SISO CONTROL, SYNTHESIS BY [T, S] SENSITIVITY FUNCTIONS

$\boxed{H_2, H_\infty \text{ CONTROL}}$

Advanced and Multivariable Control

Summary on control synthesis for SISO systems

LINEAR syst + extensions!

Riccardo Scattolini

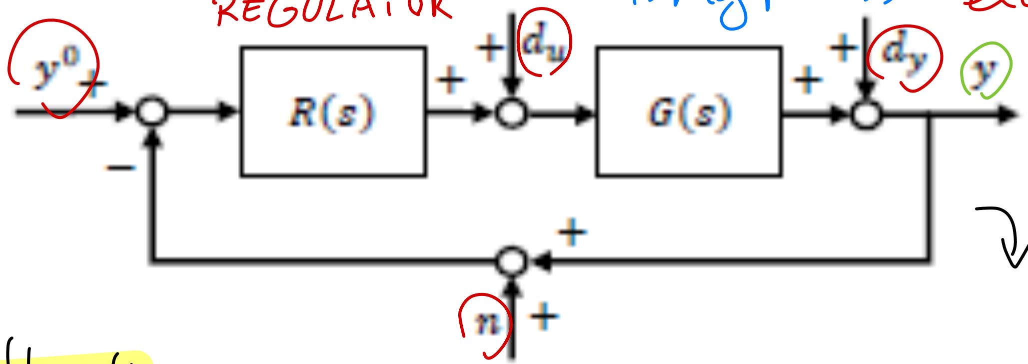
(linear feedback systems)

REGULATOR

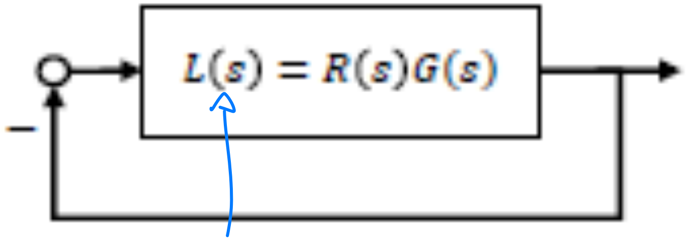
T.F of process

exogenous signals alternating system

y: output variable desired value



characteristics of feedback syst



LOOP T.F

feedback system

$$
\begin{aligned}
Y(s) &= T(s)(Y^o(s) - N(s)) + S(s)D_y(s) + S(s)G(s)D_u(s) \\
U(s) &= K(s)(Y^o(s) - N(s) - D_y(s)) + S(s)D_u(s)
\end{aligned}
$$

(T.F) that links the signals

- *Loop transfer function* $L(s) = R(s)G(s)$
- *Sensitivity* $S(s) = \dfrac{1}{1 + R(s)G(s)} = \dfrac{y}{d_y} = \quad = \dfrac{u}{d_u}$

S(s) should be small when d_y is significant

- *Complementary sensitivity* $T(s) = \dfrac{R(s)G(s)}{1 + R(s)G(s)} = \dfrac{y}{y^o} = \dfrac{y}{n}$

Important Trade off

- *Control sensitivity* $K(s) = \dfrac{R(s)}{1 + R(s)G(s)} = S(s)R(s) = \dfrac{u}{y^o} = \dfrac{u}{n}$

EXOGENOUS signals

- y_0: set point

- d_u : disturbance on Actuator (which don't work as expected! even if giving control action $\tilde{u}(t)$, due to actuator, the real $u(t)$ differs from the desired one $\tilde{u}$)

- d_y: disturbance acting on the plant .. act on output variable

- n: measurement disturbance / noise → BIAS / WHITE NOISE ...

  see how syst reacts to it!

↓ IF n polarization, prevent you to have ∅ steady state error

for example: to control room Temp @20°
but for a bias error n:

$y^o - (n+y)$ try to regulate with a modified error due to n presence!

↓

{you put more effort than required}

- we want a limited control action $u(t)$, prefer to have WORST y regulation with cheaper control action (NOT too fast control syst.)

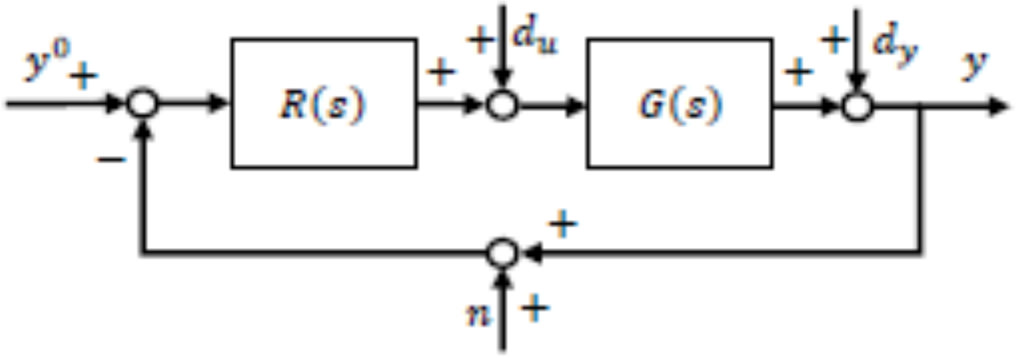
↳ $u(t)$ typically SATURATION .. || we wanna design such that slow control act. inside linearity zone ||

for LINEAR SYST : JUST consider the loop T.F $L(s)$
NOT the exog. signals

Final Goal: for linear SISO, studied stability
looking on $G(s)$: (Good for SISO)
↓
but for MIMO syst... we have more Bode diags
we cannot use Bode criterion and so on...

show how to move analysis from $L(s)$ to $S(s)$
Necessary for MIMO system
(NOT for SISO ones)

Stability **of the closed-loop system**



$$\begin{cases} Y(s) = T(s)(Y^o(s) - N(s)) + S(s)D_y(s) + S(s)G(s)D_u(s) \\ U(s) = K(s)(Y^o(s) - N(s) - D_y(s)) + S(s)D_u(s) \end{cases}$$

we wanna look also on S(s)
poles of S(s) - -

One could check the stability by looking at the poles of the functions *S(s), T(s), K(s), G(s)S(s)*

IF all poles of S(s) asymp. stable ⇒ look NOT just in one of that, because of possible critical cancellation → we have to check all!

Note: all the four transfer functions must be studied to check the presence of hidden and forbidden cancellations

$$R(s) = \frac{s-1}{s} \quad, G(s) = \frac{1}{s-1} \rightarrow S(s) = \frac{s}{s+1} \quad, G(s)S(s) = \frac{s}{(s+1)(s-1)}$$

look all T.F to be sure to NOT lose hidden poles...

hidden poles on others T.F

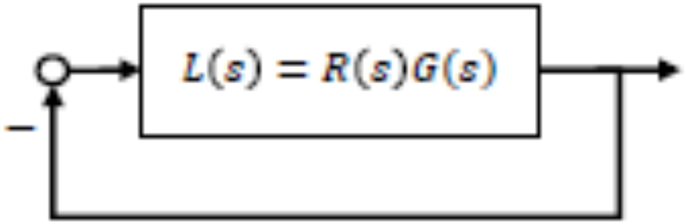
Not very practical and useful in the control design phase

consider Feedback syst looking all the poles (not only on loop T.F)

for this we introduced some criterion simpler

Stability of the closed-loop system by looking at the open-loop transfer function _L(s)_

look to L(s) to conclude stability about close loop syst !!



(complex to use, hard to modify plan
drag of L(s))

- **Nyquist criterion**: the king of analysis methods (you must know it!), but not extremely useful for the synthesis of *R(S)*

more general !

similar to Nyquist but used only if L(s) has not unstable poles!

(more restrictive cases of application)

- **Bode criterion:** equivalent to the Nyquist criterion in many problems, more practical for the synthesis → *phase (and gain) margin* (see the next slide)

↳(some index relevant RECAP: ⇒)

{get specific characteristics}

- **Root locus:** we will use only the very basic rules

gain and phase margins

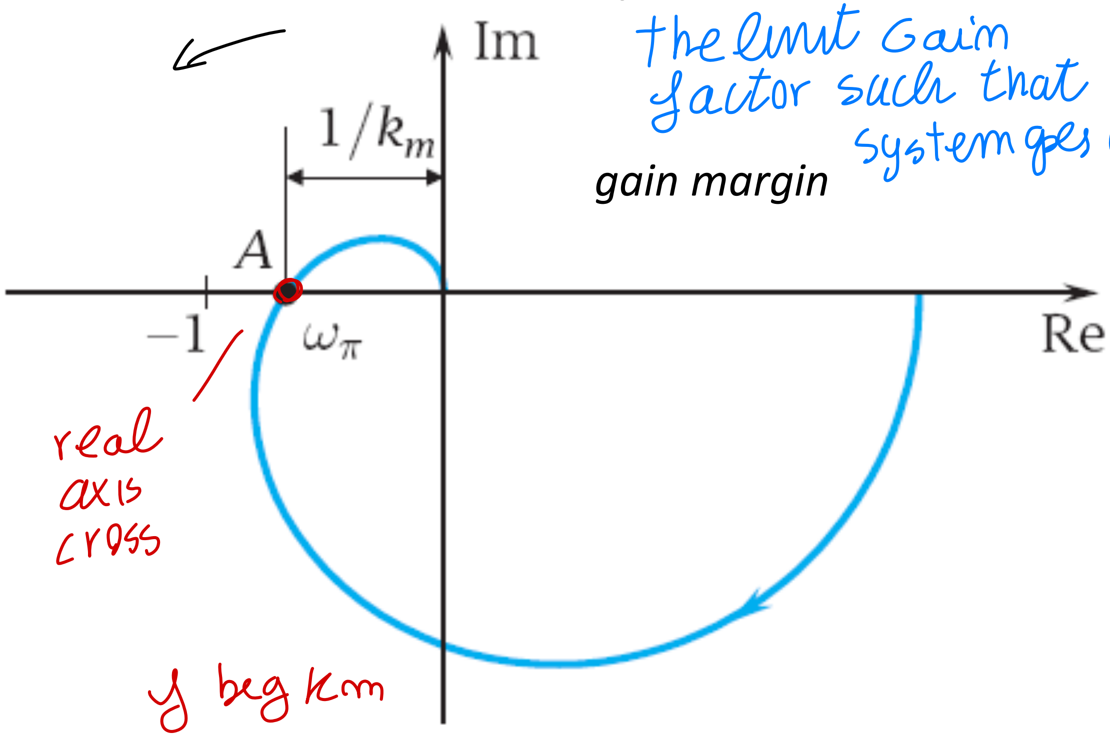
analysis of feedback syst

For many systems they represent a distance of the polar diagram of $G(j\omega)$ from point -1, in this sense a sufficiently high value is a good indicator of ***robustness*** with respect to modeling errors or variations

km is the value such that A goes to (-1+j0)

the limit Gain factor such that system goes unst.

gain margin

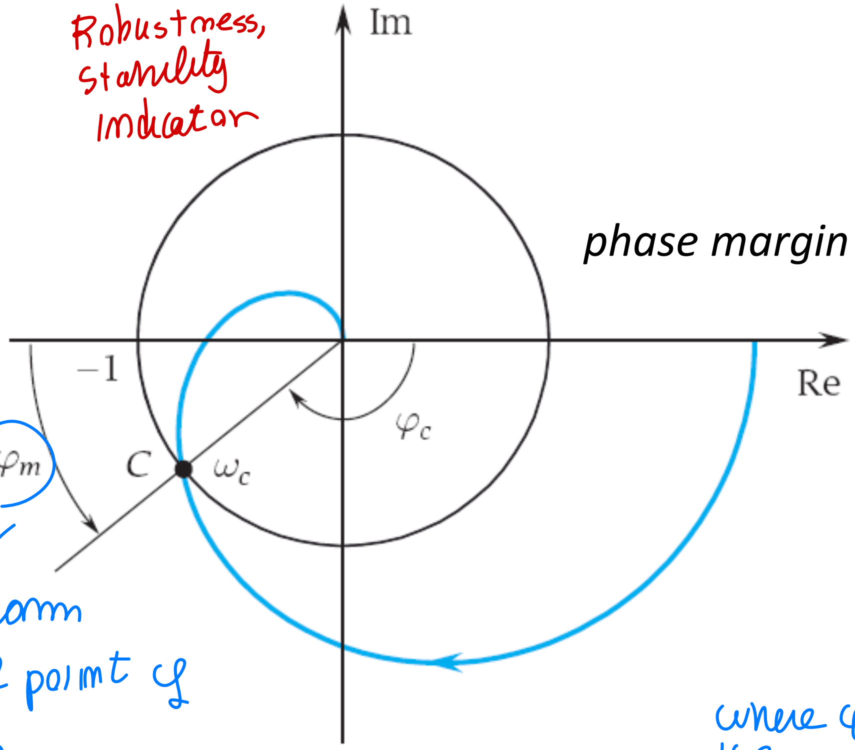


real axis cross

if big km → ROBUST PROCESS!

Robustness, stability indicator

phase margin φ_m



for from critical point if big φ_m

where φ_m, km is easier to evaluate on Bode plot

easier to use and required by the Bode criterion

If $P=0$, only one value of ω such that $|L|=1$

A.S $\Leftrightarrow$ $\varphi_m>0$ and gain $K>0$

$\left\{ \begin{array}{l} \varphi_m, K_m \text{ to} \\ \text{modify } L(s) \end{array} \right. ! \}$

give INFO about close loop syst behaviour

Phase margin and closed-loop oscillations

In many cases, the complementary sensitivity can be approximated by $T(s) = \dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ $\quad \xi \cong \dfrac{\varphi_m}{100}$

nat freq

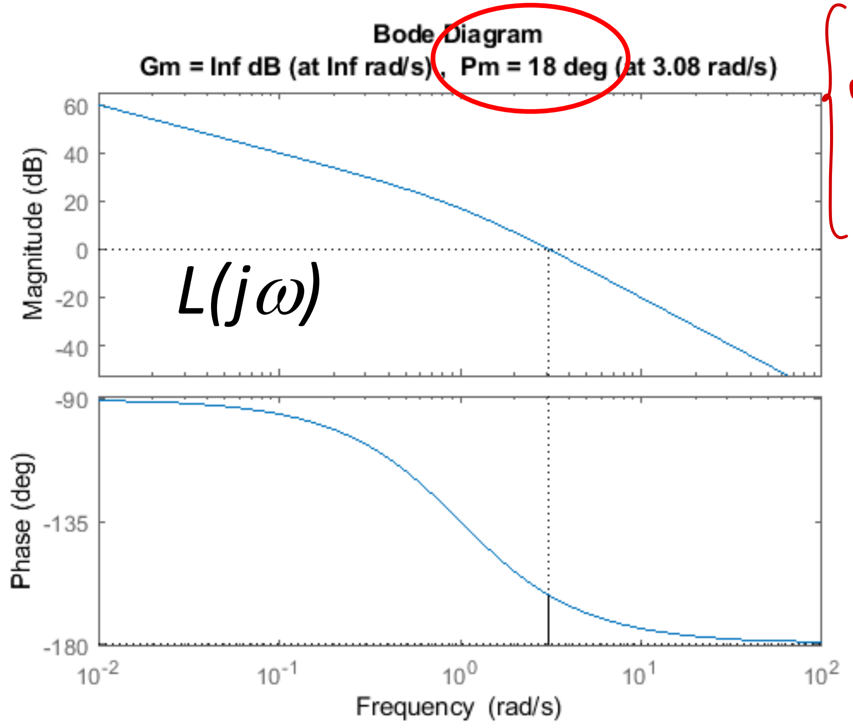
II ORD syst

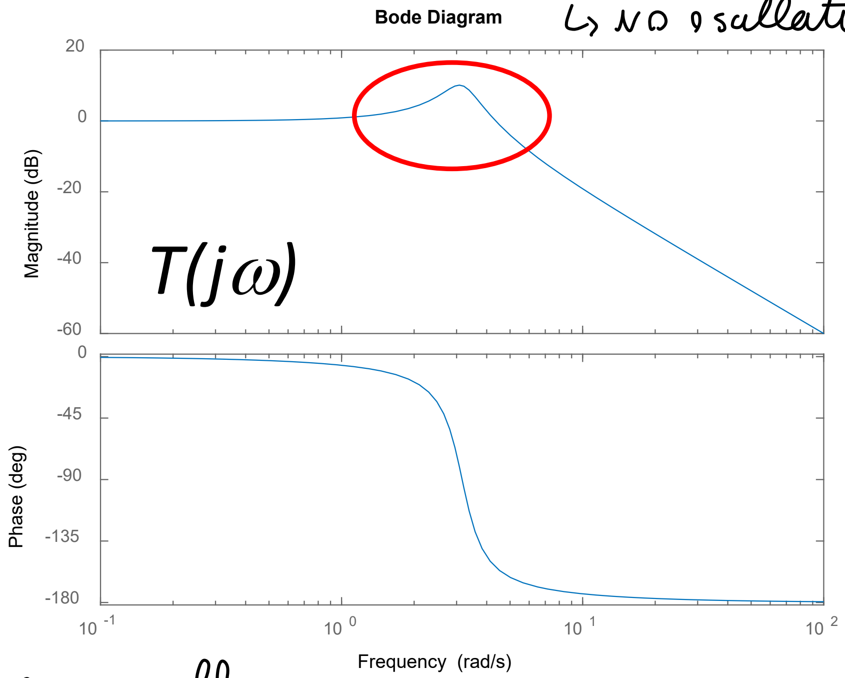
damping

In order to reduce the peak of $T(j\omega)$ one must choose a sufficiently high φ_m

- stability
- Robustness
- oscillating behav.

no significant loop
↳ no oscillation

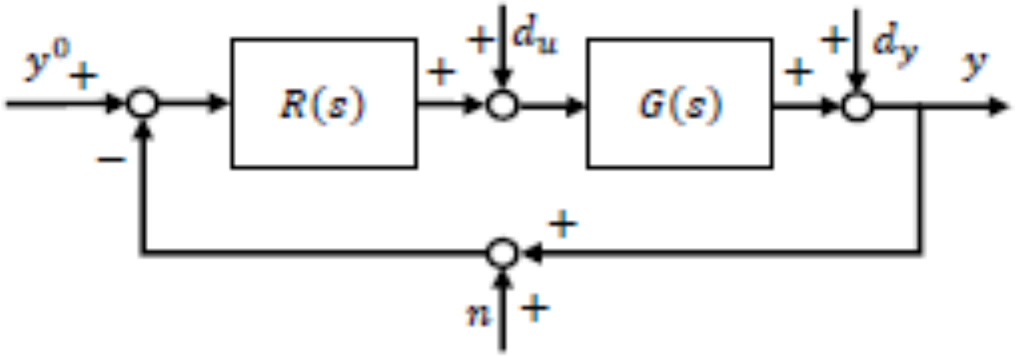




focus on T(jω): linked to φ_m ⇒

IF φ_m small, T(jω) has $\xi \downarrow\downarrow$ → pick on T(jω) → oscillation!

- **Sensitivity and performance** ?



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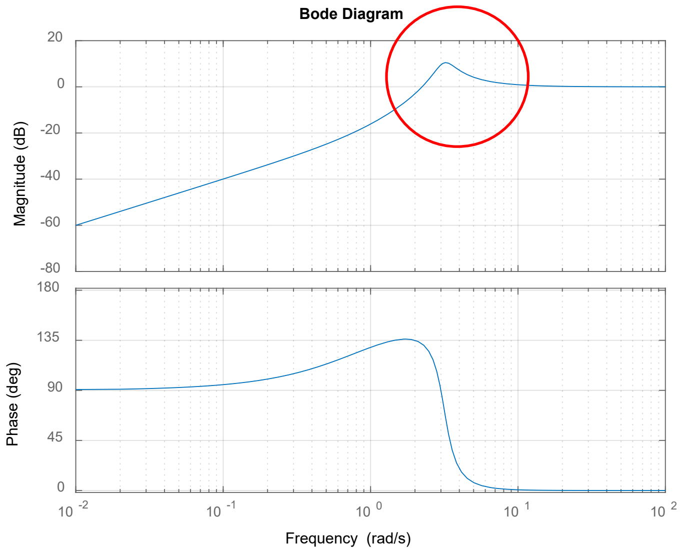
\end{aligned}
$$

$$
|S(j\omega)| = \frac{1}{|1+L(j\omega)|} \simeq \begin{cases} \dfrac{1}{|L(j\omega)|}, & |L(j\omega)| \gg 1, \quad \omega \ll \omega_c \\ 1, & |L(j\omega)| \ll 1, \quad \omega \gg \omega_c \end{cases}
$$

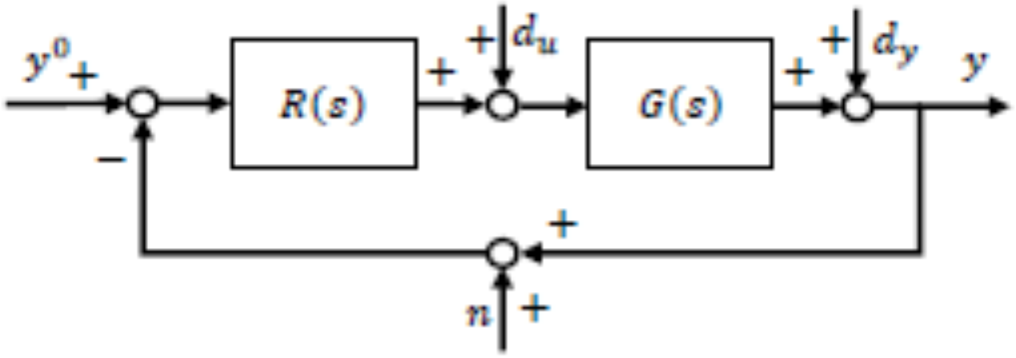
(approx!)



$|S|$ should be "small" ($|L|$ "big") where the spectrum d_y, d_u have significant harmonic components (usually at low frequencies)

↳ so attenuation !

- **Complementary sensitivity and performance** ?



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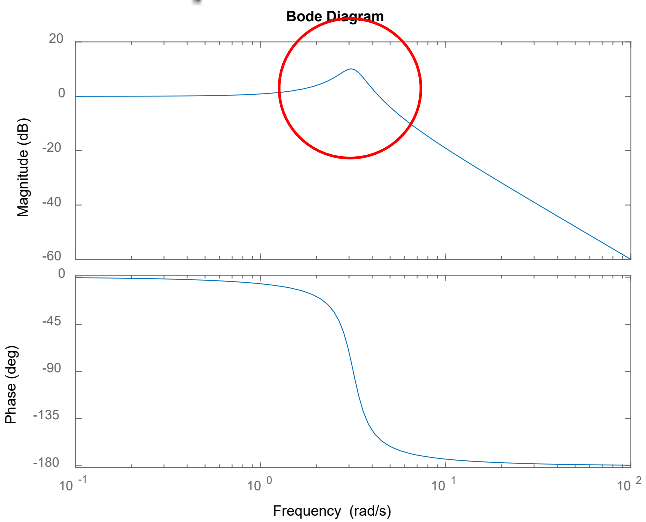
\end{aligned}
$$

$$
|T(j\omega)| = \frac{|L(j\omega)|}{|1 + L(j\omega)|} \simeq \begin{cases} 1, & |L(j\omega)| \gg 1, \quad \omega \ll \omega_c \\ |L(j\omega)|, & |L(j\omega)| \ll 1, \quad \omega \gg \omega_c \end{cases}
$$



↓

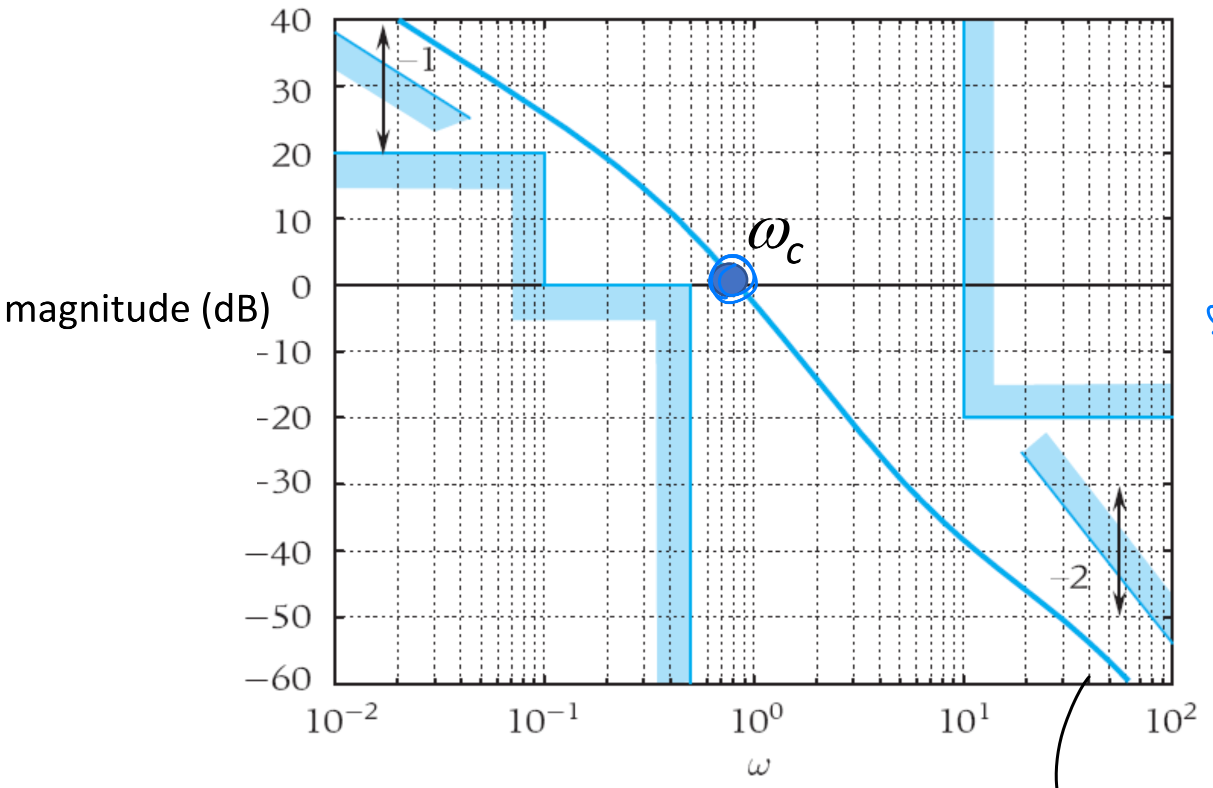
Guidelines to define |L| such that |T|, |S| behave in a certain way

→ $|T| \simeq 1$ ($|L|$ "big") in the frequency band where the spectrum of the reference signal has significant harmonic components (usually at low-medium frequencies)

→ $|T|$ "small" ($|L|$ "small") in the frequency band where the spectrum of the measurement noise n has significant harmonic components (usually at high frequencies)

Design requirements on *L(s)*

We can ask integrator so steady state good behaviour!



small |L| @ high freq so noise attenuated

high attenuation @ high freq

GOOD (mathematical P.O.V)

- $\varphi_m \geq \bar{\varphi}_m$ (stability, robustness, limited peaks of $T(j\omega)$)
- $g_m \geq \bar{g}_m$ (robustness) (Gain margin)
- $\omega_c \geq \bar{\omega}_c$ (speed of response, attenuation low frequency disturbances d_y)
- $\omega_c \leq \tilde{\omega}_c$ (limit control action, attenuation measurement noise n)

ω_c too big is NOT a wise choise due to saturation, limit of system

require a certain ω_c for speedness of syst!

Limits to performance

$$T(s) + S(s) = 1$$

Cannot be choosen independently → you need a Trade-off on requirements

– neg. phase contribution on loop

1 Systems with delay: $e^{-\tau s}G(s)$, with $G(s)$ rational transfer functions. The delay introduces a negative phase contribution at the crossover frequency equal to $-\tau\omega_c$. The cutoff frequency ω_c must be limited to guarantee a reasonable phase margin

L plant parameter

↑ Choose ω_c small to guarantee ZEROS/delays has LIMITED effect on syst. ↓

2 Systems with zeros with positive real part, such as: $(1-\tau s)G'(s)$, $\tau > 0$. The positive zero introduces a negative phase margin. In practice, it is impossible to have a crossover frequency greater than $1/\tau$ with an acceptable phase margin

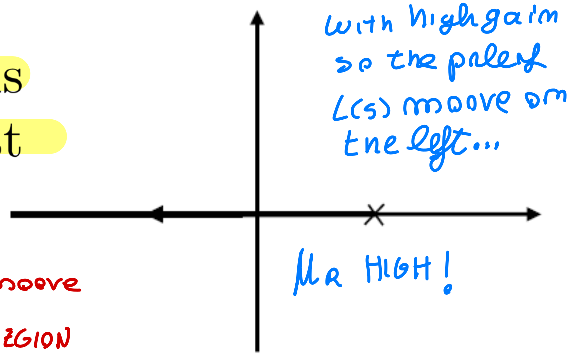
(not ω_c too large...)

3 Systems with unstable poles: a regulator with *a sufficiently high gain* is required to stabilize the system. In terms of root locus, the unstable pole must be brought to the stability region

Examples are reported in the textbook

$\frac{1}{s-1} = G(s)$

↓ close loop with high gain so the poles of $L(s)$ moove on the left...

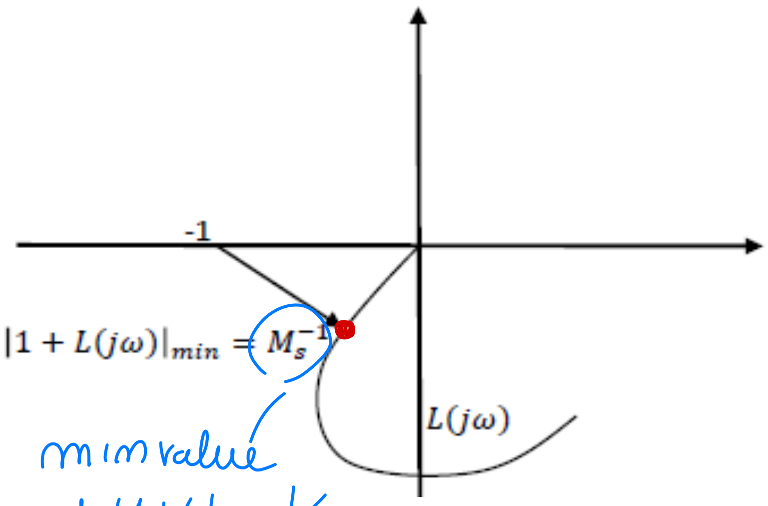


ROOT LOCUS: branches start from open loop poles.. → moove on STABILITY REGION

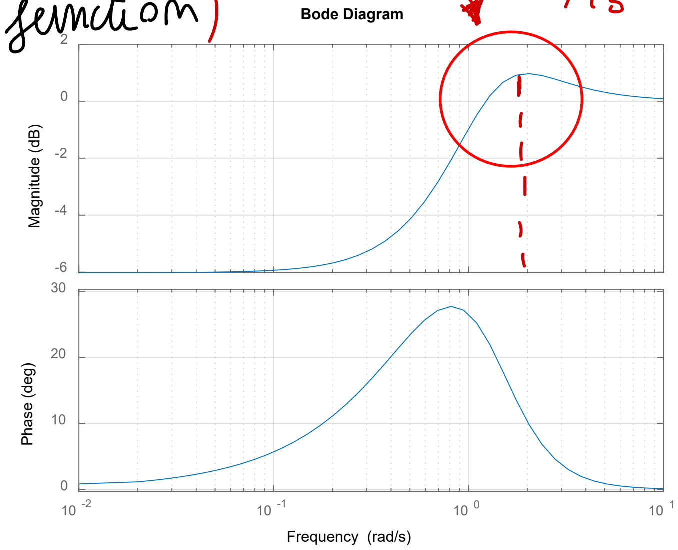
Is it truly impossible to provide specifications in terms of $\overbrace{\textit{\textbf{sensitivity functions?}}}^{T(s),\ S(s)}$

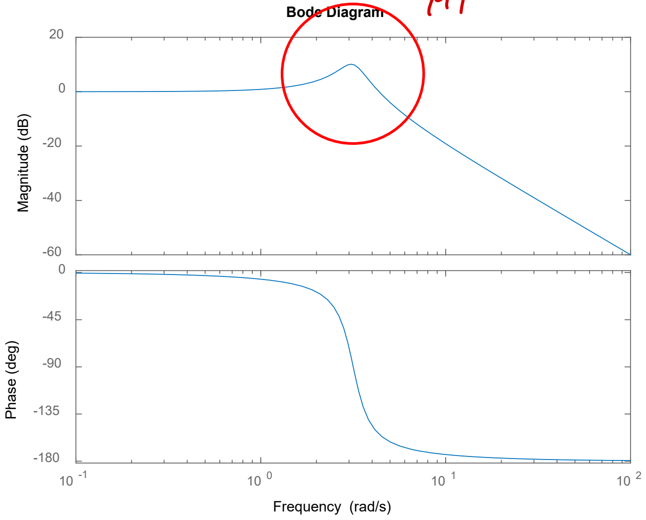
Define (H_∞ gain of sensitivity function)

M_S $\Rightarrow$ (in same way) M_T



min value of $|1+L| = 1/M_S$





$$M_S = \sup_\omega |S(j\omega)| = \|S\|_\infty \quad , \quad M_T = \sup_\omega |T(j\omega)| = \|T\|_\infty$$

It is possible to prove that (see the textbook and following slides):

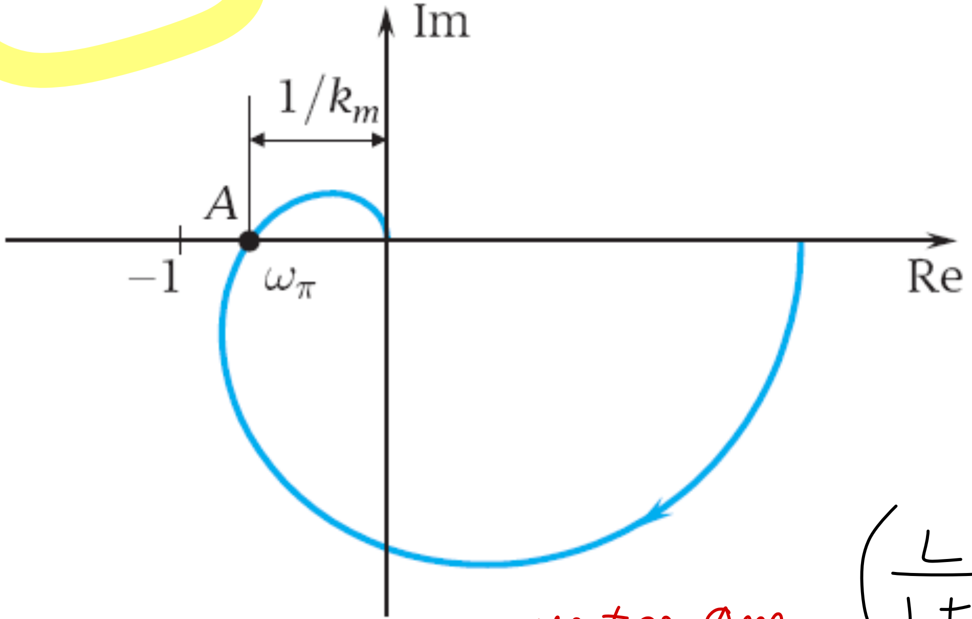
- M_S and M_T differ at most by 1 $\quad (M_S + M_T \leq 1)$
- g_m: the gain margin g_m is such that $g_m \geq 1 + \frac{1}{M_T}$, $g_m \geq \frac{M_S}{M_S - 1}$ $\leftarrow$ from M_S requirements you get g_m req.
- φ_m: $\varphi_m \geq 2 \arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T}$, $\varphi_m \geq 2 \arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S}$

- for gm requirements → solve it respect M_S!

- $1/M_T \geq \tilde{\varphi}_m$ desired

Transpose design property on $S(s)$ T.F

Gain margin



Gain margin $\boldsymbol{k_m}$, or $\boldsymbol{g_m}$ in the notes (assumed to be >1)

Requirements

$g_m \Leftrightarrow M_T$

request on g_m

$\left(\dfrac{L}{1+L}\right)$

$$L(j\omega_\pi) = -1/g_m \longrightarrow T(j\omega_\pi) = \frac{-1/g_m}{1-1/g_m} \longleftrightarrow \boxed{M_T \geq |T(j\omega_\pi)|} = \frac{1}{g_m - 1} \longrightarrow g_m \geq 1 + \frac{1}{M_T}$$

≥ 0

we want a certain $\tilde{g}_m$

$$M_S \geq |S(j\omega_\pi)| = \frac{1}{1-1/g_m} \longrightarrow g_m \geq \frac{M_S}{M_S - 1}$$

same for sensitivity function

$L(s)$ characteristics

↳ $S(s)$ characteristics

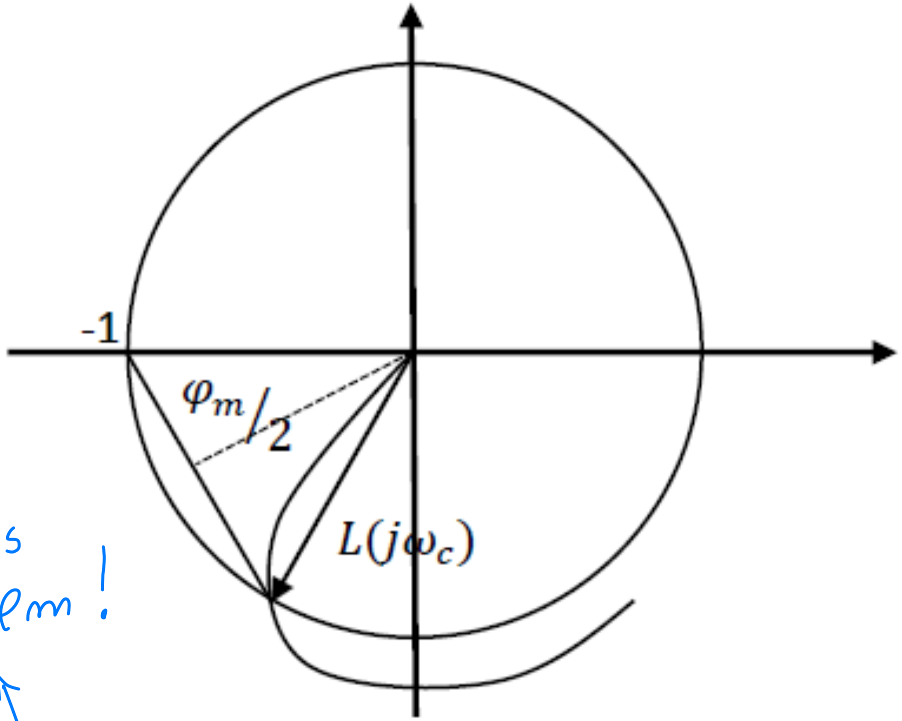
Phase margin

$$\frac{1}{M_S} \leq |1 + L(j\omega_c)| = 2\sin\varphi_m/2$$

always

$$\varphi_m \geq 2\arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S}$$

$\varphi_m \leftrightarrow M_S$
ask $S(s)$ properties an projection φ_m !



Moreover

also for T (complementary sens)

$$|T(j\omega_c)| = \frac{|L(j\omega_c)|}{|1+L(j\omega_c)|} = \frac{1}{|1+L(j\omega_c)|} = |S(j\omega_c)|$$

(same Relationship)

$$\varphi_m \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T}$$

$\varphi_m \leftrightarrow M_T$

↪ for a specific perturbation

M_S, M_T, and robustness – additive uncertainty

(φ_m, g_m)

When $\Delta G_a \neq 0$ how to check Robust.

UNKNOWN!

$$G(s) = \bar{G}(s) + \Delta G_a(s)$$

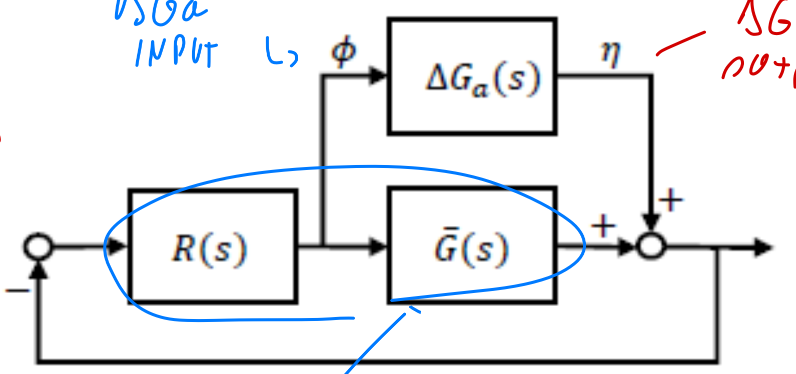
nominal — model of the system

perturbation — real T.F has some differences

(parallel)

ΔG_a INPUT ↳ ϕ

ΔG_a output



Close loop nominal syst asympst.

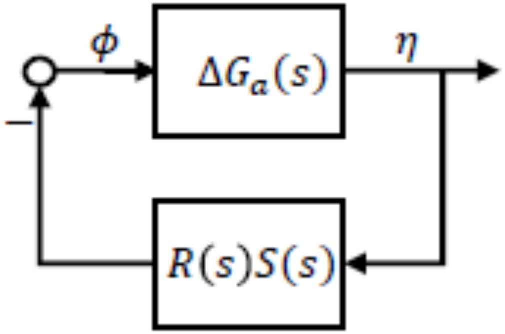
$$\phi = -R(\eta + \bar{G}\phi) \rightarrow$$
$$(1 + R\bar{G})\phi = -R\eta \rightarrow$$
$$\phi = -\frac{1}{1+R\bar{G}} R\eta$$

Assumptions

- closed-loop nominal system asymptotically stable;
- stable uncertainty hidden! $\Delta G_a(s)$ asymptotically stable. Letting $P_{\bar{G}}$ and P_G be the number of poles of $\bar{G}(s)$ and $G(s)$ with positive real part, this means $P_{\bar{G}} = P_G$;
- no cancellations of poles and zeros of $R(s)$ and $G(s)$ with non negative real part.

Guarantee I/O stability imposing:

equivalent system to study



small gain theorem

all assumptions satisfied

$$\boxed{\|RS\Delta G_a\|_\infty < 1}$$

a small M_S helps

unknown... but you know the freq of uncertainty mode

$$\| R\, S \cdot \Delta G_a \|_\infty < 1$$

take R
so that
RS small
gain

$\underbrace{\hphantom{RS\ small\ gain}}$

small

$\|RS\|_\infty$ help
to satisfy this condition

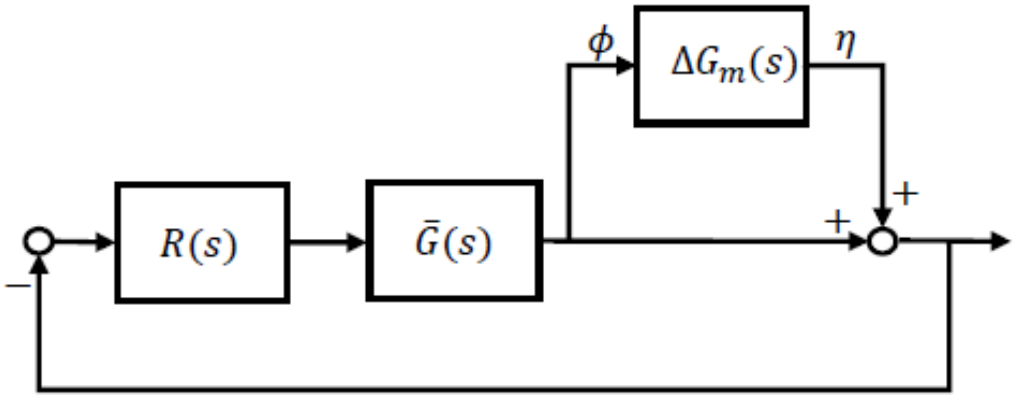
so even for
high ΔG_a it is
stabilized even
when it has relevant
freq component!

M_S, M_T, and robustness – multiplicative uncertainty

$$G(s) = \bar{G}(s)(1 + \Delta G_m(s))$$

nominal *perturbation*

unknown uncertainty



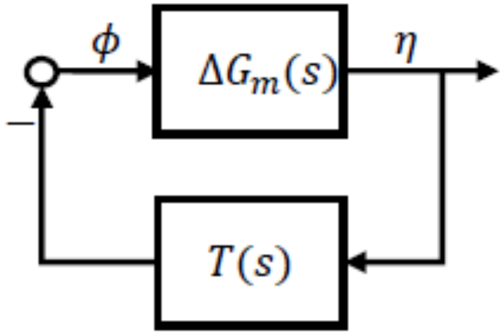
$$\phi = -R\bar{G}(\eta + \phi) \rightarrow \phi = -\underbrace{\frac{R\bar{G}}{1+R\bar{G}}}_{T(s)}\eta$$

Assumptions

- closed-loop nominal system asymptotically stable;

  unstable dyn must be all modelled on $\bar{G}(s)$ NOT LOST
- $\Delta G_m(s)$ asymptotically stable. Letting $P_{\bar{G}}$ and P_G the number of poles of $\bar{G}(s)$ and $G(s)$ with positive real part, this means that $P_{\bar{G}} = P_G$

equivalent feedback system



small gain theorem

$\longleftrightarrow$

allows you to have small T(s)

look $\|T\|_\infty$ for Robustness

$$\boxed{\|T\Delta G_m\|_\infty < 1}$$

a small M_T helps

How to model the uncertainty?

Additive or multiplicative? It depends on the problem

What kind of uncertainty can I use? depends on the problem faced

Example

$$G(s) = \frac{1}{s} e^{-\tau s} \quad , \quad \bar{G}(s) = \frac{1}{s}$$

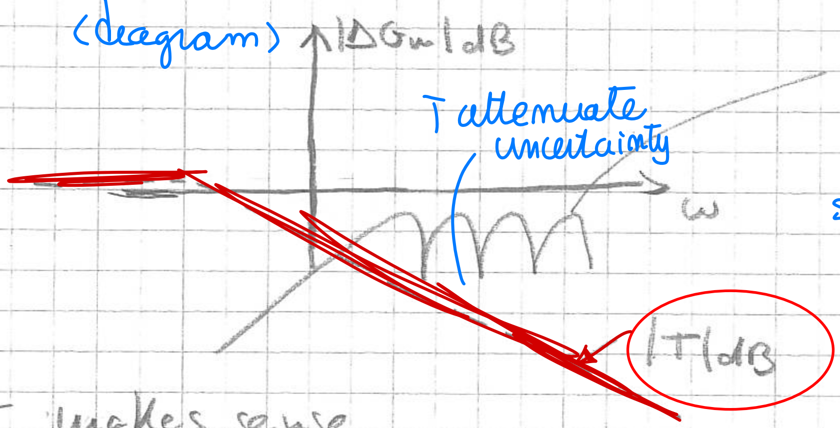
Multiplicative uncertainty

multiplicative

$$\underbrace{G(s)}_{\frac{1}{s}e^{-\tau s}} = \underbrace{\bar{G}(s)}_{1/s} \underbrace{(1 + \Delta G_m(s))}_{e^{-\tau s}}$$

so that $\left(\Delta G_m(s) = e^{-\tau s} - 1 \right)$

(diagram)



significant at high frequency

small @ low freq

small picks presents for higher ω...

↳ more meaningfull @ high freq

$$\|T \Delta G_m\|_\infty < 1$$

design properly T to attenuate uncertainty effect

It makes sense to use the multiplicative uncertainty (small at high frequency)

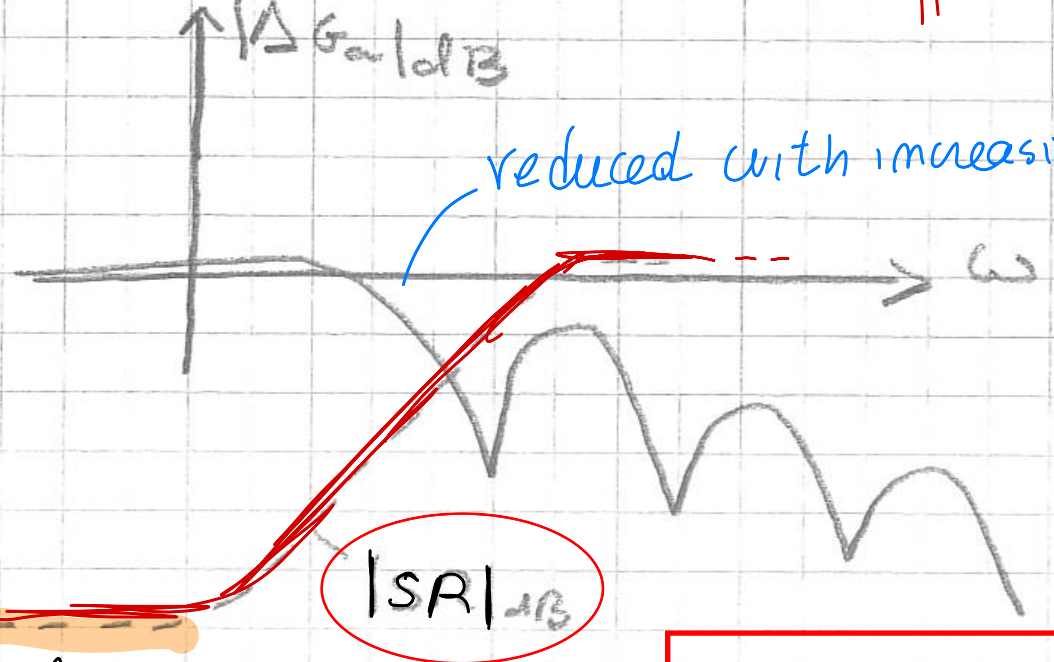
Additive uncertainty

(ZOH T.F)

"1/s"

$$G(s) = \bar{G}(s) + \Delta G_a(s) \longrightarrow \left\| \Delta G_a(s) = \frac{e^{-\tau s} - 1}{s} \right\|$$

reduced with increasing ω values



One must force RS to be small at low frequency

attenuate @ low freq, and increase for ω

$$\boxed{\|RS\Delta G_a\|_\infty < 1}$$

Example (gain uncertainty)

$$\bar{G}(s) = \frac{\bar{k}}{s+a}, \quad G(s) = \frac{\bar{k}+\Delta k}{s+a}$$

nominal model

uncertainty on the gain

↳ trying using $\boxed{\text{Multiplicative}}$ uncertainty

$$\frac{\bar{k}+\Delta k}{s+a} = \frac{\bar{k}}{s+a}\Big(1+\underbrace{\Delta G_m(s)}_{\text{multiplicative}}\Big)$$

$$\Delta G_m(s) = \frac{\Delta k}{\bar{k}}$$

constant at all freq!

constant at all frequencies (previous condition on $T(s)$ not easy to satisfy)

Example (gain uncertainty)

$$\bar{G}(s) = \frac{\bar{k}}{s+a}, \qquad G(s) = \frac{\bar{k}+\Delta k}{s+a}$$

Additive /
Multiplicative
↓
{ Choice depends on specific problem }

always reformulate and use small gain th

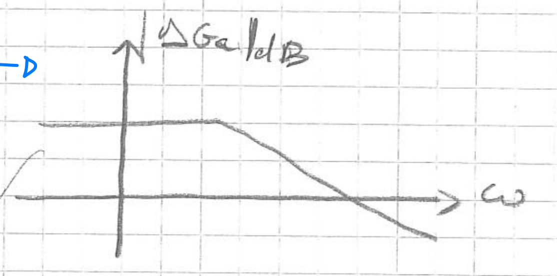
while using a different approach

Additive uncertainty

$$\frac{\bar{k}+\Delta k}{s+a} = \frac{\bar{k}}{s+a} + \Delta G_a(s)$$

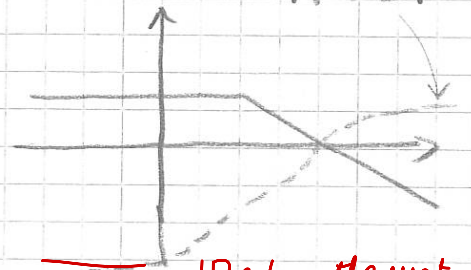
$$\boxed{\Delta G_a(s) = \frac{\Delta k}{s+a}}$$

change on freq... reduced @ high freq



centered at low frequency

Easier to force the condition on $|RS|$



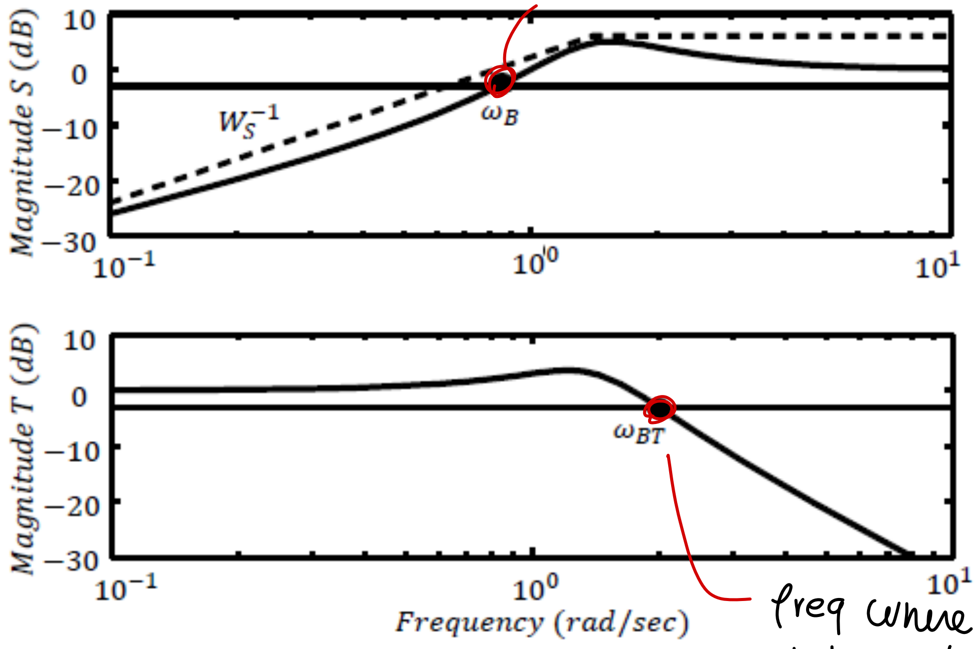
$|RS|$ attenuation @ low freq → Robustness

LINK!?

Sensitivity functions and crossover frequency

WORTH to consider!

freq where |S| valed −3dB



freq where |T| = −3dB

Define by ω_B the frequency where $|S(j\omega)|$ crosses $1/\sqrt{2}$ (-3dB) from below and by ω_{BT} the frequency where $|T(j\omega)|$ crosses $1/\sqrt{2}$ (-3dB) from above

Then, if $\varphi_m < 90^o$, one has

can be guaranteed:

$$\boxed{\omega_B < \omega_c < \omega_{BT}}$$

use NOT L(s) requirem. on ω_c → BUT requirem specification on ω_B, ω_{BT}

link between ω_c (L(s) param) and S,T data

Also in this case, specifications can be given in terms of *S(s), T(s)*

Design specifications in terms of sensitivity functions

We could specify:

- shape of $S(s)$; (including M_S: spick)
- minimum frequency ω_B;
- small or null asymptotic error for constant reference signals ($|S(j\omega)|$ small or Bode diagram of $|S(j\omega)|$ with shape +1 at low frequency);
- $M_S \leq \bar{M}_S$.

linked to $L(s)$

this defines a function $|S_{desired}(s)|$, and the function $W_S(s) = S^{-1}_{desired}(s)$ also named *(sensitivity) shaping function*

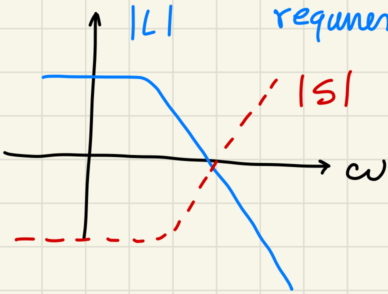
Used for MIMO approach (NOT on SISO)

then, the regulator must be designed such that

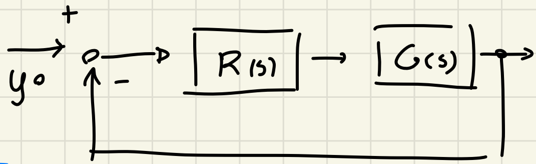
$$|S(j\omega)| < \frac{1}{|W_S(j\omega)|}, \quad \forall\omega \quad \longleftrightarrow \quad \|W_S S\|_\infty < 1$$

difficult for SISO syst!

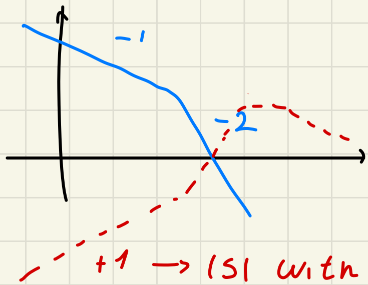
⇒ extension for MIMO usefull!



requirement on $L(s)$
to projection on $S(s)$



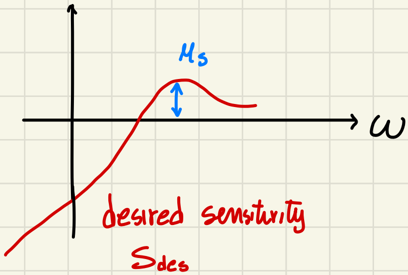
INTEGRAL ACTION? to have Good static condition



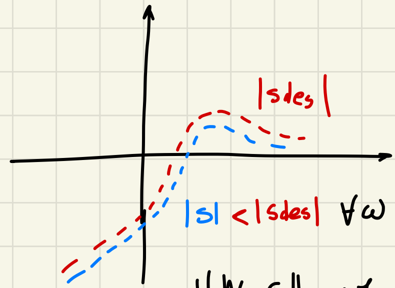
impose charact. on $S(s)$

$+1 \rightarrow |S|$ with derivative action (zero on origin)

+ ask $M_s \le \bar{M}_s$ LIMITED PICK! (for Robustness) according to Requirements



I can define $\overbrace{W_s}^{\text{sensitivity shaping function}} = 1/S_{des}$



$$\|W_s S\|_\infty < 1$$

$$|S| < \frac{1}{|W_s(j\omega)|} \quad \forall\omega$$

difficult to design REGULATOR such that it holds!

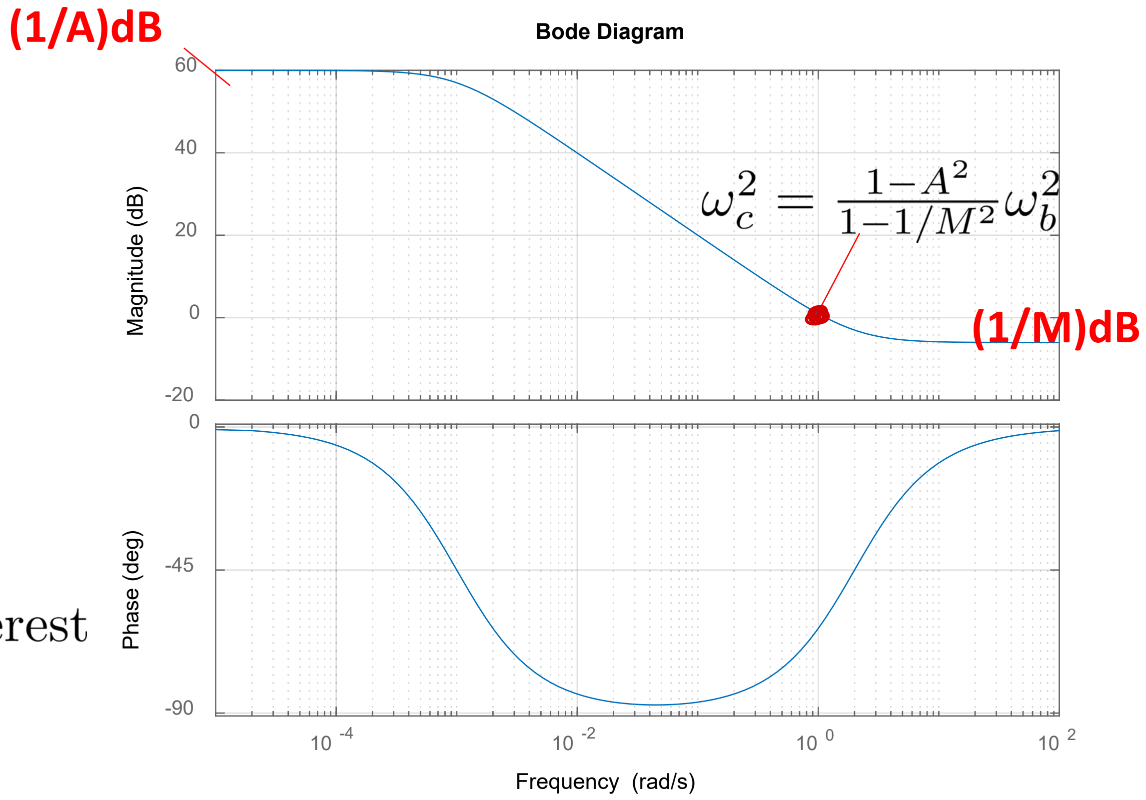
Possible choice of $W_S(s)$

$$\left(W_S(s) = \frac{s/M+\omega_B}{s+A\omega_B} \right)$$

possible easy choice example

I ORD syst

based on requirement



$A \ll 1$: desired attenuation of $S(s)$ in the band of interest

M required bound on the H_∞ norm of $S(s)$

chose M, ω_B, A according to requirement, and from here define $R(s)$ → like specify desired L shape

How to synthetise the regulator? We'll see later in the course

Design specifications in terms of complementary sensitivity function

same as S(s)

Also in this case, define a $T_{desired}(s)$, and its inverse $W_T(s) = T^{-1}_{desired}(s)$ also named *shaping function* $W_T(s)$

complementary shaping function

then, the regulator must be designed such that

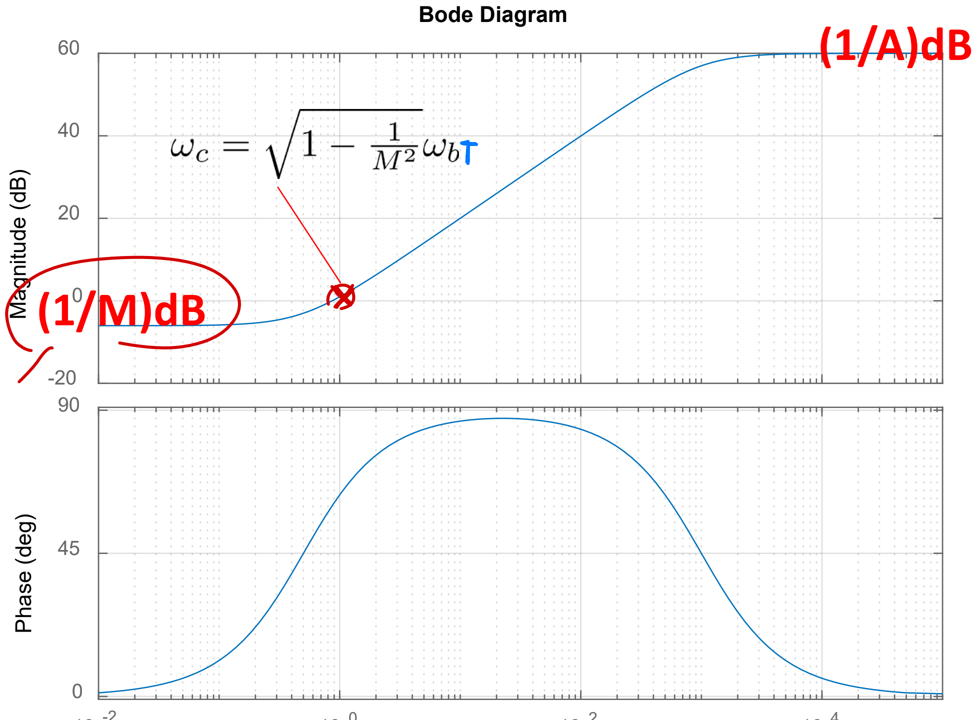
$$\boxed{|T(j\omega)| < \frac{1}{|W_T(j\omega)|}, \quad \forall\omega} \longleftrightarrow \|W_T T\|_\infty < 1$$

Example

$$W_T(s) = \frac{s + \omega_{BT}/M}{As + \omega_{BT}},$$

particular case of possible W_T choice

low freq value of W_T



Control sensitivity function $W_K(j\omega)$

once defined T, S is enough, you can avoid to specify W_K

Same approach, define the control sensitivity function *$W_K(s)$* and choose a regulator *R(s)* such that

$\to$ H∞ CONTROL ALGORITHMS, specify shaping function of T, S, K

$$\boxed{|K(j\omega)| < \frac{1}{|W_K(j\omega)|}}, \quad \forall\omega \Leftrightarrow \|W_K K\|_\infty < 1$$

The shaping function $W_S(s)$, $W_T(s)$, $W_K(s)$ must be selected as asymptotically stable systems

In summary, one must find a regulator such that

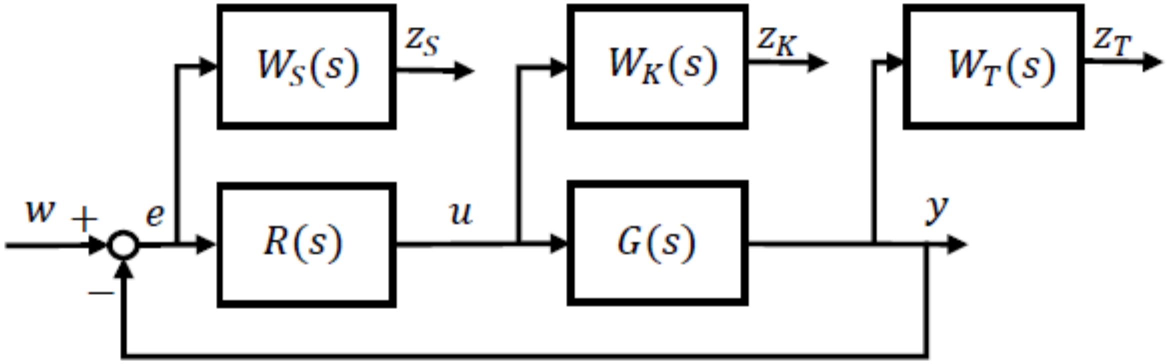
Guarantee this condition $\rightarrow$

$$\|W_S S\|_\infty < 1, \quad \|W_T T\|_\infty < 1, \quad \|W_K K\|_\infty < 1$$

basic idea of H∞ control

An interpretation

Consider the enlarged system



vector of additional variables

exogenous signals vector

Define $z = \begin{bmatrix} z_S \\ z_K \\ z_T \end{bmatrix}$, $w = y^o$, and note that $z = G_{zw} w$, $G_{zw}(s) = \begin{bmatrix} W_S(s)S(s) \\ W_T(s)T(s) \\ W_K(s)K(s) \end{bmatrix}$

T.F vector

the control synthesis can be completed by minimizing

$$\|G_{zw}\|_\infty = \sup_\omega \bar{\sigma}(G_{zw}(j\omega))$$

in principle $\|G\|<1$

(OPTIMIZ. PROBLEM)

If the resulting regulator is such that $\|G_{zw}\|_\infty < \gamma$ one has

Relax the request on γ condition

H_∞ control

$\Longrightarrow$ $$\|W_S S\|_\infty < \gamma, \quad \|W_T T\|_\infty < \gamma, \quad \|W_K K\|_\infty < \gamma$$

in **H_∞ control** you minimize H_∞ norm!

↓

minimize the T.F G_{zw} H_∞ norm instead of minimize H_∞ norm we minimize H_2 of W

H_2 control

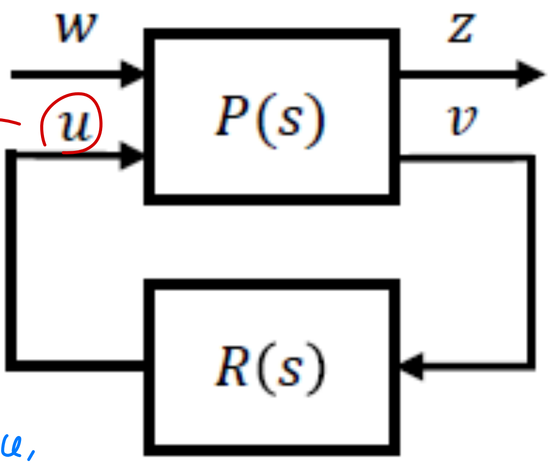
$$
\begin{aligned}
J &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(|W_S(j\omega)S(j\omega)|^2+|W_T(j\omega)T(j\omega)|^2+|W_K(j\omega)K(j\omega)|^2\right)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{+\infty}|G_{zw}(j\omega)|^2\,d\omega
\end{aligned}
$$

(2 by 2 scheme)

Formal statement of *H2 – Hinf* control

↓ you redefine the problem as follows:

real system input



Minimize the *2-norm* or *inf-norm* of $\boldsymbol{G_{zw}(s)}$ with respect to ***R(s)***

system with INPUT W: (disturbance, noise, set point)

where

z are named *performance variables*

v are named *measured variables* (measured output)

$$w = \begin{bmatrix} d \\ y^o \\ n \end{bmatrix}$$ are the *exogenous variables*

{ you want to define the Regulator R(s) such that you minimize H_∞ norm of T.F from w to z }

↓ solve this with MATLAB

Later in the course we'll study solution methods

Editor - C:\Users\Riccardo Scattolini\Documents\RICCARDO\doc\didattica\ADVANCED_MULTIVARIABLE_CONTROL\SW\First_example\first_example (2018_10_19 17_00_40 UTC).m [Read Only]

first_example (2018_10_19 17_00_40 UTC).m

```
wB=10;       %desired closed-loop bandwidth
A=1/10000;   %desired disturbance attenuation inside bandwidth
M=2 ;        %desired bound on hinfnorm(S) and hinfnorm(T)
s=tf('s'); %Laplace transform variable 's'
G=1/(s*(s+1));
numG=1;
denG=[1 1 0];
WS=(s/M+wB)/(s+wB*A); %Sensitivity weight
WK=[];                %Empty control weight
WT=(s+wB/M)/(A*s+wB); %Complementary sensitivity weight
[K,CL,GAM,INFO]=mixsyn(G,WS,WK,WT);

L=G*K;  %loop transfer function
S=inv(1+L); %Sensitivity
T=1-S;      %complementary sensitivity
figure(1)
sigma(GAM/WS,S);
title('GAM/WS and S')
figure(2)
sigma(GAM/WT,T);
title('GAM/WT and T')
figure(3)
margin(L)
% %regulator matrix A and eigenvalues
MatriceAreg=K.a
AutovReg=eig(K.a)
K.a(1,1)=0;
L1=G*K;
figure(4)
margin(L1)
[numR,denR]=ss2tf(K.a,K.b,K.c,K.d);
figure(5)
bode(numR,denR)
title('Bode diagram of the regulator')
```

Request → (inverse of static Gain)

T.F definition → II ORD system (pole in s=0, s=-1) → $\frac{1}{s^2+s}$

Ws, WT as on previous example form!

sufficent to use Ws, WT!

↳ to obtain Regulator K and Close loop (CL)

bode (PLOT)

With ALGORITHM to solve the problem, all become trivial

Beep→folder→SW→first example

[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3) or

[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3,KEY1,VALUE1,KEY2,VALUE2,...)

mixsyn H-infinity mixed-sensitivity synthesis method for robust control design. Controller K stabilizes plant G and minimizes the H-infinity cost function

$$\left\| \begin{matrix} W1*S \\ W2*K*S \\ W3*T \end{matrix} \right\|_{Hinf}$$

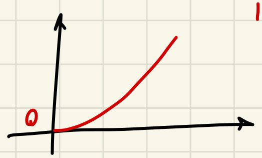
script Ln 5 Col 20

approach not so god for SISO systems
↳ good for MIMO extensions

- Guarantee a big φ_m

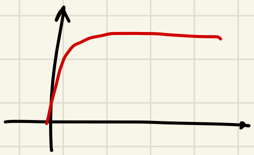
simulating the system
⇓

• open loop



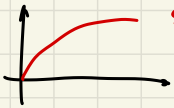
input step
↓
∞ with ramp

• close loop



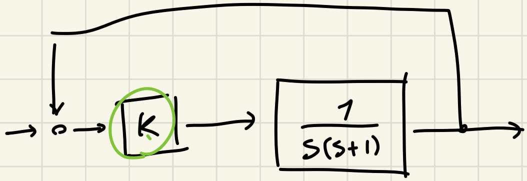
Fast responce!

IF modify ω_B
↓
$\omega_B = 1$ instead of 10...
reduce desired cross over freq

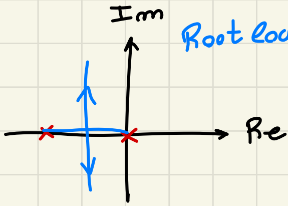


SLOWER
reduced ω_c by $\simeq 1$ decade

• H_∞ drawback: Large order of Regulator $R(s)$
Usually same order or bigger than the syst order



good result with QPRD Reg... it was not necessary an higher order R(s)



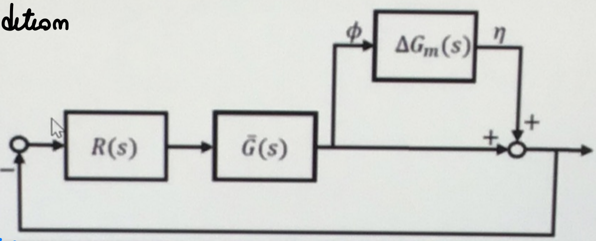
Root locus

solvable simply with PROPORTIONAL REGULATOR

MATLAB extremely usefull to understand how to solve our problems with software

Given the following feedback system, assume that in the nominal case ($\Delta G_m(s)=0$) the system is asymptotically stable and that $\Delta G_m(s)$ is asymptotically stable. Select the sufficient condition guaranteeing that in the perturbed case the asymptotic stability of the closed-loop system is maintained.

for complementary sensitivity function the condition is that $\|\bullet\|_\infty < 1$



typically unknown

- ~~$|T(jw)\ \Delta Gm(jw)| < 1\ \forall w >= 0$, T is the complementary sensitivity function~~
- $|S(jw)\ R(jw)\ \Delta Gm(jw)| < 1\ \forall w >= 0$
- $|S(jw)\ \Delta Gm(jw)| < 1\ \forall w >= 0$, S is the sensitivity function

} WRONG! NOT Related to the probleem

usually you define the control system properly

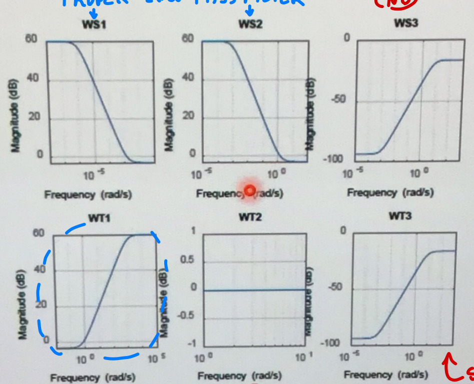
Consider the design of a *H2/Hinf* controller with shaping functions for a SISO system. Which one of the following pairs (*Wsi*, *WTj*) of the functions shown in the figure is reasonable and coherent with the goals of the control design procedure?

PROPER LOW PASS FILTER

HP filter NO

NO

[W_S, W_T, W_K can be interpreted as inverse of the desired complementary sensitivity functions]



choise looking @ complementary sensitivity (1 @ LF, small @ HF ↳ inverse graw @ HF)

this is proper choice → WS1 OR WS2 proper corresponding ?

NO don't respect

strange behav. on Low Freeq too small

- ~~WS2 - WT1~~
- WS1 - WT3
- WS3 - WT2