit static gain*
Then set

$$\boxed{Q = G_{mn}^{-1} G_f} \qquad \text{(instead of } Q = G_m^{-1} G_f \text{)}$$

where G_f is a low-pass filter, with $G_f(0)=1$, which makes Q causal (with a number of poles greater or equal to the number of zeros). G_f is also useful to provide some robustness.

Example

$$G_m(s) = \frac{5(1-s)e^{-s}}{(1+2s)^2(1+5s)}$$

$$G_{mp}(s) = (1-s)e^{-s}$$ (unit gain)

$$G_{mn}(s) = \frac{5}{(1+2s)^2(1+s)}$$

$$G_f(s) = \frac{1}{(1+10s)(1+0.1s)^2}$$

$$Q(s) = G_{mn}^{-1}(s)G_f(s) = \frac{(1+2s)^2(1+s)}{5(1+10s)(1+0.1s)^2}$$

One obtains

$$y = \frac{G_{mp}G_f + (G - G_m)G_{mn}^{-1}G_f}{1 + (G - G_m)G_{mn}^{-1}G_f} y^o + \frac{1 - G_{mp}G_f}{1 + (G - G_m)G_{mn}^{-1}G_f} d$$

If $G=G_m$

$$y = G_{mp}G_f y^o + (1 - G_{mp}G_f)d$$

Since $G_{mp}(0) = G_f(0) = 1$ at the steady state $y = y^o$

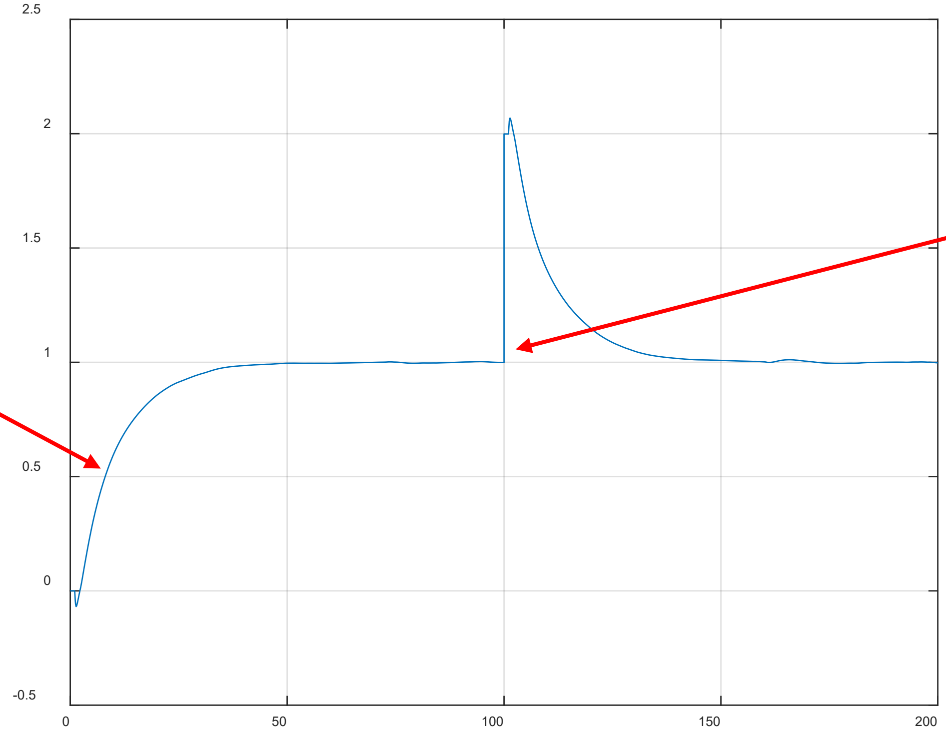
Example

$$G(s) = G_m(s) = \frac{5(1-s)e^{-s}}{(1+2s)^2(1+5s)}$$

(no modeling error)

$$G_{mn}(s) = \frac{5}{(1+2s)^2(1+s)} \quad , \quad G_{mp}(s) = (1-s)e^{-s} \quad , \quad G_f(s) = \frac{1}{(1+10s)(1+0.1s)^2}$$

Step response dominated by the slow time constant (*T=10*) of G_f



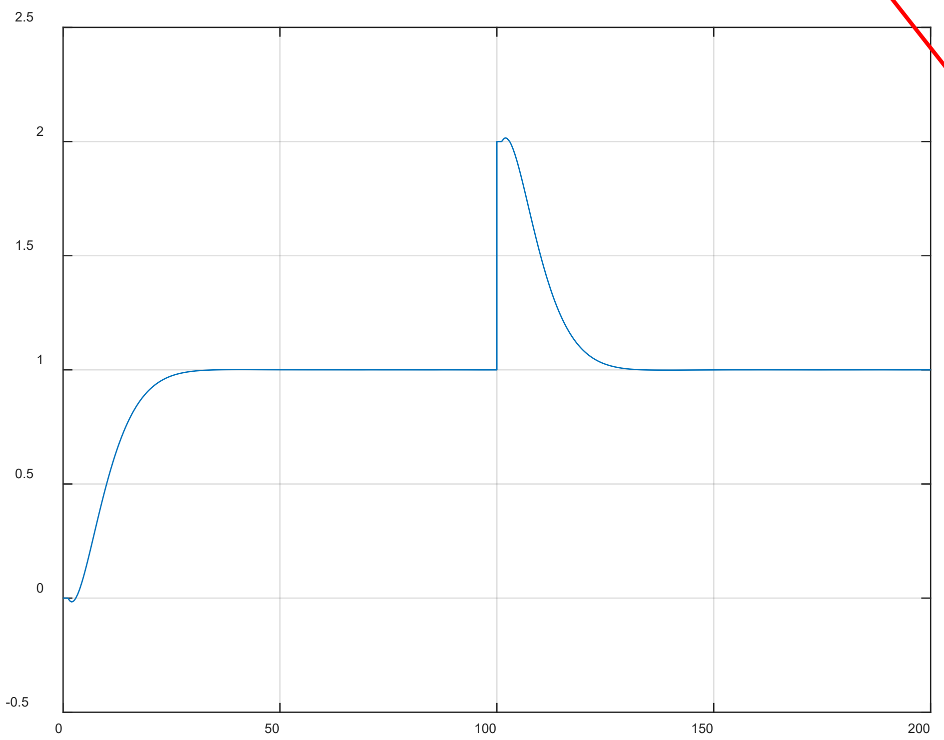
disturbance

Example

$$G(s) = \frac{5(1-s)e^{-s}}{(1+2s)^2(1+5s)} \quad , \quad G_m(s) = \frac{5(1-s)e^{-s}}{(1+2s)(1+5s)}$$

(modeling error)

$$G_{mn}(s) = \frac{5}{(1+2s)(1+s)} \quad , \quad G_{mp}(s) = (1-s)e^{-s} \quad , \quad G_f(s) = \frac{1}{(1+10s)(1+0.1s)^2}$$



"practical " robustness

Comments

- The design procedure in the case of discrete time systems is exactly the same, only with $s \rightarrow z$

- Many extensions are available, for example to unstable systems, or regulators with integral action.

- Tuning rules for PID controllers based on IMC have been proposed

- Also for MIMO systems the design procedure is conceptually the same, but the partitioning

$$G_m = G_{mn} G_{mp}$$

is more complex