July 2014

Ex 1

Equilibrium

$$\begin{cases} 0 = -\bar{x}_1 \\ 0 = (\bar{x}_1 - 1)\bar{x}_2 \end{cases} \longrightarrow \begin{cases} \bar{x}_1 = 0 \\ \bar{x}_2 = 0 \end{cases}$$

Linearized model

$$\begin{cases} \delta \dot{x}_1(t) = -\delta x_1(t) \\ \delta \dot{x}_2(t) = -\delta x_2(t) \end{cases} \longrightarrow A = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$$

Lyapunov eq.

$$A'P + PA = -2P = -Q$$

$$Q = 2I \longrightarrow P = I$$

Lyapunov function

$$V(x) = x'Px = x_1^2 + x_2^2$$

$$\dot{V}(x) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 = \underbrace{-2x_1^2}_{<0} \underbrace{- 2x_2^2(1 - x_1)}_{<0 \text{ in a neighbor of the origin } (x_1 < 1)} < 0$$

However this $\dot{V}(x)$ is negative definite only locally, so that we cannot conclude anything on the global stability of the equilibrium

$\boxed{Ex\ 2}$

$A=-1,\ B=1,\ C=1,\ Q=10,\ R=1,\ \tilde{Q}=\rho^2,\ \tilde{R}=1$

Riccati equation (control)

$$-2\bar{P}+10-\bar{P}^2=0$$

$$\bar{P}=-1+\sqrt{11}\simeq 2.32 \quad \text{(positive solution)}$$

$$\bar{K}=R^{-1}B'\bar{P}=2.32$$

$$A-B\bar{K}=-3.32$$

$$L_1(s)=\bar{K}(sI-A)^{-1}B=\frac{2.32}{s+1}$$

Riccati eq. (filter)

$$A\tilde{P}+\tilde{P}A'+\tilde{Q}-\underbrace{\tilde{P}C'\tilde{R}^{-1}}_{\bar{L}}C\tilde{P}=0$$

$$-2\tilde{P}+\rho^2-\tilde{P}^2=0 \longrightarrow \tilde{P}=-1+\sqrt{1+\rho^2}>0$$

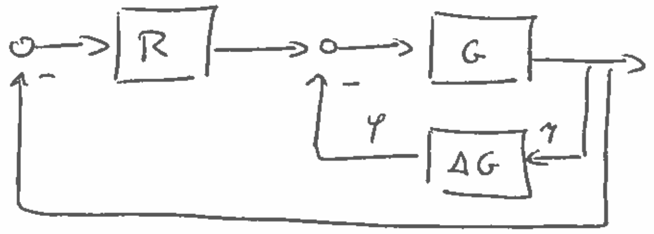
$$\bar{L}=\tilde{P}$$

$$R(s)=\bar{K}(sI-A+\bar{L}C+B\bar{K})^{-1}\bar{L}=\frac{2.32\left(-\frac{1}{\sqrt{1+\rho^2}}+1\right)}{\frac{1}{\sqrt{1+\rho^2}}s+1+\frac{2.32}{\sqrt{1+\rho^2}}}$$

$$R(s)\xrightarrow{\rho\to\infty}\frac{2.32}{\tau s+1},\quad \tau\xrightarrow{\rho\to\infty}0$$

$$L_2(s)=R(s)G(s)\xrightarrow{\rho\to\infty}\frac{2.32}{(s+1)(\tau s+1)}\simeq\frac{2.32}{s+1}=L_1(s)$$

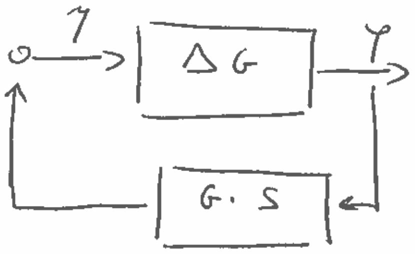
Ex 3



$$\eta = G(-\varphi - R\eta)$$

$$(1 + RG)\eta = -G\varphi$$

$$\eta = -G \cdot \underbrace{\frac{1}{1+RG}}_{S} \varphi = -GS\varphi$$



$$\| \Delta G \cdot G \cdot S \|_\infty < 1$$

Ex 4

poles of $G(s) \longrightarrow$ $s = 1$
$s = -2$
$s = -2$

zeros of $G(s) \longrightarrow$ $s = 1.5$

poles of $R(s) \longrightarrow$ $s = 0$
$s = 0$
$s = 1.5$
$s = -2$

($R(s)$ does not have any zero)

It is not possible to conclude anything with just one transfer function of the closed-loop system because there is an unstable zero of $G(s)$ coinciding with a pole of $R(s)$

Ex 5

see the notes