ADVANCED AND MULTIVARIABLE CONTROL

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Exercise 1

Consider the system

$$\dot{x}_1(\mathrm{t}) = -x_2(\mathrm{t}) - x_1^3(\mathrm{t}) - x_1(t)x_2^2(t)$$
$$\dot{x}_2(\mathrm{t}) = x_1(\mathrm{t}) - x_2^3(\mathrm{t}) - x_1^2(\mathrm{t})x_2(\mathrm{t})$$

a. Show that the origin is an equilibrium.
b. Compute the linearized system at the origin and discuss the possibility to conclude something about the asymptotic stability of the equilibrium by looking at the eigenvalues of the linearized system.
c. Study the asymptotic stability of the origin with a quadratic Lyapunov function.

Solution Exercise 1

a. Setting the state derivatives to zero it is clear that the two state equations are satisfied for $x_1 = x_2 = 0$.
b. The linearized model at the origin is

$$\dot{\delta x}_1(\mathrm{t}) = -\delta x_2(\mathrm{t})$$
$$\dot{\delta x}_2(\mathrm{t}) = \delta x_1(\mathrm{t})$$

With eigenvalues *s=±j*. So, the conditions for the asymptotic stability, or for the instability of the equilibrium are not satisfied, and no conclusion can be drawn.

c. Consider the Lyapunov function $V(x) = 0.5(x_1^2 + x_2^2) > 0$. Correspondingly,

$$\dot{V}(x) = x_1(-x_2 - x_1^3 - x_1x_2^2) + x_2(x_1 - x_2^3 - x_1^2x_2) = -(x_1^2 + x_2^2)^2 < 0$$

So that the equilibrium is asymptotically stable.

Exercise 2

Consider the MIMO system with transfer function

$$G(s) = \begin{bmatrix} \dfrac{1}{(s+1.5)} & \dfrac{2}{(s+1)} \\ \dfrac{s}{(s+1.5)} & \dfrac{1}{(s+1)} \\ \dfrac{(s+1)}{(s+1.5)} & \dfrac{(s+0.5)}{(s-0.5)} \end{bmatrix}$$

a. Compute he poles and the zeros
b. Consider if it is possible to guarantee asymptotic zero error regulation for all the three outputs and constant setpoints.

Solution Exercise 2

Poles $s=0.5$, $s=-1$, $s=-1.5$, all of order one

There are no invariant zeros

No, it is not possible because the system has only two inputs and three outputs.

Exercise 3

Consider the system

$$x(k+1) = ax^2(k)u(k) - x(k)$$

a. Assume $a=1$, and consider a constant input $u=2$. Compute the equilibria.
b. Consider the non null equilibrium, compute the corresponding linearized model and the infinite horizon LQ control law with $Q=1,\ R=1$. Compute the corresponding closed-loop eigenvalue.

Riccati equation

$$P(k) = A'P(k+1)A + Q - A'P(k+1)B(B'^*P(k+1)B + R)^{-1}B'^*P(k+1)A$$

Solution Exercise 3

Equilibria

$\bar{x} = 2\bar{x}^2 - \bar{x} \rightarrow (\bar{x} = 0, \bar{x} = 1)$

Linearized model (non null equilibrium)

$$\begin{aligned}&\delta x(k+1) = 2a\bar{x}\,\delta x(k) - \delta x(k) + a\bar{x}^2\delta u(k)\\&\delta y(k) = \delta x(k)\\&\downarrow\\&\delta x(k+1) = 3\delta x(k) + \delta u(k) \rightarrow A = 3, B = 1\end{aligned}$$

LQ control

Steady state Riccati equation

$$\bar{P} = 9\bar{P} + 1 - \frac{9\bar{P}^2}{\bar{P}+1} \rightarrow \bar{P} \approx 9.1 \rightarrow \bar{K} = (B\bar{P}B' + R)^{-1}B'\bar{P}A \approx 2.7 \rightarrow A - B\bar{K} \approx 0.3$$

Exercise 4

a. Given the discrete time system $G(s) = \frac{b}{s+a}$ design a pole placement regulator with integral action (specify the number of closed-loop poles and the set of equation to be solved).

b. With reference to a generic pole-placement problem, show the scheme to be implemented in order to avoid that the zeros of the regulator are zeros of the transfer function from the reference signal to the output.

Solution Exercise 4

Enlarged system

$$G_e(s) = \frac{b}{s(s+a)}$$

Regulator

$$R'(s) = \frac{f_1 s + f_0}{\gamma_1 s + \gamma_0} \quad , \quad R(s) = \frac{1}{s} R'(s) = \frac{1}{s} \frac{F(s)}{\Gamma(s)}$$

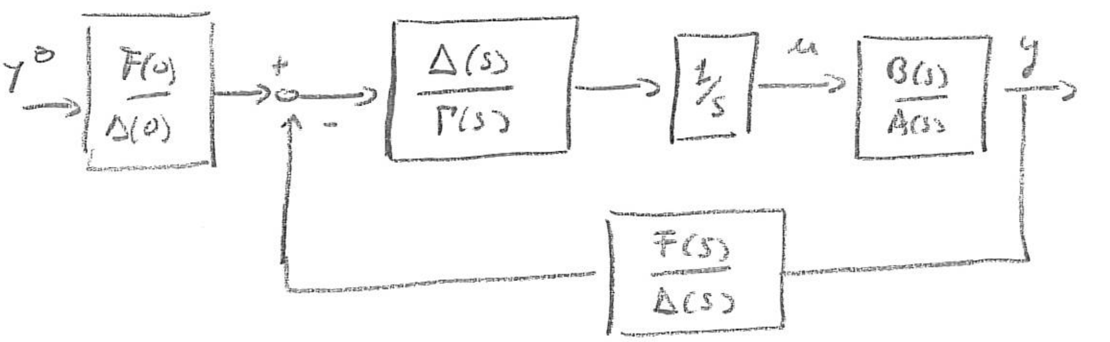
Polynomial of the closed-loop poles

$$P(s) = s^3 + p_2 s^2 + p_1 s + p_0$$

Equations to be solved

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & 0 & 0 & b \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_0 \\ f_1 \\ f_0 \end{bmatrix} = \begin{bmatrix} 1 \\ p_2 \\ p_1 \\ p_0 \end{bmatrix}$$

Control scheme



Δ(s) is of the same order of F(s), Γ(s) and with stable roots

Exercise 5

With reference to linear, continuous-time, multivariable systems describe what is the induced norm of a transfer function matrix, the directionality problem and the role played by the maximum and minimum singular values.

Solution Exercise 5

See the notes.