INTERNAL (IMC) MODEL CONTROL

Advanced and Multivariable Control

simple technique a lot used → based on using an internal model of the process

Internal Model Control

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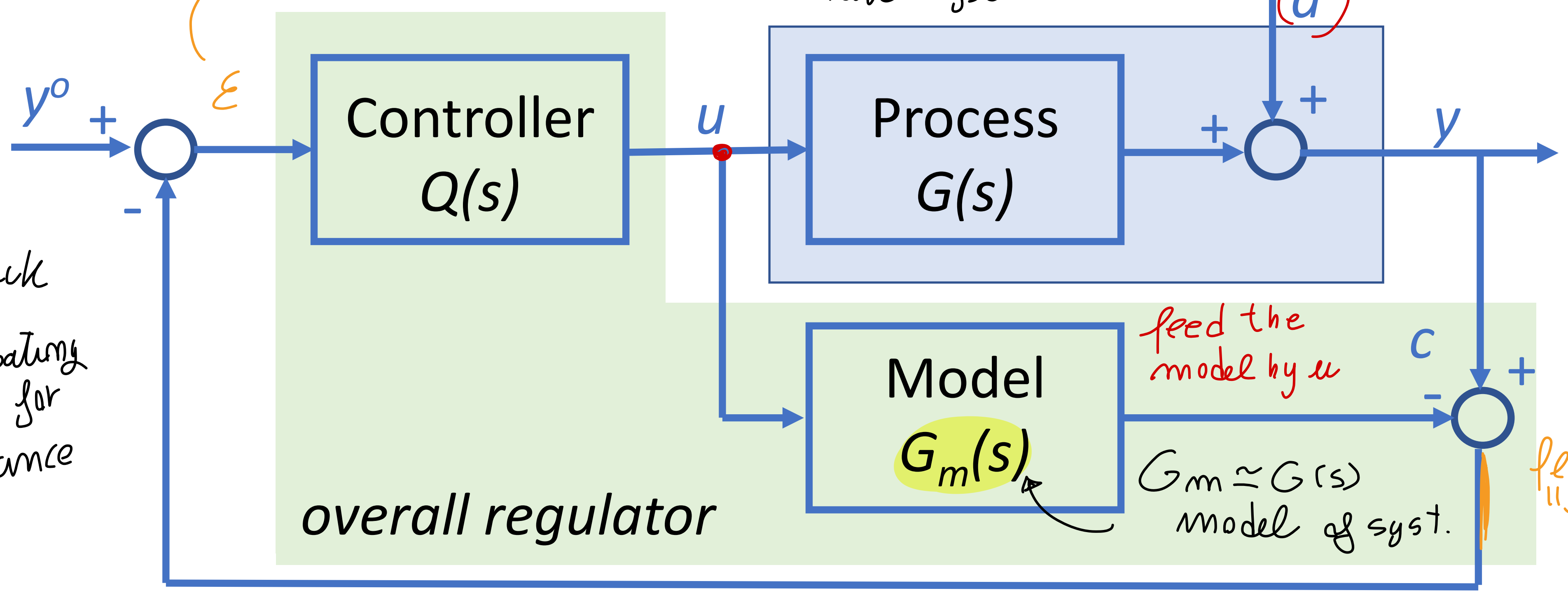
Internal Model Control scheme

different structure where controller design guaranteed by knowledge

disturbance, uncertainty

from asymp stable syst

error signal



feedback ↓ compensating signal for disturbance

including $d(t)$ ↓ feedback an "error" respect the two

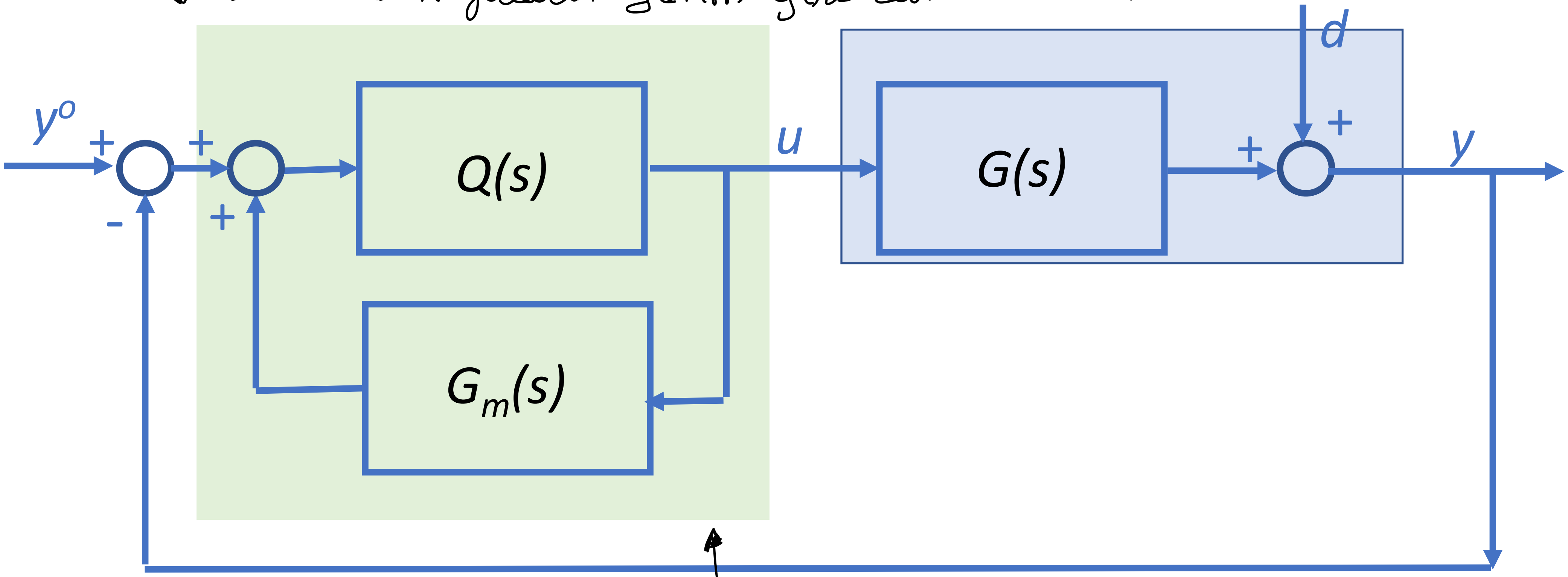
IF $G(s) = G_m(s)$, $d = 0$

$y = c$ ~> no feedback, only open loop part control

The process is *SISO*, *asymptotically stable* and described by the transfer function *G(s)*

popular approach used on process control

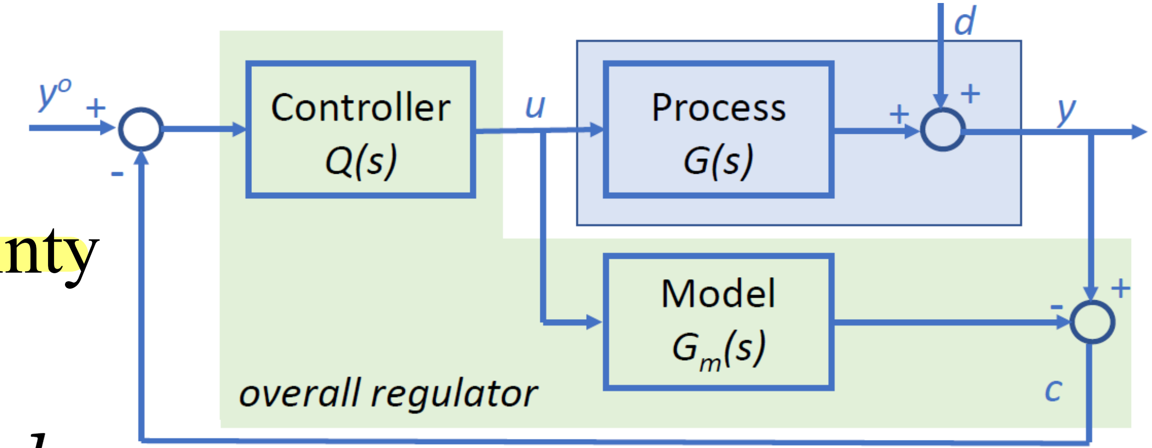
(standard Regulator form you can obtain)



Equivalent "standard " feedback regulator

$$\boxed{R(s) = \frac{Q(s)}{1-Q(s)G_m(s)}}$$

Internal Model Control scheme



$(G=G_m,\ d=0)$

$C=(G\text{-}G_m)u+d$ null if there is no uncertainty

you can compute the T.F Y/Y_o, Y/D

$$y=\frac{GQ}{1+Q(G-G_m)}y^o+\frac{1-QG_m}{1+Q(G-G_m)}d$$

$\to 1$ $\to 0$

(if G, G_m asymp. st. cascade of asymp. st!)

If $\boxed{Q=G_m^{-1}}$, then $\boxed{y=y^o}$ perfect tracking and perfect disturbance rejection

assuming ($G_m \simeq G$)

These properties hold also when $\boxed{G\neq G_m}$: Good property!

usually $G\neq G_m$ But also in this case hold..

$$y=\underbrace{\left(\frac{GQ}{\cancel{1}+Q(G-\cancel{G_m})}\right)}_{1}y^o+\left(\frac{\cancel{1}-Q\cancel{G}_m}{1+Q(G-G_m)}\right)d$$

$\to 0$

normally G, G_m are strictly proper T.F

↓ G_m^{-1} cannot be defined!

Problems

NOT possible in general ⇓

The condition $Q = G_m^{-1}$ can be critical due to

- physical realizability of G_m^{-1} (more zeros than poles)
- "unstable" zeros or delays ($e^{-\tau s}$) of G_m → cancellations with unstable zeros/poles of G and/or anticipative terms must be avoided (delays, unstable ZEROS issue!)

Solution

how to solve that problem

PARTITION... contain the other system model part

Write $G_m = G_{mn} G_{mp}$ where G_{mp} contains all the unstable zeros and the delay of G_m (*nonminimum phase terms*, **not invertible**) and has *unit static gain*

(Gmp NON INVERTIBLE!)

Then set

low pass filter of suitable order

(you invert only the one possible to invert)

(Idea to invert only the invertible part of Gm!)

$$\boxed{Q = G_{mn}^{-1} G_f}$$

(instead of $Q = G_m^{-1} G_f$)

implementable!

where G_f is a low-pass filter, with $G_f(0)=1$, which makes Q causal (with a number of poles greater or equal to the number of zeros). G_f is also useful to provide some robustness.

Gf to be choosen by modify cut-off freq of the filter

Example

non min phase

delay

$$G_m(s) = \frac{5(1-s)e^{-s}}{(1+2s)^2(1+5s)}$$

The part containing unstable ZEROS + delay

$$G_{mp}(s) = (1-s)e^{-s}$$ (unit gain) $\rightarrow G(0) = 1$

remaining Gm part

$$G_{mn}(s) = \frac{5}{(1+2s)^2(1+s)}$$

↳ G_{mn}^{-1} will have 3 ZEROS

$$G_f(s) = \frac{1}{(1+10s)(1+0.1s)^2}$$

We should include 3 poles on $G_f(s)$!

$$Q(s) = G_{mn}^{-1}(s)G_f(s) = \frac{(1+2s)^2(1+s)}{5(1+10s)(1+0.1s)^2}$$

define the dynamic

we want by dominant pole, while the others on high frequency

One obtains

$$y = \frac{G_{mp}G_f + (G-G_m)G_{mn}^{-1}G_f}{1+(G-G_m)G_{mn}^{-1}G_f} y^o + \frac{1-G_{mp}G_f}{1+(G-G_m)G_{mn}^{-1}G_f} d$$

If $G=G_m$ (perfect system model!)

cannot be removed! unstable part not cancellable

$$\boxed{y = G_{mp}G_f y^o + (1 - G_{mp}G_f)d}$$

tuning parameter in G_f to modify response!

Since $G_{mp}(0) = G_f(0) = 1$ at the steady state $y = y^o$

for constant disturbance

Example

$$G(s) = G_m(s) = \frac{5(1-s)e^{-s}}{(1+2s)^2(1+5s)}$$

(no modeling error)

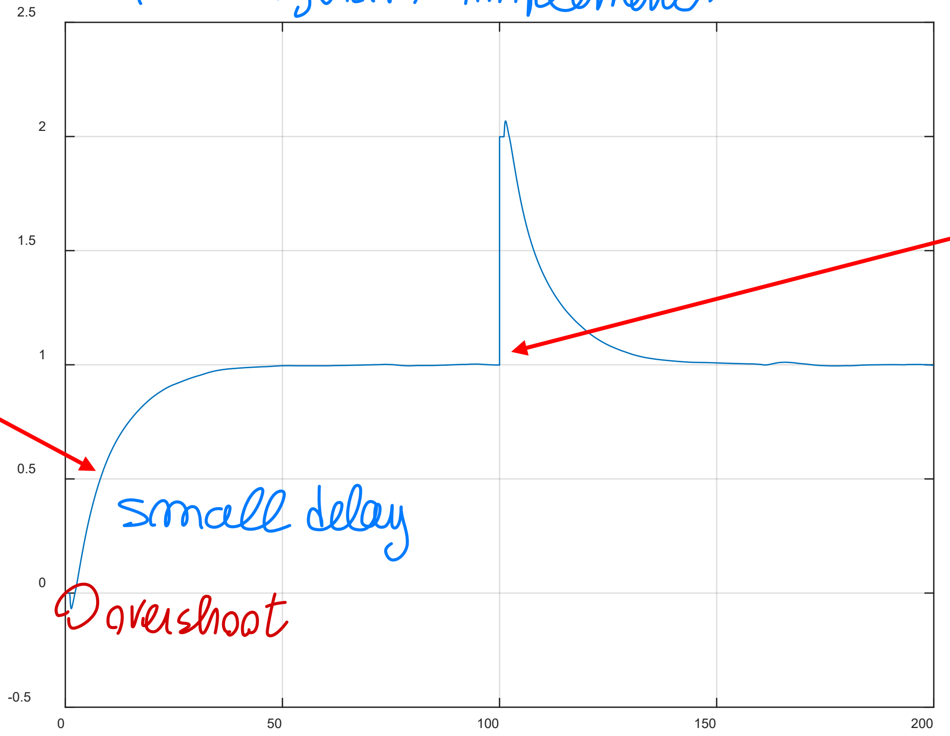
$$G_{mn}(s) = \frac{5}{(1+2s)^2(1+s)} \quad , \quad G_{mp}(s) = (1-s)e^{-s} \quad , \quad G_f(s) = \frac{1}{(1+10s)(1+0.1s)^2}$$

from system implement.

for implementation reasons

Step response dominated by the slow time constant (*T=10*) of G_f

step responce ⇒



small delay

Overshoot

disturbance

still unstable zeros as part of the system !

→ repeat analysis assuming a modelling error:

I mistake one of the pole in −0.5 !

Example

$$G(s)=\frac{5(1-s)e^{-s}}{(1+2s)^2(1+5s)}\ ,\qquad G_m(s)=\frac{5(1-s)e^{-s}}{(1+2s)(1+5s)}$$

(modeling error)

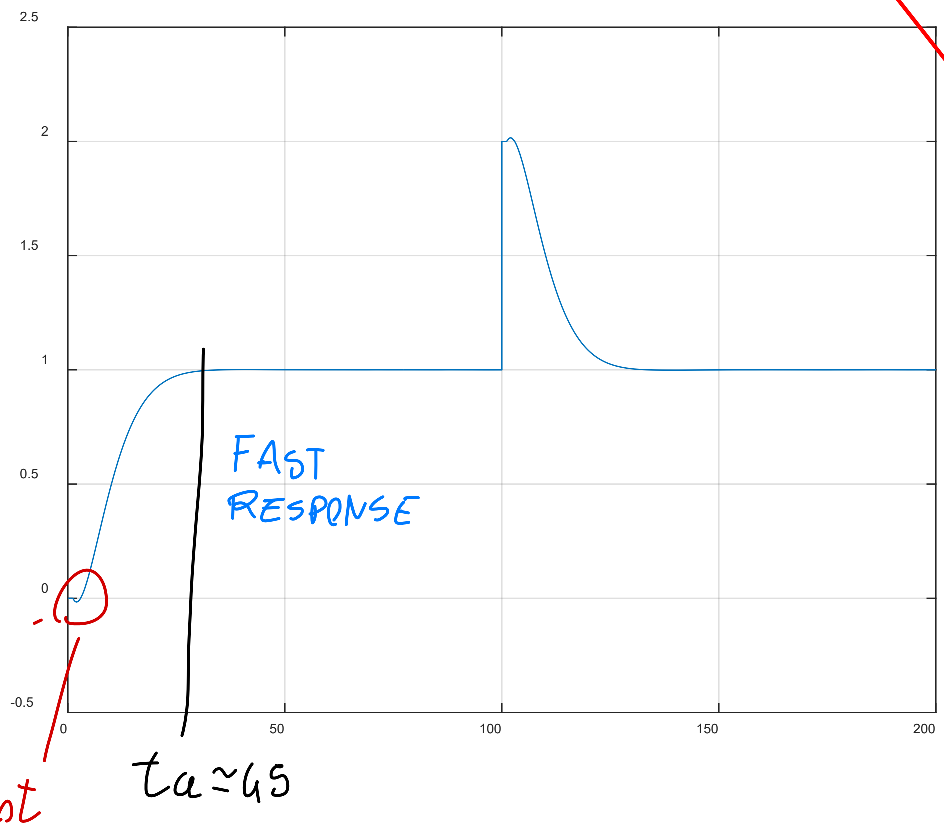
$$G_{mn}(s)=\frac{5}{(1+2s)(1+s)}\ ,\qquad G_{mp}(s)=(1-s)e^{-s}\ ,\qquad \underbrace{G_f(s)=\frac{1}{(1+10s)(1+0.1s)^2}}_{\text{free design!}}$$

Good part of G

Bad part!

system step responce ↳

Works well also for modelling error



"practical " robustness

stability Guaranteed because G(s) stable ↓ and all stability issue maintained @ numerator

Comments

miceanuseal scheme based on plant model

- The design procedure in the case of discrete time systems is exactly the same, only with $s \rightarrow z$
- Many extensions are available, for example to unstable systems, or regulators with integral action. (include integral action!)
- Tuning rules for PID controllers based on IMC have been proposed
- Also for MIMO systems the design procedure is conceptually the same, but the partitioning

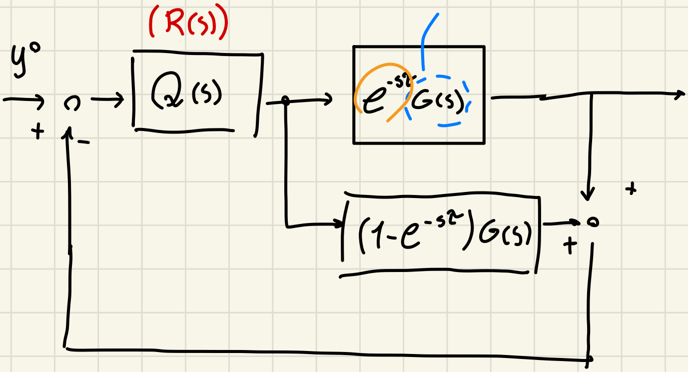
$$G_m = G_{mn} G_{mp}$$

is more complex

smith predictor

(delay constrain a lot performance due to ccc limitation)

SISO asymp stable



$\Rightarrow \boxed{L(s) = R(s)\, G(s)}$

with these scheeme using internal syst model!

on this loop T.F you can design R(s) for a system without delay!

computing $Y/Y°$ you still have the intrisic delay of the system!

↓

you can design R(s) forgetting delay for better performance

You assume perfect τ, G(s) knowledge

→ ROBUSTNESS limitation ⟷ model mismatch!

CRITICAL