ADVANCED AND MULTIVARIABLE CONTROL

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Exercise 1

Consider the system

$$\dot{x}_1(t) = x_2^5(t)$$
$$\dot{x}_2(t) = -x_1(t) - x_2^3(t)$$

A. Show that the origin is an equilibrium
B. Show that the equilibrium is asymptotically stable with the Lyapunov theory and the Krasowski La Salle principle using the Lyapunov function $V(x) = 3x_1^2 + x_2^6$.

Solution Exercise 1

a) $0 = \bar{x}_2 \rightarrow \bar{x}_1 = 0$

b)

$\dot{V}(x) = 6x_1\dot{x}_1 + 6x_2^5\dot{x}_2 = -x_2^8 \leq 0 \rightarrow$ semidefinite positive

but

$\dot{V} = 0$ for $x_2 = 0$ and from the first equation x_1 =cost=$\bar{x}_1$

from the second equation the only possible value is $\bar{x}_1 = 0$

Exercise 2

Given the system

$$\dot{x}(t) = x^3(t) - 2x(t)u(t)$$
$$y(t) = x(t)$$

a. Compute the constant input $\bar{u}$ corresponding to the equilibrium $\bar{y} = 2$.
b. Compute the linearized model corresponding to the computed equilibrium.
c. For the computed linearized system, design a regulator with the pole placement approach guaranteeing closed-loop poles in *s=-1* and asymptotic zero error for constant reference signals.
d. Draw the corresponding control scheme specifying the signals to be used (state and control variables, their variations with respect to the equilibrium values, equilibrium values).

Solution Exercise 2

A.

$$\bar{x}^3 - 2\bar{x}\bar{u} = 0 \rightarrow \bar{u} = 0.5\bar{x}^2 \rightarrow \bar{u} = 2$$

B.

$$\delta\dot{x}(t) = 8\delta x(t) - 4\delta u(t) \rightarrow G(s) = \frac{-4}{s-8}$$

C.

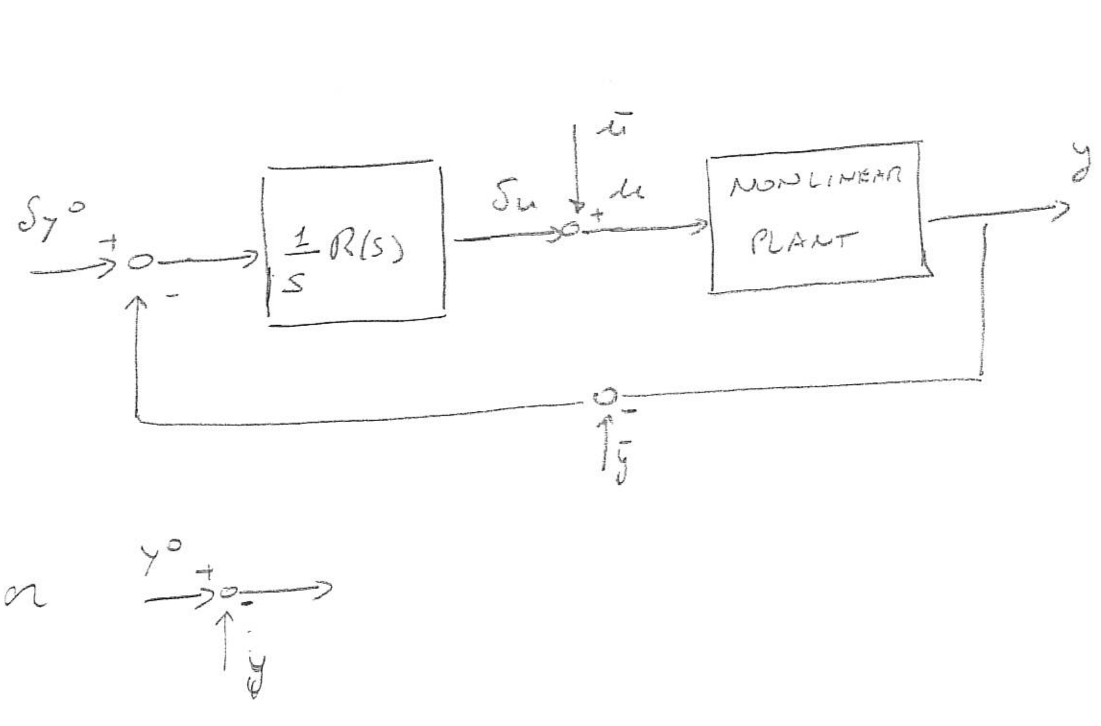
First insert an integrator

$$G'(s) = \frac{-4}{s(s-8)}$$

Then solve the pole placement approach with the solution based on transfer functions (or any other, if you prefer)

$$R(s) = \frac{f_1 s + f_0}{\gamma_1 s + \gamma_0} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ -8 & 1 & 0 & 0 \\ 0 & -8 & -4 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_0 \\ f_1 \\ f_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

D.



Exercise 3

Given the discrete-time system

$$x(k+1) = -x(k) + u(k)$$

and the cost function

$$J = \sum_{i=0}^{\infty} \left(0.5x^2(k+i) + u^2(k+i)\right)$$

A) Compute the solution of the infinite time LQ control (N=∞) and the corresponding closed-loop eigenvalue.
B) Discuss the robustness properties guaranteed by infinite-horizon LQ control of discrete-time systems.

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Riccati equation $P(k)=Q+A'P(k+1)A-A'P(k+1)B(R+B'P(k+1)B)^{-1}B'P(k+1)A$

Solution Exercise 3

A.

The steady state solution of the stationary Riccati equation with $A=B=R=1$, $Q=0.5$, is $P=1$, which corresponds to $K=(R+B'PB)-1B'PA=0.5$, therefore A-BK=0.5.

B.

See the notes, pag 171-172

Anyway, $\bar{\gamma} = \dfrac{R}{R+B'\bar{P}B} = \dfrac{1}{2}$, $\gamma = \sqrt{\bar{\gamma}} = 0.7071$

guaranteed gain margin $\left[\dfrac{1}{1+\gamma}, \dfrac{1}{1-\gamma}\right] = [0.58, 3.41]$

Exercise 4

$$\dot{x}_1(t) = -x_1(t) + u(t)$$
$$\dot{x}_2(t) = x_1(t) + x_2(t)$$
$$y_1(t) = x_1(t) + x_2(t)$$
$$y_2(t) = x_1(t)$$

A. compute the corresponding transfer function matrix;

B. show how to use, in this case, a static (order zero) observer to estimate the state from the measured outputs;

C. select the output which can asymptotically controlled to an arbitrary constant reference signal;

D. draw the scheme of a feedback controller guaranteeing that the selected output is asymptotically brought to a constant reference.

Solution Exercise 4

A.

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1} B = \begin{bmatrix} s/(s+1)(s-1) \\ (s-1)/(s+1)(s-1) \end{bmatrix}$$

B.

Since matrix C is square and nonsingular, one can set

$$x = C^{-1} y$$

This corresponds to a reduced order observer (*n=p=2, n-p=0*)

C.

By looking at the two transfer functions, it is apparent that from u to y_1 there is a derivative action, so that the zero error regulation for constant references cannot be achieved. On the contrary, it is possible to guarantee it for the second output y_2.

D.

The corresponding scheme is