Math 19 -
Homework #14
12/9/2019

Colley 6.2.4, 6.2.13, 6.3.1, 6.3.25, 7.2.1, 7.2.14, 7.3.4
T/F 6, 10, 8

Colley 6.2.4 Verify Green's theorem for the given vector field

$$\mathbf{F} = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$$

and region D by calculating both

$$\oint_{\partial D} M\,dx + N\,dy \text{ and } \iint_D (N_x - M_y)dA$$

where $\mathbf{F} = 2y\ \mathbf{i} + x\ \mathbf{j}$ and D is the semicircular region $x^2 + y^2 \leq a^2$, $y \geq 0$.

Colley 6.2.13 Evaluate $\oint_C \left(x^4y^5 - 2y\right) dx + \left(3x + x^5y^4\right) dy$, where C is the oriented curve pictured in Figure 6.29.

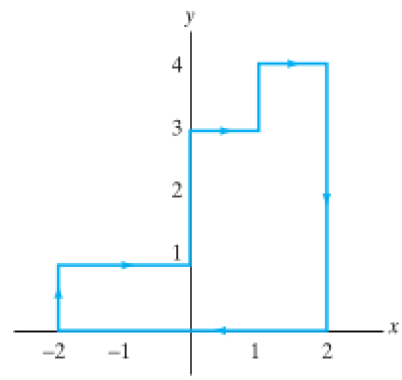

Figure 6.29 The oriented curve C of Exercise 13.

Colley 6.3.1 Consider the line integral $\int_C z^2\, dx + 2y\, dy + xz\, dz$.

(a) Evaluate this integral, where C is the line segment from $(0,0,0)$ to $(1,1,1)$.

(b) Evaluate this integral, where C is the path from $(0,0,0)$ to $(1,1,1)$ parameterized by $\mathbf{x}(t) = \left(t, t^2, t^3\right), 0 \leq t \leq 1$.

(c) Is the vector field $\mathbf{F} = z^2\, \mathbf{i} + 2y\, \mathbf{j} + xz\, \mathbf{k}$ conservative? Why or why not?

Colley 6.3.25 Let $\mathbf{F} = x^2\,\mathbf{i} + \cos y \sin z\,\mathbf{j} + \sin y \cos z\,\mathbf{k}$.

(a) Show that $\mathbf{F}$ is conservative and find a scalar potential function f for $\mathbf{F}$.

(b) Evaluate $\int_x \mathbf{F} \cdot d\mathbf{s}$ along the path $\mathbf{x} : [0,1] \to \mathbf{R}^3,\ \mathbf{x}(t) = \left(t^2+1, e^t, e^{2t}\right)$

Colley 7.2.1 Let $\mathbf{X}(s,t) = (s, s+t, t), \quad 0 \le s \le 1, \quad 0 \le t \le 2$. Find

$$\iint_{\mathbf{X}} \left(x^2 + y^2 + z^2\right) dS.$$

Colley 7.2.14 Let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 4$, and lateral surface given by the equation $x^2 + y^2 = 9$. Orient S with outward normals. Determine the given vector surface integral

$$\iint_S (x\,\mathbf{i} + y\,\mathbf{j}) \cdot d\mathbf{S}$$

Colley 7.3.4 Verify Stoke's theorem for the given surface and vector field:
S is defined by $x^2 + y^2 + z^2 = 4$, $z \leq 0$, oriented by downward normal;

$$\mathbf{F} = (2y - z)\mathbf{i} + (x + y^2 - z)\mathbf{j} + (4y - 3x)\mathbf{k}.$$

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6. If S is the unit sphere centered at the origin, then $\iint_S x^3 \, dS = 0$.

10. $\iint_S (-y\mathbf{i} + x\mathbf{j}) \cdot d\mathbf{S} = 0$, where S is the cylinder $x^2 + y^2 = 9$, $0 \le z \le 5$.

18. $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ has the same value for all piecewise smooth, oriented surfaces S that have the same boundary curve C.