Exam

5.2

$$A \rightarrow \left(\begin{array}{c|c} A_{11} & a_{12} \\ \hline a_{12}^T & a_{23} \end{array}\right) \quad U \rightarrow \left(\begin{array}{c|c} U_{11} & U_{12} \\ \hline 0 & U_{23} \end{array}\right) \quad D \rightarrow \left(\begin{array}{c|c} D_{11} & 0 \\ \hline 0 & \delta_1 \end{array}\right)$$

$$\left(\begin{array}{c|c} A_{11} & a_{12} \\ \hline a_{12}^T & a_{23} \end{array}\right) = \left(\begin{array}{c|c} U_{11} & U_{12} \\ \hline 0 & U_{23} \end{array}\right)\left(\begin{array}{c|c} D_{11} & 0 \\ \hline 0 & \delta_1 \end{array}\right)\left(\begin{array}{c|c} U_{11} & U_{12} \\ \hline 0 & U_{23} \end{array}\right)^T$$

$$= \left(\begin{array}{c|c} U_{11}D_{11} & U_{12}\delta_1 \\ \hline 0 & U_{23}\delta_1 \end{array}\right)\left(\begin{array}{c|c} U_{11}^T & 0 \\ \hline U_{12}^T & U_{23}^T \end{array}\right)$$

$$= \left(\begin{array}{c|c} U_{11}D_{11}U_{11}^T + U_{12}\delta_1 U_{12}^T & U_{01}\delta_1 \\ \hline * & U_{23}\delta_1 U_{23}^T \end{array}\right)$$

For $i=1$ to $n-1$:

- $\delta_i = A_{22}[i,i] - a_{12}^2 / D_{11}$
- $D_{22} = \delta_i$
- $U_{22} = 1$
- Compute $U_{22} = a_{12} / \sqrt{D_{11}\delta_1}$
  Update $D_{11} = D_{11} - \delta_1^2$
- Update $A_{22} = \delta_1$