AMC_July_8_2020

Dear students, in this exam you will find two types of answers:

1. closed form (choose one over four pre-defined answers)

2. open questions (questions 8 and 9) with answers to be written in sheets of paper, each one reporting your Surname, your Name, your ID Polimi, your signature, and the indication "answer to question xx". Please use the sheets of paper that have been made available to you in advance (Beep portal). At the end of the exam, you will have to prepare a unique pdf file containing all these answers and upload it in another form which will be made available to you.

During the exam, you will not be allowed to use books, notes, electronic devices (save for what you need at the end of the exam to prepare the pdf file and upload it). You cannot exchange any kind of information with anyone.

If something unusual happens, an oral exam will be required.

Good work!

* Obbligatoria

* Questo modulo registrerà il tuo nome, inserire il nome.

*

(5 punti)

Consider the discrete time system

$$x_1(k+1) = 0.5x_1(k) + 1.5x_1^2(k)x_2(k)$$
$$x_2(k+1) = 0.5x_2(k)$$

By means of the quadratic Lyapunov function $V(x) = x_1^2 + x_2^2$, discuss the stability of the origin.

- [x] The origin is a locally asymptotically stable equilibrium
- [ ] The origin is a globally asymptotically stable equilibrium
- [ ] The origin is an unstable equilibrium
- [ ] Nothing can be concluded with that Lyapunof function

$$\Delta V = x_1(k+1)^2 + x_2(k+1)^2 - x_1(k)^2 - x_2(k)^2$$

$$= (0.5x_1 + 1.5x_1^2 x_2)^2 + 0.25x_2^2 - x_1^2 - x_2^2$$

$$= 0.25x_1^2 + x_1^3 x_2 + 2.25x_1^4 x_2^2 + 0.25x_2^2 - x_1^2 - x_2^2$$

$$= \underbrace{-0.75x_1^2 - 0.75x_2^2}_{\substack{<0 \\ \text{dominates}}} + x_1^3 x_2 + 2.25x_1^4 x_2^2 < 0$$

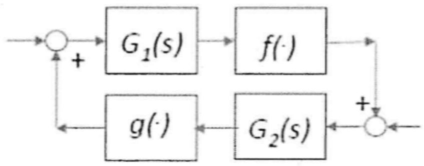
locally

2

*

(3 punti)

Consider the system



where

$$G_1(s) = \frac{a}{Ts+1}, a > 0, T > 0,\ G_2(s) = \frac{1}{s+1}$$

and *f*, *g* are sector nonlinearities uniquely defined for any input with

$$k_1 x \le f(x) \le k_2 x$$
$$h_1 x \le g(x) \le h_2 x \qquad k_1, k_2, h_1, h_2 \text{ positive values}$$

What condition guarantees Input/Output stability?

- k1 h1<1
- a k2 h2<1
- k2 h2<1
- T k2 h2 <1

3

*

(3 punti)

Consider an asymptotically stable system with static gain

$$G(0) = \begin{bmatrix} 1 & -0.5 \\ 2 & 1 \end{bmatrix}$$

and the constant inputs with 2-norm equal to 1

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

Correspondingly $\|y_1\|_2 = 2.2361$, $\|y_2\|_2 = 2.1503$, $\|y_3\|_2 = 1.1180$. Then, the singular values of $G(0)$ are

- ○ 1.2180 , 2.2361
- ○ 1.1180 , 2.1121
- ☒ 0.8508 , 2.3508
- ○ 0.8508 , 2.2120

4

Consider a standard feedback system with negative feedbak, transfer functions G(s) and R(s) of the system and the regulator, and loop transfer function L(s)=G(s)R(s). Is it possible to study the stability of the closed loop system by looking at the sensitivity function S(s)=inv(I+L(s)? *

(3 punti)

- ○ yes, always
- ○ yes, if and only if G(s) and R(s) have stable poles
- ☒ yes, if there are no cancellations of unstable poles of G(s) with (invariant) zeros of R(s) and viceversa
- ○ no never

5

The loop transfer recovery is a procedure to *
(3 punti)

- obtain a loop transfer function of a system controlled by LQG equal to the one with LQ at all the frequencies
- obtain a loop transfer function of a system controlled by LQG equal to the one with LQ in a large frequency band
- improve the robustness of LQ control
- Design a simple observer L=rho B with stability guaranteed

6

In order to guarantee that LQ control of a continuous time, linear system, provides closed-loop eigenvalues with real part smaller that -a, a>0, one must use the LQ algorithm with matrix A substituted by *
(3 punti)

- A(I+aI)
- I-aA
- aA
- aI+A

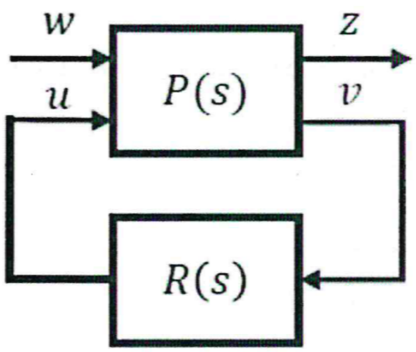
7

The gain margin guaranteed by a LQ regulator for discrete time systems is *
(3 punti)

- (0,inf)
- bounded from below and above
- (0.5,inf)
- does not exist

8

In H2 - Hinf control reference is often made to the system repoted in the figure. with reference to this figure describe the meaning of the variables, w, z, u, y and the goals of H2 - Hinf in terms of the corresponding norms *
(4 punti)



9

*
(6 punti)

Consider the system
$y(k) = \frac{1}{z-2}(u(k) + d)$
where d is a constant disturbance. For this system:

- design a pole placement regulator with integral action placing the closed-loop poles in 0.5,
- design a reduced order observer to estimate the disturbance d,
- design a compensator of the disturbance d