ADVANCED AND MULTIVARIABLE CONTROL

19/1/2023

with solutions

Surname and name ....

Signature ....

Exercise 1 (3 marks)

$V(x) = \frac{1}{2}(x_1^2 + x_2^2)$,

$\dot{V}(x) = \underbrace{(x_1^2 + x_2^2)^2}_{>0} + \cancel{(a-b)x_1 x_2}$ for $a=b$

$\dot{V}(x) > 0 \rightarrow$ unstable

Consider the continuous-time linear system:

$$\dot{x}_1(t) = ax_2(t) + x_1(t)(x_1^2(t) + x_2^2(t))$$
$$\dot{x}_2(t) = -bx_1(t) + x_2(t)(x_1^2(t) + x_2^2(t))$$

With the Lyapunov theory and a quadratic Lyapunov function it is possible to conclude that the origin is

- [x] unstable equilibrium for $a=b$
- o an asymptotically stable equilibrium for $a=b$
- o a globally asymptotically stable equilibrium for $a>b$
- o a simply stable equilibrium for $a<b$
- o No answer

Exercise 2 (3 marks)

Consider the linear discrete time 2x2 system described by the transfer function

$$G(z) = \begin{bmatrix} 1.5/(z-0.5) & (z+0.5)/(z+1) \\ a/(z+0.5) & z/(z+0.5) \end{bmatrix}$$

For this system what is the condition on the parameter *a* such that it is possible to design a regulator guaranteeing zero error regulation for constant reference signals?

- o There are no values of *a* because the transfer functions do not have integrators
- o a=2
- [x] a≠4
- o any value of *a* because one of the transfer functions has a zero at the origin
- o no answer

$$G(1) = \begin{vmatrix} \frac{1.5}{0.5} & \frac{1.5}{2} \\ \frac{a}{1.5} & \frac{1}{1.5} \end{vmatrix} \rightarrow \det G(1) \neq 0 \rightarrow a \neq 4$$

Same result if you compute the invariant zeros, which must be different from $z=1$.

Exercise 3 (3 marks)

Consider the following system

$$\dot{x}_1(t) = x_1(t) - u_1(t) + u_2(t)$$

$$\dot{x}_2(t) = x_1(t) - u_1(t)$$

For this system **it is not possible** to design a state feedback, pole placement control law if

- o both u_1 and u_2 are used
- [x] only u_1 is used
- o only u_2 is used
- o it depends on the selected position of the poles
- o No answer

Exercise 4 (3 marks)

Concerning the Loop Transfer Recovery procedure, select the correct statement

- o Can be used for discrete time systems.
- o Can be used only for systems with more inputs than outputs.
- o When applicable, is such that the bandwidth of the loop transfer function with state feedback plus observer is larger than the one with state feedback only.
- [x] Can be used only for square systems with invariant zeros with negative real part.
- o No answer

See the notes

$$A = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}, \quad B = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$$

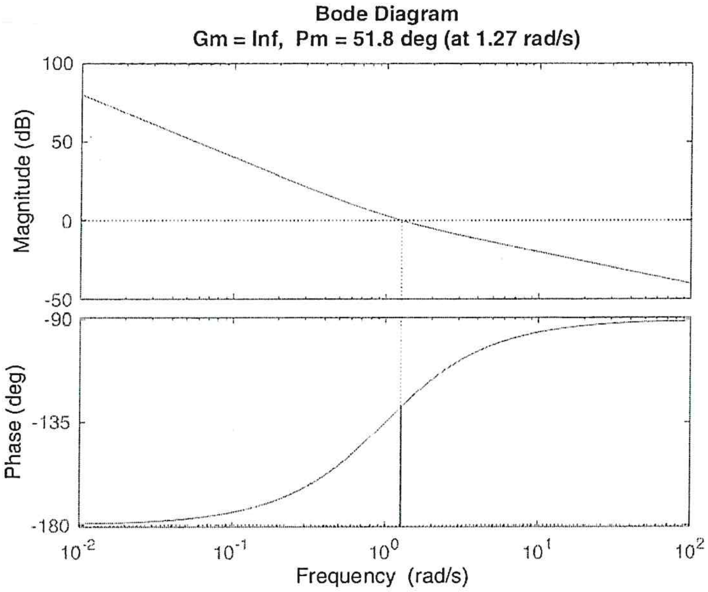
- det $B \neq 0$ → the system is reachable with u_1 and u_2

- the system is reachable only with u_2

- the system <u>is</u> <u>not</u> reachable with u_1 only.

Exercise 5 (3 marks)

Consider a feedback system with control law $u=(t)=-Kx(t)$ and the loop transfer function $L(s)=K(sI-A)^{-1}B$ with the Bode diagrams reported in the figure



Is it plausible that the gain *K* has been designed with LQ control?

- o No, because the slope at high frequency is -20dB x decade
- o No, because the gain margin is infinity
- X No, because the phase margin is too small — robustness property of LQ
- o Yes
- o No answer

Exercise 6 (8 marks)

Consider the system

$$\dot{x}(t) = ax(t) + bu(t) + v_x(t)$$

$$y(t) = x(t) + v_y(t)$$

With $a>0$, $v_x=WN(0,3a^2)$, $v_y=WN(0,1)$. For this system,

- design the Kalman Predictor
- draw the Bode diagrams (approximated) of the transfer function between the output and the estimated state,
- discuss its speed (bandwidth) in terms of the parameter a.

Steady state Riccati equation of the Kalman predictor: $0=AP+PA'+Q-PCR^{-1}C'P$

$A = a,\ B = b,\ \tilde{Q} = 3a^2,\ \tilde{R} = 1,\ C = 1$

Riccati eq.

$$2ap + 3a^2 - p^2 = 0 \longrightarrow p = \begin{cases} 3a > 0 & \checkmark \\ -a < 0 & \underline{NO} \end{cases}$$

$L = P = 3a$

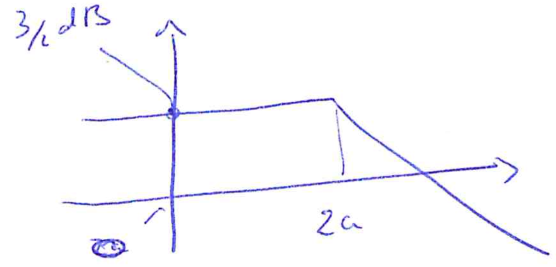
$(A - LC) = -2a$

$\dot{\hat{x}} = a\hat{x} + bu + L[y - \hat{x}]$

set $u=0$ for simplicity

$$y \longrightarrow \boxed{\frac{3a}{s+2a}} \longrightarrow \hat{x}$$

$\downarrow$



the larger a, the faster the observer.

Exercise 7 (5 marks)

Draw the approximate diagrams of the Bode plot (magnitude) of the shaping functions to be associated to the sensitivity and complementary sensitivity functions of a SISO control system to be designed with H2 or Hinf control.

See the notes of the course

Exercise 8 (5 marks)

Explain the difference between hard and soft constraints and how they can be used in the formulation of MPC control problems.

See the notes.