Homework #5

1 Let $f(x) = \alpha x^2 + \beta x + \gamma$ be an arbitrary quadratic function. Show that the number c that satisfies the conclusion of the Mean Value Theorem on the interval [a, b] is given by the midpoint $c = \frac{a+b}{c}$.

(Curiously, the value of c is independent of the quadratic's coefficients α , β , and γ !)

2 Set $f(x) = x^{-1}$, a = -1, b = 1. Verify that there is no number c for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Explain how this does not violate the Mean Value Theorem.

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3 Find, among all right circular cylinders of fixed volume V, the one with smallest surface area (counting the areas of the faces at the top and bottom).

4 Show that the sum of a positive number and its reciprocal is at least 2.

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5 The area between two varying concentric circles is at all times $9\pi^2$. The rate of change of the area of the larger circle is $10\pi^2 2$ in fact. How fast is the circumference of the smaller circle changing when it has area $16\pi^2$ in fact.

6 Suppose that f and g and are unknown differentiable functions that satisfy the relationships

$$2f'(x)c = g(x)$$
 and $g'(x) = -f(x)$ for all x (1)

- (a) Show that $f^2(x) + g^2(x) = C$ for some constant C.
- (b) Determine the value of C if we also know f(0) = 0 and g(0) = 1.
- (c) Differentiate the equations for f' and g' in (1) to show that f and g must also solve the differential equations

$$f''(x) + f(x) = 0$$
 and $g''(x) + g(x) = 0$ for all x.

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