Math 19 - 07
Homework #5
10/4/2019

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1 Let $f(x) = \alpha x^2 + \beta x + \gamma$ be an arbitrary quadratic function. Show that the number c that satisfies the conclusion of the Mean Value Theorem on the interval $[a, b]$ is given by the midpoint $c = \frac{a+b}{2}$.

(Curiously, the value of c is independent of the quadratic's coefficients α, β, and γ!)

$\blacksquare$

2 Set $f(x) = x^{-1}$, $a = -1$, $b = 1$. Verify that there is no number c for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Explain how this does not violate the Mean Value Theorem.

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3 Find, among all right circular cylinders of fixed volume V, the one with smallest surface area (counting the areas of the faces at the top and bottom).

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4 Show that the sum of a positive number and its reciprocal is at least 2.

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5 The area between two varying concentric circles is at all times $9\pi^2$. The rate of change of the area of the larger circle is $10\pi^2$ in^2/sec. How fast is the circumference of the smaller circle changing when it has area $16\pi^2$ in^2?

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6 Suppose that f and g and are unknown differentiable functions that satisfy the relationships

$$2f'(x)c = g(x) \quad \text{and} \quad g'(x) = -f(x) \quad \text{for all } x \tag{1}$$

(a) Show that $f^2(x) + g^2(x) = C$ for some constant C.

(b) Determine the value of C if we also know $f(0) = 0$ and $g(0) = 1$.

(c) Differentiate the equations for f' and g' in (1) to show that f and g must also solve the differential equations

$$f''(x) + f(x) = 0 \quad \text{and} \quad g''(x) + g(x) = 0 \quad \text{for all } x.$$

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