Advanced and Multivariable Control

7/2012

Exercise 1

Given the system

$$x(k+1) = \frac{2x^2(k)}{1+x(k)}$$

a) Compute the equilibria;
b) Study their stability with the corresponding linearized model;
c) Study the stability of the origin with the Lyapunov theory.

Exercise 2

Given the system

$$\dot{x}_1(t) = x_1^3(t) - x_2(t)$$
$$\dot{x}_2(t) = -x_2(t) + u(t)$$

Design a control law with the backstepping method such that the origini s an asymptotically stable equilibrium for the closed-loop system. White the Lyapunov function which could be used to prove the stability of the origin.

Formula for backstepping in its “basic” version:

$$u = -\frac{dV_1(x_1)}{dx_1} g_1(x_1) - k(x_2 - \phi_1(x_1)) + \frac{d\phi_1(x_1)}{dx_1}(f_1(x_1) + g_1(x_1)x_2)$$

Exercise 3

Consider the systen

$$\dot{x}_1(t) = x_2(t) + \gamma_1 d$$
$$\dot{x}_2(t) = -x_1(t) - 2x_2(t) + u(t) + \gamma_2 d$$
$$y(t) = x_2(t)$$

where d is a constant but unknown disturbance.

Show how to estimate both the state and the disturbance with a reduced order observer, compute the conditions on γ_1 and γ_2 to be fulfilled.

Exercise 4

Given the system

$$x(k+1) = -x(k) + u(k)$$

a) Compute the LQ_∞ control law with $Q=1$ and R parametric, and show the dependence of the closed-loop eigenvalue as a function of R.
b) Discuss the robustness properties of LQ control for discrete time systems and explain why it is not possible to guarantee robustness for gain variations of arbitrary amplitude.

Riccati equation for discrete time control

$$P(k) = A'P(k+1)A + Q - A'P(k+1)B[B'P(k+1)B+R]^{-1}B'P(k+1)A$$

Exercise 5

a) Define the gain of a dynamic system;
b) Describe the small gain theorem;
c) Describe the circle criterion together with the structure of the feedback system to be considered and the characteristics of the linear and nonlinear blocks.