July 2013

Ex1

1. Poles of $G(s)$ $\rightarrow$ minors $\quad M_{12} = \dfrac{\alpha - 1}{\Delta}$, $\Delta = (s+1)(s+2)$

$$M_{13} = \frac{\alpha\beta - 1}{\Delta}$$

$$M_{23} = \frac{1-\beta}{\Delta}$$

poles $s = -1$, $s = -2$

2. Since we have 2 inputs, we can guarantee a zero steady state error only for two output variables

3. By looking at M_{13}, the condition is $\alpha\beta \neq 1$ so that the steady-state gain is nonsingular

4. The state representation can be written as

$$\begin{cases} \begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & -2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} \\ \begin{vmatrix} y_1 \\ y_3 \end{vmatrix} = \begin{vmatrix} \alpha & 1 \\ 1 & \beta \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \end{cases}$$

We want to guarantee that the system does not have invariant zeros at $s=0$ (derivative actions)

Therefore the following condition must hold

$$\det \begin{vmatrix} -A & -B \\ C & 0 \end{vmatrix} \neq 0$$

$$\downarrow$$

$$\det \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ \alpha & 1 & 0 & 0 \\ 1 & \beta & 0 & 0 \end{vmatrix} \rightarrow = (-1)^6 \det \begin{vmatrix} 1 & 0 & 1 \\ \alpha & 1 & 0 \\ 1 & \beta & 0 \end{vmatrix} = \alpha\beta - 1 \neq 0$$

The condition is $\alpha\beta = 1$, as already previously noted.

5. See the notes for the design of the regulator.

In any case, a dynamic observer is not required since $p > n$ and from the output transformation one can set

$$x = C^{+} y \quad , \quad C^{+} = \text{pseudoinverse of } C$$

$\boxed{\text{EX 2}}$

1.) Equilibrium

$$\begin{cases} \sigma(\bar{x}_2 - \bar{x}_1) = 0 \rightarrow \bar{x}_1 = \bar{x}_2 \\ (r-1)\bar{x}_1 - \bar{x}_1\bar{x}_3 = 0 \rightarrow \bar{x}_1(r-1-\bar{x}_3) = 0 \\ \bar{x}_1\bar{x}_2 - b\bar{x}_3 = 0 \end{cases}$$

Then

$$\bar{x}_1 = \bar{x}_2 = \bar{x}_3 = 0 \qquad \text{equilibrium 1}$$

$$\bar{x}_3 = r-1, \quad \bar{x}_1^{\,2} = b\bar{x}_3 = b\cdot(r-1) \rightarrow \bar{x}_1 = \bar{x}_2 = \sqrt{b(r-1)}$$

(only if $r>1$)

and

$$\bar{x}_1 = \bar{x}_2 = \sqrt{b(r-1)}, \quad \bar{x}_3 = r-1 \qquad \text{equilibrium 2}$$

2.) Linearised model at the origin

$$\begin{cases} \delta\dot{x}_1 = \sigma\,\delta x_2 - \sigma\,\delta x_1 \\ \delta\dot{x}_2 = r\,\delta x_1 - \delta x_2 \\ \delta\dot{x}_3 = -b\,\delta x_3 \end{cases} \longrightarrow A = \left|\begin{array}{cc|c} -\sigma & \sigma & 0 \\ r & -1 & 0 \\ \hline 0 & 0 & -b \end{array}\right|$$

one eigenvalue is $-b < 0$ $(b>0)$

The other two eigenvalues can be computed as the roots of the characteristic equation

$$\det \begin{vmatrix} s+\sigma & -\sigma \\ -r & s+1 \end{vmatrix} = s^2 + (\sigma+1)s - r\sigma + \sigma = 0$$

Since the system is second order, the conditions are

$$\sigma + 1 > 0 \qquad \checkmark$$

$$-r\sigma + \sigma > 0 \longrightarrow r < 1$$ This is the required condition

3.)

$$V(x) = \begin{vmatrix} x_1 & x_2 & x_3 \end{vmatrix} \underbrace{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0.5 \end{vmatrix}}_{P} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$= x_1^2 + 2x_1x_2 + 2x_2^2 + 0.5\,x_3^2$$

(Note that $P>0$)

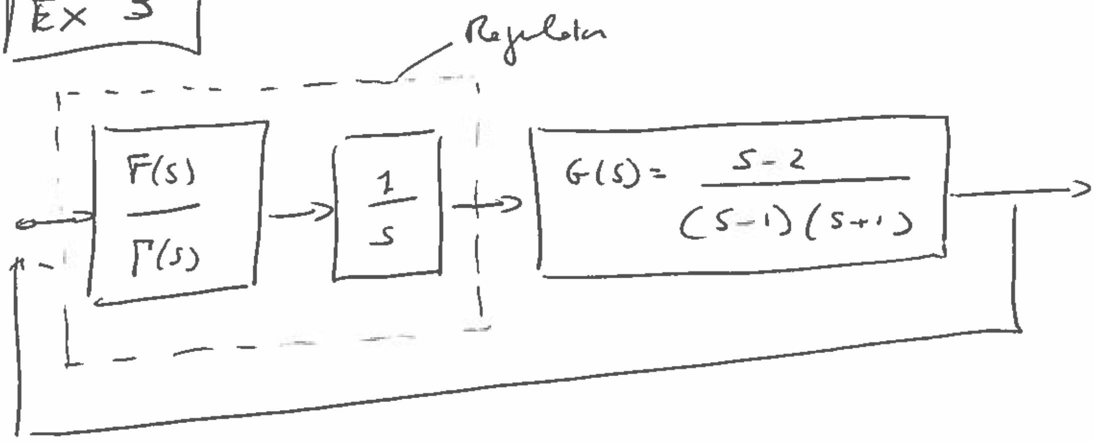
$$\dot V(x) = 2x_1\dot x_1 + 2x_2\dot x_1 + 2x_1\dot x_2 + 4x_2\dot x_2 + x_3\dot x_3$$

$$= \underbrace{-x_1^2 - 2x_2^2 - x_3^2}_{<0} + \left(-2x_1^2x_3 - 3x_1x_2x_3\right) < 0$$

these elements dominate
in the neighbor of the origin

$\boxed{\text{Ex 3}}$

1.)



$$A(s) = s(s-1)(s+1) = s^3 - s$$

$$B(s) = s-2$$

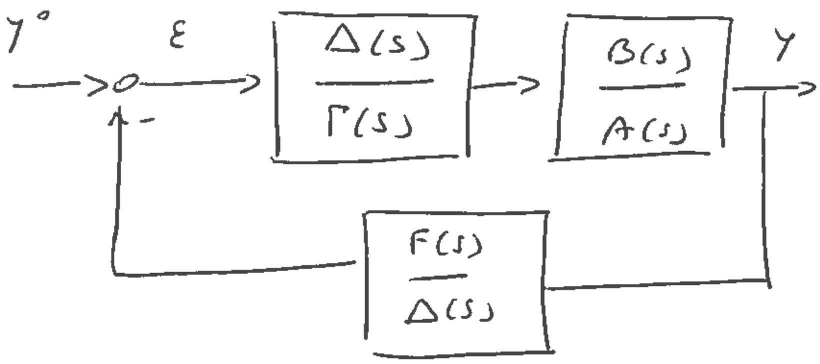
$$P(s) = (s+1)^5 = s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1$$

$$F(s) = f_2 s^2 + f_1 s + f_0$$

$$\Gamma(s) = \gamma_2 s^2 + \gamma_1 s + \gamma_0$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{vmatrix} \begin{vmatrix} \gamma_2 \\ \gamma_1 \\ \gamma_0 \\ f_2 \\ f_1 \\ f_0 \end{vmatrix} = \begin{vmatrix} 1 \\ 5 \\ 10 \\ 10 \\ 5 \\ 1 \end{vmatrix}$$

2.



$$y = \frac{\Delta B}{\Gamma A + BF} y^\circ = \frac{\Delta B}{P} y^\circ$$

$\Delta(s)$ = polynomial with the same order of $F(s)$ and $\Gamma(s)$

The roots of $\Delta(s)$ must be stable and $\Delta(s)$ should be chosen so that $\frac{F(0)}{\Delta(0)} = 1$ so that at the steady state ε is equal to the error $y^\circ - y$ and the presence of an integrator in $\Gamma(s)$ guarantees steady state zero error

$\boxed{\text{Ex } 4}$

1. See the notes

2. $k_1 = 0$, $\dfrac{1}{k_2} > 2 \longrightarrow k_2 < 0.5$

$\boxed{\text{Ex } 5}$

1.) $P(N) = 0$, $K(N-1) = 0$, $A - BK(N-1) = 4$

$P(N-1) = 1$, $K(N-2) = 1.33$, $A - BK(N-2) = 2.67$

$P(N-2) = 11.67$, $K(N-3) = 3.41$, $A - BK(N-3) = 0.58$

$N = 3$

2.) See the notes