January 2022

Exercise 1 (3 marks)

In the Lyapunov stability analysis, for a given equilibrium point, convergence and stability are equivalent (select the correct answer).

- Yes, provided that in the definition of Lyapunov stability the value of δ is sufficiently large
- Yes
- Yes, but only for asymptotic stability
- No
- no answer

Exercise 2 (3 marks)

In Model Predictive Control, the presence of constraints requires the use of slack variables in order to deal with (select the correct answer)

- only for state constraints
- the requirement of integral action in the controller
- only for input constraints
- for state and output constraints
- no answer

Exercise 3 (3 marks)

Concerning the small gain theorem, select the correct answer

- it provides necessary and sufficient conditions for the I/O stability of the feedback interconnection of linear and nonlinear systems.
- it allows one to study the I/O stability of the feedback connection of systems required to be linear.
- it allows one to study the I/O stability of the feedback connection of systems required to be I/O stable.
- In the case of feedback connection of asymptotically stable linear systems, it is more powerful than the Nyquist criterion, in the sense that its applicability is wider.
- no answer

Exercise 4 (3 marks)

Concerning the Extended Kalman filter, select the correct answer

- its dynamics is linear
- its gain is computed with the linearization around the nominal state trajectory
- its gain is time invariant
- its gain is computed with the linearization around the most recent state estimate
- no answer

February 2022

Exercise 1 (3 marks)

Consider the system described by the following transfer function matrix and select the **WRONG** statement

$$G(s) = \begin{bmatrix} \frac{s-1}{s+1} & \frac{s-2}{s+2} \end{bmatrix}$$

- o The system has a pole in *s=-1*
- o The system has an invariant zero in *s=1*
- o The system does not have invariant zeros
- o For this system it is possible to design a regulator with integral action on the output error
- o No answer

Exercise 2 (3 marks)

Given a SISO or MIMO system with one or more (invariant) zeros with positive real part, select the CORRECT statement

- o In the design of a feedback regulator, the unstable zeros can be canceled by regulator poles
- o The zeros limit the performance achievable by a closed-loop system
- o It is impossible to stabilize the system
- o Any corresponding closed-loop system cannot have any robustness property with respect to additive or multiplicative perturbations
- o No answer

Exercise 3 (3 marks)

Consider the following system

$$\dot{x}(t) = Ax(t) + Bu(t) + Md$$
$$y(t) = Cx(t) + d$$

with *n* states, *m* inputs, *p* outputs, *v* disturbances assumed to be constant but unknown. For this system select the CORRECT statement.

- o It is not possible to design an estimator of the disturbance *d*.
- o It is possible to design an estimator of the disturbance *d* provided that *m≥p*.
- o It is possible to design an estimator of the disturbance *d* provided that *m≥p* and the pair *(A,C)* is observable.
- o It is possible to design an estimator of the disturbance *d* provided that *p≥v* and the pair *(A,C)* is observable.
- o no answer

Exercise 4 (3 marks)

Concerning the Loop Transfer Recovery procedure, select the WRONG statement

- Cannot be used for discrete time systems.
- Can be used only for square systems.
- When applicable, guarantees that the loop transfer function with state feedback plus observer is equal to the one with state feedback only at any frequency.
- Can be used only for systems with invariant zeros with negative real part.
- No answer

June 2022

Exercise 1 (3 marks)

Consider the discrete-time system

$$x(k+1) = -x(k)cos^2(x(k))$$

Using a quadratic Lyapunov function, select the correct answer among the following ones

- The origin is an unstable equilibrium
- The origin is a locally stable equilibrium
- The origin is a locally asymptotically stable equilibrium
- The origin is a globally asymptotically stable equilibrium
- No answer

Exercise 2 (3 marks)

Concerning the use of the Kalman Predictor or Filter for the continuous time system

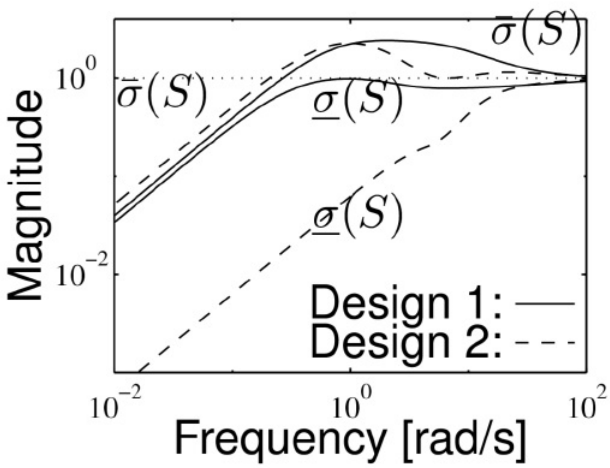
$$\dot{x}(t) = Ax(t) + Bu(t) + v_x(t)$$
$$y(t) = Cx(t) + v_y(t)$$

Select the correct answer

- It can be applied only for asymptotically stable systems
- It can be used only if v_x and v_y are uncorrelated white gaussian noises
- It can be used also when v_x is a stationary stochastic process (with suitable modifications)
- The only condition required to guarantee that the covariance of the state estimation error tends to a limiting value is that the pair (A,C) is observable
- No answer

Exercise 3 (3 marks)

Consider two control designs of a closed-loop system with the following sensitivity functions and select the answer that is surely true



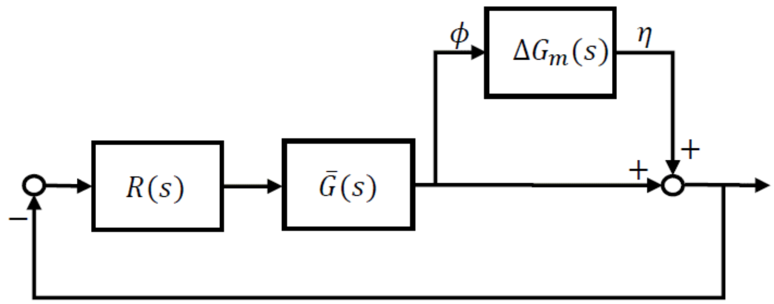
- Design 1 guarantees a faster closed-loop system
- Both designs guarantee closed-loop stability
- Design 1 guarantees slightly more attenuation of high frequency measurement noise
- Design 1 guarantees slightly more attenuation of low frequency process disturbances (d_y)
- No answer

Exercise 4 (3 marks)

In Model Predictive Control of linear systems, it is possible to guarantee robust zero steady state error regulation for constant reference signals y^o also in the case of small model uncertainties or neglected disturbances (assuming that the resulting closed-loop system is asymptotically stable) provided that

- In the cost function to be minimized it is weighted the difference between the predicted state and its asymptotic value corresponding to y^o, in nominal conditions, and the difference between the future input and its asymptotic value corresponding to y^o in nominal conditions (necessary and sufficient condition)
- The prediction horizon is chosen sufficiently long
- A model in $\delta x(k)=x(k)-x(k-1)$ and $\delta u(k)=u(k)-u(k-1)$ is used
- The prediction horizon is selected longer than the control horizon
- No answer

Exercise 5 (3 marks)
Consider the following control system with multiplicative uncertainty



where

$$\bar{G}(s) = \frac{m}{1+as},\ a > 0,\ R(s) = \frac{k(1+as)}{s},\ \Delta G_m(s) = g$$

Select the sufficient condition required to guarantee the stability of the overall system

- $||\frac{mgk}{jw+mk}||_\infty<1$
- $||\frac{gk(1+jaw)}{jw+mk}||_\infty<1$
- $|g| < 1$
- $||\frac{mk}{jw}||_\infty<1$
- No answer

July 2022

Exercise 1 (3 marks)

Concerning H_{inf} control, select the **wrong** statement among the following ones:

- LQG is a particular form of H_{inf} control,depending on the choice of the design parameters
- The structure of H_{inf} and H_2 regulators is made by a state feedback control law and a state observer
- The use of shaping functions is required to impose the form of the sensitivity functions
- the shaping functions at the process inputs or at the process outputs must be asymptotically stable
- No answer

Exercise 2 (3 marks)

Consider the following discrete-time system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.25 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathrm{u(k)}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

which output/s can be asymptotically track a given constant reference signal?

- None of them
- The first one y_1
- The second one y_2
- Both of them
- No answer

Exercise 3 (3 marks)

Consider a nonlinear discrete time system $x(k+1)=f(x(k),u(k))$, an equilibrium point, and the corresponding linearized system $\delta x(k+1)=A\delta x(k)+B\delta u(k)$. Given the Lyapunov equation

$$A'PA-P=-Q$$

with $Q>0$, it results $P>0$. Then

- Nothing can be said on the stability of the equilibrium since it should be checked that for all the possible $Q>0$ one has $P>0$
- The equilibrium is locally asymptotically stable
- The equilibrium is globally stable
- The equilibrium is globally asymptotically stable
- No answer

Exercise 4 (3 marks)

The poles of the following system are the system

$$G(s) = \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{s+3} \\ \dfrac{s-1}{(s+2)(s+3)} & \dfrac{1}{s+3} \\ \dfrac{s+2}{(s+1)^2} & \dfrac{s}{(s+1)^2} \end{bmatrix}$$

- Poles: s=-1 (double) s=-2, s=-3
- Poles: s=-1 (triple) s=-2, s=-3
- Poles: s=-1 (double) s=-2, s=-3 (double)
- Poles: s=-1 (triple) s=-2, s=-3 (double)
- No answer

September 2022

Exercise 1 (3 marks) **SELECT THE CORRECT ANSWER**

Consider the continuous-time linear system:

$$\dot{x}(t) = Ax(t)$$

and assume that $A+A'$ is definite negative. Then, with the Lyapunov theory it is possible to conclude

- Nothing about the stability of the system.
- The system is stable
- The system is asymptotically stable
- The system is unstable
- No answer

Exercise 2 (3 marks) **SELECT THE CORRECT ANSWER**

The zeros of continuous or discrete time systems are important because:

- They influence the stability of the system
- They influence the static and dynamic performances which can be achieved with the design of a closed-loop system.
- They influence only the static performances which can be achieved with the design of a closed-loop system.
- They influence the possibility to design LQ or LQG control laws.
- No answer

Exercise 3 (3 marks) **SELECT THE CORRECT ANSWER**

The design of a state-feedback pole placement control law for a reachable multi-input system

- Can be always completed only by using all the inputs
- Can be always completed by using only one input
- Can be completed using one or more inputs, it depends on the specific problem
- Can be completed only if the system does not have zeros
- No answer

Exercise 4 (3 marks) **SELECT THE CORRECT ANSWER**

Imposing the terminal constraint $x(k+N)=0$ in the formulation of MPC to guarantee stability

- Can be made only for unconstrained problems
- Can be made, but it is then impossible to use the Receding Horizon approach and all the sequence of future computed control moves $u(k),\ldots,u(k+N-1)$ must be used
- Can be made, provided that the state $x(k)$ at the current time instant k must be in the set of states where a solution exists
- Can only be made for linear systems
- No answer

Exercise 5 (3 marks) **SELECT THE CORRECT ANSWER**

Concerning the Infinite Horizon LQ control for discrete time systems:

- It guarantees at the same time gain and phase robustness margins
- It guarantees either gain or phase robustness margins
- It guarantees gain margins smaller than in continuous time
- It guarantees gain margins smaller, or equal, or larger than in continuous time
- No answer