A simple geometrical construction with 2D homogeneous coordinates

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Our goal is to complete the drawing of the **isometric projection** http://en.wikipedia.org/wiki/Isometric_projection of a cuboid when the points corresponding to a vertex and to three vertices adjacent to it are given as input.

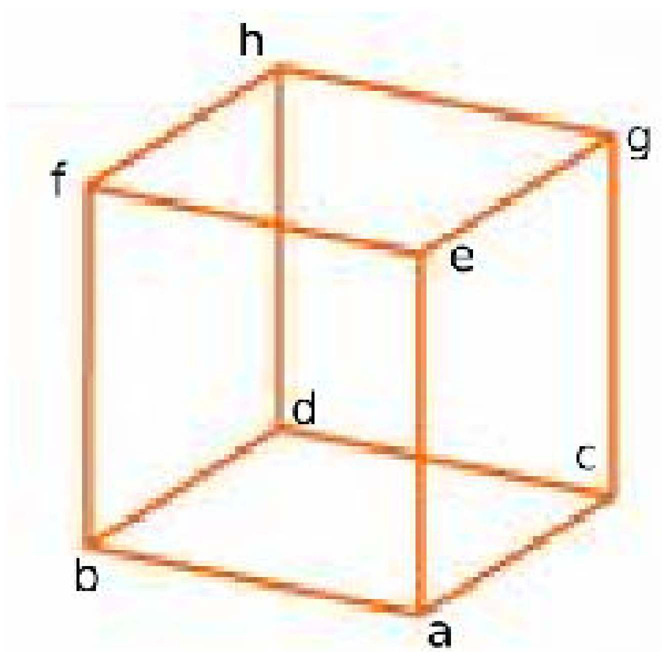
The geometrical construction only requires to find lines parallel to other lines, find lines passing through two points, and intersect lines, which is easily achieved in homogeneous coordinates.

```
clear
close all
clc
FNT_SZ = 28;
```

Naming

We will name the vertices as in the following image

```
figure(1), imshow(imread('E1_data/simplecube-letters.png'));
```



Data entering

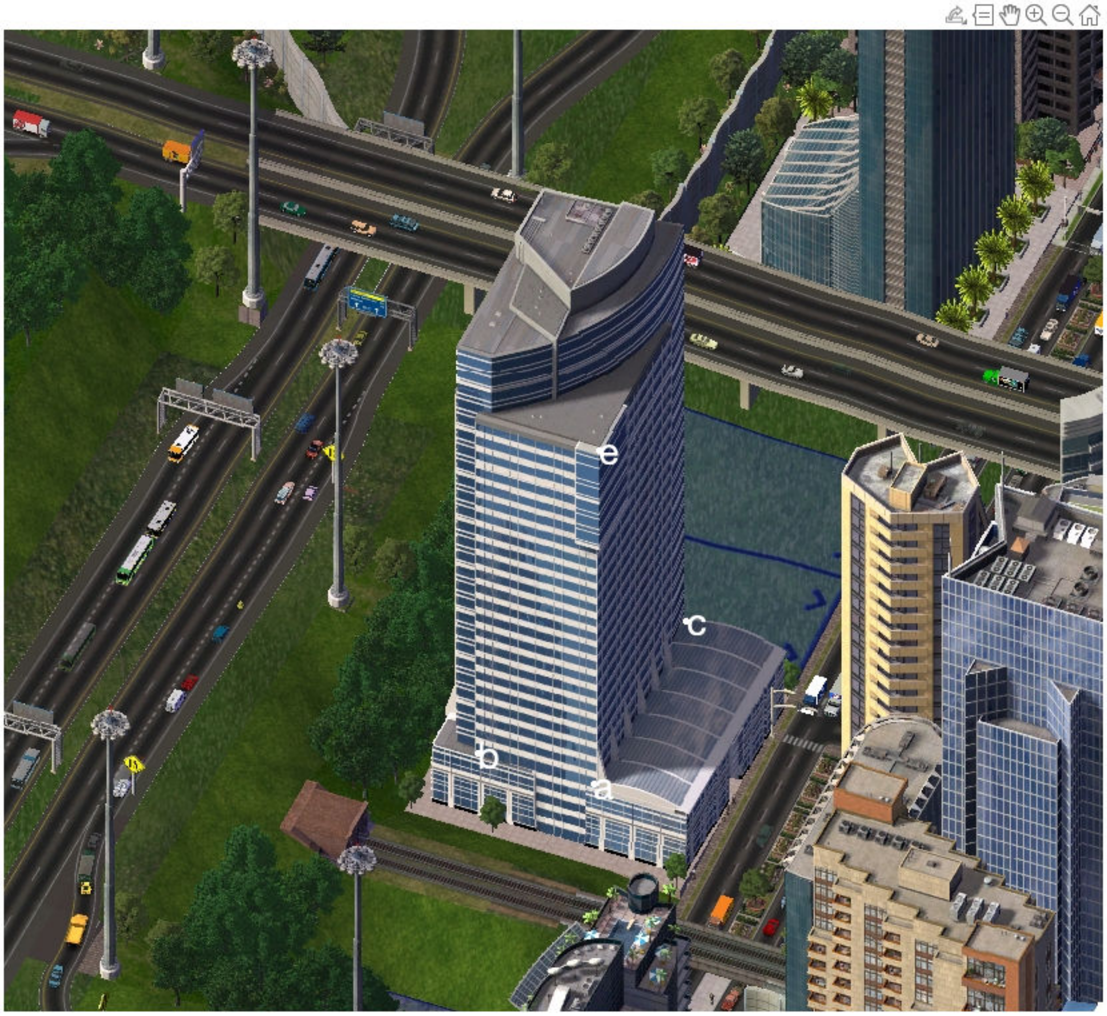
the user should click on a vertex of the cuboid, then on the three adjacent vertices

```
figure(2),
imshow(imread('E1_data/buildingSmall.png'))

hold on;
[x y]=getpts
```

```
x = 4×1
  443.0000
  356.0000
  512.0000
  447.0000
y = 4×1
  566.0000
  543.0000
  444.0000
  316.0000
```

```
plot(x,y,'.w','MarkerSize',12, 'LineWidth', 3); % plots points clicked by user with re
a=[x(1) y(1) 1]';
text(a(1), a(2), 'a', 'FontSize', FNT_SZ, 'Color', 'w')
b=[x(2) y(2) 1]';
text(b(1), b(2), 'b', 'FontSize', FNT_SZ, 'Color', 'w')
c=[x(3) y(3) 1]';
text(c(1), c(2), 'c', 'FontSize', FNT_SZ, 'Color', 'w')
e=[x(4) y(4) 1]';
text(e(1), e(2), 'e', 'FontSize', FNT_SZ, 'Color', 'w')
```



Finding some lines

Now variables a, b, c and d are 3-vectors containing the homogeneous coordinates of the 2D points. We need to find the lines passing through couples of points, using the cross product

```
lab = cross(a,b)
```

```
lab = 3×1
10^4 ×
    0.0023
   -0.0087
    3.9053
```

```
lac = cross(a,c)
```

```
lac = 3×1
10^4 ×
    0.0122
    0.0069
   -9.3100
```

```
lae = cross(a,e)
```

```
lae = 3×1
10^5 ×
    0.0025
    0.0000
   -1.1301
```

Check the incidence equation

`lab`, `lac` and `lae` now contain the homogeneous represenation of the three lines ab, ac and ae. We can easily check that the lines actually contain the points by using the incidence relation:

```
lab'*a
```

```
ans = -1.8190e-11
```

```
lab'*b
```

```
ans = -1.9099e-11
```

```
lac'*a
```

```
ans = 1.4552e-11
```

```
lac'*c
```

```
ans = 1.4552e-11
```

```
lae'*a
```

```
ans = -1.4552e-11
```

```
lae'*e
```

```
ans = -1.4552e-11
```

Parallelism

We now need to compute the line parallel to ab and passing through c. In order to do this, we create the line at infinity:

```
linf=[0 0 1]';
```

then, we find the directions of segments by ab, ac and ae intersecting their lines with the line at infinity

```
dab=cross(lab,linf)
```

```
dab = 3×1
  -87.0000
  -23.0000
         0
```

```
dac=cross(lac,linf)
```

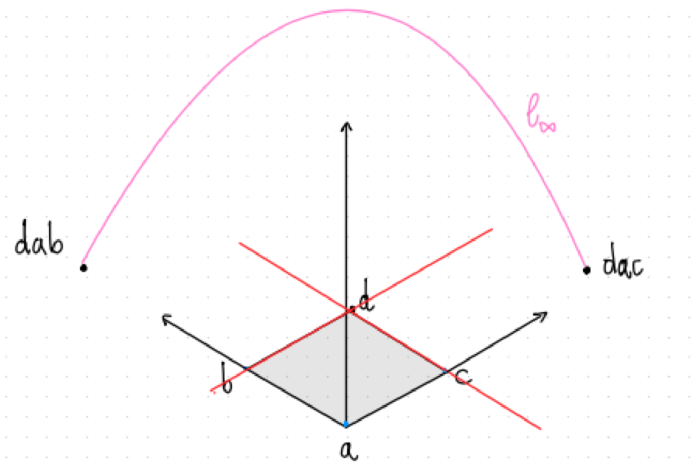
```
dac = 3×1
   69.0000
 -122.0000
```

```
         0
```

```
dae=cross(lae,linf)
```

```
dae = 3×1
    4.0000
 -250.0000
         0
```

dab, dac and dae are **points at the infinity**, which represent **directions**. All lines with a given direction pass through the corresponding point at the infinity. We can then find the lines containing segments bd and cd.



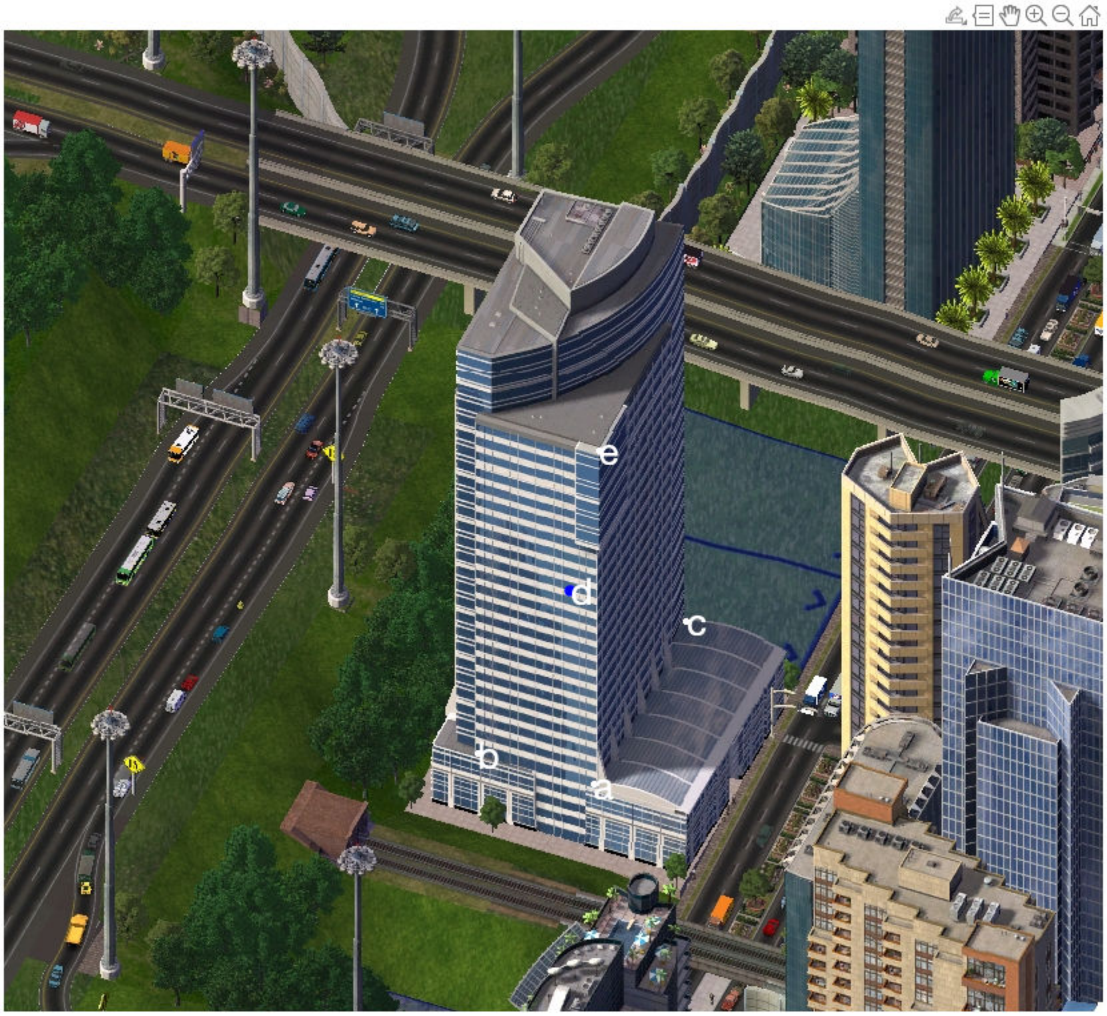

```
lbd=cross(b,dac);
lcd=cross(c,dab);
```

point d is now found by just intersecting `lbd` and `lcd`

```
d=cross(lbd,lcd);
```

we normalize d's coordinates so that we can read its cartesian coordinates in `d(1)` and `d(2)`. We plot the point with a blue circle.

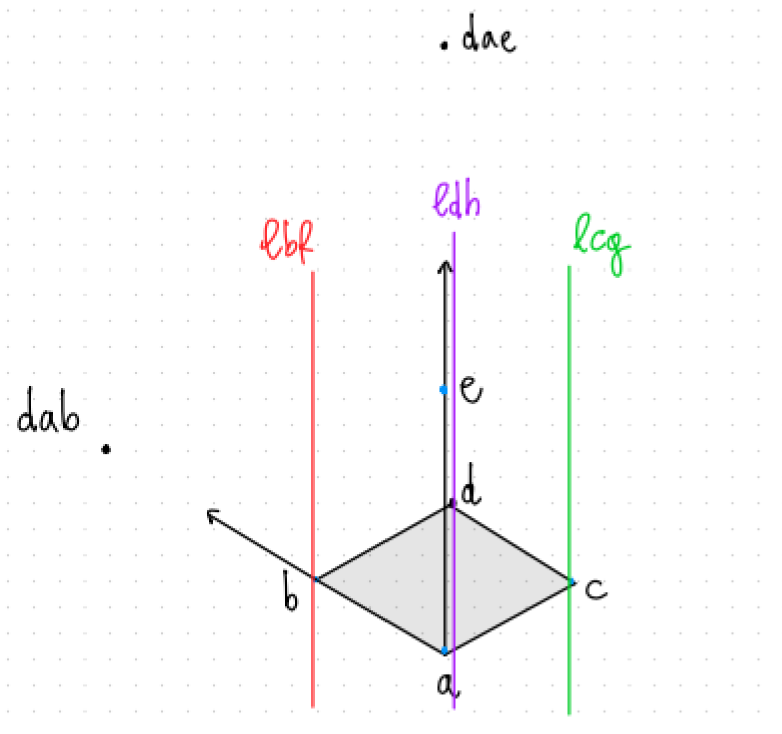
```
d=d/d(3);
plot(d(1),d(2),'.b','MarkerSize',30);
text(d(1), d(2), 'd', 'FontSize', FNT_SZ, 'Color', 'w')
```



Finding remaining points

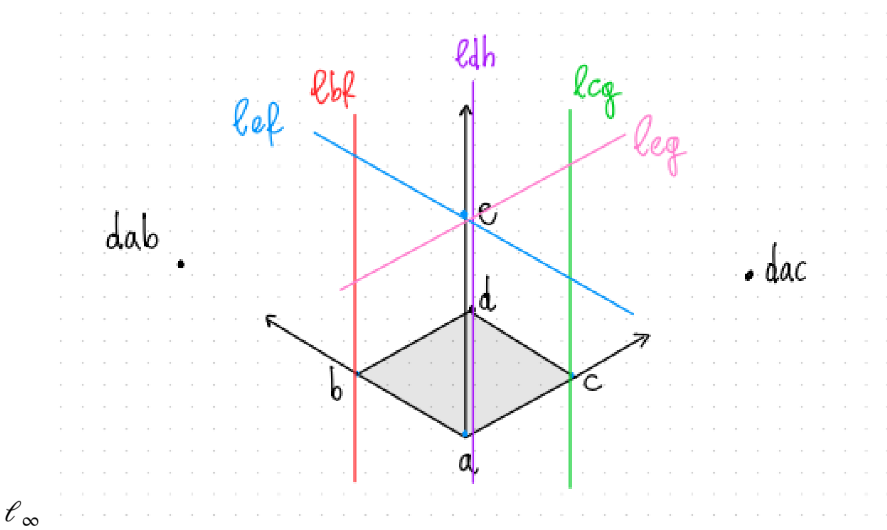
The rest of the procedure is straightforward, and follows exactly the same technique.

We first find vertical lines, that pass trough dae

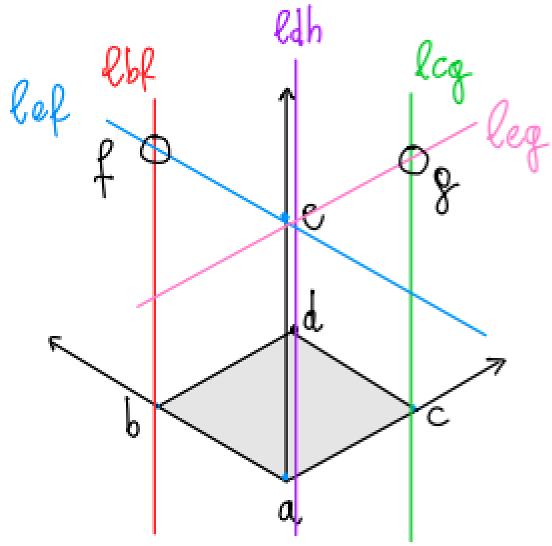



```
lbf=cross(b,dae);
lcg=cross(c,dae);
ldh=cross(d,dae);
```

We then find the lines incident to e parallel to lab and lac. Being parallel means having the same intersection at



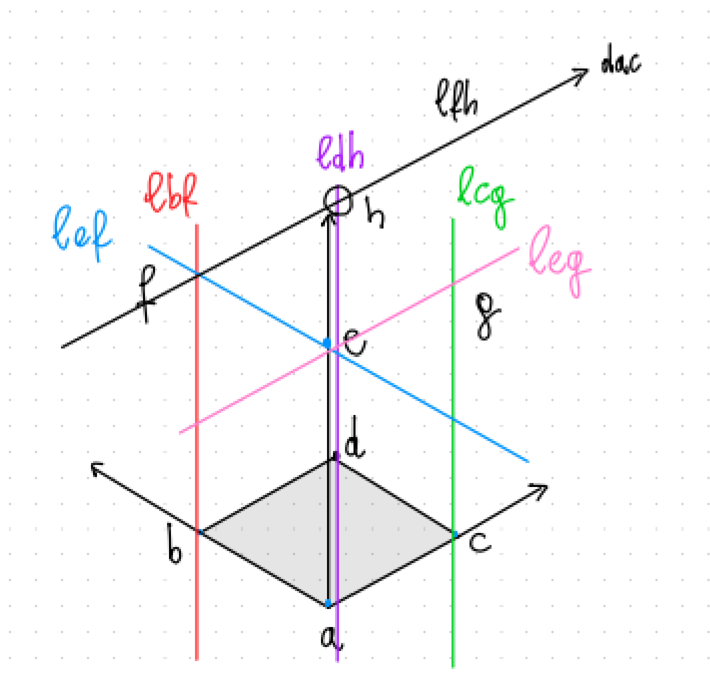

```
lef=cross(e,dab);
leg=cross(e,dac);
```



Now we can find the intersections

```
f=cross(lbf,lef);
g=cross(lcg,leg);
```

Now the line passing through f and parallel to lac. Hence we can get h.



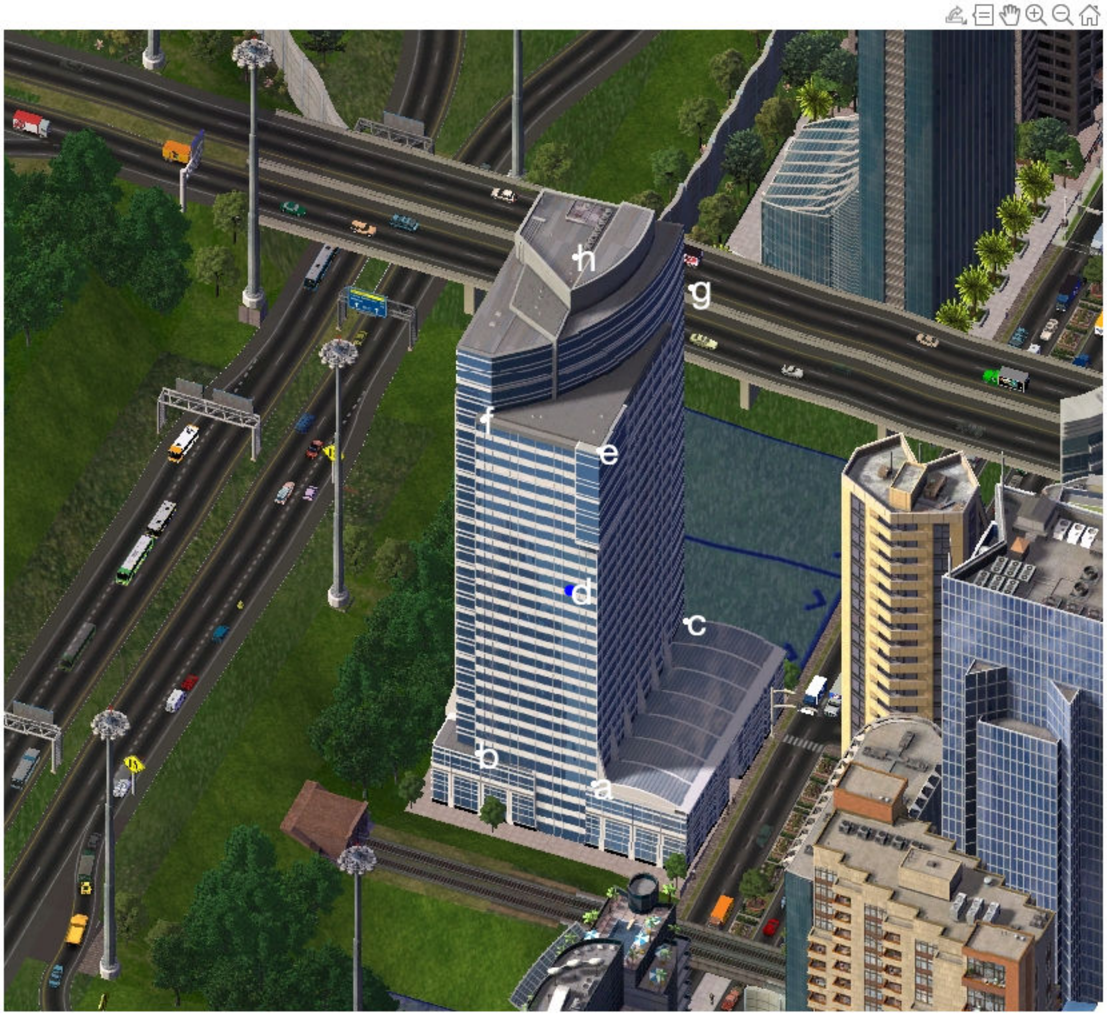

```
lfh=cross(f,dac);
h=cross(lfh,ldh);
```

Remember that points are expressed in homogenous coordinates, in order to plot them we have to convert them to cartesian.

```
f=f/f(3);
```

```
g=g/g(3);
h=h/h(3);

plot(f(1), f(2),'.w','MarkerSize',12, 'LineWidth', 3); % plots points clicked by user
text(f(1), f(2), 'f', 'FontSize', FNT_SZ, 'Color', 'w')
plot(g(1), g(2),'.w','MarkerSize',12, 'LineWidth', 3); % plots points clicked by user
text(g(1), g(2), 'g', 'FontSize', FNT_SZ, 'Color', 'w')
plot(h(1), h(2),'.w','MarkerSize',12, 'LineWidth', 3); % plots points clicked by user
text(h(1), h(2), 'h', 'FontSize', FNT_SZ, 'Color', 'w')
```

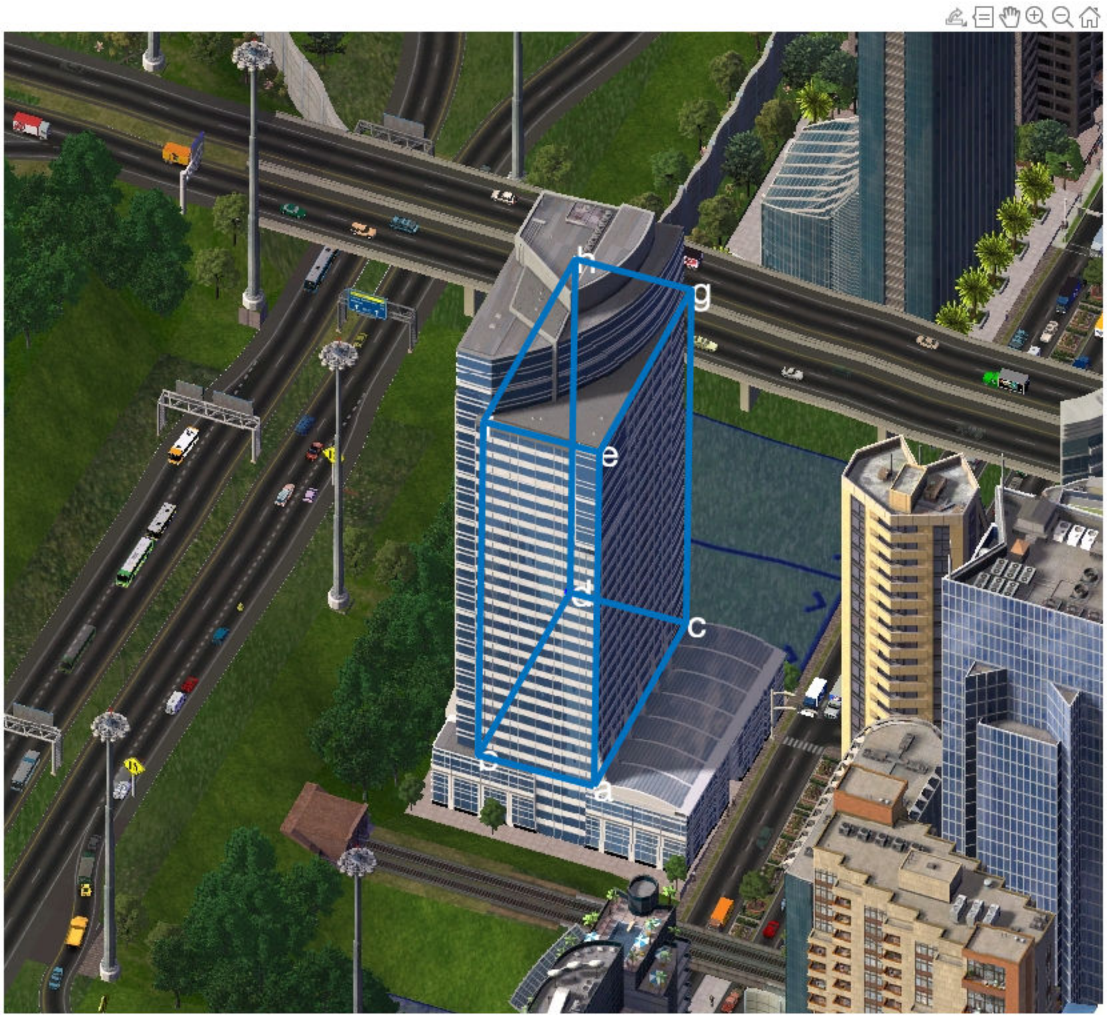


Drawing

we can now finally draw the cube.

```
myline=[a';b';d';c';a'];
line(myline(:,1),myline(:,2),'LineWidth',5);
myline=[e';f';h';g';e'];
line(myline(:,1),myline(:,2),'LineWidth',5);
myline=[a';e'];
line(myline(:,1),myline(:,2),'LineWidth',5);
myline=[b';f'];
```

```
line(myline(:,1),myline(:,2),'LineWidth',5);
myline=[c';g'];
line(myline(:,1),myline(:,2),'LineWidth',5);
myline=[d';h'];
line(myline(:,1),myline(:,2),'LineWidth',5);
hold off
```



Notes

which vertices you click is not important: the only requirement is that the first vertex is adjacent to the other three. also, note that the algorithm works, more generally, on parallelepipeds.