ADVANCED AND MULTIVARIABLE CONTROL

7/7/2022

Surname and name ....Solutions....

Signature ....

Exercise 1 (3 marks)

Concerning H_{inf} control, select the <u>wrong</u> statement among the following ones:

- ☒ LQG is a particular form of H_{inf} control,depending on the choice of the design parameters
- o The structure of H_{inf} and H_2 regulators is made by a state feedback control law and a state observer
- o The use of shaping functions is required to impose the form of the sensitivity functions
- o the shaping functions at the process inputs or at the process outputs must be asymptotically stable
- o No answer

It is a particular form of H_2 control

Exercise 2 (3 marks)

Consider the following discrete-time system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.25 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

which output/s can be asymptotically track a given constant reference signal?

- o None of them
- o The first one y_1
- ☒ The second one y_2
- o Both of them
- o No answer

You can compute the (invariant) zeros and check if they are in $z=1$ (derivative actions) or no.

Another way (the same, in practice) is to compute

$$G(z) = \begin{vmatrix} \frac{z-1}{\Delta} \\ \frac{z}{\Delta} \end{vmatrix}, \quad \Delta = (z-0.5)^2$$

you see here the derivative action.

Exercise 3 (3 marks)

Consider a nonlinear discrete time system $x(k+1)=f(x(k),u(k))$, an equilibrium point, and the corresponding linearized system $\delta x(k+1)=A\delta x(k)+B\delta u(k)$. Given the Lyapunov equation

$$A'PA-P=-Q$$

with $Q>0$, it results $P>0$. Then

- o Nothing can be said on the stability of the equilibrium since it should be checked that for all the possible $Q>0$ one has $P>0$
- X The equilibrium is locally asymptotically stable
- o The equilibrium is globally stable
- o The equilibrium is globally asymptotically stable
- o No answer

Lyapunov Theory. However, since the result is based on the analysis of the linearized system, it is only local

Exercise 4 (3 marks)

The poles of the following system are the system

$$G(s)=\begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{s+3} \\ \dfrac{s-1}{(s+2)(s+3)} & \dfrac{1}{s+3} \\ \dfrac{s+2}{(s+1)^2} & \dfrac{s}{(s+1)^2} \end{bmatrix}$$

- o Poles: s=-1 (double) s=-2, s=-3
- o Poles: s=-1 (triple) s=-2, s=-3
- o Poles: s=-1 (double) s=-2, s=-3 (double)
- X Poles: s=-1 (triple) s=-2, s=-3 (double)
- o No answer

Standard computations

Exercise 5 (3 marks)

Concerning the Extended Kalman Filter, select the correct answer among the following ones:

- o It can be applied only to discrete time systems
- o It requires the linearization along the **nominal** trajectories of the system
- o Under proper assumptions its asymptotic stability is guaranteed
- X It can be used to estimate the unknown constant parameters of a gray-box model
- o No answer

Exercise 6 (7 marks)

Consider the system

$$\dot{x_1}(t) = (1 + x_1^2(t))x_2(t)$$

$$\dot{x_2}(t) = u(t)$$

1. Design a control law with the backstepping method
2. Specify the Lyapunov function that could be used to prove the stability of the origin of the corresponding closed-loop system.

Formula backstepping

$$u = -\frac{dV_1(x_1)}{dx_1} g(x_1) - k(x_2 - \Phi_1(x_1)) + \frac{d\Phi_1(x_1)}{dx_1}(f(x_1) + g(x_1)x_2)$$

The simplest solution is to set

$$x_2 = \phi_1(x_1) := -x_1$$

The first equation becomes $\dot{x}_1 = -(1 + x_1^2)x_1$

$$V_1(x_1) = \frac{1}{2} x_1 x_1 \rightarrow \dot{V}_1(x_1) = x_1 \dot{x}_1 = \underbrace{-\underbrace{(1 + x_1^2)}_{>0} \underbrace{x_1^2}_{\geq 0}}_{<0}$$

$$u = -\underbrace{x_1}_{\frac{dV_1}{dx_1}} . \underbrace{(1 + x_1^2)}_{g(x_1)} - k(x_2 + \underbrace{x_1}_{-\phi_1(x_1)}) + \underbrace{(-1)}_{\frac{d\phi_1(x_1)}{dx_1}}(1 + x_1^2)x_2$$

Lyapunov function for the backstepping approach

$$V(x_1, x_2) = V_1(x_1) + \frac{1}{2}\underbrace{(x_2 - \phi_1(x_1))^2}_{(x_1 + x_2)^2}$$

Exercise 7 (6 marks)

Consider the following system

$$\dot{x}(t) = x(t) + u(t) + v_x(t)$$
$$y(t) = x(t) + v_y(t)$$

With $v_x = WN(0, Q = 3R), R > 0, and\ v_y = WN(0, R)$.

Compute the steady state gain of the Kalman Predictor (KP), write the formula of the KP and compute the transfer function between the output of the system and the state estimate

Steady state Riccati equation of the KP

$$0 = AP + PA' + Q - PC'R^{-1}CP$$

$A = B = C = 1, \quad Q = 3R$ (all the assumptions for KP in steady state are verified.)

$P = \begin{cases} 3R \checkmark \\ < 0 \end{cases} \Rightarrow L = PC'R^{-1} = 3$

KP $\quad \dot{\hat{x}}(t) = \hat{x}(t) + u(t) + 3\left[y(t) - \hat{x}(t)\right]$

$\Downarrow$

$\dot{\hat{x}}(t) = -2\hat{x}(t) + 3y(t) + u(t)$

$\hat{X}(s) = \dfrac{1}{s+2}\left[3Y(s) + U(s)\right]$

$\Downarrow$

transf. function $= \dfrac{3}{s+2}$

Exercise 8 (5 marks)

In order to design an MPC control law stabilizing the origin, show how to modify the optimization problem by means of suitable terminal cost and constraint. Then discuss a possible selection of these quantities which satisfy the required conditions for stability.

See the notes pag. 204-208

Specifying the zero terminal constraint approach was sufficient.