Math 19 - 07
Homework #7
10/18/2019

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1

(a) Compute the $(2n+1)$-order Taylor polynomial for $\sinh x$ using basepoint $x_0 = 0$.

(b) Compute the $2n$-order Taylor polynomial for $\cosh x$ using basepoint $x_0 = 0$.

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2 The integral

$$\int_a^b e^{-x^2}\,dx \tag{1}$$

is not directly computable since there is no known "reasonable" function $F(x)$ with $F'(x) = e^{-x^2}$. This is an issue since integrals involving the Gaussian curve e^{-x^2} are ubiquitous in statistics and science. In such cases, we can use Taylor polynomials to approximate the integrand and obtain estimates for the integral. The general idea is

$$f(x) \approx P(x) \implies \int_a^b f(x)\,dx \approx \int_a^b P(x)\,dx.$$

(a) Calculate the second-order Taylor polynomial for e^{-x^2} for $x_0 = 0$.

(b) Use your polynomial in (a) to approximate the value of $\displaystyle\int_0^{0.25} e^{-x^2}\,dx$.

(c) How does your estimate compare with a calculator estimate?

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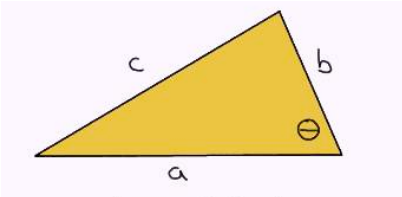
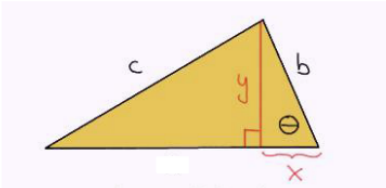
3

Consider a general triangle with side lengths a, b, and c, as in the figure. In this exercise we will prove the *Law of Cosines*:

$$c^2 = a^2 + b^2 - 2ab\cos\theta \qquad (2)$$

(when $\theta = \frac{\pi}{2}$ this agrees with the Pythagorean Theorem). Begin your proof by applying the Pythagorean theorem to the right triangle with hypotenuse c. Note that for the right triangle with hypotenuse b we know $x = b\cos\theta$ and $y = b\sin\theta$.





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4

(a) Read Colley Section 1.3. Explain the meaning of $\text{proj}_{\mathbf{a}}\mathbf{b}$ to any friend (in Math 19 or not). Write down the name of this friend for your solution to Part (a).

(b) Determine $\text{proj}_{\mathbf{a}}\mathbf{b}$ for $\mathbf{a} = (0, 1, 0)$ and $\mathbf{b} = (2, 3, 4)$.

(c) Provide a solution for Exercise 1.3.24 in Colley (Suppose that a force $\mathbf{F} = \mathbf{i} - 2\mathbf{j}\ldots$).

Colley 1.3.24: Suppose that a force $\mathbf{F} = \mathbf{i} - 2\mathbf{j}$ is acting on an object moving parallel to the vector $\mathbf{a} = 4\mathbf{i} + \mathbf{j}$. Decompose $\mathbf{F}$ into a sum of vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1$ points along the direction of motion and $\mathbf{F}_2$ is perpendicular to the direction of motion. (Hint: A diagram may help.)