ADVANCED AND MULTIVARIABLE CONTROL

June 18, 2019

Surname and Name …….

University Id……………………………………………………………..Signature………………………………………………………………….

Exercise 1

Consider the system

$$\dot{x}_1(t) = -x_2(t) - x_1^3(t)$$
$$\dot{x}_2(t) = -x_2(t) + 4x_1(t)$$

a. Show that the origin is an asymptotically stable equilibrium using the quadratic Lyapunov function $V(x_1, x_2) = \frac{1}{2}(ax_1^2 + x_2^2)$ where $a>0$ is a suitable design parameter.

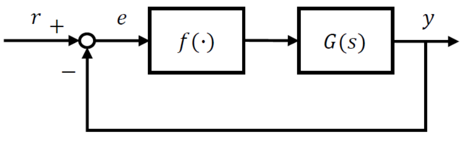
b. Discuss if the origin is a globally asymptotically stable equilibrium;

c. Describe the form of the trajectories (corresponding to a node, a focus, a saddle point …) in the neighbor of the origin by analyzing the linearized system.

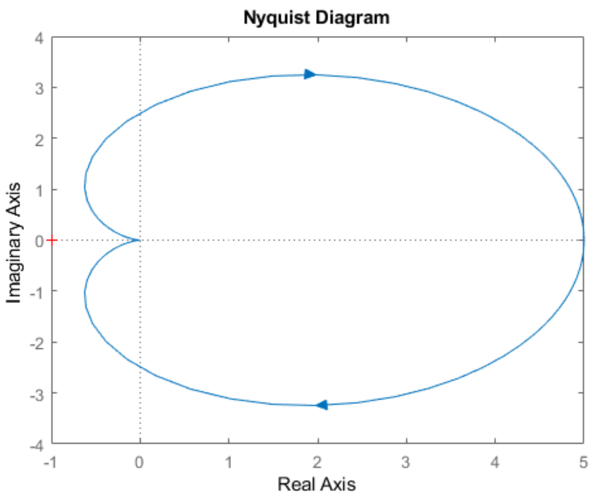
Exercise 2

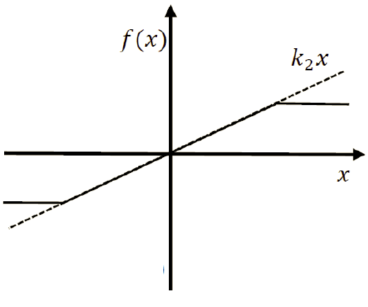
A. Given the system *y(t)=S(u(t))*, define its gain and the condition for Input/Output stability.

B. Consider the feedback system



Where G(s) is the transfer function of an asymptotically stable system with the Nyquist diagram reported below together with the form of the saturation *f(.)*.

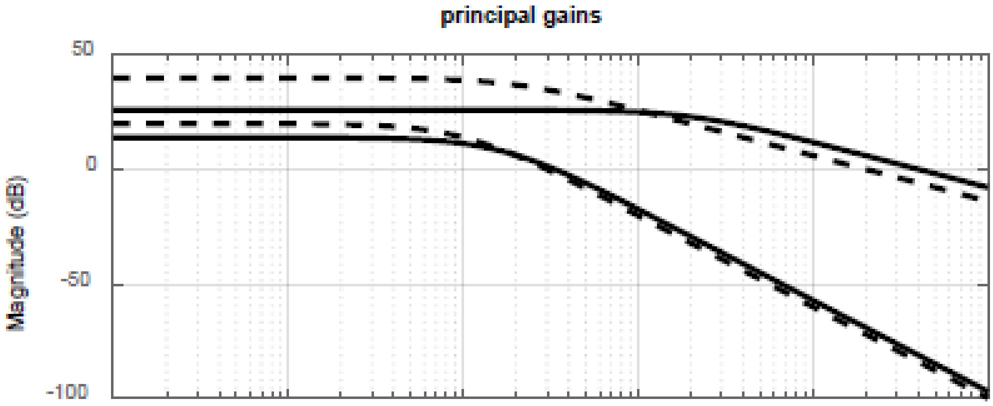




determine (qualitatively) the maximum value of *k2* guaranteeing the Input/Output stability of the system.

Exercise 3

Consider two MIMO systems with the following principal gains (system S1: continuous line, system S2: dashed line)



1. Which one guarantees more attenuation of low-frequency disturbances acting on the plant output?

2. Which one guarantees more attenuation of high-frequency disturbances acting on the measured output?

Exercise 4

Consider the system

$$\dot{x}(t) = -x(t) + 2u(t) + v_x(t)$$
$$y(t) = x(t) + v_y(t)$$

Where $v_x \sim WN(0, \tilde{q}), v_y \sim WN(0,1)$

1. Assuming that the noises are null, compute the infinite horizon LQ control law with $Q=15/4,\ R=1$

   Riccati equation

   $-\dot{P}(t) = P(t)A + A'P(t) + Q - P(t)BR^{-1}B'P(t)$

2. Compute the gain margin of the resulting feedback control law and compare it with the one guaranteed by LQ control

3. Assuming that the noises are not null, compute the Kalman Filter (KF) gain and write the expression of the KF

4. Compute the regulator transfer function obtained by combining the LQ control law and the KF

Exercise 5

Describe one (at your choice) of the methods which can be used to include an integral action in the design of a Model Predictive Control algorithm.

Solution Exercise 1

The origin is the unique equilibrium point. In fact

$$0 = -\bar{x}_2 - \bar{x}_1^3$$
$$0 = -\bar{x}_2 + 4\bar{x}_1$$

has the unique real solution $\bar{x}_1 = \bar{x}_2 = 0$

Note that V(x) is globally positive definite at the origin ad radially unbounded. Moreover,

$$\dot{V}(x_1, x_2) = a x_1 \dot{x}_1 + x_2 \dot{x}_2 = -x_2^2 + 4x_1x_2 - a x_1 x_2 - a x_1^4$$

Choose $a=4$, so that $\dot{V}(x_1, x_2) = -x_2^2 - 4x_1^4 < 0$globally, and the global asymptotic stability of the origini s proven.

The linearized system at the origin is

$$\delta\dot{x}_1(t) = -\delta x_2$$
$$\delta\dot{x}_2(t) = -\delta x_2 + 4\delta x_1$$

With eigenvalues $s = \frac{-1 \mp \sqrt{-15}}{2}$, complex conjugate. Therefore, the trajectories in a neighbor of the system are those of a stable focus.

Solution Exercise 2

Question A: see the notes

Question B:

$$\frac{-1}{k_2} \simeq -0.7$$

Solution Exercise 3

The two questions are referred to the case where S1 and S2 are the loop transfer functions of two feedback systems with unit feedback. According to paragraph 5.2.5 of the notes (pag. 86-88), the solutions are:

Question 1: System S2 since the smaller singular value is larger than the one of S1

Question 2. System S2 since the larger singular value is smaller than the one of S1

Solution Exercise 4

1. $A = -1, B = 2,$ Riccati eq. $\rightarrow \bar{P} = 0.75 \rightarrow K = 1.5$
   Control law $u(t)=-x(t)$, closed-loop eigenvalue $A\text{-}BK=\text{-}4$

2. Process transfer function $G(s) = \frac{2}{s+1}$, regulator transfer function $R(s)=1.5$, gain uncertainty term ρ, characteristic equation $s + 1 + 3\rho = 0 \rightarrow s = -1 - 3\rho$. Stability condition $\rho > -1/3$. Gain margin (-1/3,∞), guaranteed gain margin (1/2,∞).

3. $C=1, \tilde{R}=1,$ Riccati eq. solution: $\tilde{P}=L=-1+\sqrt{1+\tilde{Q}}$. Kalman Filter:

$$\frac{d\hat{x}(t)}{dt}=-\sqrt{1+\tilde{Q}}\hat{x}(t)+2u(t)+(-1+\sqrt{1+\tilde{Q}})y(t)$$

4. Regulator transfer function $R(s)=K(sI-A+BK+LC)^{-1}L=1.5\frac{-1+\sqrt{1+\tilde{Q}}}{s+3+\sqrt{1+\tilde{Q}}}$

Solution Exercise 5

See the notes, paragraph 12.2.3 (pag. 190-193), or alternatively paragraph. 12.3.2 (pag. 199-200).