July 2012

Ex 1

a) $\bar{x} = \dfrac{2\bar{x}^2}{1+\bar{x}} \rightarrow \bar{x}^2 + \bar{x} = 2\bar{x}^2 \rightarrow \bar{x}(\bar{x}-1) = 0$

$\bar{x} = 0 \,, \quad \bar{x} = 1$

b) $\delta x(k+1) = \dfrac{2\bar{x}^2 + 4\bar{x}}{(1+\bar{x})^2}\, \delta x(k)$

$\bar{x} = 0 \rightarrow \delta x(k+1) = 0$ asymptotically stable

$\bar{x} = 1 \rightarrow \delta x(k+1) = \frac{6}{4}\, \delta x(k)$ unstable

c) $V(x) = x^2 \rightarrow \Delta V(x) = (x(k+1))^2 - x^2(k)$

$\Delta V(x) = \left(\dfrac{2x^2}{1+x}\right)^2 - x^2 = \dfrac{-x^2 - 2x^3 + 3x^4}{(1+x)^2}$

in a neighbor of the origin $\dfrac{-x^2}{(1+x)^2}$ dominates

and $\Delta V < 0$

Ex 2

$f_1(x_1) = x_1^3 \,, \quad g_1(x_1) = -1 \,, \quad f_2(x_1, x_2) = -x_2 \quad g_2(x_2) = 1$

$u = \dfrac{1}{g_2}(u_c - f_2) = u_c + x_2$

The system becomes

$$\begin{cases} \dot{x}_1 = x_1^3 - x_2 \\ \dot{x}_2 = u_c \end{cases}$$

$$x_2 = \phi_1(x_1) = x_1^3 + x_1 \quad (x_2 - x_1 \text{ is another option})$$

$$\downarrow$$

$$\dot{x}_1 = -x_1 \quad (\text{linear system, asymptotically stable})$$

$$V_1(x_1) = \frac{1}{2} x_1^2 \longrightarrow \dot{V}_1(x_1) = -x_1^2 < 0$$

$$\frac{\partial \phi_1}{\partial x_1} = 3x_1^2 + 1 , \quad \frac{\partial V_1}{\partial x_1} = x_1$$

$$u_a = \underbrace{(3x_1^2+1)}_{\frac{d\phi_1}{dx_1}} \underbrace{(x_1^3 - x_2)}_{f_1 + g_1 x_2} - k \underbrace{(x_2 - x_1^3 - x_1)}_{x_2 - \phi_1(x_1)} - \underbrace{x_1}_{\frac{dV_1}{dx_1}} \underbrace{(-1)}_{g_1}$$

$$V(x) = V_1(x_1) + \frac{1}{2}\left(x_2 - \phi_1(x_1)\right)^2$$

$\boxed{\text{Ex 3}}$

a)
$$\begin{cases} \dot{x}_1 = y + \gamma_1 d \\ \dot{d} = 0 \\ \underbrace{\dot{y} + 2y - u}_{\eta} = -x_1 + \gamma_2 d \end{cases}$$

$$A = \begin{vmatrix} 0 & \gamma_1 \\ 0 & 0 \end{vmatrix} , \quad C = \begin{vmatrix} -1 & \gamma_2 \end{vmatrix} , \quad B = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$M_o = \begin{vmatrix} C \\ CA \end{vmatrix} = \begin{vmatrix} -1 & \gamma_2 \\ 0 & -\gamma_1 \end{vmatrix} \longrightarrow \gamma_1 \neq 0$$

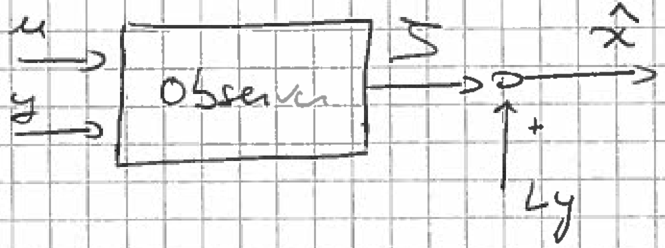
$$x = \begin{vmatrix} x_1 \\ d \end{vmatrix}$$

$$\dot{\hat{x}} = A\hat{x} + L\left[y - C\hat{x}\right] + By$$

$$\dot{\hat{x}} = (A-LC)\hat{x} + Ly + By = (A-LC)\hat{x} + L\dot{y} + 2Ly - Lu + By$$

$$\underbrace{\dot{\hat{x}} - L\dot{y}}_{\dot{\xi}} = (A-LC)\underbrace{(\hat{x} - Ly)}_{\xi} + 2Ly - Lu + (A-LC)Ly + By$$

$$\dot{\xi} = (A-LC)\xi + (B + 2L + (A-LC)L)y - Lu$$



$\boxed{\text{Ex 4}}$

a) $A = -1$, $B = Q = 1$

Stationary Riccati equation $P^2 - P - R = 0$

$$P = \frac{1 + \sqrt{1 + 4R}}{2}, \quad K = -\frac{1 + \sqrt{1 + 4R}}{2R + 1 + \sqrt{1 + 4R}}$$

$$A - BK = -1 + \frac{1 + \sqrt{1 + 4R}}{2R + 1 + \sqrt{1 + 4R}}$$

$R \to \infty$, $A - BK \to -1$

$R \to 0$, $A - BK \to 0$

b) see the notes

$\boxed{\text{Ex 5}}$

see the notes