Advanced and Multivariable Control

7/2011

Exercise 1

Design a control law with the backstepping method for the system

$$\dot{x}_1(t) = -x_1^3(t) + x_2(t)$$
$$\dot{x}_2(t) = x_2^2(t) + u(t)$$

General formula for the backstepping in its "basic" version

$$u(t) = -\frac{dV_1(x_1)}{dx_1} g_1(x_1) - k(x_2 - \phi(x_1)) + \frac{d\phi_1(x_1)}{dx_1}(f_1(x_1) + g_1(x_1)x_2)$$

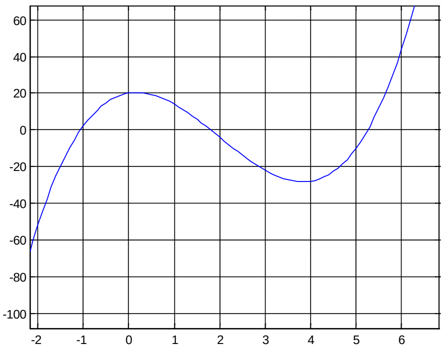
Exercise 2

Consider the first order system

$\dot{x}(t) = f(x(t))$

characterized by the function *x – dx/dt* is shown in the following picture:

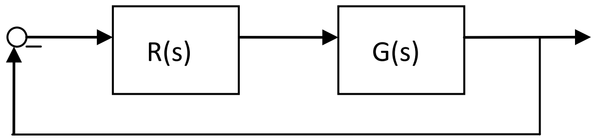
- Identify from the picture the equilibrium points;
- For each equilibrium, check its stability and, if stable, the corresponding region of attraction.



Exercise 3

Consider the feedback system shown in the following picture, where the process has two inputs and three outputs and is described by the transfer function matrix

$$G(s) = \begin{bmatrix} \dfrac{1}{s^2+3s+2} & \dfrac{-1}{s^2+3s+2} \\ \dfrac{s^2+s-4}{s^2+3s+2} & \dfrac{2s^2-s-8}{s^2+3s+2} \\ \dfrac{s-2}{s+1} & \dfrac{2(s-2)}{s+1} \end{bmatrix}$$



a. Compute the poles and the invariant zeros of *G(s)*;
b. Describe the characteristics that R(s) must not have in order to study the stability of the feedback system by just looking at the sensitivity function;
c. Is it possible to design a regulator *R(s)* such as the steady-state error $e = \begin{bmatrix} y_1^o - y^1 \\ y_2^o - y^2 \\ y_3^o - y^3 \end{bmatrix}$.

   Is asymptotically null for constant reference signals?

Exercise 4

Consider the discrete time system

$x(k+1) = 4x(k) + u(k)$

and the performance index

$$J = \sum_{i=k}^{k+N-1} x'(i)Qx(i) + u'(i)Ru(i) + x'(k+N)Sx(k+N)$$

with *Q=1, R=1, S=0*:

- Compute the LQ control with *N=∞*,
- Compute the minimum value *N* such that the RH control law stabilizes the system.

Riccati equation for LQ del control

$$P_{k-1} = A'P_k A + Q - A'P_k B(R + B'P_k B)^{-1} B'P_k A$$

Exercise 5

Consider the discrete time system

$$\begin{aligned} x_1(k+1) &= -x_1^3(k) + x_2(k) + v_{x1}(k) \\ x_2(k+1) &= x_2^2(k) + u(t) + v_{x2}(k) \\ y(k) &= x_1(k)x_2(k) + +v_y(k) \end{aligned}$$

where v_{x1}, v_{x2}, v_y are white noises with zero mean and unit variance.

1. Show the structure of the extended Kalman predictor and how it can be used on-line (describe the sequence of computations required at any time step);
2. Show the matrices to be used in the computation of the gain of the predictor;
3. Describe the difference between the extended and linearized Kalman predictor.

Exercise 6

Describe the methods for model order reduction.