Exercise 1.a

Consider the system

$$\dot{x}_1(t) = -x_1(t) + x_2(t)$$
$$\dot{x}_2(t) = -x_1^3(t) - x_2^3(t)$$

and the Lyapunov function $V(x) = bx_1^4 + ax_2^2(t)$. Select proper values of a, b such that $V(x)$ can be used to prove the stability of the origin.

Vdot$=-4bx_1^4+4bx_1^3x_2-2ax_1^3x_2-2ax_2^4$

a. *a=1, =b=1 (Vdot*$=-4x_1^4+2x_1^3x_2-2x_2^4$*)*
b. *a=2, b=1 (Vdot*$=-4x_1^2-4x_2^4<0$*)*
c. *a=0,b=1 (V is not a Lyapunov function)*

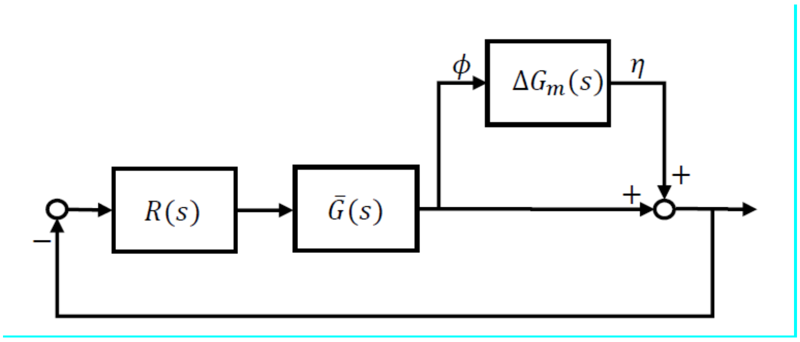
Exercise 1.b

In view of the previous answer, the origin is (***select the strongest property***)

a. stable
b. locally asymptotically stable
c. globally asymptotically stable

Exercise 2

Given the following feedback system, assume that in the nominal case ($\Delta G_m(s)=0$) the system is asymptotically stable and that $\Delta G_m(s)$ is asymptotically stable. Select the sufficient condition guaranteeing that in the perturbed case the asymptotic stability of the closed-loop system is maintained.



a. $|\mathrm{T(jw)Delta(jw)}| < 1$, for any $\mathrm{w} \geq 0$
b. $|\mathrm{S(jw)Delta}G_{\mathrm{m}}(\mathrm{jw})| < 1$, for any $\mathrm{w} \geq 0$
c. $|\mathrm{S(jw)}R(jw)\mathrm{Delta}G_{\mathrm{m}}(\mathrm{jw})| < 1$, for any $\mathrm{w} \geq 0$

Exercise 3.a

Consider the system with the following transfer function *G(s)*

$$\begin{bmatrix} \frac{a}{(s+1)} & \frac{b}{(s+1)(s+2)} \\ \frac{c}{(s+2)} & \frac{d}{(s+2)} \end{bmatrix}$$

The set of poles (with the proper multiplicity) is

a. -1,-1,-2,-2,-2

b. -1,-2,-2

c. -1, -2

$$detG(s) = \frac{ad(s+2)-bc}{(s+1)(s+2)^2}$$

Exercise 3.b

The system must not have zeros at the origin. The zero is *s=(2ad-bc)/ad*. Therefore it must be *2ad* not equal to *bc*

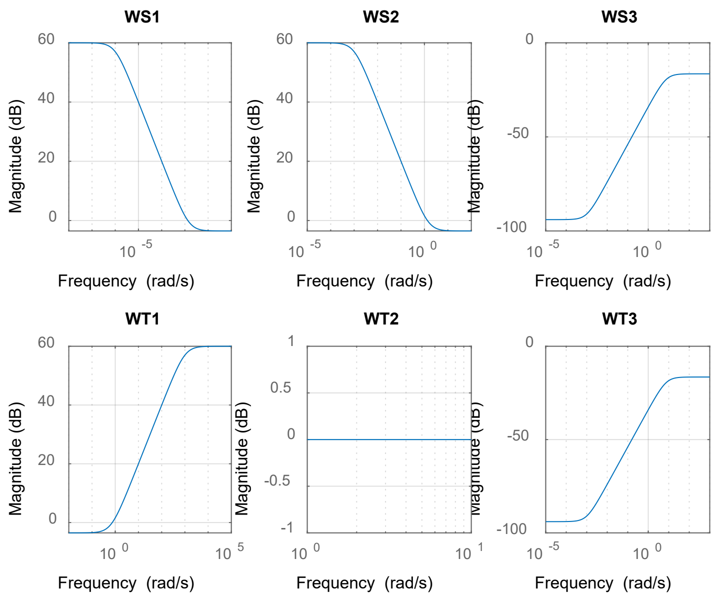
a. *a=b=c=d=1*

b. *a=b=d=1,c=2*

c. *for any value of a,b,c,d*

Exercise 4

Consider the design of a *H2/Hinf* controller with shaping functions for a SISO system. Which one of the following pairs (W_{Si}, W_{Tj}) of the functions shown in the figure is reasonable and coherent with the goals of the control design procedure?



a. WS3-WT2 WT2 does not make sense

b. WS2- WT1 acceptable

c. WS1 -WT3 WT3 is not acceptable, small gain at high frequency

Exercise 5

Consider the system

$$\dot{x}_1(t) = bu(t)$$
$$\dot{x}_2(t) = x_1(t) + u(t)$$

and assume that you want to design an infinite horizon LQ control with *$Q=diag(q_1,q_2)$, $R=1$.*

A. Compute the necessary conditions guaranteeing that the solution of the infinite horizon LQ control is stabilizing.

a. any *b, $q1>0$, $q2>0$* no, lack of reachability if *$b=0$*
b. *$b\neq 0$, $q1>0$, $q2\geq 0$* no lack of observability if *$q2=0$*
c. $b\neq 0$, any $q1\geq 0$, $q2>0$ yes

B. With *Q=I*, *b=1*, the solution of the steady-state Riccati equation is *P=I*. What are the closed-loop eigenvalues? Recall the steady state Riccati equation of LQinf control: $A'P + PA + Q - PBR^{-1}B'P = 0$

a. -1,-2
b. -2,-2
c. -1,-1

C. Assume now to implement the feedback control law $u(t) = -\rho K x(t)$ (*K* is again the solution of the LQinf problem), specify the set of values of ρ guaranteed by LQ control so that the closed-loop system remains asymptotically stable.

a. ρ in (0,∞)
b. ρ in (0.5,∞)
c. ρ in (-0.5,0.5)

A.

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} b \\ 1 \end{bmatrix} \rightarrow \text{reachability matrix } M_r = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} b & 0 \\ 1 & b \end{bmatrix} \rightarrow \text{ condition } b \neq 0$$

$$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \rightarrow Q^{1/2} = \begin{bmatrix} \sqrt{q_1} & 0 \\ 0 & \sqrt{q_2} \end{bmatrix} \rightarrow \text{observability matrix } M_o = \begin{bmatrix} Q^{1/2} \\ Q^{1/2}A \end{bmatrix} = \begin{bmatrix} \sqrt{q_1} & 0 \\ 0 & \sqrt{q_2} \\ 0 & 0 \\ \sqrt{q_2} & 0 \end{bmatrix} \rightarrow q_2 > 0$$

B.

$$K = R^{-1}B'P = \begin{bmatrix} 1 & 1 \end{bmatrix} \rightarrow A - BK = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \rightarrow \text{eigenvalues } -1, -1$$

C.

The loop transfer function is $K(sI-A)^{-1}B=(2s+1)/s^2$. In order to compute the gain margin, consider the characteristic equation $s^2+2\rho s+\rho=0$ with stable roots for any ρ>0. The gain margin is (0,inf).

Exercise 6

EKF: see the notes and the slides

Exercise 7

MPC: se the notes and the slides