Math 19 - 07
Homework #9
11/1/2019

Colley 2.2.14, 2.3.22, 2.3.33, 2.3.38, 2.3.42, 2.3.44, 2.4.29(a),(c)

Colley 2.2.14 Evaluate the limit or explain why the limit fails to exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

Colley 2.3.22 Find the gradient $\nabla f(\mathbf{a})$:

$$f(x,y) = e^{xy} + \ln(x-y), \mathbf{a} = (2,1)$$

Colley 2.3.33 Find the matrix $Df(\mathbf{a})$ of partial derivatives:

$$\mathbf{f}(s,t) = (s^2, st, t^2), \mathbf{a} = (-1, 1)$$

Colley 2.3.38 Find an equation for the plane tangent to the graph of $z = 4\cos xy$ at the point $(\pi/3, 1, 2)$.

Colley 2.3.42 Suppose that you have the following information concerning a differentiable function f:

$$f(2,3) = 12, f(1.98,3) = 12.1, f(2,3.01) = 12.2$$

(a) Give an approximate equation for the plane tangent to the graph of f at $(2,3,12)$.

(b) Use the result of part (a) to estimate $f(1.98, 2.98)$.

Colley 2.3.44

$$f(x,y) = 3 + \cos \pi xy, f(0.98, 0.51)$$

(a) Use the linear approximation $h(\mathbf{x}) = \mathbf{f}(\mathbf{a}) + D\mathbf{f}(\mathbf{a})(\mathbf{x} - \mathbf{a})$ to approximate the indicated value of the given function f.

(b) How accurate is the approximation determined in part (a)?

Colley 2.4.29(a),(c) The three-dimensional **heat equation** is the partial differential equation

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\frac{\partial T}{\partial t},$$

where k is a positive constant. It models the temperature $T(x,y,z,t)$ at the point (x,y,z) and time t of a body in space.

(a) We examine a simplified version of the heat equation. Consider a straight wire "coordinatized" by x. Then the temperature $T(x,t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial t}.$$

Show that the function $T(x,t)=e^{-kt}\cos x$ satisfies this equation. Note that if t is held constant at value t_0, then $T(x,t_0)$ shows how the temperature varies along the wire at time t_0. Graph the curves $z=T(x,t_0)$ for t_0 = 0, 1, 10, and use them to understand the graph of the surface $z=T(x,t)$ for $t\geq 0$. Explain what happens to the temperature of the wire after a long period of time.

(c) Now show that $T(x,y,z,t) = e^{-kt}(\cos x+\cos y+\cos z)$ satisfies the three-dimensional heat equation.