ADVANCED AND MULTIVARIABLE CONTROL

16/6/2022

Solutions

Surname and name ....

Signature ....

Exercise 1 (3 marks)

Consider the discrete-time system

$$x(k+1) = -x(k)cos^2(x(k))$$

Using a quadratic Lyapunov function, select the correct answer among the following ones

- o The origin is an unstable equilibrium
- o The origin is a locally stable equilibrium
- X The origin is a locally asymptotically stable equilibrium
- o The origin is a globally asymptotically stable equilibrium
- o No answer

$V(x) = x^2$

$\Delta V(x) = (-x\cos^2(x))^2 - x^2$

$= \underbrace{(\cos^4(x) - 1)}_{<0} \underbrace{x^2}_{>0}$

in a neighbor of the origin

$\Downarrow$

$\Delta V(x) < 0$ locally

Exercise 2 (3 marks)

Concerning the use of the Kalman Predictor or Filter for the continuous time system

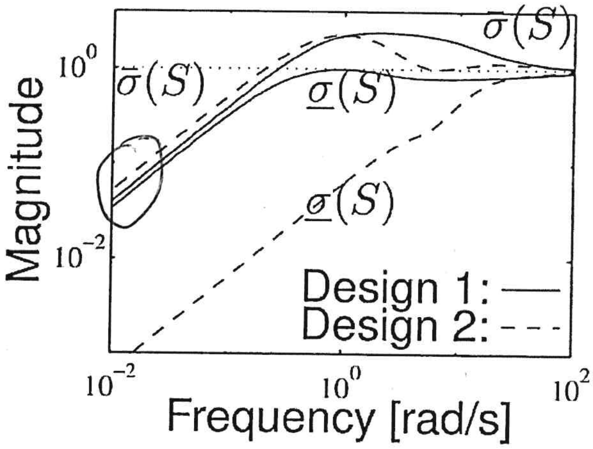
$$\dot{x}(t) = Ax(t) + Bu(t) + v_x(t)$$
$$y(t) = Cx(t) + v_y(t)$$

Select the correct answer

- o It can be applied only for asymptotically stable systems
- o It can be used only if v_x and v_y are uncorrelated white gaussian noises
- X It can be used also when v_x is a stationary stochastic process (with suitable modifications)
- o The only condition required to guarantee that the covariance of the state estimation error tends to a limiting value is that the pair (A,C) is observable
- o No answer

Exercise 3 (3 marks)

Consider two control designs of a closed-loop system with the following sensitivity functions and select the answer that is surely true



at low frequency
$\bar{\sigma}(S_1) < \bar{\sigma}(S_2)$

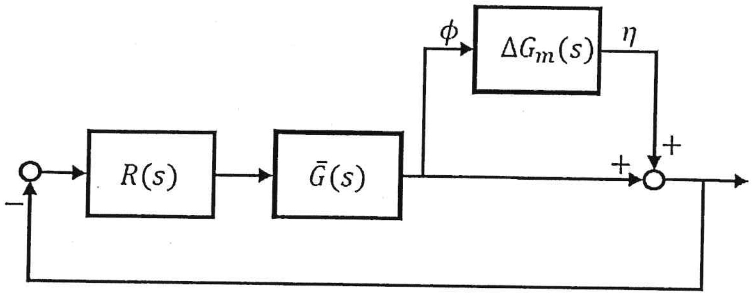
- o Design 1 guarantees a faster closed-loop system
- o Both designs guarantee closed-loop stability
- o Design 1 guarantees slightly more attenuation of high frequency measurement noise
- X Design 1 guarantees slightly more attenuation of low frequency process disturbances (d_y)
- o No answer

Exercise 4 (3 marks)

In Model Predictive Control of linear systems, it is possible to guarantee robust zero steady state error regulation for constant reference signals y^o also in the case of small model uncertainties or neglected disturbances (assuming that the resulting closed-loop system is asymptotically stable) provided that

- o In the cost function to be minimized it is weighted the difference between the predicted state and its asymptotic value corresponding to y^o, in nominal conditions, and the difference between the future input and its asymptotic value corresponding to y^o in nominal conditions (necessary and sufficient condition) — not robust for modelling errors
- o The prediction horizon is chosen sufficiently long
- X A model in $\delta x(k)=x(k)-x(k-1)$ and $\delta u(k)=u(k)-u(k-1)$ is used
- o The prediction horizon is selected longer than the control horizon
- o No answer

Exercise 5 (3 marks)
Consider the following control system with multiplicative uncertainty



where

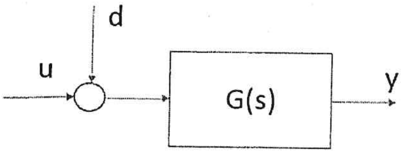
$$\bar{G}(s) = \frac{m}{1+as}, \quad a > 0, \quad R(s) = \frac{k(1+as)}{s}, \quad \Delta G_m(s) = g$$

Select the sufficient condition required to guarantee the stability of the overall system

- X $\left\|\frac{mgk}{jw+mk}\right\|_\infty<1$
- o $\left\|\frac{gk(1+jaw)}{jw+mk}\right\|_\infty<1$
- o $|g| < 1$
- o $\left\|\frac{mk}{jw}\right\|_\infty<1$
- o No answer

Exercise 6 (8 marks)

Consider the following system



where $G(s) = \frac{1}{s-1}$

For this system:

1. Design an observer (full order or reduced order) observer of the disturbance d, assumed to be constant but unknown,
2. Design a compensator of the disturbance
3. Design a feedback regulator with integral action by using the pole-placement approach
4. Show how to implement the regulator in order to avoid that the roots of the polynomial $F(s)$ are zeros of the closed-loop transfer function between the reference and the output.

$$\begin{cases} \dot{x} = x + u + d \\ y = x \end{cases} \longrightarrow \begin{cases} \begin{bmatrix} \dot{x} \\ \dot{d} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}_{\bar{A}} \begin{bmatrix} x \\ d \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\bar{B}} u \\ y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\bar{C}} \begin{bmatrix} x \\ d \end{bmatrix} \end{cases}, \quad (\bar{A}, \bar{C}) \text{ observable}$$

Full order observer

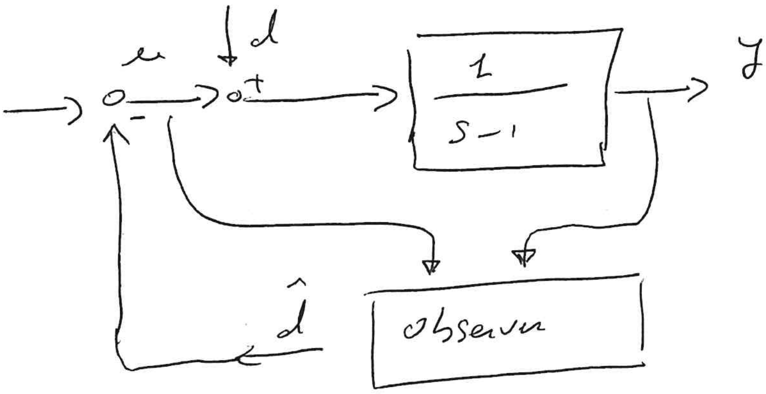
$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{d}} \end{bmatrix} = \bar{A} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix} + \bar{B}u - L\left[y - \hat{x}\right]$$

Reduced order observer

$$\dot{\hat{d}} = L\left[\underbrace{\dot{y} - y - u}_{\eta} - \hat{d}\right] \rightarrow \underbrace{\dot{\hat{d}} - L\dot{y}}_{\dot{\xi}} = -Ly - Lu - L\hat{d}$$

$$\dot{\xi} = -L\xi - L^2 y - Lu$$

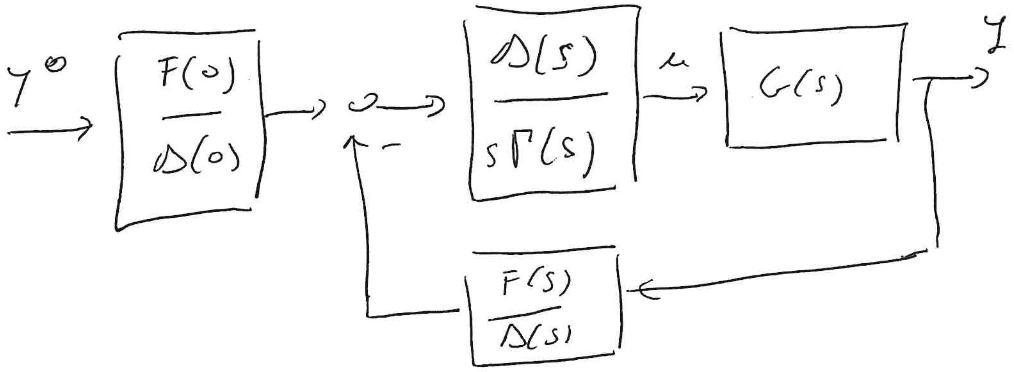
disturbance compensation



Feedback regulator + integral action

$$\tilde{G}(s) = \frac{1}{s(s-1)}, \quad R(s) = \frac{f_1 s + f_0}{\gamma_1 s + \gamma_0}, \quad P(s) = s^3 + p_2 s^2 + p_1 s + p_0$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_0 \\ f_1 \\ f_0 \end{bmatrix} = \begin{bmatrix} 1 \\ p_2 \\ p_1 \\ p_0 \end{bmatrix}$$



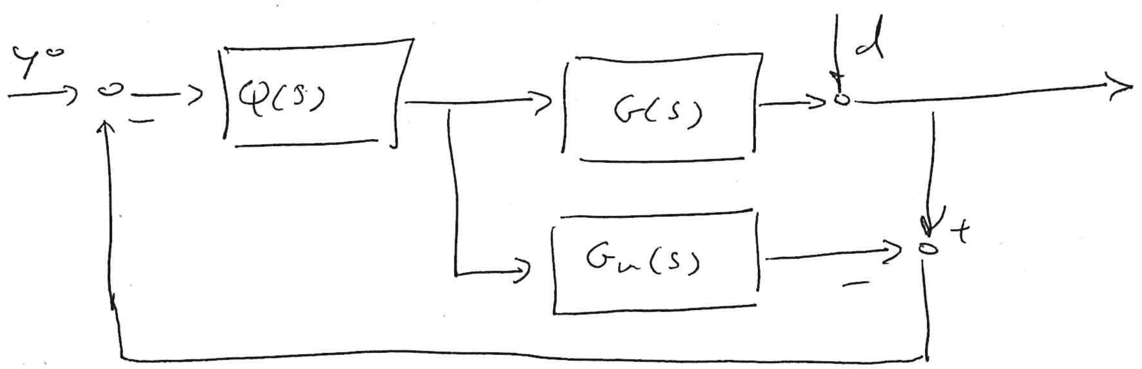
$\Delta(s) = \delta_2 s^2 + \delta_1 s + \delta_0$ (with roots " as. stable)

Exercise 7 (5 marks)

Consider a system described by the transfer function $G(s) = \frac{1-s}{(1+2s)(1+5s)}$

For this system design an Internal Model Controller.

IMC scheme



$$Q(s) = G_{mm}^{-1}(s)\, G_f(s) \quad , \quad G_m(s) = \underbrace{\frac{1}{(1+2s)(1+5s)}}_{G_{mm}(s)} \cdot \underbrace{(1-s)}_{G_{mp}(s)}$$

$$Q(s) = \frac{(1+2s)(1+5s)}{(1+sT)^2} \quad , \quad T > 0 \; ,$$

$$G_f(s) = \frac{1}{(1+sT)^2}$$

Exercise 8 (5 marks)

Explain the Loop Transfer Recovery, and in particular:

a. Its goal
b. Its applicability conditions
c. The required choice of the design parameters

a): to recover the robustness properties of LQ control (continuous time) also when an observer is used

b) ~~cont~~ conditions for stability of LQ control

and

- $m = p$
- no invariant zeros with $Re(\ ) \geq 0$

c) <u>Solution 1</u>

$L = \rho B, \quad \rho \to \infty$ (L is the observer gain)

<u>Solution 2</u>

solve KF with $\begin{cases} \tilde{Q} = \alpha BB', & \alpha \to \infty \\ \tilde{R} = I \end{cases}$