

Colley 2.2.14, 2.3.22, 2.3.33, 2.3.38, 2.3.42, 2.3.44, 2.4.29(a),(c)

**Colley 2.2.14** Evaluate the limit or explain why the limit fails to exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$$

**Colley 2.3.22** Find the gradient  $\nabla f(\mathbf{a})$ :

$$f(x, y) = e^{xy} + \ln(x - y), \mathbf{a} = (2, 1)$$

**Colley 2.3.33** Find the matrix  $Df(\mathbf{a})$  of partial derivatives:

$$\mathbf{f}(s, t) = (s^2, st, t^2), \mathbf{a} = (-1, 1)$$

**Colley 2.3.38** Find an equation for the plane tangent to the graph of  $z = 4 \cos xy$  at the point  $(\pi/3, 1, 2)$ .

**Colley 2.3.42** Suppose that you have the following information concerning a differentiable function  $f$ :

$$f(2, 3) = 12, f(1.98, 3) = 12.1, f(s, 3.01) = 12.2$$

- (a) Give an approximate equation for the plane tangent to the graph of  $f$  at  $(2, 3, 12)$ .
- (b) Use the result of part (a) to estimate  $f(1.98, 2.98)$ .

**Colley 2.3.44**

$$f(x, y) = 3 + \cos \pi xy, f(0.98, 0.51)$$

- (a) Use the linear approximation  $h(\mathbf{x}) = \mathbf{f}(\mathbf{a}) + D\mathbf{f}(\mathbf{a})(\mathbf{x} - \mathbf{a})$  to approximate the indicated value of the given function  $f$ .
- (b) How accurate is the approximation determined in part (a)?

**Colley 2.4.29(a),(c)** The three-dimensional **heat equation** is the partial differential equation

$$k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \frac{\partial T}{\partial t},$$

where  $k$  is a positive constant. It models the temperature  $T(x, y, z, t)$  at the point  $(x, y, z)$  and time  $t$  of a body in space.

- (a) We examine a simplified version of the heat equation. Consider a straight wire “coordinatized” by  $x$ . Then the temperature  $T(x, t)$  at time  $t$  and position  $x$  along the wire is modeled by the one-dimensional heat equation

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}.$$

Show that the function  $T(x, t) = e^{-kt} \cos x$  satisfies this equation. Note that if  $t$  is held constant at value  $t_0$ , then  $T(x, t_0)$  shows how the temperature varies along the wire at time  $t_0$ . Graph the curves  $z = T(x, t_0)$  for  $t_0 = 0, 1, 10$ , and use them to understand the graph of the surface  $z = T(x, t)$  for  $t \geq 0$ . Explain what happens to the temperature of the wire after a long period of time.

- (c) Now show that  $T(x, y, z, t) = e^{-kt}(\cos x + \cos y + \cos z)$  satisfies the three-dimensional heat equation.