The MATH 1210, 1220 sequence is unacceptable as a prerequisite for mathematics courses numbered 2310 and above.

# Requirements for the Major

The discipline of mathematics is present in almost every facet of our lives and in many careers. It is basic to the fields of medical imaging, sports and business data analytics, digital communication, internet search, weather prediction, finance, and actuarial work, just to name a few. Studying mathematics helps develop logical thinking, which is an essential skill for many jobs including those not directly related to mathematics. In addition, the mathematics department hopes to give the mathematics major a feel for the elegance of the subject and the excitement in learning and proving beautiful theorems.

The mathematics major at U.Va. gives students a solid foundation in the core topics of modern mathematics, while at the same time offering a great deal of flexibility to tailor a program to their interests. To this end, the mathematics department offers five concentrations: The general concentration, the graduate preparatory concentration, the probability/statistics concentration, the financial mathematics concentration, and the five-year teacher education program. Completion of one of these concentrations is required. All concentrations share a core set of essential courses and a goal of developing students' ability to reason precisely and abstractly.

Normally, the calculus sequence MATH 1310, 1320, and 2310 or its equivalent must be completed before a student can declare a major in mathematics. At least a 2.200 average in the calculus sequence and a minimum grade of C in MATH 2310 or its equivalent are required. However, the department may grant special permission to declare a major to a student who has only completed MATH 1310 and 1320, and at least one mathematics course (other than MATH 2310 or its equivalent) which could be counted toward the major in mathematics, provided the student completes MATH 2310 or its equivalent in the semester following the declaration of a mathematics major. MATH 2315 is an honors version of MATH 2310, which can be followed up with MATH 3315 (see below).

Once the calculus sequence is completed, the student proceeds to the foundational material of modern mathematics. This consists of two courses in analysis (MATH 3250 and MATH 3310), and two courses in algebra (MATH 3351 and MATH 3354). More advanced versions of these courses may be substituted. MATH 3310 and MATH 3354 are proof based courses which often represent a new level of abstraction compared to what students were previously exposed to. Students who need a more detailed introduction to proof writing should take MATH 3000 (Transition to Higher Mathematics) before or concurrently with MATH 3310 and/or MATH 3354.

Afterward, the student proceeds to more advanced electives, depending on the choice of concentration. All in all, each concentration contains a set of nine required mathematics courses all at the 3000+ level (approximately 28 credits). To graduate, a student must obtain minimum grades of C in seven of these courses and C- in the other two. Up to two courses that are being counted for another College major can also be counted for the major in mathematics. Three courses may be allowed if the other major is interdisciplinary.

Computer usage is ubiquitous in all fields where mathematics is applied, and increasingly in pure mathematics as well. Thus, to graduate with a major in mathematics the student must show computer proficiency by completing CS 1110, CS 1111, CS 1112, CS 1113, CS 1120, or PHYS 2660, or an approved equivalent course with a grade of C- or higher. This should be done as early as possible.

For students at U.Va. from the start, up to two courses that are taken from outside the University and which are equivalent to College mathematics courses may be offered for the College mathematics major. For transfer students, the allowed number of transferred mathematics courses toward mathematics majors is decided case-by-case by the Director of Undergraduate Programs with advice from the transfer credit advisor.

Certain substitutions are allowed in all options, for example, MATH 4310 for MATH 3310, MATH 4651 for MATH 3351, MATH 4652 for MATH 3354, and PHYS 5630 or APMA 5070 for MATH 4300.

The Math major who has taken MATH 2315 and MATH 3315 and achieved a B- or better in both is not required to take MATH 3351 or MATH 3250 as is required in most concentrations. MATH 3315 then counts as an elective but the total number of required courses is the same as in the concentration for which they are registered (General, Financial Math, etc.). MATH 2315 is a substitute for MATH 2310 as a requirement for declaring a major. We encourage the student who completes MATH 2315 and 3315 to take more advanced courses in Linear Algebra and Differential Equations, in particular MATH 4651 instead of MATH 3351 and MATH 4250 instead of MATH 3250. At most three APMA courses can be offered for the math major. Exceptions may be granted by the Director of the Undergraduate Program.

#### A. The General Concentration

Students fulfilling the requirements for this option have a wide range of career opportunities, from law to business to any field that requires deductive, logical reasoning skills.

This traditional program for the mathematics major provides an overview of key areas:

MATH 3250 - Ordinary Differential Equations (4 Credits)

MATH 3310 - Basic Real Analysis (3 Credits)

MATH 3351 - Elementary Linear Algebra (3 Credits)

MATH 3354 - Survey of Algebra (3 Credits)

Five mathematics courses at the 3000 level or higher. Approved courses without a MATH prefix include those listed below in the Substitutions section or courses which are listed as requirements or electives for one of the other concentrations. However, the Economics and Commerce courses listed under the Financial Mathematics Concentration are not included as allowed electives in the General Concentration. At least two electives must be MATH courses.

## **B.** The Graduate Preparatory Concentration

This concentration is for the student who plans to attend graduate school in mathematics or an allied field. The program emphasizes the fundamental ideas of mathematics with substantial work in proving and understanding the basic theorems. It consists of:

MATH 3250 - Ordinary Differential Equations (4 Credits)

MATH 3340 - Complex Variables with Applications (3 Credits)

MATH 4310 - Introduction to Real Analysis (3 Credits)

MATH 4651 - Advanced Linear Algebra (3 Credits)

MATH 4652 - Introduction to Abstract Algebra (3 Credits)

Four mathematics courses at the 3000 level or higher. Courses without a MATH prefix, and not listed below as an approved substitution or elective, are generally not allowed. At least two electives must be MATH courses.

(Students may wish to take  $\underline{\text{MATH } 3310}$  in preparation for  $\underline{\text{MATH } 4310}$ ,  $\underline{\text{MATH } 3351}$  in preparation for  $\underline{\text{MATH } 4651}$ , and  $\underline{\text{MATH } 3354}$  in preparation for  $\underline{\text{MATH } 4652.}$ )

This constitutes the minimum expected of an incoming graduate student in most programs nationwide. The department strongly recommends MATH 4330 (Advanced Multivariate Calculus), as well as courses in differential geometry (MATH 4720) or topology (MATH 4770) or both. The department may recommend access to its 7000-level graduate courses for undergraduates with particularly strong capabilities.

## C. The Probability and Statistics Concentration

This concentration is designed to give the student a good theoretical underpinning in probability and statistics, as well as the opportunity to go deeper in these fields. The program can lead to a Master of Science in Statistics with one additional year of course work, if additional courses in statistics are taken in the fourth year. (Those interested in the M.S. in Statistics should contact the graduate advisor in the Department of Statistics prior to the beginning of their fourth year.) The requirements for the concentration are the following:

MATH 3100 - Introduction to Probability (3 Credits)
STAT 3120 - Introduction to Mathematical Statistics (3 Credits)

MATH 3250 - Ordinary Differential Equations (4 Credits)

MATH 3310 - Basic Real Analysis (3 Credits)

MATH 3351 - Elementary Linear Algebra (3 Credits)

MATH 3354 - Survey of Algebra (3 Credits)

MATH 4110 - Introduction to Stochastic Processes (3 Credits)

Two additional courses chosen from:

MATH 4310 - Introduction to Real Analysis (3 Credits)

STAT 3130 - Design and Analysis of Sample Surveys (3 Credits) or STAT 5180

STAT 5120 - Applied Linear Models (3 Credits)

STAT 5170 - Applied Time Series (3 Credits)

#### D. The Financial Mathematics Concentration

This program provides the student with a broad background of basic mathematics, which is essential for an understanding of the mathematical models used in the financial markets. The mathematics of modern finance includes probability, statistics, regression, time series, partial differential equations, stochastic processes, stochastic calculus, numerical methods, and analysis. The program consists of:

MATH 3100 - Introduction to Probability (3 Credits)

STAT 3120 - Introduction to Mathematical Statistics (3 Credits)

MATH 3250 - Ordinary Differential Equations (4 Credits)

MATH 3310 - Basic Real Analysis (3 Credits)

MATH 3351 - Elementary Linear Algebra (3 Credits)

MATH 3354 - Survey of Algebra (3 Credits)

MATH 4140 - Mathematics of Derivative Securities (3 Credits)

Two additional courses chosen from:

MATH 4220/APMA 3140 - Partial Differential Equations and Applied

Mathematics (3 Credits)/Applied Partial Differential Equations (3 Credits)

APMA 5070 - Numerical Methods (3 Credits)

MATH 4110 - Introduction to Stochastic Processes (3 Credits)

STAT 5120 - Applied Linear Models (3 Credits)

STAT 5170 - Applied Time Series (3 Credits)

SYS 3021 - Deterministic Decision Models (3 Credits) SYS 3060 - Stochastic Decision Models (3 Credits)

In addition to the nine required MATH courses, choose two from: (completing all four courses is recommended)

COMM 2010 - Introduction to Financial Accounting (3 Credits)

COMM 2020 - Introduction to Management Accounting (3 Credits)

ECON 2010 - Principles of Economics: Microeconomics (3 Credits)

ECON 2020 - Principles of Economics: Macroeconomics (3 Credits)

# E. The Five-year Teacher Education Program

This option leads to both Bachelor of Arts and Master of Teaching degrees after five years. The program is for both elementary and secondary teachers and is administered by the Curry School of Education.

The following are the required mathematics courses for this program (the Curry School has additional requirements):

MATH 3100 - Introduction to Probability (3 Credits)

STAT 3120 - Introduction to Mathematical Statistics (3 Credits)

MATH 3250 - Ordinary Differential Equations (4 Credits)

MATH 3310 - Basic Real Analysis (3 Credits)

MATH 3351 - Elementary Linear Algebra (3 Credits)

MATH 3354 - Survey of Algebra (3 Credits)

MATH 4040 - Discrete Mathematics (3 Credits)

MATH 5010 or MATH 5030 - The History of Mathematics (3 Credits)

MATH 5700 - Introduction to Geometry (3 Credits)

# The Distinguished Major Program in Mathematics

The Distinguished Major Program (DMP) is a special option within the math major that provides advanced training in mathematics by combining extensive course work (at the level of the Graduate Preparatory Track and beyond) with active involvement in various aspects of mathematical research. Successful completion of the DMP is required to receive high/highest honors. The centerpiece of the program that sets it apart from any track of the math major is the requirement/opportunity for a participating student to work on the Distinguished Major Thesis under the supervision of a faculty member (typically) in the 4th year of his/her undergraduate studies and then present the findings in a public defense of this work.

Students interested in the DMP should first declare a math major, choose a concentration, and have a plan to fulfill all the requirements of this concentration (see additional course requirements below). Students apply for admission to the DMP no later than in the spring semester of their third year, and should have completed at least two of the required courses below by the time of application. Criteria for acceptance into the program include letters of recommendation from mathematics instructors, the GPA in mathematics, and the cumulative College GPA (3.400 or higher). Because of the importance of the research component in the program, the individual programs of studies of the students interested in the DMP should include the completion of Math 4840 Introduction to Mathematical Research at an early stage - typically, by the time of application and certainly no later than the Fall semester of the fourth year.

A complete application will include a letter of application addressed to the DUP (Director of Undergraduate Programs), a copy of the transcript, and two letters of recommendation. One of these letters should be from the prospective thesis advisor confirming his or her readiness to supervise the project and outlining the general topic of the thesis. While the applicant could request one more letter of recommendation from a UVA math faculty member, another possibility might be, for example, the supervisor of an REU project (Research Experiences for Undergraduates) taken outside UVA. A letter from a Math 4840 instructor (if this course either has already been completed or is being taken by the student at the time of application) can also be helpful in the decision-making process (in addition to or as one of the two letters required for application).

The decision on admission to the DMP is made by the DUP in consultation with the prospective thesis advisor.

Students are expected to complete the following courses with a GPA of at least 3.4 and a minimum grade of B- in each course:

Math 3340 Complex Variables with Applications

Math 4310 Introduction to Real Analysis

Math 4651 Advanced Linear Algebra

Math 4652 Introduction to Abstract Algebra

Math 4770 General Topology

Math 4330 Advanced Multivariable Calculus, or Math 4720 Introduction to Differential Geometry

In addition, students must complete at least two math electives at the 4000 level and above. Furthermore, Math 4840 Introduction to Mathematical Research, and the two semester sequence, Distinguished Major Thesis I and II, Math 4900 and 4901 (see below) are required. Certain substitutions such as graduate level versions of the courses listed above are possible at the discretion of the DUP.

All these courses assume the ability to understand and write proofs. So students potentially interested in the DMP but having insufficient prior exposure to proof-based mathematical instruction should discuss their situation with the DUP in order to determine the best way of acquiring the necessary skills before taking the courses required for the DMP. (This can be accomplished, for example, by taking the

Advanced Calculus sequence Math 2315-3315 and/or some of the following courses: Math 3000, Math 3310 and Math 3354, but there are other possibilities).

Distinguished Major Thesis is an original essay containing an exposition of results in advanced mathematics. It is written by a student under the supervision of a faculty advisor who guides the student through all stages of the process, from formulating the topic and determining the scope of the project to putting the finishing touches on the final product and presenting it at the public defense. For bookkeeping purposes, all these activities will be framed as taking Math 4900 and Math 4901, Distinguished Major Thesis I and II, in the fall and spring of the 4th year; each semester will carry 3 credits. In preparation for the work on the thesis, students are expected to acquire some initial skills of mathematical research through taking Math 4840, which is the reason why students interested in the DMP should consider enrolling in this class early on.

The work on the thesis is a multi-stage process, which should begin no later than the end of the third year, soon after the application for the DMP has been approved. At the initial stage the faculty advisor discusses with the student the general topic of the project, determines its parameters and recommends the materials for the student to work with over the summer to get introduced to the chosen area. The precise topic of the thesis can be formulated in the beginning of the fourth year based on the student's report on the work done in the summer. Depending on the availability of funds, the department will try to help DMP students stay in residence at UVA for several weeks during the summer to facilitate an early start on the work on the thesis through frequent consultations with the advisor. As the project takes shape, the department may also help the DMP students to travel to suitable venues to present the results of their work if recommended by the faculty advisor.

The almost year-long process of preparation of the thesis culminates in a public defense of the work. The defense includes a presentation of the main findings in front of an audience consisting of undergraduate and graduate students, faculty and guests, open discussion of the results in a Q&A format, and a closed to the public examination with the defense committee (thesis advisor and two more faculty members). In the case of a successful defense, the student is awarded a special departmental certificate. This grade (in conjunction with the GPA in the required math classes) will be a major factor in nominating the student for high/highest honors.

While the Distinguished Major Thesis is a significant investment of time and effort, it has several important benefits for a student in addition to qualifying him/her for the high/highest honors. First and foremost, it creates a unique opportunity for a student to work one-on-one with a faculty advisor for a period of about one calendar year on a topic in advanced mathematics of mutual interest. This work will help to develop the student's analytical, research and expository skills, and can be expected to boost his/her application for graduate admission as well as for jobs in industry. It can also be a basis for the student's presentations at various venues and can sometimes lead to publications.

### **SEAS Students**

Students in SEAS who wish to earn a bachelor's degree in mathematics must complete:

- 1. All courses required for a major in mathematics, in a chosen concentration, as listed in the undergraduate record and including minimum grade requirements.
- At least 7 MATH courses (6 courses for Systems Engineering students) numbered 3000 or above, or approved electives from other departments, that are NOT listed as required courses by their SEAS specialization.

### **Echols Mathematics Club**

**Echols Mathematics Club** is an undergraduate club for mathematics students that sponsors lectures, mathematics films, problem-solving sessions for the Putnam Mathematical Competition and other similar activities.

## **Additional Information**

You can also find more information on the department web site: <a href="http://www.math.virginia.edu/">http://www.math.virginia.edu/</a>.

### **Course Information**

## **Elementary Courses in Mathematics**

The entering College student has a variety of courses in mathematics from which to choose. Among those that may be counted toward the College area requirement in natural science and mathematics, are several options in calculus, elementary (non-calculus based) courses in probability and in statistics, and courses dealing with computer techniques in mathematics. Pre-commerce students are required to take a statistics course and one other mathematics course, usually MATH 1110, 1210, 1220, or 1310.

Students planning to major in the social sciences, arts, or humanities who wish to take a mathematics course but omit the study of calculus may choose from <u>MATH 1110</u> (Elementary Probability Theory) and <u>MATH 1140</u> (Financial Mathematics). Even though it is not a prerequisite, <u>MATH 1110</u> is frequently taken prior to Introductory Statistics. <u>MATH 1150</u> and <u>1160</u> are introductory courses that investigate familiar areas of elementary mathematics at a deeper level and are intended for first- and second-year nonmajors, especially those preparing to teach in elementary and middle schools.

In <u>MATH 1140</u> the students learn the mathematics needed to understand and answer a variety of questions that arise in everyday financial dealings. The emphasis in this course will be on applications, including simple and compound interest, valuation of bonds, rates of return on investments, and more. Although the topics in this course are drawn primarily from business and economics, students of all majors are welcome and should find the applications interesting and relevant.

## Calculus Sequence

The study of calculus is the foundation of college mathematics for students planning to major in mathematics or the physical sciences or anticipating a career or graduate study in any of the natural sciences, engineering, or applied social sciences (such as economics). There are three programs of study available in calculus:

MATH 1210, 1220 is a terminal one-year sequence intended for business, biology, and social science majors;

MATH 1310, 1320, 2310 is the traditional calculus sequence intended for students of mathematics and the natural sciences, as well as for students intending to pursue graduate work in the applied social sciences;

 ${\underline{\sf MATH~2315}}$  is the honors calculus program for advanced students, and it is usually offered in the Fall semester

The MATH 1210, 1220 sequence is unacceptable as a prerequisite for mathematics courses numbered 2310 and above. Students anticipating the need for higher mathematics courses such as MATH 3250 (Differential Equations), MATH 3100 (Probability) or STAT 3120 (Statistics) should instead elect the MATH 1310, 1320, 2310 sequence. Credit is not allowed for both MATH 1210 and 1310 (or its equivalent). MATH 2310 is the prerequisite for many advanced mathematics courses.

Students who need a remedial review of algebra and trigonometry may elect <u>MATH 1190</u> Applied Calculus I with Algebra which is a 4-credit hour course and includes a review of algebra and trigonometry. Credit is not allowed for both <u>MATH 1190</u> and <u>1210</u> (or its equivalent).

#### **Advanced Placement**

Students who have previously passed a calculus course in high school may elect MATH 1220, 1310, 1320, or 2310 as their first course, depending on placement, preparation, and interest. A strong high school calculus course is generally adequate preparation for MATH 1320 as a first calculus course, even if advanced placement credit has not been awarded for MATH 1310. Students planning to take any advanced course in mathematics should not take MATH 1220, because credit for that course must be forfeited if the student takes MATH 1320 (or its equivalent). Well-prepared students (who place out of both MATH 1310 and 1320) may choose either MATH 2310 or 3250 (Differential Equations) as their first course. First and second year students have the option of taking MATH 3000 Transition to Higher Mathematics, which is offered in the Spring semester. MATH 3000 is designed for students who wish some preparation before taking MATH 3310 Basic Real Analysis and/or MATH 3354 Survey of Algebra. Students with a grade of B or better in MATH 3310, 3354, or any 5000-level Math course are not eligible to enroll in MATH 3000.

Advanced placement credit in the calculus sequence is granted on the basis of the College Entrance Examination Board Advanced Placement Test (either AB or BC). A score of 4 or 5 on the AB test or on the AB subscore of the BC test gives the student credit for MATH 1310 . A score of 4 or 5 on the BC test gives the student credit for both MATH 1310 and 1320.

#### **Substitutions**

There are numerous instances of equivalent courses offered by the Department of Mathematics as well as by the Department of Applied Mathematics in the School of Engineering and Applied Science. A student may not offer for degree credit two equivalent courses (e.g., MATH 1310 and APMA 1090, or MATH 1210 and MATH 1310). The following are equivalent courses from the School of Engineering and Applied Sciences:

APMA 1090 and MATH 1310 - Calculus I (4 Credits)

APMA 1110 and MATH 1320 - Calculus II (4 Credits)

APMA 2120 and MATH 2310 - Calculus III (4 Credits)

APMA 2130 and MATH 3250 - Ordinary Differential Equations (4 Credits)

APMA 3080 and MATH 3351 - Elementary Linear Algebra (3 Credits)

APMA 3100 and MATH 3100 - Introduction to Probability (3 Credits)

APMA 3120 and STAT 3120 - Introduction to Mathematical Statistics (3 Credits)

APMA 3140 and MATH 4220 - Partial Differential Equations and Applied Mathematics (3 Credits)

APMA 3340 and MATH 3340 - Complex Variables with Applications (3 Credits)

APMA 5070 and MATH 4300 - Elementary Numerical Analysis (3 Credits)

As noted above, certain substitutions are allowed for all options:

MATH 2315 - Advanced Calculus and Linear Algebra I (4 Credits) for MATH 2310 - Calculus III (4 Credits)

MATH 4310 - Introduction to Real Analysis (3 Credits) for MATH 3310 - Basic Real Analysis (3 Credits)

MATH 4651 - Advanced Linear Algebra (3 Credits) for MATH 3351 - Elementary Linear Algebra (3 Credits)

MATH 4652 - Introduction to Abstract Algebra (3 Credits) for MATH 3354 - Survey of Algebra (3 Credits)

PHYS 5630 - Computational Physics I (3 Credits) or APMA 5070 - Numerical Methods (3 Credits) for MATH 4300 - Elementary Numerical Analysis (3 Credits)

#### **Standard Allowed Electives**

CS 3102 - Theory of Computation (3 Credits)

CS 4102 - Algorithms (3 Credits)

ECON 4010 - Game Theory (3 Credits)

- PHIL 5420 Advanced Logic (3 Credits)
- PHIL 5470 Philosophy of Mathematics (3 Credits)
- STAT 3120 Introduction to Mathematical Statistics (3 Credits)
- STAT 5265 Investment Science I (3 Credits)
- SYS 3060 Stochastic Decision Models (3 Credits)
- SYS 3021 Deterministic Decision Models (3 Credits)

# **Course Descriptions**

- MATH 1140 Financial Mathematics
- MATH 1150 The Shape of Space
- MATH 1160 Algebra, Number Systems, and Number Theory
- MATH 1190 Applied Calculus I with Algebra
- MATH 1210 Applied Calculus I
- MATH 1220 Applied Calculus II
- MATH 1310 Calculus I
- MATH 1320 Calculus II
- MATH 1330 Calculus Workshop I
- MATH 1340 Calculus Workshop II
- MATH 2310 Calculus III
- MATH 2315 Advanced Calculus and Linear Algebra I
- MATH 2700 Euclidean and Noneuclidean Geometry
- MATH 3000 Transition to Higher Mathematics
- MATH 3100 Introduction to Probability
- MATH 3250 Ordinary Differential Equations
- MATH 3255 Ordinary Differential Equations
- MATH 3310 Basic Real Analysis
- MATH 3315 Advanced Calculus and Linear Algebra II
- MATH 3340 Complex Variables with Applications
- MATH 3350 Applied Linear Algebra
- MATH 3351 Elementary Linear Algebra
- MATH 3354 Survey of Algebra
- MATH 4040 Discrete Mathematics
- MATH 4080 Operations Research
- MATH 4110 Introduction to Stochastic Processes
- · MATH 4140 Mathematics of Derivative Securities
- MATH 4210 Advanced Calculus with Applied Mathematics
- MATH 4220 Partial Differential Equations and Applied Mathematics
- · MATH 4250 Differential Equations and Dynamical Systems
- MATH 4300 Elementary Numerical Analysis
- MATH 4310 Introduction to Real Analysis
- MATH 4452 Algebraic Coding Theory
- MATH 4595 Undergraduate Research Seminar
- MATH 4651 Advanced Linear Algebra
- MATH 4652 Introduction to Abstract Algebra
- MATH 4657 Bilinear Forms and Group Representations
- MATH 4658 Galois Theory
- MATH 4660 Algebraic Combinatorics
- MATH 4720 Introduction to Differential Geometry
- MATH 4750 Introduction to Knot Theory
- MATH 4770 General Topology
- MATH 4830 Seminar
- · MATH 4840 Introduction to Mathematical Research
- MATH 4993 Independent Study