Exercises session 7: Model Predictive Control

Ex. 1: Given the discrete time system

$$\begin{cases} x(k+1) = 5x(k) + u(k) \end{cases} \tag{1}$$

and the cost function

$$J(x,u) = \sum_{i=0}^{N-1} \left[(x(k+i))^2 + (u(k+i))^2 \right] + \mathcal{S}x(k+N) \tag{2}$$

1. With $\mathcal{S} = 1$, find the minimum prediction horizon N which ensure the closed-loop stability.
2. If $N = 1$, which is the minimum value of $\mathcal{S}$ which guarantees the closed loop stability?
3. Consider $N = 2$, $S = 1$ and $x(k) = 0.2$. Moreover, assume the following constraints

$$\begin{cases} 0 \leq x(k+i) \leq 2 \\ -1 \leq u(k+i) \leq 1 \end{cases} . \tag{3}$$

   Show how to set-up a Quadratic programming optimization problem.

Ex. 2: Given the step $u(k) = step^*(k)$, consider the system whose response to that step is

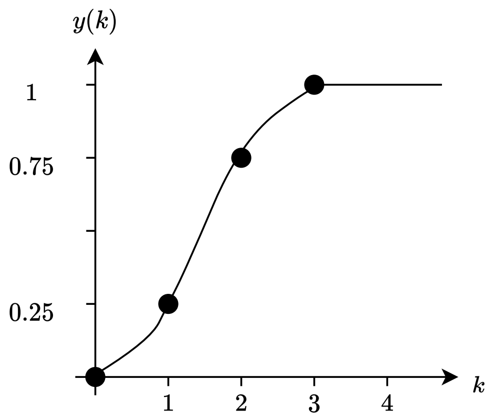

Figure 1

1. Define the step response model and the impulse response model of the system.
2. Formulate the closed-loop impulse-response predictive model with horizon N=2.
3. Formulate the closed-loop step-response predictive model with horizon N=2.

Ex. 2: Given following dynamical system

$$x(k+1) = 0.5x(k) + 0.5u(k) \tag{4}$$

1. Define the predictive model with horizon N=3 using the impulse response.
2. Show how to modify the approach to have a closed-loop control scheme.