Exercises session 4: MIMO Analysis, Pole Placement and Zero Error regulation

Ex. 1: Given the discrete-time system

$$G(z) = \begin{bmatrix} \dfrac{z-1}{(z-0.5)^2} \\ \dfrac{z}{(z-0.5)^2} \end{bmatrix} \tag{1}$$

and the state space representation

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.251 & \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \tag{2}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{3}$$

Find

1. Find the poles and zeros of G(s),
2. Determine how many outputs can be regulated to constant references,
3. Verify there are no invariant zeros in z=1, using the state space model.
4. Assume the states are measurable, show how to design a pole placement scheme guaranteeing zero error robust regulation (refer to Figure 1)
5. In case the state are not measurable, can we use a static observer?

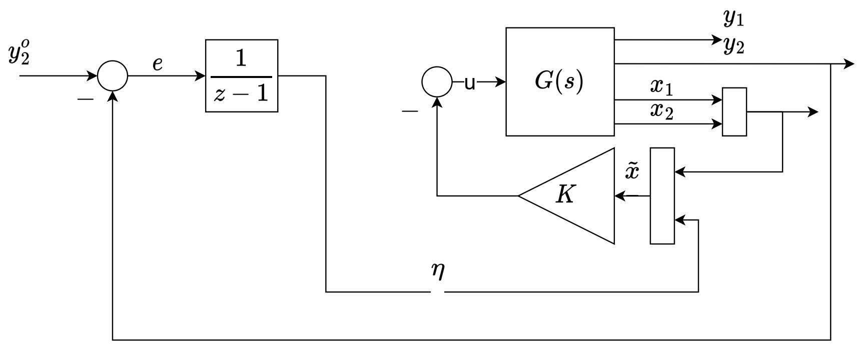

Figure 1

Ex. 2: Given the regulated system in Figure 2, the regulator matrix

$$R(s) = \begin{bmatrix} 1 & \dfrac{1}{s-0.5} \\ 0 & -\dfrac{s-0.5}{s+1} \end{bmatrix} \tag{4}$$

and transfer function

$$G(s) = \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{s-0.5} \\ 0 & \dfrac{1}{s+2} \end{bmatrix} \tag{5}$$

1. Compute the poles and zeros of $G(s)$,
2. Is it enough to check the stability of $S(s) = (I + G(s)R(s))^{-1}$ to check the stability of the closed loop?
3. Specify which transfer functions should be considered to asses the stability of the closed-loop system.

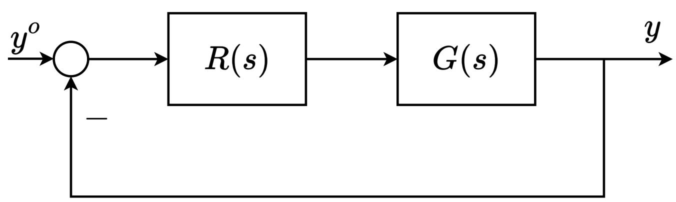

Figure 2

Ex. 3: Consider the continuous time system

$$\begin{cases} \dot{x} = x^3 - x \cdot u^2 \\ y = x \end{cases} \tag{6}$$

and the equilibrium point $\bar{x} = 1,\ \bar{u} = 1$. Design a pole placement controller that allows asymptotically stable equilibrium in two ways:

1. Pole placement to stabilize and a cascade PI for performances,
2. Pole placement on the enlarged system.

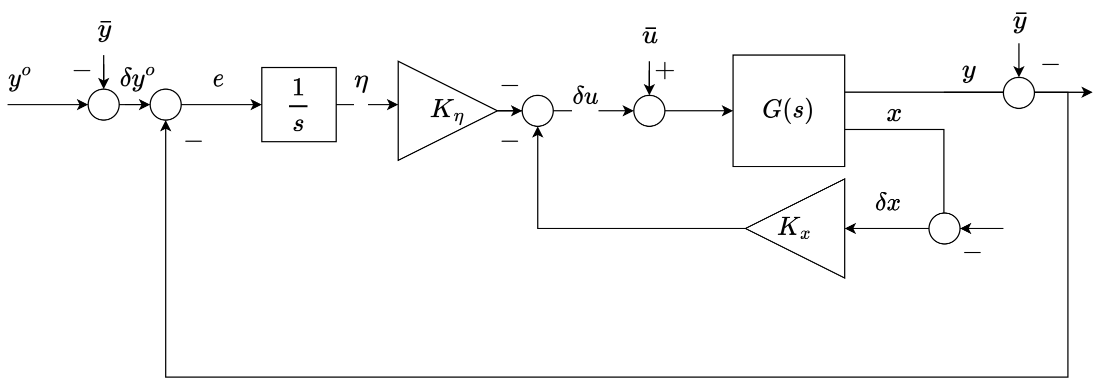

Figure 3

Ex. 4: Consider the following open loop transfer functions

$$L(s) = \begin{bmatrix} \dfrac{1}{\alpha s + 1} & \dfrac{\beta}{s + 2} \\ \dfrac{\gamma}{\alpha s + 1} & \dfrac{\delta}{s + 1} \end{bmatrix} \tag{7}$$

1. Compute the poles and zeros of the system
2. Which conditions must be imposed on α, β , γ and δ to study the stability of the closed loop system just analyzing $S(s) = (I + L(s))^{-1}$?
3. Which conditions must be imposed on α, β, γ and δ to design a closed loop controller with integral action?