Electric systems – load flow

Basics! description

- Preliminaries
- An introductory example
- Problem statement
- A solution example
- Conclusions

Foreword

- Prior to entering the subject, we need to recall/review two basic concepts:
  - how power is transferred in the network $\Rightarrow$ machine angle,
  - and how the effects of generators are combined $\Rightarrow$ network admittance matrix.
- Moreover, we need to abandon the purely energy-centred approach (no voltages or currents) taken so far, and adopt a phasor-oriented vision.
- Without impairing the conveyed message, we also recall/adopt the following simplifications:
  - single-voltage network (no transformers),
  - single-phase (or equivalently, perfectly balanced multiphase) system,
  - amplitude of generator voltages ideally controlled ,
  - *prevailing network*, i.e., all the generators are individually so small compared to the union of the others that each of them sees the network voltage as a fixed phasor.
- Of course the network frequency is controlled (now we know how).

Generator to network power transfer and machine angle (review)

- Let $\underline{V}_n = V$ (phase 0) be the network voltage phasor.
- Let the generator voltage amplitude be V, thus its phasor be

$$\boxed{\underline{V}_g = V\left(\cos\delta_{gn} + j\sin\delta_{gn}\right)}$$

Voltage phasor of generator

  where δ_{gn} is the generator *machine angle* (w.r.t. the network).
- Let finally $\underline{Y}_{gn} = G_{gn} - jB_{gn}$ (mind the minus!) be the admittance of the generator–network connection.
- We already computed the active and reactive power (P_{gn} and Q_{gn}) flowing from generator to network, that is,

when $B_{gn} \gg G_{gn}$!

$$\begin{array}{rcl} P_{gn} & = & V^2\left(\cancel{G_{gn}(1-\cos\delta_{gn})} + B_{gn}\sin\delta_{gn}\right) \\ Q_{gn} & = & V^2\left(B_{gn}(1-\cos\delta_{gn}) - G_{gn}\sin\delta_{gn}\right) \end{array}$$

Generator to network power transfer and machine angle

(review)

- Thus, varying δ_{gn} one can control P_{gn}, and Q_{gn} will follow as a consequence.
- To control *both* P_{gn} and Q_{gn} one may for example act on the excitation voltage. To see that, we can redo our computations with $|\underline{V}_g|$ set to V_{gm} instead of V.

→ control both & consider different magnitude

- We already did this as well, and got

$$
\begin{cases}
P_g = G_{gn}V_{gm}^2 - V_{gm}V\left(G_{gn}\cos\delta_{gn} - B_{gn}\sin\delta_{gn}\right) \\
Q_g = B_{gn}V_{gm}^2 - V_{gm}V\left(B_{gn}\cos\delta_{gn} + G_{gn}\sin\delta_{gn}\right)
\end{cases}
$$

- In this course we do not deal with reactive power control. → focus on ACTIVE POWER
- Suffice thus to say that to govern its power transfer to the network, a generator varies δ_{gn} with *transient* accelerations/decelerations...
- ...that however do not influence the network frequency, thus $\underline{V}_n$, owing to the prevailing network hypothesis.
- Actually there is a *local* influence at the swinging time scale, but this is not relevant for the problem we are treating now,

Node to node power transfer

and the network admittance matrix

- Obviously, if there is an electric connection between two nodes in a network, we can apply the same reasoning.
- The power transfer from node i to node j is computed exactly as we did for a generator to the network, coming to depend on $|\underline{V}_i|$, $|\underline{V}_j|$ and the angle δ_{ij} between the voltage phasors at the two nodes.
- Indicating the generic k-th node voltage magnitude $|\underline{V}_k|$ with V_k we thus have

$$\begin{cases} P_{ij} = g_{ij}V_i^2 - V_iV_j\left(g_{ij}\cos\delta_{ij} - b_{ij}\sin\delta_{ij}\right) \\ Q_{ij} = b_{ij}V_i^2 - V_iV_j\left(b_{ij}\cos\delta_{ij} + g_{ij}\sin\delta_{ij}\right) \end{cases}$$

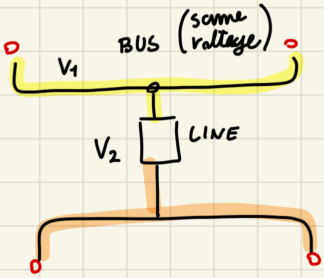
where $g_{ij} - jb_{ij}$ is the complex admittance of the line connecting nodes i and j (the reason for the lowercase g and b will be clarified soon).

Node to node power transfer

and the network admittance matrix

- Consider a network with n_B nodes or, to adopt the specific jargon of the addressed problems, n_B busses.
- Let $\underline{V}_i$ be the voltage at bus i and $\underline{I}_i$ the current injected in it by the locally connected generator(s) or drawn from it by the (local) bus load(s).
- In general each bus is connected to others via lines.
- Let $\underline{y}_{ij} = g_{ij} - jb_{ij}$ be the complex admittance of the line connecting busses i and j, of course with $i \neq j$.

gen and utilizer : inject + take current on that node and each nod (bus) considering a network of different elements



ideally same voltages,
assuming V_A = to any other V on that line

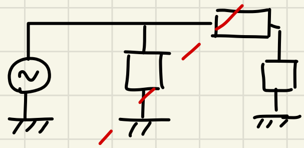
Node to node power transfer

and the network admittance matrix

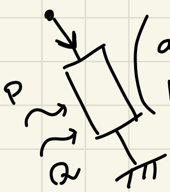
- Also, a bus may exhibit an admittance to ground.
- If this is true for bus i, we shall denote that admittance by $\underline{y}_{ii} = g_{ii} - jb_{ii}$.
- Some busses have at least one generator attached to them, and will be termed Generator (G) busses.
- The other busses carry only loads that absorb a certain amount of active power P and reactive power Q; these are called *Load* (L) busses or, more frequently, PQ busses.
- We need to describe the network by a matrix that will be useful for the problem we are discussing, and is called the admittance matrix.

↳ to be defined..

typically Gen + impedences on
a network



mod for
load flow, different approach!



(as a variable
resistor / inductance) → or as an inverted,
unconventional power

The basics

Admittance matrix

like injecting current on each Bus...

and indicate $\underline{Y}_{ij}$ as the ratio

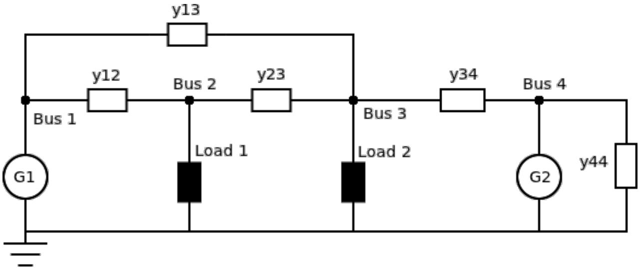
- The admittance matrix $\underline{\mathbf{Y}}$ is created by
  - starting from the usual representation of a network containing *voltage* generators and *impedances* (there is a more synthetic formalism used in power network engineering, called the *one-* or *single-line diagram*, but we do not have the time to treat it),
  - injecting a *current* $\underline{I}_i$ in each bus i and computing the so induced voltages $\underline{V}_j$, in all the nodes,
  - indicating by $\underline{Y}_{ij}$ (uppercase, notice) the $\underline{I}_i/\underline{V}_j$ ratio,
  - and finally assembling $\underline{\mathbf{Y}}$ as

$$\underline{\mathbf{Y}} = [\underline{Y}_{ij}] = \begin{cases} \underline{y}_{ii} + \sum_{j=1, j\neq i}^{n_B} \underline{y}_{ij} & \text{for the diagonal elements,} \\ -\underline{y}_{ij} & \text{for the other elements,} \end{cases}$$

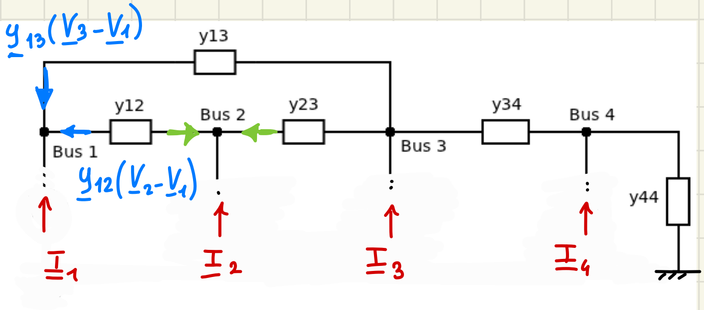
  - It should now be clear why we have used lowercase letters. Let us see an example to confirm our comprehension.

The admittance matrix
(an example)

- Consider the 4-busses network shown below:



- Remove all generators and loads, denote by $\underline{V}_i$ the voltage at the i-th bus, inject in each bus a current $\underline{I}_i$, write the nodal equations (KCL), and solve for the injected nodal currents.



first remove load and gen and inject current

↳ Writing equations:

- $I_1 = y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$
- $I_2 = y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)$

...

↳

The admittance matrix

(an example)

- Network with bus voltages and injected currents:

- Nodal equations (KCL) for busses 1–4:

$$\left\{\begin{aligned}
\underline{I}_1 &= \underline{y}_{12}\left(\underline{V}_1-\underline{V}_2\right)+\underline{y}_{13}\left(\underline{V}_1-\underline{V}_3\right)\\
\underline{I}_2 &= \underline{y}_{12}\left(\underline{V}_2-\underline{V}_1\right)+\underline{y}_{23}\left(\underline{V}_2-\underline{V}_3\right)\\
\underline{I}_3 &= \underline{y}_{23}\left(\underline{V}_3-\underline{V}_2\right)+\underline{y}_{13}\left(\underline{V}_3-\underline{V}_1\right)+\underline{y}_{34}\left(\underline{V}_3-\underline{V}_4\right)\\
\underline{I}_4 &= \underline{y}_{34}\left(\underline{V}_4-\underline{V}_3\right)+\underline{y}_{44}\underline{V}_4
\end{aligned}\right.$$

- Note that obviously $\underline{y}_{ij}=\underline{y}_{ji}$, thus $\underline{\mathbf{Y}}$ is symmetric.

organize it into
a matrix → admittance matrix from KCL

The admittance matrix

matrix that multiplied by node voltage provides the current

IF voltage known I can obtain current and so the $\underline{Y}$ MATRIX!

(an example)

- Now start from the KCLs

$$
\begin{aligned}
\underline{I}_1 &= \underline{y}_{12}(\underline{V}_1-\underline{V}_2)+\underline{y}_{13}(\underline{V}_1-\underline{V}_3)\\
\underline{I}_2 &= \underline{y}_{12}(\underline{V}_2-\underline{V}_1)+\underline{y}_{23}(\underline{V}_2-\underline{V}_3)\\
\underline{I}_3 &= \underline{y}_{23}(\underline{V}_3-\underline{V}_2)+\underline{y}_{13}(\underline{V}_3-\underline{V}_1)+\underline{y}_{34}(\underline{V}_3-\underline{V}_4)\\
\underline{I}_4 &= \underline{y}_{34}(\underline{V}_4-\underline{V}_3)+\underline{y}_{44}\underline{V}_4
\end{aligned}
$$

- and express $\underline{\mathbf{Y}}$, obtaining

(symmetric)

$$
\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\\ \underline{I}_3\\ \underline{I}_4\end{bmatrix}
=
\begin{bmatrix}
\underline{y}_{12}+\underline{y}_{13} & -\underline{y}_{12} & -\underline{y}_{13} & 0\\
-\underline{y}_{12} & \underline{y}_{12}+\underline{y}_{23} & -\underline{y}_{23} & 0\\
-\underline{y}_{13} & -\underline{y}_{23} & \underline{y}_{13}+\underline{y}_{23}+\underline{y}_{34} & -\underline{y}_{34}\\
0 & 0 & -\underline{y}_{34} & \underline{y}_{34}+\underline{y}_{44}
\end{bmatrix}
\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\\ \underline{V}_3\\ \underline{V}_4\end{bmatrix}
$$

- Verify the rule and convince yourselves:

$$
\underline{\mathbf{Y}} = [\underline{Y}_{ij}] = \begin{cases} \underline{y}_{ii} + \sum_{j=1, j\neq i}^{n_B} \underline{y}_{ij} & \text{for the diagonal elements,}\\ -\underline{y}_{ij} & \text{for the other elements,}\end{cases}
$$

The admittance matrix

- The admittance matrix can also be used to compute the power injected in all busses if voltages are known, as apparently

$$\underline{V} = \underline{\mathbf{Y}}^{-1}\underline{I}$$

  where $\underline{V}$ and $\underline{I}$ are respectively the vectors of bus voltages and *injected* currents; matrix $\underline{\mathbf{Y}}^{-1}$ is also called the network (or nodal, or bus) *impedance* matrix, and denoted by $\underline{\mathbf{Z}}$.

- Therefore, knowing the bus voltage phasors, one can obtain the complex power injected at each bus as

$$\underline{S} = \underline{V} \circ \underline{I}^* = \underline{V} \circ (\underline{\mathbf{Y}}\,\underline{V})^*$$

  where $\circ$ denotes the Schur product, and * the complex conjugate.

- We shall soon go through another example, to help us formulate the Load Flow (LF) problem.

Network-related problems

Load Flow and Optimal Power Flow

(typically small power loss on the net, but it can happen that a trasmission line is overloaded!)

until now..

- So far, we have been dealing with optimising the generation of the required power.
- However, generating here or there entails using the transmission lines differently.
- We need to ensure that no line gets overloaded, and if possible to account – when computing the generation cost – also for the power lost in the transmission process itself.

- In other words, we need to check how power "flows" in the network.
- This problem has been historically given two names:
  - the Load Flow (LF) problem, on which we are saying some words,
  - and the Optimal Power Flow (OPF) problem, that in this course we shall just mention.

Load Flow

An introductory example

- Consider a network with two busses 1 and 2.
- Take $\underline{V}_2$ as phase reference (i.e., assume its phase is zero).
- Let busses 1 and 2 be connected by a line of complex admittance $\underline{y}_{12} = g_{12} - jb_{12}$.
- Denote by δ_{12} the difference between the phases of $\underline{V}_1$ and $\underline{V}_2$ – i.e., the phase of $\underline{V}_1$ is δ_{12} – and go for Maxima:

```
y12 : g12-%i*b12;
Y   : matrix([y12,-y12],[-y12,y12]);
V   : matrix([V1*(cos(d12)+%i*sin(d12))],[V2]);
I   : Y.V;
S   : V*conjugate(I);
P   : trigsimp(realpart(S));
Q   : trigsimp(imagpart(S));
```

running it on Maxima...

We evaluate admittance matrix..

↓

and we can obtain the current from $\underline{Y}$ and vector of voltage

↳ from here we can compute the S, P, Q values of system (active / reactive powers at each node)

Load Flow

An introductory example

- We obtain

$$
\begin{aligned}
P_1 &= g_{12}V_1^2 - V_1V_2\left(g_{12}\cos\delta_{12} - b_{12}\sin\delta_{12}\right) \\
Q_1 &= b_{12}V_1^2 - V_1V_2\left(b_{12}\cos\delta_{12} + g_{12}\sin\delta_{21}\right) \\
P_2 &= g_{12}V_2^2 - V_1V_2\left(g_{12}\cos\delta_{12} + b_{12}\sin\delta_{12}\right) \\
Q_2 &= b_{12}V_2^2 - V_1V_2\left(b_{12}\cos\delta_{12} - g_{12}\sin\delta_{12}\right)
\end{aligned}
$$

- These four equations provide $P_{1,2}$ and $Q_{1,2}$, i.e., the (signed) active and reactive power injected at each of the two busses, based on the knowledge of their voltage phasors (and of course of the network parameters).

  ↑ impedence value

- Now, let us view the problem in another way.

Load Flow

The problem *per exemplum*

- Suppose that bus 2 is a Load (PQ) bus, and bus 1 a Generator (G) one.
- Suppose to know the active power demand from the load (remember the forecasts for tertiary control?) and that loads are managed so that the reactive demand is maintained within a prescribed power factor.
- Suppose, in one word, to know P_2 and Q_2.
- Suppose then that the *active* power generation at bus 1 is controlled (remember how P_m was managed to match P_e via power/frequency control?) and the same is true for the voltage *magnitude* V_2—not for the phase, as this is the means to release power.
- Again, take the phase of $\underline{V}_2$ as reference, and – remember – assume ideal (or almost ideal) frequency control.

Load Flow

The problem *per exemplum*

I know P, Q and voltage → unknowns remain (Q_2, δ_{21}, V_2) and I compute the following balance equations:

- Given all the above, we are dealing again with the equations

$$
\left\{
\begin{array}{rcl}
P_1 & = & g_{12}V_1^2 - V_1V_2\left(g_{12}\cos\delta_{12} - b_{12}\sin\delta_{12}\right) \\
Q_1 & = & b_{12}V_1^2 - V_1V_2\left(b_{12}\cos\delta_{12} + g_{12}\sin\delta_{21}\right) \\
P_2 & = & g_{12}V_2^2 - V_1V_2\left(g_{12}\cos\delta_{12} + b_{12}\sin\delta_{12}\right) \\
Q_2 & = & b_{12}V_2^2 - V_1V_2\left(b_{12}\cos\delta_{12} - g_{12}\sin\delta_{12}\right)
\end{array}
\right.
$$

  but in the three unknowns Q_2, δ_{21}, V_2.
- Note that also in the previous view the free quantities were in fact three, as V_1, V_2 and δ_{12} decided all of the rest.
- To determine all the nodal voltage phasors, thus, we could write
  - the balance equation for active and reactive power at bus 1 (PQ),
  - and the balance equation for active power at bus 2 (G).
- This is a nutshell-size example of LF problem. Let us now abstract and generalise.

LOAD FLOW

...

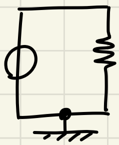
Load Flow

Problem statement

- Given a network composed of
  - n_G generator (G) busses,
    where the (injected) active power and the voltage amplitude are known,
  - plus n_{PQ} load (PQ) busses,
    where the (drawn) active and reactive powers are known,
  - plus one bus, called the *slack* (S) bus, (Reference !) $\Rightarrow$
    where the voltage amplitude and phase are known,

  determine all the voltage phasors (magnitudes and phases). in all the nodes !

- Note: of course $n_G + n_{PQ} + 1$ equals the total number n_B of busses,
  but in the following we shall preferably count busses by type.



from a circuit, you need a ground to compute it! otherwise only ΔV can be computed

{ Reference necessary ! }

Load Flow

Unknowns, equations, and solution

- Unknowns: magnitude + phases
  - $2(n_G + n_{PQ} + 1)$, i.e., all voltage phasors' magnitudes and phases
  - minus n_G because at G busses the voltage amplitude is known
  - minus 2 because at the S bus both voltage amplitude and phase are known,

  $\Rightarrow$ for a total of $n_G + 2n_{PQ}$.
- Equations:
  - n_G balances of active power at G busses,
  - plus n_{PQ}, balances of active power at PQ busses
  - plus n_{PQ}, balances of reactive power at PQ busses,

  $\Rightarrow$ for a total of $n_G + 2n_{PQ}$. → same numb of unkn!
- Solution:
  - the problem is not dynamic, apparently, ⇝ algebraic problem
  - but at the same time highly nonlinear; (cos, sin terms)
  - many numerical methods were proposed and are continuously studied,
  - ranging from standard ones (e.g., Newton-Raphson) or modifications of these, up to completely *ad hoc* ones (not the matter of this course).

Load Flow

The slack bus

2 viewpoints:

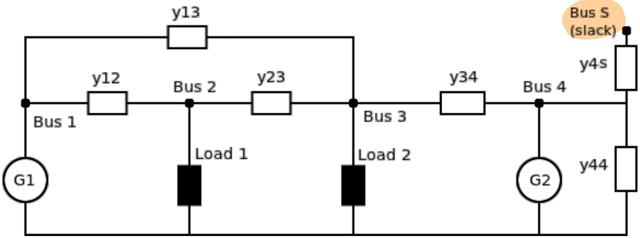
- The slack bus, can be just considered a reference, *mutatis mutandis* pretty much like the ground when solving an electric circuit,
- Alternatively, it can be viewed as the connection to a larger – e.g., cross-national – network, that is seen as a fixed phasor since such a network can be assumed to have prevailing power w.r.t. the considered – e.g., national – one, like the considered one is assumed to have w.r.t. any individual generator in it.

Load Flow

An example

We write LF-equations, not solving

- Let us write the LF equations for the network



Load Flow

An example – solution

- First (although not strictly necessary, one could directly reason with its elements) write the admittance matrix:

$$\underline{\mathbf{Y}} = \begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} & \underline{Y}_{13} & \underline{Y}_{14} & \underline{Y}_{1s} \\ \underline{Y}_{12} & \underline{Y}_{22} & \underline{Y}_{23} & \underline{Y}_{24} & \underline{Y}_{2s} \\ \underline{Y}_{13} & \underline{Y}_{23} & \underline{Y}_{33} & \underline{Y}_{34} & \underline{Y}_{3s} \\ \underline{Y}_{14} & \underline{Y}_{24} & \underline{Y}_{34} & \underline{Y}_{44} & \underline{Y}_{4s} \\ \underline{Y}_{1s} & \underline{Y}_{2s} & \underline{Y}_{3s} & \underline{Y}_{4s} & \underline{Y}_{ss} \end{bmatrix}$$

$$= \begin{bmatrix} \underline{y}_{12}+\underline{y}_{13} & -\underline{y}_{12} & -\underline{y}_{13} & 0 & 0 \\ -\underline{y}_{12} & \underline{y}_{12}+\underline{y}_{23} & -\underline{y}_{23} & 0 & 0 \\ -\underline{y}_{13} & -\underline{y}_{23} & \underline{y}_{13}+\underline{y}_{23}+\underline{y}_{34} & -\underline{y}_{34} & 0 \\ 0 & 0 & -\underline{y}_{34} & \underline{y}_{34}+\underline{y}_{44} & -\underline{y}_{4s} \\ 0 & 0 & 0 & -\underline{y}_{4s} & \underline{y}_{4s} \end{bmatrix}$$

Load Flow

An example – solution (Maxima, we are not doing this by hand)

solve with software...

```
y12   : g12-%i*b12;
y13   : g13-%i*b13;
y23   : g23-%i*b23;
y34   : g34-%i*b34;
y44   : g44-%i*b44;
y4s   : g4s-%i*b4s;
Y     : matrix([ y12+y13, -y12    , -y13        ,  0      ,  0 ],
               [-y2     ,  y12+y23, -723        ,  0      ,  0 ],
               [-y13    , -y23    ,  y13+y23+y34, -y34    ,  0 ],
               [ 0      ,  0      , -y34        ,  y34+y44, -y4s],
               [ 0      ,  0      ,  0          , -y4s    ,  y4s]);
V1    : V1m*(cos(d1)+%i*sin(d1)); /* unknown  d1, V1m is known (G bus) */
V2    : V2m*(cos(d2)+%i*sin(d2)); /* unknowns V2m and d2 (PQ bus)      */
V3    : V3m*(cos(d3)+%i*sin(d3)); /* unknowns V3m and d3 (PQ bus)      */
V4    : V4m*(cos(d4)+%i*sin(d4)); /* unknown  d4, V4m is known (G bus) */
Vs    : Vsm;                      /* phase 0, slack is reference       */
V     : transpose(matrix([V1,V2,V3,V4,Vs]));
I     : Y.V;                 /* matrix product (dot) here              */
S     : V*conjugate(I);      /* element by element product (star) here */
P     : trigreduce(realpart(S));
Q     : trigreduce(imagpart(S));
LFeq1 : P1 = P[1,1];
LFeq2 : P4 = P[4,1];
LFeq3 : P2 = P[2,1];
LFeq4 : Q2 = Q[2,1];
LFeq5 : P3 = P[3,1];
LFeq6 : Q3 = Q[3,1];
```

admittance

admittance matrix

voltages

← compute current

compute powers on the net

Load Flow equations! →

Balance of P @ BUS1 (eq 1)
Balance of P @ BUS4 (eq 2)
Balance of P,Q @ BUS2,3 (eq 3,4,5,6)

...but given a network, you are expected to know *which* are the LF equations.

Solution by MAXIMA:

define parameters and $\underline{Y}$ admittance matrix,

than the voltages vector

Load Flow

Role in the overall network control

check no overloading!
+ forecast to use
obtain the BIAS to give

standard practice

- Once primary/secondary control is in place and tertiary optimisation is done, use LF to check that no line is overloaded by also computing currents. If said check fails, modify the optimised solution to a suboptimal one "near" to the optimal but fulfilling the overload avoidance constraints.
- Express the mentioned overload avoidance conditions and plug them into tertiary optimisation as additional constraint. Note that this *significantly* complicates the optimisation problem on the constraint side. → highly non lin constraints! →

  know what admissible and not!
- Use LF to compute a transmission cost in terms of power lost over the lines, and plug this into tertiary optimisation. Doing so further complicates the optimisation, this time also by modifying the cost function.

still frontier

A general *panorama*

on control in electric (AC) systems

- We now have a reasonably complete *panorama* of network control and optimisation for the purpose of this course.
- Our boundaries are generator internals "below" and management optimisation "above": automation and control, that we discussed, lies in the middle:
  - primary control – proportional to frequency error, act in seconds or tens of, local, stop frequency drift;
  - secondary control – integral, steer frequency back to nominal, act in minutes, computed at network level and distributed;
  - tertiary control: optimise generation distribution and transmission, account for economic facts (when it is to control frequency, physics rules), computed and actuated by trading periods (in the order of 1/2 hour).

+LF problem