MIMO SYST.

ANALYSIS,

ENLARGED SYST.

Advanced and Multivariable Control

Multivariable systems analysis → similar to SISO case...

(MIMO)

Riccardo Scattolini

Block schemes definitions:

same connections

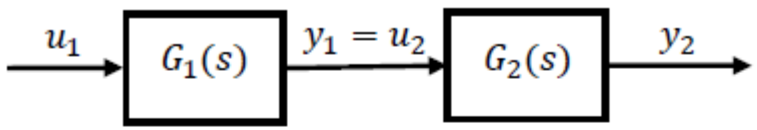


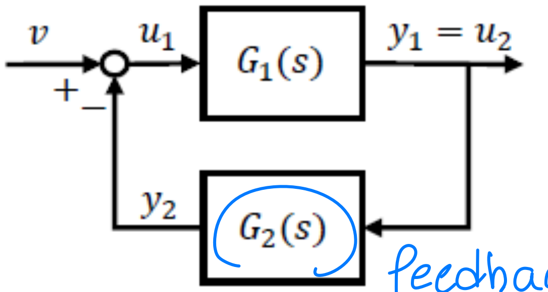
Figure 5.1: Systems in series configuration.

carefully on writing in the proper sequence the matrix product

$$Y_2(s) = G_2(s)U_2(s) = G_2(s)G_1(s)U_1(s)$$

(respect sign, here the ORDER matters)
(dimensional issue)

direct path



feedback T.F

Figure 5.2: Systems in feedback configuration.

from standard feedback procedure:

$$Y_1(s) = (I_p + G_1(s)G_2(s))^{-1}G_1(s)V(s)$$

but also … ↕ same result! T.F

$$Y_1(s) = G_1(s)\,(I_m + G_2(s)G_1(s))^{-1}V(s)$$

it is possible to compute it from u_1, u_2, same Result!

Check the previous results!

Frequency response

(Linear dynamic system)

Assumption: asymp. stable

SISO

$$u(t) = U sin(\bar{\omega}t) \longrightarrow \boxed{G(s)} \longrightarrow y(t) = Y sin(\bar{\omega}t + \alpha)$$

sin(ωt) input after transient gives a sin(ωt) output

$$Y = U\,|G(j\bar{\omega})| \text{ and } \alpha = \arg(G(j\bar{\omega})) \qquad |G(j\bar{\omega})| = \frac{|Y(j\bar{\omega})|}{|U(j\bar{\omega})|}$$

«gain» @ the given frequency

you can (removing stability issue...)

MIMO

$$\boxed{\frac{\|Y(j\omega)\|_2}{\|U(j\omega)\|_2} = \frac{\|G(j\omega)U(j\omega)\|_2}{\|U(j\omega)\|_2}}$$

gain depends on INPUT direction

equivalent result related to gain of $\|G\|$

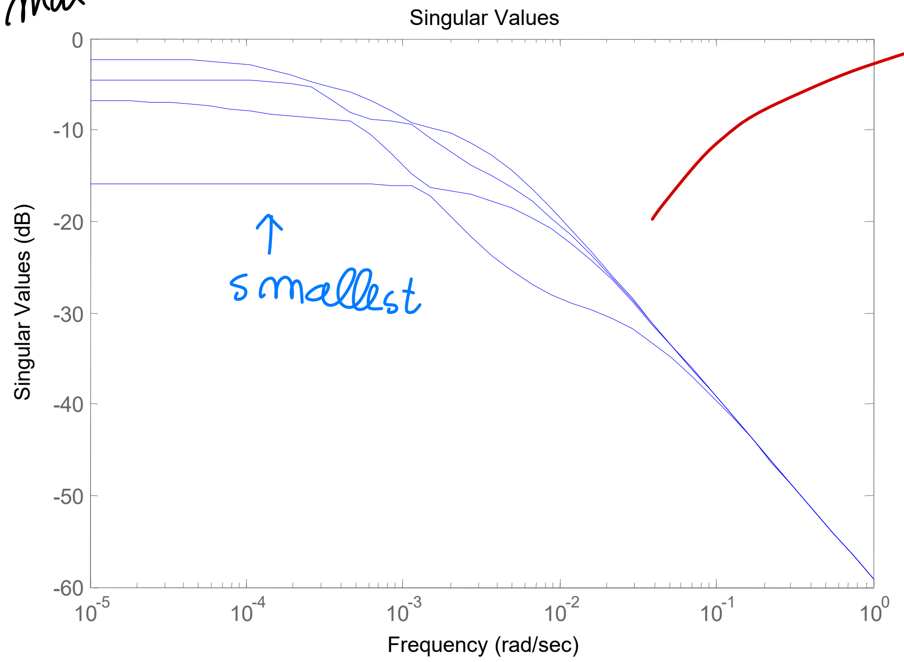
Gain and input directions in MIMO systems

contained between (smallest, largest) sing value @ that freq

$$\underline{\sigma}(G(j\omega)) \leq \overbrace{\frac{\|G(j\omega)U(j\omega)\|_2}{\|U(j\omega)\|_2}} \leq \bar{\sigma}(G(j\omega))$$

The singular values $\underline{\sigma}(G(j\omega))$ and $\bar{\sigma}(G(j\omega))$ of the system are called $\boxed{\text{“principal gains”}}$

$\underline{\sigma}$ min
$\bar{\sigma}$ max
sing value



4 sing values σ (equal to the min between IN/OUT number)

$\Downarrow$

$\gamma(G(j\omega))$? sing values almost same @ high freq while in low freq → very different behavior,

@ high freq basically one line → same effect on system well defined but used index :

The condition number is defined as

$$\gamma(G(j\omega)) = \frac{\bar{\sigma}(G(j\omega))}{\underline{\sigma}(G(j\omega))} \geq 1$$

A system with a condition number close to 1 is "easy" to control, since it is not sensitive to the direction of the input

Example

Consider a static system with two inputs, two outputs, and described by the matrix

$$G = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

differently directed on (u_1, u_2) space!

and the following inputs with norm equal to one $(\|U_i\|_2 = 1)$

$$U_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, U_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, U_3 = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}, U_4 = \begin{bmatrix} 0.707 \\ -0.707 \end{bmatrix}, U_5 = \begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix}$$

Good for Analysis

Bad for synthesis where you have to care about how input acts to different inputs!

The corresponding outputs are

$$Y_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, Y_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, Y_3 = \begin{bmatrix} 2.12 \\ 4.95 \end{bmatrix}, Y_4 = \begin{bmatrix} -0.707 \\ -0.707 \end{bmatrix}, Y_5 = \begin{bmatrix} -0.4 \\ 0 \end{bmatrix}$$

with norms → very different norm values!

$$\|Y_1\|_2 = 3.16, \|Y_2\|_2 = 4.47, \boxed{\|Y_3\|_2 = 5.38}, \|Y_4\|_2 = 1.00, \boxed{\|Y_5\|_2 = 0.4}$$

same syst amplify/attenuate in different way depending on how U_i is directed on space

$$\bar{\sigma}(G) = 5.47; \quad \underline{\sigma}(G) = 0.37$$

all $\|Y_i\|_2$ included inside $(\underline{\sigma}, \bar{\sigma})$

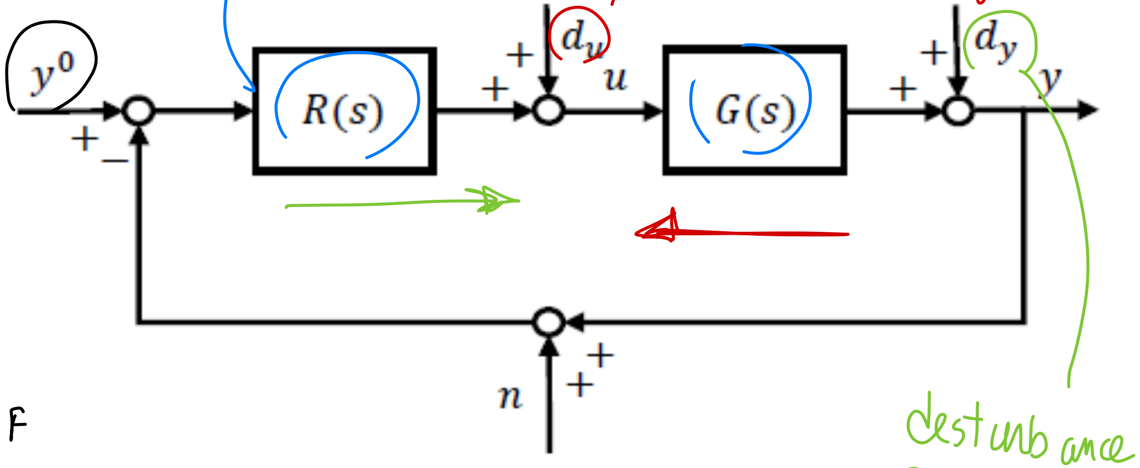
T.F MATRIX

actuator uncertainty
=
disturbance acting on u

MIMO analysis of

Feedback systems

set points



disturbance on output

Loop transfer functions

going back in the loop direction

$$L(s) = G(s)R(s), \quad L_u(s) = R(s)G(s)$$

[p×m] [m×m]

↑ you should consider two loop T.F

Sensitivity functions

with MATRIX form!

$$S(s) = (I + L(s))^{-1}$$ (same as SISO case) *sensitivity*

$$T(s) = (I + L(s))^{-1} L(s) = L(s)(I + L(s))^{-1}$$ *complementary sensitivity*

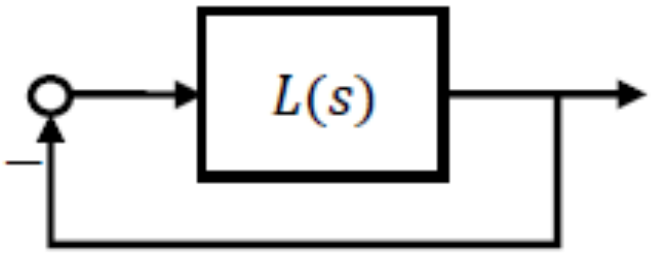
Closed-loop functions

disturbances to reject

$$Y(s) = T(s)(Y^o(s) - N(s)) + S(s)D_y(s) + S(s)G(s)D_u(s)$$

designing properly the loop

No conceptual differences with respect to the SISO case



Stability of feedback systems

Linear

A system composed by several blocks is asymptotically stable if: (a) *there are no hidden cancellations of unstable modes, i.e. cancellations between poles and zeros with nonnegative real part inside its blocks, and* (b) *bounded inputs applied at any point of the system produce bounded outputs at any point of the system*)

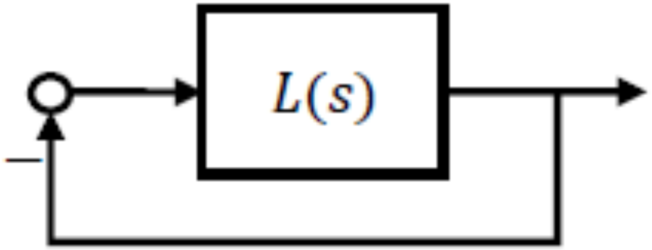
alternative def of syst

No hidden cancellation inside G(s), R(s) → NON Reach / NON obs part

neither G(s)·R(s) cancel

from this Def ...

Guarantee no hidden cancellations inside the blocks



(Bode theorem does NOT exist!) → more sing values to check

Nyquist theorem for MIMO systems

Let P_{ol} be the number of poles of $L(s)$ with positive real part. Then, the closed-loop system with loop transfer function $L(s)$ and negative feedback is asymptotically stable if and only if the Nyquist plot of $\boxed{\det(I + L(s))}$: (i) does not pass through the origin, (ii) the number of its encirclements (positive in the anticlockwise direction) around the origin is P_{ol}.

same of SISO Nyquist.. plotting det(..) with critical point 0+J0 (ORIGIN)

here you look at origin

- **Not easy to understand how to modify L(s) to get stability** →

not usefull for synthesis of regulator!

In practice, on SISO we shape properly L(s) placing poles/zeros... here without Bode criterion we use Automatic regulator criterion

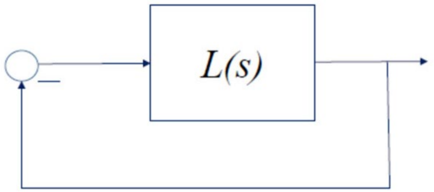
Bode criterion not available (remember the singular values!)

{ AUTOMATIC SYNTHESIS PROCEDURE } needed

{ Freq domain analysis NOT for synthesis! }

Exercise (exam)

Pole = ∅ → we need NO circle around origin



L(s) is a 2x2 matrix with no unstable poles

plotted by SOFTWARE

Consider the following Nyquist plots of $\boxed{\det(I+L(s))}$ and specify which case (A), (B) corresponds to asymptotically stable closed-loop systems. Motivate your answer.

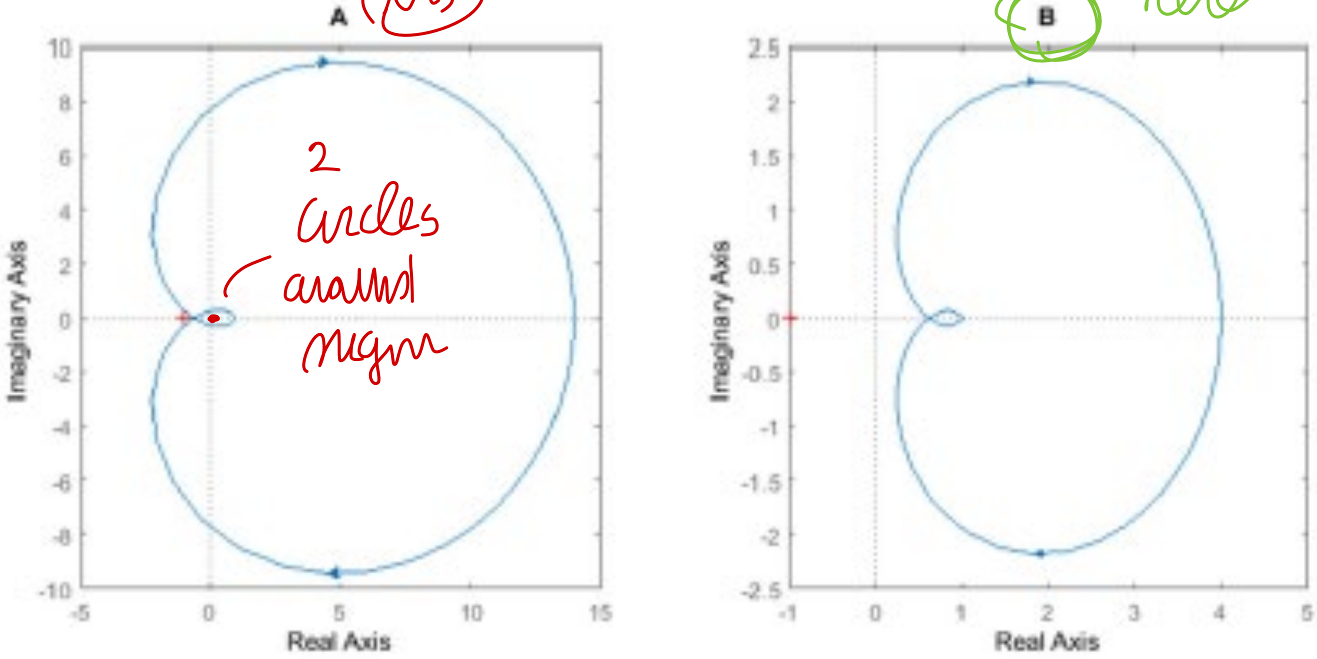
NO

here

usefull for analysis

2 circles around origin

to modify the Nyq plot I can change the gain of the T.F!

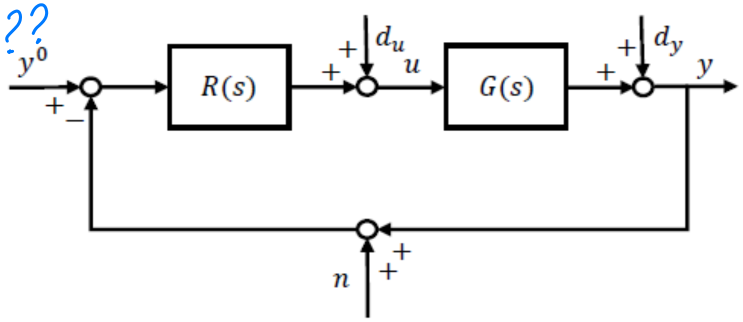


Case B, since in view of the Nyquist criterion for MIMO systems, and in view of the fact that *L(s)* does not have unstable poles, the Nyquist plot must not encircle the origin.

Same results for ROOT LOCUS → complex Algorithms!!

Nyquist very usefull looking @ L(s)... consider S(s), T(s) I can conclude something ??

Stability from the closed-loop transfer functions



$$Y(s) = T(s)(Y^o(s) - N(s)) + S(s)D_y(s) + S(s)G(s)D_u(s)$$

all should be asymp stable ...

The feedback system is internally stable if and only if the transfer functions

you need to look to all of that T.F stability property

$$\begin{cases} K_1(s) = (I + L(s))^{-1} L(s) \\ K_2(s) = (I + L(s))^{-1} \\ K_3(s) = (I + L(s))^{-1} G(s) \\ K_4(s) = (I + L_u(s))^{-1} R(s) \\ K_5(s) = (I + L_u(s))^{-1} \end{cases}$$

here I can have hidden cancellation unstable !

→ looking that one I can detect unstable issues

are asymptotically stable

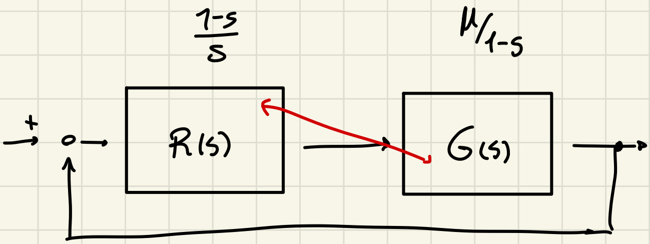
study stability in terms of S(s), T(s) looking to that T.F !

Why do we need to look at all these transfer functions? Because in some of them forbidden cancellations could be hidden

Zeros on MIMO
↳ invariant zeros !

hidden cancelletion between R, G remain unstable
is unst. sing

cancellation ⇒ you should look other T.F to see if there was a lost unstable poles in $L(s)$



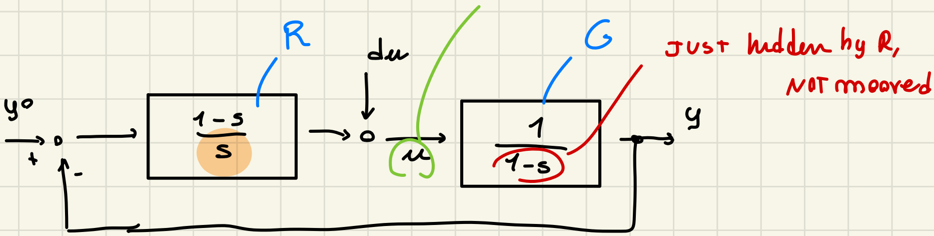
$$y = \frac{RG}{s} y^\circ =$$

$$= \frac{\mu/s}{1+\mu/s} y^\circ =$$

$$= \frac{\mu}{s+\mu} y^\circ$$

poles cancellation

μ is the input to the system with also the disturbance d_u



$$y = \frac{RG}{1+RG} y^\circ + \frac{G}{1+RG} d_u = \frac{1}{s+1} y^\circ + \frac{s}{(s+1)(s-1)} d_u$$

on MIMO case to be sure of NOT having hidden singularits you have to look all T.F

an integrator on R causes a derivator on d_u → so cancel out d_u on my insteady state

Cancellations and stability

(moove singularities in unreach/unobs part)

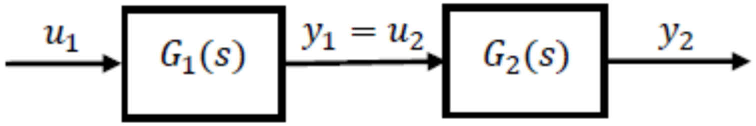


Figure 5.1: Systems in series configuration.

If $G_1(s)$ and $G_2(s)$ have a common pole/zero, it can happen that the cancellation does not occur

formally we should have cancellation

(independently)

A fake MIMO system: poles and zeros are the ones of the single transfer functions

zeros in +1 · pole in +1 · cancellation?

$$G_1(s) = \begin{bmatrix} \dfrac{4}{s+3} & 0 \\ 0 & \dfrac{-4(s-1)}{s} \end{bmatrix}, \quad G_2(s) = \begin{bmatrix} \dfrac{4}{s-1} & 0 \\ 0 & \dfrac{1}{s+8} \end{bmatrix} \longrightarrow G_2(s)G_1(s) = \begin{bmatrix} \dfrac{16}{(s+3)(s-1)} & 0 \\ 0 & \dfrac{-4(s-1)}{s(s+8)} \end{bmatrix}$$

Diag matrix ... ⇒ poles are single T.F poles, same for zeros! like 2 SISO systems!

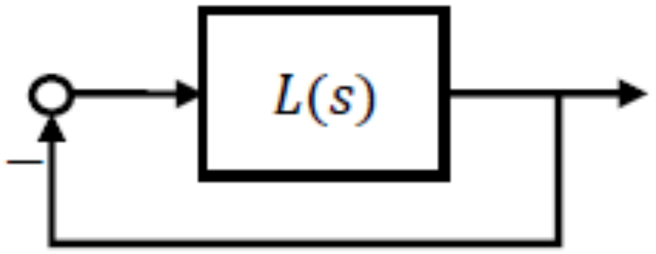
completely independent syst still diag

A cancellation between $G_1(s)$ and $G_2(s)$ exists only if the poles of $G_1(s)$ and/or of $G_2(s)$ are not included in the poles of $G_1(s)G_2(s)$ (or $G_2(s)G_1(s)$)

Looking set of poles/zeros on T.F is NOT enough to check

→ compute poles/zeros of G_1, G_2 then check G_1G_2, G_2G_1 to see if cancellation

Small gain theorem for MIMO systems



A closed-loop system made by asymptotically stable linear systems and with loop transfer function $L(s)$ is asymptotically stable if the loop gain is less than 1, that is if

$$\boxed{\|L\|_\infty < 1}$$

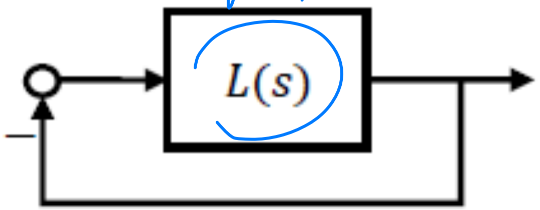
↳ as on siso case...

Also in this case:

- only a sufficient condition
- very conservative condition

Static properties – let's recall some results for SISO systems

(extend SISO results to MIMO!)



Assuming that the feedback system is asymptotically stable, given a constant set-point with Laplace transform $Y^0(s) = \frac{A}{s}$, the output asymptotically tends to

$$L(s) = \frac{\mu \prod (1+\tau_i s)}{s^g \prod (1+T_j s)}$$

g singularities @ origin
g>0: poles
g<0: zeros

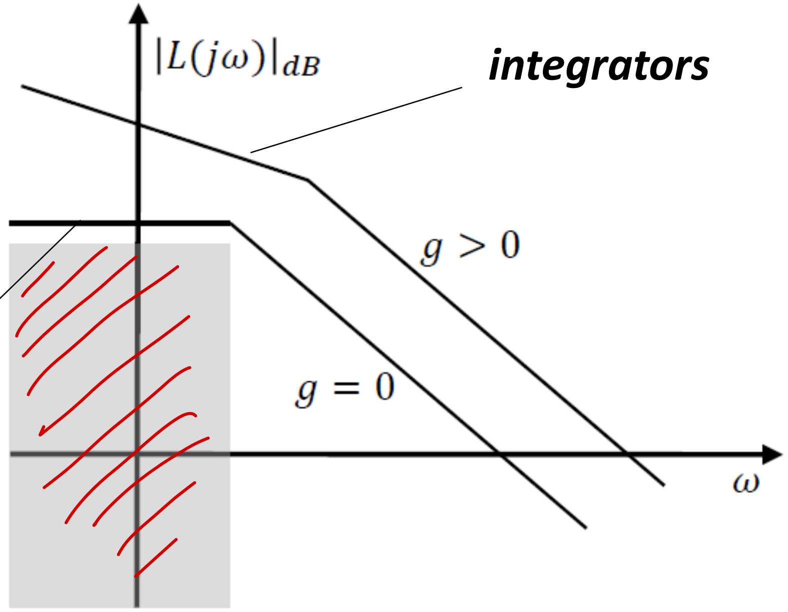
μ ↑↑ Request! no integrator / derivative

$$y_\infty = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s \frac{L(s)}{1+L(s)} \frac{A}{s} = \begin{cases} \frac{\mu}{1+\mu} A & g=0 \\ A & g>0 \end{cases}$$

FINAL VALUE theorem

integrator into loop

↳ set point amplitude static gain asymp!



No integrators-> high value of μ

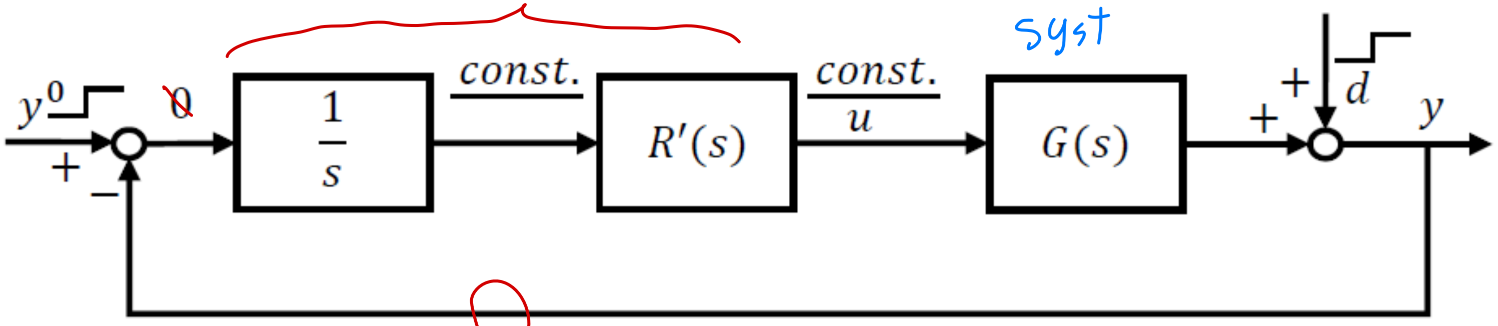
we ask @ least an high low freq gain μ

IF integrator on G(s) does not remove const disturb...

syst

Interpretation – how the integrator works

R(s)



here it can act a disturbance dm

BIAS on meas signals!

If the closed-loop system is asymptotically stable, for constant exogenous signals, the input of the integrator must be asymptotically zero

1/s guarantee STATIC performance

(respecting design requirement!)

Classical design approach: first put an integrator to guarantee static performance, then choose *R'(s)* to stabilize *(1/s)G(s)*. The overall regulator is *(1/s)R'(s)*

for asymp stable syst.. integrator will produce 0 as its input

When it works? *G(s)* must not have derivative actions

The same approach can be used for MIMO systems

y^0, d constant → input of 1/s asymp → 0

this is why we includ (I) action

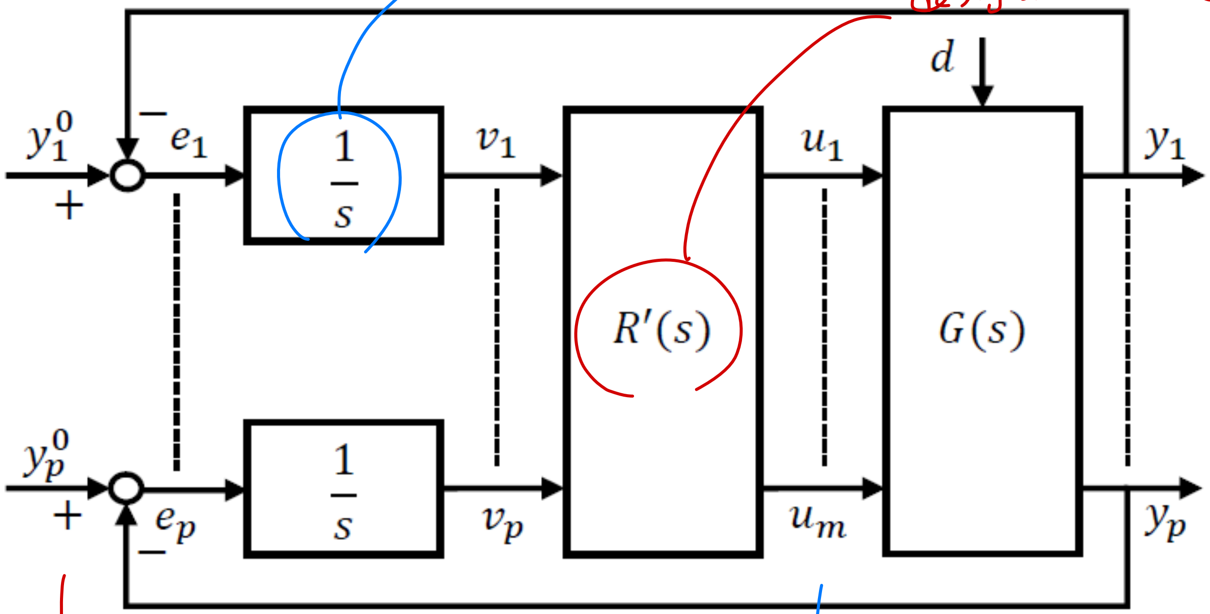
0 steady state

asymp. $e_i \to 0$

Static performance for MIMO systems – basic idea

firstly we put integral action ∀ error element

design overall R' to stabilize the system!



carefully when adding integrator to NOT destroy stability!

(track p set points)

m inputs

mathematically avoid cancellation integrator/derivative action

One integrator for each error

R'(s) stabilizes *G(s)* + integrators T.F

Required conditions for MIMO system

from s.s descriptions

constant disturbance $d(t) = d \ \forall t$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Md \\ y(t) = Cx(t) + Nd \end{cases}$$

$x \in R^n$, $u \in R^m$, $y \in R^p$, and $d \in R^r$

If we want that, for constant *d*, the output reaches a constant reference value *y°*, it must hold that at the steady state

$(\dot{x} = 0) \rightarrow (x = \bar{x}, y = y^o)$

REWRITING the syst in MATRIX FORM!

$$\begin{cases} 0 = A\bar{x} + B\bar{u} + Md \\ y^0 = C\bar{x} + Nd \end{cases} \longrightarrow \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \begin{bmatrix} 0 & -M \\ I & -N \end{bmatrix} \begin{bmatrix} y^0 \\ d \end{bmatrix}$$

↳ I can compute $\bar{x}, \bar{u}$ Given y^o, d? ⇒ can be solved under proper conditions!

System matrix *P(0)*

TO SOLVE this set of equations ⇒

$p \leq m$: to my p out to have constant output y_0 you need MORE inputs!

$$\Sigma = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \in R^{n+p,n+m} \longrightarrow$$

Conditions:

1. $p \leq m$ ← less-eq than unknowns!
2. $rank(\Sigma) = n + p$

to properly be solvable!

Conditions:

To allow R'(s) design... → ?

1. $p \leq m$ → **At least as many inputs as outputs**

2. $rank(\Sigma) = n + p$ → **No invariant zeros s=0, i.e. no derivative actions**

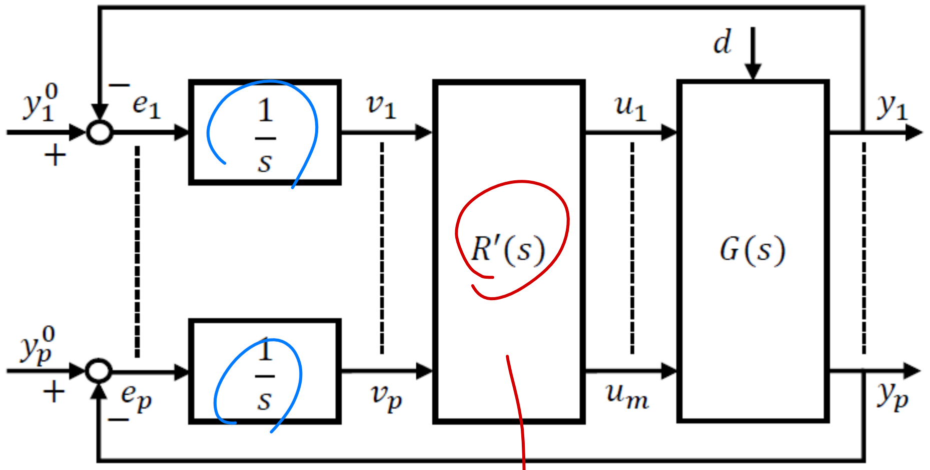
if derivative actions, bring the OUTPUT to 0 asymptotically

R'(s) must stabilize the plant + the integrators

$\in \mathbb{R}^p$

Integrators

$$\begin{aligned}\dot v(t) &= e(t)\\ &= y^0 - y(t)\\ &= y^0 - Cx(t) - Nd\end{aligned}$$



STATIC conditions

design R'(s) for our request

Plant + integrators

(enlarged state)

dynamic of enlarged syst + integrator

$$\begin{aligned}
\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 & M \\ I & -N \end{bmatrix} \begin{bmatrix} y^0 \\ d \end{bmatrix} \\
v(t) &= \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}
\end{aligned}$$

syst.

integr.

This is the system that must be considered in the design of the stabilizing regulator (with pole-placement, LQ,…)

Letting $\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & I \end{bmatrix}$

↳ enlarged system may be

$\boxed{(\bar{A}, \bar{B}) \text{ must be reachable}}$

$\boxed{(\bar{A}, \bar{C}) \text{ must be observable}}$

use reachability / observability TEST

if this condition holds

OR we can ask it to be STABILIZABLE / ...

conditions on ORIGINAL system!

$$\boxed{\text{observability of } (\bar{A}, \bar{C})}$$

(if initially nomods, obviously enlarging I can lose obs)

The pair is observable ***iff*** the original pair is observable (see the textbook)

$$\boxed{\text{reachability of } (\bar{A}, \bar{B})}$$

The pair is reachable ***iff*** the original pair is reachable ***and*** the system under control does not have invariant zeros in ***s=0,*** i.e. no derivative actions (see the textbook)

plant + integrator

↳ adding 1/s you must guarantee no cancellation on s=0 for zeros of syst @ origin

This simply means that the added integrators must not cancel with zeros of the system in *s=0*

- Be carefully on the proper order of multiplication when dealing with MIMO systems ...

Dynamic performance (*sketch, see the textbook*)

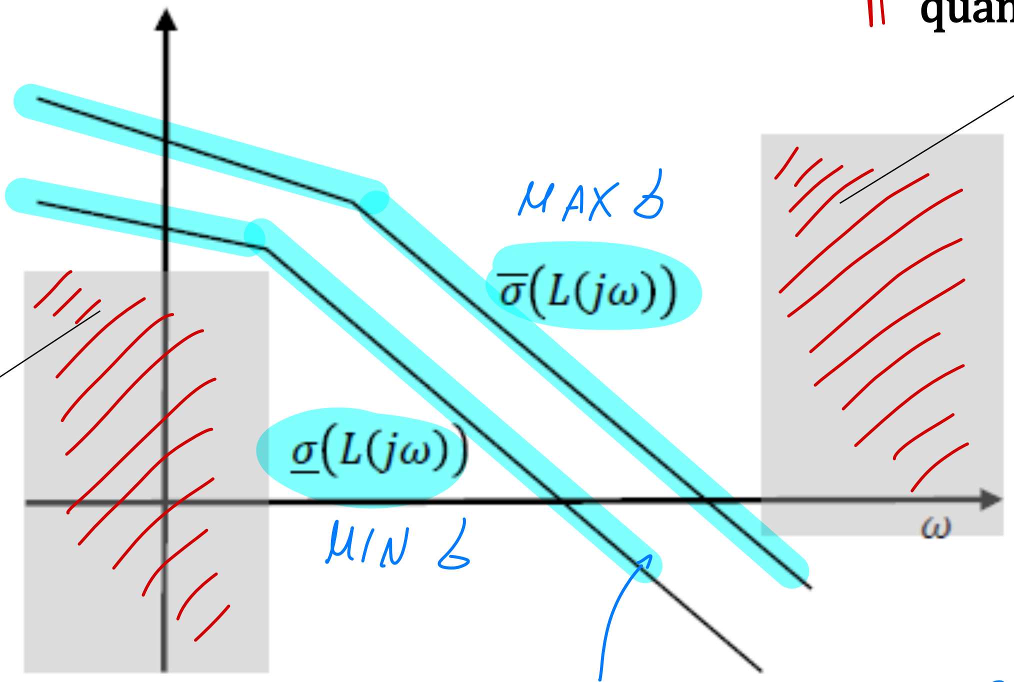
relevant result rely on SISO case property L_D

$$\begin{cases} Y(s) = T(s)\left(Y^o(s) - N(s)\right) + S(s)D_y(s) \\ U(s) = \left(I + R(s)G(s)\right)^{-1} R(s)\left(Y^o(s) - D_y(s) - N(s)\right) \end{cases}$$

Playing with singular values, requirements on *T(s)*, *S(s)* etc. can be transformed into requirements on the minimum and maximum singular values

→ design shaping function and set R(s) to obtain request

Loop singular values of L(s)



***T* small** at high frequency

↓

Maximum s.v. of *L(s)* smaller than a given quantity

high freq request of attenuation!

look smallest sing value

T* ≃*I at low frequency
S* ≃*0 at low frequency

↓

Minimum s.v. of *L(s)* greater than a given quantity

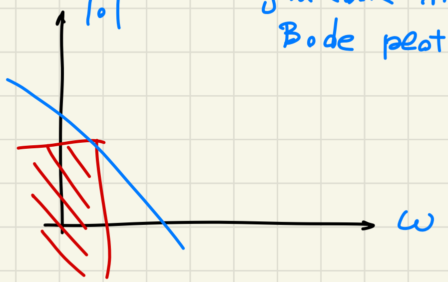
(sufficient attenuation)

consideration to design a shaping function

on MIMO case more lines respect SISO where you have just one bode plot line!

While on SISO case

You look magnitude Bode plot!



Summary of the main problems and facts related to the design of regulators for MIMO systems

There is not a unique loop transfer function *L(s)*, it is not at all clear how to modify the loop transfer function (matrix) by a proper selection of the regulator

The Bode criterio does not exist, multivariable Root Locus is an extremely complex approach, and synthesis in the frequency domain turns out to be difficult

Automatic tuning

These are good motivations for the use of automatic synthesis procedures (pole placement, optimal contrl…)

Anyway, choosing a priori the proper controller structure allows one to solve the static problem (asymptotic tracking of constant reference signals, asymptotic rejection of constant disturbances)

What about discrete-time systems? Most of the arguments should be reconsidered. However the structure with integrators can be used as well (next slide)

Integrators of discrete time systems

same consideration of cont. time syst

- Form 1

$$v(k+1) = v(k) + e(k)$$

$$e(k) \rightarrow \boxed{\frac{1}{z-1}} \rightarrow v(k)$$

integrator model

We'll use this form only because is slightly easier to manage

- Form 2

$$\begin{cases} \varphi(k+1) = \varphi(k) + e(k) \\ v(k) = \varphi(k) + e(k) \end{cases}$$

better! provides advance features

$$e(k) \rightarrow \boxed{\frac{z}{z-1}} \rightarrow v(k)$$

also a z @ num !

Pole in 1

like considering a PI T.F on cont time

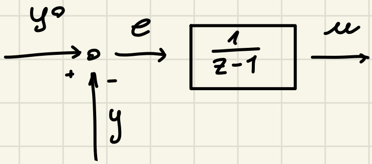
I) K/s — origin pole

PI) $K\frac{(1+sT)}{s}$ — usefull zero

The zero in *z=0* guarantees ***faster response*** (anticipative term), but it can cause ***implementation problems***. In zero time you should sample the error with A/D converter, compute *v*, and write it on the D/A port

read the error

true for all digital controller implementation



$$u(k) = \frac{1}{z-1} e(k) \Rightarrow z\,u(k) = u(k) + e(k)$$

$$u(k) = z^{-1} u(k) + z^{-1} e(k) = u(k-1) + e(k-1)$$

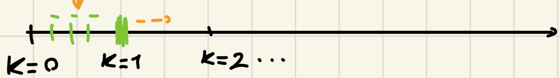
z^{-1} . time shift

"integrator" = summ on discrete time

compute $u(k)$ from past values!

disturb (you see it later)

sampling past values



(transform past values..)

IF you have $u(k) = \frac{z}{z-1} e(k)$

$$z\,u(k) = u(k) + z\,e(k)$$

$$\boxed{u(k) = u(k-1) + e(k)}$$

with logical st to perform it!

in 0 time: read error $e(k)$, compute $u(k)$ and convert!

FASTER!

⇓

BUT on controller design for slow system, the delay is negligible

⇑

in phase contribution, anticipative effect, help!

IF disturbance acting between $(k, k+1)$ you see the disturbance and compensate it on that time instant without delay!

if instead fast system → troubles on computing time of controller

TRUE both for strictly proper OR NOT strictly proper Regulator

↓

IF $R(z) = \dfrac{z^{\nu-1} + \dots}{z^{\nu} + \dots}$ more poles than zeros

- strictly proper

delay time...

↳ $u(k) = f(u(k-i), e(k-i))$ $i > 0$

- NON STRICTLY proper

$$R(z) = \frac{z^{\nu} + \dots}{z^{\nu} + \dots}$$

$\| u(k) = f(u(k-i), e(k-i+1)) \quad i > 0 \|$

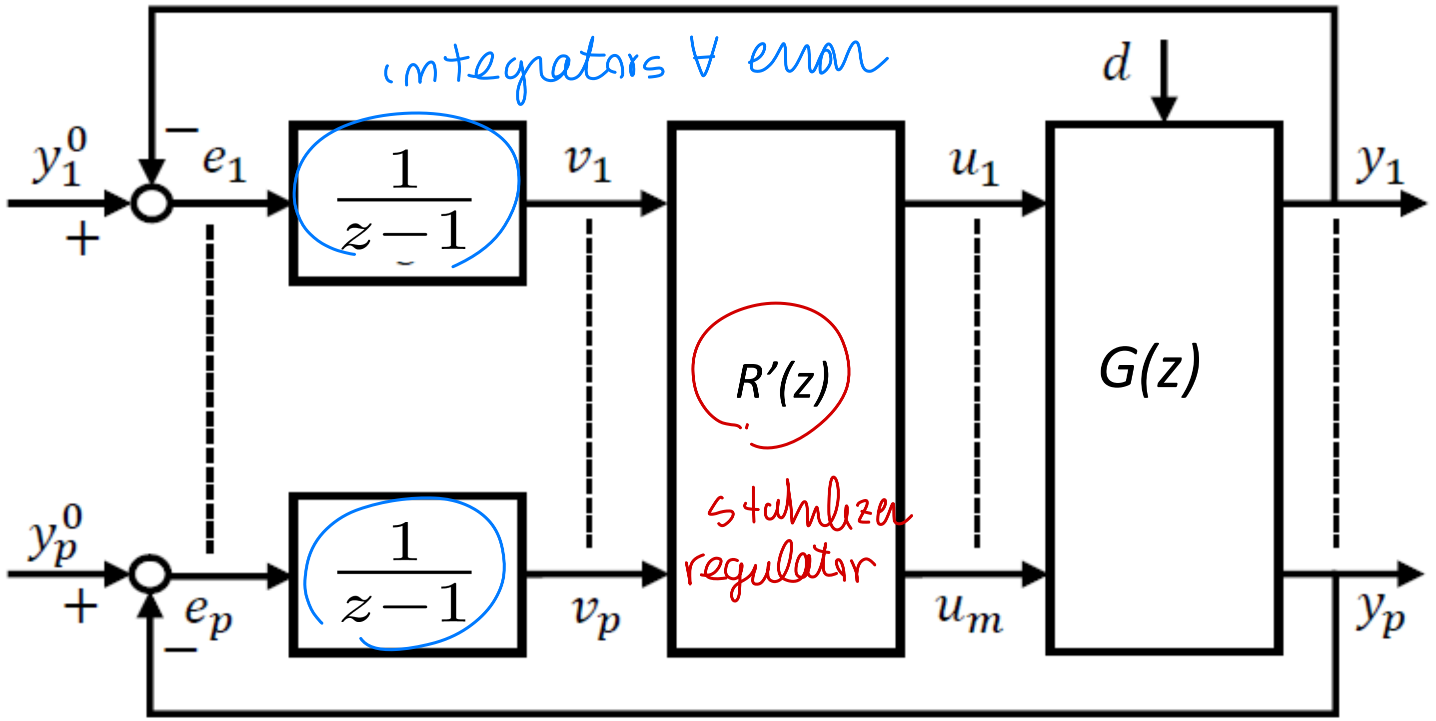
all in time ∅

↑

for small delay... usually ignored respect some syst.. IF on fast syst could be meaningfull delay Δt

⇒ on CRITICAL cases

if Δt required to run $R(z)$ is longer than step time... unable to produce proper u on time → PROBLEM!



like on cont. time

Plant + integrators

small diff than cont. time

block diag matrix

$$\left[\begin{array}{c} x(k+1) \\ v(k+1) \end{array}\right] = \left[\begin{array}{cc} A & 0 \\ -C & I \end{array}\right] \left[\begin{array}{c} x(k) \\ v(k) \end{array}\right] + \left[\begin{array}{c} B \\ 0 \end{array}\right] u(k) + \left[\begin{array}{cc} 0 & M \\ I & -N \end{array}\right] \left[\begin{array}{c} y^0 \\ d \end{array}\right]$$

$$v(k) = \left[\begin{array}{cc} 0 & I \end{array}\right] \left[\begin{array}{c} x(k) \\ v(k) \end{array}\right]$$

Enlarged system

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & I \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 & M \\ I & -N \end{bmatrix} \begin{bmatrix} y^0 \\ d \end{bmatrix}$$

$$v(k) = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix}$$

we must guarantee that R' can be designed

The system is reachable and observable iff the original system is reachable and observable and does not have invariant zeros in *z=1*

OBS/REACH to guarantee!

R'(s)

Static performance are guaranteed for constant y^o, *d*. Stabilization algorithms will be based on pole placement, optimal control, Model Predictive Control, …

(done for discrete time)

… some (exam) exercises …

Consider an asymptotically stable system with static gain

$$G(0) = \begin{bmatrix} 1 & -0.5 \\ 2 & 1 \end{bmatrix}$$

and the constant inputs with 2-norm equal to 1

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

Correspondingly $\|y_1\|_2 = 2.2361$, $\|y_2\|_2 = 2.1503$, $\|y_3\|_2 = 1.1180$. Then, the singular values of $G(0)$ are

- 1.2180 , 2.2361
- 1.1180 , 2.1121
- ✗ 0.8508 , 2.3508
- 0.8508 , 2.2120

$$\underline{\sigma} \leq \|y_i\|_2 \leq \bar{\sigma} \quad \forall i$$

$$\left.\begin{aligned} \|y_i\|_{max} &= 2.2361 \\ \|y_i\|_{min} &= 1.1180 \end{aligned}\right\} \Rightarrow \text{from here you find } \underline{\sigma}, \bar{\sigma}$$

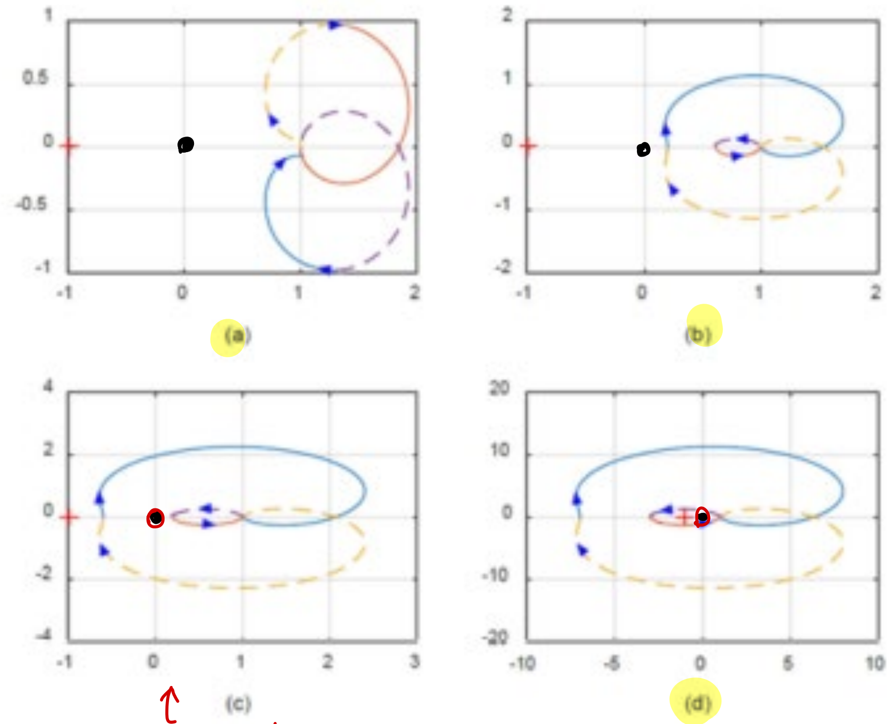
Consider a standard feedback system with negative feedbak, transfer functions G(s) and R(s) of the system and the regulator, and loop transfer function L(s)=G(s)R(s). Is it possible to study the stability of the closed loop system by looking at the sensitivity function S(s)=inv(I+L(s)? *
(3 punti)

- ○ yes, if and only if G(s) and R(s) have stable poles → NOT enough!
- ○ no never wrong
- ○ yes, always wrong → in principle we should study 5 T.Fs
- ☒ yes, if there are no cancellations of unstable poles of G(s) with (invariant) zeros of R(s) and viceversa

↑ proper answer

Consider a feedback system with loop transfer function $L(s)$ without unstable poles.

Consider the following Nyquist plots of $det\ (I+L(s))$ and specify which cases (a), (b), (c), (d) correspond to asymptotically stable closed-loop systems.

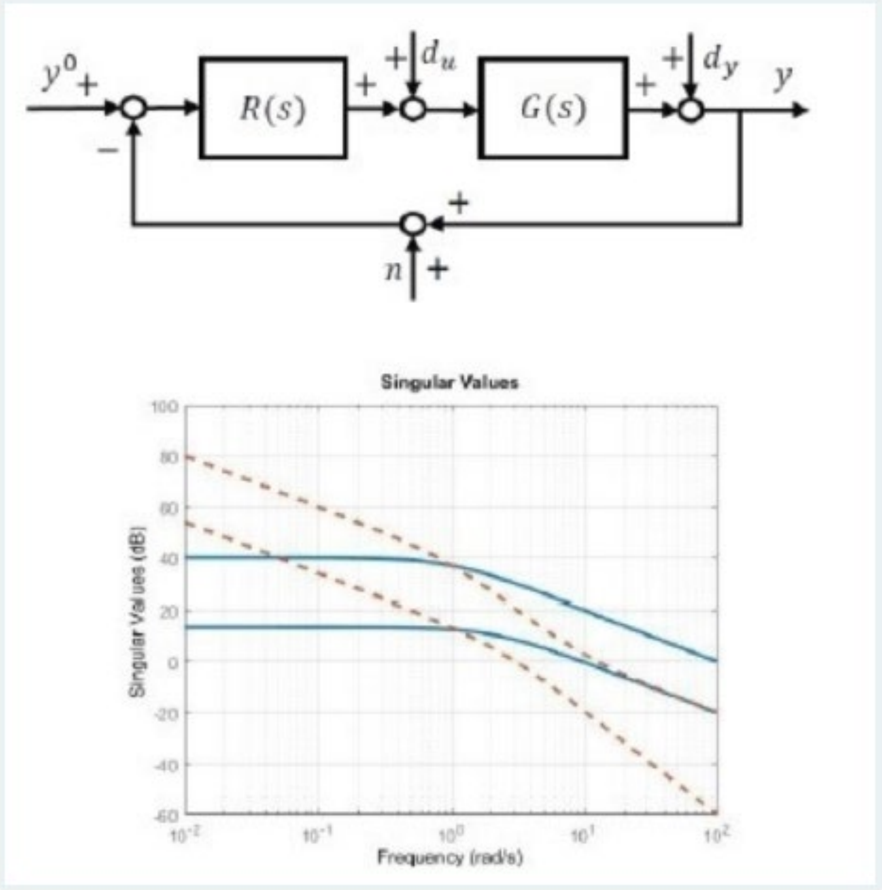


a, b does NOT do circle around the origin!

made 1 circle around 0!

overall one clockwise and one anti-clockwise!

- b, c, d
- a, b
- a, c, d
- X a, b, d



Consider the feedback system reported in the block diagram and two possible loop transfer functions L1(s)=R1(s)G1(s) and L2(s)=R2(s)G2(s) with the principal gains reported in the figure. Let L1(s) be associated to the dashed lines and L2(s) to the continuous ones. Select the true answer among the following ones:
(3 punti)

NO

- Assuming that at low and high frequency all the singular values diagrams have the same slope shown in the figure, it is likely that the two loop transfer functions do not have poles at the origin. NOT true because y sing value @ low freq... integration

NO

- L1(s) and L2(s) have roughly the same crossover frequency. meaningless state. on MIMO at least one crossover & gain

NO

- At high frequency (ω > 20) the guaranteed attenuation of the noise disturbance n provided by L2(s) is always greater than the one guaranteed by L1(s). → @ high freq largest sing value sufficiently small!

- ~~At low frequency (ω < 1) the guaranteed attenuation of the disturbance dy provided by L1(s) is always greater than the one guaranteed by L2(s).~~

apply to smallest sing values --- has smallest than —