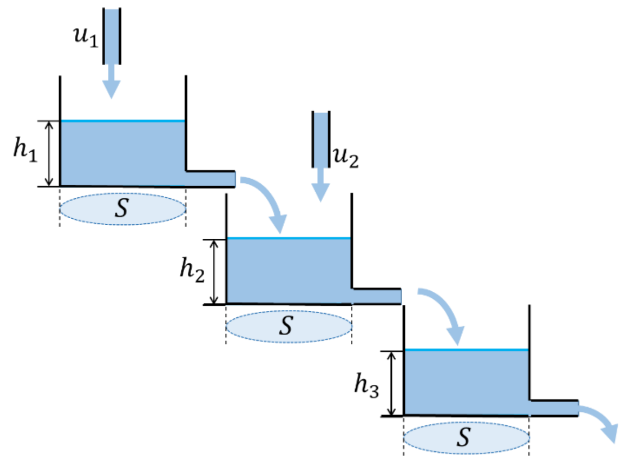
Three-tanks system

Consider the system illustrated in the following figure, consisting of a cascade interconnection of three tanks.



The linearized model is

$$\begin{cases} S\dot{h}_1 = & -kh_1 + u_1 \\ S\dot{h}_2 = & kh_1 - kh_2 + u_2 \\ S\dot{h}_3 = & kh_2 - kh_3 \end{cases}$$

where $S = 1\ \mathrm{m}^2$ and $k = 1\ \mathrm{m}^2/s$. Note that, since the model is linearized around a nominal condition, all the variables of the model above should be regarded as differences with respect to nominal values. All water levels are measurable. Defining $x = [h_1, h_2, h_3]^T$ and $u = [u_1, u_2]^T$, the system's dynamics is described by the model

$$\dot{x} = Ax + Bu$$

where the matrices are defined in the corresponding MATLAB file.

Problem:

1. Decompose the state and input vectors into subvectors, consistently with the physical description of the system. Obtain the corresponding decomposed model.
2. Generate the system matrices (both continuous-time and discrete-time, the latter with a sampling time selected compatibly with the continuous-time dynamics). Perform the following analysis:
    a. Compute the eigenvalues and the spectral abscissa of the (continuous-time) system. Is it open-loop asymptotically stable?
    b. Compute the eigenvalues and the spectral radius of the (discrete-time) system. Is it open-loop asymptotically stable?
3. For different state-feedback control structures (i.e., centralized, decentralized, and different distributed schemes) perform the following actions
    a. Compute the continuous-time fixed modes
    b. Compute the discrete-time fixed modes

c. Compute, if possible, the CONTINUOUS-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.
d. Compute, if possible, the DISCRETE-TIME control gains using LMIs to achieve the desired performances. Apply, for better comparison, different criteria for computing the control laws.
e. Analyze the properties of the so-obtained closed-loop systems (e.g., stability, eigenvalues) and compute the closed-loop system trajectories (generated both in continuous-time and in discrete-time) of the water levels starting from a common random initial condition.