EXAM ADVANCED AND MULTIVARIABLE CONTROL 1 SEPT. 2020

EXERCISE 1

Consider a second order system with dynamic matrix A, and a matrix Q equal to the 2x2 identity, the solution of the Lyapunov equation is $P = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1.5 \end{bmatrix}$. Then, the system is

Matrix P is positive definite, therefore, by applying the Lyapunov theory, we can conclude that the system is asymptotically stable

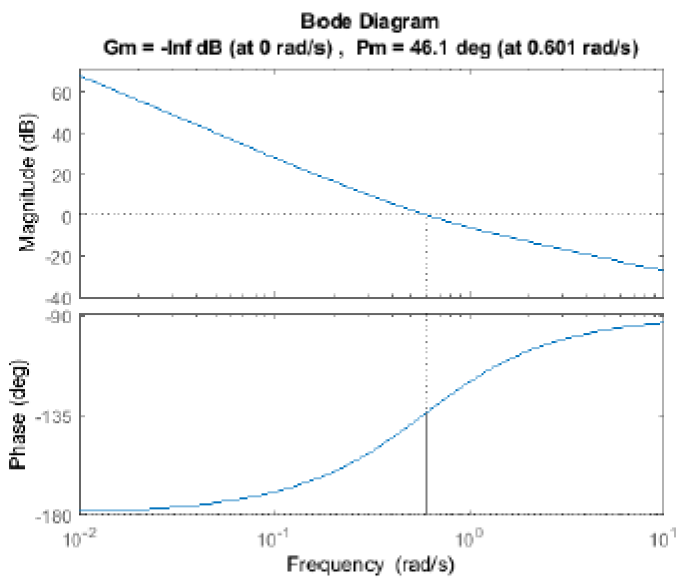
EXERCISE 2

Consider a nonlinear system and an equilibrium. If the eigenvalues of the corresponding linearized system have negative and null real part, the equilibrium is

Nothing can be concluded (see the notes)

EXERCISE 3

Consider a feedback system with control law $u(t) = -Kx(t)$ and loop transfer function $L(s) = K(sI - A)^{-1}B$ characterized by the Bode diagram reported in the following. Specify if the value of K can have been computed with LQ control.



The phase margin is smaller that 60 degrees, so it cannot be a regulator designed with LQ

EXERCISE 4

Given a system in balanced realization form with transfer function $G(s)$ and controllability gramian $P = diag\{2, 1, 0.8, 0.25, 0.15, 0.01, 0.005\}$ select the order of a reduced model $G_a(s)$ such that $\|G(s) - G_a(s)\|_\infty \leq 0.35$

Summing the last three singular values one obtains 0,165, while summing the four ones one obtains 0,415. Therefore it is possible to neglect 3 singular values and the solution is that the order of the reduced system is n=4

EXERCISE 5

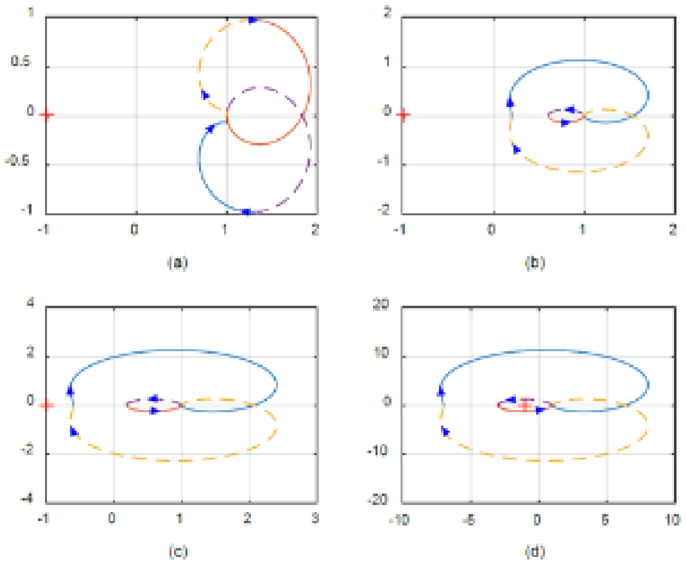
In Linear Quadratic control, is it possible to weight the outputs of the system?

YES, take Q=C'C

EXERCISE 6

Consider a feedback system with loop transfer function $L(s)$ without unstable poles.

Consider the following Nyquist plots of $det\ (I+L(s))$ and specify which cases (a), (b), (c), (d) correspond to asymptotically stable closed-loop systems.



a,b,d where the number of encirclements (positive if anticlockwise and negative if clockwise) around the origin is null

EXERCISE 7

Consider the following discrete-time system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.25 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathrm{u(k)}$$

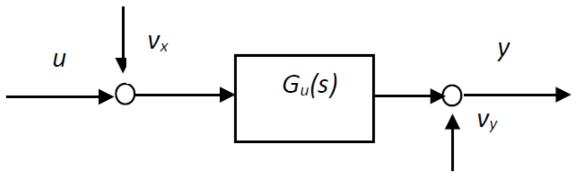
$$\mathrm{y(k)} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

which output variable can be forced to be asymptotically equal to a given arbitrary constant reference value?

Output 2. In fact, only one output can be controlled to a given setpoint, since there is only one input. But, if you consider output y1, the transfer function from the input to y1 has a zero in z=1 (derivative action), while the transfer function fom u to y2 has zero at z=0

EXERCISE 8

Given the system



where $G_u(s) = \dfrac{1}{s+a}$, $a > 0$

a. Assuming that v_x and v_y are null, compute the *LQ* control law with *R=1, Q=3a*^{2} and the corresponding closed-loop eigenvalue.
b. Assuming that *y* (not *x*) is measurable and $v_x = WN(0, 3a^2),\ v_y = WN(0,1)$ design the Kalman predictor and write its formula.
c. Compute the overall regulator transfer function with LQ + KP and the closed-loop poles.

Stationary Riccati equation of LQ control

$$0 = A'P + PA + Q - PBR^{-1}B'P$$

Solution question a

A=-a, B=1, solution of the stationary Riccati equation P=a, control gain K=a

A-BK=-2a (closed loop eigenvalue with state feedback)

Solution question b

For duality, the Kalman gain is L=a, and the observer gain is A-LC=-2a

The formula of the Kalman filter is reported in the notes.

Solution question C

As reported in the notes R(s)=K(sI-A+BK+LC)$^{-1}$L and the closed-loop eigenvalues are those of A-BK and A-LC

EXERCISE 9

Given a discrete time, linear, and asymptotically stable system, show how to use its impulse response coefficients to design a Model Predictive Controller with output feedback.

See the notes