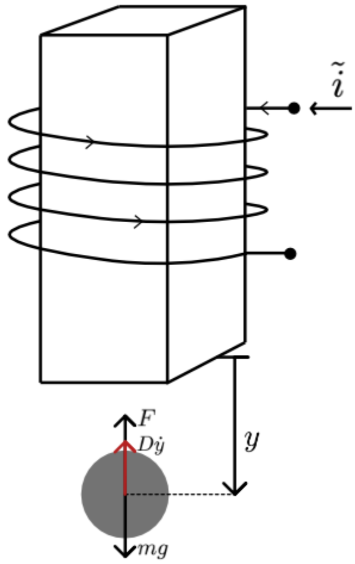
Laboratory 1 – Magnetic Levitation System



The simplified model of the system is

ball weight   viscous coefficent

$$m\,\ddot{y} = mg - D\dot{y} - \frac{\tilde{\imath}^2}{2(1+y)^2}$$

force generated by the coie

Where $m = 1$, $D = 1$. Denoting by $i = \tilde{\imath}^{\,2}$

assumptions

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = g - y_2 - \dfrac{i}{2(1+y_1)^2} \end{cases}$$

{s.s representation of the systhem}

Tasks

backstapping control.. usefull for NONLIN systems

1. Compute the constant input $\bar{\imath}$ such that $\bar{y}_1 = 1$ and $\bar{y}_2 = 0$ is an equilibrium.
2. Apply a change of coordinates which translates the equilibrium to the origin:

$$\begin{aligned} x_1 &= y_1 - \bar{y}_1 \\ x_2 &= y_2 - \bar{y}_2 \\ u &= i - \bar{\imath} \end{aligned}$$

backstep work on the origin equilibrium

   Then, draw the phase plane to study the stability of the origin.
3. Compute the backstepping control law.
   Test the closed-loop in Simulink, considering different initial condition of the system.
   Then, inspect the closed-loop phase plane.

→ by Gain block

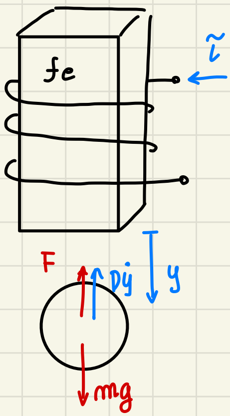
4. Assuming that only the position is measurable (y_1), design a Proportional controller.
   Test the closed-loop in Simulink, considering different initial condition of the system.
   Then, inspect the closed-loop phase plane.
5. Design a PI controller and test the closed-loop in Simulink.

Available files

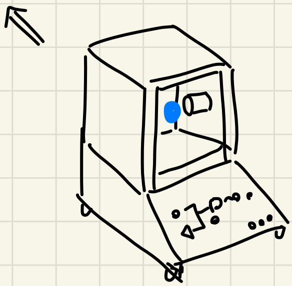
- `levitation_openloop` (open-loop model of the system)
- `levitation_ol` (phase-plane equations)

Magnetic Levitation system

(model simplified) machine able to stabilize spherical ball in a magnetic field → CONTROL position



$\tilde{i}$ create magn field on fe material



simplified model

$$m\ddot{y} = mg - D\dot{y} - \overbrace{\frac{\tilde{i}^2}{2(1+y)^2}}^{F \text{ (ferromagn force)}}$$

$m = 1, \quad D = 1$

$\Downarrow$ $\quad [i = \tilde{i}^2]$

state space model

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = g - y_2 - \dfrac{i}{2(1+y_1)^2} \end{cases}$$

$\Downarrow$

back stepping method for design REGULATOR

WORKS if $\bar{x} = \underline{0}$ (eq on origin) → otherwise change variable

↓ to define control action u which globally stabilize the system

1) compute equilibrium

from $\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = g - y_2 - \dfrac{i}{2(1+y_1)^2} \end{cases} \Rightarrow \bar{i} = 2g(1+\bar{y}_1)^2 = 8 \cdot g$

2) change of cohordinates

$$\begin{cases} x_1(\tau) = y_1(\tau) - \bar{y}_1 \\ x_2(\tau) = y_2(\tau) - \bar{y}_2 \\ u(\tau) = i(\tau) - \bar{i} \end{cases} \Rightarrow \begin{cases} y_1(\tau) = 1 + x_1(\tau) \\ y_2(\tau) = x_2(\tau) \\ i(\tau) = u(\tau) + 8g \end{cases}$$

↓ replace on s.s model

$$\begin{cases} \dot{x}_1(\tau) = x_2(\tau) \\ \dot{x}_2(\tau) = g - x_2(\tau) - \dfrac{u(\tau)+8g}{(2+x_1(\tau))^2 2} \end{cases}$$

than to compute pplane → system symulation

⇒ pplane10b

File > load system > levitation_ol

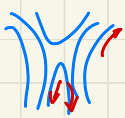
proceed

⇓

show phase plane

Simulate evolution from a given selected initial point (trajectory)

saddle point ⇒ unstable equilibrium



3) Backstepping formulation

from our system → use formule to obtain the state feedback controller...

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1) \cdot x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2) \cdot u \end{cases} \rightarrow \begin{cases} \dot{x}_1 = f(x_1) + g_1(x_1) \cdot x_2 \\ \dot{x}_2 = u_a \end{cases}$$

↳ control law $\quad u_a = -\dfrac{dV_1(x_1)}{dx_1} \cdot g_1 - K(x_2 - \phi(x_1)) + \dfrac{d\phi(x_1)}{dx_1}(f_1 + g_1 x_2)$

$$\begin{cases} f_1 = \emptyset & g_1 = 1 \\ f_2 = g - x_2 - \dfrac{8g}{2(2+x_1)^2} \\ g_2 = -\dfrac{1}{2(2+x_1)^2} \end{cases} \longrightarrow$$

$u_a \triangleq f_2 + g_2 \cdot u \Rightarrow u = (u_a - f_2)\dfrac{1}{g_2}$

$\Downarrow$

$x_2 = \phi(x_1) \quad \phi(0) = \emptyset$

$V(x_1) > 0 \quad \dot{V}(x_1) \leq 0$

$\Rightarrow \begin{cases} \dot{x}_1 = f_1 + g_1 \phi(x_1) = \phi(x_1) \\ \dot{x}_2 = u_a \end{cases}$

$V_1(x_1) = \frac{1}{2} x_1^2$ Lyapunov $\rightarrow$ $\dot{V}_1(x_1) = x_1 \dot{x}_1$

function of the states which goes to $\emptyset$... energy dissipation

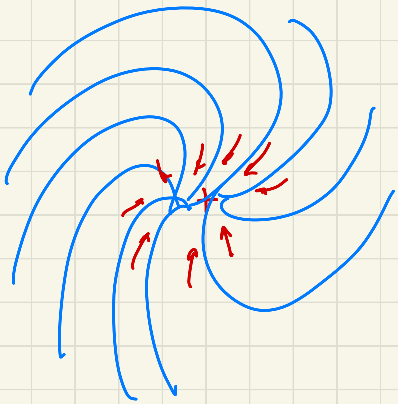
$\dot{V}(x_1) = x_1 \phi(x_1)$

$\hookrightarrow$ choosing $\phi(x_1) = -x_1$

$\dot{V}(x_1) = -x_1^2 < 0$ neg def

$\hookrightarrow u_a = -x_1 - K(x_1 + x_2) - x_2 = -x_1(1+K) - x_2 - Kx_2$

by simulation



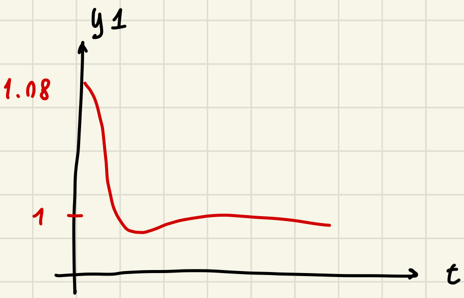
spiral traj

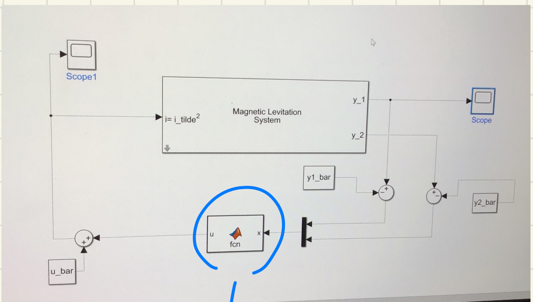
$\Downarrow$

global asymp. stable solution

$\downarrow$

simulink modelling of syst





$$u = 2(2+X(1))^2((1+k)x(1)+K\,x(2)+g)-8g$$

In this way we can't take into account saturation issue

4) P controller

↓

by Gain block on simulink

considering the original system to tune the P controller

→ it can be usefull to use the root locus from the initial system: Linearization

$$\begin{cases} \delta \dot{y}_1 = \delta y_2 \\ \delta \dot{y}_2 = -\delta y_2 + \dfrac{\cancel{4}(1+\bar{y}_1)\bar{i}}{\cancel{4}(1+\bar{y}_1)^{\cancel{4}\,3}} \delta y_1 - \dfrac{1}{2(1+\bar{y}_1)^2} \delta i \end{cases}$$

$$\begin{cases} \delta \dot{y}_1 = \delta y_2 \\ \delta \dot{y}_2 = g\, \delta y_1 - \delta y_2 - \frac{1}{8} \delta i \end{cases}$$

$\downarrow$

$A = \begin{bmatrix} 0 & 1 \\ g & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ -1/8 \end{bmatrix}$ $\underbrace{C = [1 \quad 0]}$ $D = [0]$

we measure as output y_1 look I state

↑ proper system

implement on Matlab

evaluate system TF to tune proportional controller $\rightarrow$ $G = tf(ss(A, B, C, D))$

poles = eig(A): eigvalues of the systhem

because $G(0) = -0.125$

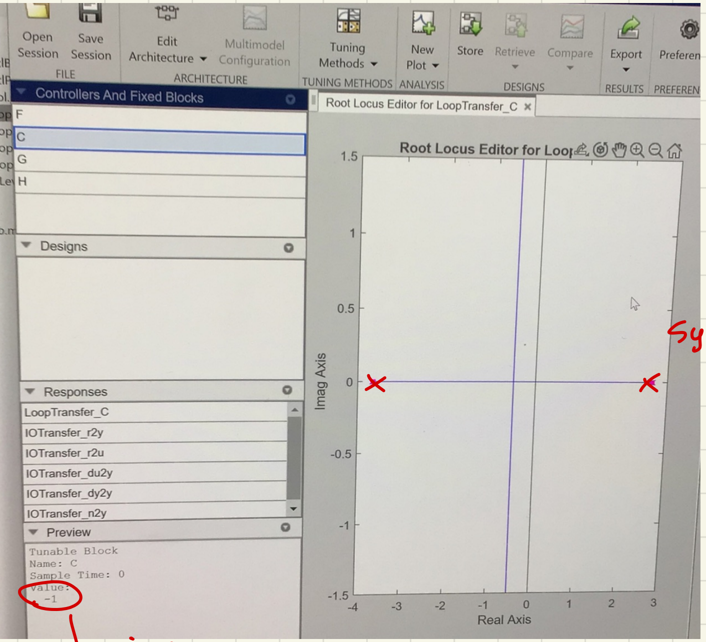
$\Downarrow$

try with $\boxed{K = -1}$ proportional Gain

$\Downarrow$

with rltool (G, Reg) → inspect Regulator

look C to change poles C ⇒ value = -1



syst poles

↓

we choice the Gain with $K=-80.4$

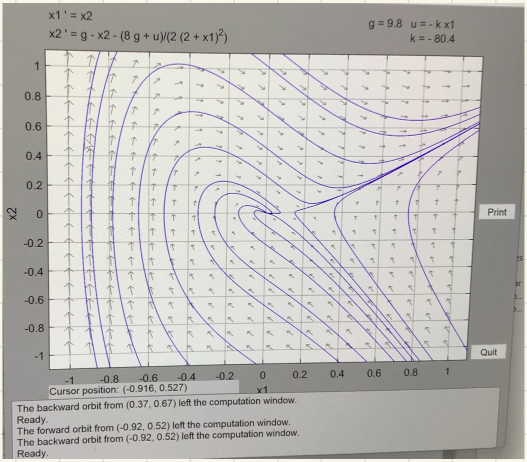
gain of controller

check also by pplane

↓

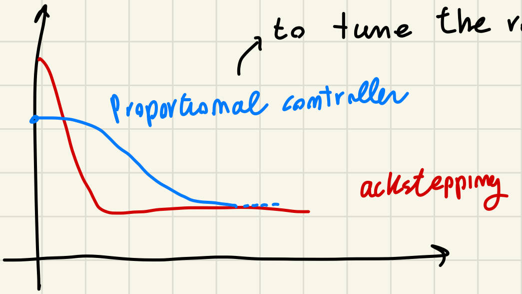
we specify $K = -80.4$

No more globally asymp. stable



stable only around the equilibrium

on simulink implementation



5) we need I action to ensure $\emptyset$ error

→ we implement PI controller

{ tuned by the controllers to obtain a less violent responce }