Roots of random polynomials with arbitrary coefficients

We consider the following model of random polynomials

$$P_n(x) = c_n \xi_n x^n + \cdots + c_1 \xi_1 x + c_0 \xi_0 x^0,$$

where the ξ_i are independent random variables with bounded $2 + \epsilon$ moments, and the c_i are deterministic coefficients. One of the simplest examples is the Kac polynomial in which $c_i = 1$ for all i. Classical theorems by Kac (1943), Wilkins (1988), and Edelman-Kostlan (1996) show that for Kac polynomial with ξ_i being iid standard Gaussian, the expectation of number of real roots of P_n is $\frac{2}{\pi} \log n + O(1)$. It took considerable effort to prove the same asymptotics for the Kac polynomial with more general distributions of ξ_i; for example, the works by Erdős-Offord (1956), Edelman-Kostan (1995), Nguyen-N-Vu (2015), Do-Nguyen-Vu (2015). What about other non-Kac polynomials?

In this talk, we discuss optimal local universality for roots of P_n when the c_i have polynomial growth and as an application, we derive sharp estimates for the number of real roots of this polynomial. Our results also hold for series; in particular, we prove local universality for random hyperbolic series. This is joint work with Yen Do and Van Vu.