Math 19 -
Homework #13
12/6/2019

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1 Consider the integral

$$\int_0^2 \int_{x^2}^{2x} (4x+2)\, dy\, dx.$$

(a) Sketch the region R in the xy plane over which the integration takes place.

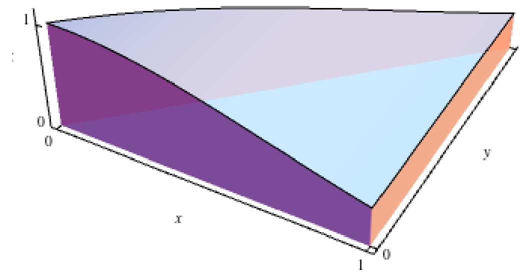
(b) Write an equivalent integral with the order of integration reversed.

(c) Evaluate the integral using either one of the integrals.

2 Compute

$$\iint_R e^{-x^2}\,dx\,dy$$

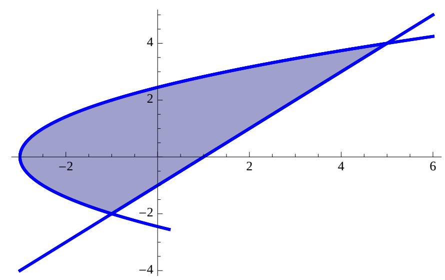
where R is the region bound by the x-axis, the line $y = x$, and the line $x = 1$.



3 Consider the integral

$$\iint_R xy\, dA$$

where R is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.



1. Setup, but do not evaluate, the integral using $dA = dy\, dx$.

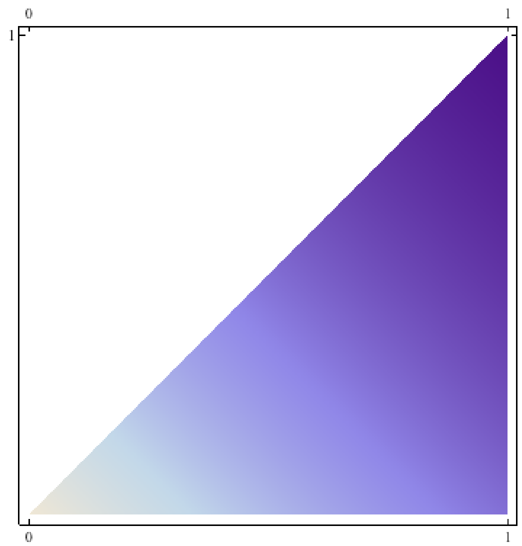
2. Setup, but do not evaluate, the integral using $dA = dx\, dy$.

4 The center of mass for a planar region R with density $\rho(x,y)$ is $(\bar{x},\bar{y}) = \left(\dfrac{M_y}{M}, \dfrac{M_x}{M}\right)$ where

$$M = \iint_R \rho(x,y)\, dA,\ M_y = \iint_R x\,\rho(x,y)\, dA,\ \text{ and }\ M_x = \iint_R y\,\rho(x,y)\, dA$$

(M is the mass, M_y and M_x are the moments with respect to the y- and x-axis, respectively). Determine the center of mass for a triangular plate defined by the region between $y = 0, x = 1$, and $y = x$ with density $\rho(x,y) = 2e^{x+y}$. The figure shows a *density plot* of the region, where higher densities are indicated by darker shading.

Note: You will need to use *integration by parts* in this exercise. For a review and discussion of this technique see Problem Set 01 in the HMC Math Resources 2019 Sakai folder.



5 Let W be a right circular cone with base radius a and height h.

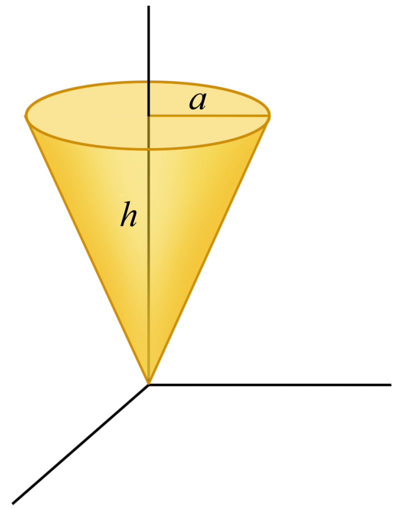
Determine its volume

$$V = \iiint_W 1 \, dV$$

by using cylindrical coordinates with

$$dV = r \, dr \, d\theta \, dz.$$

Assume the tip of the cone is at the origin.



Colley 5.5.25

Evaluate

$$\iint_D \cos\left(x^2 + y^2\right)\, dA,$$

where D is the shaded region in Figure 5.106.

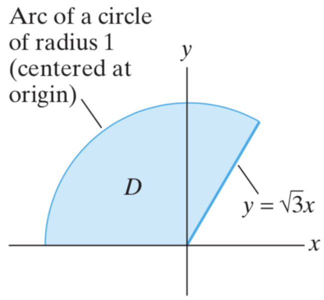


Figure 5.106 The region D of Exercise 25.

Colley 5.5.31 Determine

$$\iiint_W \left(x^2 + y^2 + 2z^2\right)\, dV,$$

where W is the solid cylinder defined by the inequalities $x^2 + y^2 \leq 4$, $-1 \leq z \leq 2$.

Colley 5.5.34 Determine the value of the given integral, where W is the region bounded by the two spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, for $0 < a < b$.

$$\iiint_W \frac{dV}{\sqrt{x^2 + y^2 + z^2}}$$