

Automation of Energy Systems

Project for the academic year 2022/2023

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Assumptions:

- Rigid network in case of multiple generators (we don't have any mechanical parameter to build a swinging model)
- generators can be activated/deactivated at any time, useful assumption when solving the optimization problem. (Even if real system behavior can be different)

PROBLEM STATEMENT:

AC Grid, $f_0 = 50 \text{ Hz}$, Three generators:

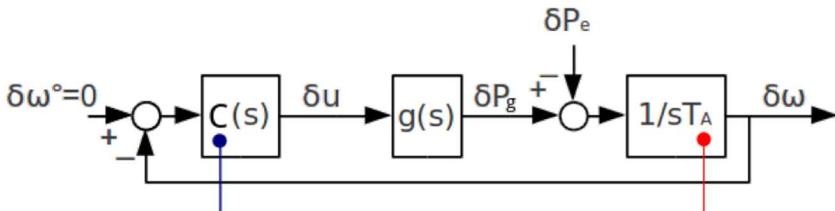
$$\mathbf{G1:} P_{n1} = 100 \text{ MW} , \tau_1 = 10 \text{ sec}, J_1 = 20 \frac{KJ}{\left(\frac{r}{s}\right)^2} , c_1(P_{g1}) = 0.5 + 1.5 * (P_{g1}-95)^2$$

$$\mathbf{G2:} P_{n2} = 50 \text{ MW} , \tau_2 = 5 \text{ sec}, J_2 = 10 \frac{KJ}{\left(\frac{r}{s}\right)^2} , c_2(P_{g2}) = 1 + 3 * (P_{g2}-48)^2$$

$$\mathbf{G3:} P_{n3} = 150 \text{ MW} , \tau_3 = 20 \text{ sec}, J_3 = 10 \frac{KJ}{\left(\frac{r}{s}\right)^2} , c_3(P_{g3}) = 1.5 + 4 * (P_{g3}-140)^2$$

1) Configuration A, control

To set up and tune the control scheme for configuration A we can work on the two sub-grids separately, starting with the **islanded generator (G3 feeding L3+L4) grid**:



from the problem data we can compute all blocks:

$$\tau_3=20 \text{ sec}, J_3=10 \text{ kJ/(r/s)}^2, P_{n,3}=150 \text{ MW}$$

$$g(s) = \frac{1}{1+20s} \quad T_A = \frac{J_3 * \omega_0^2}{P_{n,3}} = 6.573 \text{ sec}$$

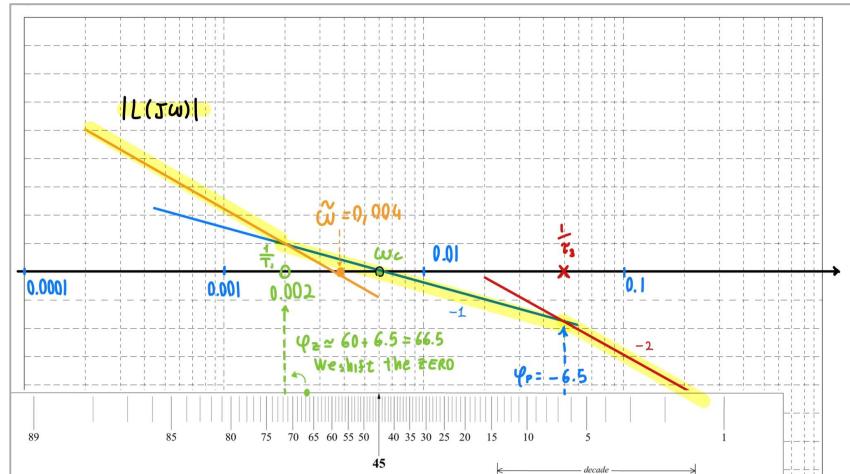
If we **start tuning C(s) as a PI controller** by loop shaping, we find our loop transfer function specification from the problem request:

$\delta\omega \leq 0.05$: this will become a request on the gain and on the speed of the system to avoid too much frequency loss
 $t_{set} \approx 15 \text{ min} = 900 \text{ sec}$: correspond to system speed, so request on the cut-off frequency ω_c

$$5/\omega_c \leq 900 \rightarrow \omega_c \geq 0.006 \text{ rad/s}$$

Then we ask to have a good non oscillatory response, corresponding to an high phase margin $\Phi_m \geq 60^\circ$

Doing our loop shaping procedure by hands, we set all our specification (phase one with the help of a phase ruler)



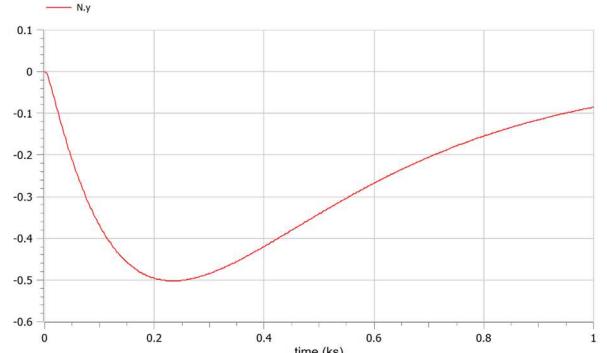
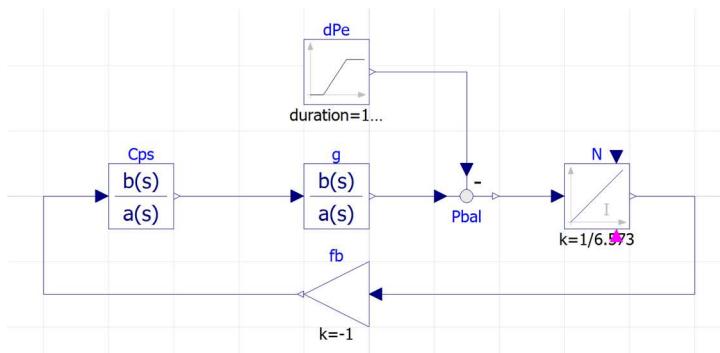
From L(s) shape we can compute the regulator TF as:

$$C(s) = \frac{1+500s}{500s} * 0.052$$

Simulating on Modelica our control scheme we can check our solution, with a ramp δPe input

(this solution is very bad just looking on the response, we don't even look on the feasibility of the control action)

MODELICA:



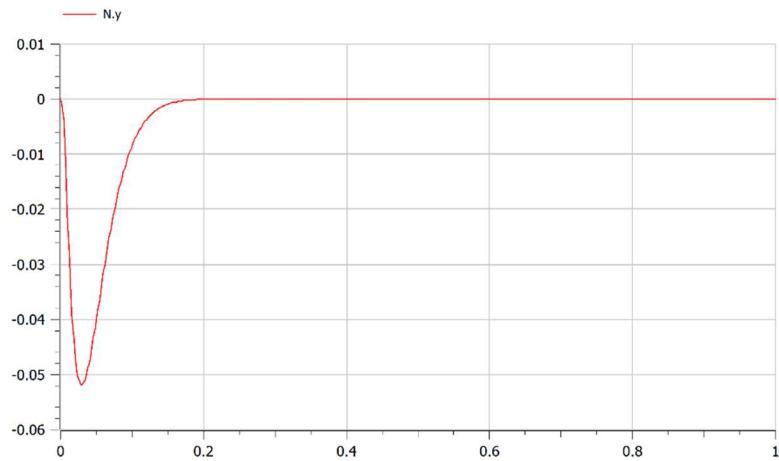
As we can see $\delta\omega$ with a simple PI control does not respect the request on max 0.05 phase loss, and also the settling time is not respected well. This is probably because setting up the controller to have 15min settling time we obtain an extremely slow control action, so the frequency error is not well compensated in time.

To solve this problem we can try to use a PID controller, this allow us to increase the gain and speed up the system, without loosing too much phase margin.(in fact the initial phase of the response is given by the proportional (primary) contribution of the controller, that we want to make stronger)

Now we set up $\omega_c = 0.06$, one PID zero is used to cancel the generator pole, and place the PID pole in 0.6, and we place the other zero in 0.022 to obtain $\varphi_m \approx 60^\circ$. We obtain:

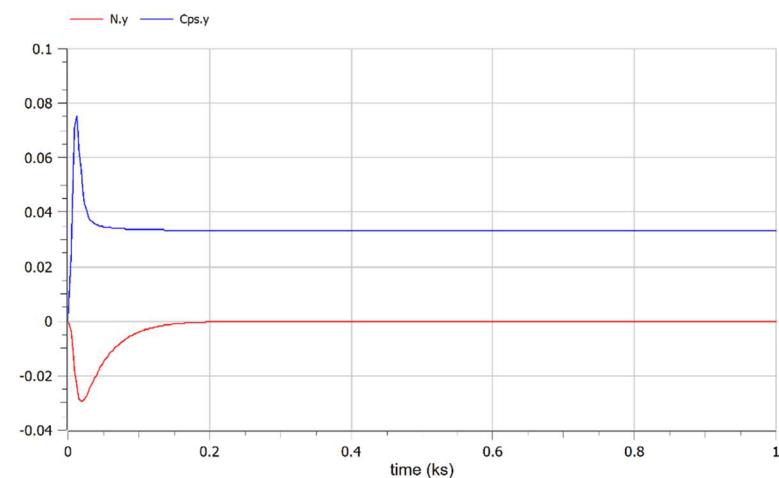
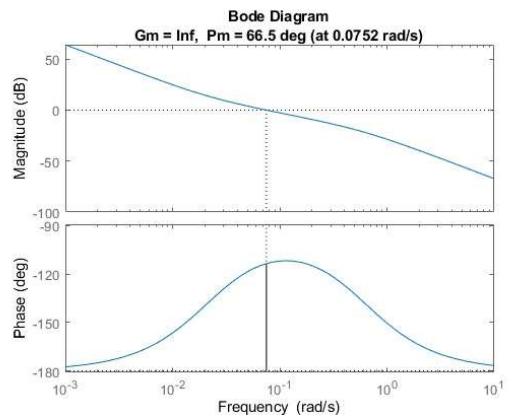
$$C(s) = 6.573 * (0.04)^2 * \frac{(1+45.45s)*(1+20s)}{(1+1.67s)*s}$$

using the same Modelica scheme as before, we simulate our response



We try simply by doubling the control gain, and as we can see from Modelica simulation it is our final Control C(s)

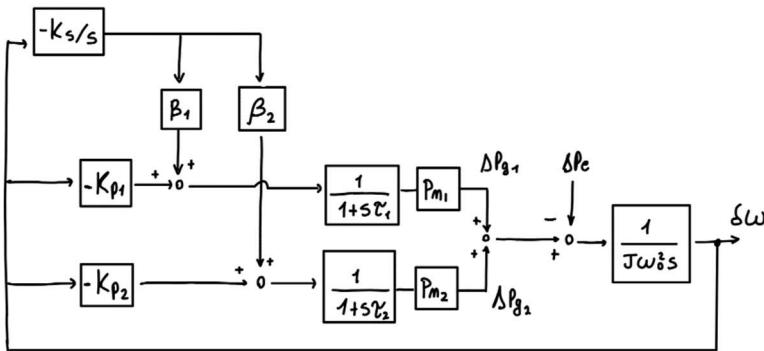
This time we have a better settling time but still not enough robustness respect $\delta\omega$ limits. Furthermore, looking the Bode plot of L(s) magnitude and phase, we prefer to increase the gain and shift on the right ω_c to stay in the better region of the phase, where a gain increase cause a phase loss (while now we are in the other side). With this design choice we also increase the proportional action and allow for less frequency error.



As we can see from the simulation, now the response is perfect. We check if the control action was reasonable, and as we can see it is ok!

$$C(s) = 2 * 6.573 * (0.04)^2 * \frac{(1+45.45s)*(1+20s)}{(1+1.67s)*s}$$

Network generators (G1,G2 feeding L1+L2) grid:



from our data:

$$g_1(s) = \frac{1}{1+10s} \quad P_{n,1}=100\text{MW}$$

$$g_2(s) = \frac{1}{1+5s} \quad P_{n,2}=50\text{MW}$$

We find the equivalent alternator dynamic from power to freq. error considering an overall inertia of the parallel generators as the sum of the two inertias:

$$J_1=20 \text{ KJ}/(\text{r/s})^2, \quad J_2=10 \text{ KJ}/(\text{r/s})^2$$

$$\frac{10^6}{(J_1+J_2)*\omega_0^2} = \frac{1}{2960.88}$$

(the 10^6 factor is because we are using data in MW)

To set up the secondary contribution proportional to generator nominal powers as $\beta_1=1/3, \beta_2=2/3$

We have to guarantee the same specification of the islanded case. Proceed by imposing a desired $L(s)$ and computing $C(s)$ as a PID, then invert the dynamic (loop shaping technique as before). In particular K_{p1}, K_{p2} and K_s are our degrees of freedom to obtain an equivalent parallel control scheme as the desired transfer function.

We can solve this in **Maxima**:

$$\begin{aligned} \rightarrow L &= (((ks \cdot 0.66/s + kp1) \cdot 100/(1+s \cdot 10)) + (ks \cdot 0.33/s + kp2) \cdot 50/(1+s \cdot 5)) \cdot 1/2960/s; \\ \text{(%)10} \quad L &= \frac{100 \left(\frac{0.66 ks}{s} + kp1 \right)}{10 s + 1} + \frac{50 \left(\frac{0.33 ks}{s} + kp2 \right)}{5 s + 1} \\ &= \frac{2960 s}{2960 s} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{ratsimp}(\%) \\ \text{rat: replaced } 0.33 \text{ by } 33/100 = 0.33 \\ \text{rat: replaced } 0.66 \text{ by } 33/50 = 0.66 \end{aligned}$$

$$\text{(%)15} \quad L = \frac{(200 kp2 + 200 kp1)s^2 + (198 ks + 20 kp2 + 40 kp1)s + 33 ks}{59200 s^4 + 17760 s^3 + 1184 s^2}$$

$$\rightarrow F = 10 \cdot ((0.0041^2/s^2) \cdot (1+s \cdot 333.33)/(1+s \cdot 5));$$

$$\text{(%)24} \quad F = \frac{1.68110^{-4} (333.33 s + 1)}{s^2 (5 s + 1)}$$

$$\frac{(200 kp2 + 200 kp1)s^2 + (198 ks + 20 kp2 + 40 kp1)s + 33 ks}{59200 s^4 + 17760 s^3 + 1184 s^2} = \frac{0,1163 s + 0,0003601}{s^2 \cdot (5s + 1)}$$

$$\frac{(200 kp2 + 200 kp1)s^2 + (198 ks + 20 kp2 + 40 kp1)s + 33 ks}{1184 \cdot s^2 \cdot (1+10s) \cdot (1+5s)} = \frac{0,11634 + 0,0003601}{s^2 \cdot (1+5s)}$$

$$\frac{(200 kp2 + 200 kp1)s^2 + (198 ks + 20 kp2 + 40 kp1)s + 33 ks}{1184 \cdot (1+10s)} = (0,1163 s + 0,0003601) \cdot$$

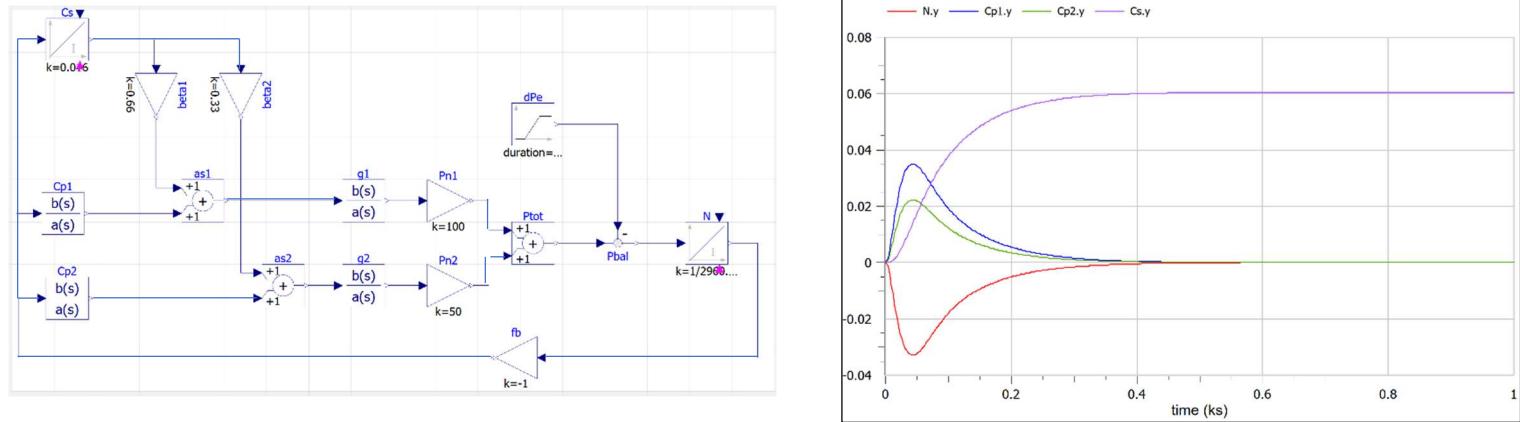
$$(200 kp2 + 200 kp1)s^2 + (198 ks + 20 kp2 + 40 kp1)s + 33 ks = 1377 s^2 + 142 s + 0,4264$$

$$\begin{aligned} \rightarrow \text{EQSYS3:} & [200 \cdot kp2 + 200 \cdot kp1 - 1377, 198 \cdot ks + 20 \cdot kp2 + 40 \cdot kp1 - 142, 33 \cdot ks - 0.4264]; \\ \text{(%)16} \quad & [200 kp2 + 200 kp1 - 1377, 198 ks + 20 kp2 + 40 kp1 - 142, 33 ks - 0.4264] \end{aligned}$$

$$\begin{aligned} \rightarrow \text{solve(EQSYS3, [kp1,kp2,ks])}; \\ \text{rat: replaced } -0.4264 \text{ by } -533/1250 = -0.4264 \end{aligned}$$

$$\text{(%)17} \quad \left[kp1 = \frac{2177}{25000}, kp2 = \frac{42487}{6250}, ks = \frac{533}{41250} \right]$$

After the initial mathematical computation of the gains, we adapt it repeating the simulation multiple times in the attempt to properly load each generator, so we weight the k_p depending on the nominal power of each generator, and we look to the control action of each controller to verify our goal.



$$K_p1 = 1.064, \quad K_p2 = 0.677327, \quad K_s = 0.016$$

Notice that during our tuning procedure (here and in future when approaching point 3) we don't limit our checks on the behavior of the frequency error. We focus on the overall system, taking care of obtaining a feasible control action, without a nervous actuation signal, and properly partitioning the generator load with respect to their capability (P_n).

So the results shown are the outcome of several attempts done with the knowledge of the internal system behavior and the influence of each control gain on the final response (and with a little bit of magic an tons of errors).

2) Configuration A, Optimization

To fulfill the request we should approach an optimization problem. Which in case of 2 generators is easy to solve, having few possible combinations to compare. From our data:

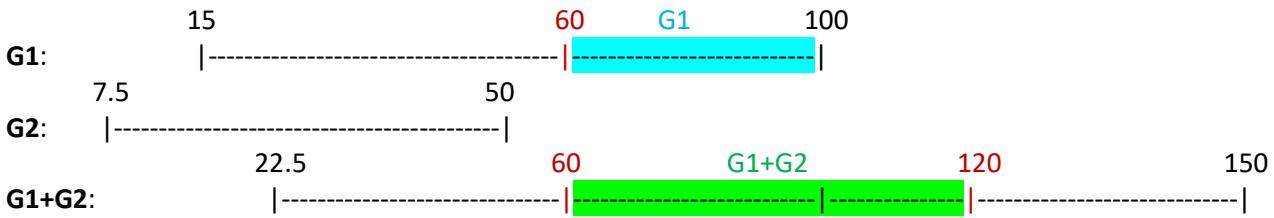
$$\begin{array}{ll} P_{g1,min} = 15 \text{ MW} & P_{g1,max} = 100 \text{ MW} \\ P_{g2,min} = 7.5 \text{ MW} & P_{g2,max} = 50 \text{ MW} \end{array} \quad \begin{array}{l} c_1(P_{g1}) = 0.5 + 1.5 * (P_{g1} - 95)^2 \\ c_2(P_{g2}) = 1 + 3 * (P_{g2} - 48)^2 \end{array}$$

Being on configuration A, with G1,G2 feeding L1,L2 (on the other subnet there is nothing to optimize), we can consider our network demand P_e as the sum of the two loads request, for each interval S_i .

(Notice: all power numbers in [MW] from now on..)

Obtaining:	S1	S2	S3	S4	S5	S6	
P_e :	60	70	120	120	110	60	$[P_{e,min}=60, P_{e,max}=120]$

We can graphically represent our feasible generator combinations for each net demand:



It is clear that G2 alone is never a valid solution, while for our 2 possible intervals

- I₁: [60,100] → (G1, G1+G2) [here I have to compare between the two solutions which one has the min cost rate]
- I₂: [100,120] → (G1+G2) [while here I just need to find the optimal pool of P_{g1} and P_{g2}]

When dealing with just G1 we need to compute the cost rate using a power of $P_{g1}=P_e$ (All net request satisfied by G1). While in the G1+G2 case, we should properly use the two generator, respecting their constraints and the overall net demands. So it is necessary to set up an optimization problem from our data that can be solved with KKT equations. (to have the total minimum expenditure we want to minimize the overall cost rate for each interval S)

Let's start considering the case for $P_e=60$ (S1 and S6):

Discuss step by step how to find the optimal solution and then trivially report the optimal result for the other P_e . $P_e \in I_1$ so we have to compare the two possible solutions of G1 or G1+G2, choosing the one with minimum cost:

G1:

$P_{g1} = P_e = 60$ (the only constrain is to fulfill all the net request, possible because in the range of P_{g1})
 $c_1 = 0.5 + 1.5 * (60 - 95)^2 = 1838$

G1+G2:

Setting up the optimization problem:

Cost function: $f(P_{g1}, P_{g2}) = c_1(P_{g1}) + c_2(P_{g2})$

$$\text{s.t. } \begin{cases} P_{g1} + P_{g2} = P_e \\ P_{g1,min} \leq P_{g1} \leq P_{g1,max} \\ P_{g2,min} \leq P_{g2} \leq P_{g2,max} \end{cases} \rightarrow \text{as KKT formulation:}$$

General formulation (here $P_e=60$)

$$\begin{aligned} f &= 1.5 + 1.5 * (P_{g1} - 95)^2 + 3 * (P_{g2} - 48)^2 \\ g &= P_{g1} + P_{g2} - P_e \\ h_1 &= P_{g1} - P_{g1,min} \\ h_2 &= P_{g1,max} - P_{g1} \\ h_3 &= P_{g2} - P_{g2,min} \\ h_4 &= P_{g2,max} - P_{g2} \end{aligned}$$

$$L = f + \lambda * g + \mu * h \quad (\text{Lagrangian})$$

we can solve our problem using Maxima (or any software capable of computing derivates and solve a system of equation).

So the next step is to properly write the KKT equations, solve it and understand the meaning of each solution to select the optimum (min).

MAXIMA:
(using our data)

```
Pg1min : 15;
Pg1max : 100;
Pg2min : 7.5;
Pg2max : 50;
c1 : 0.5+1.5·(Pg1-95)^2;
c2 : 1+3·(Pg2-48)^2;
f : c1+c2;
Pe : 60;
g : Pg1+Pg2-Pe;
h1 : Pg1-Pg1min;
h2 : Pg1max-Pg1;
h3 : Pg2-Pg2min;
h4 : Pg2max-Pg2;
L : f+lambda·g+mu1·h1+mu2·h2+mu3·h3+mu4·h4;
KKTEqs : [diff(L,Pg1),diff(L,Pg2),
diff(L,lambda),
mu1·diff(L,mu1),mu2·diff(L,mu2),
mu3·diff(L,mu3),mu4·diff(L,mu4)];
S : solve(KKTEqs,[Pg1,Pg2,lambda,mu1,mu2,mu3,mu4]);
for i:1 thru length(%rnum_list) do S:subst(t[i],%rnum_list[i],S);
float(S);
fvals : float(makelist(subst(S[i],f),i,1,length(S)));
```

Just reading the final output (so the cost function values for the optimal solutions), we are able to tell that for $P_e=60$ the best choice is to use just G1 to feed all the Net. In fact the list of possible cost function values is:
[6890.5, 9628.5, 23271.0, 7631.625, 10851.0]

So even if we have to properly discard the unfeasible one and local optimum, the smallest cost is 6890.5 which is bigger than the cost $c_1=1838$ found using only G1!

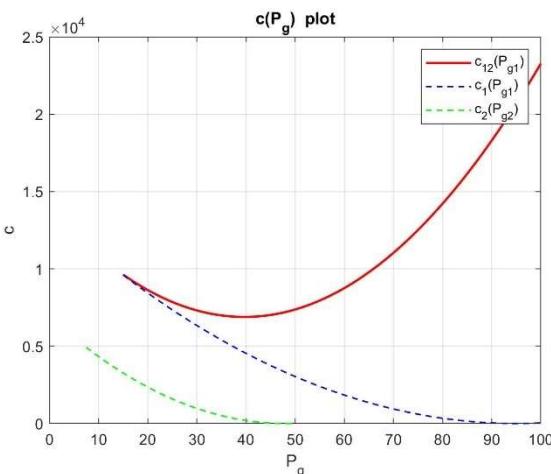
(this is because of the cost rate function shape, which is of second order in P_g , so a parabola. This means that increasing P_g the cost rate decreases until the minimum is reached, so sometimes it can be better to feed all the net just using one generator instead of dividing the request on more Generators).

For the seek of clarity we can anyway discuss the solutions and try to understand their meaning:

P_{g1}	P_{g2}	λ	μ_1	μ_2	μ_3	μ_4	f	optimal sol.
39.67	20.33	166	0.00	0.00	0.00	0.00	6890.5	
15.00	45.00	18.0	222	0.00	0.00	0.00	9628.5	left extr of P_{g1} ($\mu_1 \neq 0$)
100.0	-40.0	528	0.00	543	0.00	0.00	23271	unfeasible ($h_3 < 0$)
52.50	7.50	127.5	0.00	0.00	115.5	0.00	7631.625	left extr of P_{g2} ($\mu_3 \neq 0$)
10.0	50.0	255	0.00	0.00	0.00	267	10851	unfeasible ($h_1 < 0$)

We could also check our results **graphically**:

In fact, plotting in 2D $f(P_{g1}) = c_{12}$ for $P_e=60$ (we can substitute in $f(P_{g1}, P_{g2})$, $P_{g2}=P_e-P_{g1}$ for our constraint)
[in case of 2 generators is easy to plot the cost rate as function of just one generator power]



From c_{12} plot we can confirm that in case of two generators combination G1+G2, the optimal solution correspond to a value of $P_{g1}=39.66..$ (min of the curve c_{12}) But using of only G1, looking to the curve c_1 , setting $P_{g1}=60$ provide a better solution!
(in the plot there is also $c_2(P_{g2})$, which highlight how the usage of only G2 could be an advantage on a range where $P_e \leq 50$)

So for $P_e=60$ the optimal solution is using only G1:
[$P_{g1}=60$, $P_{g2}=0$] (G2 not even activated, or it must stay on its range).

Same reasoning is for $P_e=70$ (S2):

Still $P_e \in I_1$

G1:

$P_{g1} = P_e = 70$

$c_1 = 0.5 + 1.5 * (70 - 95)^2 = 938$ (this result can be seen from the plot above)

G1+G2:

Same KKT formulation and maxima script, except for the value of $P_e=70$

Taking directly the best solution (discharging extremum and unfeasible ones): $P_{g1}=46.34$, $P_{g2}=23.66$

Which correspond to a cost $f = 5330.5$

Also for $P_e=70$ the optimal solution is using only G1: $[P_{g1}=70, P_{g2}=0]$

For Pe=120 (S3 and S4):

In this case instead we have to use both G1 and G2 (only G1 is not enough), so we have to reason as before through the KKT to find the optimal solution, with same formulation as before but with Pe=120

Also in this case is easy to choose the best one:

[$P_{g1}=79.67$, $P_{g2}=40.33$] corresponding to $f = 530.5$

For Pe=110 (S5):

Same reasoning give us:

[$P_{g1}=73.00$, $P_{g2}=37.00$] corresponding to $f = 1090.5$

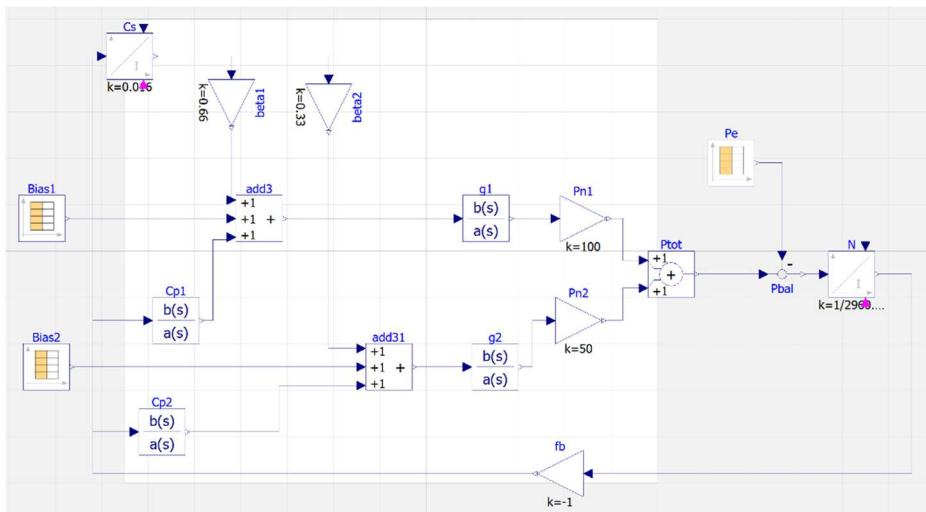
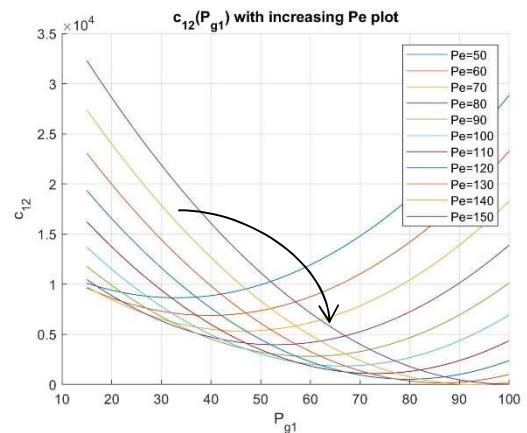
Now that we have all the optimal generator pool for the six time slots S, we can use it as the tertiary biases for each S. Then using the control scheme of point (1) [primary + secondary]

we add our tertiary control to simulate in Modelica and verify our results. (normalizing it or not respect P_n depending on where they are summed)

Summing up the results and tertiary bias:

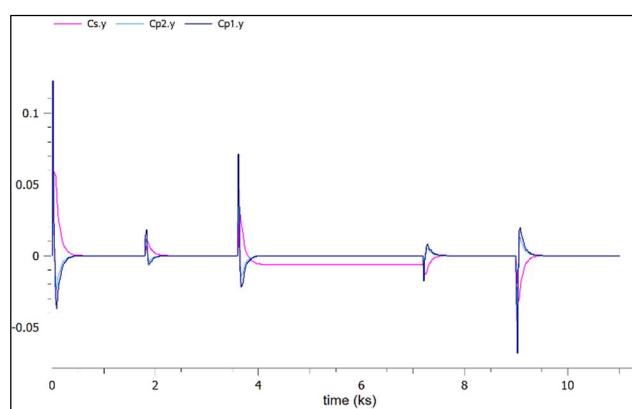
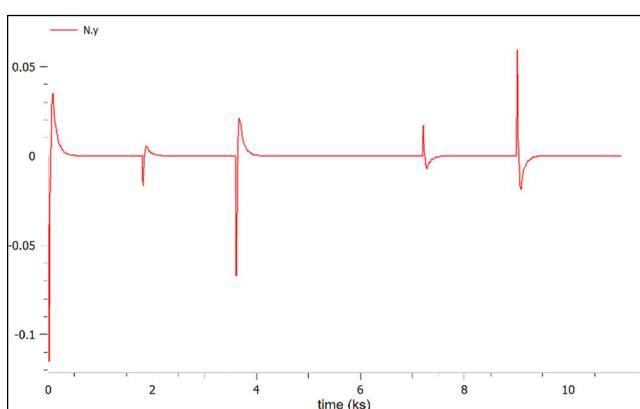
	P_{g1}	tBias1	P_{g2}	tBias2	cost rate	combination
S1:	60	0.6	0	0	1838	G1
S2:	70	0.7	0	0	938	G1
S3:	79.67	0.8	40.33	0.81	530.5	G1+G2
S4:	79.67	0.8	40.33	0.81	530.5	G1+G2
S5:	73	0.73	37	0.74	1090.5	G1+G2
S6:	60	0.6	0	0	1838	G1

We can observe that for P_e rising the cost rate curve move on the left and to the bottom, this is because we use in a better way our generator after the start up interval, as the following plot show us. The initial cost is higher but then we have a gain.



Implementing the tertiary bias and the time varying net request with a combination Time Table block , it is possible to simulate our system behavior in Modelica.

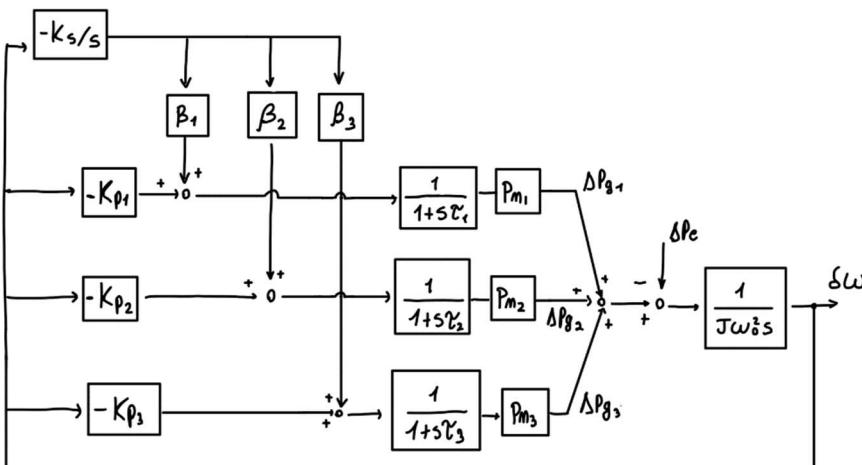
The response would normally be a bit over-damped because of the switch on the optimal generator pool between some intervals. This cause an initial settling dynamic



3) Configuration B, control

This time we are working in configuration B, so when all Generators are connected to all the Loads.

Anyway the set up of primary and secondary control can be done in the same way of point 1, just with a little bit of generalization to treat more generators in parallel.



$$g_1(s) = \frac{1}{1+10s} \quad P_{n,1}=100\text{MW} \quad J_1=20\text{KJ}/(\text{r/s})^2$$

$$g_2(s) = \frac{1}{1+5s} \quad P_{n,2}=50\text{MW} \quad J_2=10\text{KJ}/(\text{r/s})^2$$

$$g_3(s) = \frac{1}{1+20s} \quad P_{n,3}=150\text{MW} \quad J_3=10\text{KJ}/(\text{r/s})^2$$

We compute the alternator dynamic from total power to freq. error as done for point 1:

$$\frac{10^6}{(J_1+J_2+J_3)*\omega_0^2} = \frac{1}{3947.8}$$

We weight the secondary contribution β as the requirement $\beta_1 = \beta_3 = 0.4$, so the rest is provided by G2 $\beta_2=0.2$

Proceeding by loop shaping, we adapt the script of point 1 to our new settings, now with an additional gain degree of freedom given by the gain $Kp3$.

After solving through Maxima the system of equations obtained by equaling the coefficients of the parametric $L(s)$ and the desired one (kept the same as the previous point) we get the following solutions.

$$(%o4) L=((37500 kp3+50000 kp2+50000 kp1) s^3 + (45000 ks+11250 kp3+7500 kp2+12500 kp1) s^2 + (11000 ks+750 kp3+250 kp2+500 kp1) s+550 ks)/(19739000 s^5 + 6908650 s^4 + 690865 s^3 + 19739 s^2)$$

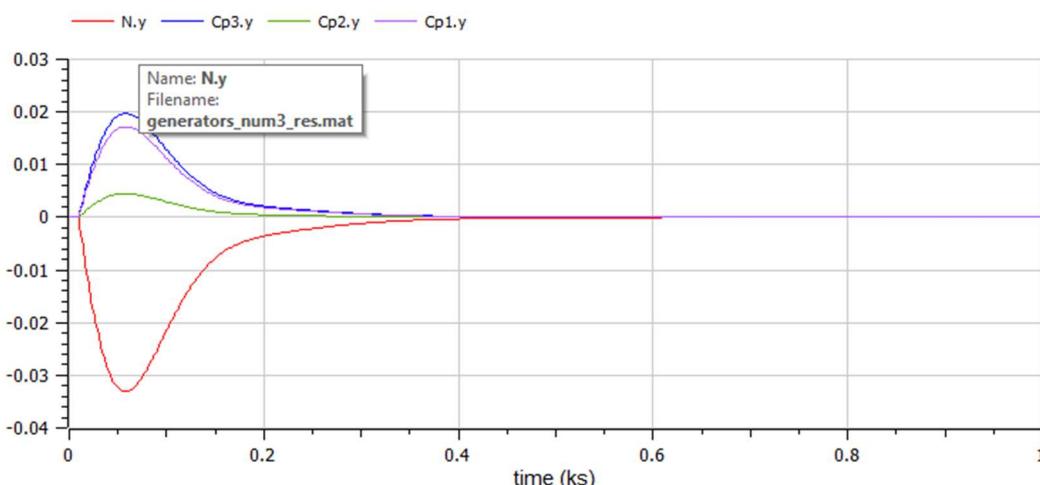
$$(%o5) \text{EQSYS}: [37500 \cdot kp3+50000 \cdot kp2+50000 \cdot kp1-1.148 \cdot 10^5, 45000 \cdot ks+11250 \cdot kp3+7500 \cdot kp2+12500 \cdot kp1-3.479 \cdot 10^4, 11000 \cdot ks+750 \cdot kp3+250 \cdot kp2+500 \cdot kp1-2402, 550 \cdot ks-7.108]$$

$$(%o5) [37500 kp3+50000 kp2+50000 kp1-114800.0, 45000 ks+11250 kp3+7500 kp2+12500 kp1-34790.0, 11000 ks+750 kp3+250 kp2+500 kp1-2402, 550 ks-7.108]$$

```
(%o6) solve(EQSYS,[kp1,kp2,kp3,ks]);
rat: replaced -114800.0 by -114800/1 = -114800.0
rat: replaced -34790.0 by -34790/1 = -34790.0
rat: replaced -7.108 by -1777/250 = -7.108
```

$$(%o6) \left[kp1=\frac{1788}{34375}, kp2=\frac{1396}{103125}, kp3=\frac{184012}{61875}, ks=\frac{1777}{137500} \right]$$

These solutions however produce once again an imbalanced response of the control variable, we therefore modified them to get a better-distributed stress in the network while still fulfilling the requirements. The final response and the control variable of the generators are plotted here. (As we can see we distribute a primary control action proportional to the generator nominal power capacity)



$$Kp1 = 0.520$$

$$Kp2 = 0.135$$

$$Kp3 = 0.595$$

$$Ks = 0.0129$$

4) Configuration B, optimization

The solution should be approached as done for point (2), but this time considering all the network in configuration B.

Reporting the useful data:

$$P_{g1,\min} = 15 \text{ MW} \quad P_{g1,\max} = 100 \text{ MW}$$

$$c_1(P_{g1}) = 0.5 + 1.5 * (P_{g1} - 95)^2$$

$$P_{g2,\min} = 7.5 \text{ MW} \quad P_{g2,\max} = 50 \text{ MW}$$

$$[c_2(P_{g2}) = 1 + 3 * (P_{g2} - 48)^2]$$

$$P_{g3,\min} = 22.5 \text{ MW} \quad P_{g3,\max} = 150 \text{ MW}$$

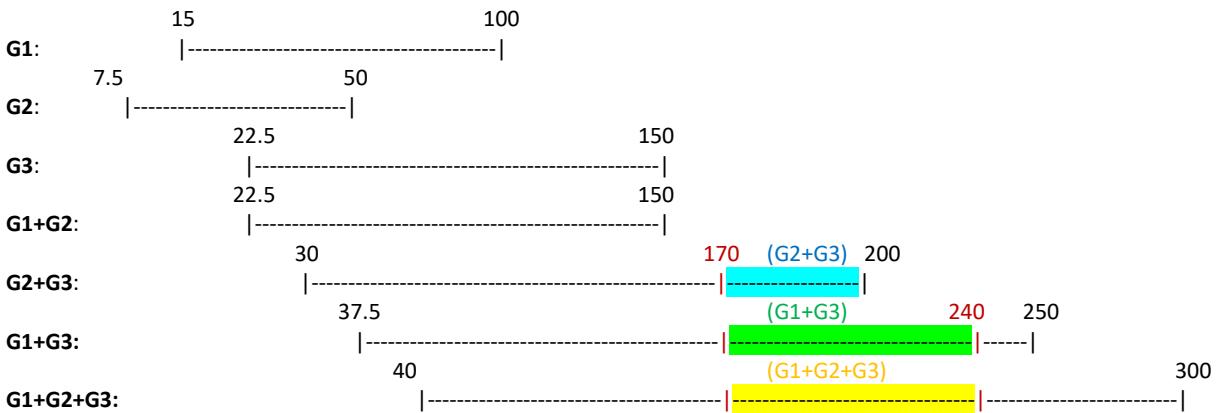
$$c_3(P_{g3}) = 1.5 + 4 * (P_{g3} - 140)^2$$

In this configuration the three Generators G1,G2,G3 feed the overall network composed of the four loads

	S1	S2	S3	S4	S5	S6	
P_e:	170	180	230	210	240	190	[P _{e,min} =170, P _{e,max} =230]

After computing the net demand we can proceed as before, but in this case being careful in setting up the optimization KKT equations. In fact we are requested to minimize the expenditure of the generator pool made of G1 and G3. So it is the point of view of a company providing generation using G1 and G3 from his side, knowing there is a third generator G2 available with its specification (P_{g2,min},P_{g2,max}). the client give us the overall Net demand P_e for each S. The producer (us), want to minimize his cost properly partitioning the power among three generators, but carrying only the cost of G1 and G3, so it can freely assign P_{g2} on its feasible range, maybe given to the client by another producer. (Obviously our solution will not be so realistic, because maybe it can lead to a solution highly disadvantageous for the other producer) This means that in our optimization procedure P_{g2} will be a variable chosen by us but c₂ does not influence the overall cost function f !

First we represent graphically the possible Generators combinations and the one to optimize respect net demand:



So analyzing the feasible combinations and dividing into intervals:

$$I_1: [170, 200] \rightarrow (G2+G3, G1+G3, G1+G2+G3)$$

$$I_2: [200, 240] \rightarrow (G1+G3, G1+G2+G3)$$

In this case we can try to find again a general formulation of the KKT setting, but properly adapting it to the generator combination considered.

$$f = 0.5 + 1.5 * (P_{g1} - 95)^2 + 1.5 + 4 * (P_{g3} - 140)^2 \quad (\text{neglect } c_2 \text{ on the cost function, as we pointed out})$$

$$g = P_{g1} + P_{g2} + P_{g3} - P_e$$

$$h_1 = P_{g1} - P_{g1,\min}$$

$$h_2 = P_{g1,\max} - P_{g1}$$

$$h_3 = P_{g2} - P_{g2,\min}$$

$$h_4 = P_{g2,\max} - P_{g2}$$

$$h_5 = P_{g3} - P_{g3,\min}$$

$$h_6 = P_{g3,\max} - P_{g3}$$

$$L = f + \lambda * g + \mu * h \quad (\text{Lagrangian})$$

(blue parts only if G3 is active, green one when G1 is active)

Now it's time to optimize all possible feasible generator combination for each net demand, using Maxima (the script is very similar to the one used in point 2, we just need to use the right h and rewrite the cost function f depending on the combination considered, also deriving respect the proper variables).

Summing up the optimal results for each S

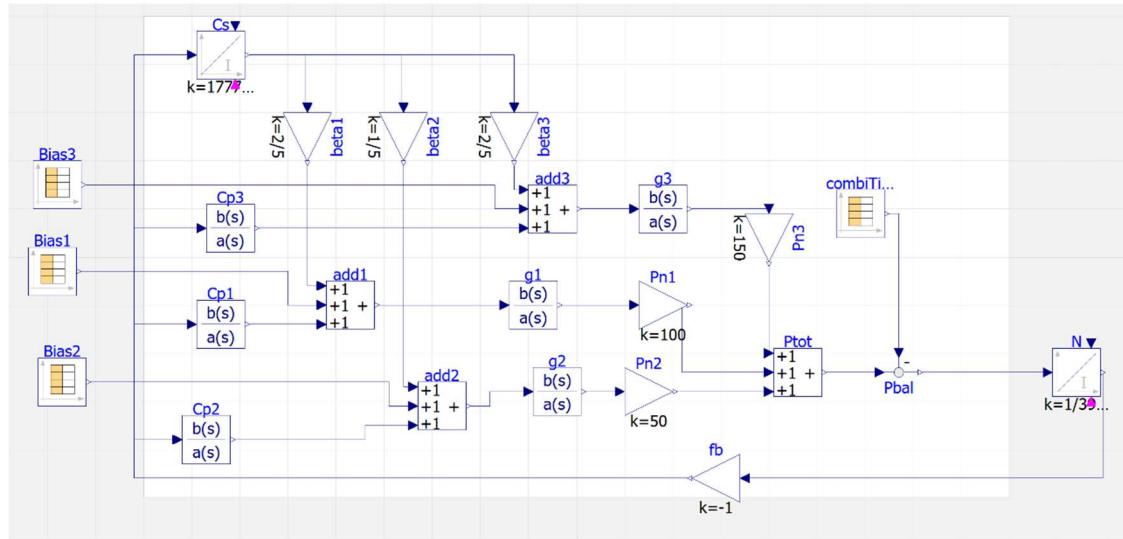
(to find it we just need to proceed as done for point 2, so compute the optimum for each P_e request respect the feasible combinations, and take the one with minimum cost. When computing the optimum in maxima we remove unfeasible, extremum and local optimum solution checking the values and parameters μ_i)

	P_{g1}	$tBias1$	P_{g2}	$tBias2$	P_{g3}	$tBias3$	cost rate	combination
S1:	0	0	30	0.6	140	0.93	1.5	G2+G3
S2:	0	0	40	0.8	140	0.93	1.5	G2+G3
S3:	91.36	0.91	0	0	138.64	0.92	29.27	G1+G3
S4:	76.82	0.77	0	0	133.18	0.89	683.2	G1+G3
S5:	93.18	0.93	7.5	0.15	139.32	0.93	8.82	G1+G2+G3
S6:	0	0	50	1	140	0.93	1.5	G2+G3

We can highlight how each time we can use G2+G3, this is the best choice, by working on the minimum of G3 ($P_{g3}=140$, minimizing c_3) and leaving the difference to G2 which does not affect our overall cost!

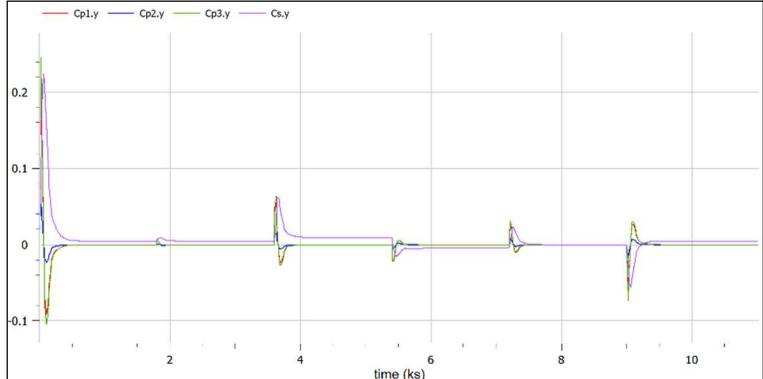
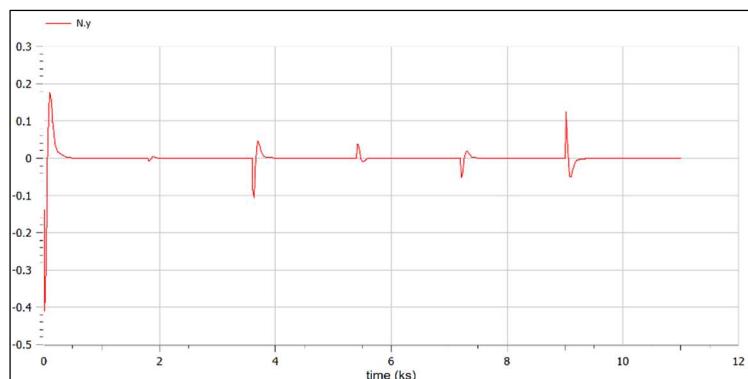
Notice also that even if during S6 we let G2 work with its nominal Power, this is not an issue, in fact by definition the nominal power of a generator is related to a 24/7 working condition without fault.

The table above show also the tertiary bias needed for the tertiary control on our control scheme (computed dividing each Generator power respect its nominal one). So we can set up our control scheme adding the bias to the scheme of point 3:



(Properly defining Time Table respect each interval S , and setting up the blocks using the same values of point 3)

The response behavior is very similar to the one of point 2, with an initial over-dump due to the high G3 request, but then remain enough stable even in between generator pool switch.



5)