Symplectic 4-manifolds with $b_2^+ = 1$ versus complex surfaces with $p_g = 0$

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One of the fundamental problems in the study of 4-manifolds is to find a new family of simply connected smooth (symplectic, complex) 4-manifolds. Though many interesting 4-manifolds have been constructed using techniques such as fiber sum, rational blow-down, knot surgery, Luttinger surgery and so on, it is still very hard to find a new family of 4-manifolds with small Euler characteristic.

Since I discovered a new simply connected symplectic 4-manifold with $b_2^+ = 1$ and $c_1^2 = 2$ in 2004 using a rational blow-down surgery, many new simply connected 4-manifolds with small Euler characteristic have been constructed and now it is one of most active research areas in 4-manifolds theory to find a new family of 4-manifolds with $b_2^+ = 1$ (equivalently $p_g = 0$ in complex category).

The aim of this talk is to review recent development in the construction of 4-manifolds with small Euler characteristic. In particular, I'd like to survey the existence problems of simply connected symplectic 4-manifolds with $b_2^+ = 1$ and complex surfaces of general type with $p_g = 0$.