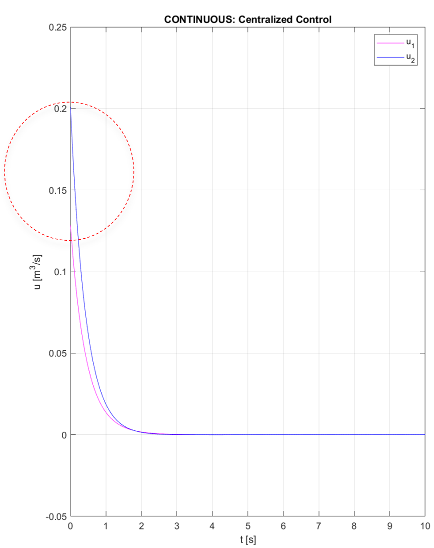
Alternative Control Design
Limit control action rate

Notice that, we are dealing with a **real phisycal system**…

In our control framework we observe the system (linearized) and for a variation of the tanks' height respect the nominal conditions Dh, we act by a variation on the water input inside the tanks (to avoid any confusion, we call u that variation of u respect nominal value $\bar{u}$), in order to bring that variations Dh to 0.

Let's look to any control action (same reasoning in Discrete time):



Control variables vary from 0 to 0.2 (and 0.12) in 0 sec, and in 1 sec we are again back to 0. Even if an electric valve can be instantaneus, we can **assume to have some control rate limitation** (actuator dynamic), taking into account that Δu is limited, and that control action at t=0 start from 0, not from the instantaneus value chosen by the control law.

This constraint allow us to simulate better a real system, but also (sometimes) to improve the perfomance through a minimization of Δu.

Which leads to a distributed action over time, converging without aggressive behaviour.

Alternative Control Design
Limit control action rate

OPTION 1, LMI: (discrete time)

$\Delta u_k = K(F + GK - I)x_k$

To minimze Δu_k , we should minimize $\|K(F + GK - I)\|$

But if we also implement the minimization of the control effort, $\|K\|$ will be already minimized. We need to minimize 2-norm of $(F + GK - I)$

Proceeding similarly to the procedure for the minimization of control effort:

$\|F + GK - I\| = \|(FP + GL - P)P^{-1}\| \le \|FP + GL - P\|\|P^{-1}\|$

Where $\|P^{-1}\|$ again is minimized by the control effort minimization.

So we focus on $\|FP + GL - P\|$:

$\min\{\alpha_D\}$

s.t. $\|FP + GL - P\| \le \alpha_D$ (This can be casted by the shur complement into an LMI)

So overall, use this additional LMI and rewrite the overall cost as:

$J = a_D\alpha_D + a_Y\alpha_Y + a_L\alpha_L$ (with the proper trade-off choise of the weigth)

Alternative Control Design
Limit control action rate

Similarly, **continuous time**:

$u(t) = Kx(t)$ (state-feedback)

$\dot{u}(t) = K\dot{x}(t) = K\big(Ax(t) + Bu(t)\big) = K(A + BK)x(t)$

Same reasoning as before, we want to minimize $\|A + BK\|\ldots$

Anyway, this option doesn't capture our modelling goal, not obtaining the proper cost trade-off to minimize well the control rate toghether with the control effort.

That's why we opted for

OPTION 2, State expansion:

Consider the control vector u as part of the state vector, expanding the state-matrix, and take as input of the system a virtual input v, representing the derivative of u.

(nothing forbid us to apply the virtual input v computed by our control scheme, and integrate it numerically to compute the real actuation input!)

Alternative Control Design
state expansion

From the initial system:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

We expand the dynamic:

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{u} = v \\ y = Cx \end{cases}$$

A = $\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

C = I_3

And, redefine the system vectors as:

$x = [x_1 \quad u_1 \quad x_2 \quad x_3 \quad u_2]^T$

u= $[\, v_1 \quad v_2]^T$

(Exactly the same reasoning is done for discrete time state-space description)

So the final state-space model become:

$$\begin{cases} \dot{x} = A_e x + B_e u \\ y = C_e x \end{cases}$$

$$A_e = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_e = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad C_e = I_5$$

Alternative Control Design
state expansion

At this point, the system decomposition comes straigth foreward

$$A_e = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_e = \begin{array}{c} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \\ B_1 \quad B_2 \end{array} \quad C_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \left.\vphantom{\begin{matrix}0\\0\end{matrix}}\right\} C_1 \\ \left.\vphantom{\begin{matrix}0\\0\\0\end{matrix}}\right\} C_2 \end{array}$$

Finally, we use exactly the same script as before, but with this new system matrix both in continuous and dicrete time. Through the control action minimization, this time we are minimizing u= $[\, v_1 \quad v_2]^T$ control effort, exactly $\dot{u}$ as desired (or Δu in discrete).

Remember also to impose as initial condition:

x_0 = $[h_{1,0}$, 0,$h_{2,0}$,$h_{3,0}$,$0]^T$ (to start from 0 control action).

The tuning procedure to chose the best control design parameter is exactly as before, we can even try using the same optimal trade-off found previously.

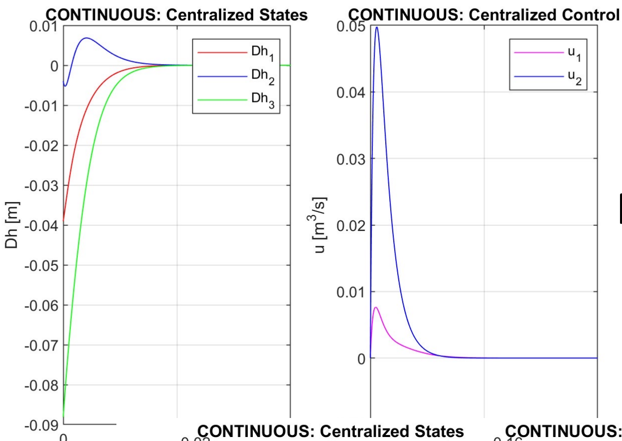
Simulation - Continuous time
Limit control action rate

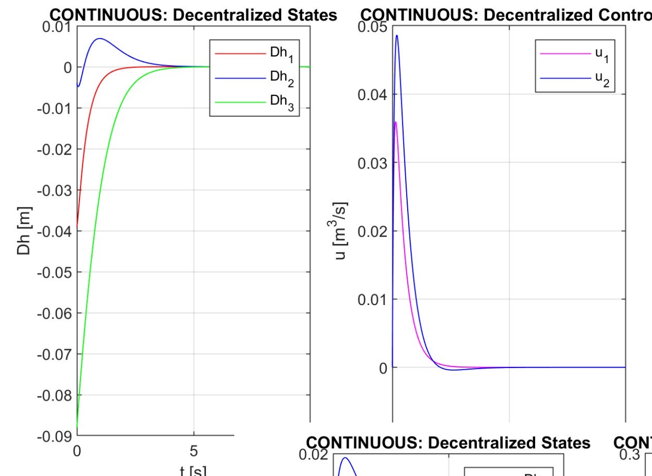
$\alpha_{max}=1.7$, $\vartheta_{max}=10$ $x_0=[-0.039,-0.04,-0.088]^T$

CENTRALIZED

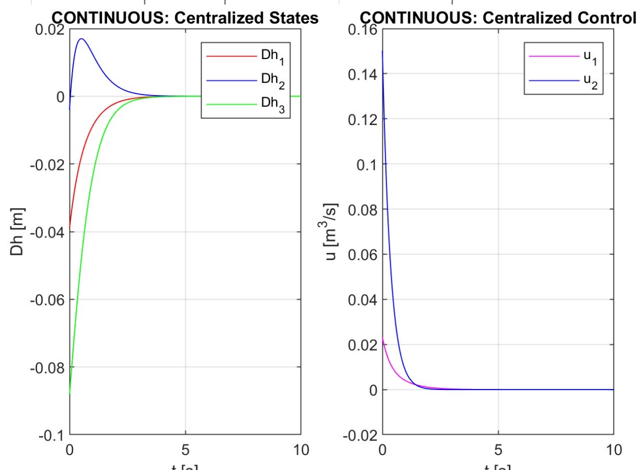
DECENTRALIZED

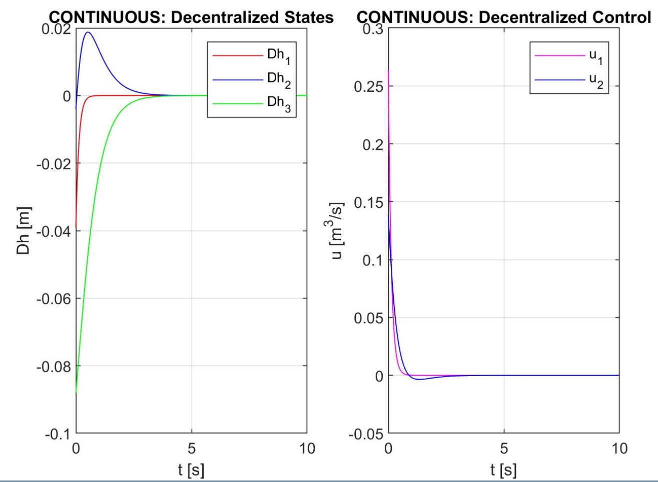
Rate Limit





NO Rate Limit





Simulation - Continuous time
Limit control action rate

$\boldsymbol{\alpha_{max}}$ =1.7, $\boldsymbol{\vartheta_{max}}$=10 x_0=[−0.039,−0.04,−0.088]^T

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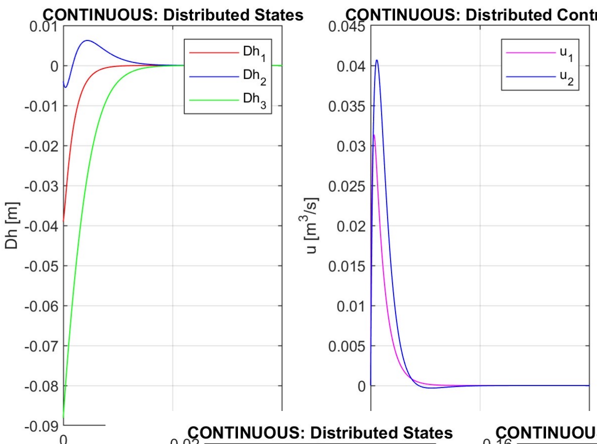

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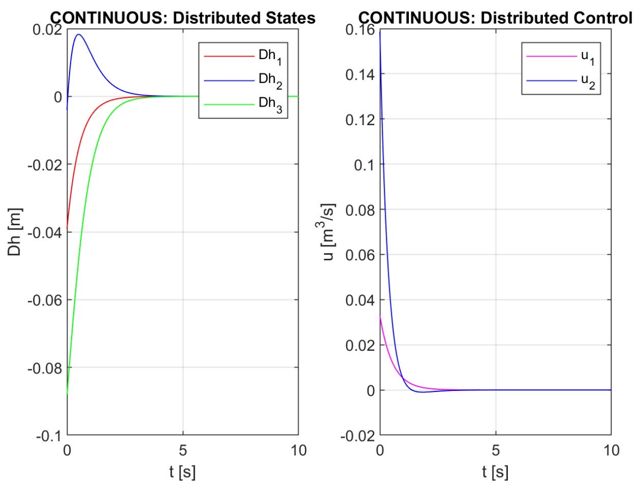

NO Rate Limit

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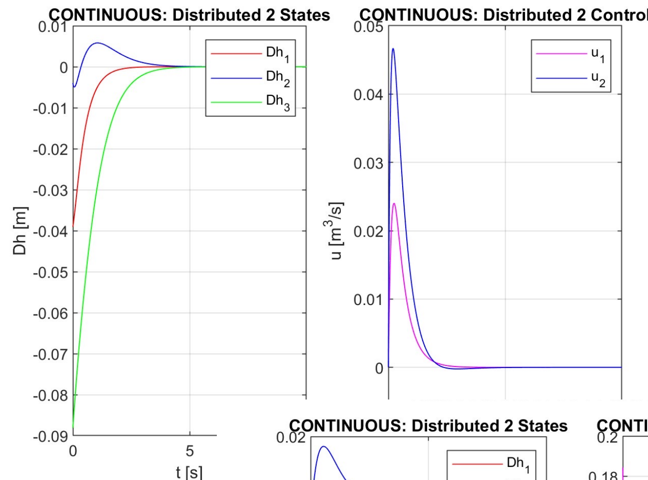
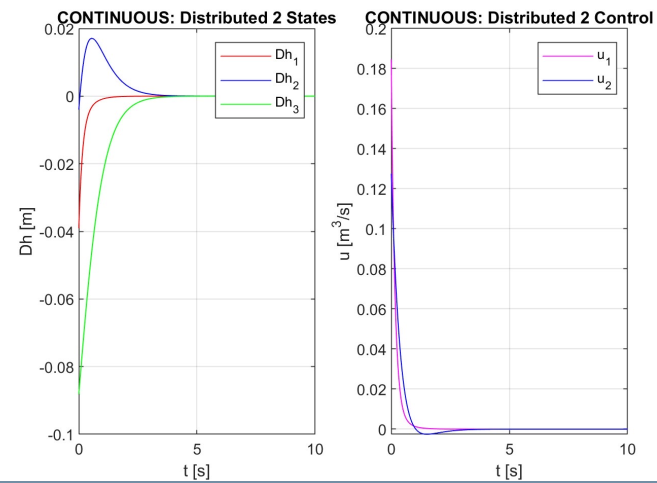
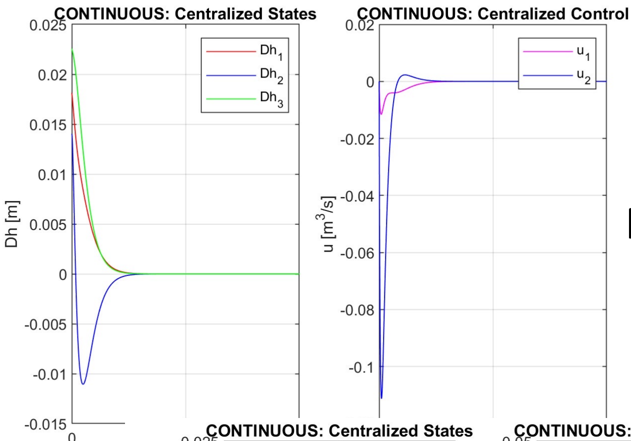




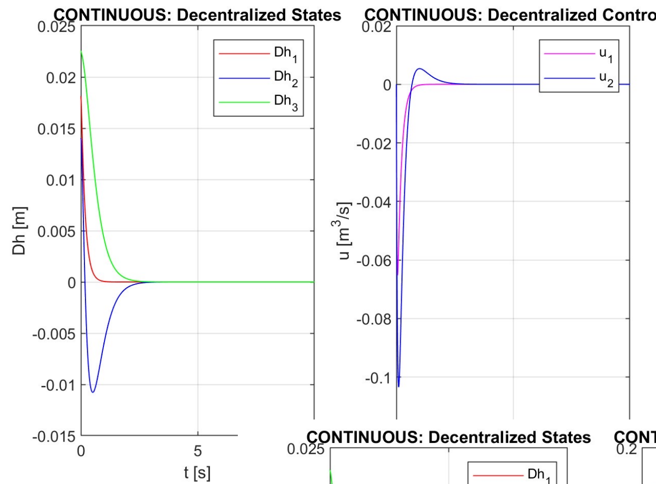
Simulation - Continuous time
Limit control action rate

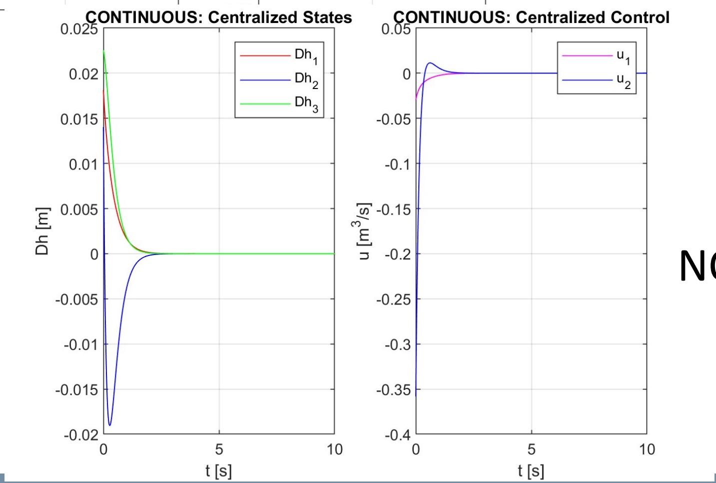
$\alpha_{max} = 3$, $\vartheta_{max}=10$ $x_0=[0.018,0.014,0.023]^T$

CENTRALIZED **DECENTRALIZED**

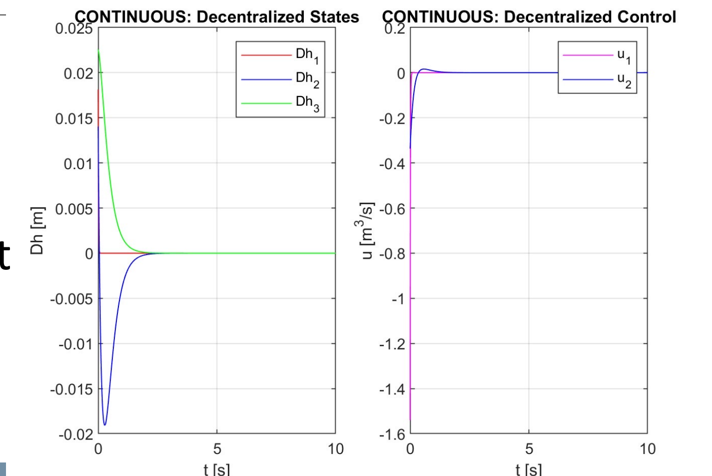


Rate Limit





NO Rate Limit



Simulation - Continuous time
Limit control action rate

$\boldsymbol{\alpha_{max}}$ **= 3,** $\boldsymbol{\vartheta_{max}}$**=10** x_0=[0.018,0.014,0.023]^T

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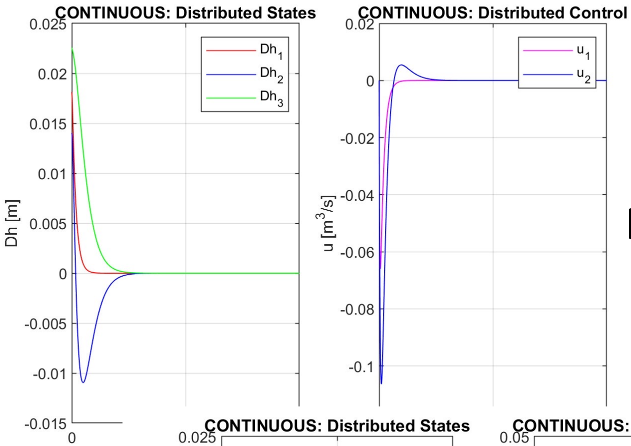
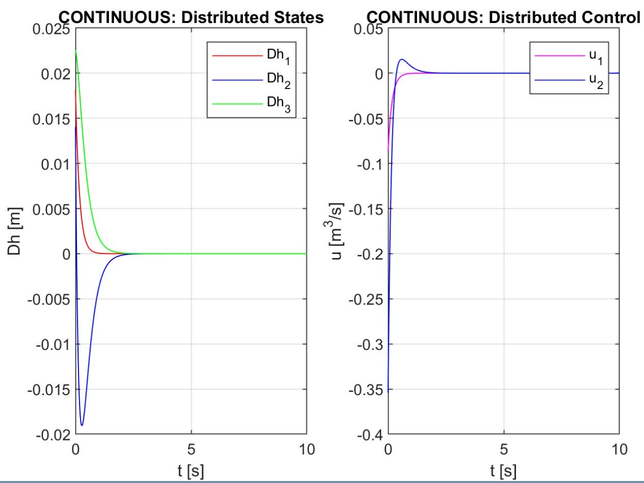
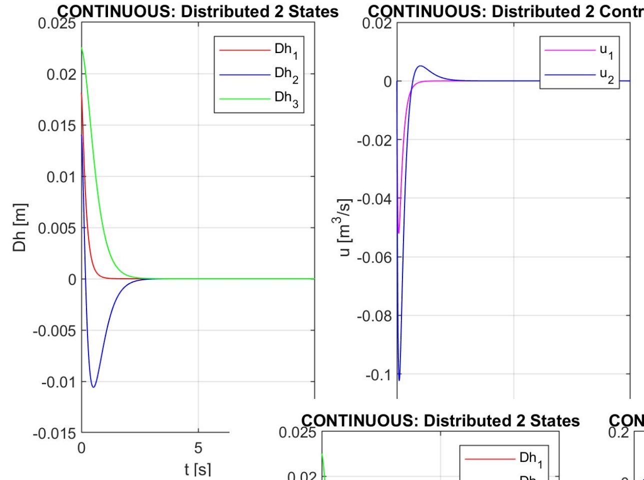


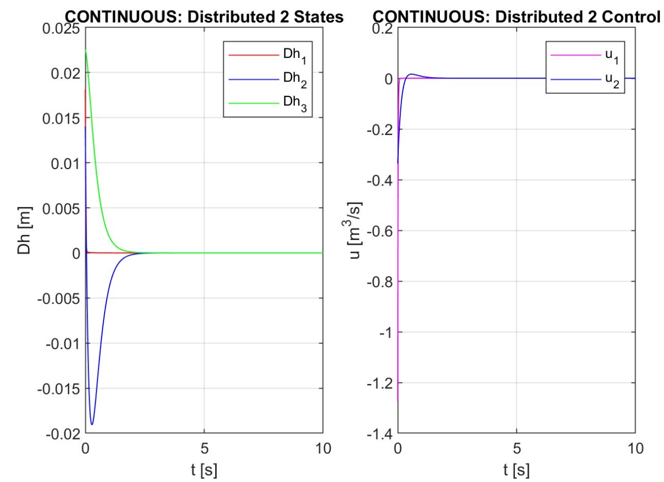
Rate Limit



NO Rate Limit

DISTRIBUTRED 2





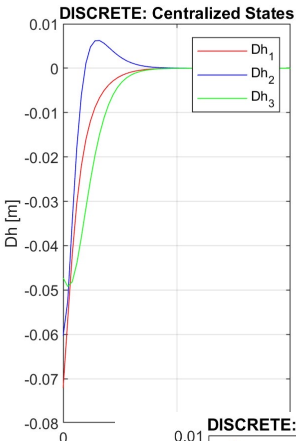
Comments - Continuous time
Limit control action rate

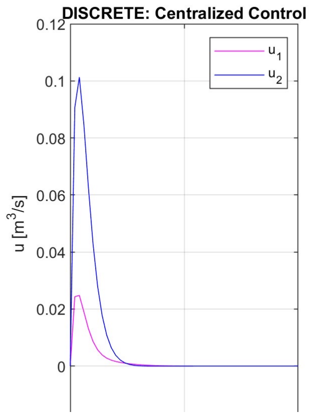
- Simulating by pushing even further the performance (for those cases when we need a really short settling time but with a feasible action), for $\alpha_{max} > 3$, the previous evidence of better performance of Centralized and Distributed 1 control structure here disappears. With this design, in the previous ‘worst’ schemes (Dec, Dist 2) in many cases we are able to reduce the control action peak by a factor of 30. So even the decentralized scheme has a good performance/control trade-off.
- Even asking for the same performance of before, we are able to reduce a lot the control effort, so this solution is perfect for those cases where we have saturation limits on actuators
- Notice that in most of the cases we are even able to reduce the maximum amount of overshoot

Simulation - Discrete time
Limit control action rate

α_{max} =−0.55 , ρ_{max} = 0.15, h = 0.2, x_0=[−0.072,−0.060,−0.047]^T

CENTRALIZED | **DECENTRALIZED**





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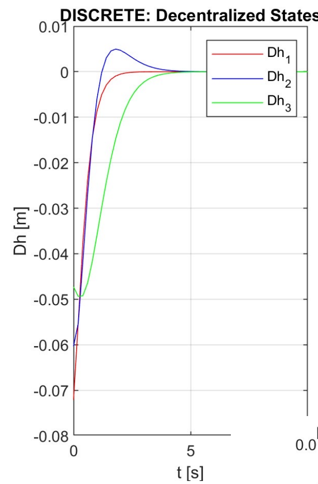

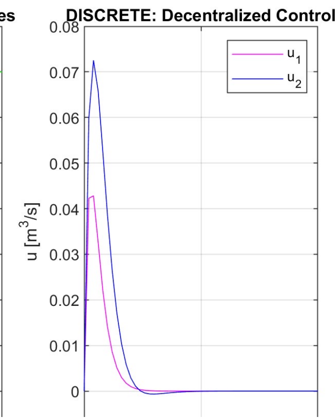

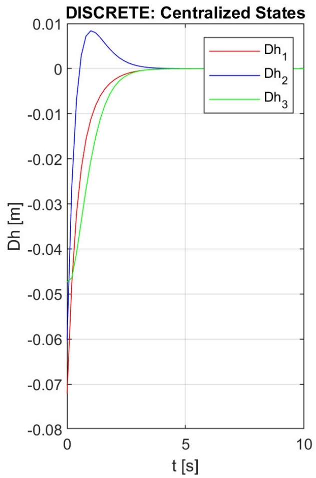

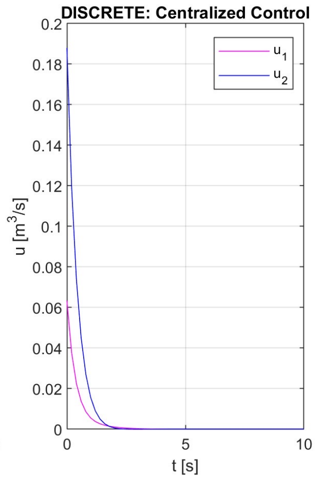

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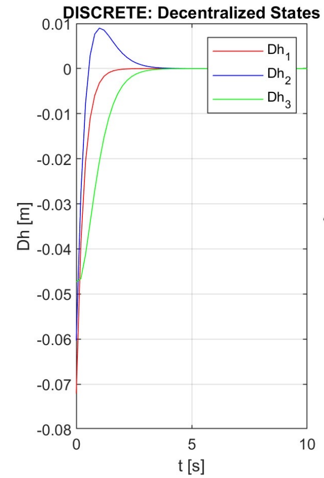
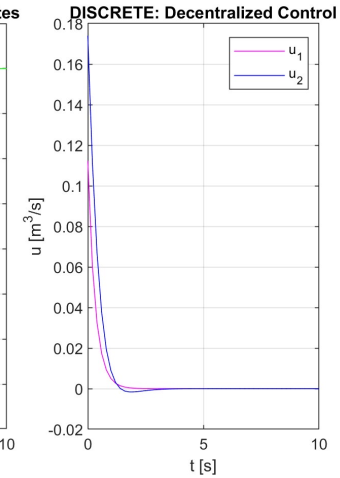


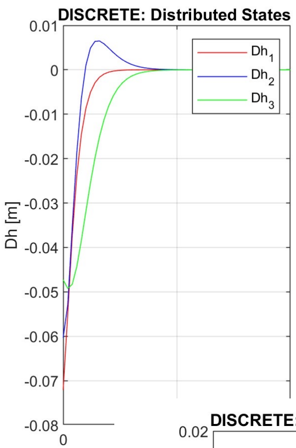


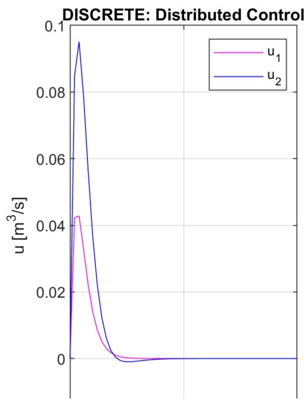
Simulation - Discrete time
Limit control action rate

α_{max} =−0.55 , ρ_{max} = 0.15, h = 0.2, x_0=[−0.072,−0.060,−0.047]^T

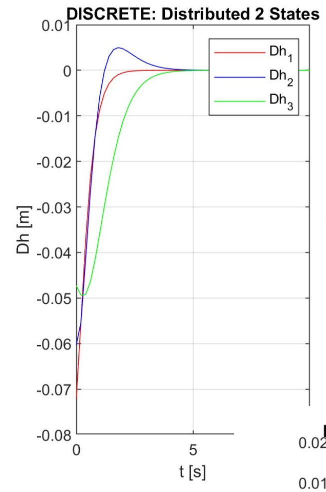
DISTRIBUTED

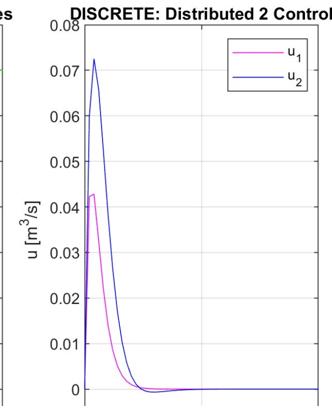
DISTRIBUTRED 2

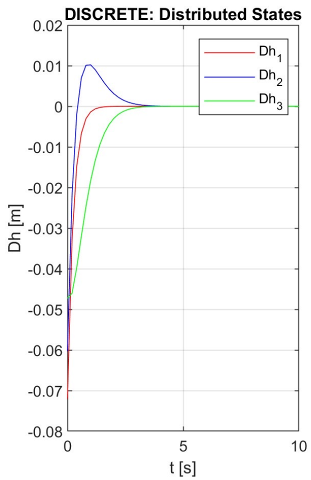


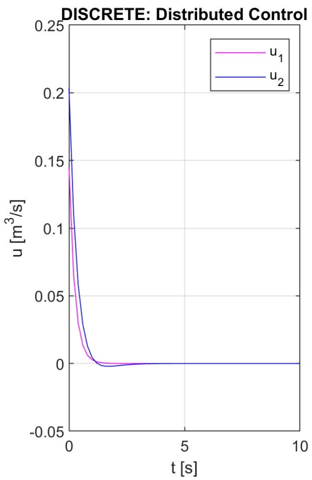


Rate Limit









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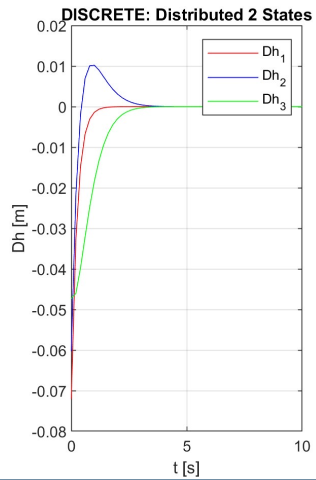
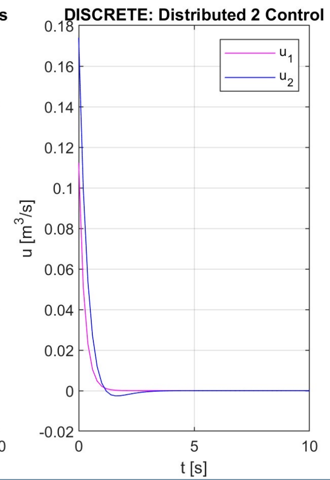


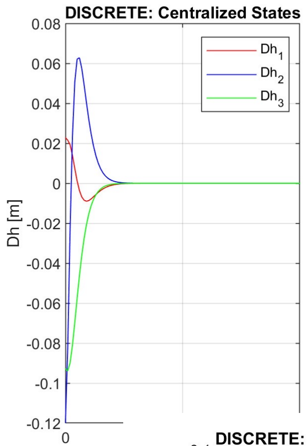


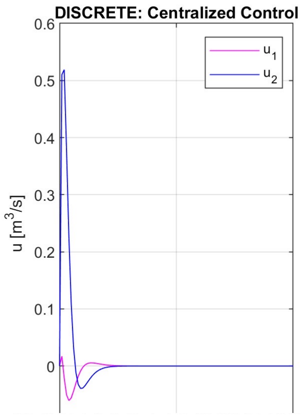
Simulation - Discrete time
Limit control action rate

α_{max} =−0.6 , ρ_{max} = 0.1, h = 0.1, x_0=[−0.023,0.12,−0.093]^T

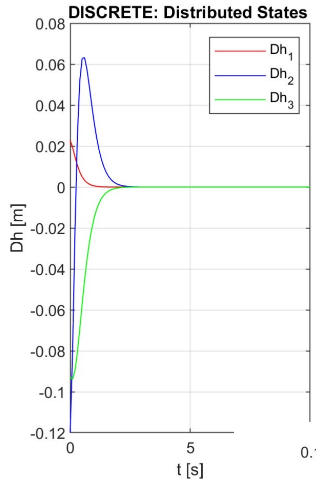
CENTRALIZED

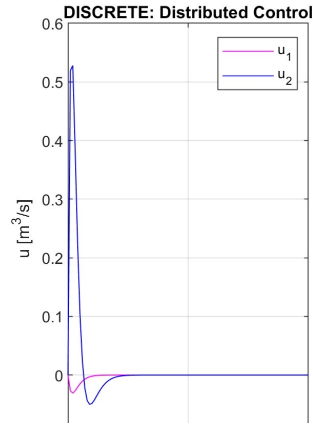
DISTRIBUTRED

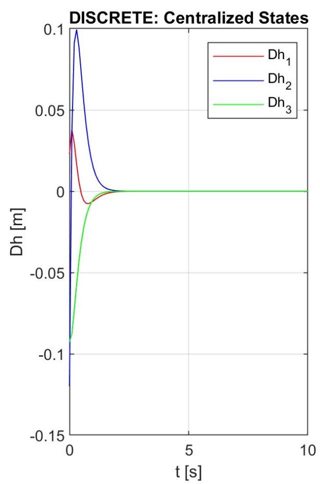


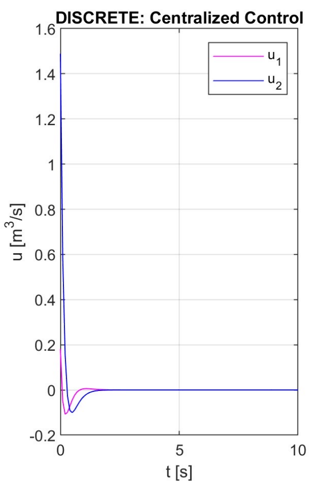


Rate Limit









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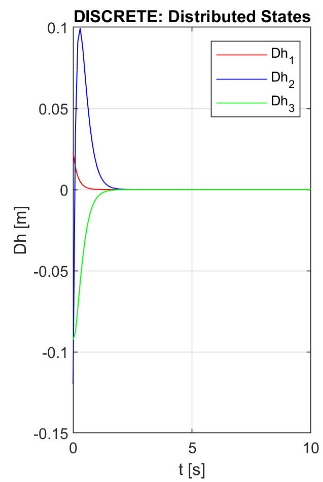

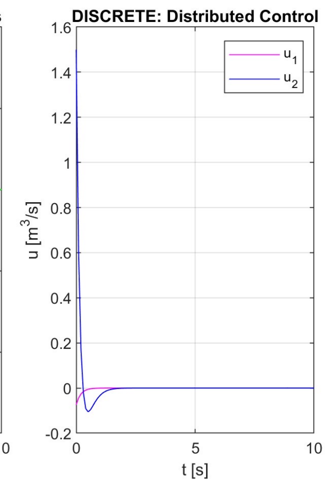

Comments - Discrete time
Limit control action rate

- With other simulations, improving the performance in terms of α_{max} we observe that we are able in most of the cases to reduce a lot the control variable peaks, making the requested control action feasible in case of actuator's limitations.
- There are some cases where this solution underperforms respect to the ones described before. Because of the control action rate limitations, it can become a bit slower
- This control strategy being less reactive can work even for h=0.1, in fact it guarantees the same performance of classical controllers, but with less effort. Anyway the control effort is still not negligible, this is not a good advantage.
- Again as in the previous discrete control design, there is not so much difference between the performance and effort of each control structure.
- Overall this strategy is very good if we have huge limitations on the actuators, in terms of max/min saturation and rate limitations, but it doesn't improve too much the performance that we could already obtain, except for some cases.

The results obtained in continuous time were more performing

(maybe with a better tuning something better can be obtained even in discrete)