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$\boxed{\text{Ex 1}}$

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)\, x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)\, u \end{cases}$$

$$f_1 = -x_1^3, \quad g_1 = 1, \quad f_2 = x_2^2, \quad g_2 = 1$$

$$u = \frac{1}{g_2}\left(u_a - f_2\right) = \left(u_a - x_2^2\right)$$

The new system is

$$\begin{cases} \dot{x}_1 = -x_1^3 + x_2 \\ \dot{x}_2 = u_a \end{cases}$$

Consider

$$x_2 = \phi(x_1) = -k_1 x_1$$

So that

$$\dot{x}_1 = -x_1^3 - k_1 x_1 \quad , \quad k_1 > 0$$

$$V_1(x_1) = \frac{1}{2} x_1^2, \quad \dot{V}_1(x_1) = x_1 \dot{x}_1 = -x_1^4 - k_1 x_1^2 < 0$$

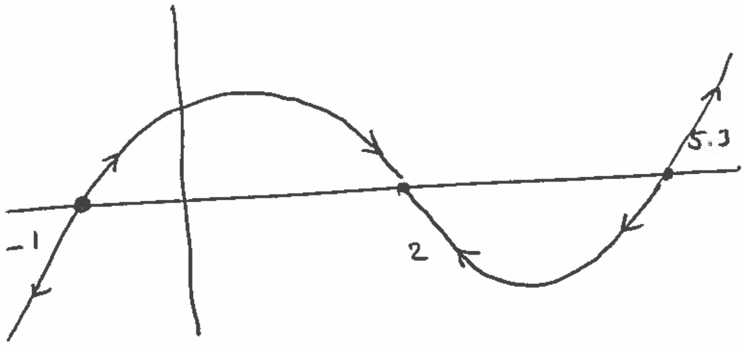
By applying the backstepping formula

$$u_a = -\frac{dV_1}{dx_1} g_1 - h\left(x_2 - \phi(x_1)\right) + \frac{d\phi}{dx_1}\left(f_1 + g_1 x_2\right)$$

$$u_a = -x_1 - h\left(x_2 + k_1 x_1\right) - k_1\left(-x_1^3 + x_2\right)$$

$$u_a = -x_1 - h x_2 - h k_1 x_1 + k_1 x_1^3 - k_1 x_2$$

EX 2



equilibria $\rightarrow$ $x = -1$, $x = 2$, $x = 5.3$

$x = -1$ unstable

$x = 2$ asymptotically stable

$x = 5.3$ unstable

EX 3

a) M_{G12} (minor, rows 1, 2) $= \dfrac{3(s-2)}{(s+1)^2(s+2)}$

$$M_{G13} = \frac{2(s-2)}{(s+1)^2(s+2)}$$

$$M_{G23} = \frac{3s(s-2)}{(s+1)^2(s+2)}$$

poles $s = -1$, $s = -1$, $s = -2$

zeros $s = 2$

b)

$R(s)$ must not have poles in $s = 2$

c) NO, $y \in R^3$, $u \in R^2$

We do not have enough control variables.

Ex 3

$LQ_\infty \rightarrow \bar{P} = 16.06$

$\bar{K} = 3.76$

$A - B\bar{K} = 0.236$

$$RH \rightarrow \begin{cases} P_N = 0 \\ K_{N-1} = 0 \\ A - BK_{N-1} = 4 \end{cases}, \quad \begin{cases} P_{N-1} = 1 \\ K_{N-2} = 2 \\ A - BK_{N-2} = 2 \end{cases}, \quad \begin{cases} P_{N-2} = 9 \\ K_{N-3} = 3.6 \\ A - BK_{N-3} = 0.4 \end{cases},$$

$N = 3$

The next iteration is

$$\begin{cases} P_{N-3} = 15.4 \\ K_{N-4} = 3.75 \\ A - BK_{N-4} = 0.25 \end{cases}$$

almost equal to the LQ_∞ solution

Ex 4

$(k|k$

$$\begin{cases} \hat{x}_1(k+1|k) = -\hat{x}_1^3(k|k-1) + \hat{x}_2(k|k-1) + l_1(k)\left[y(k) - \hat{x}_1(k|k-1)\hat{x}_2 \right. \\ \hat{x}_2(k+1|k) = \hat{x}_2^2(k|k-1) + u(k) + l_2(k)\left[y(k) - \hat{x}_1(k|k-1)\hat{x}_2(k|k \right. \end{cases}$$

$$L(k) = \begin{vmatrix} l_1(k) \\ l_2(k) \end{vmatrix}$$ is obtained with the Riccati equation where the matrices to be used are

$$A(k) = \begin{vmatrix} -3x_1 & 1 \\ 0 & 2x_2 \end{vmatrix}_{\begin{array}{l} x_1 = \hat{x}_1(k/k-1) \\ x_2 = \hat{x}_2(k/k-1) \end{array}}$$

$$C(k) = \begin{vmatrix} x_2 & x_1 \end{vmatrix}_{\begin{array}{l} x_1 = \hat{x}_1(k/k-1) \\ x_2 = \hat{x}_2(k/k-1) \end{array}}$$