

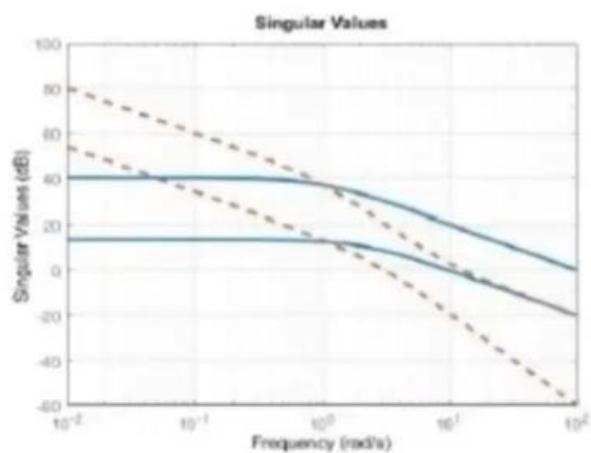
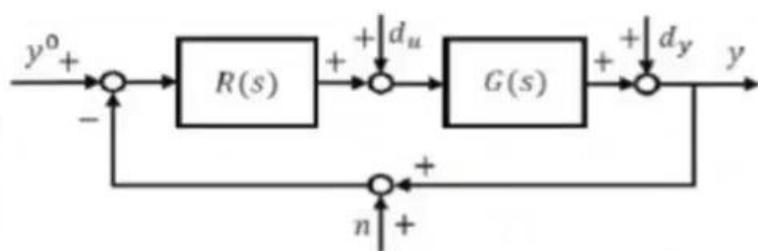
(3 punti)

Consider a second order dynamic system with state  $x = [x_1 \ x_2]'$ , described by  $\dot{x} = f(x)$  and  $f(0) = 0$ . For this system, take a Lyapunov function  $V(x)$  globally positive definite and radially unbounded. Assume also that  $\dot{V}(x) = (x_2^2 - 1)x_1^2$ . The origin is an equilibrium:

- unstable
- locally asymptotically stable
- globally asymptotically stable
- stable



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Consider the feedback system reported in the block diagram and two possible loop transfer functions  $L_1(s) = R_1(s)G_1(s)$  and  $L_2(s) = R_2(s)G_2(s)$  with the principal gains reported in the figure. Let  $L_1(s)$  be associated to the dashed lines and  $L_2(s)$  to the continuous ones. Select the true answer among the following ones:

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- Assuming that at low and high frequency all the singular values diagrams have the same slope shown in the figure, it is likely that the two loop transfer functions do not have poles at the origin.
- $L_1(s)$  and  $L_2(s)$  have roughly the same crossover frequency.
- At high frequency ( $\omega > 20$ ) the guaranteed attenuation of the noise disturbance  $n$  provided by  $L_2(s)$  is always greater than the one guaranteed by  $L_1(s)$ .
- At low frequency ( $\omega < 1$ ) the guaranteed attenuation of the disturbance  $d_y$  provided by  $L_1(s)$  is always greater than the one guaranteed by  $L_2(s)$ .

Assume to have a linear system with  $n$  states and  $n-1$  outputs. Then, it is possible to design with pole placement a regulator of order 1 such that all the eigenvalues of the closed-loop system are in prescribed positions.

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- No, the regulator must be of order  $n$
- yes provided that the system is stabilizable and detectable
- yes, provided that an estimator of unknown disturbances is also used
- Yes, provided that the system is reachable and observable

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Consider the system described by

$$G(s) = \begin{bmatrix} \frac{(s+1)(s+\alpha)}{(s+5)^3} \\ \frac{2}{(s+1)(s+5)} \end{bmatrix}$$

and select the *wrong* statement

- the zeros are  $s=-1, s=-\alpha$
- for a generic value of  $\alpha$ , it is not possible to jointly bring the two outputs to arbitrary constant values
- the poles are  $s=-1, s=-5$  (multiplicity 3)
- for  $\alpha=0$ , only the second output can be asymptotically regulated to an arbitrary constant value

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Consider the system

$$\dot{x}(t) = 2x(t) + u(t)$$

For this system compute the infinite horizon LQ control law with  $Q = 2.25$ ,  $R = 1$ .

The Riccati equation for LQ control is

$$-\dot{P}(t) = A'P(t) + P(t)A + Q - P(t)BR^{-1}B'P(t)$$

The corresponding closed-loop system is characterized by:

- pole in -2.5, gain margin (0.445,inf)
- pole in -2.5, gain margin (4.5,inf)
- pole in -4.5, gain margin (2,inf)
- pole in -2.5, gain margin undefined since the open loop system is unstable

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In model order reduction methods:

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- It is possible to guarantee a prescribed approximation error expressed in terms of the  $H_{\infty}$  norm of the difference between the real and the approximate transfer function
- with balanced residualization the approximation error is smaller than in balanced truncation (for a fixed order of the approximant)
- the procedure can be applied only to transfer functions of a regulator computed with  $H_2$  or  $H_{\infty}$  control
- both balanced truncation and balanced residualization guarantee that the steady state gain is preserved



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In Model Predictive Control stability of the origin can be obtained by including in the problem formulation

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- a zero terminal constraint and no weight on the final state
- a quadratic terminal constraint and a quadratic weight on the final state
- a zero terminal constraint and a control horizon smaller than the prediction horizon
- a quadratic terminal constraint and a linear weight on the final state

Domanda  
(5 punti)

Consider the system

$$\begin{aligned}x(k+1) &= x(k) + ax(k)u(k) + v_x(k) \\y(k) &= x(k) + v_y(k)\end{aligned}$$

where  $a$  is an unknown, but constant parameter, and  $v_x$ ,  $v_y$  satisfy the standard assumptions of Kalman filtering. Show how to implement an Extended Kalman Predictor to estimate  $x$  and  $a$ , including the computation of the matrices  $A(k)$ ,  $C(k)$  (Riccati equation not required).



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Consider the discrete time system



$$\begin{aligned}x(k+1) &= x(k) - 2u(k) + d(k) \\y(k) &= x(k)\end{aligned}$$

for this system

- (1) design with pole placement a stabilizing regulator guaranteeing closed-loop poles in  $z = 0.5$ ;
- (2) design with  $LQ_\infty$  a stabilizing controller. Consider a generic  $Q$  and  $R = 8Q$ .
- (3) Show how to estimate the constant, but unknown disturbance  $d$ .

steady state Riccati equation:

$$P = A'PA + Q - A'PB(R + B'PB)^{-1}B'PA$$