ADVANCED AND MULTIVARIABLE CONTROL

July 9, 2021

Solutions

Surname .... Name ....

Identification code ....

Signature ....

AMC - July 2021

Dear students, in this exam you will find two types of answers:

1. closed form (questions 1-7,choose one over pre-defined answers), +3 for any correct answer, -0.33 for any wrong answer

2. open questions (questions 8 and 9) with answers to be written in sheets of paper, each one reporting your Surname, your Name, your ID Polimi, your signature, and the indication "answer to question xx". Please use the sheets of paper that have been made available to you in advance (Beep portal). At the end of the exam, you will have to prepare a unique pdf file containing all these answers and upload it in another form which will be made available to you.

Answers to questions 8 and 9 will not be corrected if the overall grade for closed form answers is lower than 8.2

During the exam, you will not be allowed to use books, notes, electronic devices (save for what you need at the end of the exam to prepare the pdf file and upload it). You cannot exchange any kind of information with anyone.

If something unusual happens, an oral exam will be required.

Good work!

1

(3 punti)

Consider the system

$$\dot{x}_1(t) = x_1(t)x_2(t) - x_1(t)$$
$$\dot{x}_2(t) = -x_1^2(t) - x_2^2(t) - x_2(t)$$

For this system it is possible to say that:

- ○ The origin is the only one equilibrium and it is asymptotically stable
- ○ The origin is a globally asymptotically stable equilibrium
- ☒ The origin is an equilibrium and it is asymptotically stable
- ○ The origin is a locally unstable equilibrium
- ○ no answer

equilibria $(\bar{x}_1 = \bar{x}_2 = 0)$, $(\bar{x}_1 = 0, \bar{x}_2 = 1)$

So, the origin cannot be <u>globally</u> asymptotically stable

Linearized system at the origin

$$\begin{cases} \delta\dot{x}_1 = -\delta x_1 \\ \delta\dot{x}_2 = -\delta x_2 \end{cases}$$

eigenvalues at $s=-1$ (double) $\rightarrow$ asymptotic stability

2

(3 punti)

Consider the discrete time system

$$x_1(k+1) = x_1(k) + 2x_2(k) + b_1 u(k)$$
$$x_2(k+1) = 3x_2(k) + b_2 u(k)$$
$$y(k) = x_1(k)$$

For this system compute the values b_1, b_2 such that it is **NOT** possible to design a regulator guaranteeing zero error regulation for constant reference signals.

- ○ b2 equal to 1.5*b1
- ○ b2=3 , b1=1
- ☒ b2 = b1
- ○ no conditions
- ○ no answer

The system must NOT have zeros in $z = 1$

System matrix (alternatively you can compute the transfer function)

$$P(z) = \begin{vmatrix} zI - A & -B \\ C & 0 \end{vmatrix} = \begin{vmatrix} z-1 & -2 & -b_1 \\ 0 & z-3 & -b_2 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\det P = 2b_2 - 3b_1 + b_1 z \to \Delta$$

$$z = 1 \to \Delta(1) = 2b_2 - 2b_1 \neq 0 \to b_1 \neq b_2$$

3

In order to study the robust stability of closed-loop systems with plants characterized by multiplicative perturbations, it is worth analysing the: (3 punti)

- [x] nominal complementary sensitivity function T(s)
- [ ] nominal sensitivity function S(s)
- [ ] nominal control sensitivity function K(s)
- [ ] both T(s) and S(s) because they are not independent
- [ ] no answer

See the notes

4

(3 punti)

Consider the system

$$\dot{x}(t) = x^5(t) + x(t)u(t)$$

and two control laws

$$u_{c1}(t) = -x(t)$$
$$u_{c2}(t) = -x^4(t) - x(t)$$

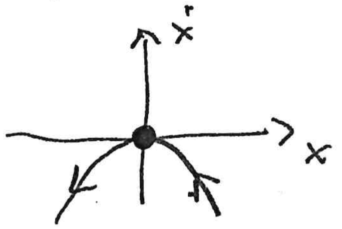
Select the correct answer:

- [ ] uc1 guarantees local asymptotic stability and uc2 global asymptotic stability
- [ ] both the control laws guarantee local asymptotic stability
- [x] none of them guarantee stability
- [ ] both guarantee global asymptotic stability
- [ ] no answer

1) $\dot{x} = x^5 - x^2$ no way to check the stability with Lyapunov function $V(x) = \frac{1}{2}x^2$

2) $\dot{x} = -x^2$ as before

Moreover



(Solution 2)

unstable equilibrium

As for solution 1 → higher order terms (x^5) do not modify the conclusions

5

Select the correct statement:
(3 punti)

○ The small gain theorem is a necessary and sufficient condition for the I/O stability of the closed-loop system

○ It can be applied to open-loop and closed-loop systems

○ It can be applied to feedback systems with integrators along the loop

⊗ If its applicability conditions are not verified, the feedback system can be I/O stable

○ no answer

It is just a sufficient condition

6

Concerning the Loop Transfer Recovery procedure, select the wrong answer
(3 punti)

○ can be applied only to continuous time systems

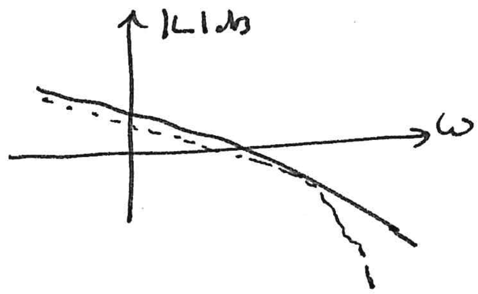
⊗ allows one to recover the performance of LQ control at all frequencies

○ is useless if the state of the system is measurable

○ consists in the design of a suitable observer

○ no answer

↳ no, in a limited frequency range.



example discussed in the lessons.

7

The Hamilton-Jacobi-Bellman equation for finite horizon problems:
(3 punti)

- [ ] Requires to compute on-line the solution of the HJB equation at any time instant in the considered horizon

not necessary to compute on-line

- [ ] Is based on necessary conditions for optimality
- [ ] no answer
- [x] Leads to a state feedback control law defined over the considered horizon
- [ ] Guarantees the asymptotic stability of the corresponding closed-loop system

8

(5 punti)

Consider the system

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

Formulate an MPC problem where you penalize the future error between the output and a given reference signal y^o, the future control variables and the future increments. Include in the problem formulation hard constraints on the future controls and control increments and soft constraints on the future errors. Use different control (N_u) and prediction ($N_y \geq N_u$) horizons.
Specify if it is possible to compute an explicit solution of the resulting optimization problem.

See the notes.

(7 punti)

Given the system

$$\dot{x}(t) = x^2(t) - x(t)u(t)$$
$$y(t) = x(t)$$

a. Compute the constant input $\bar{u}$ corresponding to the equilibrium $\bar{y} = 1$.
b. Compute the linearized model corresponding to the computed equilibrium.
c. For the computed linearized system, design a regulator with the pole placement approach guaranteeing closed-loop poles in $s = -1$ and asymptotic zero error for constant reference signals.
d. Draw the corresponding control scheme specifying the signals to be used (state and control variables, their variations with respect to the equilibrium values).

$\dot{x} = 0 \rightarrow \quad \bar{x}^2 - \bar{x}\bar{u} = 0$. For $\bar{x} = \bar{y} = 1 \rightarrow \bar{u} = 1$

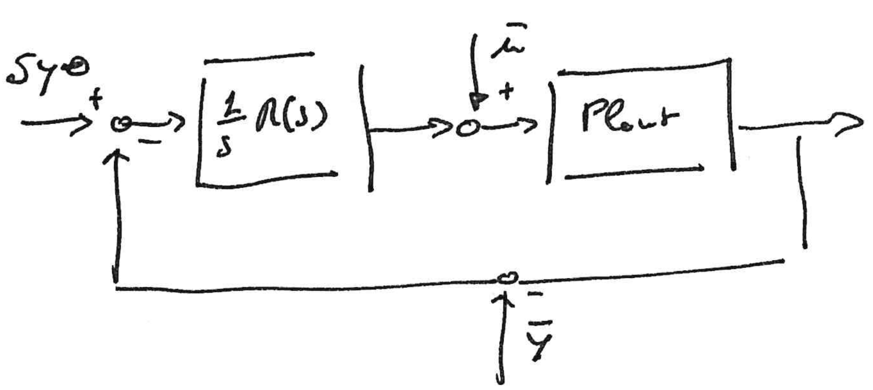
Linearized model

$\delta\dot{x} = \delta x - \delta u \rightarrow \quad G(s) = \dfrac{-1}{s-1}$

There are different ways to solve the problem.
The simplest one is $G(s) \longrightarrow \tilde{G}(s) = \dfrac{-1}{s(s-1)}$ add an integrator

$R(s) = \dfrac{f_1 s + f_0}{\gamma_1 s + \gamma_0}$, $P(s) = (s+1)^3 = s^3 + 3s^2 + 3s + 1$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \cdot \begin{vmatrix} \gamma_1 \\ \gamma_0 \\ f_1 \\ f_0 \end{vmatrix} = \begin{vmatrix} 1 \\ 3 \\ 3 \\ 1 \end{vmatrix}$$



But it is also possible to remove $(\bar{u}, \bar{y})$ because

$$y^o - y \Rightarrow \bar{y}^o + \delta y^o - \bar{y} - \delta y = \delta y^o - \delta y$$

and the contribution of $\bar{u}$ can be given by the integrator.

II solution

$$\delta \dot{x} = +\delta x - \delta u$$

$$\delta \dot{v} = y^o - x = \delta y^o - \delta y = \delta y^o - \delta x$$

$$\begin{vmatrix} \delta \dot{x} \\ \delta \dot{v} \end{vmatrix} = \underbrace{\begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix}}_{A} \begin{vmatrix} \delta x \\ \delta v \end{vmatrix} + \underbrace{\begin{vmatrix} -1 \\ 0 \end{vmatrix}}_{B} \delta u$$

For this system compute the pole placement control law

$$\delta u = -\underbrace{|K_x \quad K_v|}_{K} \begin{vmatrix} \delta x \\ \delta v \end{vmatrix}$$

such that $(A - BK)$ has eigenvalues in $s = -1$

Then