Solutions partielles

ADVANCED AND MULTIVARIABLE CONTROL

8/9/2022

Surname and name ....

Signature ....

Exercise 1 (3 marks)

Consider the continuous-time linear system:

$$\dot{x}(t) = Ax(t)$$

and assume that $A+A'$ is definite negative. Then, with the Lyapunov theory it is possible to conclude

- o Nothing about the stability of the system.
- o The system is stable
- X The system is asymptotically stable
- o The system is unstable
- o No answer

(Consider it as a Lyapunov equation with $P=I$)

Exercise 2 (3 marks)

The zeros of continuous or discrete time systems are important because:

- o They influence the stability of the system
- X They influence the static and dynamic performances which can be achieved with the design of a closed-loop system.
- o They influence only the static performances which can be achieved with the design of a closed-loop system.
- o They influence the possibility to design LQ or LQG control laws.
- o No answer

Exercise 3 (3 marks)

The design of a state-feedback pole placement control law for a reachable multi-input system

- o Can be always completed only by using all the inputs
- o Can be always completed by using only one input
- X Can be completed using one or more inputs, it depends on the specific problem
- o Can be completed only if the system does not have zeros
- o No answer

Exercise 4 (3 marks)

Imposing the terminal constraint $x(k+N)=0$ in the formulation of MPC to guarantee stability

- o Can be made only for unconstrained problems
- o Can be made, but it is then impossible to use the Receding Horizon approach and all the sequence of future computed control moves $u(k),\dots,u(k+N-1)$ must be used
- X Can be made, provided that the state $x(k)$ at the current time instant k must be in the set of states where a solution exists
- o Can only be made for linear systems
- o No answer

Exercise 5 (3 marks)

Concerning the Infinite Horizon LQ control for discrete time systems:

- o It guarantees at the same time gain and phase robustness margins
- o It guarantees either gain or phase robustness margins
- X It guarantees gain margins smaller than in continuous time
- o It guarantees gain margins smaller, or equal, or larger than in continuous time
- o No answer

Exercise 6 (6 marks)

Consider the system

$$\dot{x_1}(t) = x_2(t)$$

$$\dot{x_2}(t) = \left(3x_2^3(t) + x_2(t)\right) + u(t)$$

1. Design a control law with the backstepping method
2. Specify the Lyapunov function that could be used to prove the stability of the origin of the corresponding closed-loop system.

Formula backstepping

$$u = -\frac{dV_1(x_1)}{dx_1} g(x_1) - k\left(x_2 - \Phi_1(x_1)\right) + \frac{d\Phi_1(x_1)}{dx_1}\left(f(x_1) + g(x_1)x_2\right)$$

$f_1(x_1) = 0$, $g_1(x_1) = 1$, $f_2(x_1, x_2) = 3x_2^3 + x_2$, $g_2(x_1, x_2) = 1$

$u = u_a - (3x_2^3 + x_2)$, $x_2 = -kx_1 \rightarrow \begin{cases} \dot{x}_1 = -kx_1 \\ \dot{x}_2 = u_a \end{cases}$

$\phi_1(x_1) = -Kx_1$ $(K > 0)$, $V_1(x_1) = \frac{1}{2}x_1^2$, $\dot{V}_1(x_1) = x_1\dot{x}_1 = -Kx_1^2$

$\frac{dV_1}{dx_1} = x_1$, $\frac{d\phi_1}{dx_1} = -K$

$u_a = -x_1 - h(x_2 - \phi_1(x_1)) + \frac{d\phi_1}{dx_1}(f_1 + g_1 x_2)$, $h > 0$

$u_a = -x_1 - hx_2 + hk\, x_1 - kx_2$

$V_2(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}(x_2 - \phi_1(x_1))^2$

Exercise 7 (7marks)

Consider the following system

$$
\begin{aligned}
\dot{x}(t) &= ax(t) + bu(t) + v_x(t) \\
y_1(t) &= x(t) + v_{y1}(t) \\
y_2(t) &= x(t) + v_{y2}(t)
\end{aligned}
$$

Where $a = \frac{r+1}{2r}$, $b = \sqrt{r}$, $v_x \sim WN\left(0, 2\frac{R+1}{R}\right)$, $v_y = \begin{bmatrix} v_{y1} \\ v_{y2} \end{bmatrix} \sim WN\left(0, \begin{bmatrix} 1 & 0 \\ 0 & r \end{bmatrix}\right)$

and design a LQG regulator minimizing the following performance index

$$
J = \lim_{T\to\infty} \frac{1}{T} E\left[\int_0^T \left(2\frac{(r+1)^2}{r^2} x^2(\tau) + ru^2(\tau)\right) d\tau\right]
$$

Steady state Riccati equation of LQ control

$$
0 = A'P + PA + Q - PBR^{-1}B'P
$$

$\boxed{LQ_\infty}$, $A = a$, $B = \sqrt{r}$, $a = \frac{r+1}{2r}$, $Q = 2\left(\frac{r+1}{r}\right)^2$, $R = r$

$A'P + PA + Q - PBR^{-1}B'P = 0 \longrightarrow P = \begin{cases} 2\frac{r+1}{r} \\ * < 0 \end{cases}$

$K = R^{-1}B'P = \frac{2(r+1)\sqrt{r}}{r^2}$, $A - BK = -\frac{3}{2}\frac{r+1}{r} < 0$

$\boxed{KP}$

$\tilde{Q} = \frac{2(r+1)}{r}$, $\tilde{R} = \begin{vmatrix} 1 & 0 \\ 0 & r \end{vmatrix} \to \tilde{R}^{-1} = \begin{vmatrix} 1 & 0 \\ 0 & 1/r \end{vmatrix}$

$AP + PA' + \tilde{Q} - PC'\tilde{R}^{-1}CP = 0 \longrightarrow P^2 - P - 2 = 0$, $P = \begin{cases} 2 \\ * < 0 \end{cases}$

$L = PC'\tilde{R}^{-1} = \begin{vmatrix} 2 & 2/r \end{vmatrix}$, $A - LC = -\frac{3}{2}\frac{r+1}{r}$

$\dot{\hat{x}} = A\hat{x} + Bu + L[y - C\hat{x}]$

Exercise 8 (5 marks)

Shortly describe what are the control horizon and the minimum prediction horizon in MPC and why are used in many applications.

See pages 193-195 of the notes.