Exercises session 5: Reduced order observer, LQ control, LQG and loop transfer recovery

Ex. 1: Given the scheme block in Figure 1

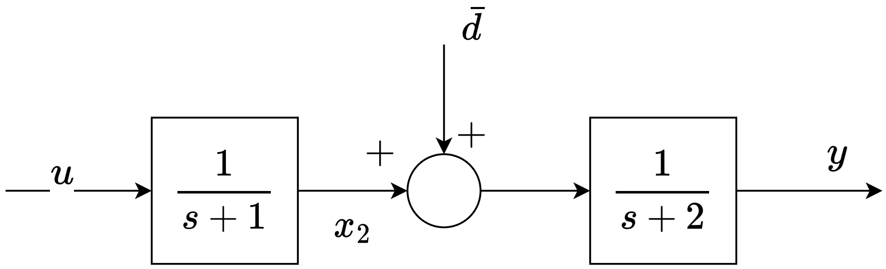

Figure 1

1. Design a reduced order observer in case $\bar{d}$ is a constant unknown disturbance.

Ex. 2: Given the continuous time system

$$\begin{cases} \dot{x}(t) = -x(t) + u(t) \\ y(t) = x(t) \end{cases} \tag{1}$$

and the Riccati Differential Equation

$$\dot{P}(t) + A^T P(t) + Q - P(t)BR^{-1}B^T P(t) + P(t)A = 0 \tag{2}$$

1. Find the LQ_∞ control law with $Q = 1$, $R = 1$.
2. Find the corresponding closed-loop poles, the closed loop T.F., the maximum gain variation and evaluate the phase margin.
3. Design a steady-state Kalman Filter with $\tilde{Q} = \rho^2$, $\tilde{R} = 1$.
4. Compute the overall LQG regulator T.F.
5. Show how to apply the loop transfer recovery procedure (LTR).

Ex. 3: Given the system

$$\dot{x}(t) = 0.5x(t) + u(t) \tag{3}$$

1. Find the LQ_∞ control law with $Q = 1$, $R = 1$.
2. Find the corresponding closed-loop poles.
3. Given $u(t) = -\rho K_{LQ} x(t)$, find the set of ρ for which the closed loop system is A.S.
4. Find the phase margin.
5. Which is the maximum time-delay that allows to maintain the asymptotic stability?
6. Enforce a closed loop pole faster than $s = -2$