Math 19 -
Homework #11
11/5/2019

Colley 3.1.3, 3.1.10, 3.1.17, 3.2.10,
3.3.9, 3.3.18, 3.4.4, 3.4.12(b)(c), 3.4.13, 3.4.16

Colley 3.1.3 Sketch the images of the following path, using arrows to indicate the direction in which the parameter increases:

$$\begin{cases} x = t\cos t \\ y = t\sin t \end{cases} \quad , -6\pi \leq t \leq 6\pi$$

Colley 3.1.10 Calculate the velocity, speed, and acceleration of the path:

$$\mathbf{x}(t) = \left(e^t, e^{2t}, 2e^t\right)$$

Colley 3.1.17 Find an equation for the line tangent to the given path at the indicated value for the parameter:

$$\mathbf{x}(t) = \left(t^2, t^3, t^5\right), \quad t = 2$$

Colley 3.2.10 If f is a continuously differentiable function, show how Definition 2.1 may be used to established the formula

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Definition 2.1: The **length** $L(\mathbf{x})$ of a C^1 path $\mathbf{x} : [a, b] \to \mathbf{R}^n$ is found by integrating its speed:

$$L(\mathbf{x}) = \int_a^b ||\mathbf{x}'(t)||\, dt$$

Colley 3.3.9 Sketch the given vector field on $\mathbf{R}^3$:

$$\mathbf{F} = (0, z, -y)$$

In addition to the sketch, provide a description of the vector field.

Colley 3.3.18 Verify that the path given is a flow line of the indicated vector field. Justify the result geometrically with an appropriate sketch.

$$\mathbf{x}(t) = (\sin t, \cos t, 2t), \quad \mathbf{F} = (y, -x, 2)$$

Colley 3.4.4 Calculate the divergence of the vector field given:

$$\mathbf{F} = z\cos\left(e^{y^2}\right)\mathbf{i} + x\sqrt{z^2+1}\mathbf{j} + e^{2y}\sin 3x\mathbf{k}$$

Colley 3.4.12(b)(c)

(b) Use geometry to determine $\nabla \times \mathbf{F}$, where $\mathbf{F} = \frac{(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})}{\sqrt{x^2+y^2+z^2}}$.

(c) For $\mathbf{F}$ as in part (b), verify your intuition by explicitly computing $\nabla \times \mathbf{F}$.

Colley 3.4.13 Can you tell in what portions of $\mathbf{R}^2$, the vector fields shown in Figures 3.43-3.46 have positive divergence? Negative divergence?

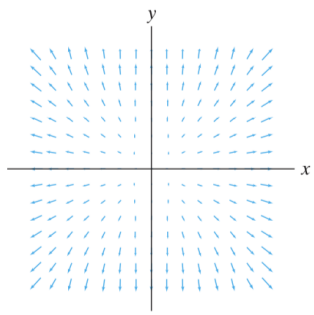

Figure 3.43 Vector field for Exercise 13(a).

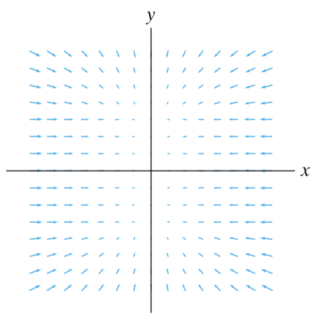

Figure 3.44 Vector field for Exercise 13(b).

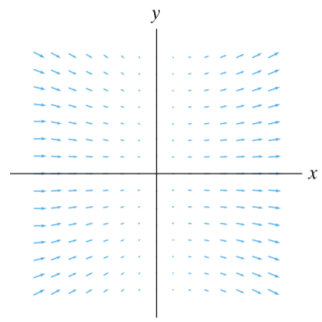

Figure 3.45 Vector field for Exercise 13(c).

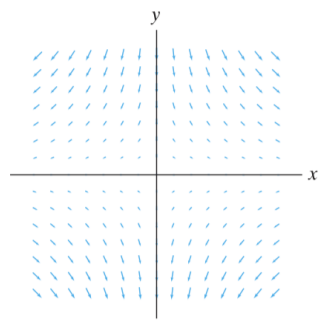

Figure 3.46 Vector field for Exercise 13(d).

Colley 3.4.16 Prove Theorem 4.4.

Theorem 4.4: Let $F : X \subseteq \mathbf{R}^3 \to \mathbf{R}^3$ be a vector field of class C^2. Then $\operatorname{div}(\operatorname{curl} \mathbf{F}) = 0$. That is, curl $\mathbf{F}$ is an incompressible vector field.