Thermal systems – control problems (part 1)

of thermal syst.
Related to thermal energy

Part 1 of control problems..

- Flow/pressure control with liquids
- Temperature control in pipes
- Joint temperature and flow control

Foreword

- In thermal energy systems flow control is related to "moving energy around" properly, by transporting fluids. (Flow control)

  avoid override! → sp don't exceed a limit!
- Pressure control is mostly introduced as a means to not exceed equipment limits.
- In this course we restrict the scope to liquid thermovector fluids.
- The involved process components are piping, $\underbrace{\text{pumps and valves}}_{\text{actuators}}$ (the latter two as actuators).
- Our main point is that these introduce nonlinear characteristics in control loops.
- We need to manage this properly (and say a few words on sizing).

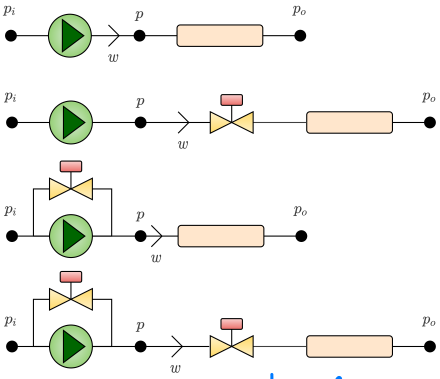
  determining equipment size! relevant control problem, correct size!

Problem statement

→ control the flow of a liquid, to carry heat between two places and some pressure (Δp)

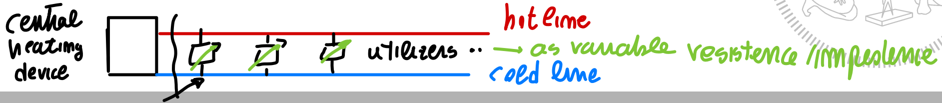
(major configurations)

possible connections



- The main schemes build upon
  - variable-speed pump,
  - fixed-speed pump with series valve,
  - fixed-speed pump with recirculating valve.
- The main sources of disturbance are
  - variations in the inlet and/or the outlet pressure (for the shown open circuit case),
  - variations in the load hydraulic impedance.

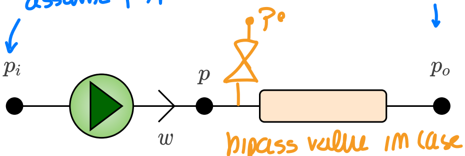
typical case:



Problem statement

Typical configuration (a)

assume p_i, p_o as boundary conditions: taking fluid from distribution line fixed @ p_i, connecting to a fixed p_o by tank etc..



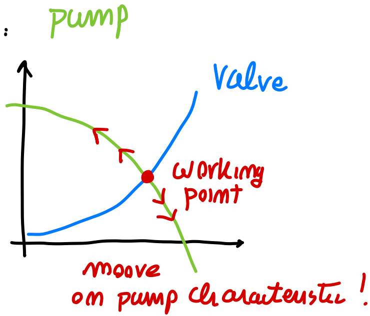
bypass valve in case of dangerous situation

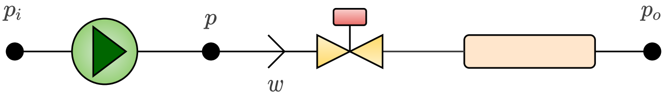
- Need variable-speed pump:
  - can control either p or w; or possible control override → provide $p < p_{lim}$!
  - relief lines are provided if p could – but must not – exceed some limits.

Problem statement

Typical configuration (b)

Idea:



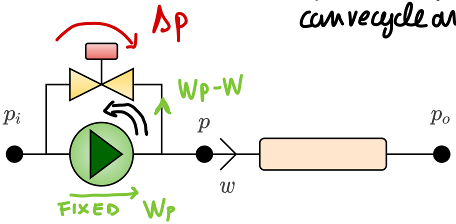


- Fixed-speed pump (most commonly used):
  - valve opening can control either p or w;
  - sometimes override control is used: keep w if p does not exceed a maximum, else keep p. (dangerous!)
- Variable-speed pump:
  - use pump speed to control p and valve opening to control w;
  - possibly adjust the p set point to keep the valve opening in range (say 20–80%) while keeping pump speed – thus power – as low as possible.

Problem statement

Typical configuration c

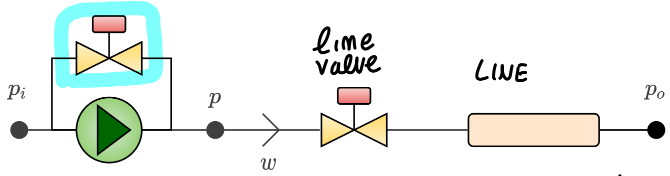
IF you change valve opening, you can recycle and reuse



- Fixed-speed pump (most frequent case):
  - valve opening typically used to control w;
  - relief lines are provided for p if needed.
- Variable-speed pump (quite uncommon):
  - use pump speed and valve opening to control both p and w; ← complex coupling NO control both!
  - coupling between the two is a bit complex, however.

Problem statement

Typical configuration d)



→ yet 2 actuators! not a third one necessary

- Fixed-speed pump (hardly any use for a variable-speed one):
  - recirculating valve opening typically used to control p,
  - while line valve opening controls w.

once p controlled!

[main {w, p} control configuration using variable / Fixed-speed pump]

Problem statement

Common facts and wrap-up

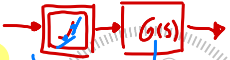
↓ control problems!

with liquids (incompressible)

↑ inertia only for large motors

- All the relevant dynamics is in the actuator (pump motor and/or valve positioner);
- the said dynamics is reasonably linear at least in the small (rate limits in the large); NOT FAST motion
- hence the main issue is the nonlinearity of pump and/or valves, as well as of the hydraulic load, ↳ quadratic
- which collectively result in the controller(s) seeing a variable differential gain cascaded to a quite low-order linear dynamics (1st–2nd normally suffices).

you can have huge # of combinations, we see some cases particular...



depend on input value → different gain (tangent to curve)

(G(0)=1)

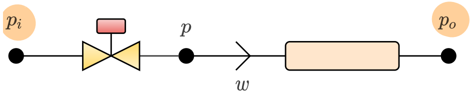
- For space/time reasons we are thus studying here just a couple of representative cases: generalisation to the variety of possible ones is up to the engineer.
- Then we are solving a few control problems for completeness.

Case 1 (simplest)

Setting

prescribed (fixed) pressure p_i, p_o

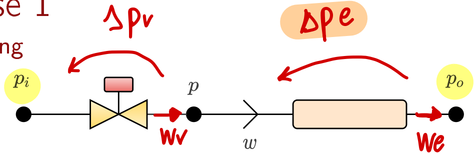
⇓ we wanna...



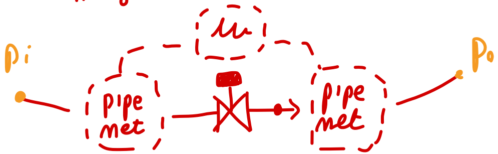
- Assignment:
  1. size the valve flow coefficient;
  2. determine the valve installed characteristic;
  3. make the said characteristic linear (physically or in software).
- We exploit this first case to also provide the necessary definitions "on the job".

Case 1

Setting



in general case:



some boundary conditions always present!
<- here simplest case!

- Hypotheses:
  - p_i and p_o are prescribed boundary conditions;
  - the valve (for liquid, v subscript) is ruled by

$x \in [0,1] \Rightarrow \begin{cases} \phi(0) = 0 \\ \phi(1) = 1 \end{cases}$

opening function

$$w_v = C_v \phi(x) \sqrt{\rho \Delta p_v}$$

valve flow rate relationship

flow coeff. command

  where w_v is the inlet to outlet mass flowrate, ρ the liquid density, Δp_v the inlet minus outlet pressure drop; $x \in [0,1]$ is the valve opening and $\Phi(x)$ the intrinsic characteristic, with $\Phi(0) = 0$ and $\Phi(1) = 1$; C_v is the flow coefficient;
  - the hydraulic load (l subscript) is ruled by

(for simplicity, neglect gravity)

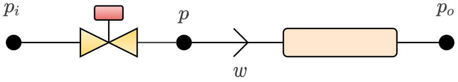
$$\Delta p_l = K_l \rho w_l^2$$

  with the same notation and conventions; K_l is a known parameter, we assume unidirectional flow (reasonable) and neglect gravity for simplicity.

Case 1

Assignment 1 — sizing C_v

it can be $p_i \in [p_{iMIN}, p_{iMAX}]$
$p_o \in [p_{oMIN}, p_{oMAX}]$ $\quad p_{iMIN} < p_{oMAX}$
↳ worst case
(size on worst case condition) ← using min Δp for sizing



transport w_n kg/s on the line

- We assume a nominal flowrate w_n is required, hence in nominal (sizing) conditions

$$\underbrace{p_o}_{\text{KNOWN}} = \ldots \quad p = p_o + K_l \rho w_n^2.$$

in nominal conditions, p KNOWN

- As such with valve fully open we get

$\phi(1)=1$

$$C_v \sqrt{\rho \left(\overbrace{p_i - \underbrace{(p_o + K_l \rho w_n^2)}_{p}}^{\Delta p}\right)} = w_n$$

Flow rate

nom

- Solving for C_v yields the nominal value for C_v as cleverly !

$$C_{vn} = \frac{w_n}{\sqrt{\rho (p_i - p_o - K_l \rho w_n^2)}}.$$

various cases possible, like sizing near nominal, then size 20% more towards 80% valve! ←

needed to transfer w_n with that p_i, p_o fixed

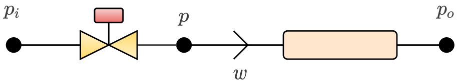
- In general we then take $C_v = \alpha C_{vn}$, with $\alpha > 1$ (not too much).

Case 1

Assignment 2 — "installed characteristic"

definition :=

[in our case of valve control W then installed characteristic is W(x)]
IF control p → inst. char is p(x)!



- Definition: variable to govern expressed as a function of the valve opening (x).
- In our case, $w(x)$ or $p(x)$ depending on the purpose of the control system.
- In the w case we start from $w = C_v\Phi(x)\sqrt{\rho(p_i - p_o - K_l\rho w^2)}$, and solving for w (recall the assumed signs) we obtain

NOT linear form

Can I find φ(x) such that W(x) = WMAX · x ←

$$w(x) = \frac{C_v\Phi(x)\sqrt{\rho(p_i - p_o)}}{\sqrt{1 + K_l\rho\ C_v^2\Phi^2(x)}}.$$

≠ Wmax x

I want to see actuator dynamic · x linear form

- In the p case we just need to bring in $p(x) = p_o + K_l\rho w^2(x)$, whence

Substitute $w^2(x)$ here!

or Find φ(x) such that p(x) = po − μx ←

$$p(x) = p_o + \frac{K_l C_v^2\Phi^2(x)\rho^2(p_i - p_o)}{1 + K_l\rho\ C_v^2\Phi^2(x)}.$$

$$W(x) = C_v\,\phi(x)\sqrt{\rho\,(p_i - p_o - K_e W^2(x))}$$

installed characteristic

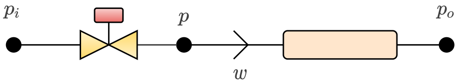
$$W^2(x) = C_v^2\,\phi^2(x)\,\rho\,(p_i - p_o - K_e W^2(x))$$

$$W^2(x)\left(1 + C_v^2\,\phi^2(x)\rho K_e\right) = C_v^2\,\phi^2(x)\rho\,(p_i - p_o)$$

$$W(x) = \frac{C_v\,\phi(x)\sqrt{\rho\,(p_i - p_o)}}{\sqrt{1 + C_v^2\,\phi^2(x)\rho K_e}}$$

Case 1

Assignment 3 — linearisation, physical version



- Purpose: select $\Phi(x)$ so that the installed characteristic be linear (or affine) in x.
- We address the w case. With the selected C_v (no matter α) setting $x = 1$ gives

$$w_{max} = \frac{C_v\sqrt{\rho\,(p_i - p_o)}}{\sqrt{1 + K_l\rho\; C_v^2}}.$$

w_{max} for $x=1$

I want C_v from here!

  while obviously $x = 0$ gives $w = 0$.
- We want to determine $\Phi(x)$ such that

$$w(x) = w_{max}\cdot x$$

  i.e., in this case, a truly linear installed characteristic.

$$\underbrace{\frac{\cancel{C_v}\sqrt{\cancel{\rho(p_i-p_0)}}}{\sqrt{1+K_e\rho C_v^2}}}_{W_{MAX}} \cdot x = \underbrace{\frac{\cancel{C_v}\,\phi(x)\sqrt{\cancel{\rho(p_i-p_0)}}}{\sqrt{1+K_e\rho C_v^2\phi^2(x)}}}_{W(x)}$$

Find $\phi(x)$ such that
$W(x) = W_{MAX} \cdot x$

$$\Downarrow$$

$$x^2\left(1+K_e\rho C_v^2\phi^2(x)\right) = \phi^2(x)\left(1+K_e\rho C_v^2\right)$$

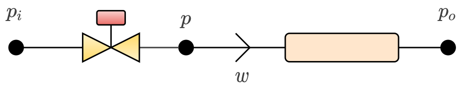
$$\phi(x)\left[1+K_e\rho C_v^2 - x^2 K_e\rho C_v^2\right] = x^2$$

$\Rightarrow$ knowing the signs (as we do..) $\phi(x) = \dfrac{x}{\sqrt{1+K_e\rho C_v^2(1-x^2)}}$

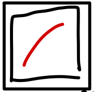
(we should be
able to do this also for a general system)

Case 1

Assignment 3 — linearisation, physical version



≃ linear



controller works good anyway

We need to know how relevant for control P.O.V is non linearity ⇒

- We rewrite $w(x) = w_{max} \cdot x$ expressing $w(x)$ and w_{max}, that is

$$\frac{C_v \Phi(x) \sqrt{\rho(p_i - p_o)}}{\sqrt{1 + K_l \rho \, C_v^2 \Phi^2(x)}} = \frac{C_v \sqrt{\rho(p_i - p_o)}}{\sqrt{1 + K_l \rho \, C_v^2}} x,$$

we need to check if this NON linearity is relevant!

and solve for $\Phi(x)$, which gives

$$\Phi(x) = \frac{x}{\sqrt{1 + K_l \rho \, C_v^2 (1 - x^2)}}.$$

- Now, we just need to find a valve with this intrinsic characteristic in some supplier catalogue ☺

similar to this $\phi(x)$ value !

Case 1

Assignment 3 — linearisation, physical version

- First, however, let us consider for simplicity

$$\Phi(x) = \frac{x}{\sqrt{1+\beta(1-x^2)}}$$

similar to $w(x)$ expression

$\Updownarrow$

and take its inverse

$$\left\| \, x(\Phi) = \frac{\Phi\sqrt{1+\beta}}{\sqrt{1+\beta\Phi^2}}. \right\| \simeq \frac{\text{const}\cdot\phi}{\sqrt{1+\text{const}\,\phi^2}}$$

- Note the similarity of $x(\Phi)$ to the expression we derived before for $w(x)$: $x(\Phi)$ in fact represents the *shape* of the installed flow characteristic if the intrinsic one were linear. → replacing ϕ with x!

$\Downarrow$

- The question is: how relevant is the above nonlinearity for control? And as a consequence, how precisely does one need to linearise?

knowing: ρ, $\phi(x)$ ± tollerance, K_e → how you need to linearise?! ⇒

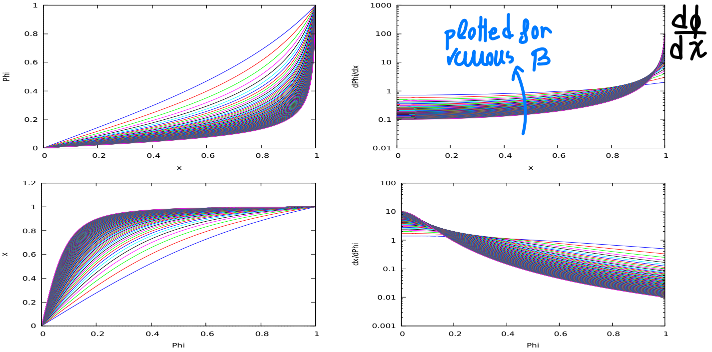
Case 1

Assignment 3 — linearisation, physical version

- wxMaxima: plotting

```
Phix   : makelist(x/sqrt(1+beta*(1-x^2)),beta,1,100); xPhi   : makelist(Phi*sqrt((beta+1)/(beta*Phi^2+1)),beta,1,100);
plot2d(Phix,[x,0,1],[xlabel,"x"],[ylabel,"Phi"],[legend,false]);
plot2d(diff(Phix,x),[x,0,1],[xlabel,"x"],[logy],[ylabel,"dPhi/dx"],[legend,false]);
plot2d(xPhi,[Phi,0,1],[xlabel,"Phi"],[ylabel,"x"],[legend,false]);
plot2d(diff(xPhi,Phi),[Phi,0,1],[xlabel,"Phi"],[logy],[ylabel,"dx/dPhi"],[legend,false]);
```

- Note: we make the $d\Phi/dx$ and $dx/d\Phi$ axes logarithmic for better readability.

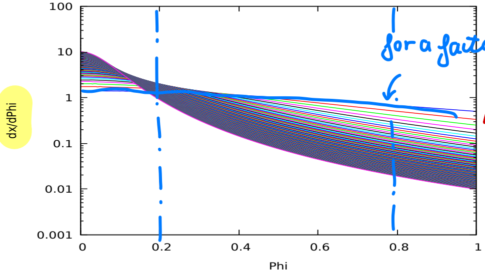


Case 1

Assignment 3 — linearisation, physical version

which determine the gain change! respect relevance of den. wrt linear num.

- Let us concentrate on $dx/d\Phi$:



for a factor of 4

3 ORD of magn. depending on $\beta = K_\ell \rho C_v^2$

$$X(\phi) = \frac{\phi\sqrt{1+\beta}}{\sqrt{1+\beta\phi^2}}$$

← lin (numerator)

← non lin (denominator)

- Depending on how relevant the denominator is wrt the (linear) numerator, the differential gain change seen by a flow controller can range from a factor 4–5 up to 3 orders of magnitude.
- However things are better if we focus on the typical opening range of a valve for smooth operation, say $\boxed{0.2\text{–}0.8}$.

Case 1

Assignment 3 — first lessons learnt (and software moving in)

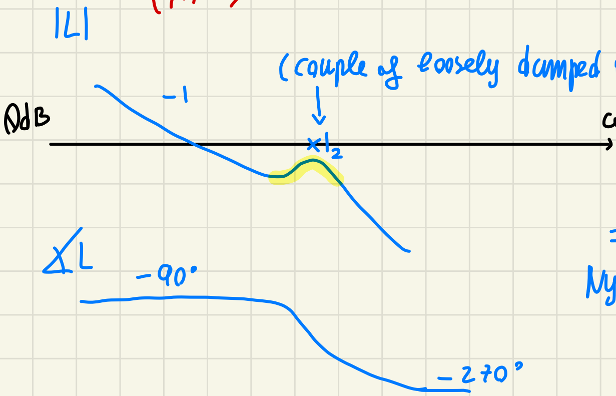
NON terrible oscillatory dyn.
(normally well damp)

- As the actuator dynamics is low-order and normally decently damped, achieving good gain margins is generally no big deal. (*) ⇒
- Thus, once linearisation reduces the intrinsic valve nonlinearity to an $x \to w$ gain variation within a factor well below the gain margin in the range of interest, its duty can be taken for accomplished.
- If necessary to achieve tight control, the "rest" of the linearisation can be done in software, by cascading a nonlinear compensating characteristic to the controller (a task once devoted to mechanical cams). (*)(*) ⇒
- Methodological hint for the enthusiast: absolute stability.

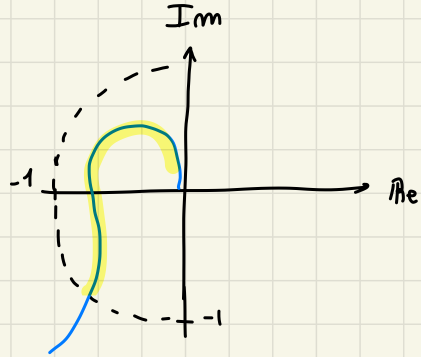
refers to guarantee stability
different shape depending on β of φ(x)
↓
(fitting linearization)

- Anyway, this is why on the market one finds valves with standard intrinsic characteristics (linear, equi-percent, quick-opening...) designed to fit most needs. Throw a glance at some catalogue on the net if curious, there are many.

(*) ⇒



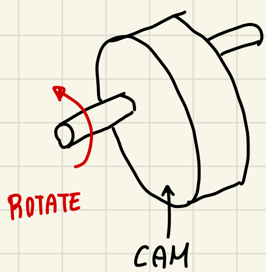
⇒ Nyquist

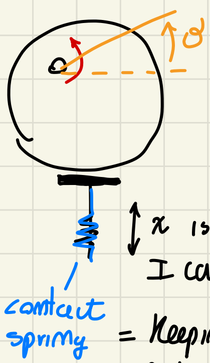


Bad phase margin due to damping law

(*)(*) ⇒ from mech shaft

FRONT VIEW





x is a NON LIN function of ϑ that I can chape by shaping the CAM

contact spring = keeping piston connected to cam → NOW all done IN SOFTWARE!

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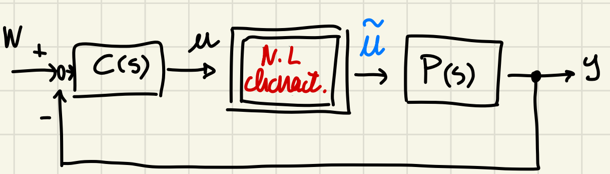
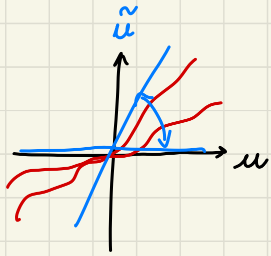
absolute stability





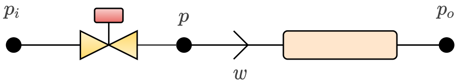
(guarantee it lays in a sector)

⇓

guarantee absolute stability!

Case 1

Assignment 3 — back to linearisation, physical and/or software (now we know)



IF valve closed $w=0$,
$(p_o = p)$

- Now (briefly) for the p case. For this installed characteristic we got

$$p(x) = p_o + \frac{K_l C_v^2 \Phi^2(x) \rho \, (p_i - p_o)}{1 + K_l \rho \, C_v^2 \Phi^2(x)}$$

$W^2(x)$ from previous expression!

Affine characteristic (NOT linear!) → NOT pass through origin

- Setting $x=0$ and $x=1$ yields respectively

you cannot increase p above a certain limit! ←

$$p_{x0} = p_o, \qquad p_{x1} = p_o + \frac{K_l C_v^2 \rho \, (p_i - p_o)}{1 + K_l \rho \, C_v^2}.$$

- We want to determine $\Phi(x)$ such that

difference..

$$p(x) = \underbrace{p_{x0}}_{\text{for } (x=0)} + (p_{x1} - p_{x0}) \cdot x$$

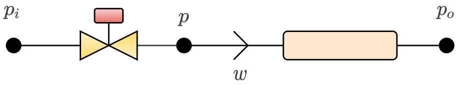
i.e., in this case, an "*affine* installed characteristic."

$$\begin{cases} p(0) = p_{x0} \\ p(1) = p_{x1} \end{cases}$$

Case 1

Assignment 3 — linearisation...

{ how you treat CASE 1 ! }



- We rewrite $p(x) = p_{x0} + (p_{x1} - p_{x0}) \cdot x$ expressing $p(x)$, p_{x0} and p_{x1}, that is

$$p_o + \frac{K_l C_v^2 \Phi^2(x) \rho \ (p_i - p_o)}{1 + K_l \rho \ C_v^2 \Phi^2(x)} = p_o + \frac{K_l C_v^2 \rho \ (p_i - p_o)}{1 + K_l \rho \ C_v^2} x,$$

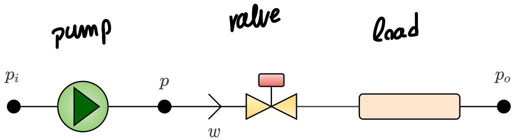
  and solve for $\Phi(x)$, which gives

$$\Phi(x) = \sqrt{\frac{x}{1 + K_l \rho \ C_v^2 (1 - x)}}.$$

{• Analogous considerations as for the w case apply. }

Case 2

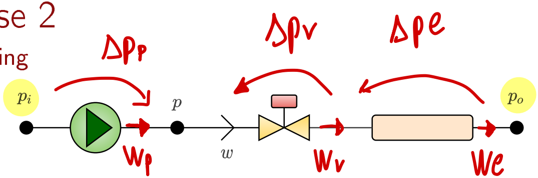
Setting



- Assignment:
  1. size the valve flow coefficient; C_v
  2. determine the valve installed characteristic;
  3. make the said characteristic linear.
- Definitely quite similar to Case 1: for compactness and clarity we (concentrate on the essentials.)

Case 2

Setting



assume flow unidirectional + neglect gravity

- Hypotheses:
  - p_i and p_o are prescribed boundary conditions;
  - valve and hydraulic load are ruled respectively by

$$w_v = C_v \phi(x) \sqrt{\rho \Delta p_v}, \qquad \Delta p_l = K_l \rho w_l^2$$

  with the same notation and conventions we used for Case 1;
  - the pump (subscript p) is centrifugal, fixed-speed and ruled by

$$\Delta p_p = H_0 - H_1 w_p^2$$

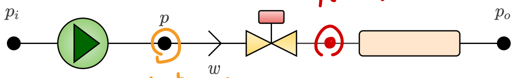
  where H_0 and H_1 are known positive parameters while (beware) Δp_p equals the *output* minus the *input* pressure; we still assume flows to be unidirectional and neglect gravity effects.

Case 2

Assignment 1 — sizing C_v

downstream

$p_o + K_l \rho w^2$



upstream

- From the component constitutive laws and the connections we have

$$p = p_i + H_0 - H_1 w^2, \qquad w = C_v \Phi(x) \sqrt{\rho \left(p - \left(p_o + K_l \rho w^2\right)\right)}$$

whence

$$w = C_v \Phi(x) \sqrt{\rho \left(p_i - p_o + H_0 - (H_1 + K_l \rho) w^2\right)}$$

- Requiring a nominal flowrate ($w = w_n$) with valve fully open ($\Phi = 1$) we thus get the nominal C_v as

$$C_{vn} = \frac{w_n}{\sqrt{\rho \left(p_i - p_o + H_0 - (H_1 + K_l \rho) w^2\right)}}.$$

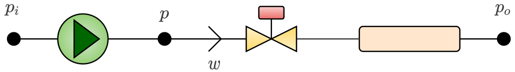
$\phi(x) = 1$
solve respect Cv:
(C_{vn})

Case 2

Assignment 2 — installed characteristic

$\Delta p = H_0 - H_1 W^2(x)$ (pump char.)

What as $H_1 \simeq 0$, why?



- We see only the w case. Solving

$$w = C_v \Phi(x) \sqrt{\rho \left(p_i - p_o + H_0 - (H_1 + K_l \rho) w^2\right)}$$

for w we get

$$w(x) = \frac{C_v \Phi(x) \sqrt{\rho (p_i - p_o + H_0)}}{\sqrt{1 + C_v^2 \rho \Phi^2(x) (H_1 + K_l \rho)}} = \frac{\beta \phi(x)}{\sqrt{1 + \alpha \phi^2(x)}}$$

β, α constant! similar to case 1

→ analogous linearization, select $\phi(x)$ such that $W(x)$ LINEAR!

- Very similar to Case 1; assignment 3 left as an exercise.
- Suggested question: what happens as H_1 approaches zero? Why?

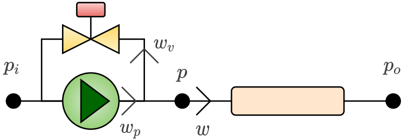
Modelica → course library

↓

different possible interpolations

Case 3

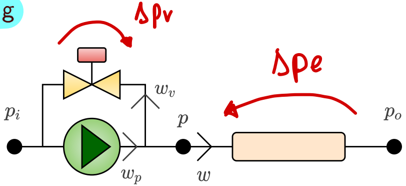
Setting



- Assignment:
  1. size the valve flow coefficient;
  2. determine the valve installed characteristic;
  3. make the said characteristic linear.
- This is a bit different from Cases 1 and 2: again, we concentrate on the essentials.

Case 3

Setting



- Hypotheses:
  - p_i and p_o are prescribed boundary conditions;
  - valve and hydraulic load (same conventions as above) are ruled respectively by

$$w_v = C_v \phi(x)\sqrt{\rho \Delta p_v}, \qquad \Delta p_l = K_l \rho w_l^2.$$

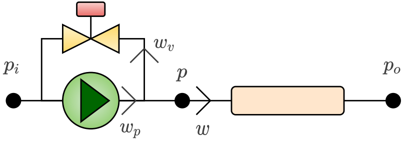
  - the pump (subscript p) is fixed-speed and volumetric (typical choice), hence ruled by

$$w_p = w_{p0}.$$

  - We still assume flows to be unidirectional, and neglect gravity effects.

Case 3

Assignment 1 — sizing C_v



- From the component constitutive laws and the connections we have

$$p = p_o + K_l \rho w^2, \quad w = w_{p0} - w_v, \quad w_v = C_v \Phi(x) \sqrt{\rho (p - p_i)},$$

whence

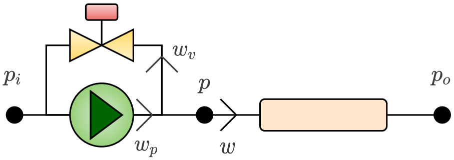
$$w = w_{p0} - C_v \Phi(x) \sqrt{\rho (p_o - p_i + K_l \rho w^2)}$$

- Requiring a nominal flowrate ($w = w_n$) with valve fully open ($\Phi = 1$) we get the nominal C_v as

$$C_{vn} = \frac{w_{p0} - w_n}{\sqrt{\rho (p_o - p_i + K_l \rho w_n^2)}}.$$

Case 3

Assignment 2 — installed characteristic



- We see again only the w case.
- Solving

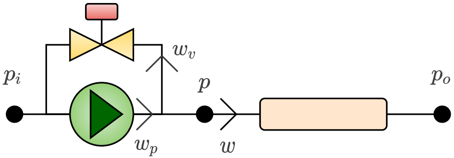
$$w = w_{p0} - C_v \Phi(x) \sqrt{\rho \left(p_o - p_i + K_l \rho w^2\right)}$$

for w, we get

$$w(x) = \sqrt{\frac{w_{p0}^2 - 2C_v\sqrt{\rho\left(p_o - p_i + K_l \rho w^2\right)}w_{p0}\Phi(x) + C_v^2 \rho\left(p_o - p_i\right)\Phi^2(x)}{1 - K_l \rho^2 C_v^2 \Phi^2(x)}}.$$

Case 3

Assignment 2 — installed characteristic

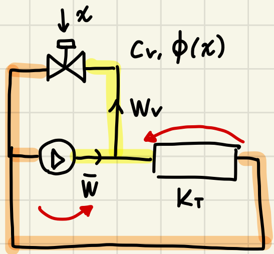


- Substituting 0 and 1 for x we respectively have

$$w_{x0} = w_{p0}, \quad w_{x1} = \sqrt{\frac{w_{p0}^2 - 2C_v\sqrt{\rho(p_o - p_i + K_l\rho w^2)}w_{p0} + C_v^2\rho(p_o - p_i)}{1 - K_l\rho^2 C_v^2}}.$$

- Intuitively, and subject to the obvious physical feasibility conditions we omitted throughout for brevity, w_{x1} must be less than w_{x0}.
- Hence in this case we have an affine installed characteristic with a negative slope in x.
- We do not discuss further for space reasons.

Es 1



Given C_v, $\phi(x)$, quadratic friction coefficent K_T. Determine $W(x)$?

equal pressure drop → taken between same connections!

$$W_v = \bar{W} - W = C_v \phi(x) \sqrt{\rho \underbrace{(K_T W^2)}_{\Delta p}}$$

$$(\bar{W} - W)^2 = C_v^2 \phi^2(x) \rho K_T W^2$$

$$\bar{W}^2 - 2\bar{W}W + W^2 = C_v^2 \phi^2(x) \rho K_T W^2$$

$$W^2 (1 - C_v^2 \phi^2(x) \rho K_T) - 2\bar{W}W + \bar{W}^2 = 0 \qquad \text{2nd order equation}$$

solved easly as:

$$\cancel{\bar{W}^2} - \cancel{\bar{W}^2} + \bar{W}^2 C_v^2 \phi^2 \rho K_T$$

$$W = \frac{\bar{W} \mp \sqrt{\bar{W}^2 - \bar{W}^2(1 - C_v^2 \phi^2(x) \rho K_T)}}{1 - C_v^2 \phi^2(x) \rho K_T} = \frac{\bar{W} \mp \bar{W} C_v \phi(x) \sqrt{\rho K_T}}{//} =$$

$$= \bar{W} \frac{(1 \mp C_v \phi(x) \sqrt{\rho K_T})}{1 - C_v^2 \phi^2(x) \rho K_T}$$

which one ? $\mp$?

↓

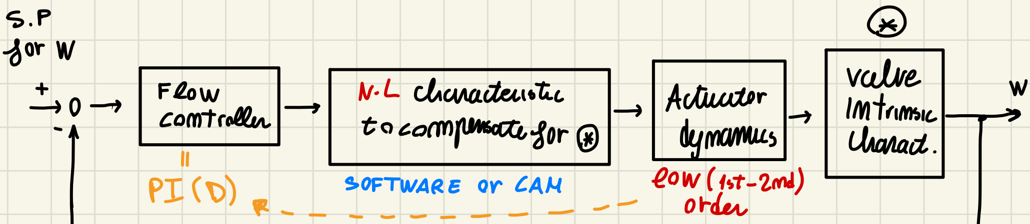
opening value, the flow W such that

- $x = 0 \quad \phi(0) = 0 \qquad W = \bar{W}$
- IF $x \simeq 0 : \Rightarrow \quad 1 - C_v^2 \phi^2(x) \rho K_T \simeq 1$

W cannot be $> \bar{W}$!

must be $-$

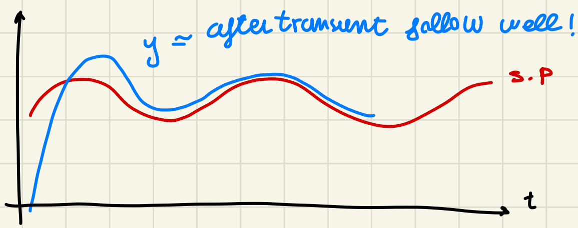
Summing up for FLOW CONTROL



so simple controller is enough!

- simulating in Modelica...

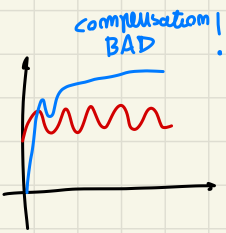
>> valve_linearization_case004_LTI



good work even with different frequency until

I wanna compensate valve intrinsic characteristic through a compensator → [in between I have some dynamic!]

so there is a delay → IF you goo too fast → you have BAD compensation



check that actuator is fast enough

I/O delay must be small!

↓

you need proper tracking

this is a small thresholds $\longrightarrow$ strange behav. of non linear systems!

$\Downarrow$

$\left\{ \begin{array}{l} \text{small changes causes Bad} \\ \text{catastrofic effect on the system !} \end{array} \right\}$

Flow/pressure control with liquids

✓ solved.. ⇒ PROBLEM 2 ? →

Wrap-up

- The scheme is composed of pumps, valves, piping (and tanks).
- One actuates with combinations of pumps and valves (we saw examples).
- In closed circuits $p_i = p_o$ and there is typically some pressure reference (such as vents to atmosphere, large tanks or pressurisers), hence our examples apply.
- Sizing and control synthesis are intertwined owing to inherent nonlinearity.
- Dynamics are of low order, concentrated in the actuators, while differential gains can vary a lot.
- The typical control structure is a dynamic controller, most often of PI/PID type, with possibly a static output characteristic if physical linearisation is not enough.
- Once linearised, controller tuning is straightforward.

Foreword

moving heated fluid..

- We stick to liquids as thermovectors of election.
- Our point is to control temperature at some point in a pipe (e.g., the outlet of a heating station).
- We have two major ways to do that:
  1. release thermal power to the pipe upstream the controlled point (generally to heat);
  2. mix in another liquid stream at different temperature (generally to cool).
- We shall see when dealing with heat networks that the two actions may need combining — as one can already guess, incidentally.
- Action 1 is quite straightforward to implement. (Just open fuel valve, command)
- Action 2 is conversely a bit more tricky owing to sensor dynamics; we are now investigating this.

↳...

Controlled system

consider a mix-up of two temperatures

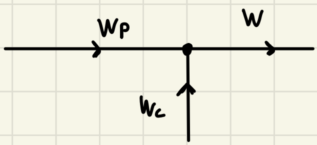
- Let w_p be the incoming "process" flowrate at temperature T_p, to join with the "control" flowrate w_c at temperature T_c so as to get a mix temperature T°.
- Assume T_c to be ideally controlled — for simplicity; even if not so the main point in the following treatise holds.

  ↑ main point holds
- Assume that w_c obeys to a control input u through a dominantly 1st-order dynamics (reasonable in the case u acts on a valve positioner, possibly with an inner flow loop).
- Assuming instantaneous mixing (reasonable in turbulent flows), the resulting temperature T – also interpretable as the equilibrium one with constant inputs – is

$$T = \frac{w_p T_p + w_c T_c}{w_p + w_c}.$$

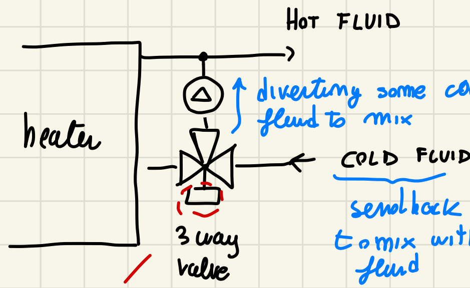
(regime Temp)

T control in pipes



{ to make it faster.. }

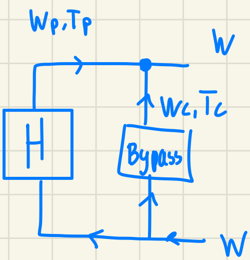
for example



diverting some cold fluid to mix

send back to mix with fluid

- in this case:

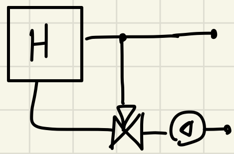


By acting on valve /pump one can change the MIX temperature very fast!

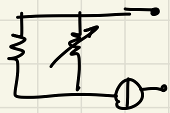
(using heater you need more time)

⇓

ya heat more slowly less fluid (without loosing energy!) → NO WASTE

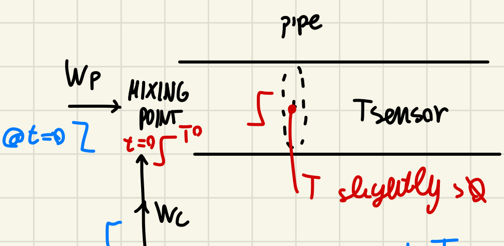


like..



current gen with impedence

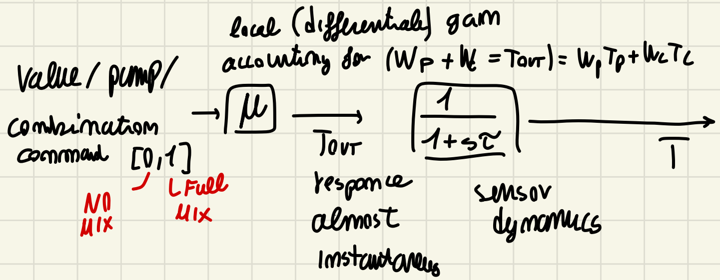
What does a controller see?



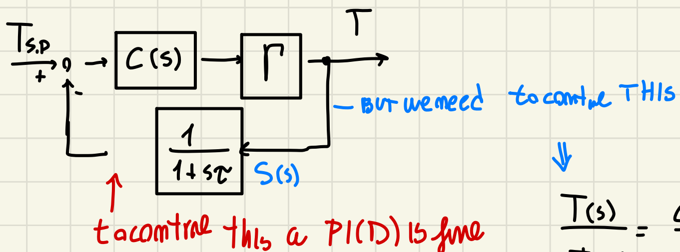
← idealT may respond in fraction of seconds..

measured T

into Block diagram:



⇓ NOW close the loop

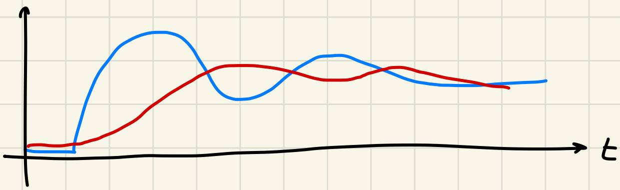


⇓

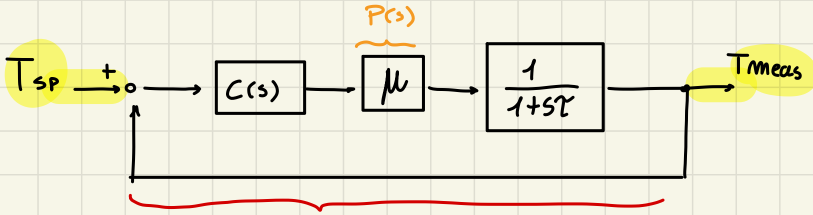
$$\frac{T(s)}{T_{sp}(s)} = \frac{C(s)\mu}{\underbrace{1+C(s)\mu S(s)}_{\text{main dynamic in a feedback path}}}$$

modelica simulation.

good model in case of faster sensor !



• POSSIBLE **Re-formulation**



We can make this equal to $\frac{1}{1+s\lambda}$ by

$$C\mu \frac{1}{1+s\tau} = \frac{1}{s\lambda} \Rightarrow \left(C = \frac{1}{s\lambda} \frac{1+s\tau}{\mu} \right)$$

And doing so... we get

$$\frac{T(s)}{T_{sp}(s)} = \frac{1+s\tau}{1+s\lambda}$$ What I got

hence, IF you want

$$\frac{T(s)}{T_{sp}(s)} = \frac{1}{1+s\lambda}$$ $\neq$ What I want

Using set point prefiltering! $\longrightarrow$ it suffices to filter T_{sp} through $\frac{1}{1+s\tau}$

{ (Note): must know sensor time constant }

Library case: $P(s) = \dfrac{\mu_P}{1+s\tau_P}$ $S(s) = \dfrac{1}{1+s\tau_s}$

$$C(s) = \frac{\omega_c}{s} \frac{(1+s\tau_P)(1+s\tau_s)}{1+s/10\omega_c}$$

+ Prefilter on set point $F(s) = \dfrac{1}{1+s\tau_s}$

- simulating in modelica...

Controlled system

- Linearising and assuming all inputs but w_c to not vary for brevity, we have

$$\delta T = \mu \delta w_c, \qquad \mu = \frac{\overline{w}_p(\overline{T}_c - \overline{T}_p)}{(\overline{w}_p + \overline{w}_c)^2}.$$

  where bars denote equilibrium values.

- Assuming now $u \in [0,1]$ and denoting by $w_{c,max}$ and τ_c the maximum w_c and the $u \rightarrow w_c$ time constant we obtain

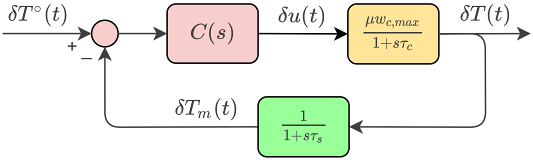
$$\delta T = \frac{\mu w_{c,max}}{1 + s\tau_c} \delta u$$

- and finally, denoting by τ_s the time constant of the required T sensor we can express the measured mixing temperature (variation) as

$$\delta T_m = \frac{\mu w_{c,max}}{(1 + s\tau_c)(1 + s\tau_s)} \delta u.$$

Control block diagram

- Indicating by $C(s)$ the controller for δT acting on δu leads to



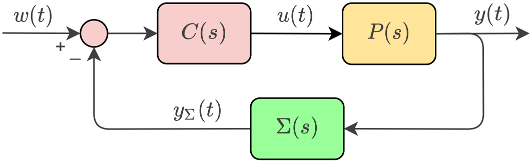
- Problem: there are cases where τ_s is remarkably larger than τ_c.
- As a result, achieving a good control of δT_m in general does not mean controlling well δT, which is the real objective.

- WARNING: an autotuning controller (often used for mass tuning of "simple" loops) would exactly aim for a good control of δT_m: any "auto-" or "self-something" object needs instructing... ☺

needed instructions (intelligence on system)

Control block diagram

- Let us study the problem with a lighter notation for simplicity:



- We have

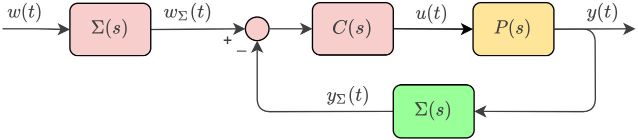
$$\frac{Y(s)}{W(s)} = \frac{C(s)P(s)}{1+C(s)P(s)\Sigma(s)}$$

so that aiming for $Y(s)/W(s) = F(s)$ results in

$$C(s) = \frac{F(s)}{P(s)(1-\Sigma(s)F(s))}.$$

Control synthesis

- Idea: require y_Σ to track not w but instead w filtered through Σ, *i.e.*,



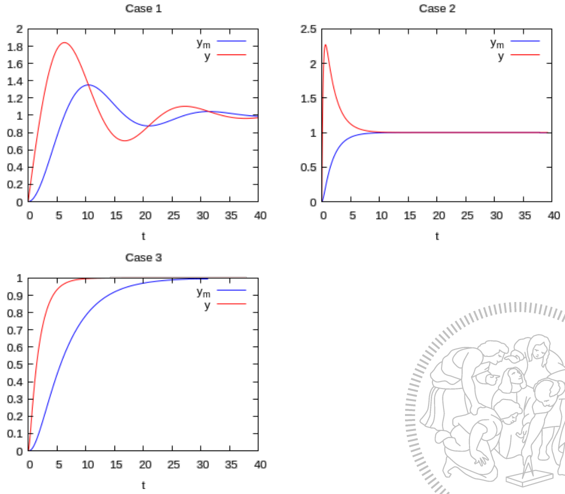
  where the controller includes a sensor *replica*.
- This is sensible, as if y_Σ tracks the output of Σ then y will track its input, that is w...
  ...as long as Σ *replica* in the controller matches the physical Σ in the process (closely enough in the band of interest).
- Doing so, one can tune $C(s)$ by shaping the loop frequency response $C(j\omega)P(j\omega)\Sigma(j\omega)$ as usual.

Example

— try changing Ts/Tp and ω_c, making the set point filter Sf (slightly) differ from sensor S,...

wxMaxima: simulation example!

```
data : [mu=1,Tp=1,Ts=5,wc=0.5];

P    : mu/(1+s*Tp);
S    : 1/(1+s*Ts);

C1   : wc/s/P;                  /* PI              */
C2   : wc/s/P/S/(1+s/(10*wc));  /* Real PID        */
Sf   : S;                       /* S filter for w  */

/* Closed-loop responses of y_m and y to a w step */

yc1  : ilt(subst(data,C1*P/(1+C1*P*S))/s,s,t);
yc2  : ilt(subst(data,C2*P/(1+C2*P*S))/s,s,t);
yc3  : ilt(subst(data,Sf*C2*P/(1+C2*P*S))/s,s,t);

ymc1 : ilt(subst(data,C1*P*S/(1+C1*P*S))/s,s,t);
ymc2 : ilt(subst(data,C2*P*S/(1+C2*P*S))/s,s,t);
ymc3 : ilt(subst(data,Sf*C2*P*S/(1+C2*P*S))/s,s,t);

wxplot2d([ymc1,yc1],[t,0,40],[legend,"y_m","y"]);
wxplot2d([ymc2,yc2],[t,0,40],[legend,"y_m","y"]);
wxplot2d([ymc3,yc3],[t,0,40],[legend,"y_m","y"]);
```



Wrap-up, remarks, lesson learnt

(prefilter)

- Clearly, whatever choice for Sf, preserve the unit gain (no need to explain why).
- We did not investigate the idea of tuning a feedback C by cancelling the (dominant) dynamics of S instead of P $\Rightarrow$ exercise. : better control on sensor BUT bad upstream control
- But anyway, is this matter energy-relevant?
- It can be. Overtemperatures (transiently) increase losses, see for example $y(t)$ in case 2 of the previous example. losses depends on average temp! y hotter, more loss on env.

  ↳ heating up too much, then you need to heat up less!
- In any case, most of the times the sensor dynamics is not the main point, however as we just saw this is not completely general. Keep in mind.

Foreword

Joint Temp/Flow control

when deliver heat.. Flow & heater temp. control independent? YES&NO! depending on POV

- Suppose you have to provide a certain heat rate to a body or ambient by means of a thermovector fluid (here, liquid).
- This means in general that you need to govern
  - a temperature or a temperature difference across some exchanger,
  - and possibly a fluid flowrate.

  in some cases to control independently...
- Are the two controls independent?
- Which are the energy-relevant aspects of the problem?
- Let us investigate.

{ there is some optimum point to control } ?

Controlled system

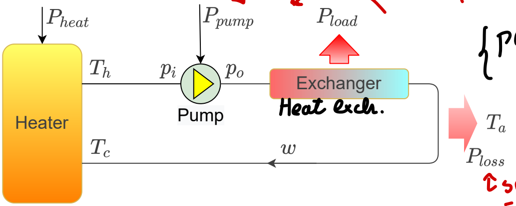
(SET-UP)

representative example

ΔP

ΔP same pressure drop, quadratic

{pump delivers to a load}



some losses to external environment) proportional to fluid flow

- Assuming no pump heating nor gravity effects (for simplicity and without impairing the following treatise) the heating and pumping powers are respectively

$$
\begin{cases}
P_{heat} = wc(T_h - T_c) \\
P_{pump} = w\underbrace{\frac{p_o - p_i}{\rho}}_{\Delta h \text{ no temp increases...}} = w\underbrace{K_{piping}w^2}_{\text{"}\Delta p/\rho = \Delta h} = K_{piping}w^3
\end{cases}
$$

cubic in the flow rate!

- pressure increase 1 bar : $\Delta h = \Delta p / \rho$

  - water $10^5/10^3 \simeq 100$
  - air $10^5/1 \simeq 10^5$ >> water

  $\simeq 1000$

- Temperature increase 10° : $\Delta h = c \Delta T$

  factor...

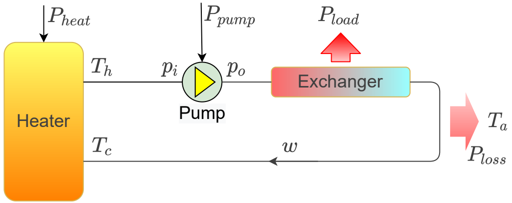
  - water $4186 \cdot 10 = 41860$
  - air $1020 \cdot 10 = 10200$

  $\simeq 4$

↓

to size heat network, pumping energy less than heating one...

Controlled system



- As for the loss power, we simplistically – yet here adequately – refer it to the average fluid temperature, and assume a constant-conductance exchange with an exogenous ambient temperature T_a; this yields

$$P_{loss} = G\left(\frac{T_h + T_c}{2} - T_a\right).$$

Control requirements

(assume we can control both)

{ so decoupling is not needed
band separation → so we can
control separately! }

- Let us assume to control w and T_h.
- This can be accomplished with two independent loops quite straightforwardly.
- But how to select the set points w°, T_h°?
- A reasonable criterion is to minimise the total power $P_{tot} = P_{heat} + P_{pump} + P_{loss}$, that is, given the load to fulfil,

Power delivered by heater...

$$\min_{w^\circ, T_h^\circ} P_{tot} \qquad s.t. \; P_{heat} = P_{load} + P_{loss}.$$

taking by load

- As in T-project years the KKT equations (already known at this point in E-project years) come later, we exploit the presence of only two variables.
- Just recall – in particular if we had more decision variables – that this is clearly constrained optimisation.

Set point determination

- wxMaxima:

```
e1     : Ploss = G*((Th+Tc)/2-Ta);
e2     : Pheat = Pload+Ploss;
e3     : Pheat = w*c*(Th-Tc);
s1     : solve([e1,e2,e3],[Ploss,Th,Pheat]);
sTh    : rhs(s1[1][2]);
sPheat : rhs(s1[1][3]);
Ppump  : Kpiping*w^3;
Ptot   : subst(s1,Pheat+Ppump);
```

→ which is Ploss

→ set Pheat

→ say another Pheat expression

solving the equations

extract interested results

substituting the solutions..

- We obtain (reintroducing the set point mark omitted in the script)

$$P_{tot} = K_{piping} w^{\circ 3} + \frac{2w^{\circ}c\,(P_{load} + G(T_c - T_a))}{2w^{\circ}c - G}.$$

In Maxima

(%i4) s1 : solve([e1,e2,e3],[Ploss,Th,Pheat]);

(%o4) $[[Ploss=-\frac{2\,G\,Tc\,c\,w-2\,G\,Ta\,c\,w+G\,Pload}{G-2\,c\,w}, Th=-\frac{Tc\,(2\,c\,w+G)-2\,G\,Ta+2\,Pload}{G-2\,c\,w}, Pheat=-\frac{2\,G\,Tc\,c\,w-2\,G\,Ta\,c\,w+2\,Pload\,c\,w}{G-2\,c\,w}]]$

↓

taking Th, Pheat solution as matrix formulation

⇓ @ end I get

(%i10) Ptot : subst(s1,Pheat+Ppump);

(%o10) $Kpiping\,w^3-\frac{2\,G\,Tc\,c\,w-2\,G\,Ta\,c\,w+2\,Pload\,c\,w}{G-2\,c\,w}$

Set point determination

Example 1 — just to give an idea of the orders of magnitude

with numbers..

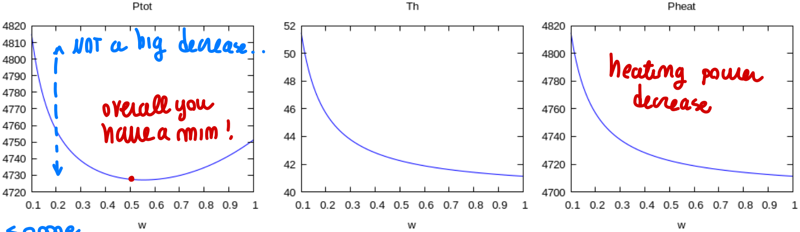
on average base, small house

- Small residential case:

DATA:
  - pressure drop $1m_{H_2O} \approx 10^4 Pa$ for a nominal flowrate of $0.5kg/s$ and $\rho = 1000kg/m^3$ $\Rightarrow K_{piping} = 40$; (on flow rate equation)
  - $c = 4186J/kg°C$, $P_{load} = 4000W$, $T_c = 40°C$ (must ensure a minimum fluid T); through the building
  - $T_a = 5°C$ (winter), $G = 20W/°C$ (quite poor insulation).

$\Downarrow$

- Result: depending on Flow rate W



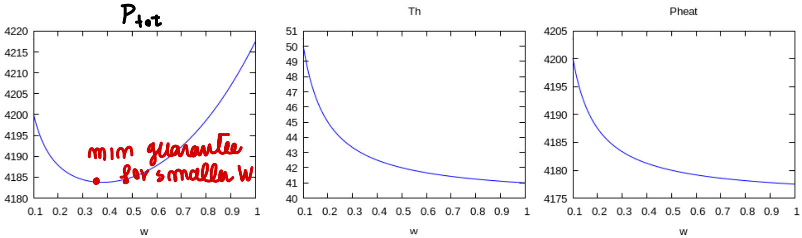
some percentage gain → you should stay with $W \simeq 0.5$

Set point determination

Example 2 — same purpose

→ simulation, quantities as order of magnitude

- Same as example 1 but with $G = 5W/°C$ (quite good insulation).
  with good insulation (less conductance)
- Result:

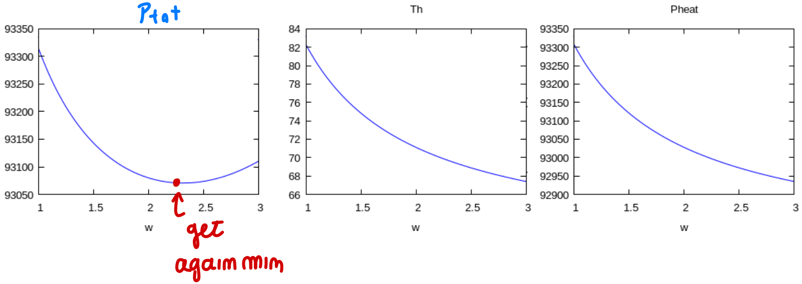


Set point determination

Example 3 — same purpose

- Slightly larger case:
    - pressure drop $5m_{H_2O} \approx 0.5bar$ for a nominal flowrate of $10m^3/h \approx 2.8kg/s$ and $\rho = 1000kg/m^3 \Rightarrow K_{piping} = 6.4$;
    - $c = 4186J/kg°C$, $P_{load} = 90kW$, $T_c = 60°C$; more thermal load
    - $T_a = 5°C$ (winter), $G = 50W/°C$ (good insulation).

- Result:



Wrap-up and lessons learnt

- We do not delve into mathematics to determine the optimum set points, as the assumptions we made are fine for evidencing the issue to face but not that much for precise computations (you have a library with realistic models for that).
- However:
  simulate different flow rate
  - though often w cannot be modulated and T_h° is computed based on ambient conditions with "climatic curves", if w control is available joint set point optimisation can be an option;
  - with liquid thermovectors P_{pump} should be far smaller (hardly ever above 1–2% or so) of P_{heat}, but in general pumps are electric, and an electric kWh (to date) costs more than a thermal one.
    ↳ depending on political choices
- In any case, when the objective to attain comes from the combined effect of two or more set points, and for these there are multiple choices that for the said objective are in fact equivalent, at least consider the possibility of introducing some set point optimisation.
  ↳ find optimal choice