Advanced and Multivariable Control

6/2014

Exercise 1

Consider the system

$$\dot{x}_1(t) = -2x_1(t) + x_2(t)$$
$$\dot{x}_2(t) = 2x_1(t) - x_1^3(t) - x_2(t)$$

and check the asymptotic stability of the origin with the Lyapunov function

$$V(x) = (x_1 + x_2)^2 + 0.5x_1^4$$

Exercise 2

Given the system

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = u(t)$$

and the infinite LQ control problem with performance index

$$J = \int_0^\infty (qx_1^2(\tau) + u^2(\tau))d\tau$$

1. compute the condition to be imposed on *q* to guarantee the stability of the feedback system;
2. given the solution of the stationary Riccati equation

$$P = \begin{bmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{bmatrix}$$

compute the eigenvalues of the closed-loop system

3. compute the phase and gain margins and compare their values to the ones guaranteed by LQ control (as for the phase margin, just show how to draw some qualitative conclusions from the analysis of the Bode diagrams of the loop transfer function).

$$\dot{P}(t) = A'P(t) + P(t)A + Q - P(t)BR^{-1}B'P(t)$$

Exercise 3

Given a continuous time system with *m* inputs and *p* outputs, describe the conditions required to design a regulator guaranteeing the asymptotic zero error regulation for constant reference signals.

Show how to design this regulator, specifying its structure and the (enlarged) system to be considered to guarantee the closed-loop stability.

Exercise 4

Given the discrete time system with transfer function

$$G(z) = \frac{z-1.2}{(z-2)(z+1)}$$

1. design a pole placement regulator with integral action and such that the closed-loop poles are in $z=0.5$ (it is enough to describe the regulator structure and the linear system to be solved to compute the regulator's parameters).
2. Show how to implement the regulator so as to avoid that the zeros of the regulator are not zeros of the closed-loop transfer function between the reference signal and the output.

Exercise 5

Describe at least one MPC formulation guaranteeing that the origin is an asymptotically stable equilibrium of the closed-loop system.