Exercises session 6: LQ control, KF, LQG, loop transfer recovery

Ex. 1: Given the continuous time system

$$\begin{cases} \dot{x}(t) = -x(t) + u(t) \\ y(t) = x(t) \end{cases} \tag{1}$$

and the Riccati Differential Equation

$$\dot{P}(t) + A^T P(t) + Q - P(t)BR^{-1}B^T P(t) + P(t)A = 0 \tag{2}$$

1. Find the LQ_∞ control law with $Q = 1$, $R = 1$.
2. Find the corresponding closed-loop poles, the closed loop T.F., the maximum gain variation and evaluate the phase margin.
3. Design a steady-state Kalman Filter with $\tilde{Q} = \rho^2$, $\tilde{R} = 1$.
4. Compute the overall LQG regulator T.F.
5. Show how to apply the loop transfer recovery procedure (LTR).

Ex. 2: Given the system

$$\dot{x}(t) = 0.5x(t) + u(t) \tag{3}$$

1. Find the LQ_∞ control law with $Q = 1$, $R = 1$.
2. Find the corresponding closed-loop poles.
3. Given $u(t) = -\rho K_{LQ} x(t)$, find the set of ρ for which the closed loop system is A.S.
4. Find the phase margin.
5. Which is the maximum time-delay that allows to maintain the asymptotic stability?
6. Enforce a closed loop pole faster than $s = -2$

Ex. 3: Given discrete time system

$$\begin{cases} x(k+1) = -x(k) + u(k) + \nu_x(k) \\ y(k) = x(k) + v_y(k) \end{cases} \tag{4}$$

where $\nu_x \sim WGN(0, \tilde{Q})$, $\nu_y \sim WGN(0, \tilde{R})$

1. Find the LQ_∞ control law with $Q = 1$, $R = 2$.
2. Compute the maximum gain variation allowed by the LQ_∞.
3. Design a Kalman Filter using $\tilde{Q} = 2$, $\tilde{R} = 1.5$.
4. Compute the regulator T.F.
5. Compute the closed loop poles.

The steady-steate Riccati equation for discrete time systems is

$$\bar{P} = Q + A^T \bar{P} A - A^T \bar{P} B (R + B^T \bar{P} B)^{-1} B^T \bar{P} A \tag{5}$$

Ex. 4: Given the system

$$\begin{cases} \dot{x}_1 = u \\ \dot{x}_2 = x_1 \\ y = x_2 \end{cases} \tag{6}$$

1. Design a LQ_∞ control law with
$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$
and $R = 1$.
2. Compute the closed loop poles.
3. Design a Kalman Filter and apply the LTR procedure.