Part A

Principles and background

Open Modellica.org → Download (LAB SW)
and copy from github (from Terminal)
(>code → copy) than from Terminal git clone (... link...)
→ create the folder $\underbrace{\text{"autom. of en syst"}}$

- modellica
  - > Licence
  - > readme.md
- .git don't touch!

DON'T touch ".git" file ... $\underbrace{\text{"git pull"}}_{\text{to update}}$ → to download update

(after » git install)

"OMEdit" launcher for Modellica → Aut of En syst
↓
package. mo

Libraries → "coursework" → models [used]

Modelling principles

- Modelling and control
- First-principle *versus* data-based modelling
- Balance equations
- An explanatory example
- Lessons learnt

Preliminary definitions

possible saturation to handle

to hand this
control

- Modulating control:
  - the controller outputs are real numbers (within given intervals);
  - the natural formalism is continuous- or discrete-time dynamic systems.
- Logic control:
  - the controller outputs are lexical variables (on/off, forward/stop/backward...);
  - the natural formalism is discrete-events dynamic systems.

Modelling and – or better, *for* – control (in energy systems)

A look at the encountered problems

on engineering POV

control

- Control problems are classically divided into process and motion ones.
- We shall now
  - first discuss the system-theoretical side of the above division,
  - then point out that the so introduced viewpoint on a control problem has consequences
    - on the characteristics that a model has to possess to be "good for control",
    - on the way a control scheme needs structuring,
    - and on the *rationale* to follow for tuning the blocks in that scheme.
  - and finally focus for this part of the course on models (the other items above shall be addressed later on).
- We shall also start introduce simple examples with an explanatory purpose.

(with equations to run)

Process *versus* motion control

Focus on modulating control — by abstraction from an example

- Process control — typical case, a chemical plant:
  - set points are seldom – and most often slowly – modified (if ever)
    — think e.g. of the desired temperature profile on a distillation column;
  - the main role of control is to reject disturbances, and these can be of large and unpredictable entity, sometimes up to exceeding short-term actuator capability (exceed actuator limits)
    — e.g., weather on the distillation column above; ↳ miss the set point because under actuate
    - in the long run if their effect is "averaged" (e.g., the quality of a product flow is averaged by filling tanks), ↳ all mixed ≈ averaging (temporary ok)
    - or also in the short run if this counts (e.g., when even a brief over-temperature may generate spurious species and compromise a reaction); No way back
  - hence, multiple time scales are almost ubiquitous ans span a wide band
    — e.g., seconds or less for flows, hours or days for temperatures; or days for start-up ↳ [s], [h] ... on huge reaction you can have $\tau \approx 2h$ (long transient)
  - somehow correspondingly, the control system tends to show a clear hierarchy and schemes composed of quite numerous blocks (Not overall but in a plant control)
    — e.g., a plant-wide MPC could optimise set points for plant units, where in turn local MPCs determine local set points for further low-level loops or structures, mostly based on simple blocks like PIDs.
    (Model Predictive control)

Process *versus* motion control

Focus on modulating control — by abstraction from an example

- Motion control — typical case, a machining center:
  - set points are modified continuously and in general rapidly — think e.g. of the desired trajectory and speed of a tool;
  - the main role of control is to track set points, and these are quite often known *a priori* — e.g., many operations are repetitive; → known traj to follow
  - disturbances are present, though in general quite well known/predictable, but controllers and actuators must recover as fast as required — e.g., variable payloads or material characteristics;

    no temporary violation of request!
  - there can be multiple time scales, but not much widespread (say ms to s) and in general not so dependent on the particular application — e.g., not as heterogeneous as reactor residence times; or generator τ

    like electro-mech syst
  - the control system hierarchy, as well as the individual schemes, is in general quite simple (not exactly the same for *logic* control, but details on this would stray from our scope).

    ↑ complex

    ↳ nested control for pos, vel, current

Process *versus* motion control

Focus on logic control

dispatching

You can change producer power, (rare) and slow

- Process control:
  - the system maintains an operating regime thanks to modulating control;
  - logic control takes care of startup, shutdown, emergencies, and little else.

different than maintain a regime

- Motion control:
  - the system carries out working sequences governed by logic control;
  - modulating control is mostly confined within machines to take care of positions, speeds and the like.

↳ most on machine

- Observing that of course the scheme we are discussing is a crude simplification, let us now move the focus to the context of energy systems.

↳ FOCUS!

Focus on energy systems

and on models as a consequence of problems

- In energy systems, "process-type" control problems tend to dominate...
- ...but nonetheless, from the abstracted system-theoretical viewpoint, those systems offer a mix of "process-type" and "motion-type" control problems, our domine as mix of these
- and most relevant, the balance is changing
  - as the traditional situation of a few large generators *versus* many small loads (we shall return on this) is becoming less general,
  - and as renewables are moving in with increasing importance;
- problems (thus models) are multi-physics — most typically hydraulic, thermal, and electrical (mechanics appears essentially in rotating masses); on same model all toghether
- distributed-parameter systems, hence Partial Differential Equations (PDEs) appear, but generally in one spatial coordinate — e.g., for piping.

seldom 3D model, with complex syst you should use higher Dim models (like on air-condition of a stadium) → 3D case

Preliminary definitions

when our model:

$\Downarrow$

- First-principle models:
  - rely on physical laws and/or empirical correlations;
  - natively continuous-time, possibly switching (the latter fact not of particular interest for us in this course);
  - discrete-time version of course possible; (discretization)
  - parameters have a direct physical interpretation;
  - can describe objects that do not yet exist; (without data) → you can build simulator of a future phisical obj $\rightarrow$ (VIRTUAL PROTOTYPE) $\Rightarrow$ {build on simulated word}
  - can be built with components validated individually prior to being assembled together;
  - the interfaces of the said components being suitable for attributing a physical meaning as well (pins, flanges,...); ← physical interfaces
  - may have to be complex if physics call for this.

    ↳ NON trivial model

Preliminary definitions

uses data, NOT physics knowledge

- Data-based models:
  - rely on reproducing measured data; (HIDA)
  - hence of course cannot model the not yet existing;
  - inherently discrete-time as so is data; (data collected on district)
  - structure either "partially suggested" by the phenomenon to describe (grey box)
  - or just chosen as the best to fit the data (black box);

    structure best fitting the data
  - parameters have in general no physical meaning, (ai, bi ... of model)
  - hard to make modular unless orientation or causality (what is input and what is output) is decided *a priori*,
  - which is not always possible if the same component needs connecting with others in different or even time-varying manners;
  - may be able of "summarising" complex physics with simple structures (on a per-case basis, however).

No general result

Foreword

We use I principle models

needing equations to build such models

- We need
  - mass, energy and momentum equations for thermo-hydraulic networks (for space reasons we only treat incompressible fluids with a single species); — approx! — we can deal with steam someway
  - energy equations for solid bodies such as walls; (room or pipe boundaries)
  - (semi)empirical equations for flow/pressure relationships and heat transfer;
  - equations for electric networks (phasor-based and reduced to a single phase since this simplification does not hinder explaining the required concepts); — multi-phase can be reduced to single one
  - energy equations for rotating masses (e.g., in alternator-based electric generators);
- Of course we deal with *dynamic* balance equations; ⟹
- we now proceed to a quick illustration by (main) application area, then discuss an example, and finally abstract the lessons learnt.

RECAP of all equations of balance!

Thermo-hydraulic networks

Mass equation

- The equation is written with reference to a *control volume*.
- Let M be the (incompressible, single species fluid) mass contained in the volume.
- Let w_i, $i = 1 \ldots n_m$, be the n_m mass flowrates exchanged by that volume with the external environment, considered positive if entering the volume.
- The equation then simply reads

$$\boxed{\frac{dM(t)}{dt} = \sum_{i=1}^{n_m} w_i(t)}$$

where t is obviously the (continuous) time.

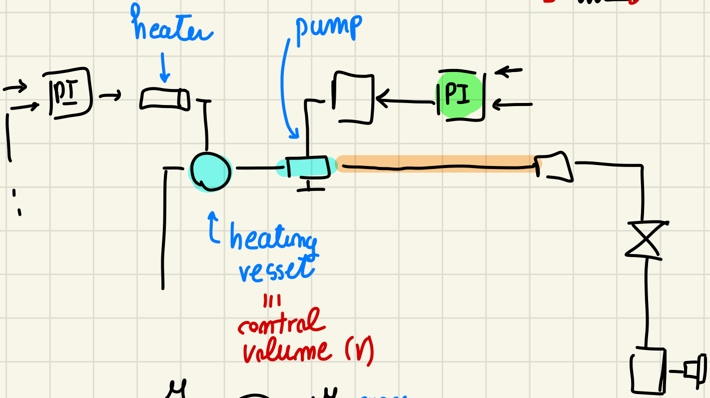
$\rightarrow$ (>0) V $\rightarrow$ (<0)

We are assembling:

our models are based on components and phisical connections

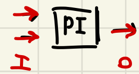
||

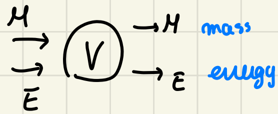
for example



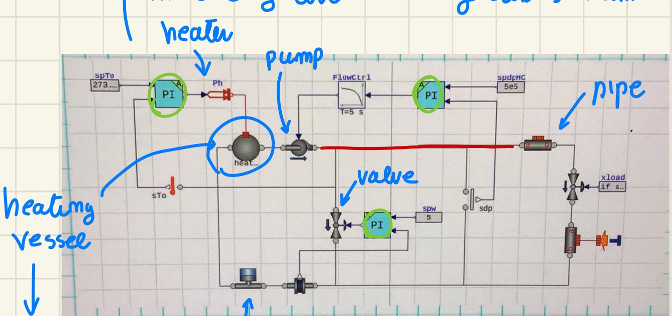
toghether with blocks with I/O connections

for example



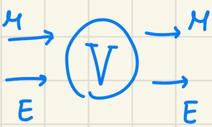


model of an heating sub-station



pressure expansion vessel

control volume



In / out

Mass Energy

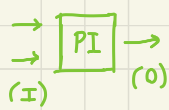
- phisical components
- phisical connection (not I/O)

for example

MIX the two usage

- I/O connections

I/O blocks



Thermo-hydraulic networks

Energy equation

- Also this equation is written with reference to a control volume containing a total energy E and where n_m mass flowrates w_i and n_h heat rates Q_j enter (if positive).
- The time derivative of E is the sum of the heat rates Q_j, not associated to any mass transfer, and of the energy contribution yielded by the mass flowrates.

NO Mass transf

- The latter contributions are of two *coexisting* types:
  1. (signed) heat transfers inherent to mass transfers, taking the form
(fluid brings in energy because its hot)
mass flowrate $\times$ fluid specific energy ($[kg/s] \times [J/kg] = [J/s] = [W]$)
  2. and work exerted by the entering fluid on that contained in the volume or *vice versa*, that in differential and specific form reads

introduce mechanical work on volume →

$$d\mathscr{L} = d(pv) = d(p/\rho)$$

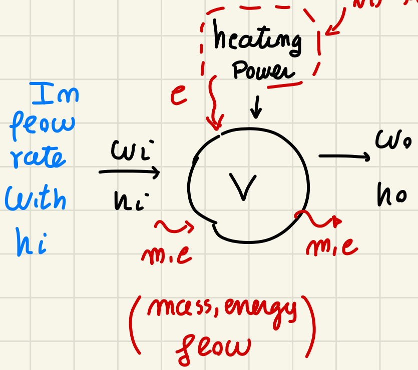
compression work

pushing work

where v is specific volume, ρ is density, and $d\mathscr{L} = pdv + vdp$; pdv is "compressing work" and vdp "impelling work" (both signed).

pdV : apply pressure, V↓ / vdp : volume and fluid passing requiring pushing, pressure heating Vessel (NO mass transfer)

NO MASS flow with that energy flow!



Out flow rate with h_o

Thermo-hydraulic networks

Energy equation

- Thus, the thermodynamic variable characterising the energy contribution of an entering/exiting mass flowrate to a volume, is the fluid specific enthalpy

  (entalpy) $\longrightarrow$ $$\boxed{h = \underbrace{e}_{\text{thermal}} + \underbrace{\frac{p}{\rho}}_{\text{work and heat}}}$$

  {quantity that the flow rate introduce in to the volume}

  work and heat ⇐ ($w_i \cdot h_i$ is what is introduced!)

  where e is the specific internal energy.

- The energy equation then takes the form

  $$\boxed{\frac{dE(t)}{dt} = \sum_{i=1}^{n_m} w_i(t)h_i(t) + \sum_{j=1}^{n_h} Q_j(t).}$$

  heat rates injected or lost on the volume towards ext env. without mass lost

- For incompressible fluids, at pressures and temperatures of interest for us, e largely dominates p/ρ; hence, *in these conditions* we can approximately take $h \overset{\sim}{=} e = cT$, where c is the fluid specific heat (assumed here constant) and T its temperature. (NOT true for ideal gas)

  $e >> p/\rho$

  $h \simeq e = cT$

  acceptable!

  on gas $h \simeq c_p \cdot T$

Thermo-hydraulic networks

Energy equation

all same fluid (no mixture!)

- Furthermore here we only deal with single-species fluids, thereby having always to do with a single specific heat c. no mixing fluid, only one c

so... ⇓

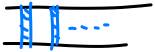
- Given all the above, for our purposes the energy equation for fluids is

$$\boxed{c\frac{dM(t)T(t)}{dt} = c\sum_{i=1}^{n_m} w_i(t)T_i(t) + \sum_{j=1}^{n_h} Q_j(t).}$$

where T is the control volume temperature, assumed uniform (hence adopting a finite-volume approach to distributed-parameter systems, though we do not further discuss this matter) and M the fluid mass.

finite volume approach, assume uniform quantities on each volume

pipe = seq. of volume



Thermo-hydraulic networks

Energy equation

heating vessel full of fluid always, than $M \simeq$ const

- Sometimes the mass is constant, like in a tube section always filled, whence

$$cM\frac{dT(t)}{dt} = c\sum_{i=1}^{n_m} w_i(t)T_i(t) + \sum_{j=1}^{n_h} Q_j(t).$$

in general cases

- In other cases, such as tanks, this is not true. It is then convenient to expand the derivative on the left hand side, and subtract the mass equation multiplied by cT; this provides (TRICK) from energy equation

subtract

$$\begin{array}{rcrcl}
cM(t)\frac{dT(t)}{dt} & + & \cancel{cT(t)\frac{dM(t)}{dt}} & = & c\sum_{i=1}^{n_m} w_i(t)T_i(t) + \sum_{j=1}^{n_h} Q_j(t) \\
 & - & \cancel{cT(t)}\frac{dM(t)}{dt} & = & -\, cT(t)\sum_{i=1}^{n_m} w_i(t) \\
\hline
cM(t)\frac{dT(t)}{dt} & & & = & c\sum_{i=1}^{n_m} w_i(t)\boxed{\left(T_i(t)-T(t)\right)} + \sum_{j=1}^{n_h} Q_j(t)
\end{array}$$

(mass equation) multiplied by $cT(t)$

flow rate ≥ 0

(simpler to solve)

that is sometimes called the "net energy" equation.

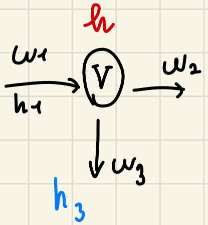
composed of one derivative less, so simpler to write!

depends on T size

flow rate depending on relative T if hotter or colder than local fluid

Notice... Flow Reversal can change causality, hence equation structure

suppose to have a certain Volume



other terms

$$\frac{dE}{dt} = (\dots)\; w_1 h_1 - w_2 h - W_3 h$$

But if W_3 changes sign it must be

↳ becoming negative

$$\frac{dE}{dt} = w_1 h_1 - w_2 h - W_3 h_3$$

↑ if the flow reverses, I'm not giving out the internal one but an external Input

We must deal with flow Reversal!

NET-energy equation

$$\boxed{c\, M(t) \frac{dT(t)}{dt} = c \sum_{i=1}^{n_m} w_i(t) \left(T_i(t) - T(t)\right) + \sum_{j=1}^{n_h} Q_j(t)}$$

Thermo-hydraulic networks

Momentum equation ("quantità di moto")

- This equation is used primarily for modelling tubes (valves are a somehow analogous case briefly treated later on).
- Consider a tube (element) and write that the time derivative of the fluid momentum is the sum of the forces acting on it, that is,
  - pressure forces at the two ends,
  - gravity force,
  - and friction force on the lateral surface,

  all projected onto the tube *abscissa* x $\Rightarrow$
- In fact, other components do not act on the fluid motion on the *prevailing dimension* x and merely result in constraint reaction forces, not relevant for the energy-related aspects on which we focus.

Thermo-hydraulic networks

Momentum equation

$W = \rho A u$ ← velocity (uniform)

flowrate (uniform) ↑ since incompressible fluid: Flow Rate is spatially uniform (NO STORAGE)

if $A\downarrow$, $u\uparrow$ to preserve w

- To keep complexity at a level compatible with the course, consider a tube with uniform section A (and recall that we only deal with incompressible fluids).

  ↓ no storage, all flow along tube axis

- This yields

  ↳ {mom. derivative} ⇒ constant! @ pressure output

  GRAVITY force

$$M\frac{du(t)}{dt} = Ap_i(t) - Ap_o(t) + Mg\sin(\alpha) - f_a(t)$$

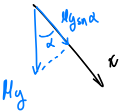
(friction forces)

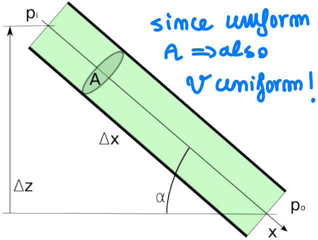
uniform section A, incompressible fluid!

only one vel. u

@input pressure, area

Σ Forces = d(momentum)/dt





since uniform A ⇒ also v uniform!

- Note that $\sin(\alpha) = \Delta z/\Delta x$, Δz being the initial altitude minus the final one
- while the friction force f_a, always contrasting motion, is

  model of fluid ⇓ interaction

  ↳

$$f_a = K_f A_\ell \rho u|u|,$$

{simple correlation}

opposite to velocity direction, proportional to u^2

where A_ℓ is the lateral surface.

↳ Friction coeff depending on material (tabulated) physical parameter!

Thermo-hydraulic networks

Momentum equation

- The quantity K_f is called *friction coefficient*, depends on the fluid/wall contact characteristics, and is tabulated for most cases of interest based on experiments and empirical correlations.
- The equation obviously contains an inertia term, that in our models can however be omitted. → omitted
- This is possible because hydraulic phenomena are much faster than thermal ones, which are our main subject. hydraulic ≃ sound speed << thermal ≃ fluid speed (few m/s)
- In other words, since thermal variables (such as temperatures) propagate at the fluid speed while hydraulic ones (such as pressures and flowrates) propagate at the speed of sound in the fluid, we can safely assume that for our purposes "hydraulics is always at steady state". $d/dt \simeq 0$ → momentum derivative on time almost 0
- Thus, we can write the momentum equation as the algebraic one

[steady state]

$$A\left(p_i(t)-p_o(t)\right)+Mg\frac{\Delta z}{\Delta x}-K_f A_\ell \rho u(t)|u(t)|=0.$$

Thermo-hydraulic networks

Momentum equation

- Summing up, denoting by A the tube (uniform) section, by L its length, by ω its internal perimeter, simplifying the notation a bit and recalling that $w = \rho A u$, we have

$$A(p_i - p_o) + \rho A L g \frac{\Delta z}{L} - K_f \omega L \rho u|u| = 0$$

$$\Downarrow$$

$$p_i - p_o = K_f \frac{\omega L}{\rho A^3} w|w| - \rho g \Delta z.$$

no flow reversal usually on energy syst

- In addition, if the tube is installed in such a way that w has always the same sign, taken positive when going from the higher- to the lower-pressure end, we can write

(simplify model)

$$p_i - p_o = \underbrace{K_f \frac{\omega L}{\rho A^3}}_{K_T} w^2 - \rho g \Delta z,$$

that we shall often further summarise as

$$\boxed{p_i - p_o = \frac{K_T}{\rho} w^2 - \rho g \Delta z.}$$

{equation used}

Thermo-hydraulic networks

Momentum equation

- The momentum equation can also represent the typical valve behaviour.
- This means neglecting more than one phenomenon, that however is of interest only for sizing, or operating conditions not advised for “good” plant management—i.e., not of interest here.
- From our point of view, consider a valve like a variable-section short tube installed so that the flow does not reverse. Thus, take

$$p_i - p_o = \frac{K_T}{\rho} w^2 - \rho g \Delta z,$$

small valve, irrelevant elevation

set $\Delta z = 0$ (short component, hardly any gravity effect) and rewrite as

$$w = C_v \Phi(x) \sqrt{\rho (p_i - p_o)}$$

role of $\Phi(x)$

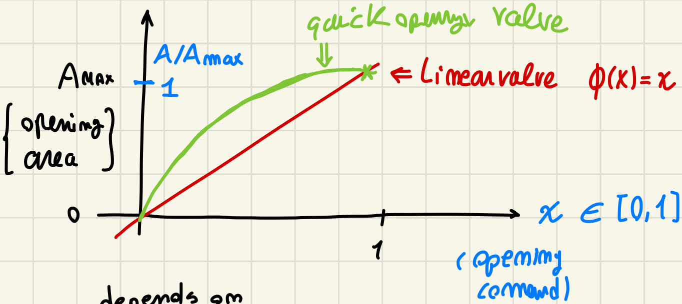
where C_v is the *flow coefficient*, $x \in [0,1]$ the command, and $\Phi(x), \Phi(0) = 0, \Phi(1) = 1$ the *opening* or *intrinsic characteristic*.

$\phi(x)$ Role:

[adimensional]

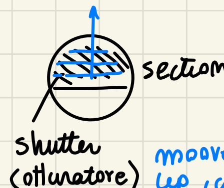
$\phi(x)$

$$\begin{cases} \phi(0) = 0 \\ \phi(1) = 1 \end{cases}$$

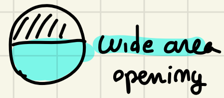


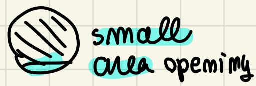
depends on
geom constr of the valve

+ other $\phi(x)$ characteristics



moove up create a varying area ΔA





Solid bodies

Energy equation

c ≃ uniform

- In this case we naturally neglect any volume effect, and consider the specific heat spatially uniform (non-homogeneous walls will be treated with at least one equation per material layer). wall as seq of layers uniform
- Since there is no mass transfer, the equation simply reads

$$\boxed{cM\frac{dT(t)}{dt} = \sum_{j=1}^{n_h} Q_j(t).}$$

where symbols have the same meaning as in the fluid case, and M is of course constant.

Heat transfer equations

Foreword

- These are algebraic equations, as they describe no storage.
- We need to model
  - conduction within solids (and sometimes fluids),
  - convections between a solid and a fluid,
  - and radiation.
- In all cases we shall adopt simplified concentrated-parameter descriptions right from the beginning.

Heat transfer equations

Conduction

- We shall only use a simplified planar descriptions, as more detailed ones would stray from our scope.
  useless
- The heat rate from the a to the b side of a solid layer is

$$Q_{ab} = G(T_a(t) - T_b(t))$$

  where T_a and T_b are the side temperatures.
- The thermal conductance G is

$$G = \lambda \frac{A}{s}$$

  where λ is the material's thermal conductivity, A the layer surface, and s its thickness. of material

Heat transfer equations

Convection

- The convective heat rate from a solid wall (subscript w) to a fluid (subscript f) is

$$\boxed{Q_{wf} = \gamma A(T_w(t) - T_f(t))}$$

related to

where T_w and T_f are the wall and a fluid "bulk" temperature, while A is the contact surface.

↳ because usually there are layers of exchange effect, NO model

- The thermal exchange coefficient γ can be considered constant (as we shall almost always do) or made dependent on the fluid and motion conditions, typically with relationships involving the Reynolds (for forced convection) or Grashof (for natural one), Nusselt and Prandtl numbers.

Heat transfer equations

Convection

- A common, somehow intermediate refinement is to have γ just depend on the fluid velocity tangent to the wall.
- To this end, taking a reference heat exchange coefficient value γ_0 as corresponding to a reference velocity u_0, a widely used relationship is

$$\boxed{\gamma(t) = \gamma_0 \left(\frac{u(t)}{u_0}\right)^{0.8}}$$

where $u(t)$ is the fluid velocity.

Heat transfer equations

Radiation

↘ different phenomenon

{ for convention/conduction I need a counter part body exchanging heat, some contact ↘ radiation is different, just depends on Temp)

- At a simplified level, the radiative heat transfer from a body a to a body b depends on the difference of their absolute (Kelvin) temperatures to the fourth power, i.e.,

(BOLTZMAN theory, if high T you speed up photons!)

↳ RADIATIONS

$$Q_{ab} = K\left(T_a^4 - T_b^4\right).$$

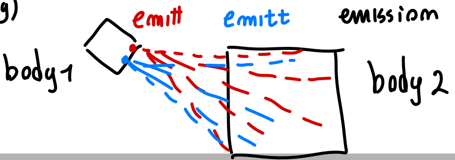
⇐ tables of emission typical situation empirically

- The radiative heat transfer coefficient K depends on several things, including the bodies' emissivity and their view factors.
- However in this course the only relevant case will be that of solar radiation, that can be very naturally viewed as a prescribed power flux $[W/m^2]$. $\simeq K\left(T_{sun}^4 - \cancel{T_{body}^4}\right)$

(practically impressed)

Tbody <<< Tsun

emission as integrals... surface



Electric networks

Foreword

⇓

- In this course we shall deal essentially with AC power networks.
- The matter is vast, and simplifications are introduced so as to transmit the necessary concept with the minimum complexity sufficient to explain them.
- In detail (and somehow anticipating) we shall assume
  - a single-frequency synchronous network (quite reasonable if frequency is well controlled, and we do not have the time to deal with the connected "stability" problems), (automation layer) → phasor
  - all generators described by a constant voltage behind their internal reactance,
  - linear behaviour of transmission lines, (impedence)
  - no transformers (we only spend some words on reactive power control) as doing so significantly reduces computations,
  - a one-phase (or equivalently, a *balanced* multiphase) system.
- We shall thus adopt a *phasor-based* modelling approach.

Electric networks

Phasors

- Any quantity varying (co)sinusoidally with constant frequency ω can be represented as

$$A\cos(\omega t+\theta)=\Re\left(Ae^{j(\omega t+\theta)}\right)=\Re\left(\overset{\text{(phasor)}}{Ae^{j\theta}}\,e^{j\omega t}\right)$$

  where j is the imaginary unit, $e^{j\omega t}$ yields time dependence, and the phasor $Ae^{j\theta}$ magnitude and phase with respect to a convenient reference.
- This allows for a phasor arithmetic to handle AC networks with frequency "hidden"...
- ...in the term $e^{j\omega t}$ and in the value of frequency-dependent impedances.
- Recall – for the last time – that phasor analysis is for synchronous networks with constant frequency (whence its alternative name as "static analysis").

Electric networks

Basic equations (essentially to agree notation...)

- Ohm's law $\underline{V} = \underline{Z}\underline{I}$ or $\underline{I} = \underline{Y}\underline{V}$, where $\underline{V}, \underline{I}$ are voltage and current (phasors) and $\underline{Z}, \underline{Y}$ the complex impedance and admittance, respectively (underline indicates complex numbers). We shall typically express $\underline{Z}$ as $R + jX$ $R, X \geq 0$, and $\underline{Y}$ as $G - jB$ ($G \geq 0$, and mind the minus to have $B \geq 0$), where R, X, G, B are respectively called resistance $[\Omega]$, reactance $[\Omega]$, conductance $[S]$, and susceptance $[S]$.
- Kirchoff's laws (nothing to say here).
- Power (* denotes the complex conjugate):

(do NOT confuse it!)

  - complex $\quad \underline{S} = \underline{V}_{RMS}\underline{I}^*_{RMS} = P + jQ = Ae^{j\phi}$
  - apparent $\quad A = |\underline{S}| = V_{RMS}I_{RMS} = V_{max}I_{max}/2 \quad [VA]$, ← magnitude of complex one
  - active $\quad P = \Re(\underline{S}) = V_{RMS}I_{RMS}\cos\phi \quad [W]$,
  - reactive $\quad Q = \Im(\underline{S}) = V_{RMS}I_{RMS}\sin\phi \quad [VAR]$,

  $\cos\phi$ power factor.

For a sinusoidal signal F, $F_{RMS} = F_{max}/\sqrt{2}$.

Energy equation for rotating masses

(it was the only energy storage)

- Many electric generators contain rotating masses, like turbine and alternator rotors.
- Their angular velocity affects the generated frequency (to be controlled).
- The energy equation states that the time derivative of the kinetic energy equals the algebraic sum of powers, i.e.,

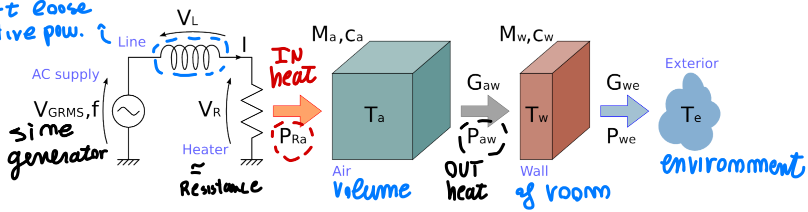
$$\boxed{\frac{d}{dt}\left(\frac{1}{2}J\omega_r^2\right) = P_m - P_e}$$

where J is the inertia, ω_r the angular velocity, P_m the mechanical power applied to the shaft (positive if entering the considered machine) and P_e the *active* electric power (positive if generated).

Our first multi-physics model

Example on MODELLICA

- Crude simplification of an electrically heated room:

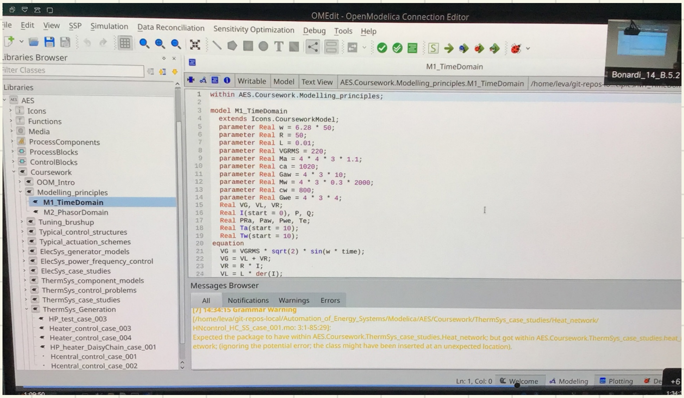


- Data:
  - room dimensions $4m \times 4m \times 3m$ height;
  - only one side wall (30cm thick) exchanges heat, the others are adiabatic;
    ↳ same T room on other side
  - no openings, external temperature $5°C$;
  - no heater losses (all of its power is released to air);
  - air density $1.1 kg/m^3$, specific heat $1020 J/kg°C$;
  - wall density $2000 kg/m^3$, specific heat $800 J/kg°C$; ≃ concrete
  - air–wall heat exchange coefficient $10 W/m^2°C$;
  - wall–exterior heat exchange coefficient $4 W/m^2°C$; ← convection more efficent inside
  - AC supply voltage $220V\ RMS$, $f = 50Hz$, $R = 50\Omega$, $L = 10mH$.

(SI UNIT)

model defined on Modelica

on modelling Libraries → coursework > M1_time Domain
M2_Phasor domain

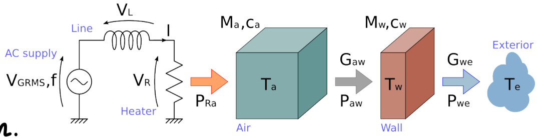


on "Text view"

First model (M1)

with the electric part described in the time domain

Time domain descr.



$V_G(t) = V_{GRMS}\sqrt{2}\sin(2\pi f t)$	Source generator
$V_R(t) = RI(t)$	Heater resistor voltage
$V_L(t) = LdI(t)/dt$	Line inductor Voltage
$V_R(t) + V_L(t) = V_G(t)$	KVL (VOLT LAW)
$P(t) = V_R(t)I(t)$	Active power formula
$Q(t) = V_L(t)I(t)$	Reactive power (addendum)
$P_{Ra}(t) = P(t)$	All *active* power to air
$M_a c_a dT_a(t)/dt = P_{Ra}(t) - P_{aw}(t)$	Air energy balance
$M_w c_w dT_w(t)/dt = P_{aw}(t) - P_{we}(t)$	Wall energy balance
$P_{aw}(t) = \gamma_{aw} A_w (T_a(t) - T_w(t))$	Air–wall heat transfer
$P_{we}(t) = \gamma_{we} A_w (T_w(t) - T_e(t))$	Wall–exterior heat transfer
$T_e(t) =$	Exogenous temperature

EQUATIONS, NOT alghoritms, random order

electrical part provides Boundary condition between air - elect. part of syst

air room eq

Note the partition into electric and thermal equations; each has its boundary conditions, and one such condition is presented by the former to the latter set.

Second model (M2)

with the electric part described in the phasor domain

- First let us compute P with Maxima by issuing the commands

```
IRMS : VGRMS/(R+%i*w*L);
S    : VGRMS*conjugate(IRMS);
P    : realpart(S);
```

- This yields

$$P = \frac{RV_{GRMS}^2}{R^2 + (\omega L)^2},$$

where obviously $\omega = 2\pi f$, hence M2 is

$P(t) = RV_{GRMS}^2/(R^2+(\omega L)^2)$	Active power
$M_a c_a \dot{T}_a(t) = P_{Ra}(t) - P_{aw}(t)$	Air energy balance
$M_w c_w \dot{T}_w(t) = P_{aw}(t) - P_{we}(t)$	Wall energy balance
$P_{Ra}(t) = P(t)$	All *active* power to air
$P_{aw}(t) = G_{aw}A_w(T_a(t) - T_w(t))$	Air–wall heat transfer
$P_{we}(t) = G_{we}A_w(T_w(t) - T_e(t))$	Wall–exterior heat transfer
$T_e(t)$	Exogenous temperature

Note: from now on we shall often use a dot to indicate time derivatives.

Time scales for the thermal part

↓ from phasor domain

- Compute the transfer function $G(s)$ from P_{Ra} to T_a: (linear relation)

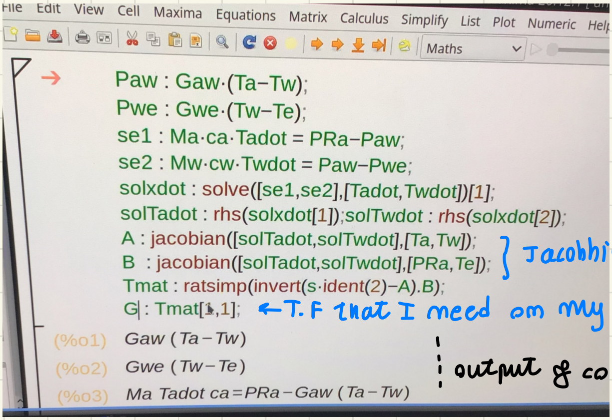
```
Paw      : Gaw*(Ta-Tw);
Pwe      : Gwe*(Tw-Te);
se1      : Ma*ca*Tadot = PRa-Paw;
se2      : Mw*cw*Twdot = Paw-Pwe;
solxdot  : solve([se1,se2],[Tadot,Twdot])[1];
solTadot : rhs(solxdot[1]);
solTwdot : rhs(solxdot[2]);
A        : jacobian([solTadot,solTwdot],[Ta,Tw]);
B        : jacobian([solTadot,solTwdot],[PRa,Te]);
Tmat     : ratsimp(invert(s*ident(2)-A).B);
G        : Tmat[1,1];
```

paste this commands on Maxima, it compute the TF

- Result:

$$G(s) := \frac{T_a(s)}{P_{Ra}(s)} = \frac{c_w M_w s + G_{we} + G_{aw}}{c_a c_w M_a M_w s^2 + (c_w G_{aw} M_w + c_a (G_{we} + G_{aw}) M_a) s + G_{aw} G_{we}}$$

simulate system by Modellica
(with defferent model kind) → [deffrent times requared for complete simulation]



model pasted on Maxima

":" defenition of relation

} Jacobbian Matrix, A, B

← T.F that I need on my relation

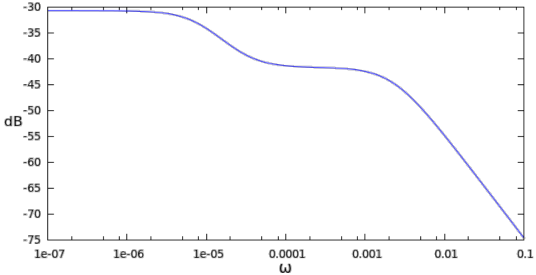
output of computation

Time scales for the thermal part

- Put numbers in and plot the Bode magnitude diagram:

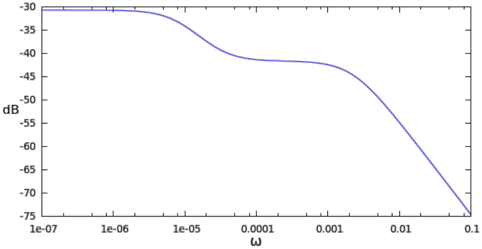
```
load("bode");
Gnum(s)  := ev(subst([Ma=4*4*3*1.1,ca=1020,Gaw=4*3*10,
                      Mw=4*3*0.3*2000,cw=800,Gwe=4*3*4],G));
bode_gain(Gnum(s),[w,1e-7,1e-1]);
```

- Result:



Time scales for the thermal part

- Interpretation:



Two time scales:

- one ("fast") relative to the energy storage in the air – order of magnitude, $1000\,s$,
- the other ("slow") relative to wall storage – order of magnitude, some $10^5\,s$, i.e., some days.

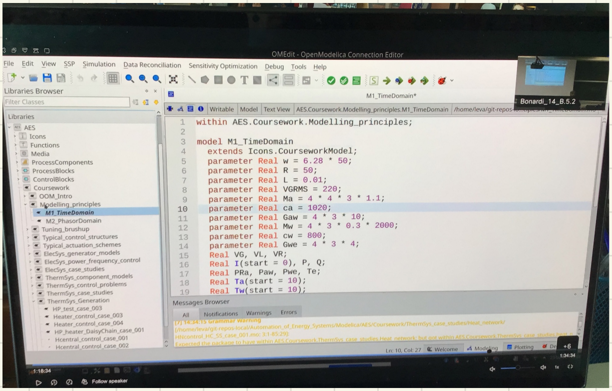
- Hence, electric phenomena (time scale dominated by $L/R = 0.2\,ms$) are about 10^5 times faster than the faster thermal phenomena, which are in turn about 10^4 times faster than the slower thermal ones.

Modelica implementation of M1 and M2

```
model M1
  parameter Real w=6.28*50;
  parameter Real R=50;
  parameter Real L=0.01;
  parameter Real VGRMS=220;
  Real VG,VL,VR;
  Real I(start=0),P,Q;
  parameter Real Ma=4*4*3*1.1;
  parameter Real ca=1020;
  parameter Real Gaw=4*3*10;
  parameter Real Mw=4*3*0.3*2000;
  parameter Real cw=800;
  parameter Real Gwe=4*3*4;
  Real PRa,Paw,Pwe,Te;
  Real Ta(start=10);
  Real Tw(start=10);
equation
  VG             = VGRMS*sqrt(2)*sin(w*time);
  VG             = VL+VR;
  VR             = R*I;
  VL             = L*der(I);
  P              = VR*I;
  Q              = VL*I;
  Ma*ca*der(Ta) = PRa-Paw;
  Mw*cw*der(Tw) = Paw-Pwe;
  PRa            = P;
  Paw            = Gaw*(Ta-Tw);
  Pwe            = Gwe*(Tw-Te);
  Te             = 5;
end M1;
```

```
model M2
  parameter Real w=6.28*50;
  parameter Real R=50;
  parameter Real L=0.01;
  parameter Real VGRMS=220;
  parameter Real Ma=4*4*3*1.1;
  parameter Real ca=1020;
  parameter Real Gaw=4*3*10;
  parameter Real Mw=4*3*0.3*2000;
  parameter Real cw=800;
  parameter Real Gwe=4*3*4;
  Real P,PRa,Paw,Pwe,Te;
  Real Ta(start=10);
  Real Tw(start=10);
equation
  P              = R*VGRMS^2/(R^2+w^2*L^2);
  Ma*ca*der(Ta) = PRa-Paw;
  Mw*cw*der(Tw) = Paw-Pwe;
  PRa            = P;
  Paw            = Gaw*(Ta-Tw);
  Pwe            = Gwe*(Tw-Te);
  Te             = 5;
end M2;
```

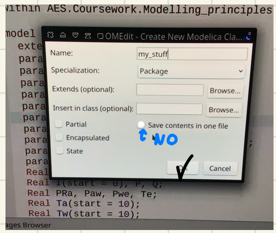
Modellica



to create a new model

File > New > New modellica class

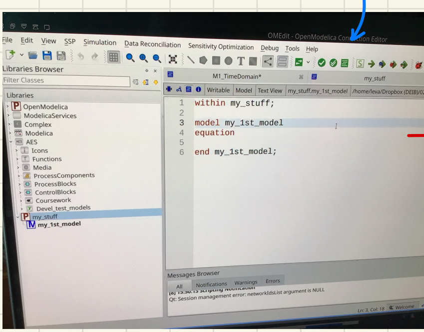
than save it from here on a certain folder...



and package.mo created on > my-stuff folder

↳ here on my-stuff > create > new modellica class "Model"

created on that directory



starting point

(between row 3,4 :)

parameter Real a=1;
parameter Real b=0.5;
parameter Real xstart=1;

Real x(start=xstart, fixed=true);
Real u;

equation

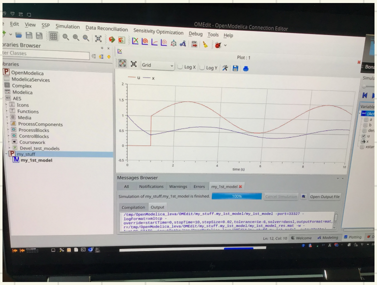
der(x)= a*x+b*u;

u = if time<1 then 0 else 1+0.5 sin(time-than save > check model ✓

and simulate

$\boxed{S}$ selecty the seconds of simulation for

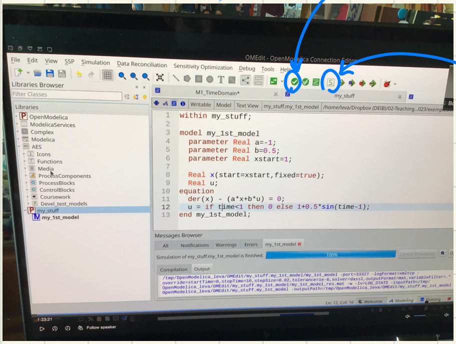
u, x on a plot



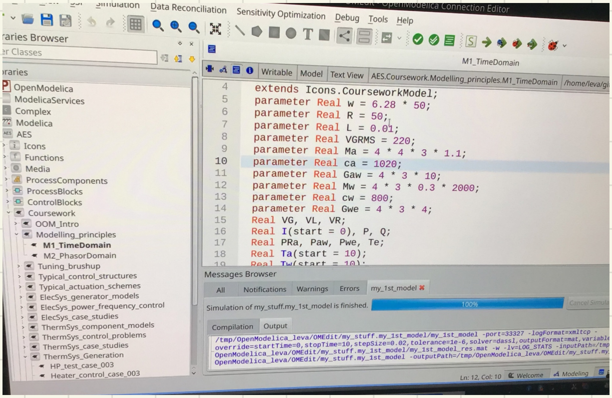
example

check model

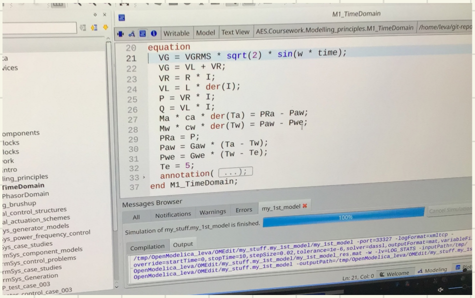
simulation



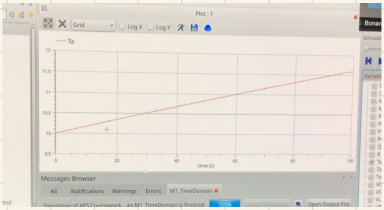
IN OUR MODEL



same equation as written on paper...



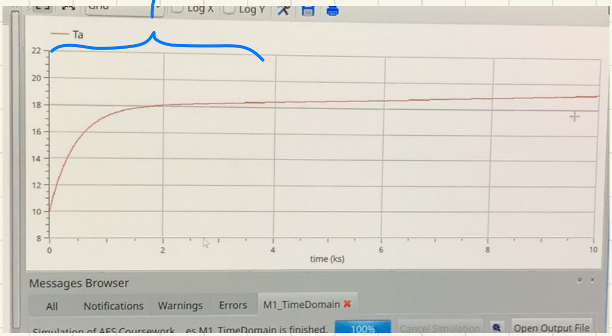
simulating 100 s showing only Ta(t)



→ it took 0.451735 s in total for simulate

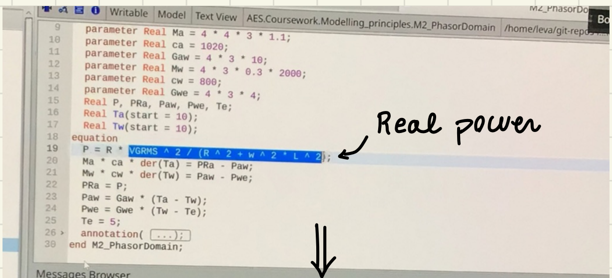
If simulating 10K seconds

transient



↖ on previous domain in time I also simulate the sim waves $i(t)$ useless signal!

- using instead phasor domain model



```
parameter Real Ma = 4 * 4 * 3 * 1.1;
parameter Real ca = 1020;
parameter Real Gaw = 4 * 3 * 10;
parameter Real Mw = 4 * 3 * 0.3 * 2000;
parameter Real cw = 800;
parameter Real Gwe = 4 * 3 * 4;
Real P, PRa, Paw, Pwe, Te;
Real Ta(start = 10);
Real Tw(start = 10);
equation
  P = R * VGRMS ^ 2 / (R ^ 2 + w ^ 2 * L ^ 2);
  Ma * ca * der(Ta) = PRa - Paw;
  Mw * cw * der(Tw) = Paw - Pwe;
  PRa = P;
  Paw = Gaw * (Ta - Tw);
  Pwe = Gwe * (Tw - Te);
  Te = 5;
  annotation( ... );
end M2_PhasorDomain;
```

Real power

here I don't compute $i(t)$ current signal

⇓

instead simulating here 10Ks its faster!
with exactly same result
tuned complexity of model

(FASTER simulation)

⇙

different speed simulation

Simulation of M1 and M2

for comparison (OpenModelica 1.17.0 on Ubuntu 18.04, i7-3520M CPU @2.90GHz, 16GB RAM)

- CPU time for simulating two hours (DASSL variable-step solver):

Model M1	11.79	s
Model M2	3.95	ms

- Average errors on air temperature and heating power:

$$\bar{e}_{Ta} = \frac{1}{7200}\int_0^{7200}\left(T_{a,M1}(t) - T_{a,M2}(t)\right)dt = 2.5\cdot 10^{-4}\,{}^{\circ}C$$

$$\bar{e}_{PRa} = \frac{1}{7200}\int_0^{7200}\left(P_{Ra,M1}(t) - P_{Ra,M2}(t)\right)dt = 2.8\cdot 10^{-2}\,W$$

- With an average power of approximately 1 kW, *from the thermal viewpoint* the two models are practically equivalent; however, thanks to not representing the electric time scale in detail, model M2 simulates almost 3000 times faster than M1.

!

Model analysis for control

through electrical part the reaction time constant as $\tau = L/R$ is [ms] for Temp

fast relation, phasor approach, istant reaction, electrical part has ≃ ms τ

With phasor approach you assume istantaneus reaction of power

- The relationship between heater command (control signal) and heater power is very fast, hence from the thermal viewpoint algebraic.
- The transfer function from heater command to air temperature (controlled variable) can be written as

$$\boxed{G(s) = \mu \frac{1 + s\tau_z}{(1 + s\tau_1)(1 + s\tau_2)}.}$$

different τ

- Let us see a less extreme example of spread dynamics than the room case, namely $\underbrace{\mu = 1,\ \tau_1 = 100,\ \tau_2 = 1,\ \tau_z = 10.}_{\text{data suppose}}$ ⇒ model (*)
- Question: which is the "best" 1st order approximation in this case for setting up a controller?

Model analysis for control

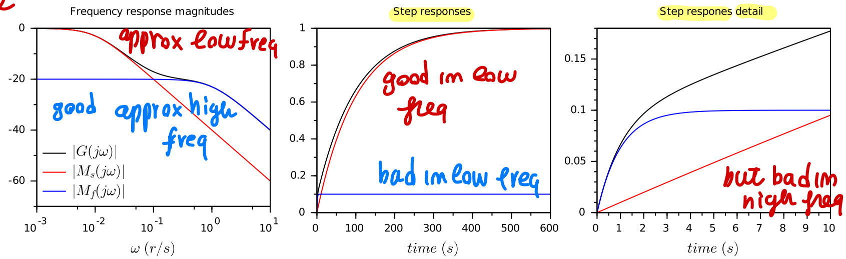
- Answer: it depends on which dynamics you want to control (in the room case, air or walls).
- Consider two 1st order models – $M_s(s)$ and $M_f(s)$ to name them – focusing on the slow and the fast dynamics respectively in $G(s)$, i.e.,

(*)

$$M_s(s) = \frac{1}{1+100s}, \quad M_f(s) = \frac{0.1}{1+s}.$$

- Observe the Bode diagrams and the step responses of $G(s)$, $M_s(s)$ and $M_f(s)$:

Themal part



Model analysis for control

- Answer: it depends on which dynamics you want to control (in the room case, air or walls).
- Consider two 1st order models – $M_s(s)$ and $M_f(s)$ to name them – focusing on the slow and the fast dynamics respectively in $G(s)$, i.e.,

(elect part soo fast that can be neglected)

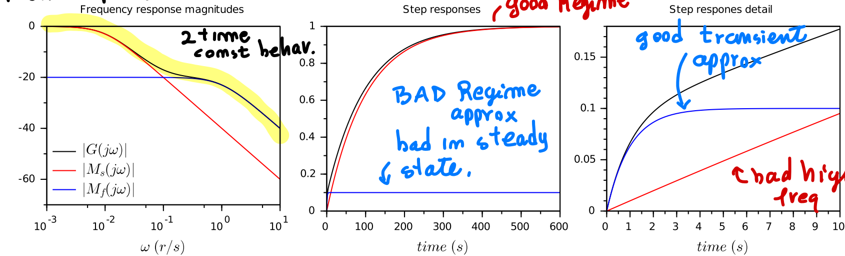
$$M_s(s) = \frac{1}{1+100s}, \quad M_f(s) = \frac{0.1}{1+s}.$$

- Observe the Bode diagrams and the step responses of $G(s)$, $M_s(s)$ and $M_f(s)$:

I can approx in 2 ways...

True model

Thermal part



The CONTROLLER is focused on $\left\{\begin{array}{l}\text{air}\\ \text{wall}\end{array}\right.$ dynamic ?

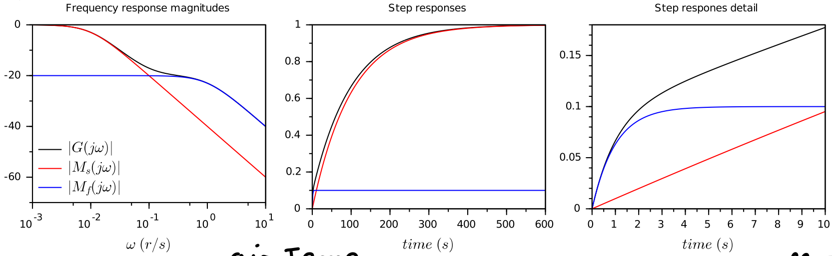
$\Downarrow$

Model analysis for control

⇐ depending on interested dynamic I choose which model to use

the model to use depends on controller focus → air/walls dynamic?

bad open loop approx

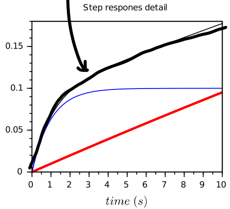


- Interpretation: a $\underbrace{\text{“fast” state variable}}_{\text{air Temp.}}$ moving relatively to a $\underbrace{\text{“slow” one}}_{\text{wall Temp.}}$.
- Model $M_f(s)$ is apparently unfit to reproduce the step response of $G(s)$...
- ...but what if you want a controller for a critical frequency in the vicinity of the fast dynamics?

Model analysis for control

it's a I ORD response with slowly moving base line (≃ constant) (tune controller on dynamics)

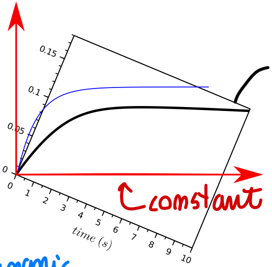
mooving respect



{transform Reference}

Rotate red & black, copy blue

suppose to rotate and copy



its like the blue mooving respect red one

constant pratically

tune controller on blue dynamic

- Further interpretation: at the required time scale I have to govern how T_a moves w.r.t. T_w — at that scale, T_w is not moving.
- In this respect, what does the figure above INTUITIVELY suggest?
- Exercise: set up a controller for $G(s)$ aiming at a closed-loop settling time around 10s, using the two approximated models, and comment.

→ which provide better result

- We shall come back to this matter when discussing tuning policies.

Lessons learnt

NOT electrical ones
if I'm interested on Thermal

good model
fast to
simulate

- Models must represent the phenomena of interest, and be efficient.
- Models must catch the control-relevant dynamics and time scales.
- Consequences:
  - one has invariantly to use sets of models (and data), or said otherwise, Model and Data Bases; → to deal with complex design
  - one must keep such knowledge bases consistent — research topic here, volunteers? 🤓
  - no control engineering culture without modelling culture.