Part C

Thermal systems

Thermal systems – generalities

Thermal part of the course ...

↳ motivations
+ relevant aspects ⇒

- System characteristics
- Main problems

Foreword

(abstract theoretical aspects!)

↱ there is a big difference between E/T systems

- As we saw (in E years) or shall see later on (in T years), electric (AC) systems
  - are characterised by network-wide interaction, as frequency is an inherently network-wide quantity; → all is coupled! ↱ electric phenomenon (immediates)
  - exhibit very fast coupling (wrt control bands) among the network nodes;
  - allow for a modest role of storage, at least at the network level; → storage @ home / build... level, hard to achieve good storage
  - are governed with highly standardised control schemes, that involve a few strong central authorities. ↳ primary control + signals...

(centralized controller..)

- Thermal systems are radically different: ↱ slow coupling respect control var.
  - interaction is much looser, and mostly localised to well identifiable parts of the system; → loose coupling, just if sharing same heat NET
  - tendentiously slow coupling (again, wrt control bands) is observed;
  - the role of storage can be paramount in several system components;
  - control schemes are very heterogeneous, and often contain numerous local authorities with limited power (*lato sensu*). ↳ heat net control scheme very different between different syst. (no uniform of central autorities)

different control problems

electric heater has a storage... to heat up you need high KW of power ⇒ easy to store heat than electricity! ⇒ today we use on elect net different storage type

(store electricity is harder)

Foreword

Focus on Thermal..

- As a first consequence, wrt the electric case,
  - there are more concurrent time scales to consider, [from sec to hours time scales]
  - which helps establish some hierarchy in the systems to address → as a cascade control
  - but same time results in a higher structural complexity. → many compensation/decoupler ecc.. lots schemes
- As a second consequence, and again wrt electric systems,
  - we are encountering a wider variety of problems → NOT only control problems
  - and thus we need a larger zoo of models.

↓

- This motivates the co-existence of the two system types (E and T) in this course... (coordination of E/T systems)
- ...though we can devote just a limited amount of time to talk about their coordination.

Thermal systems – component models

⇓

Introduction and hypotheses

mooving heat around
≡ mooving fluid around
(so we control fluids)

Fluid transport and flow control components

Heat exchangers

Thermal machines

Containment elements → building walls,
others container like tubes

Main elements to be modelled

- Fluid (liquid) transport, motion and flow control (pipes, pumps, valves);
- heat exchangers (liquid/liquid and liquid/air);
- thermal machines (boilers, chillers, heat pumps);
- containment elements and associated thermal exchanges (walls, openings, conductive/convective/radiative exchanges).
- Note: for simplicity, unidirectional fluid flow (quite realistic, anyway) will be assumed.

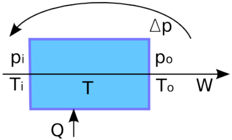
reasonable,
pump which allow
unidirectional flow
of Fluid

Involved substances and materials

- We shall treat thermo-vector fluids (e.g., water flowing in heating elements) as incompressible, and with constant specific heat;
- the same will be done for the air contained in buildings (pressure is practically the atmospheric one): trick!
- thus, we shall not treat centralised air treatment systems like AHUs (Air Handling Units) and air-based heat distribution — if not very marginally.
- Solid materials will come into play for containments, and here too simple descriptions (constant and uniform properties) will be used:
- always recall our system-level attitude.

Pipes

model through momentum equation
we relate Δp across pipe to friction term (lumping coefficients)



on datasheet, data frequently specified by bars/km loss @ nominal flow rate (easy modelling of K_T)

- Hydraulic equation (review, Δz is the height difference (inlet minus outlet), ρ the fluid density):

Friction losses

gravity terms

$$\Delta p = \frac{K_T}{\rho} w^2 - \rho g \Delta z$$

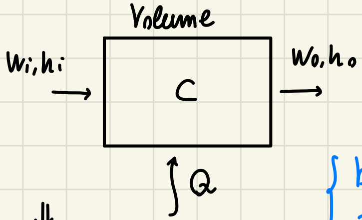
$\Delta z \simeq h_i - h_o$ heigh in-out

- Energy balance (T assumed uniform and equal to T_o, V is the pipe volume, c the fluid specific heat):

it depends if we consider 0/1 DIM element

$$\rho V c \dot{T}_o = c w (T_i - T_o) + Q$$

considering a volume with heat capacity c



Hyp: $e = cT$

$h = e + \cancel{\frac{P}{\rho}} \simeq e$ neglect!

{ because for fluids @ normal P, e >> P/ρ dominates }

$\Downarrow$

$$\frac{dE}{dt} = W_i c h_i - W_o c h_o + Q \underset{(W_i = W_o)}{=} Wc\,(T_i - T_o) + Q$$

$\Downarrow$

$c\,\dfrac{dT}{dt}$: but which T ? (in/out/average ?!)

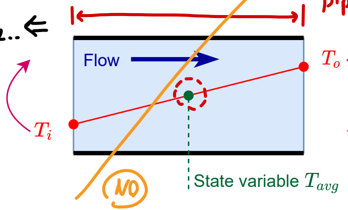
↳ normally if no flow reversal it is more sensate to take outlet temp $\Rightarrow$

Pipes

Why T_o as state variable — connected to our unidirectional flow hypothesis

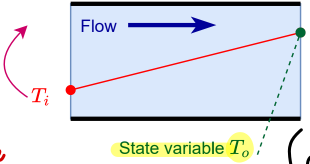
taking Tavg seems reasonable.. ⇐ because evaluating themal exchange I consider surface temp to compute exchanges → Tavg

pipe section



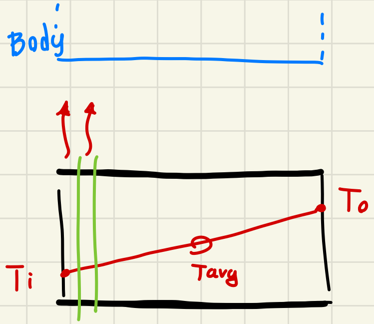
If taking Tavg as state

It is better to use To as state var



so To does not change in short time ↑ (knowing outlet side)

- Which T shall we take to represent the lump energy content?
- Suppose we take the average T_{avg} (left) and then apply a step-like variation to the inlet one T_i:
  - *natura non facit saltus — ergo nec faciunt status variabiles* ☺ (nature does not make jumps — hence neither state variables do)
  - ⇒ we would see a transient negative variation of the outlet temperature T_o, i.e., a non-physical nonminimum-phase behaviour.
- Apparently this does not happen (right) if we take T_o as state variable (but we have to know/assume the flow direction).



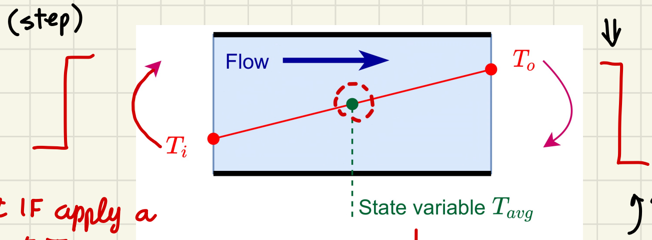
take slice
and compute heat
exchange

IF using Tavg, I should integrate

I integrate T(x) and take
Tavg → reasonable state var!

(simulation behaviour!) NOT GOOD

(step)



{NOT FEASIBLE!}

⇑ against real physics

but IF apply a
step to Ti:
(enter heated hot fluid)

↓ (cannot change immediately!)

so to have
same Tavg,
a step in Ti
→ leads To↓

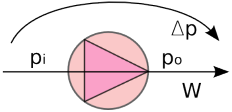
avg does not change
in short time frame!

inverse response on
To, NOT FEASIBLE!

Pumps

↓ 2 kinds of pumps

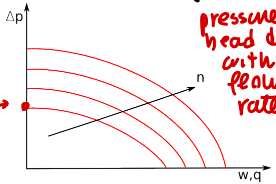
(NON LINEAR behaviour)

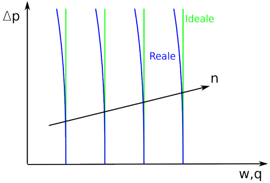


characteristic

pressure head decrease with flow rate

@Q FLOW RATE



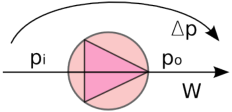


Centrifugal ≃ parabolic curve

Volumetric → [more used on high pressure hydraulic]

- The command n prescribes the pump speed, typically in rpm.
- Mass and volume flowrate (here proportional by ρ) are denoted by w and q, respectively.

Pumps



- Centrifugal pump hydraulics (given the fluid):

{depend on rotational speed of the pump}

$$\boxed{\Delta p = H_0(n) - H_1(n)w^2}$$

where, given a nominal rpm n_0 and correspondingly $H_0 = \overline{H}_0$ e $H_1 = \overline{H}_1$,

$$H_0(n) = \overline{H}_0\, n/n_0, \quad H_1(n) = \overline{H}_1\, n/n_0.$$

- Volumetric pump hydraulics (given the fluid):

n: [rev/min]

$$\boxed{w = Kn}$$

where K is a characteristic parameter.

- We can neglect mechanical heating, thus in both cases $T_o = T_i$.

on Modelica > process components > liquid > ...

$$\Delta p = \Delta p_0 * max(0, ..) - K_p \cdot W^2$$ ... (pump)

on > coursework > Thermsys_component_models

{ tons of models of thermal component

centrifugal vs volumetric pump flow rate }

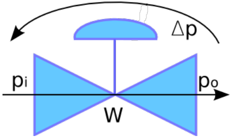
combination of pump/valves
depends on what I want to control

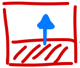
Valves

$$\phi(x): \begin{cases} \phi(0)=0 \\ \phi(1)=1 \end{cases}$$

flow surface increase while increase opening command

IF EXAMPLE surface valve square





$x\uparrow \Rightarrow \phi\uparrow$ linear

non lin.. different shape

no cavitation / no strange phenomenon

- Hydraulic equation (we assume proper sizing, the curious can refer e.g. to ANSI/ISA-75.01.01 or IEC 60534-2-1):

opening function

flow rate

$$w = Cv_{max}\boxed{\Phi(x)}\sqrt{\rho\Delta p}$$

valve coeff

pressure drop across

- Isenthalpic transformation:

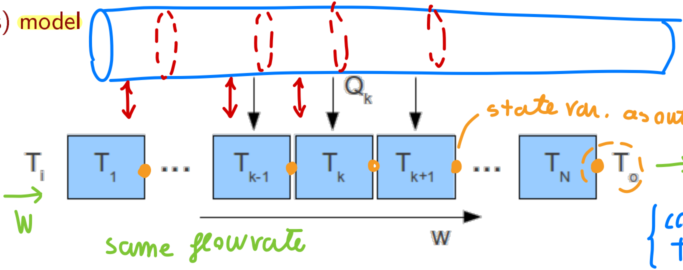
$$h_o = h_i.$$

Fluid streams

↳ we have to do with incompress. fluid

Lumped (Finite Volumes) model

↓ you model in a finite volume approach ⇓ divide fluid stream in lumps



like a sliced tube..

{ coupling of hydraulic/thermal eq if density change with T }

- For incompressible fluids, thermal equations are decoupled from hydraulics (constant density). (coupled when density depends on temperature)
- Having decided the flow direction, the equation for each element of the FV description is thus

ρV := mass ∀ element

heat capacity [J/K] — heat comes laterally — assumptions

$$\rho c V_k \dot{T}_k = \underbrace{wcT_{k-1}}_{\text{IN}} - \underbrace{wcT_k}_{\text{OUT}} + Q_k, \qquad \boxed{T_{-1} = T_i,\ T_o = T_N},$$

where the V_k are the lump volumes, often (for us, always) equal.

om Modelica → Thermal > Liquid > $\underbrace{\text{Tube stream}}$

$\Downarrow$

$C_{lump} * \frac{d}{dt} T_1 = \dots$

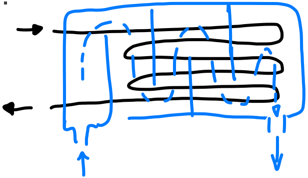
Main geometries

heat exchangers components are very important on thermal systems
⇓
used to transfer heat from one fluid to another without mixing the two

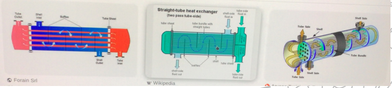
- Plate, shell-tube...
- All composed of two fluid streams separated by a wall typically made of well-conducting metal.

that can have complex geom

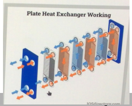
many geometries!
(EX)
- shell_tube



shell tube heat exchanger



- plate heat exchanger



↳ (what you typically see on heat network sub station)

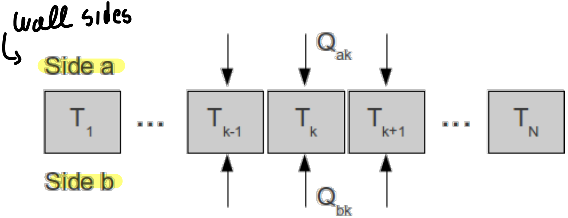
Wall

→ characterized by heat capacity (which can be significant respect fluid heat capacity)

(we account for heat capacity of metal, because for energy menage, is a storage usable)

⇓

[storage in metal that can be used]



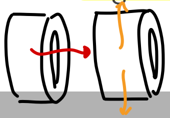
- Dividing the wall in the same way as the stream(s),

heat capacity

$$\rho_w c_w V_{wk} \dot{T}_k = \underbrace{Q_{ak} + Q_{bk}}_{\text{what comes from the side}}$$

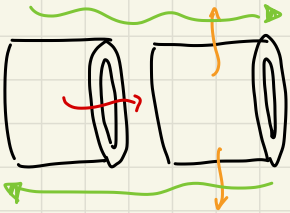
where the "w" subscript stands for "wall".

- Axial conduction in the wall is neglected, which is realistic enough for our purposes. ⇒



irrelevant conduction!

each tube piece "K"



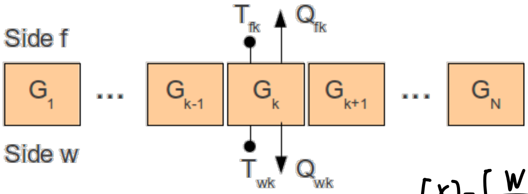
this fluid is more efficent to transfer heat than the metal!

↓

(experimental evidence)

Convective exchanges

↳ no through heat capacity...



$[\gamma] = \left[\frac{W}{m^2 K}\right]$

- Finally comes the (convective) exchange element, that adopting the same discretisation as before yields

opposite rate [W/K]

$$Q_{fk} = -Q_{wk} = \gamma S_k (T_{fk} - T_{wk}) \quad [W]$$

where the f and w subscript stand for "fluid" and "wall".

- The S_k are the lump surfaces while γ is the thermal exchange coefficient, either constant or dependent on the flow conditions.

↳ depends on conditions...

Convective exchanges

Correlations for γ

- In the literature many correlations to conpute γ were proposed.
- These typically distinguish
  - natural convection, caused by buoyancy, vs. forced convection, induced by movers like pumps and fans;
  - laminar flow, with particles moving "by parallel lanes" and prabolic velocity profile on the duct cross surface, vs. turbulent flow, with particles mixing randomly and a quasi-flat velocity profile;
  - intuitively, forced turbulent convection is most efficient.
- In the forced case the kind of flow is discriminated by the Reynolds number

$$Re = \frac{\rho u D}{\mu}$$ → laminant / turbolent ?

where ρ is density, u velocity, D (hydraulic) diameter, and μ dynamic viscosity.

- Low Re means laminar flow, high Re turbulent flow (there is debate on thresholds, will show some data in exercises).

Convective exchanges

Correlations for laminar and turbulent flow

- A notorious example is the Dittus-Bölter correlation for turbulent flow

$$Nu = 0.023\, Re^{4/5}\, Pr^{n}$$

where Nu and Pr are respectively the Nusselt and Prandtl numbers

$$Nu = \frac{\gamma L}{\lambda}, \quad Pr = \frac{c_p \mu}{\lambda}$$

with L characteristic dimension (in tubes, stream diameter), λ thermal conductivity, and c_p specific heat; n is 0.3 for the fluid being heated, 0.4 if the fluid is cooled.

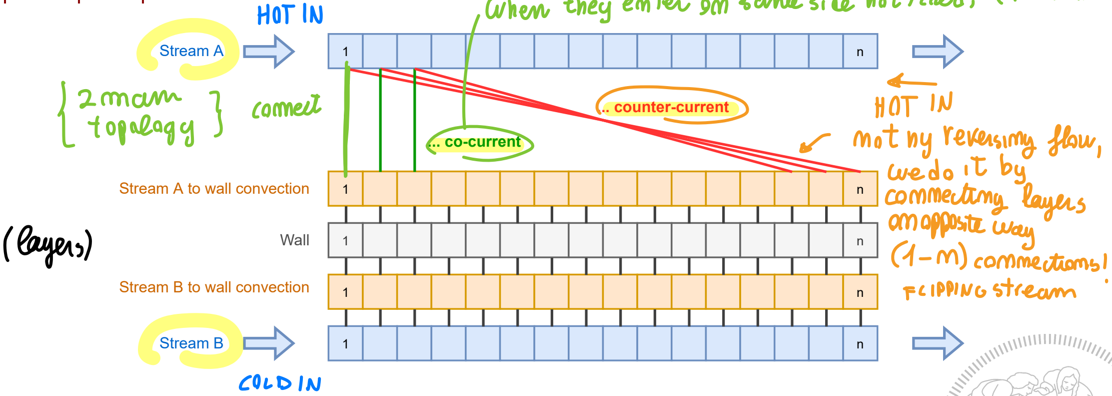
- Further details would stray from our scope; the curious can check out e.g. `https://en.wikipedia.org/wiki/Nusselt_number` for the variety of cases.

Complete models of a heat exchanger

typical liquid-liquid case

(so they exchange parallel)

When they enter on same side hot/cold, (1-1) connect



- The connections realise the heat exchanger configuration (e.g., as shown, co-current or counter-current).
- Complex geometries may require more articulated interconnections (not our subject).

Complete models

One-volume liquid-liquid case

on district level heat net... (more interactions) lot exchangers

- In large models (e.g., a complete heat network) one may sometimes not be interested in the detailed dynamics of every (substation) heat exchanger.
- On the other hand, in addition, one may want to parametrise exchanger models with minimal – most typically, datasheet – information, such as
  - nominal hot and cold side flowrates,
  - nominal hot and cold side inlet temperatures,
  - nominal transferred power and efficiency.

↳ frequently you don't have detailed physical knowledge, so INFO used for sizing (design) are usefull!

- In such cases, always keeping an eye on validity limits as already noted, very simple models can be devised.
- We show a quite extreme case, with one volume (per side) only.

Complete models

One-volume liquid-liquid case (same fluid on both sides)

hot/cold flow rate   power transfer   ~> given this I compute model parameter

- Maxima:

efficiency

equations:

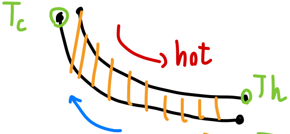
```
/* Given c,wh,wc,Thi,Tci,P,eta     (sp. heat, flowrates, inlet Ts, nom. P, efficiency)  */
e1: c*wh*(Thi-Tho) = P;            /* heat taken from (h)ot side                         */
e2: c*wc*(Tco-Tci) = P*eta;        /* heat released to (c)old side (1-eta is p.u. loss) */
e3: P=G*(Thi-Tci);                 /* G is equivalent conductance -- question: why "i"? */
solve([e1,e2,e3],[G,Tho,Tco]);
```

inlet temperature (never reverse)

- whence, introducing hot and cold side volumes V_h, V_c to compute heat capacities,

not cell related to cold size, you have lost!

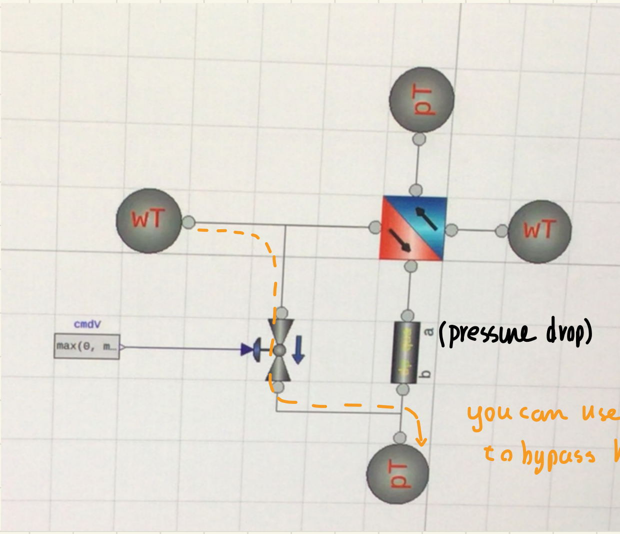
(counter current)



cold (sliding)   Tc > Th can be higher! → I use inlet T to guarantee feasibility

$$
\begin{aligned}
P_h &= G(T_{hi}-T_{ci}) \\
P_c &= \eta P_h \\
\rho c V_h \dot{T}_{ho} &= c w_h (T_{hi}-T_{ho}) - P_h \\
\rho c V_c \dot{T}_{co} &= c w_c (T_{ci}-T_{co}) + P_c
\end{aligned}
$$

You may have more physical situation..



(pressure drop)

you can use value to bypass heat exchanger

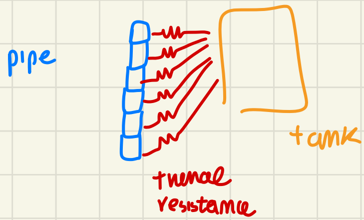
Complete models

Another extreme case (just some words)

residence time := time requested to renew completely the fluid contained in the volume, respect flow rate
↓ if small ≃ uniform properties

- In some cases it can be convenient to have only one volume on one side, while the other is still represented by a sequence of volumes along its stream.
- A couple of examples: ~ model volume along serpentine!
  - when one side is a.g. a serpentine and the other a storage tank that can be taken as fully mixed; (typical in thermal solar system)
  - when one side has a very small fluid residence time (contained mass over traversing flowrate) like e.g. the air volume in a fan coil.
- In such cases the model still has the structure shown previously for the two-stream exchanger, however with the convective conductances on the single-volume stream side all connected to that volume.

all exchang with a single volume



A broad classification and two hypotheses

I'm coursework on modelica (Thermal syst comp. model)

{ modelling of components... }

- For our purposes, we distinguish two types of machines:
  1. those that inject heat into a fluid by combustion or from analogous sources, such as solar radiation, that can possibly be modulated (e.g., by operating a fuel valve or focusing/defocusing a mirror); these include boilers, thermal solar captors, and similar objects;
  2. those that employ work to transfer heat from a cold to a hot source; these include all kinds of heat pumps.

  using work to transfer (NOT violation of II principles)
- Hypotheses:
  1. at a system level, thermal machines can be described as static relationships coupled to a simple (for us, 1st-order) dynamics to represent the internal mass/energy storage, or in other words connected to the fluid residence time (contained mass over traversing flowrate, recall);
  2. at the same level, the fluid(s) perceive the machine operation as impressed heat rates: possible relationships of those heat rates with temperatures can be represented as static characteristics.

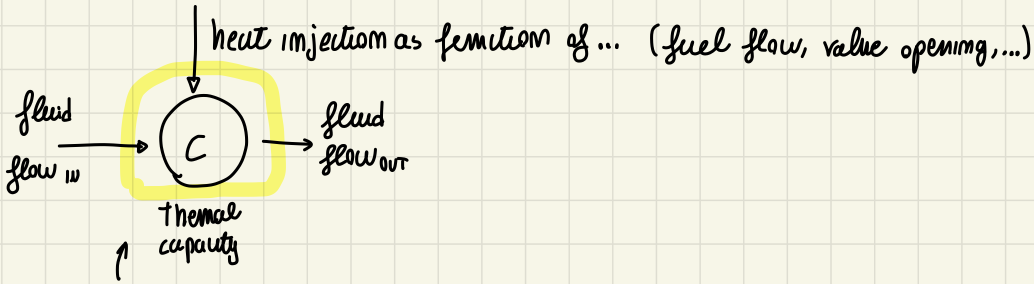
  (heat as function of ...) , "C" concentrated → I ORD dynamic!

We need (for control purposes) → simple system level models

(Based on a share scheme)

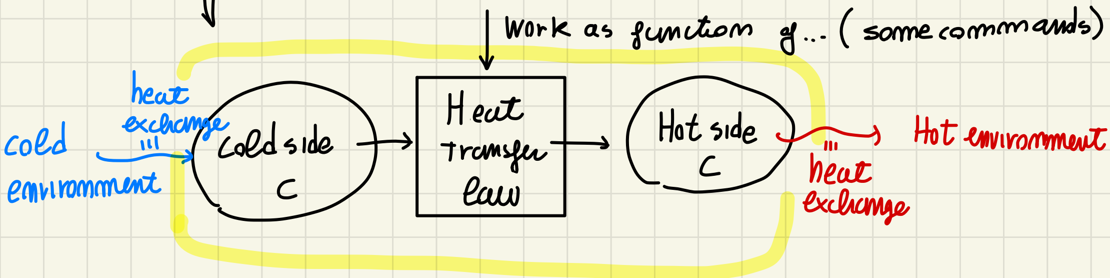
"simple model for a thermal machine"

(1)



machine model limits

(2)



Thermal machines

Type 1 machines – example: heater, "direct" model

(1) "direct model"

(flow on boiler) [Kg/s] * [J/Kg] = [W]

- Denoting by w the fluid flowrate, by w_f the fuel one, by HH its calorific power, by V the volume of contained fluid and by η_c a "combustion" efficiency, one can simply write with self-explanatory notation

(simple heat balance)

what comes from

dE/dt IN-OUT combustion

$$\rho c V \dot{T}_o = wc(T_i - T_o) + w_f HH \eta_c(w, T_o)$$

efficency, which depends on operation condition

where $\eta_c(w, T_o)$, or just $\eta_c(w)$ depending on the detail level, provides the machine efficiency curve. → less fuel = different η → machine efficency curve

- Suggestion: try to reformulate for a thermal solar captor where the captured radiative flux can be partialised.

Thermal machines

Type 1 machines – example: heater, "inverse control-based" model

- Sometimes models like the above are too simplistic, yet physically detailed ones are too demanding and/or the information required for them is not available.
- In such cases, if the object to model has controls aboard, one can exploit them...

  ... in the way that we show her for a heater, but is clearly more general.

- Idea: trust the control designer and assume that the controlled variable y tracks the set point w through a reasonably low-order dynamics; the ideal case is

track set point with I ORD dym

$$\boxed{\frac{Y(s)}{W(s)} = \frac{1}{1+s/\omega_c}}$$

Ideally we know ω_c, set-point/controlled var relationship

where to know ω_c it suffices to know the closed-loop settling time (from technical specs or even just logged signals). ↳ {apply a step and watch transient}

- Now let us apply the idea to the water heater.

Thermal machines

Type 1 machines – example: heater, "inverse control-based" model

- Heater on ⇒ outlet temperature T_o follows set point T_{set} with closed-loop time constant τ_{cl}, thus

follow set point

$$T_o + \tau_{cl}\dot{T}_o = T_{set}.$$

$\xrightarrow{\mathcal{L}} (1+s\tau_{CL})T_O = T_{SET} \rightarrow \frac{T_O}{T_{set}} = \frac{1}{1+s\tau_{CL}}$

- Heater off ⇒ outlet temperature T_o approaches inlet one T_i with free-cooling time constant τ_{fc}, thus

(I can measure τ_{fc}) sperimentally

$$T_o + \tau_{fc}\dot{T}_o = T_i.$$

using as input T_i instead T_{set}

- Summing up, since both T_i and the flowrate w will be dictated by the connected thermo-hydraulic network, the model just requires τ_{cl} and τ_{fc}, and reads

SWITCHING system

↓

(state vector) doesn't change BUT right hand side of eq changes!

$$\dot{T}_o = \begin{cases} -\frac{1}{\tau_{cl}}T_o + \frac{1}{\tau_{cl}}T_{set} & \text{heater on} \\ -\frac{1}{\tau_{fc}}T_o + \frac{1}{\tau_{fc}}T_i & \text{heater off} \end{cases}$$

$$P_h = \begin{cases} wc(T_o - T_i) & \text{heater on} \\ 0 & \text{heater off} \end{cases}$$

writing it as:

$$\dot{T}_o = A_\delta T_o + B_\delta \begin{bmatrix} T_{set} \\ T_i \end{bmatrix} \qquad \delta = on/off$$

$$\left\{ \begin{matrix} \text{linear} \\ \text{switching} \\ \text{dyn syst.} \end{matrix} \right\}$$

simple model!

$$A_{on} = -\frac{1}{\tau_{cl}} \qquad A_{off} = -\frac{1}{\tau_{fc}}$$

$$B_{on} = \begin{bmatrix} 1/\tau_{cl} \\ 0 \end{bmatrix} \qquad B_{off} = \begin{bmatrix} 0 \\ 1/\tau_{fc} \end{bmatrix}$$

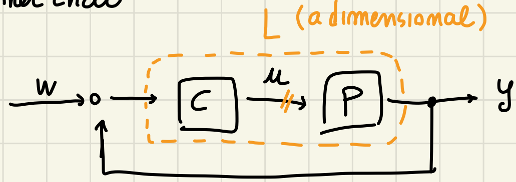
Thermal machines

Type 1 machines – example: heater, “inverse control-based” model

limit concerning control signal representation and consumed power!

- Evident limits:
  - knowing τ_{cl} provides information about the loop transfer function, not on C and P, so that we may represent well the controlled temperature but not the control signal, hence power P_h;
  - in addition, the idea just introduced and exploited lives entirely in a linear context, so that we have no means to account for saturation limits on P_h to the further detriment of its representation.
- We could however do better with some knowledge of the controlled process: let us therefore assume to know the maximum heater power $P_{h,max}$, the contained fluid volume V, and a nominal/design fluid flowrate w — not too tall an order at all with heater datasheet and installation conditions at hand.

remember that



however, to
use this model to compute u (consumption) → I can have problems
⇓

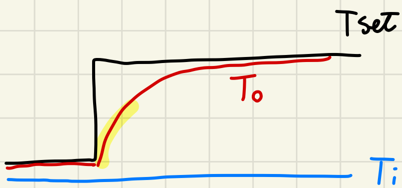
$\boxed{\text{@s.state}}$ IF I know $\left.\begin{matrix} W \\ T_o, \; T_i \end{matrix}\right\}$ → can compute consumption

$$P = \frac{cW(T_o - T_i)}{\eta}$$

BUT
IF completely linear syst.

During transient it might be inaccurate...

Example

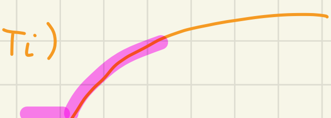


T_i (constant) ≃ big storage upstream, (long time)

good means to simplify thermal models, BUT we don't properly evaluate consumptions! so behaviour of control not properly represented! (ISSUE)

⇓

means that
$(T_o - T_i)$



with this I don't notice control logic ... ⇐

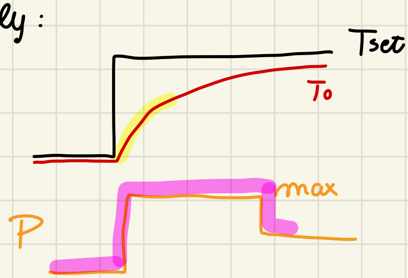
≃ (P) with cw constant
as estimated

too different

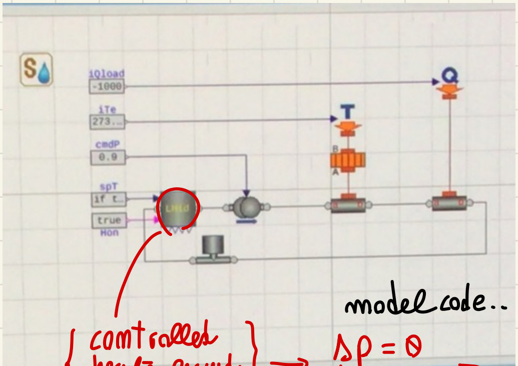
But in reality, most likely:

{ using actuator, goto MAX then exit saturation }

Best controller try to go to set point then exit SAT to avoid overshoot



Idea of "inverse model" → ideal case >> controlled_heater_case003_ideal



{ controlled heater equip ideal } → $\Delta p = 0$

$h_{ao} = c_p T_{fo} = h_{bo}$

model code.. ⟹

```
// no pressure drop
  dp = 0;
// enthalpy boundary conditions
  hao = cp*Tfo;
  hbo = cp*Tfo;
// definition of the input and output temperatures
  cp*Tfi = hai;
  if ON then
    Tfo + Tcl * der(Tfo) = To;
  else
    Tfo + Thc * der(Tfo) = Tfi;
  end if;
// Power consumption
  Pc = noEvent(max(cp*w*(Tfo-Tfi), 0.0));
  Pc = der(Ec);
// Temp meas
  oTi = Tfi;
  oTo = Tfo;
  annotation( ...);
end ControlledLiquidHeater_ideal;
```

← set point T_0

← opposite case, cooling τ, tends to inlet T_{fi}

input power

this can be transiently (<0) giving heat out! No relevant for us! We compute only expense

← no gain IF <0 term

{ avoid discontinuity caused by max, events on simulation NO! }

avoid issue

$$\left\{ \frac{T_{fo}}{T_0} = \frac{1}{1+s\tau_L} \right\}$$

But we have limitations!

Thermal machines

Type 1 machines – example: heater, "inverse control-based" model

What Flowrate comes

- Energy balance in the "heater on" case:

IN − OUT

mass

$$c\rho V \dot{T}_o = wc(T_i - T_o) + P_h$$

heat power!

W·To (out∗state), W·Ti (in∗out)

- This is nonlinear (w times T), but let us assume w controlled (i.e., input) and seldom changed, so as to take it as a parameter and write for the process

W≃constant (through proper control scheme)

w ≃ parameter make model linear!

$$T_o(s) = \frac{1}{1 + s\rho V/w} T_i(s) + \frac{1/wc}{1 + s\rho V/w} P_h(s).$$

heater always traversed by same flow rate, independently by utilizer draw

- Note that increasing w decreases both the $P_h \to T_o$ gain $1/wc$ and the process time constant $\rho V/w$ — not surprisingly, the residence time.

With W↑ ⇒ { gain ↓ (Ph/To), residence time ↓ }

$W \simeq$ parameter → model linear:

T_{fo}, T_{fi} coherent to modellica library

from equation $$c\rho V \frac{dT_{fo}}{dt} = Wc\,T_{fi} - cW\,T_{fo} + P_h$$

↓ $\mathcal{L}$

$$(s\,c\rho V + Wc)\,T_{fo} = Wc\,T_{fi} + P_h$$

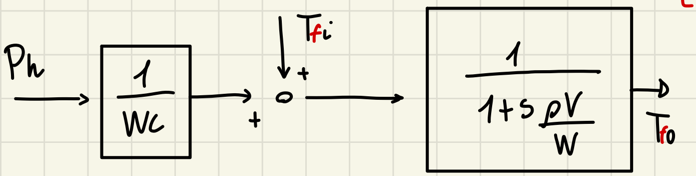
$$\left(1 + s\,\frac{\cancel{c}\rho V}{W\cancel{c}}\right) T_{fo} = T_{fi} + \frac{P_h}{Wc}$$

"Residence time"

$$\left[\frac{\rho V}{W}\right] = \left[\frac{\frac{kg}{m^3}\cdot m^3}{kg/s}\right] = [s]$$

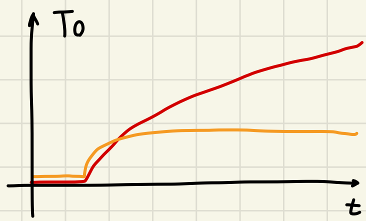
|||

time required to renew the mass contained on heater



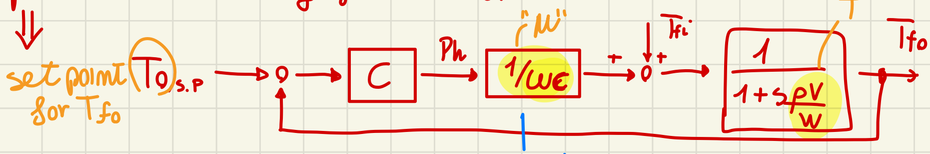
same P_h step

produces ⟹ different T_o transients for different (constant) W



We want a model that takes as INPUT the T_o set point and computes P_h as coming from the embedded controller

⇓



1st order syst. → C as (PI) controller

Thermal machines

Type 1 machines – example: heater, "inverse control-based" model

to represent control signal behaviour ... (Ph) we have to introduce controller, to complete the model with "fake" controller

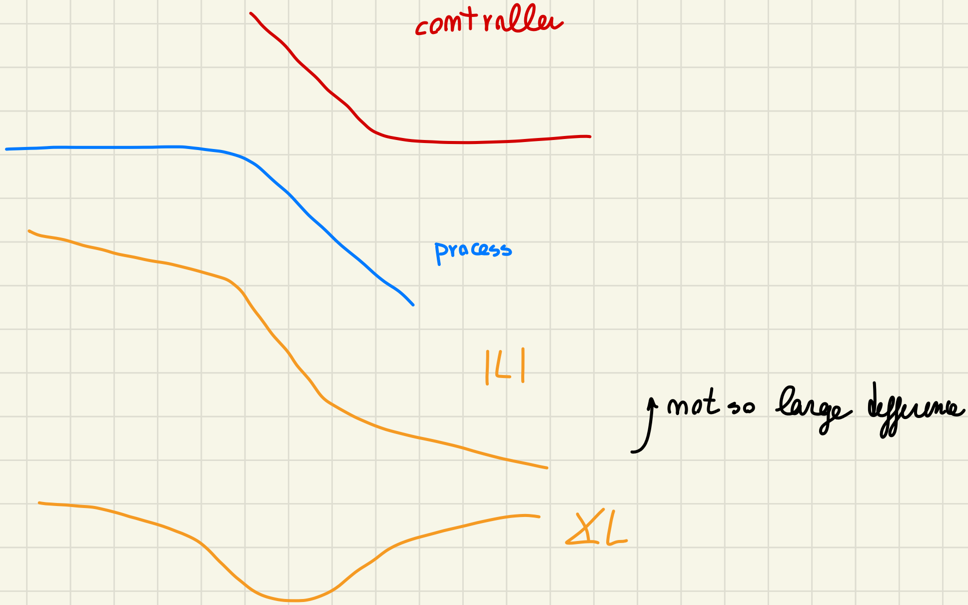
- Up to here, almost objective. ← {all objective, introducing JUST some simplification inside heater}
- Now comes the controller, and here we have to make some further assumptions:
  - we take a PI – very reasonable – and assume the integral time equal to the process time constant (it might be a little smaller, we are not discussing this now),
  - while the gain is easy to guess as we know the closed-loop time constant;
  - most important, then, we have to choose an antiwindup mechanism;
  - this is in fact really guessing and we are not deepening the issue here, if not for noticing that in the absence of datasheet information recorded data can help;
  - to exemplify, in the following we use the internal feedback technique.

↳ PI through non lin charact feed foreward + dynamic feedback!

for strong feedback, smaller time...

result depend on antiwindup to introduce!

as we see on Tuning: strenght feedback setting controller ZERO at higher freq. than process pole! (NO time by cancellation) tuning...



easy to know!

given a process $P = \frac{\mu}{1+sT}$ $\quad C = K\,\frac{1+sT}{sT} \Rightarrow L = \frac{K\mu}{sT}\,,\quad \omega_c = \frac{K\mu}{T}$

closed-loop time constant: $T/K\mu$

knowing it...

connected to the gain..

$\left\{ \begin{array}{ll} \mu \simeq \frac{1}{wc} & \text{"flow rate"} \\ T \simeq \frac{\rho V}{w} & \end{array} \right.$

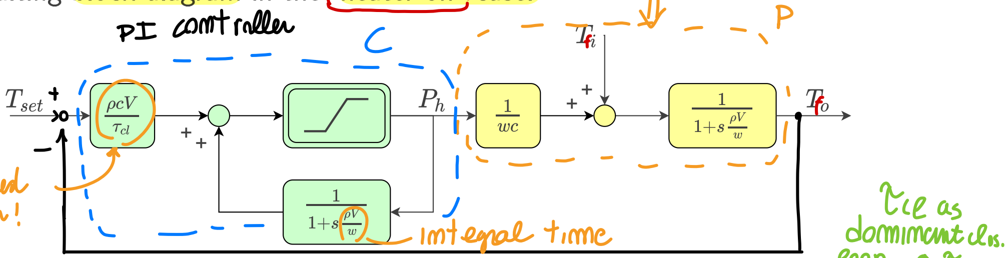
(parameters from datasheet)

Thermal machines

when to follow Tset

Type 1 machines – example: heater, "inverse control-based" model

- Resulting block diagram in the "heater on" case:

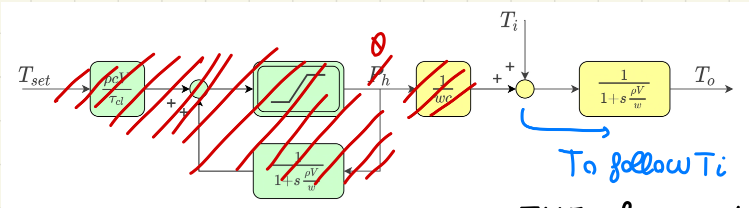


- The saturation block has limits 0 and $P_{h,max}$.
- The model – including the "heater off" case – is easy to write in the form of conditional equations: in the "off" case it suffices to set the "fake PI" (green blocks) to tracking at zero output, while the process state is always T_o.

Good!

$$L = C\,P = \frac{\cancel{\rho c V}}{\tau_{cl}} \cdot \frac{\cancel{1+s\rho V/w}}{s\cancel{\rho V/w}} \cdot \frac{1}{\cancel{wc}} \cdot \frac{1}{\cancel{1+s\rho V/w}} = \frac{1}{s\tau_{cl}}$$

in "heater off" → IF you set controller to tracking mode and put 0 on P_h → constraining the output of controller → you kill the feedback and scheme



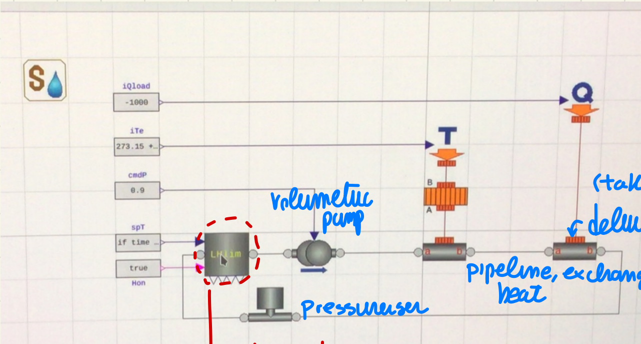
To follow T_i you can JUST change the time constant!

⇐ close/open loop time constant are different because

- open loop τ: residence time
- closed loop: $\tau_{cl} \neq \tau_{ol}$

τ comes from physics, while τ_{cl} from controller tuning!

looking in Modelica



volumetric pump

(taking fixed heat rate) delivery, fake utiliser

pipeline, exchanging heat

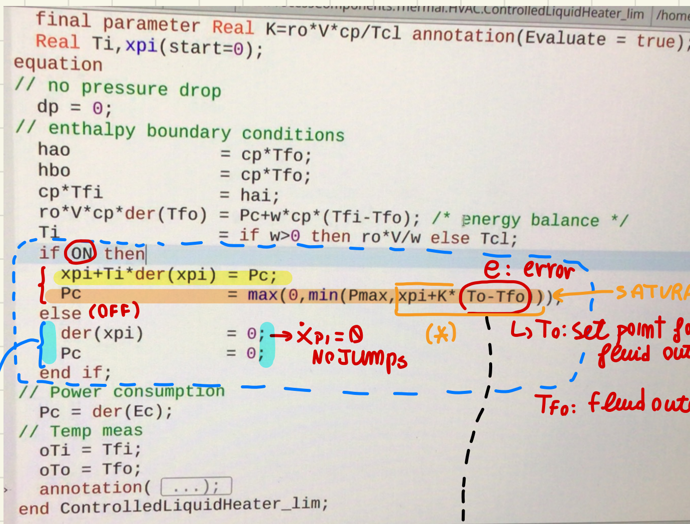
pressuriser

ideal heater

limited machine {analogous BUT different PI controller}

⇓ code...

↳ simulating... ⇒



```
final parameter Real K=ro*V*cp/Tcl annotation(Evaluate = true);
Real Ti,xpi(start=0);
equation
// no pressure drop
  dp = 0;
// enthalpy boundary conditions
  hao                 = cp*Tfo;
  hbo                 = cp*Tfo;
  cp*Tfi              = hai;
  ro*V*cp*der(Tfo) = Pc+w*cp*(Tfi-Tfo); /* energy balance */
  Ti                  = if w>0 then ro*V/w else Tcl;
  if ON then
    xpi+Ti*der(xpi) = Pc;
    Pc              = max(0,min(Pmax,xpi+K*(To-Tfo)));
  else
    der(xpi)        = 0;
    Pc              = 0;
  end if;
// Power consumption
  Pc = der(Ec);
// Temp meas
  oTi = Tfi;
  oTo = Tfo;
  annotation( ...);
end ControlledLiquidHeater_lim;
```

(OFF)

$\dot{x}_{PI} = 0$ NO JUMPS

e: error

SATURATION

(*)

↳ To: set point for fluid outlet Temp.

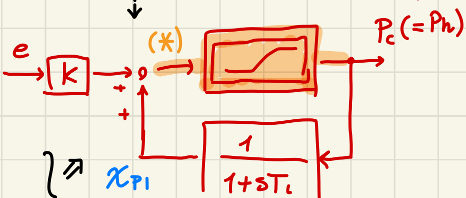
Tfo: fluid outlet Temp

PI with tracking to zero

(see "limited heated model" in the course libraries)

tracking to zero

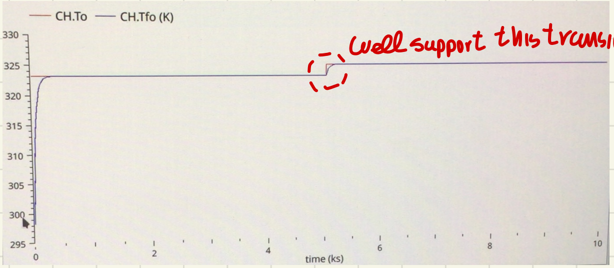
combustion ≃ heating



{this modelica code correspond to that PI controller}

$$\left\{ \frac{x_{PI}}{P_c} = \frac{1}{1+sT_i} \right\} \rightarrow \text{hence...} \quad x_{PI} + T_i \dot{x}_{PI} = P_c$$

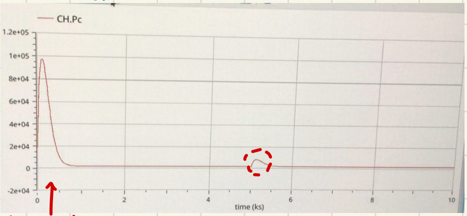
... simulating



well support this transient

initialization

consumed Power

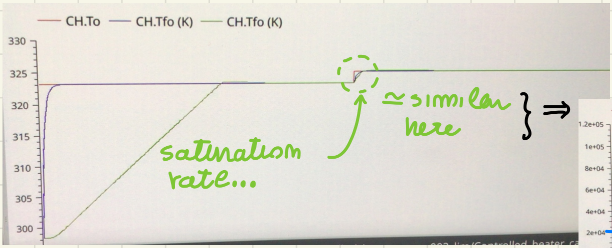


high heat rate pick initially

, too much required power for initial pick!

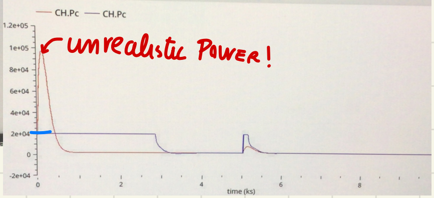
on limited case, modelling

$\boxed{\text{CHlim}}$ (same scheme) comparing Ideal vs limited



saturation rate...

≃ similar here



unrealistic Power!

but we can approx $T_{fo\,id} \simeq T_{fo\,real}$ ⇐ { different consumption } ← SATURATION limit on real case..

BUT not respect P_c which are VERY different → different energy outcome!

we need a good representation of control

- you can't simplify models too much loosing a well representation of the controller signal

↓

how much we could do trusting the "control embedded on the system" ↳ significantly simplify the model (neglecting some phisic aspects) ⟶ system level model

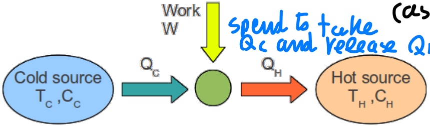
@ end we realistically represent the system in a simple way

Thermal machines

there are multiple types of heat pump... depending on fluid used on the hot/cold side + many technology to implement heat pump (as reversed refrigerator..)

Type 2 machines – example: heat pump

we model heat pump as: cold/hot side



spend to take Q_C and release Q_H to hot source

- The steady-state balance is apparently $Q_H = Q_C + W$, where W can come e.g. from a compressor (we do not treat more articulated cases such as absorption cycles).
- Machines like the one here schematised are statically described by the so called *Coefficient Of Performance* (COP), defined as "useful effect over needed work". As one may use the machine for heating or cooling, we have

$$COP_{heat} = \frac{Q_H}{W}, \qquad COP_{cool} = \frac{Q_C}{W}$$

heat pump

air conditioner

← depending on the usefull effect of the machine

where

$$\frac{Q_H}{W} = \frac{Q_C + W}{W} \quad \Rightarrow \quad COP_{heat} = COP_{cool} + 1.$$

adimensional number

Thermal machines

"maximum efficiency": reversible transformation, NO entropy generation

Type 2 machines – example: heat pump

- If the machine operates at the maximum theoretical (Carnot) efficiency

put it on def..

$$\left\| \frac{Q_H}{T_H} = \frac{Q_C}{T_C}. \right\|$$

← (generating NO entropy)

- Thus, theoretical (Carnot) values for COP_{heat} and COP_{cool} can be defined as

$$COP^C_{heat} = \frac{T_H}{T_H - T_C}, \quad COP^C_{cool} = \frac{T_C}{T_H - T_C},$$

where, remember, absolute (Kelvin) temperatures are to be used. !!

multiply by η

- To represent real machines efficiencies are introduced, hence

↳ manufacturer can provide COP curve.. (as operating T function) OR represent by a RATIO ≃ guess

$$COP_{heat} = \eta_h \frac{T_H}{T_H - T_C}, \quad COP_{cool} = \eta_c \frac{T_C}{T_H - T_C},$$

where η_h and η_c, $0 < \eta_{h,c} < 1$, can be considered constant (as we shall do) or be made dependent on temperatures.

Thermal machines

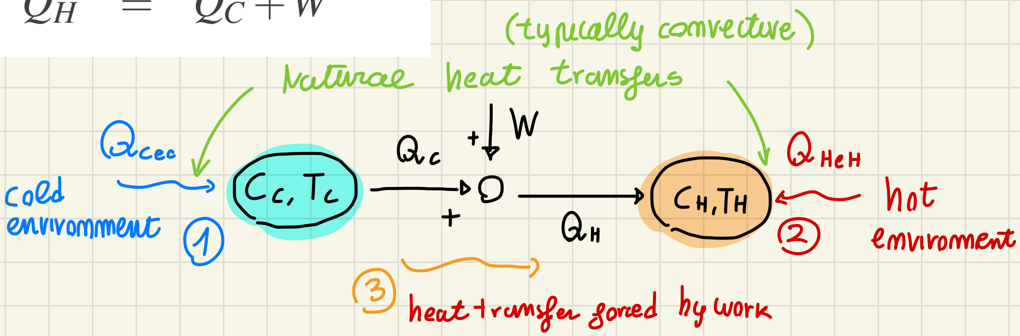
Type 2 machines – example: heat pump

- To obtain a simple (system-level) dynamic model, associate two thermal capacities $C_{H,C}$ to the hot and cold sources (i.e., considering the classical refrigerator cycle as a representative example, to the condenser and the evaporator respectively).
  HOT COLD
- This, together with the previous COP considerations, yields for the heating case

$$
\begin{aligned}
C_H\dot{T}_H &= Q_H + Q_{Heh} \\
C_C\dot{T}_C &= Q_C + Q_{Cec} \\
COP_{heat} &= \eta_h \frac{T_H}{T_H - T_C} \\
Q_H &= COP_{heat} W \\
Q_H &= Q_C + W
\end{aligned}
$$

where Q_{Heh} and Q_{Cec} are the heat rates exchanged by the H and C sides (typically by convection) with the environments to which either of the two is exposed; W is here an input.

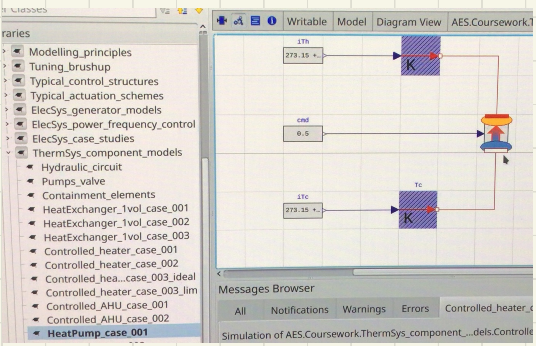
$$
\begin{aligned}
C_H \dot{T}_H &= Q_H + Q_{Heh} \\
C_C \dot{T}_C &= -Q_C + Q_{Cec} \\
COP_{heat} &= \eta_h \frac{T_H}{T_H - T_C} \\
Q_H &= COP_{heat} W \\
Q_H &= Q_C + W
\end{aligned}
$$



↓

on Modelica... on "Process components" >> HP

↳ typically you have an upper limit! + ΔT dependent COP!



IF air instead of water!

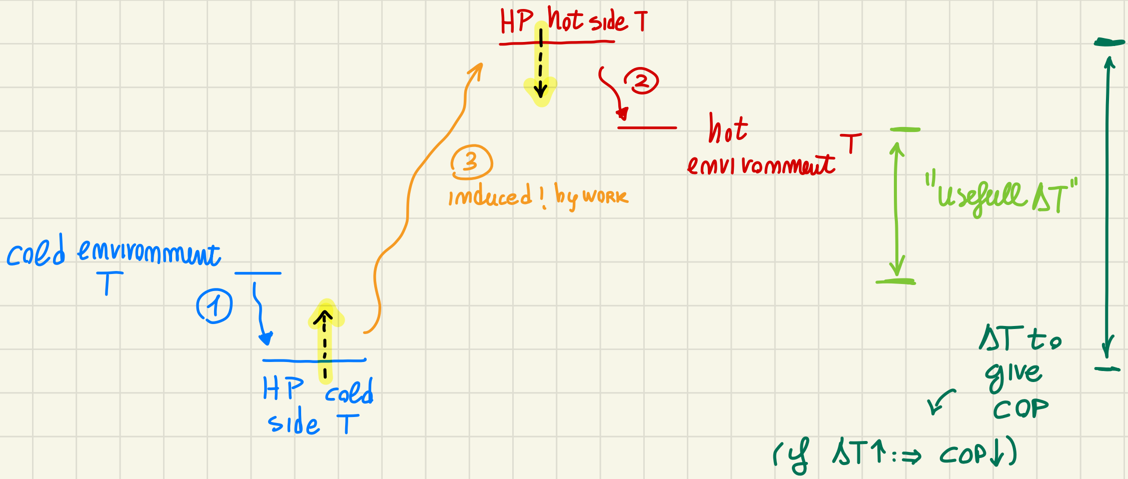
↓

air circulating on an heat pump dryer

{ heats up air from bring it to condense water and release.. }

↳ heat pump to condense heat

So...



one may want to make temperature differences across 1 and 2 smaller ⇒ larger heat exchanges

⇒ larger expenditure

⇒ trade off

(design and control problems)

Preliminaries

last elements we need to describe..

(piping in thermal systems/ ambient boundaries (walls..) ecc..)

- Containment elements [in the absence of openings] (where flow happen) are simple conduction ones.
- Such elements are possibly (and in fact most frequently) composed of a series of layers of different materials (i.e., with different specific heats and conductivities)...
- ...so that their electric equivalent is a series of RC cells.
- Glazing in buildings entails many particular cases and may require specific modelling (for example, windows often need to account for the thermal power transmitted by radiation), but we do not deal with such details in this course.

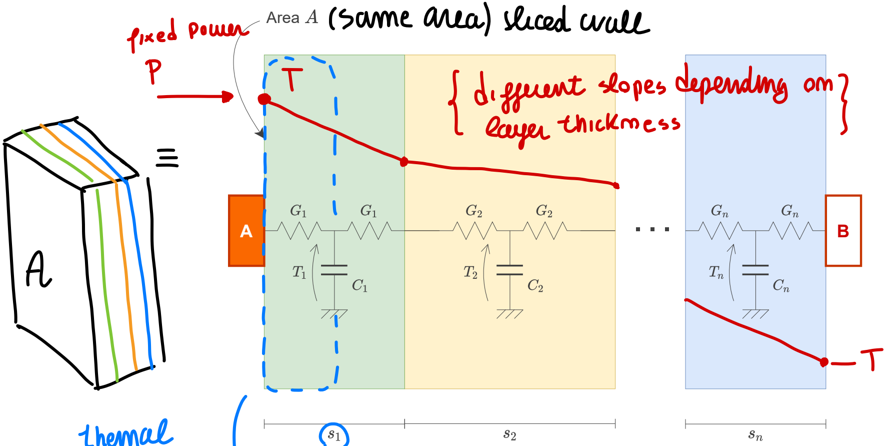
{ disturbance generation
physical details... }

A multilayer wall

"walls"

— nonhomogeneous case, planar geometry only for simplicity

represent each layer of contained element as homogeneous solid ≃ so characterized by λ, s, C, G



statical model representing thermal profile inside

thermal conductance of half layer G_i

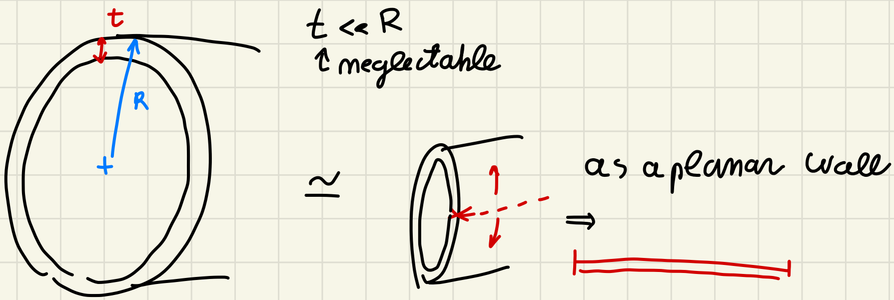
thickness of layer

$$C_i = c\rho_i A s_i, \qquad G_i = \lambda_i \frac{A}{s_i/2}$$

heat capacity

thermal conductivity

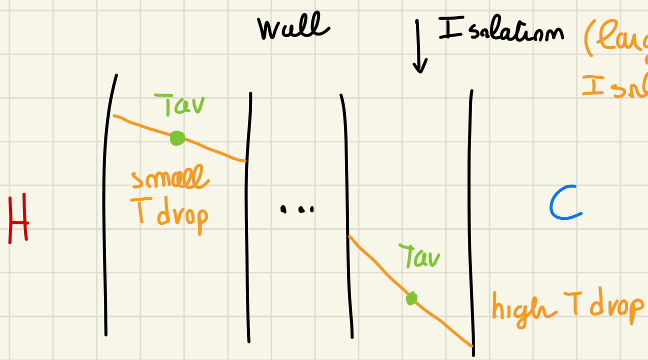
OR cylindrical case (≠ planar case)



cylindrical ≃ planar

- quantity of energy stored in a layer depends on the heat capacity and on temperature

statically: isolation side doesn't matter
while dynamically isolation side matter!



(large drop on Isolated term)
↳ very different energy storage!

average T: for energy stored :⇒ [Tav representative of energy amount]

An interesting particular case

- A specific case of through-containment heat exchange is air renovation, on which we conversely spend some words.
- Denoting by w_r the renovation air flow rate, and with T_i and T_e the internal and external temperatures, respectively, we have

$$\begin{array}{rcl} Q_{e\to i} & = & w_r c_a T_e \\ Q_{i\to e} & = & w_r c_a T_i \end{array}$$

{ heat exchanger equations.. }

where c_a is the air specific heat.

- Summing the above with the due signs, the net e→i heat rate is

$$Q_{ei} = G_{ei}(T_e - T_i), \quad G_{ei} = w_r c_a$$

where G_{ei} plays the role of an equivalent thermal conductance, governed by the renovation flowrate.

"air renovation"

Boundary (WALL)

internal environment | external environment

Renovation flow rate W → W ← (same balance air IN/OUT)

(Temperature T_i) | (temperature T_e)

energy flows → wcT_i

wcT_e ←

$\Rightarrow$ net flow $(i \to e)$: $\underbrace{Wc}_{G}(T_i - T_e)$ (positive if in → ext)

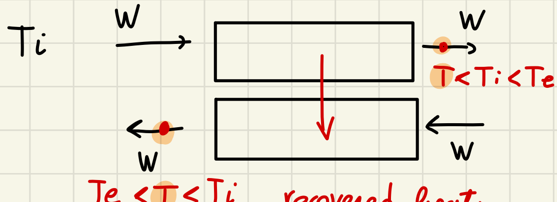
equivalent of a conductance

(tuning by flow rate) → "variable resistor"

(without heat recovery..)

If this is NOT acceptable.. one can introduce heat recovery systems in the form of heat exchanges

internal (warm) (heat exch.) external (cold)



recovered heat

$\Downarrow$

[Way to recover heat without mixing, which preserves renovation]