Advanced and Multivariable Control

6/2013

Exercise 1

Given the system

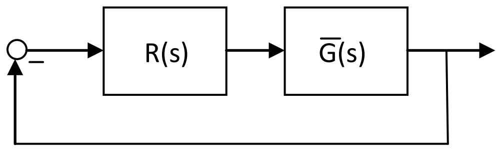
$$\dot{x}_1(t) = -\alpha x_1(t) - \beta x_1^2(t) - \gamma x_1(t)x_2(t) + \theta x_2(t)$$
$$\dot{x}_2(t) = -\gamma x_1(t)x_2(t) - \varepsilon x_2^2(t) - \theta x_1(t) - \eta x_2(t)$$
$$\dot{x}_3(t) = \gamma x_1(t)x_2(t) - \varsigma x_3(t) + \varepsilon x_2^2(t)$$

Compute the conditions to be imposed to *α, β, γ, θ, ε, η, ζ*, to guarantee that the origin is an asymptotically stable equilibrium by means of:

1. the analysis of the linearized system,
2. the Lyapunov theory.

Eexercise 2

Consider the feedback system



where the nominal transfer function of the system under control is

$$\overline{G}(s) = \frac{1}{1+sT} \ , \ T>0$$

Assume that the true transfer function is

$$G(s) = \frac{1}{(1+sT)(1+as)} \ , \ a>0$$

- model the uncertainty as an additive and multiplicative one;
- show how to choose the Bode diagram of the magnitude of the complementary sensitivity to guarantee the stability also in perturbed conditions;
- assuming to use the H_∞ synthesis technique, show (Bode diagrams are enough) how to choose the weighting functions $W_S(\omega)$, $W_T(\omega)$ in view of the previous results.

Exercise 3

Consider the system

$$\dot{x}(t) = 2x(t) + u(t)$$

1. Compute the asymptotic LQ control law with *$Q=5$, $R=1$.*
2. Compute the real gain margin of the feedback system and compare it with the one guaranteed by the LQ control.
3. Assuming to have the output $y(t)=x(t)$ and using an observer with gain $L=\rho B$, compute the transfer function of the overall regulator and show that, for $\rho \to \infty$, the resulting loop transfer function tends to the one with the state feedback (LTR procedure).

RICCATI equation (LQ control)

$$\dot{P} = A'P + PA + Q - PBR^{-1}B'P$$

Exercise 4

Show how to setup a model predictive controller for an asymptotically stable system described by its impulse response, it is required that the computed control action explicitly depends on the output variable.
Describe the meaning of the control horizon and the minimum prediction horizon and why their use can enhance the performances.

Exercise 5

Consider a dynamic system described by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Md + v(t) \\ y(t) &= Cx(t) + Nd + w(t)\end{aligned} \qquad x \in R^n,\ y \in R^p,\ u \in R^m,\ d \in R^r,$$

where d is a constant disturbance.

1. First assume that $v=w=0$ and show how to estimate the state and the disturbance, also specifying the observability requirements and the maximum possible number of components r of the disturbance.

2. Assuming that the system is scalar with $d=0$, $n=m=p$, v and w white uncorrelated noises, and A is an unknown parameter, show how to use the extended Kalman filter for the joint estimation of the state and of the parameter A.