Complexity of Neural Networks

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Preliminaries

Neuron's representation : $y = \sigma(\sum_{j=1}^{n} w_j x_j - w_0)$
Transfer functions :

- $sign(x) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$
- $\sigma(t) = (1 + e^{-t})^{-1}$
- $Pr(\sigma_T(t) = 1) = (1 + e^{-t/T})^{-1}$

Preliminaries

Neural nets can be

- cyclic or acyclic
- symmetric or asymmetric w.r.t. interconnections
- continuous or discrete w.r.t. updates
- synchronous or asynchronous w.r.t. updates in discrete neural nets

We're going to consider

- back-propagation nets
- Hopfield nets

Parameters

- size
- depth
- weight

Theorem

Any threshold function on n variables can be computed by a thresholdgate with integer weights w_i st :

$$|w_i| \leq \frac{(n+1)^{(n+1)/2}}{2^n}, \forall i = 0, ..., n.$$

Theorem

For infinitely many n, there are threshold functions on n variables whose computation by a single threshold gate requires weights as large as $\frac{n^{n/2}}{2^n}$.

Definition (Threshold circuit loading)

Given $\{(\vec{x_1}, b_1), ..., (\vec{x_m}, b_m)\}$, where each $\vec{x_i} \in \{0,1\}^n$ and each $b_i \in \{0,1\}$, and a directed acyclic graph, can we find a weight assignment st given f the function computed by the resulting threshold circuit : $f(\vec{x_i}) = b_i, \forall i = 1, ..., m$?

Definition (Threshold circuit loading)

Given $\{(\vec{x_1}, b_1), ..., (\vec{x_m}, b_m)\}$, where each $\vec{x_i} \in \{0,1\}^n$ and each $b_i \in \{0,1\}$, and $K \in \mathbb{N}$, does exists a threshold circuit of at most K neurones st given f the function computed by the resulting threshold circuit : $f(\vec{x_i}) = b_i, \forall i = 1, ..., m$?

Definition (Hopfield network)

Neural net in which

- all arcs are bidirectional
- each neuron is interconnected to all other neurons
- each neuron has a state
- each neuron's state is updated asynchronously

Theorem

In a symmetric Hopfield net with w_{ij} integer weights and n neurons, the convergence is obtain in

$$3\sum_{j<i}|w_{ij}| = O(n^2 \cdot \max_{i,j}|w_{ij}|)$$

asynchronous changes of neuron state.

Definition (Attraction domain)

Let $\vec{x_i}$ be a stable global state for a given Hopfield net. The *attraction domain* of $\vec{x_i}$ is the set of all vectors which are guaranteed to converge to $\vec{x_i}$.

Definition (Attraction radius)

The *attraction radius* of x_i is the largest Hamming distance from which all other vectors are guaranteed to converge to $\vec{x_i}$[2].

Definition (Hopfield Net Loading Problem)

Given a set of vectors $X = \{\vec{x_1}, .., \vec{x_m}\}$, where each $\vec{x_i} \in \{-1, 1\}^n$, and a constant ρ, is there a symmetric net of n neurons that has X as stable states with attraction radii $\geq \rho$?

- Decide if a given symmetric, simple network has more than one stable state.
- Determine if a symmetric simple network converges to a given stable state from any other initial state.

- Decide if a given asymmetric net under synchronous updates will converge from any (or all) initial state.
- Computeing the attraction radius of a given stable state in a symmetric simple net.

Thank you for your attention.