Advanced and Multivariable Control
7/2013

Exercise 1

Given the system with transfer function matrix

$$G(s) = \begin{bmatrix} \dfrac{\alpha}{s+1} & \dfrac{1}{s+2} \\[2mm] \dfrac{1}{s+1} & \dfrac{1}{s+2} \\[2mm] \dfrac{1}{s+1} & \dfrac{\beta}{s+2} \end{bmatrix}$$

1. Compute the poles.
2. How many output variables can asymptotically track their constant reference signals (motivations are required)?
3. Compute the conditions required to asymptotically track constant reference signals for the outputs 1 and 3.
4. The system can be written in state-space form as

$$\dot{x}_1(t) = -x_1(t) + u_1(t)$$
$$\dot{x}_2(t) = -2x_2(t) + u_2(t)$$
$$y_1(t) = \alpha x_1(t) + x_2(t)$$
$$y_2(t) = x_1(t) + x_2(t)$$
$$y_3(t) = x_1(t) + \beta x_2(t)$$

   Show the enlarged state (with integrators) to be used in the design of a stabilizing regulator guaranteeing asymptotic zero error regulation for constant reference signals (outputs 1 and 3).
5. Briefly describe possible synthesis techniques for the design of this regulator, specifying if it is necessary to use a dynamic state observer.

Exercise 2

Consider the system (Lorentz eqns.)

$$\dot{x}_1(t) = \sigma(x_2(t) - x_1(t))$$
$$\dot{x}_2(t) = r x_1(t) - x_2(t) - x_1(t) x_3(t)$$
$$\dot{x}_3(t) = x_1(t) x_2(t) - b x_3(t)$$

where $\sigma, r, b > 0$.

- Compute the equilibria
- Analyze the stability of the origin by means of the linearized system.
- Let $\sigma = 1, r = 0.5, b = 1$ and study the stability of the origin of the nonlinear systems with the Lyapunov function

$$V(x) = x' \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} x$$
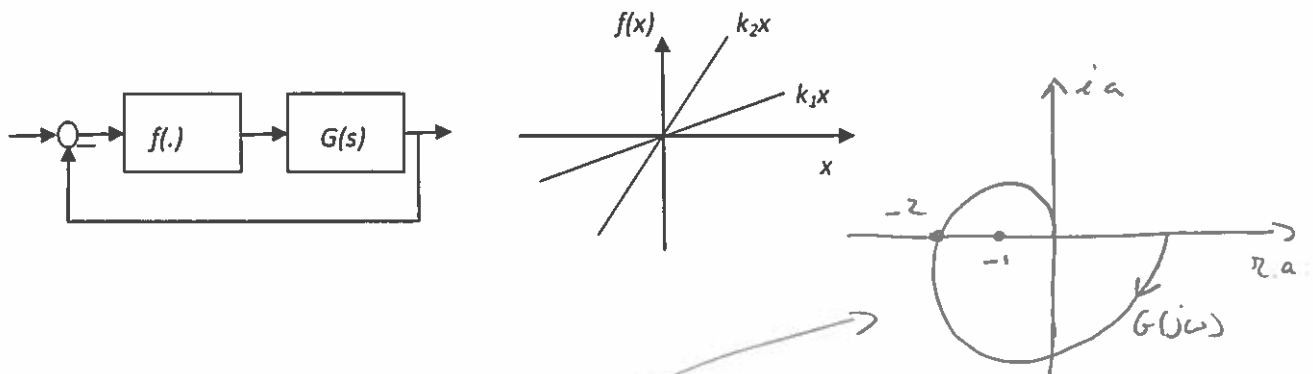
Exercise 3

Given the system with transfer function $G(s) = \dfrac{s-2}{(s-1)(s+1)}$

1. Show how to design a regulator with integral action such that the closed-loop poles are in $s = -1$. (write the system of linear equations to be solved).

2. Considering a general case (not the previous $G(s)$), show how to implement the regulator in order to avoid that the zeros of the regulator transfer function are zeros of the closed-loop transfer function from the reference signal to the output.

Exercise 4

1. Define the I/O (input/output) stability of a dynamical system
2. Given the feedback system

where $G(s)$ is asymptotically stable with polar diagram, compute the minimum value of $k_1 \geq 0$ and the maximum value of $k_2 > 0$ to guarantee I/O stability (a graphical solution is enough).

Exercise 5

Given the discrete-time system

$x(k+1) = 4x(k) + u(k)$

and the performance index

$$J = \sum_{i=0}^{N-1}(x^2(k+i) + 2u^2(k+i))$$

by using the Receding Horizon strategy, show how to compute the minimum value of N guaranteeing the stability of the closed-loop system

(Riccati eq.: $P(k) = A'P(k+1)A + Q - A'P(k+1)B[B'P(k+1)B + R]^{-1}B'P(k+1)A$)

Explain when, in predictive control, it is better (in some cases mandatory) to use an "open-loop" approach rather than relying on the Riccati equation.