ADVANCED AND MULTIVARIABLE CONTROL

10/2/2023 Solutions

Surname and name ....

Signature ....

Exercise 1 (3 marks)

Consider the continuous-time linear system **(select the correct answer)**:

$$\dot{x}_1(t) = x_2^3(t)$$
$$\dot{x}_2(t) = u(t)$$

and the control law

$$u(t) = -x_2(t) - x_1(t)x_2^2(t)$$

With the Lyapunov theory and Krasowski La Salle theory (if necessary) and a quadratic Lyapunov function it is possible to conclude that the origin is

- o an unstable equilibrium
- o an asymptotically stable equilibrium
- o a globally asymptotically stable equilibrium
- X a simply stable equilibrium
- o no answer

$V(x) = \frac{1}{2}(x_1^2 + x_2^2)$

$\dot{V}(x) = (x_1\dot{x}_1 + x_2\dot{x}_2) =$

$= \cancel{x_1 x_2^3} - x_2^2 - \cancel{x_1 x_2^3} \leq 0$

With KLS it is not possible to conclude anything because $\dot{x}_1 = 0 \rightarrow x_1 = \text{const.}$ is compatible

Exercise 2 (3 marks)

The Nyquist criterion for MIMO systems **(select the correct answer)**

- o does not exist
- o exists only for open-loop asymptotically stable systems
- X exists both for continuous time and discrete time systems
- X exists for continuous time systems
- o can be applied only when the conditions of applicability of the small gain theorem are satisfied
- o no answer

(also this answer accepted even though { not discussed in the course)

Exercise 3 (3 marks)

The Extended Kalman filter can be used (**select the correct answer**):

- [ ] only if the system is linear
- [ ] only if the linearized system is asymptotically stable
- [ ] when convergence of the estimated state to the real one must be guaranteed
- [x] when there are unknown constant parameters entering the system's equations
- [ ] no answer

Exercise 4 (3 marks)

Recursive feasibility of the MPC problem (**select the wrong answer**)

- [x] is not strictly required in an on-line implementation of the MPC control law
- [ ] is always guaranteed when there are only input constraints
- [ ] can be obtained for state and output constraints by means of slack variables
- [x] it is not required when terminal cost and terminal constraints are included into the problem formulation
- [ ] No answer

(Two answers were correct, only one was required)

Exercise 5 (3 marks)

Given a SISO, closed-loop asymptotically stable system with perturbation, the H_{inf} norm of the nominal complementary sensitivity function is useful to (**select the correct answer**)

- [ ] Study the stability of the closed-loop system with additive perturbations
- [x] Study the stability of the closed-loop system with multiplicative perturbations
- [ ] Study the stability with parametric perturbations
- [ ] None of the previous answers
- [ ] No answer

Exercise 6 (7 marks)

Consider the system

$$\dot{x}_1(t) = x_1(t) - x_2(t)e^{-x_2(t)}$$
$$\dot{x}_2(t) = -2x_2(t) - x_1(t)e^{-x_2(t)} + u(t)$$

a) Verify that, for *u(t)=0*, the origin is an equilibrium point.
b) Considering the linearized model at the origin, specify if it possible to conclude that the equilibrium of the nonlinear system is asymptotically stable.
c) Design a state feedback control law such that the linearized closed-loop system has eigenvalues *s=-1* (double).
d) Specify if it is possible to conclude that the corresponding nonlinear feedback system is globally asymptotically stable?

a) $\dot{x}_1 = 0, \dot{x}_2 = 0 \longrightarrow \left.\begin{array}{l} \bar{x}_1 - \bar{x}_2 e^{-\bar{x}_2} = 0 \\ -2\bar{x}_2 - \bar{x}_1 e^{-\bar{x}_2} = 0 \end{array}\right\}$ Satisfied for $\bar{x}_1 = \bar{x}_2 = 0$

b) Linearized model (at the origin)

$$\begin{cases} \delta\dot{x}_1 = \delta x_1 - \delta x_2 \\ \delta\dot{x}_2 = -\delta x_1 - 2\delta x_2 + \delta u \end{cases} \qquad A = \begin{vmatrix} 1 & -1 \\ -1 & -2 \end{vmatrix}, \quad B = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

$\det(sI - A) = s^2 + s - 3 \Leftarrow$ "unstable" eigenvalue

c) reachability $|B \ AB| = \begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} \rightarrow \det \neq 0 \rightarrow$ system reachable

$\delta u = -K\delta x = -|K_1 \ K_2| \delta x$

$\det(sI - (A - BK)) = s^2 + (-3 + K_1)s + (1 - K_1 - K_2) = 0$

$= s^2 + 2s + 1$

$K_1 = -5, \ K_2 = 1$

d) no, global stability cannot be derived by a local analysis like those based on linearized model

Exercise 7 (6 marks)

Consider a continuous time linear system of order *n*, with *m* inputs and *p* outputs, *m*≥*p*.

Discuss the main steps to design a regulator made by an observer of minimal order plus a state feedback control law and with an integral action on the output error variables.

See the notes. However:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad , x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad y \in \mathbb{R}^p$$

+ <u>integrators</u>

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{v} = y^{o} - y = y^{o} - Cx \\ y = Cx \end{cases} \Rightarrow \begin{vmatrix} \dot{x} \\ \dot{v} \end{vmatrix} = \underbrace{\begin{vmatrix} A & 0 \\ -C & 0 \end{vmatrix}}_{\bar{A}} \begin{vmatrix} x \\ v \end{vmatrix} + \underbrace{\begin{vmatrix} B \\ 0 \end{vmatrix}}_{\bar{B}} u$$

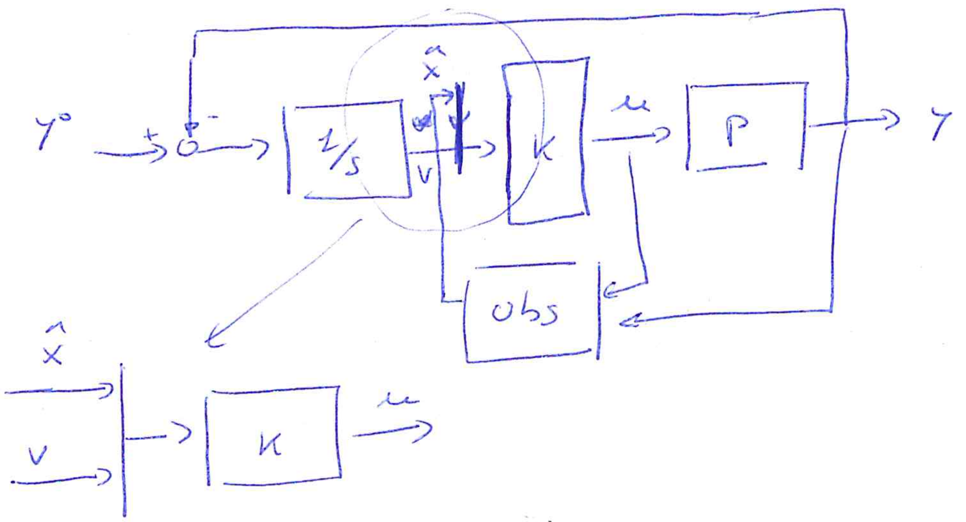
<u>State feedback</u>

$u = -K \begin{vmatrix} \hat{x} \\ v \end{vmatrix}$, $\hat{x}$ computed with observer

$K \to$ pole placement or LQ for $\bar{A}, \bar{B}$

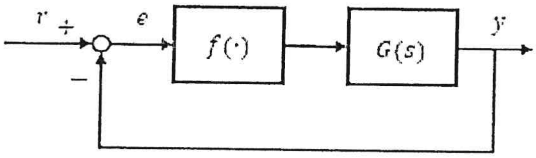
<u>Observer</u>

reduced order observer of ~~...~~ order $n-p$

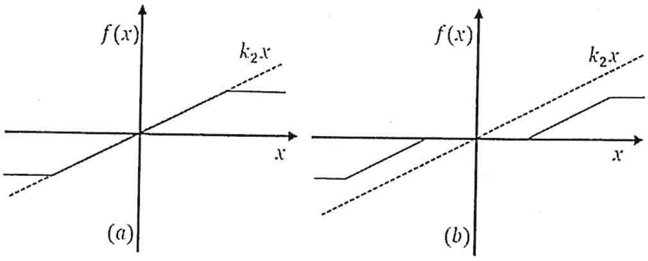


Exercise 8 (5 marks)

Consider the system

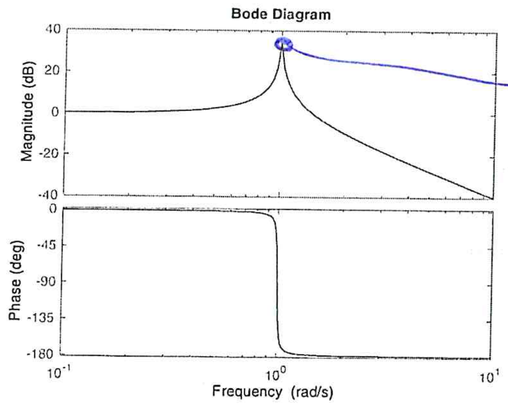


Where the function *f(.)* can take one of the following two forms,



$(k_2 > 0)$

while G(s) is the transfer function of a SISO, asymptotically stable system with Bode diagrams



$(|G|_\infty)_{dB}$

Specify the set of values of k_2 such that the I/O stability of the closed-loop system is guaranteed.

Specify if the above condition is necessary, sufficient, necessary and sufficient.

Small gain theorem $\longrightarrow$ $K_2 \cdot |G(j\omega)|_\infty < 1$

only a sufficient condition