CEEN241 TOC

> CEEN241 - 2025-01-06
>
> #notes #meche #ceen241
>
> ## some quick housekeeping stuff
>
> - prefers either Professor Guerra (Geh-ra) or Dr. Guerra
>
> ## What do we already know about statics?
>
> ### Group Members
>
> - Alec Malcangio, He/Him
> - Bliss O'Shea, She/Her
>
> ### What we know
>
> - Bliss doesn't like it
> - Everything go to zero
>   - Things don't move!
> - Lots of shenanigans with moments
>   - Which is force times distance!
> - Mohr's Circle is a thing that exists and is "not fun" according to higher MechE majors
> - Much drawing.
> - You do a lot of multiplying stuff together
> - Bridge type shit?
>
> ### Stuff other people said
>
> - High homework load
>
> ## More General Housekeeping Stuff

- We went on about how learning and such works for, like, a *while*
- "There's no such thing as a bad question"
  - this is not true.

More Specific (?) Housekeeping Stuff

- "Apply, Arrange, Solve"
- We're - generally - going to assume that lots of things are Rigid Bodies unless otherwise mentioned
- There's a neat document on Canvas going over learning objectives for every day. I don't intend to really use it, but neat that it exists
- Homework late policy is 2% per hour late
  - Using Canvas / Gradescope timestamp for that
- Handwritten homework is probably going to be annoying at best.
- Office Hours are M/W/F 8:30 to 9:30 as well as by appointment
  - I still haven't gone to office hours ever, but, y'know, good to know
- Historical Student Performance
  - Homeworks and quizzes both average above 90, exams are typically somewhere in the 70 range

Technically actually statics talk

- We use two systems of units, we don't convert between the two, but we do use two different systems
- Some problems are going to be SI, some problems are going to be in US Customary

SI	US Customary
Meter (m)	Foot (ft)
Second (s)	Second (s) they're the same! wow
Kilogram (kg)	Slug ($\frac{\mathrm{lb}\cdot\mathrm{s}^2}{\mathrm{ft}}$)
Newton (N) $\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$	Pound (lb)

come back next time for

- Vector operation shenanigans
-

CEEN241 - 2025-01-08

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Big topic of the day is vectors

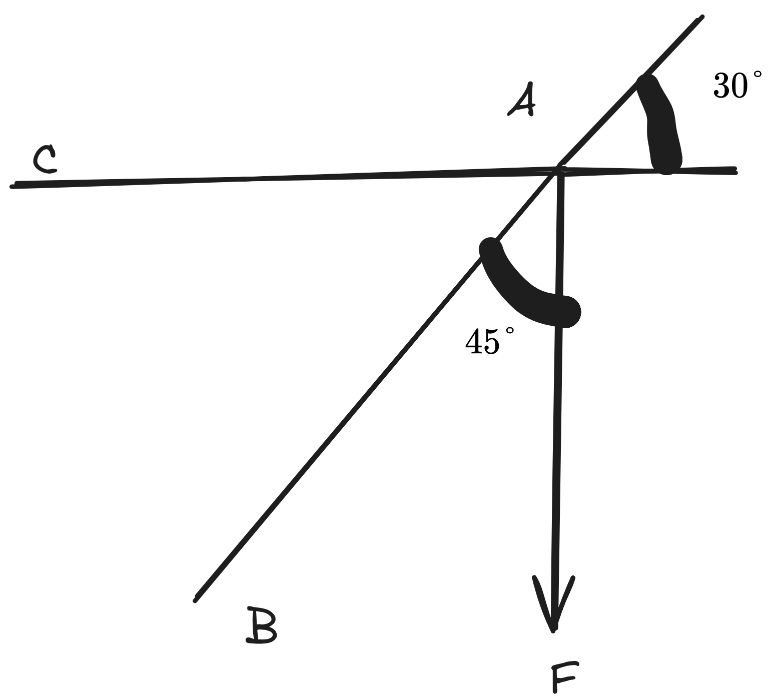
we start with a quick clicker(s)?

- Pounds are a force. Crazy.

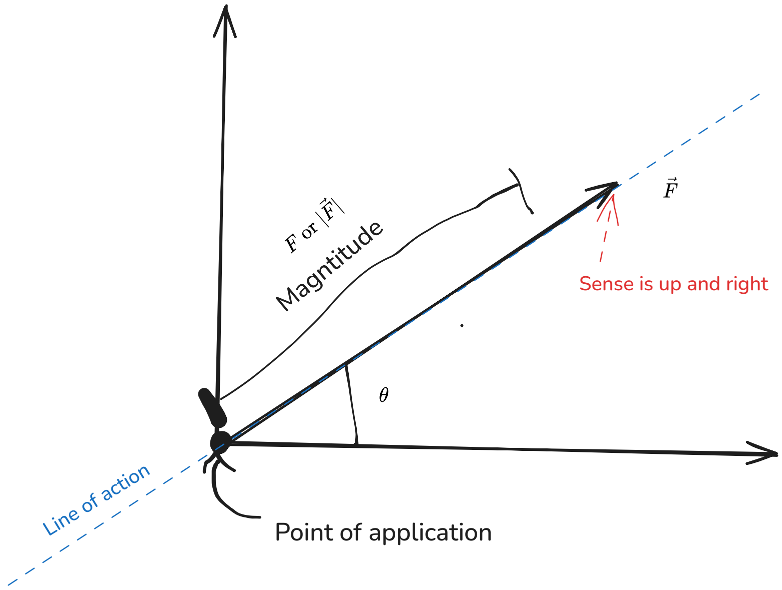
Housekeeping stuff

- Homework exists, and I'm probably going to need to go get in an argument.
- Go to any recitation, hands on activities start next week, they're basically office hours+

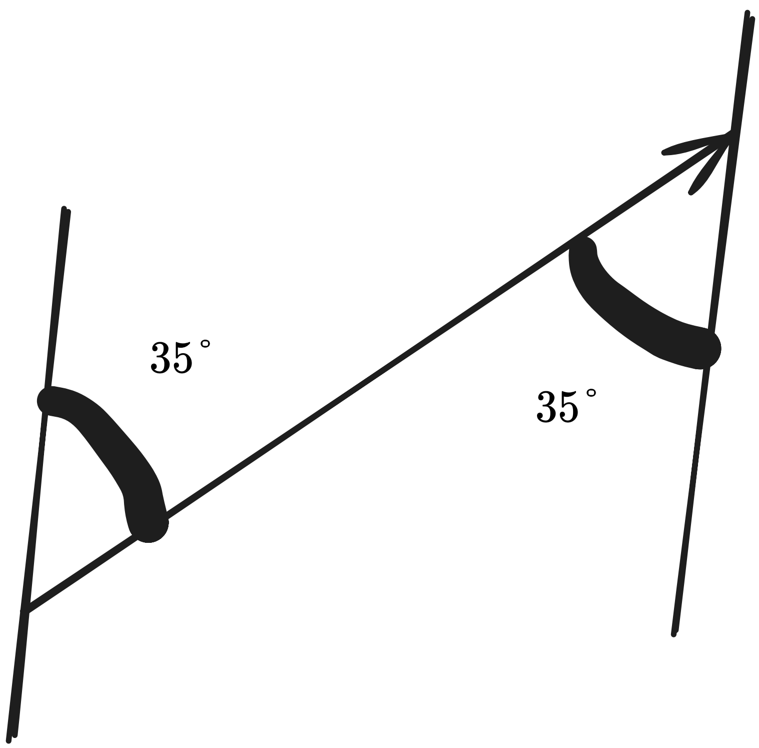
okie vector time



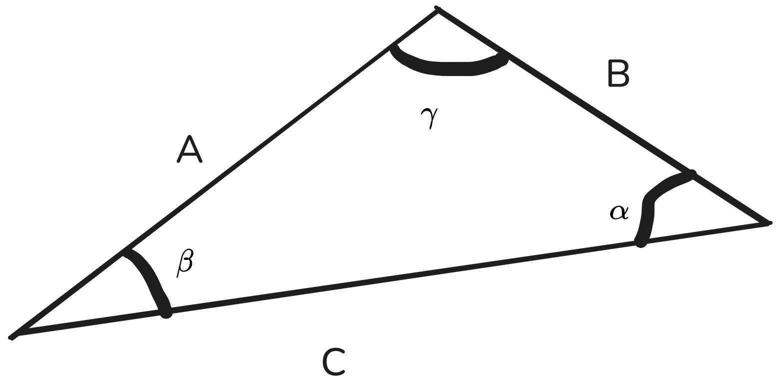
- Oh, we weren't even doing anything. Brutal.
- Delightful news! Everything is 2d. For right now. Literally just right now. Friday is 3d.
  - So, what's going on with vectors?



- The textbook doesn't use hat or vector signs! They use bold font, because fuck you, that's why.
- Talking about "a couple other tools" with respect to vectors
  - Corresponding angles



    - On that alternate interior angles type shit
  - Law of Sines and Cosines (for when we have a general triangle)
    - By "general" we mean anything that's not a right triangle. We can go suck a toe or something for right triangles.



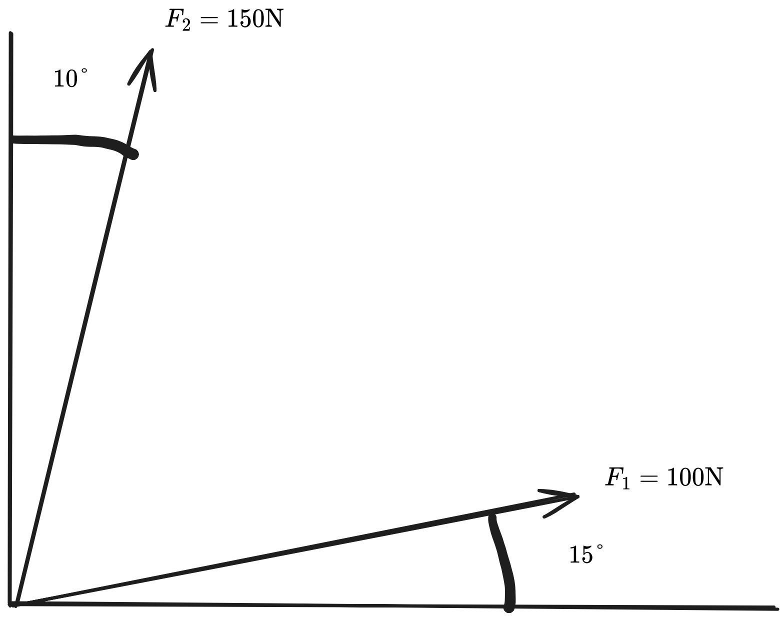
- Law of sines is just saying that

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B}$$

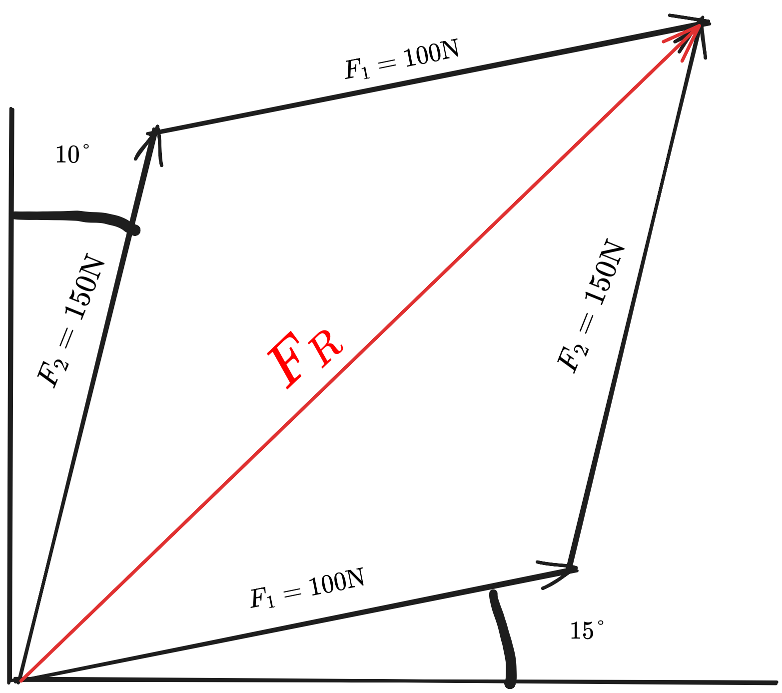
  - You can do the same thing for any of them (ie $\frac{\sin\alpha}{A} = \frac{\sin\gamma}{C}$)
    - For completeness's sake, you can also do $\frac{\sin\gamma}{C} = \frac{\sin\beta}{B}$
- Law of Cosines is a bit more of a pain
  - $A^2 = B^2 + C^2$
    - We start off looking like the Pythagorean theorem. Easy. Reliable. Got it. Love it.
    - However.
  - $A^2 = B^2 + C^2 - 2(BC)\cos(\alpha)$
    - This is essentially our pythagorean theorem, times two times (what's on the right) multiplied by the cosine of the angle on the left.
      - If you go crazy and start reorganizing, you end up with
        - $B^2 = A^2 + C^2 - 2ACC\cos(\beta)$
        - $C^2 = A^2 + B^2 - 2AB\cos(\gamma)$

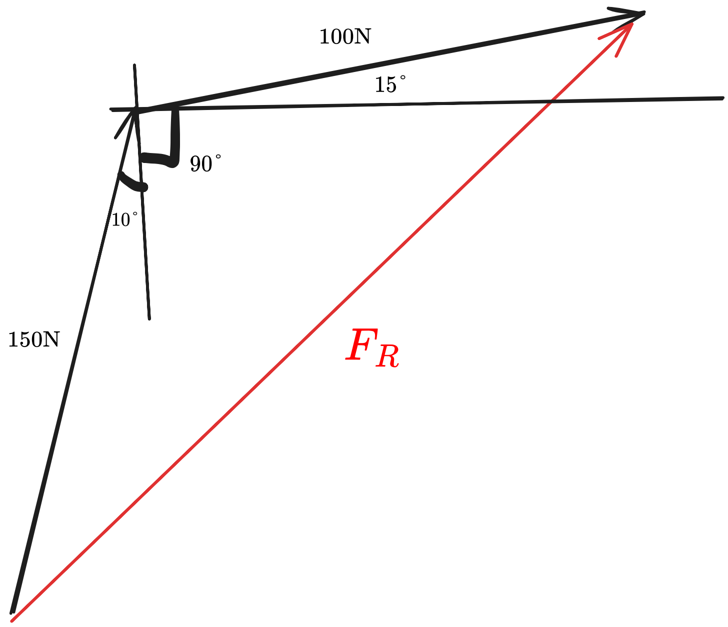
- "We want to have the least number of math to do"
  - Neither math nor english is our strong suit.

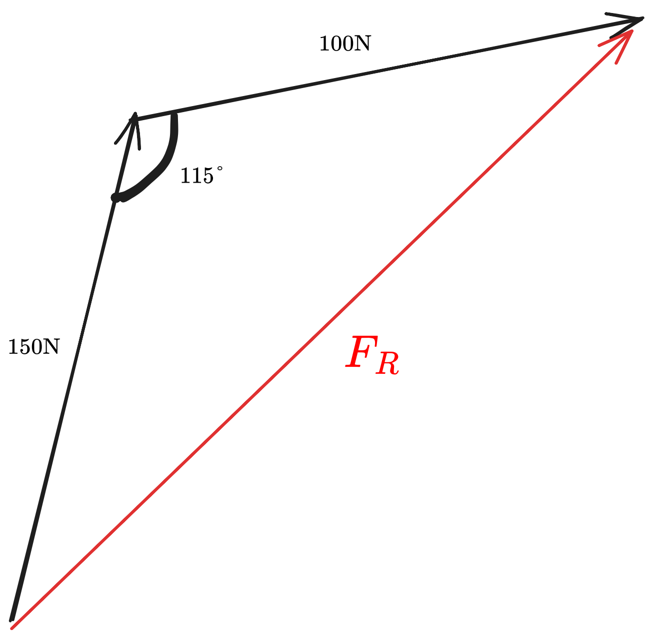
Alright example problem time



- Given all that, find the resultant vectors.
- We want to find the magnitude and direction (maybe we can get to stealing the moon at some point) of our resultant vector
  - We have a couple different solutions
  - You could go tip to tail.
  - I personally fuck way more with components, especially for this problem, but regardless.







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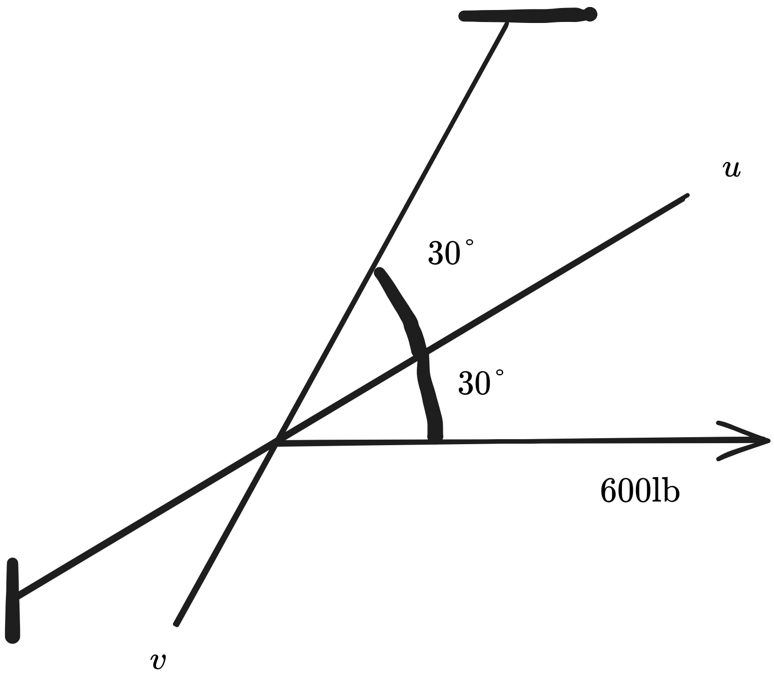
okie new problem (2.2 in the textbook)



- Go crazy with parallelograms, is how you go about doing this one
  - The video reportedly does finish it

CEEN241 - 2025-01-10

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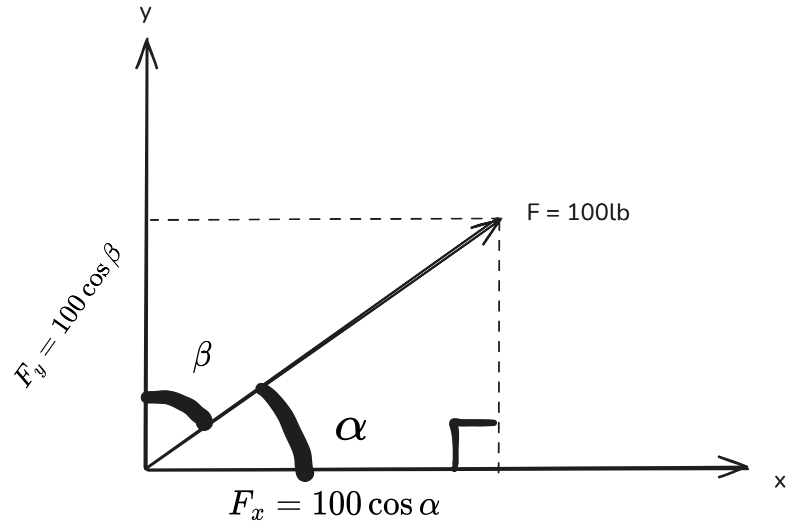
y'know that bit about moving to 3d? we're moving to 3d.

First up is dealing with angles and cosines and such

I'm a liar, we're doing cartesian vectors in 2d. We'll get there.



$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

$$= 100\cos\alpha\hat{\imath} + 100\cos\beta\hat{\jmath}$$

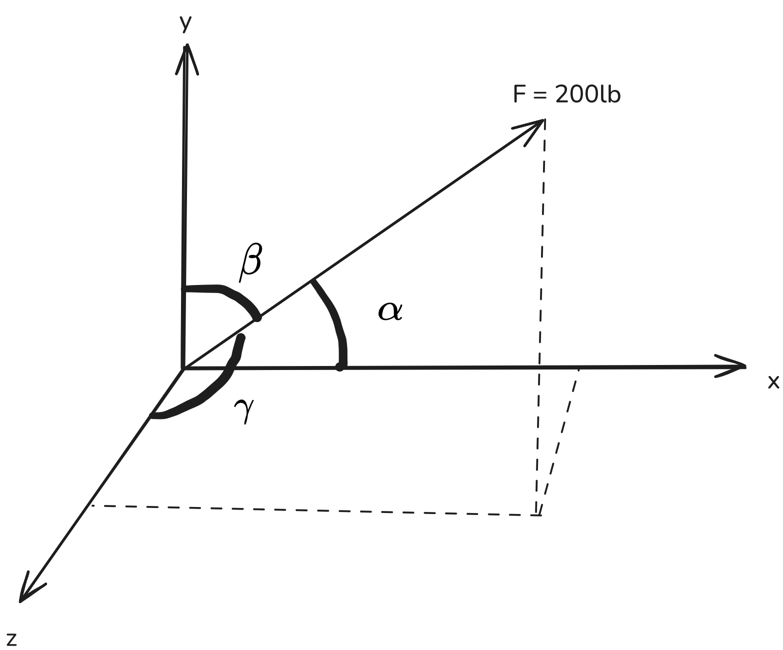
$$= 100(\cos\alpha\hat{\imath} + \cos\beta\hat{\jmath}$$

Mag ↑ Unit vector ↑

$$\vec{F} = F \cdot \vec{u}$$

Ok *now* we're moving to 3d. I promise.

- why in the sweet kentucky fried hell is y up. we're not minecraft. what is happening here.



- This makes it so that we end up with something like

$$\vec{F} = 200\cos\alpha\hat{\imath} + 200\cos\beta\hat{\jmath} + 200\cos\gamma\hat{k}$$

$$\vec{F} = 200(\cos\alpha\ \hat{\imath} + \cos\beta\ \hat{\jmath} + \cos\gamma\ \hat{k})$$

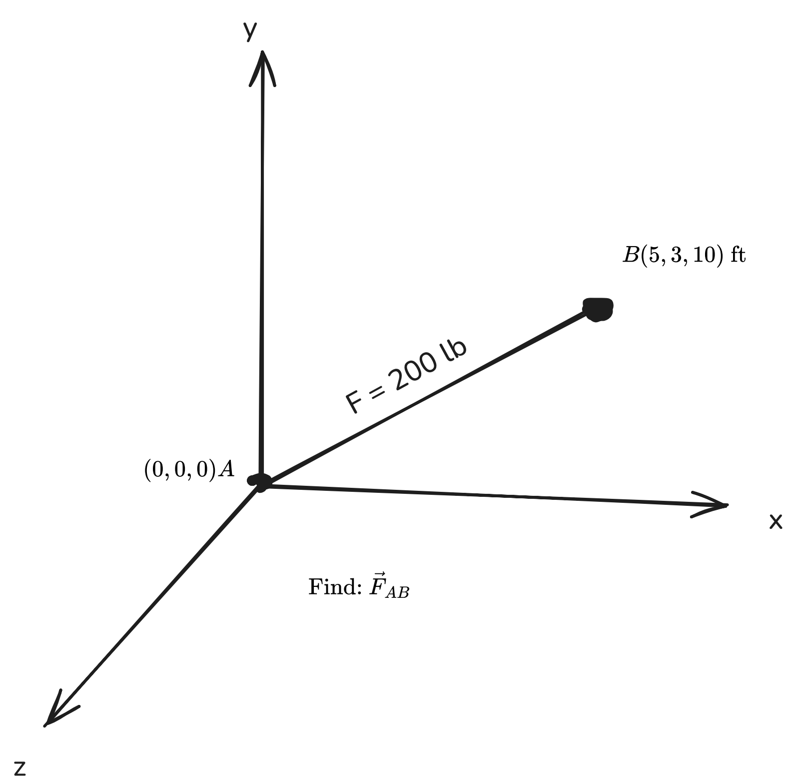
- Other important bits
    - $|\vec{u}| = 1.0$ (the magnitude of a unit vector is 1)
        - And, a unit vector is unitless! How silly.
    - $\vec{F} = 200\text{lb}\ \cdot \vec{u} \rightarrow \text{lb}$
- I dunno where we're going with this, so

$$\cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{F_x}{F}, \cos\beta = \frac{F_y}{F}, \cos\gamma = \frac{F_z}{F}$$

$$\vec{u} = \frac{F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}}{F}$$

    - Wow, just wrote the same thing like three times. Shit crazy.

Position Vector Time (Coordinates)



- Just as a note here, there's no way the z coordinate is correct. If we're just throwing sense out the window, we should call the x coordinate 2.

$$\vec{F} = F \cdot \vec{u}$$

$$\vec{u} = \frac{\vec{r}_{AB}}{r_{AB}}$$

$$\vec{r}_{AB} = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}$$

$$\vec{r}_{AB} = (5 - 0)\hat{\imath} + (3 - 0)\hat{\jmath} + (10 - 0)\hat{k}$$

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \quad < 5, 3, 10 >$$

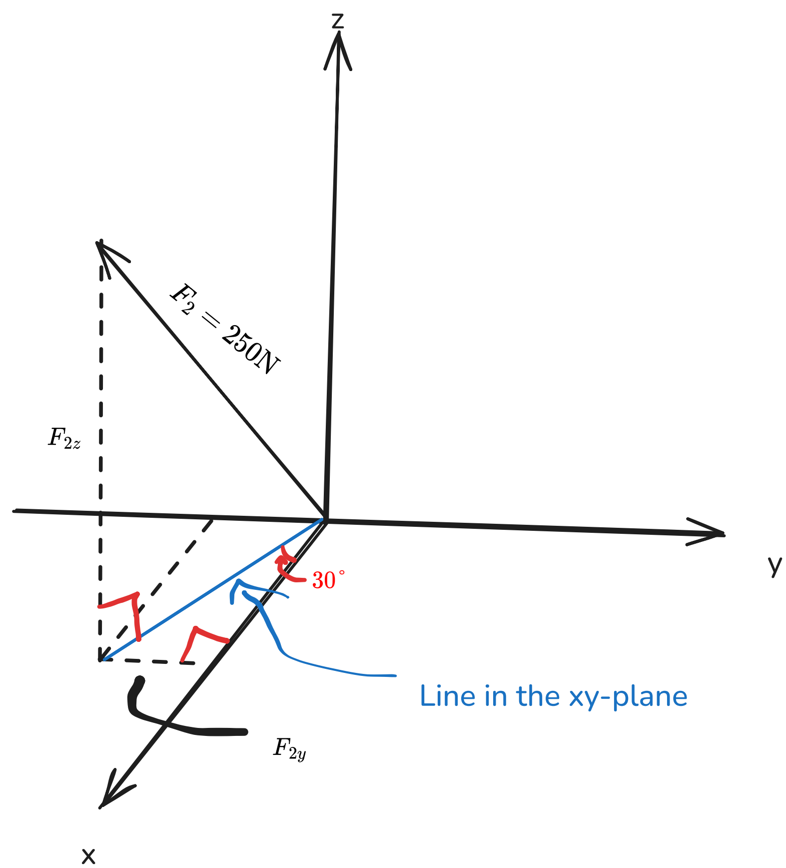
$$= \frac{< 5, 3, 10 >}{\sqrt{5^2 + 3^2 + 10^2}} = \frac{< 5, 3, 10 >}{\sqrt{134}}$$

$$\vec{u}_{AB} = 0.43\hat{\imath} + 0.26\hat{\jmath} + 0.86\hat{k}$$

$$\vec{F} = 200\, \vec{u}_{AB} = 86\ \hat{\imath} + 52\ \hat{\jmath} + 172\ \hat{k}\ \mathrm{lb}$$

- Y'know what. Fuck you. Also find α, β, γ
  - This is actually really easy - just $\arccos$ of what we got for our components over the big force
    - $\alpha = \arccos(0.43) = 64.5^\circ$
    - $\beta = \arccos(0.26) = 74.9^\circ$
    - $\gamma = \arccos(0.86) = 30.7^\circ$

Planar Projections_{the one I don't actually like}



On that stair shit (some steps)

1. Find lines parallel to x, y, z
2. Find right angles (oh hey, we made two right triangles)
    - That will always be two right triangles for planar projection.
3.

CEEN241 - 2025-01-13

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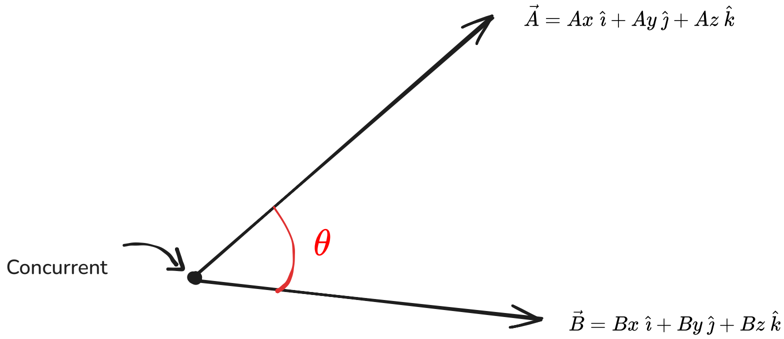
shenanigans with the dot product

- also, when did he pick up a stake? did I miss a vampire event? has that been here the whole time? does he just walk around with a stake in his backpack? who does he think he is, andres van helsing?

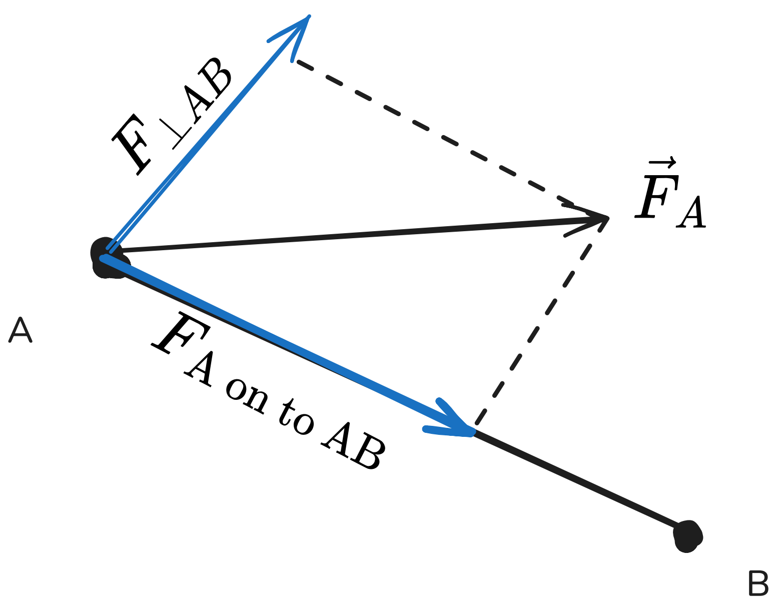


- Given that they're concurrent, we can do the dot product, which, mathematically, is

$$\vec{A} \cdot \vec{B} = A * B \cos(\theta)$$

  - Standard dot product bits, A and B are magnitudes, we get a scalar out, you do $A_x * B_x + A_y * B_y + A_z * B_z$, and that works out nice and neat.
- Reportedly, two important applications:
  1. We can find θ! $\theta = \cos^{-1}(\frac{\vec{A} \cdot \vec{B}}{A * B})$
     - That literally just comes from solving the existing dot product equation for theta. Sure is neat.
       - Magnitudes are standard magnitude business.

Parallel and Perpendicular Components

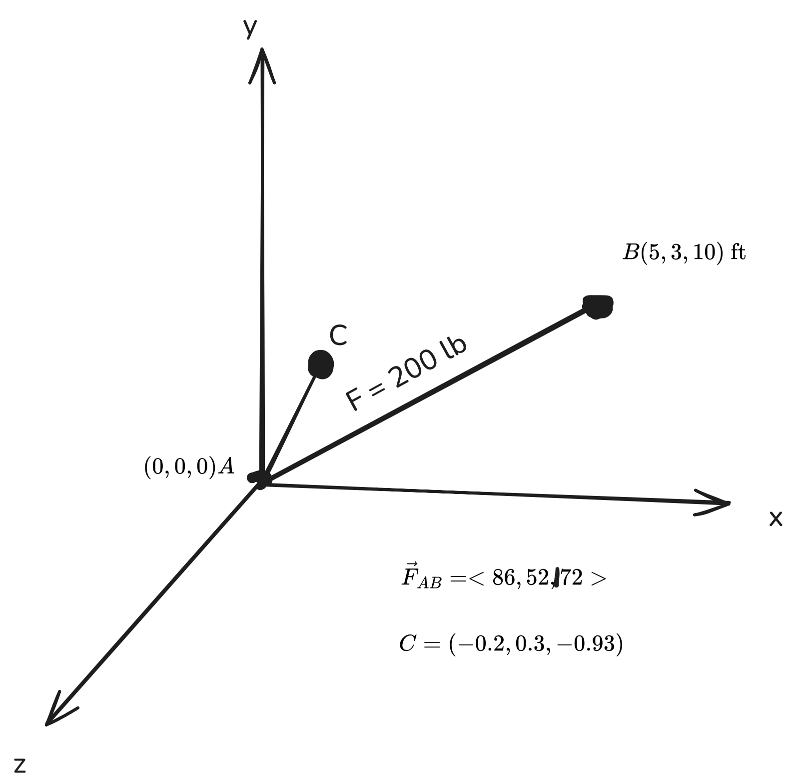


- If we want to actually get this, $F_{\text{AontoAB}} = \vec{F}_A \cdot \vec{u}_{AB} = \text{Scalar!}$
  - If we really want to get $\vec{F}_{\text{AontoAB}}$, you just multiply by the unit vector $(F_{\text{AontoAB}} \cdot \vec{u}_{AB})$, and that spits out your vector

- Alright alright, that was the parallel part, now get $\perp$ with it.
  - Note: You always find the parallel first. Sorry bub, just how it goes.
  - $F_A^2 = (F_{\text{AontoAB}})^2 + (F_{\perp AB})^2$
    - I love the pythagorean theorem.
    - So you solve, and $F_{\perp AB} = \sqrt{F_A^2 - (F_{\text{AontoAB}})^2}$
  - To get the vector, you just have $\vec{F}_A = \vec{F}_{\text{AontoAB}} + \vec{F}_{\perp AB}$
    - So you solve, $\vec{F}_{\perp AB} = \vec{F}_A - \vec{F}_{\text{A onto AB}}$
- The general vibe is here is that the components add back to the original F_A

egg sample time

Given:



- Point C is Chekov's Stake, driven in to the wall.
  - I am being threatened.

Find:

- Components of AB parallel and perpendicular to AC. (Scalar and Vector)

Solution:

$$F_{\text{ABontoAC}} = \vec{F}_{AB} \cdot \vec{u}_{AC}$$

$$\vec{u}_{AC} = \frac{\vec{r}_{AC}}{r_{AC}} = \frac{< -0.2, 0.3, -0.93 >}{\sqrt{0.2^2 + 0.3^2 + 0.93^2}} =< -0.2, 0.3, -0.93 >$$

$$F_{\text{ABontoAC}} =< 82, 52, 172 > \cdot < -0.2, 0.3, -0.93 >= -17.2 + 15.6 - 159.96 = -161.6 \text{ lb}$$

- That silly little negative number is telling us that we're going in the exact opposite of the direction of the unit vector
- If we hang on to the sign, and do

$$\vec{F}_{\text{AontoAB}} = -161.6\vec{u}_{AC} = 32.32 \hat{\imath}, -48.48 \hat{\jmath}, 150.3 \hat{k}$$

- Now get perpendicular with it

$$F_{\perp AC} = \sqrt{200^2 - (161.6)^2} = 117.84$$

$$\vec{F}_{\perp AC} = \vec{F}_{AB} - \vec{F}_{\text{ABontoAC}}$$

CEEN241 - 2025-01-14

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$$< 250\cos(35)\sin(25), 250\cos 35\cos 25, -250\sin(35) >$$

$$< 86.6, 185.6, -143.3 >$$

$$490 < \cos(120), \cos(45), \cos(60) >$$

$$< -245, 346.48, 245$$

$$< -158.4, 532.08, 101.7 >$$

$$< -2.5, y, z >$$

$$< -2.5, 4, 6 >$$

$$7.6321687612$$

that over that, times 80

$$< 0.33, 0.524, 0.786 >$$

$$< 2, 4, -6 >$$

$$7.4833147735$$

$$< 0.267, 0.534, 0.801 >$$

$$< -12.8, -68.6, 22.8 >$$

CEEN241 - 2025-01-15

#notes #meche #ceen241

two dimensional particle equilibrium

$$\sum F = 0$$

- "Steeper Angle, Bigger Tension"

Get Springy Wit it

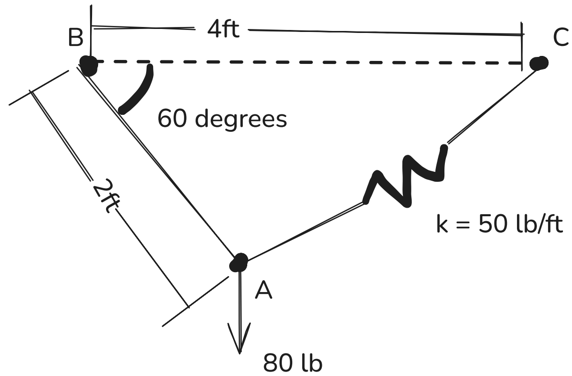
- $F = ks$, it's Hooke's Law, you know and love it.
  - (s is the stretch in the spring, k is the spring constant {stiffness} [usually lb/in or N/m])
- Ya want some springs, kid?
- Springs don't change equilibrium

Pulleys

- Pulleys have your average component shenanigans, the tension on both ends of the pulley are always the same and the sum of the forces has to manage to equal zero.

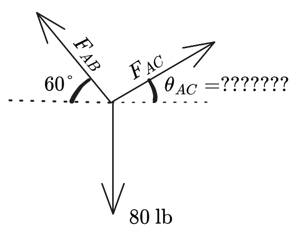
Exampling Time

Given:



Find: Unstretched length of spring AC

Solution: FBD at A



- Hey, at current we have a bit of a problem: three unknowns but only two equations
- Two potential ways to remedy this:
  - Get up to geometric *shenanigans*
  - Or draw another free body diagram
- Oh hey, we sure do have a triangle with SAS given, and we're trying to find some funky other things

$$\ell_{AC}^2 = 2^2 + 4^2 - 2 * 2 * 4 \cos 60$$

$$\ell_{AC} = 3.46 \text{ ft}$$

$$\theta_C = 30^\circ$$

- I mean we used law of cosines, but we also have adjacent and hypotenuse and 2/4 is in fact 0.5... y'know nevermind. Don't make assumptions.
- Oh hey, now we've chopped down to two unknowns with two equations. Ain't that neat.

$$\sum F_x = 0(\rightarrow_+)$$

$$-F_{AB}\cos(60) + F_{AC}\cos(30) = 0$$

$$\sum F_y = 0(\uparrow_+)$$

$$F_{AB}\sin(60) + F_{AC}\sin(30) - 80 = 0$$

- Do Algebra and solve that. I have faith.
  - Reportedly, F_{AC} is 40 lb.

- Can get Hooke-y with it, $\frac{40lb}{50\frac{lb}{ft}} = 0.8ft$, and then the unstretched length is going to be actual minus stretched, or $\ell_o = 3.46 - 0.8 = 2.66ft$

CEEN241 - 2025-01-16

#notes #meche #ceen241

$$F_z = 692.8203230276$$

692.8203230276

$$F_{xy} = 400$$

$$F_x = 346.4101615138$$

346.4101615138

$$F_y = 200$$

200

$$F_u =< 0.866025, 0.4330125, 0.25 >$$

$$A_u =< \frac{4}{6}, \frac{4}{6}, \frac{2}{6} >$$

$$BC =< 6, 4, -2.5 >$$

$$BC =< 0.786, 0.524, 0.327 >$$

$$F = 130 < 6, 8, 2.5 >$$

$$F = 130 < -0.58, 0.776, 0.24 >$$

CEEN241 - 2025-01-17

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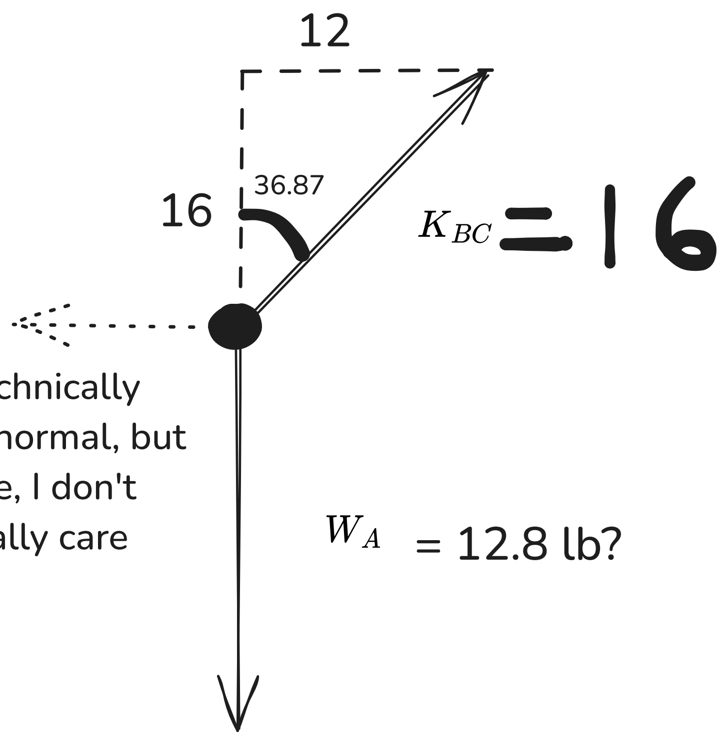
Clicker

- I was starting to do work but it's literally just 400 - - 175 and that's the only one with 575
  - One of the sillier parts of statics is watching everyone move over to the correct answer

particle equilibrium example

- I believe the only other people who say "free that body from the rest of the world" are drug dealers



- wait i did math wrong hang on (maybe)
  - yeah I forgot how triangles work but I did have the right vibe
- "Cables are all pulling forces *right now*"
  - Right now? The fuck do you push with a cable? What?

ok work adventure for realsies

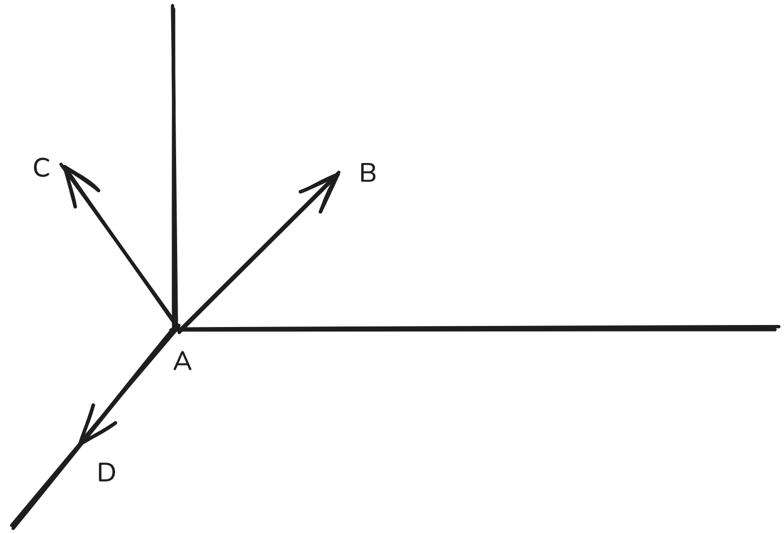
- crate adventure thing or whatever

$$A = < 0, 0, 0 >$$

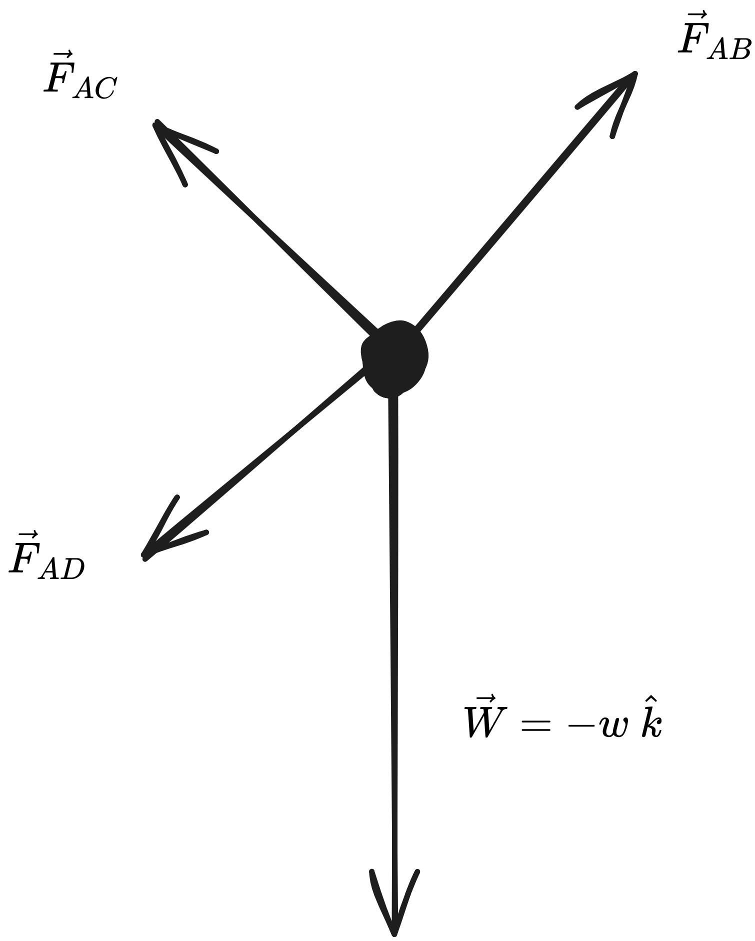
$$B = < -2, 1, 2 >$$

$$C = < -2, -2, 1 >$$

$$D = < 3, 0, 0 >$$



- Find the maximum W so that no cable exceeds 450 lb of tension



- Strictly speaking you can write a vector as its magnitude times a unit vector (I answered the sum of the squares because that's like relevant kinda for AD balancing with the rest but w/e)

$$|B| = 3, |C| = 3, |D| = 3$$

$$B_u = < -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} >$$

$$C_u =< -\frac{2}{3}, -\frac{2}{3}, \frac{1}{3} >$$

$$D_u =< 1, 0, 0 >$$

$$\vec{F}_{AB} = F_{AB} B_u$$

.... yadayadayada.

- See, here's Chekov's Components I was talking about.

$$\sum F_x = 0 = -\frac{2}{3} F_{AB} - \frac{2}{3} F_{Ac} + F_{AD}$$

$$\sum F_y = 0 = \frac{1}{3} F_{AB} - \frac{2}{3} F_{AC}$$

$$\sum F_z = 0 = \frac{2}{3} F_{AB} + \frac{1}{3} F_{AC} - w$$

- ok now we go on an algebra expedition (or just guess)
  - Given that B_u has the steepest z component, we're going to assume that's our limiting
  - (I haven't written that phrase since chem 1)

$$\frac{1}{3}(450) = \frac{2}{3} F_{AC}$$

$$F_{AC} = 225$$

$$\frac{2}{3}(450) + \frac{2}{3}(225) = F_{AD} = 450$$

$$\frac{2}{3}(450) + \frac{1}{3}(225) = W = 375 \text{ lb}$$

CEEN241 - 2025-01-19

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$$200 \cos(45) x_1 = 300 \cos(53.13) x_2$$

$$x_2 = 0.786 x_1$$

$$200 \sin(45) x_1 + 300 \cos(53.13) x_2 = 196.2$$

$$200 \sin(45) x_1 + 300 \cos(53.13) 0.786 x_1 = 196.2$$

$$x_1 (200 \sin(45) + 300 \cos(53.13) 0.786) = 196.2$$

CEEN241 - 2025-01-22

#notes #meche #ceen241

housekeeping stuff

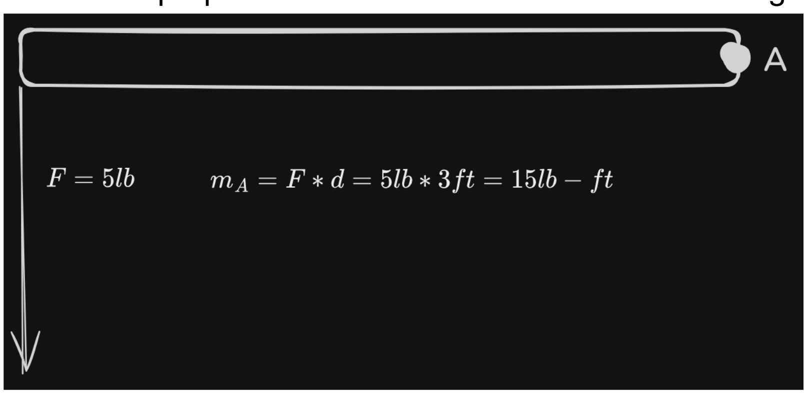
- hands on 1 & 2 are available before feb 4th, so, probably go do that
- recitations end 5pm on exam days, fun fact
    - also the exam is feb 4th

hey wait a sec, gimme a moment

- i am going insane.
    - please see Year 1/Semester 2/MATH112/MATH112 - 2024-01-26 and PHGN 100 - 2024-02-28
- if I had a nickel for every british crane we saw dunk itself into the water, I'd have two nickels, which isn't a lot, but, y'know
- we now have rigid objects that resist compression (ie, you push on it) and bending amon type shit
- moments cause a tendency for objects to rotate

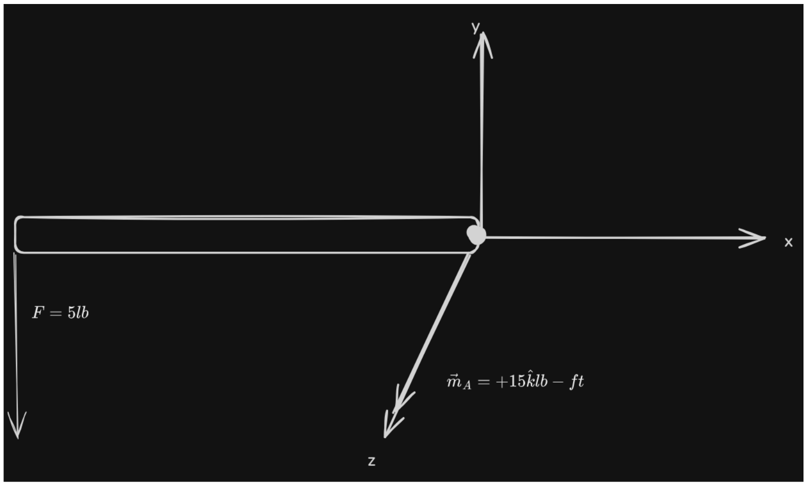
$$m = F * d$$

- where d is perpendicular to the line of action of F and goes to the point of interest



moment vectors

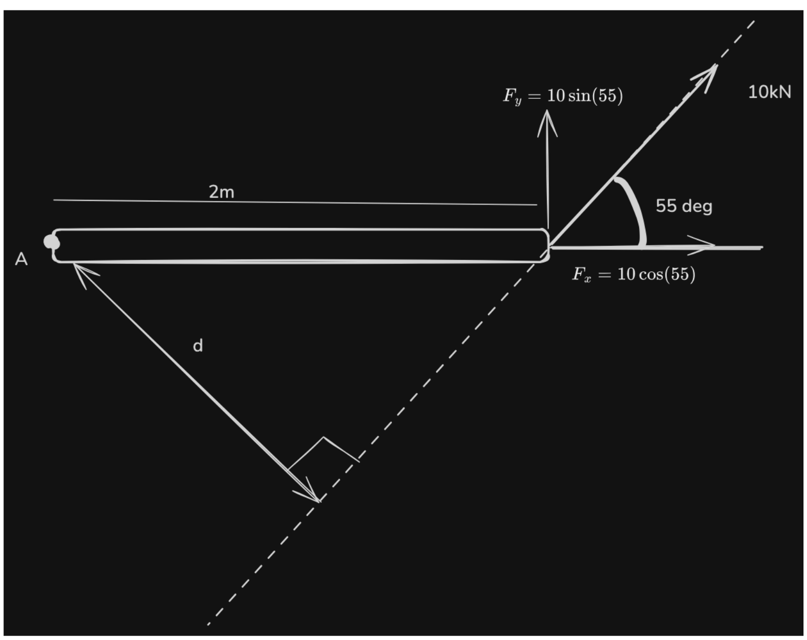
- these point in the direction of the axis of rotation



- We use the right hand rule to figure this out (the curly variation)
  - fairly certain it also works out using the perpendicular one if you put one finger on line of action of the force and the other along the distance, but, anyways
- "fingers towards your palm will never do you wrong" is a nice rhyme
  - so help me if someone has not heard of lefty loosey righty tighty in their sophomoreish year of an engineering program i have QUESTIONS

ohhhh nooo! just broke my coordinate system!

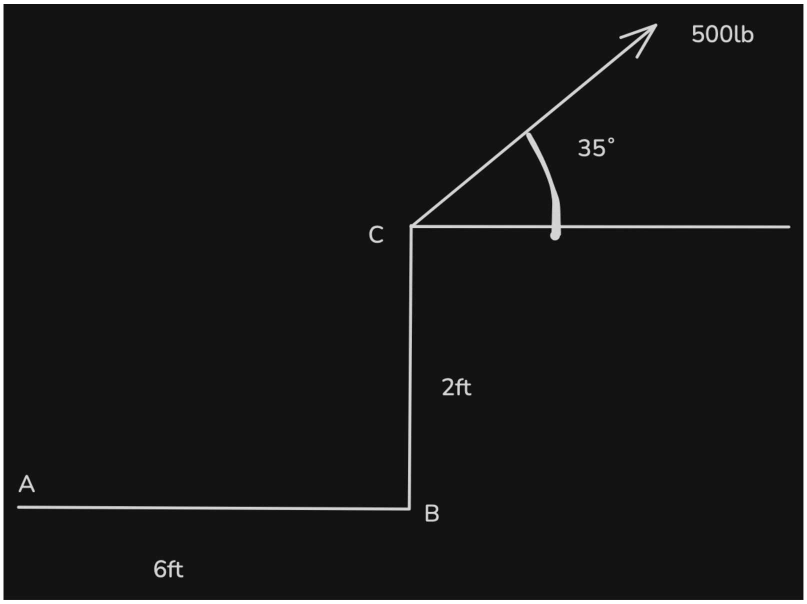
ok jokes aside, principle of transmissibility

- we can use any point on the line of action to calculate the moment
- 

- Fun fact! F_x ain't doing shit to cause us to rotate (it's acting right along the axis. this is regular ol' torque/moment stuff)
- F_y, on the other hand, is getting up to some grand old shennanigans of $F_y(2m)$, which then just means the moment about A is just going to be $10\sin(55) * 2m = 16.4kN - m$

- Counterclockwise is always positive in 2D

egg sample

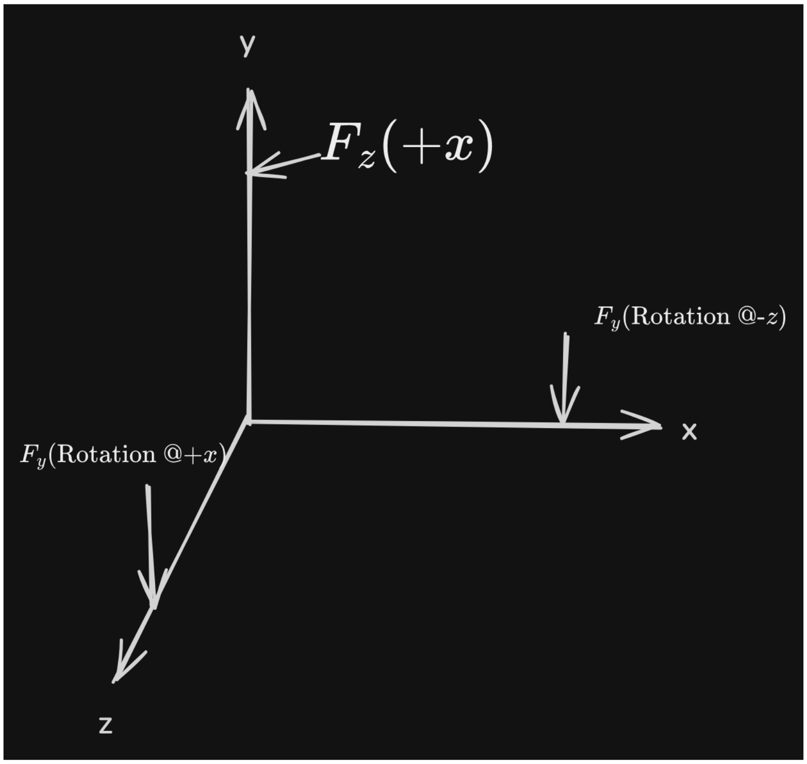


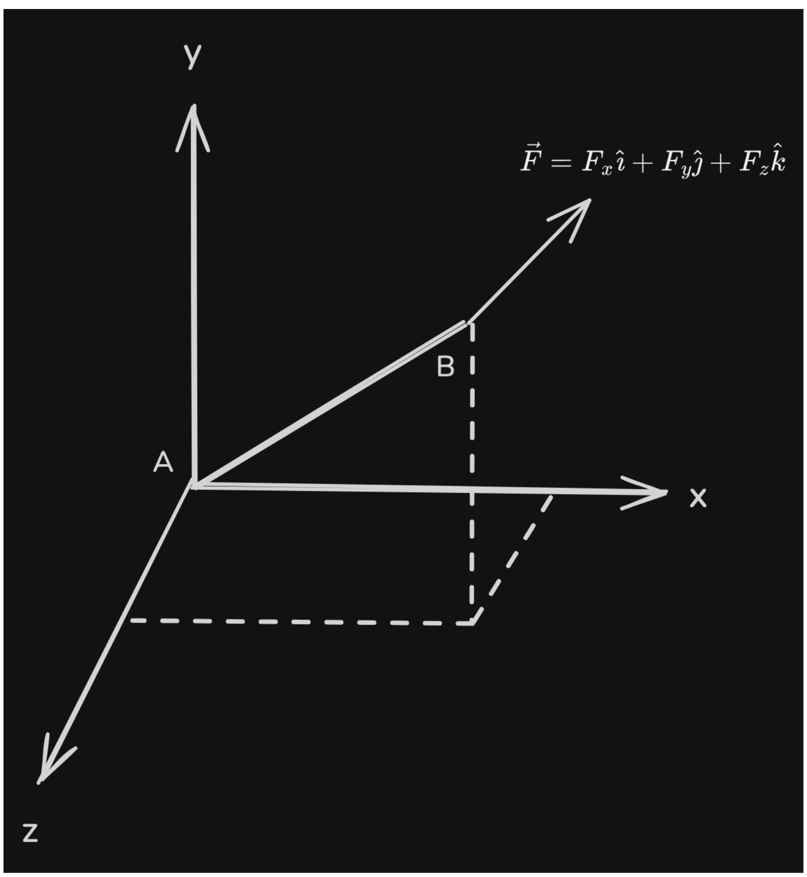
- Find the moment about A
- Okies, so you break into components
  - $F_x = 500\cos(35)$
  - and $F_y = 500\sin(35)$
- From your components, you can pick your force ($500\cos(35)$, in our case), and then you just have to pick your distance (which is just going to be our 2ft, per the transmissibility shenanigans)
- minor problem, we do also have the other component kicking it around, and that does matter, so that'll be $500\sin(35) * 6ft$
  - y forces will always have an x distance and vice versa, and that will *always* be the case
    - signs with tendency to rotate (right hand rule or CCW convention)
- If you run the numbers, m_A is going to be $-819.2 + 1720.7 = 901.6$ft, which is positive, so clockwise
- Even when you get up to some geometric shenanigans, you really well and truly just care about what's happening perpendicular relative to the point - if you have a big ol' elbow getting up to some mischief, that really doesn't matter
-

CEEN241 - 2025-01-24

#notes #meche #ceen241

moments in all of three dimensions





- Alright, so if we're doing this problem, we have two real ways of thinking about the moment

Scalar Approach

- x-axis: $-F_y * r_z + F_z * r_y = mA_x$
- y-axis: $F_x * r_z - F_z * r_x = mA_y$
- z-axis: $F_y r_x - F_x r_y = mA_z$
- So let's go about thinking of the board as $\vec{r}_{ab}$, and then the components are going to be r_y, r_x, and r_z respectively
- $\vec{m}_A = m_{Ax}\hat{\imath} + m_{Ay}\hat{\jmath} + m_{Az}\hat{k}$
- You can get jiggy with it and create a plane given that you have a couple points and a vector or two, which your moment vector is going to be perpendicular to, similarly making the axis of rotation perpendicular to plane $\vec{r} - F$
  > - Hey hold your ass up. That sounds an awful lot lik-

THE CROSS PRODUCT (vector approach)

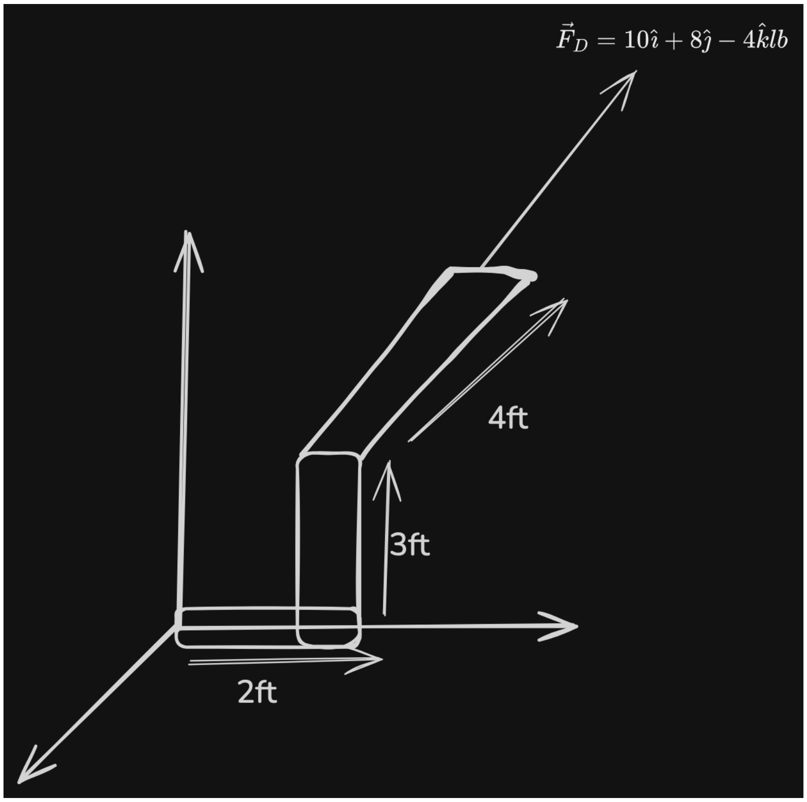
$$\vec{m}_A = \vec{r}_A \times \vec{F}$$

- You gotta crossy the product between your endpoints, r is going to end at ***any*** point along the line of action of the force
  > - It starts, however, at the point you care about determining the moment around
- Of note if you just do $\vec{F} \times r$, all of your signs will be wrong. Ain't that quirky

$$\vec{m}_A = \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{matrix} = \hat{\imath}(r_y * F_z - F_y r_z) - \hat{\jmath}(r_x F_z - F_x r_z) + \hat{k}(r_x F_y - F_x r_y)$$

- Don't break the news to the math department that we found a use for the cross product (please ignore any previous instances of the phrase "crossy the product" or "do the cross product" outside of a math class. those aren't real. nuh-uh-uh.)

exampling time



- Given that, find $\vec{m}_A$ and direction cosines (α, β, γ)

$$\vec{m}_A = r\vec{_{AD}} \times \vec{F}_D$$

$$\vec{r}_{AD} =< 2, 3, -4 >$$

$$\vec{F}_D =< 10, 8, -4 >$$

- Now you crossy the product

$$\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & -4 \\ 10 & 8 & -4 \end{matrix} = \hat{\imath}(-12 + 32) - \hat{\jmath}(-8 + 40) + \hat{k}(16 - 30)$$

$$\vec{m_A} =< 20, -32, -14 >$$

- Magnitude would just be $\sqrt{400 + 1024 + 196} = 40.2$ ft-lb
- α, β, γ are all gotten as you would reasonably expect with arccosines
-

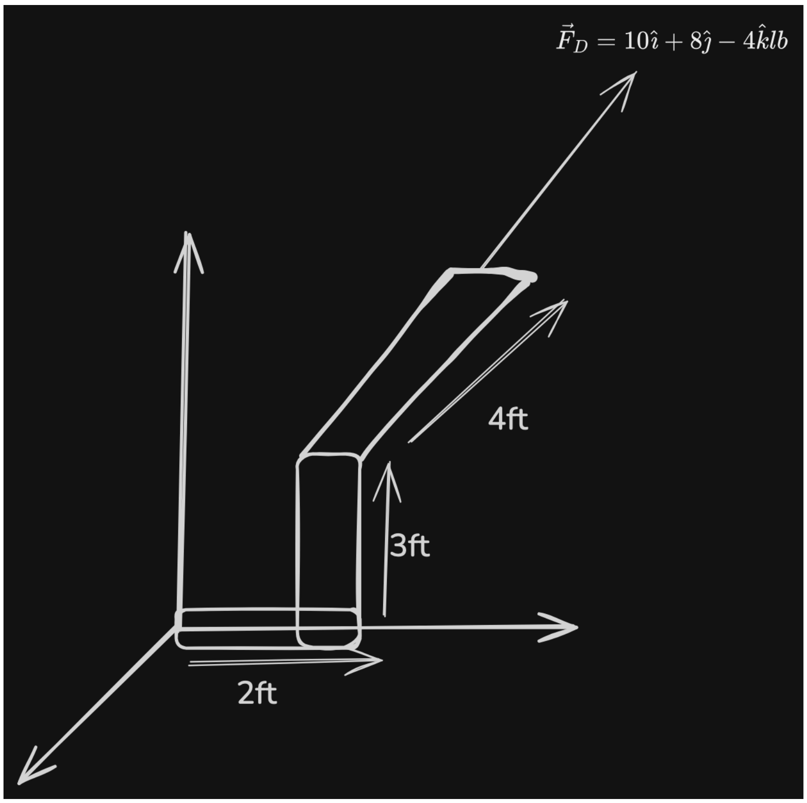
CEEN241 - 2025-01-27

#notes #meche #ceen241

moment at an axis

- Find the moment at any point along the axis of interest
- Which is going to be $\vec{m} = \vec{r} \times \vec{F}$
- Once we've got that, we can find the projection of $\vec{m}$ onto Axis of Interest, which is $m_{axis} = \vec{m} \cdot \vec{u}_{axis}$ =scalar
- Combine these steps into the "Triple Scalar Product" $m_{axis} = (\vec{r} \times \vec{F}) \cdot \vec{u}_{axis}$

example vibes



- So we're using this same general pipe assembly vibe
  - Except there's going to be a new pipe segment from the middle of the bottom (segment AB, except I didn't label it), and the new point F is going to be out at (2,-1,5) (I'm feeling lazy and don't want to draw
- Find the moment vector from $\vec{m}_{EF}$ from $\vec{F}_D$
  - So our position vector $\vec{r}$ can be any point on the axis of interest
  - We only have one force
  - and we want the axis of interest
  - soooo $m_{EF} = (\vec{r}_{ED} \times \vec{F}_D) \cdot u_{EF}$
  - $\vec{r}_{ED} = (2, 3, -4) - (1, 0, 0) =< 1, 3, -4 >$
  - $\vec{u}_{EF} =< 0.192, -0.192, 0.962 >$

$$m_{EF} = \begin{array}{ccc} 0.192 & -0.192 & 0.962 \\ 1 & 3 & 4 \\ 10 & 8 & -4 \end{array} = -10.4 \text{ ft-lb}$$

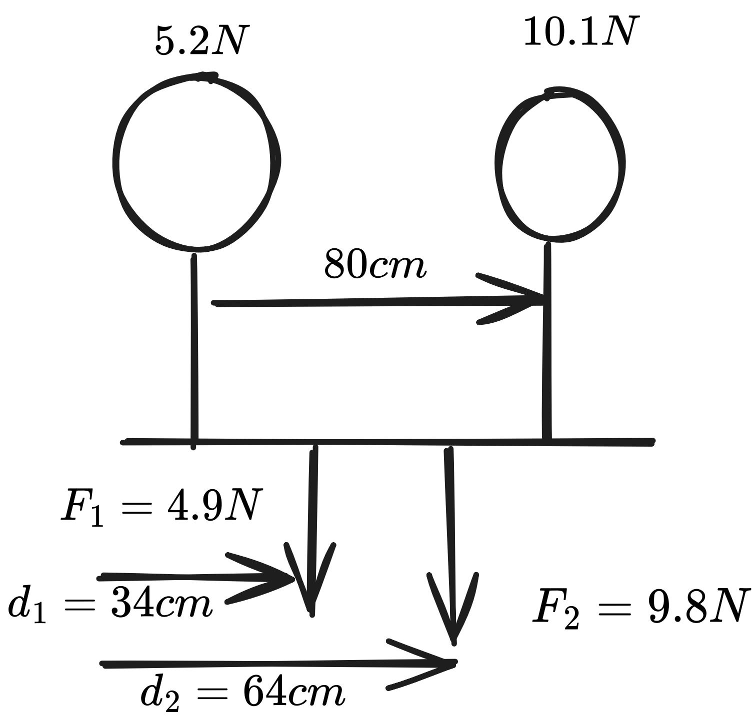
- Which is negative because it'll be the opposite of the unit vector we used
- So then when you multiply through by $\vec{u}_{EF}$, you end up with $-2\hat{\imath} + 2\hat{\jmath} - 10\hat{k}$ ft-lb as your final answer

CEEN241 - 2025-01-31

#notes #meche #ceen241

Equivalent Systems

also, housekeeping, who steals a semi assigned seat at this point in the semester?



- So we want to take this system, and replace those multiple forces with an equivalent force somewhere in the middle

Two Rules for Equivalency

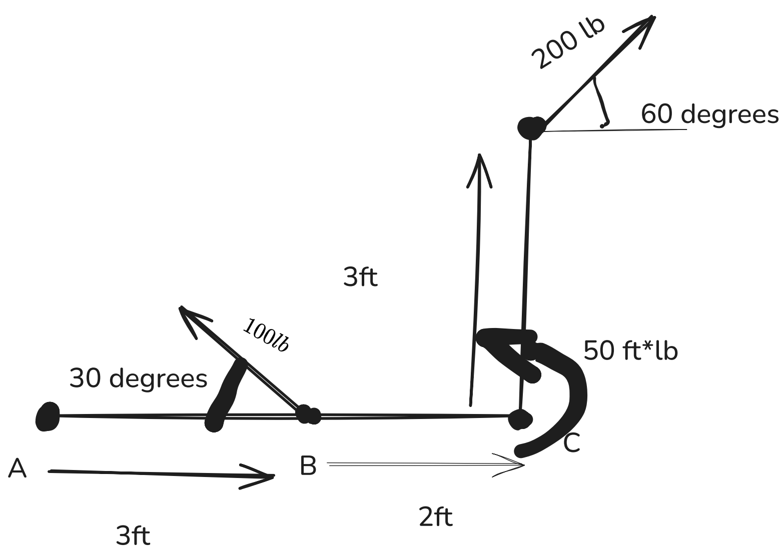
1. Applied forces in original must be equal to applied forces in resultant ($F_R = F_1 + F_2$)
    - In our case, that means F_R is 14.7N

2. Moments from the *applied* forces in original about any point must be the same as the moment from the applied forces in the equivalent about the same point
    - This means in our case we should get 54cm

Second Way of doing things

- Get asked to replace OG with a force and a moment at a point
    - "Replace F_1 and F_2 with F_R and M at point A"
    - End up needing to add some couple moment, M_a, in order to make the systems equivalent

Example



- replace with an equivalent force and specify its location along AC measured from A.
  - Alrighty, combination time
  - And by combination, I really mean we're going to split things apart

$$F_{Rx} = -100\cos(30) + 200\cos(60) = 13.4lb$$

$$F_{Ry} = 100\sin(30) + 200\sin(60) = 223.21lb$$

- What I reaaaally care about is the moments, to be so real.

$$M_{og} = 100\sin(30) * 3 - 200 * \cos(60) * 3 + 200\sin(60) * 5$$

$$M_{og} = \cancel{716.025} = 100 * \sin(30) * 3 - 200 * \cos(60) * 3 + 200 * \sin(60) * 5 + 50 = 766.025$$

- And then solving for the distance we only really care about the F_{Ry} because of how rotation works, so $\frac{766.025}{223.21} = 3.43$

-

CEEN241 - 2025-02-03

#notes #meche #ceen241

$$900N = \left(\frac{4}{5}\right) * AD$$

$$900/\frac{4}{5} = 1125N$$

---

force? i was a force once. they put me in a plane. an xy plane. an xy plane with cosines. the cosines made me a force. force? i was a force once

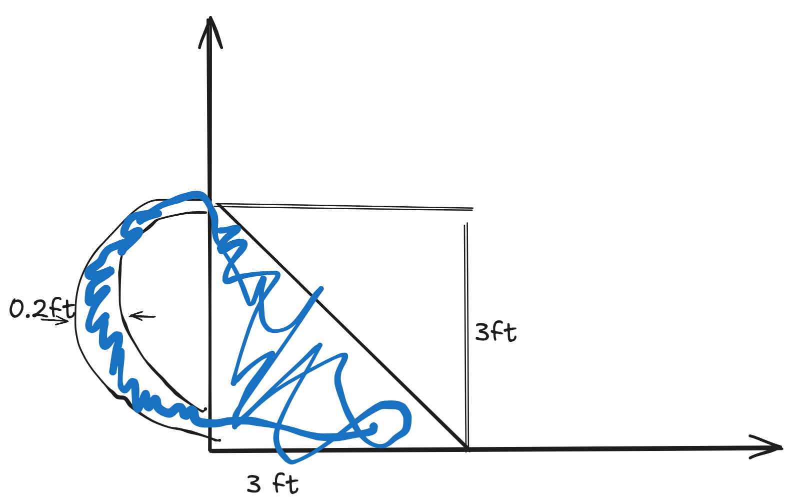
- Text me about this when you find this, whoever is reading my notes.

CEEN241 - 2025-02-10

#notes #meche #ceen241

Dealing with centroids and other such fun

- We often care about the average between two things
- Center of a rectangle is, quite shockingly, half the radius and half the height.



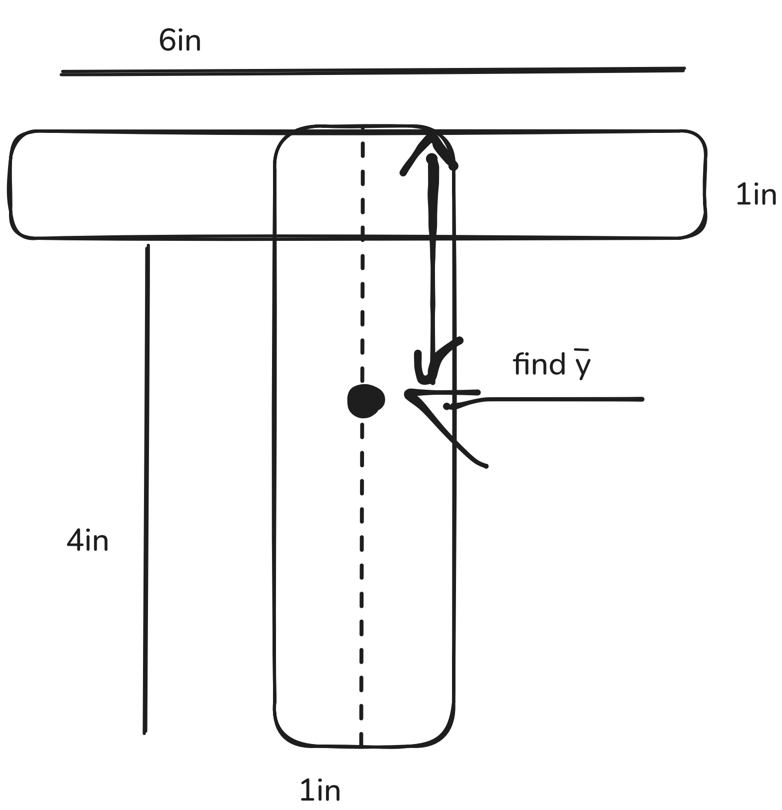
- We want to find $\bar{x}$ and $\bar{y}$

- The trick is really just to break this into components (I'm working backwards, and was busy doing comp sci)
- For that circle with the funky negative area, let's just worry about the outside
    - $r = 1.5ft$
    - $A_2 = \pi \frac{r^2}{2} = 3.53ft^2$
    - $\tilde{x} = -\frac{4r}{3\pi} = -0.64$ft
    - $\tilde{y} = +r = +1.5$
- For the inside bit of it,
    - $\tilde{x}_3 = \frac{-4r}{3\pi} = -0.53ft$
    - $\tilde{y}_3 = 1.5$ft
    - $A_3 = -\frac{\pi r^2}{2} = -2.65$ft
- For the triangle, it's just $\frac{1}{3}$ of our bits and bobs, meaning $\tilde{x}$ and $\tilde{y}$ are both 1ft

$$\bar{x} = \frac{\tilde{x}A_1 + \tilde{x}_2A_2 + \tilde{x}_3A_3}{A_1 + A_2 + A_3} = \frac{1 * 4.5 + -0.64 * 3.53 + -0.53 * -2.65}{4.5 + 3.53 - 2.65} = 0.678ft$$

$$\bar{y} = \frac{\tilde{y}_1A_1 + \tilde{y}_2A_2 + \tilde{y}_3A_3}{A_1 + A_2 + A_3} = \frac{1(4.5) + 1.5(3.53) + 1.5(-2.65)}{4.5 + 3.53 - 2.65} = 1.08ft$$

- New example, given:



- Line of symmetry down the middle was added, that's where $\bar{x}$ is going to be (at 3, by the by)

- I like splitting this fine fella into two rectangles
- Alright, so I care about the y coordinates, measured from the top, essentially, so that top one has a $\tilde{y}$ of 0.5 and an area of 6
  - The bottom rectangle, on the other hand, would have a $\tilde{y}$ of 3 and an area of 4
- So we're looking at

$$\frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2} = \frac{0.5(6) + 3(4)}{6 + 4)} = \frac{3 + 12}{10} = 1.5in = \bar{y}$$

-

CEEN241 - 2025-02-11

$$\bar{x} = \frac{\int \tilde{x} dA}{\int dA}$$

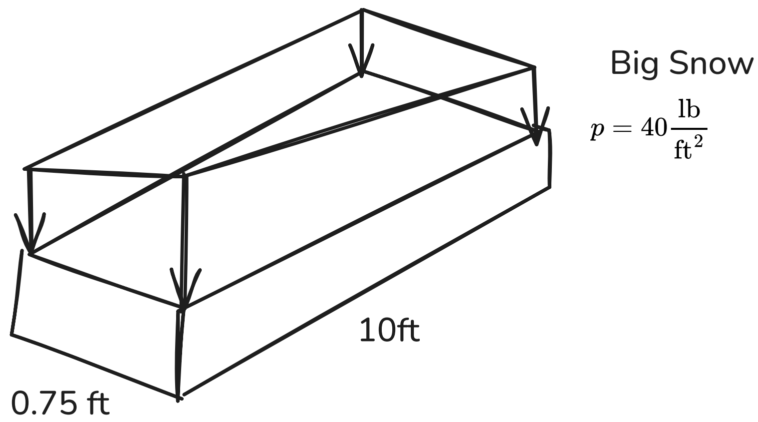
$$y = \frac{\int \tilde{y} dA}{\int dA}$$

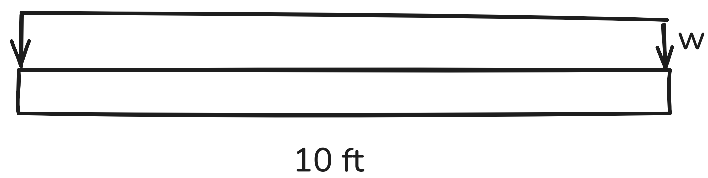
$$dA = y * dx$$

CEEN241 - 2025-02-12

#notes #meche #ceen241

Distributed Loads





w is a distributed load

$$w = 40\frac{\text{lb}}{\text{ft}^2}(0.75\text{ft})$$

$$w = 30\ \text{lb}/\text{ft}$$

- This is equivalent to some concentrated force, and, quite unfortunately, we need some concentrated force in order to do moments or any of the other fun stuff that we generally do.
  - The centroid here is just going to be the middle, and F_r is just going to be the area of our shape.

$$F_R = 30\ \text{lb}/\text{ft}\ (10\ \text{ft}) = 300\ \text{lb}$$

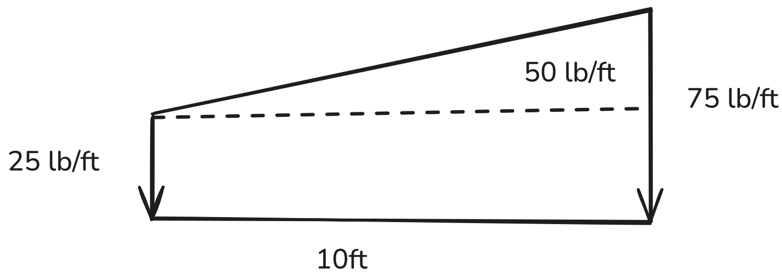
Triangular Load

- If our snow was instead a triangular setup, with some w equal to 100 lb/ft on the upper end of our triangle, and 0 lb/ft on the lower end of our triangle, with an L still of ten feet, it's still just the area, right?

$$\frac{1}{2}10\ \text{ft} * 100\ \text{lb}/\text{ft}\ = 500\ \text{lb}$$

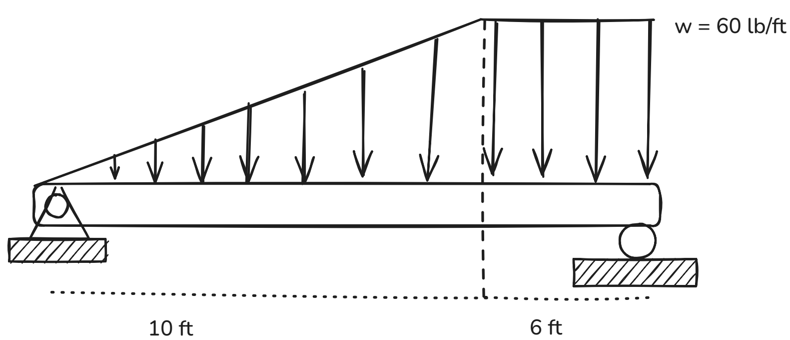
- The force would be acting itself over at $\frac{2}{3}10\text{ft}$

Trapezoidal Load



- That's, essentially, a rectangle and a triangle snipped together.
  - So the triangular bit is just going to be $F_{R1} = \frac{1}{2}(10\text{ft})(50\text{lb}/\text{ft})$
  - And then the rectangle is going to be $F_{R2} = 10\text{ft}(25\text{lb}/\text{ft})$
  - And then ya smack em together and bob's your uncle.
    - Also they're acting at their respective centroids, so it's marginally more complicated, but don't worry too much.

Egg



- Wow, I biffed this scale *bad*
- We're looking for equivalent resultant force and where it's acting from relative to A
- Alrighty, so actually solving this:
  - Sure looks awfully like we have a rectangle and a triangle.
    - Rectangle on the right would just be 60 lb/ft * 6ft, giving us a balmy 360 lb acting right around 13 ft from A
    - Triangle on the left would be a delightful $\frac{1}{2}b * h$, where we have $\frac{1}{2}10 * 60 = 300$lb, acting at 6.67 ft from A
  - So the total magnitude of our force is just going to be 660 lb.
    - We need our distance though, so the sum of our initial moments was 6681 lb*ft, so, you go and divide that by 660, and you get that the resultant force is acting right about 10.12 ft from A

-

CEEN241 - 2025-02-14

#notes #meche #ceen241

Rigid Body 2D FBDs

- We're isolating the system from the rest of the world, and replacing any physical supports with forces & moments.
- In order to do this, we gotta ask:
  - What does the support prevent?
  - What does the support allow?
- If the support prevents translation, then we replace it with a force reaction.
- If the support prevents rotation, then we replace it with a moment reaction.

Connections and Supports, oh my

Smooth Pin / Hinge

- Does... fuck all to rotation. Spin like crazy.
- Translation though? We stopping translation.

Roller

- Does nothing to translation parallel to it, but prevents translation down through it. (Perpendicular)

Rocker

- Basically the same as a roller

Fixed Support

- They just stick beams into the ground really hard. It can't go anywhere. It's crazy.

Connections

Cable

- Can only support pulling forces

Weightless Link (Two Force Member)

- Same vibe as a cable, line of action has to be along the thingy.

Frictionless (Smooth) Contact

- Allowed to move along, not allowed to move in, same vibe as a roler/rocker

Weird Shit

Roller or Pin in Slot

- Allowed to move up and down in slot, but can't exit the slot

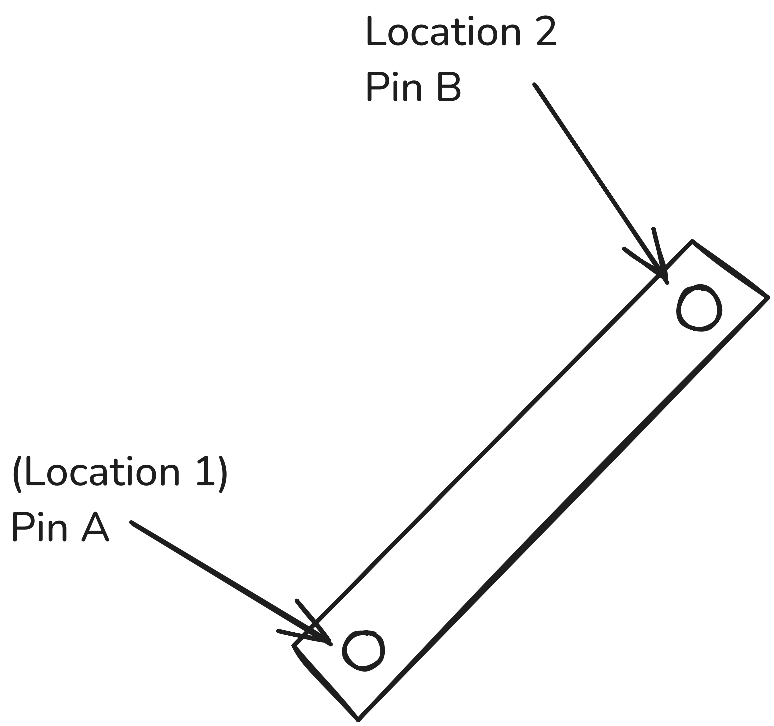
Member Pin on Smooth Rod

- Can move up and down on rod, but can't go through the rod

Moving on

- We're maintaining the same counterclockwise being positive sign convention.
- Lots of other discussion that i took no notes from

Two Force Members



- If there are only two locations on a rigid object with forces and no moments, you've got yourself a two force member.
- Line of action of reaction force always goes from Location 1 to Location 2
- If there are more than two locations, it's what we in the business call a

Multiforce Member

- More than two locations with forces *or* a moment applied anywhere on the system
-

CEEN241 - 2025-02-24

#notes #meche #ceen241

Tippy tippy outriggers

- At the limit for tipping, the faraway outrigger is going to have a whopping force of... zero!
- We, actually, only care about sum of moments. You can solve for the force if you feel like it, but that moment is going to be zero.

Other Notes

- Neil DeGrasse Tyson jumpscare?

One Unknown Force Supports

- Cables are just one force in the direction of the support
- Smooth surface and rollers are just a single force perpendicular to the surface

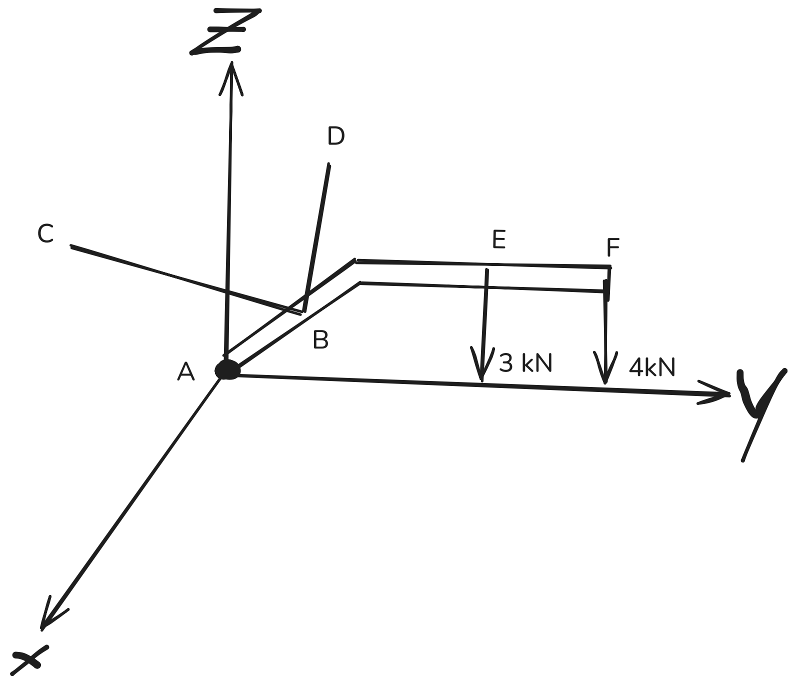
Other Supports

- Ball and sockets allow all rotation but do have F_x, F_y, F_z
- Single smooth pin
  - Can't rotate in one direction, allows rotation in the other two directions and prevents movement in all of them
- Single Thrust Bearing & Single Journal Bearing
  - Thrust bearings are designed to handle a thrust onto the shaft
  - Journal bearings, on the other hand, are not designed for that, and as such you discount that force
  - Also, they're bearings, they don't prevent rotation along the axis
- Shockingly, bearings with a square shaft in a square hole can't rotate.
  - That's.... not a bearing. Go fish.
- Single Hinge
  - Made to rotate along the axis of its shaft, no other movement allowed
- Fixed Support
  - Allows fuck all. It's fixed. It doesn't move.
- Moment reactions have a habit of poofing away
  - If you count up your unknowns and have more than six (our number of equations), your moments will end up poofing.
- Properly aligned bearings are important, fully rigid members are more important.
- You can do moment about axes in 3D! Ain't that neat.
-

CEEN241 - 2025-02-26

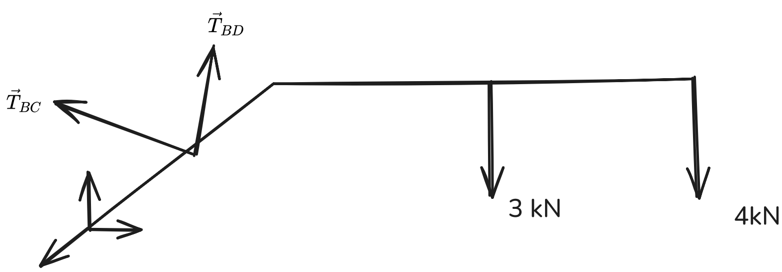
#notes #meche #ceen241

3D Rigid Body Equilibrium



$$A(0,0,0), B(0,1,1), C(2,0,3), D(-2,0,3), E(0,4,2), F(0,5.5,2)$$

Find: Reactions at A and in cables BC and BD



- All of our unknowns are $A_x, A_y, A_z, T_{BC}, T_{BD}$
- I mean, there's a whole problem solving approach where you can like roadmap and find all the unknowns
  - $\sum F_x = 0$, unknowns A_x, T_{BC}, T_{BD}
  - $\sum F_y = 0$ unknowns A_y, T_{BC}, T_{BD}
  - $\sum F_z = 0$ unknowns A_z, T_{BC}, T_{BD}
  - $\sum M_{A,x} = 0$ unknowns $M_{T_{BC}}, M_{T_{BD}}$
  - $\sum M_{A,y} = 0$ unknowns $M_{T_{BC}}, M_{T_{BD}}$
  - $\sum M_{A,z} = 0$ unknowns $M_{T_{BC}}, M_{T_{BD}}$
- Alright mathing time

$$\vec{M}_A = \vec{r}_{AB} \times \vec{T}_{BC} + \vec{r}_{AB} \times \vec{T}_{BD} + \vec{r}_{AE} \times -3\vec{k} + \vec{r}_{AF} \times -4\vec{k}$$

$$\vec{r}_{AB} =< 0,1,1 >,\ \vec{r}_{AE} =< 0,4,2 >,\ \vec{r}_{AF} =< 0,5.5,2 >$$

$$\vec{u}_{BC} = \frac{\vec{r}_{BC}}{r_{BC}} =< \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} >$$

$$\vec{u}_{BD} =< -\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} >$$

$$\boxed{\vec{T}_{BC} =< \frac{2}{3}T_{BC}, -\frac{1}{3}T_{BC}, \frac{2}{3}T_{BC} >}$$

$$\boxed{\vec{T}_{BD} =< -\frac{2}{3}T_{BD}, -\frac{1}{3}T_{BD}, \frac{2}{3}T_{BD} >}$$

\vec{r}_{AB}\times \vec{T}_{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0

\end{vmatrix} = T{BC}\hat{\imath}+ \frac{2}{3}T{BC}\hat{j}- \frac{2}{3}T_{BC}\hat{k}

$$\$\$\vec{r}_{AB} \times T_{BD} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 1 \\ -\frac{2}{3}T_{BC} & -\frac{1}{3}T_{BC} & \frac{2}{3}T_{BC} \end{vmatrix} = T_{BD}\hat{\imath} - \frac{2}{3}T_{BD}\hat{\jmath} + \frac{2}{3}T_{BD}\hat{k}$$

$$\vec{r}_{AE} \times -3k = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 4 & 2 \\ 0 & 0 & -3 \end{vmatrix} = -12\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

$$\vec{r}_{AF} \times -4k \begin{vmatrix} = \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 5.5 & 2 \\ 0 & 0 & -4 \end{vmatrix} = -22\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

- Wow, that's all a pain in the ass.

$$T_{BC} = T_{BD} = 17kN$$

- Awful.
  $$

CEEN241 - 2025-02-28

#notes #meche #ceen241

Straight Trussing It.

- Cheap, easy, *and* strong. Holly jolly.

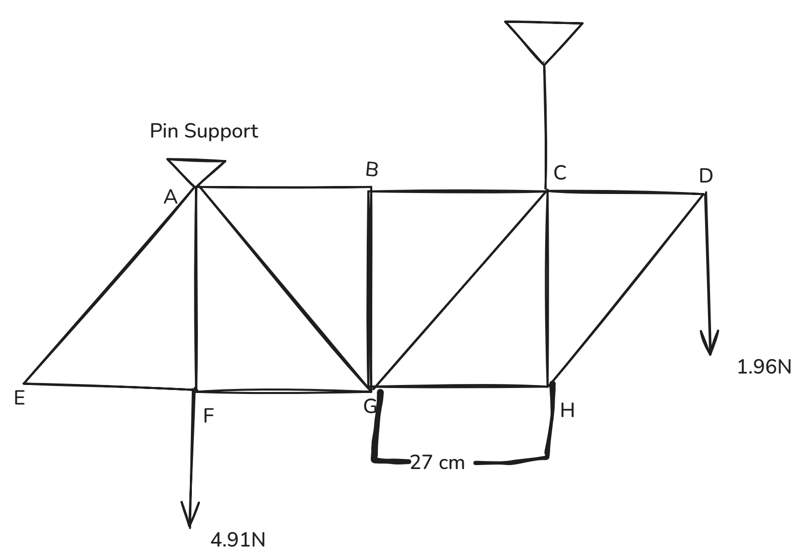
Special Properties of Trusses

- △ are massive.
  - I mean like we love △ up in here
    - Why do anything else when you could have ⊿
- It's 📌s all the way down.
  - Note from later in the lecture: This whole "all pin" thing is bullshit. They just gotta act pin-ish as long as they're two force.
- All loads are applied at the joints.
  - If you apply a load at the beam, it becomes a multiforce member
  - Also, we're ignoring the weight of the truss members.
- All two force members in trusses.
- If forces pull a member apart it's under tension, it's under compression otherwise.

How do we do these things?

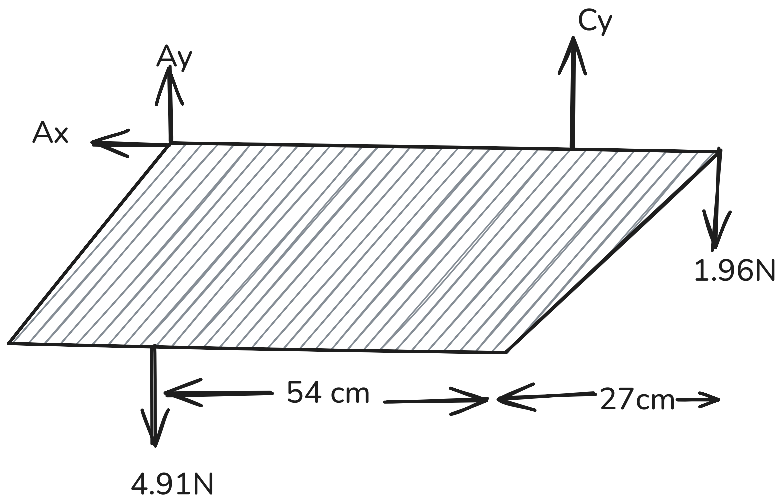
- That's right. You guessed it. We do an FBD of the whole damn thing.
  - You're never gonna guess. $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0, \sum M = 0$
- Do FBD of a joint to find forces in the members of the truss.
  - Assume any unknown is in tension.
    - You like, could assume that everything's under compression, but this creates a sign convention where positive is tension is negative is compression.
    - Each joint only has $\sum F_x = 0, \sum F_y = 0$
      - It's basically a particle, there are no distances, so ya can't do moments. Womp to the womp.

Example Time



- We want to find force in DH, CD, CH, CG, CB, GH, and find all zero force members.

First up is an FBD of the whole thing to find support reactions.



$$\sum M_A = 0$$

$$C_y(54) - 1.96(81) = 0$$

$$\boxed{C_y = 2.94N}$$

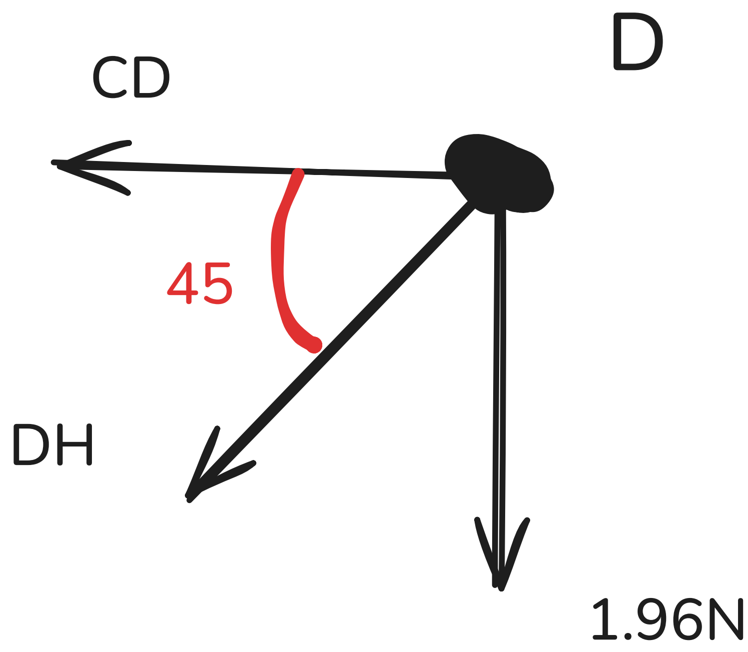
$$\sum F_y = 0$$

$$A_y + C_y - 4.91 - 1.96 = 0$$

$$\boxed{A_y = 3.93N}$$

$$\sum F_x = 0, \sum F_x = A_x = 0$$

Pick an FBD of a Joint with Knowns and not two many unknowns.



- How howdy. Doesn't it look like we only two unknowns? Ain't that neat

$$\sum F_y = 0, 0 = 1.96 - DH\sin(45)$$

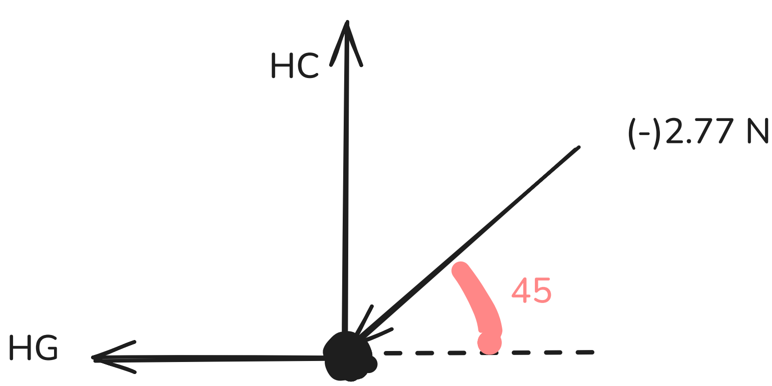
$$DH = -2.77N$$

- Oh hey, a negative means we're under compression. Neato.

$$\sum F_x = 0, \quad -2.77\cos(45) = CD$$

$$CD = 1.96N$$

Now we keep on joint chopping. I want a nap.

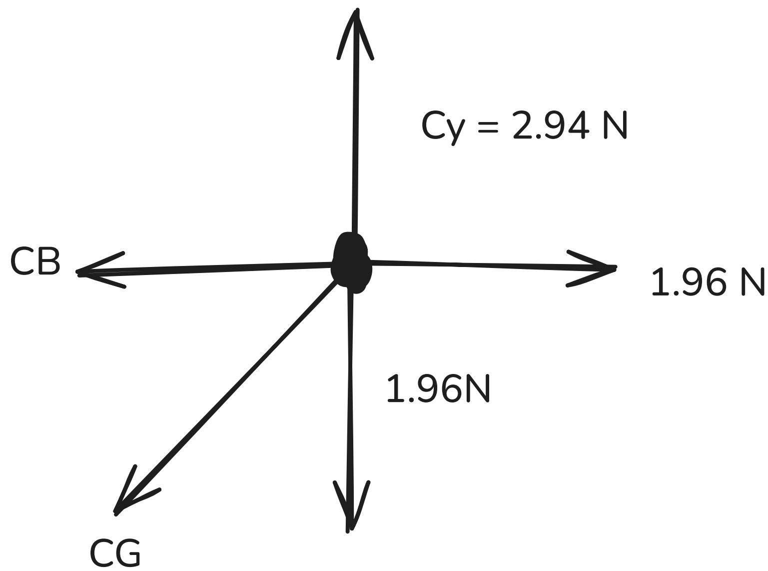


$$2.77\cos(45) = HG$$

$$HG = -1.96N$$

$$HC = 2.77\sin(45)$$

$$HC = 1.96N$$



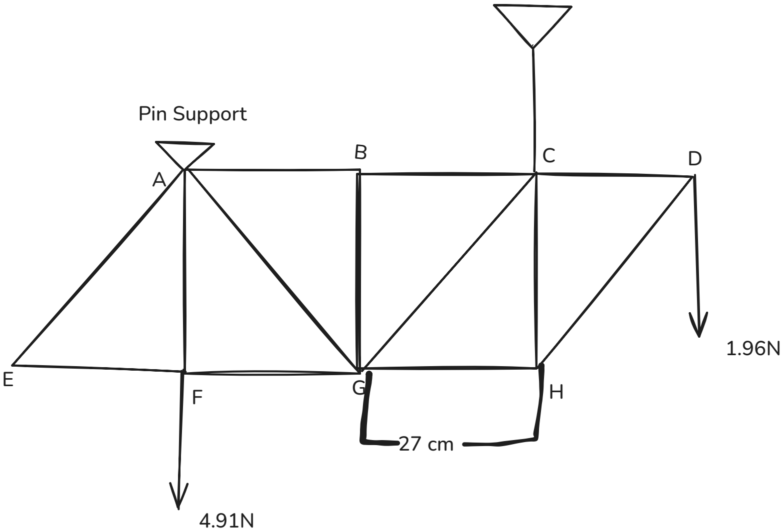
CEEN241 - 2025-03-03

#notes #meche #ceen241

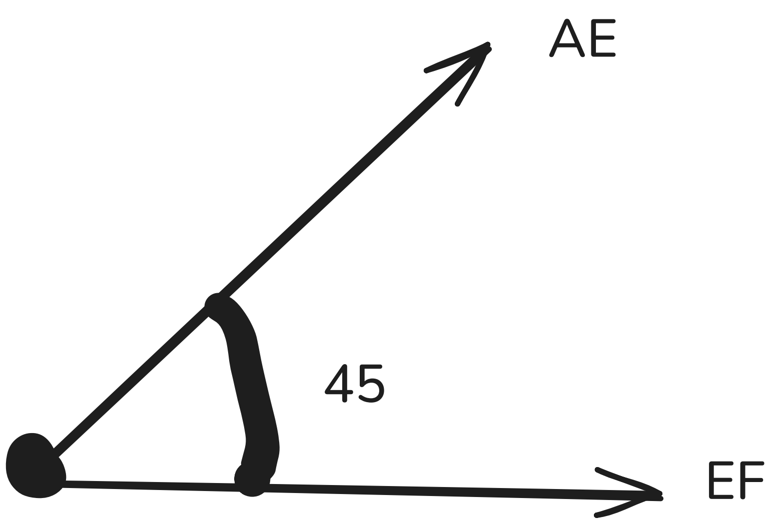
continuing work with our yardstick truss from CEEN241 - 2025-02-28

Zero Force Members

- Draw an FBD at a joint that meets the following criteria
  1. Only two members meet at a joint
  2. (or) Three members meet and two are colinear
- Why do we have zero force members?
  - Different loading conditions (ie, it's zero force *in this case*)
  - Overall stability so it doesn't get all wiggly wonkly
- Sooooo, back to our example



- Alright, so our conditions give us two possibilities
  Where only two meet, in which case we're looking at D & E
- D is out for sure.
- At E, on the other hand



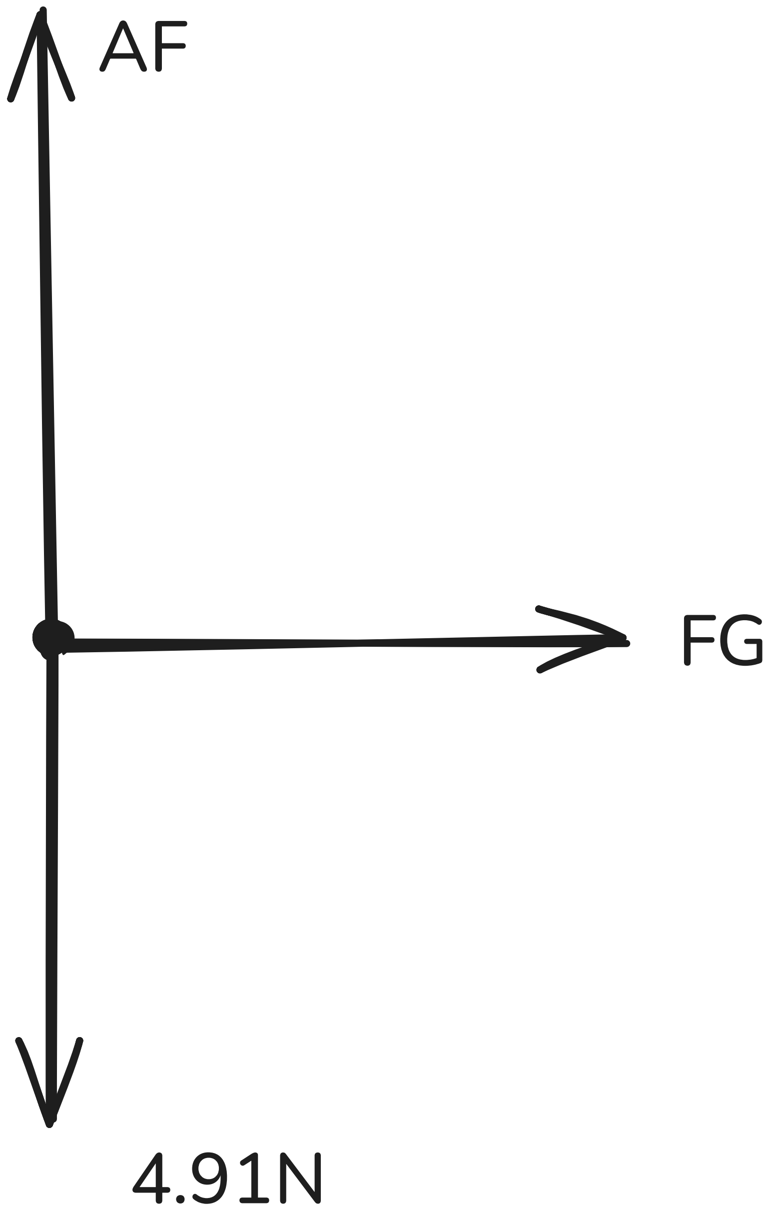
$$\sum F_y = 0$$

$$AE \sin(45) = 0, AE = 0$$

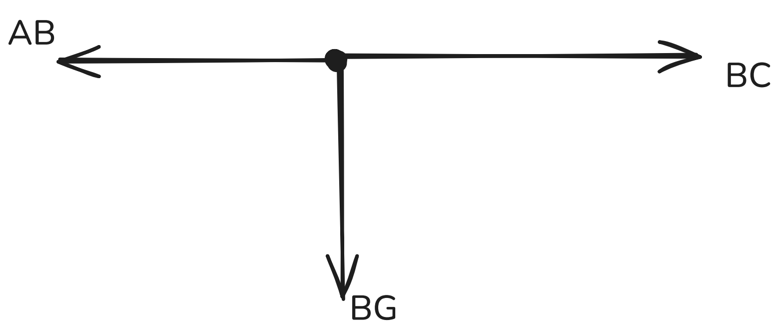
$$\sum F_x = 0$$

$$AE \cos_4 5 + EF = 0, EF = 0$$

- Oh hey, they're both zero, we've searched and destroyed. Now F is a member that only has two, ain't that neat



- That sure looks like FG is going to have to be zero.



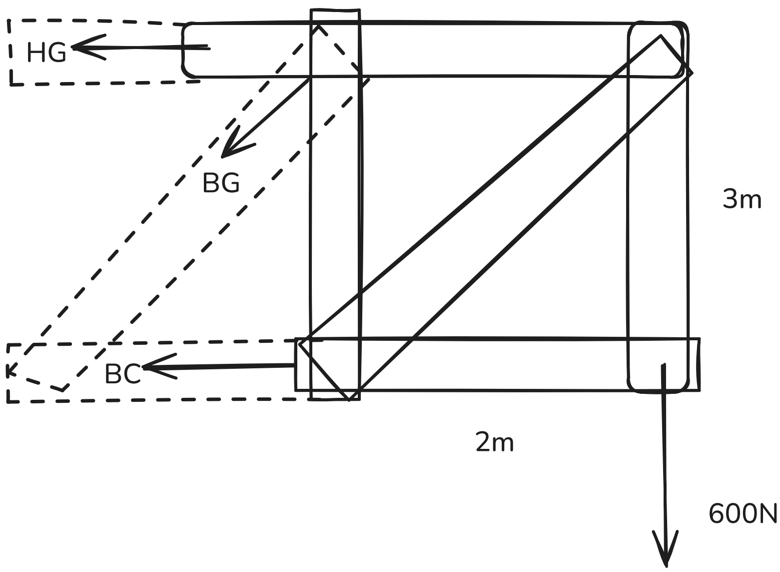
$$\sum F_y = 0 = BG$$

- Oh boy, another zero force, get that shit *OUTTA HERE!*
- One FBD will never contradict another FBD

There's really two ways of doing trusses

- We just covered method of joints
- And now we're doing

Method of Sections



$$\sum F_y = 0, -BG\sin(56.31) - 600 = 0, BG = -721.1N$$

$$\sum M_B = 0 = HG(3) - 600(4), HG = 800$$

$$\sum F_x = 0 = -HG - BG\cos 56.31 - BC = 0$$

$$-800 - (-721.1)\cos_5 6.31 - BC = 0, BC = -400N$$

CEEN241 - 2025-03-06

#notes #meche #ceen241

Hands on 3

$$m_1 = 0.1kg$$

$$m_2 = 0.k2$$

- 13 on the way right
- 37.9 for m2
- 63.5 for m1

8.5, 54, 95.5

m=550g, w = 0.7 N

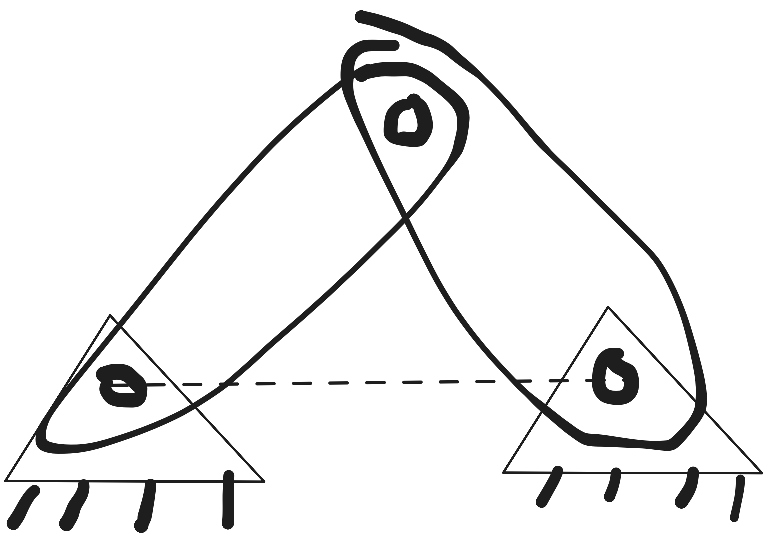
CEEN241 - 2025-03-10

#notes #meche #ceen241

Frames (and comparing them to Trusses)

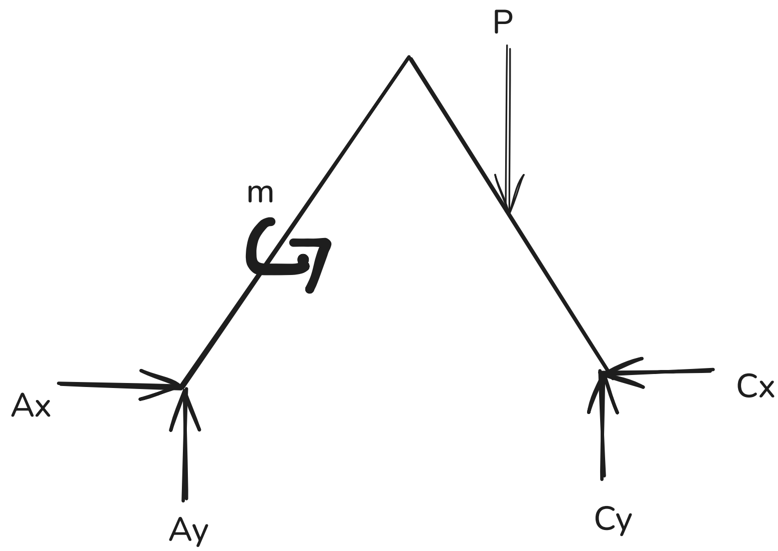
- We now have loading in between joints (trusses just had them at the joints)
- Here we look at FBDs of individual members or of groups of members
- No cuts (method of sections) through multiforce members

Textbook Example 6.9

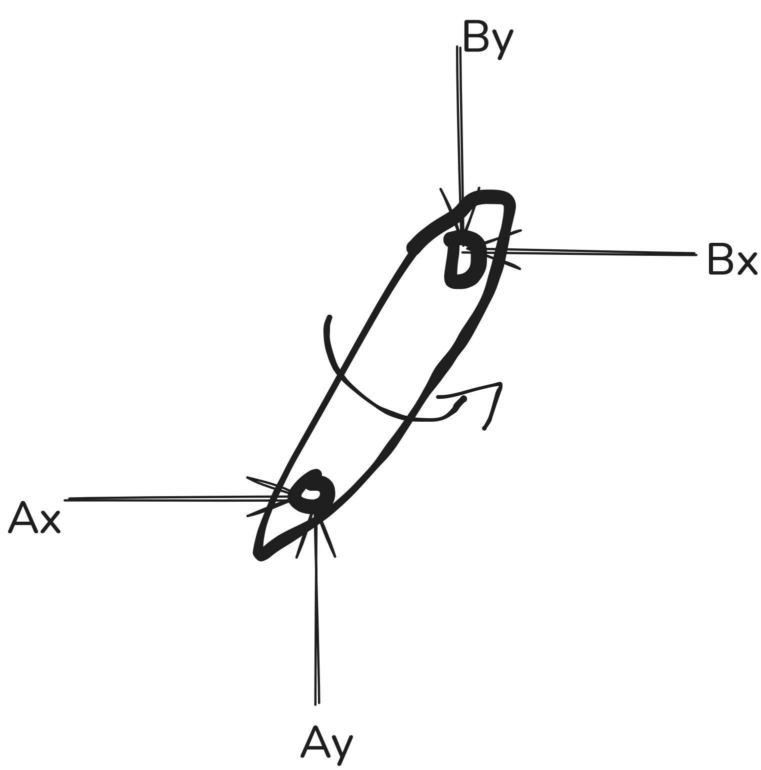


1. ID all member types
2. ID all possible FBDs
3. Use equilibrium equations to solve

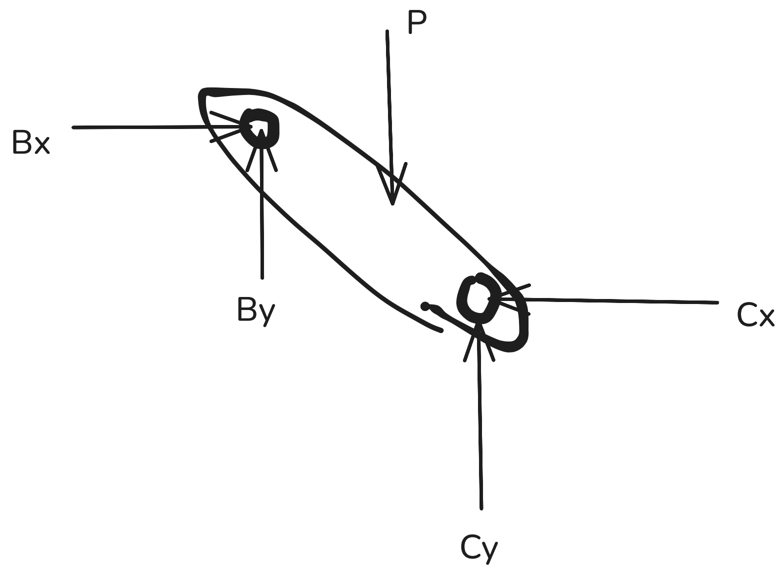
FBD of the whole



FBD of AB

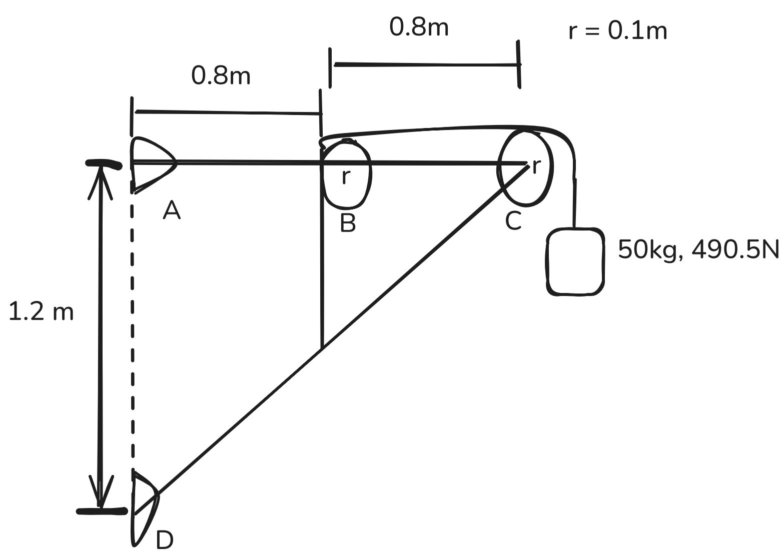


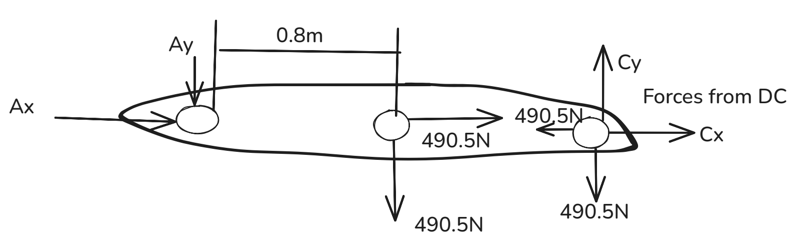
FBD of AC



- Solution is to go off the FBD of the whole, find $\sum M_A$, find C_y, find $\sum F_y = 0$, find A_y
  - Just go FBD by FBD, knocking out what you can with the knowns you got until you get more knowns and can keep going

Textbook Problem 6.102



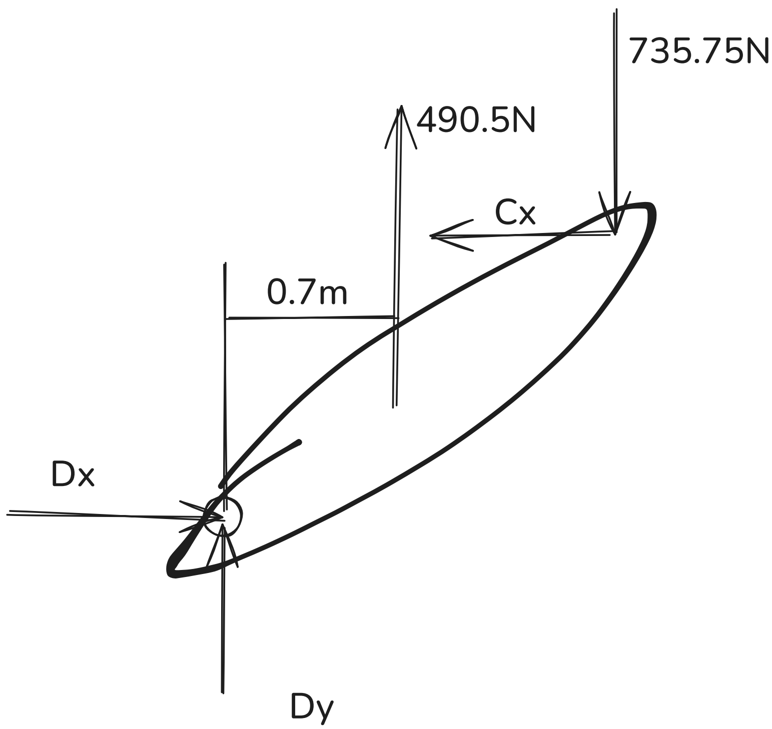


$$\sum M_A = 0$$

$$-490.5(0.8m) - 490.5(1.6)m - C_y(1.6m) = 0$$

$$C_y = -735.75N$$

- Negative just means the opposite of how it's drawn



$$\sum M_D = 0 = 490.5(0.7) - 735.75(1.6) + C_x(1.2)$$

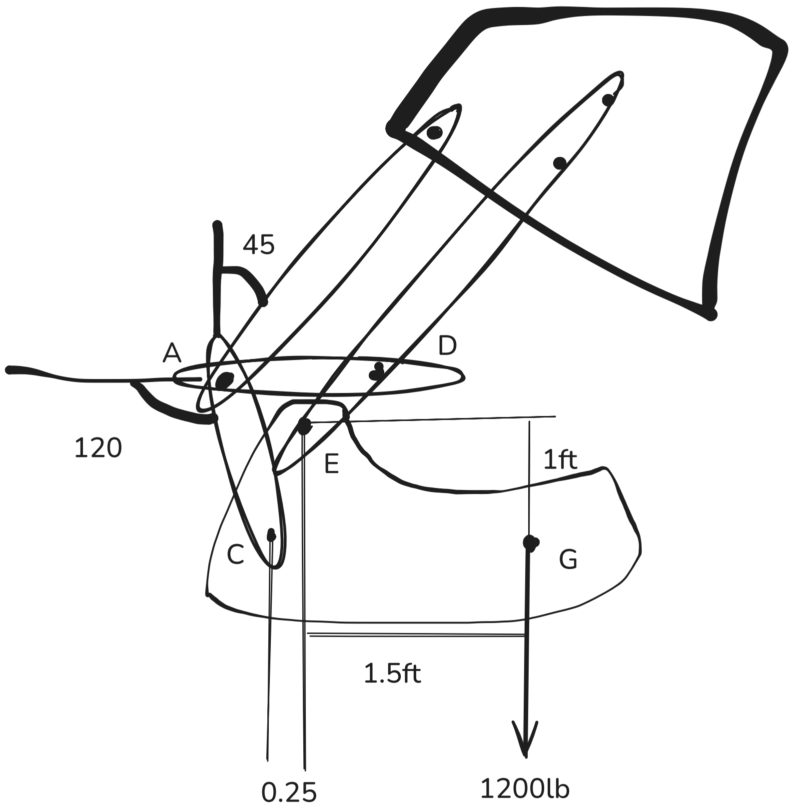
$$C_x = 694.88N$$

CEEN241 - 2025-03-14

#notes #meche #ceen241

Doing Machines, which are basically just frames

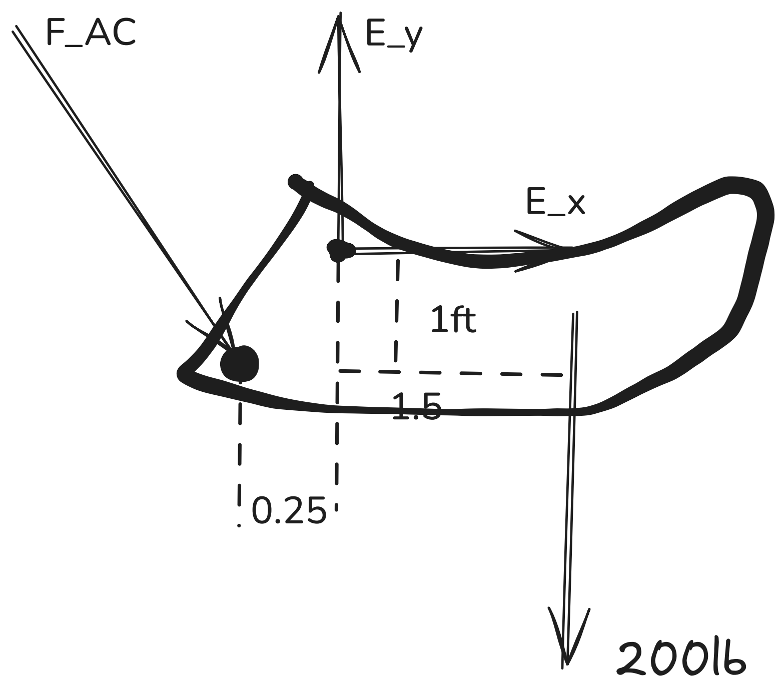
- The bucket of a backhoe and it's contents have a weight of 1200 lb and a center o fgravity at G. Determine the forces of the hydraulic cylinder AB and in links AC and AD in order to hold the load in the position shown. The bucket is pinned at E



- Find AB, AC, and AD
    - Wow, I hate that drawing, a lot

Rigid Object	Type
AC	2 force member
AD	2 force member
AB	2 force member
ECG	Multi
EDB	Multi

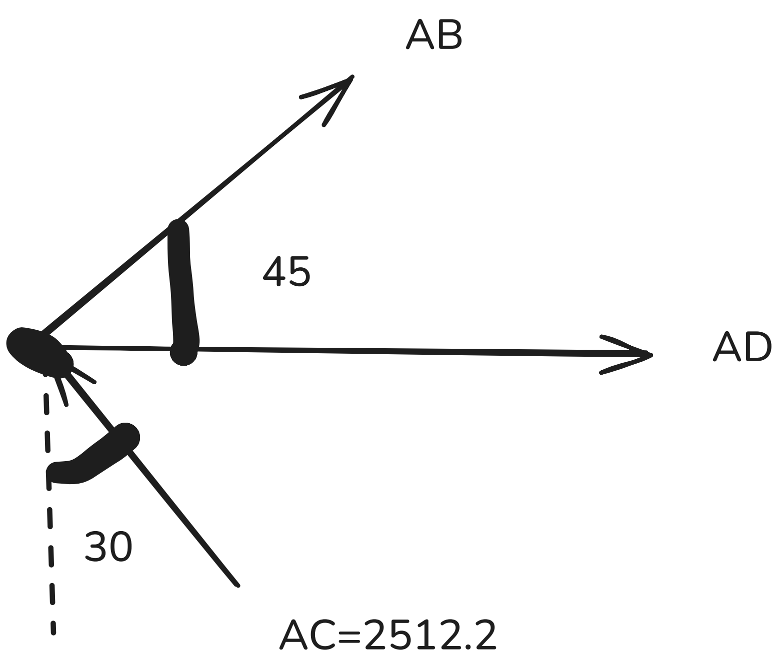
- FBD of ECG has a known and makes a great place to start (ok how the fuck do we actually draw it now)



$$F_{AC}\sin(30)(1ft) + F_{ac}\cos(30)(0.25ft) - 1200(1.5)ft = 0$$

$$F_{AC} = +2512.2\ lb$$

- We could solve for E_x and E_y by just using $\sum F_x = 0$, but they're not asking, so lowkey.... don't care.
- FBD at joint A (this is what I tried to do and did wrong on one of the homeworks. jo comprendo.)



$$\sum F_y = 0$$

$$AB\sin(45) + AC\cos(30) = 0$$

$$AB = -3076.8\ lb$$

$$\sum F_x = 0$$

$$AB\cos(45) + AD - AC\sin(30) = 0$$

$$-3076.8\cos(45) + AD - 2512.2\sin(30) = 0$$

$$AD = 3431.72\ lb$$

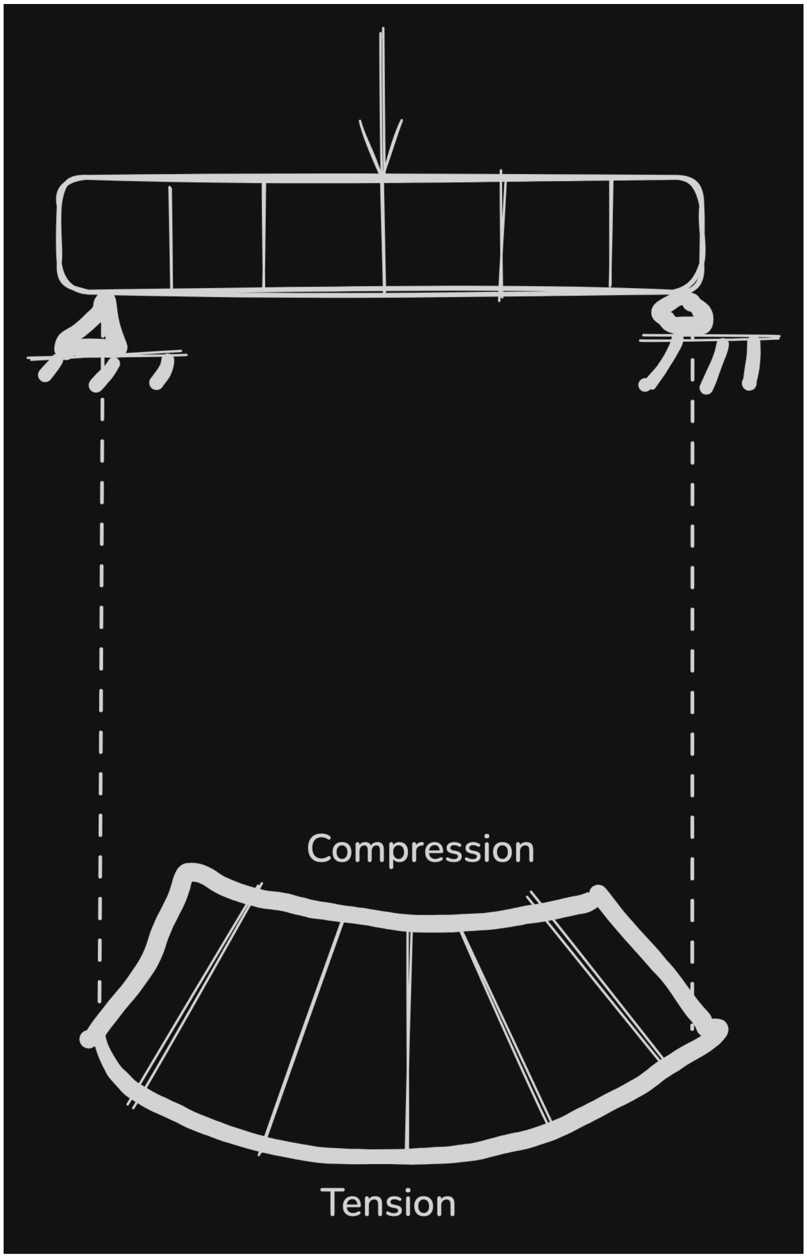
CEEN241 - 2025-03-24

#notes #meche #ceen241

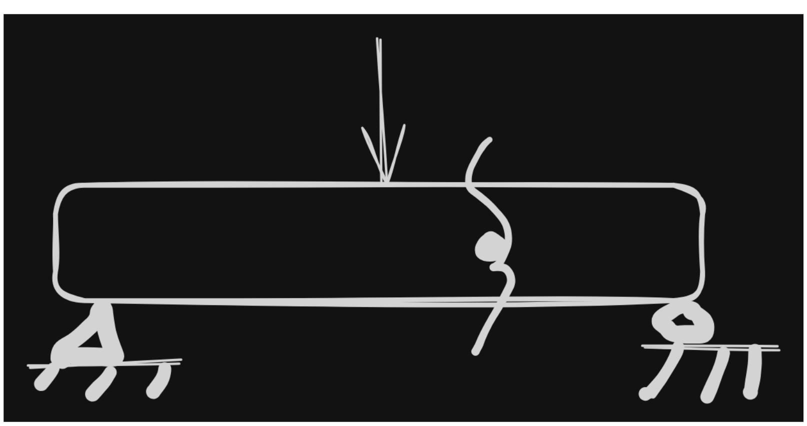
Internal Force Adventure

- Find the internal forces when you cut things, draw an FBD, other such fun adventures

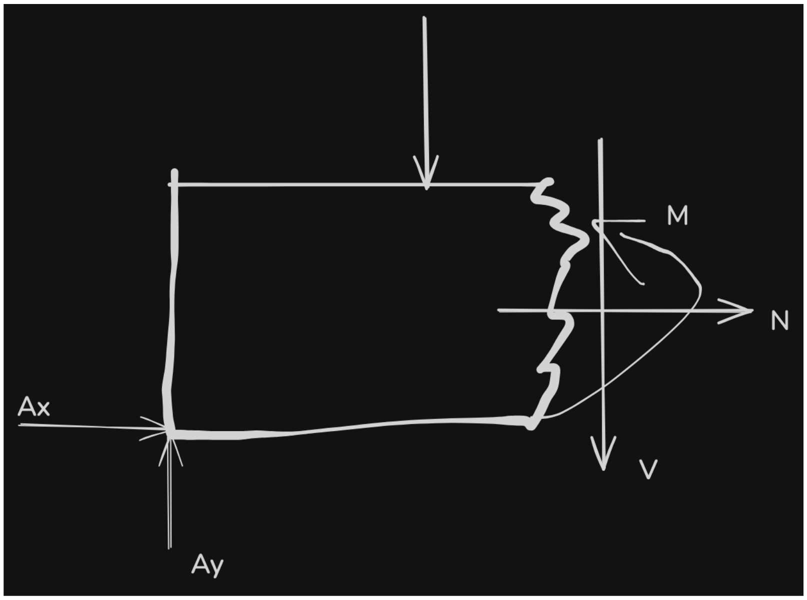
- Some beam under tension would spread apart, beam under compression would squish together
- So if you had some concrete beam, where'd you stick the rebar?
- I voted for the top, let's see how it goes
- That was wrong. The bottom is under tension



- This is referred to as a bending moment, with compression on one side and tension on the other
- New question: What is exposed at a cut?
  -

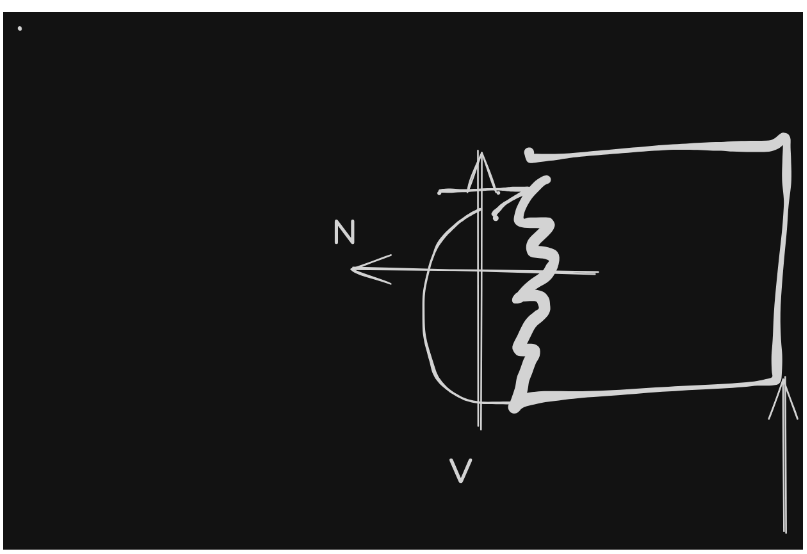


- If we're looking for internal forces at that point, we're going to have to do an FBD on both sides of the cut

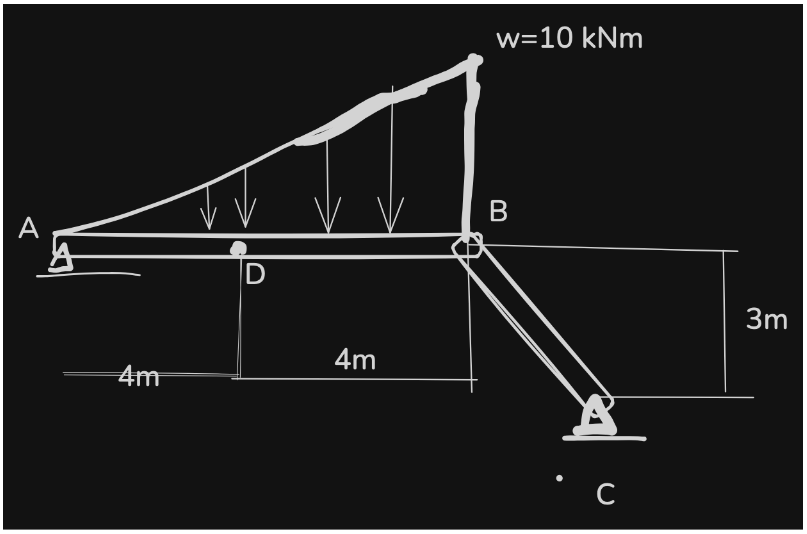


Sign convention for internal forces

- N is the normal force (perp to cut) and is positive in tension
- V is the shear force which is positive
    - Shear force is positive when it rotates the object clockwise
- M, the bending moment is positive when in tension on bottom and compression on top
    - Gotta "pull" on the bottom and "push" on top

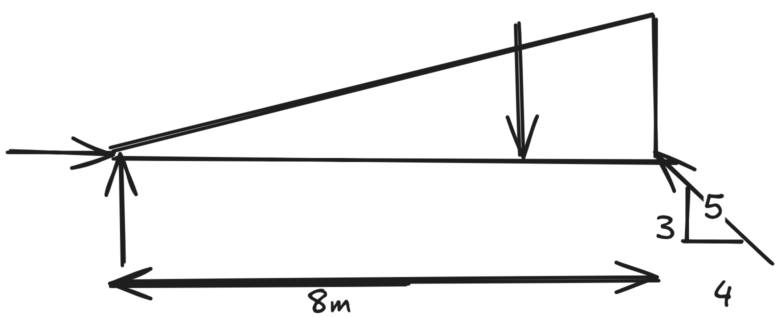


Moving In to an Example



- Looking for internal forces at D and in BC
- Solution:
  - Going to need an FBD of AB to find support reactions

F_R = 40kN



$$
\sum M_{A}=0, F_{BC}=44.4kN

\sum F_{x}=0,A_{x}=35.5kN

\sum F_{y}=0,A_{y}=13.3kN

$![[CEEN241-2025-03-242025-03-2410.40.55.excalidraw]]$

\sum F_{x}=0, 35.5+N_{D}=0,N_{D}=-35.5kN
$$

- Combined loading is where you combine all the compression loads, and that's a mechanics problem

$$\sum F_y = 0,\ \ 13.3 - 10 - V_D = 0, V_D = 3.3kN$$

$$M_D + 10kN\left(\frac{1}{3}(4)\right) - 13.3(4) = 0,\ \ m_D = 40kN - m$$

- For the internal forces in BC, a cut would be perpendicular to the member, a force coming from the other beam of 44.4 kN, perpendicular, and N acting along it
-

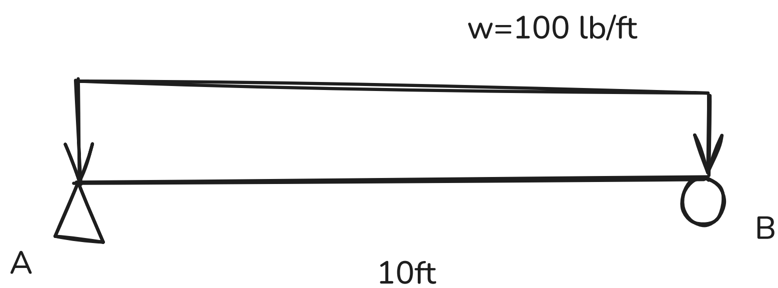
CEEN241 - 2025-03-26

#notes #meche #ceen241

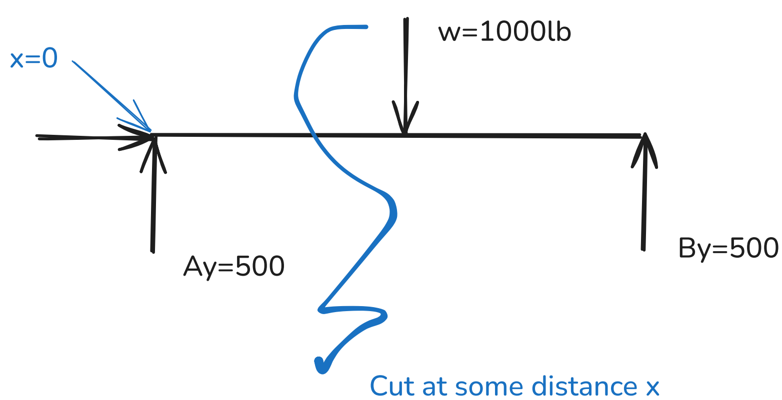
V And M (Shear and Bending) Diagrams

- So what we're doing is we're making cuts at some distance x

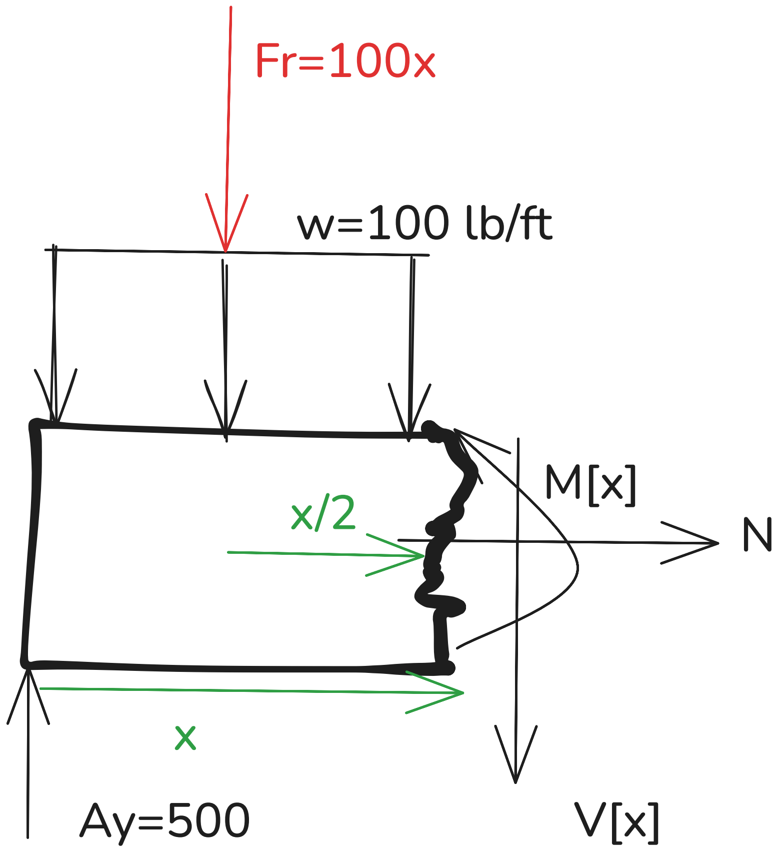
Posthaste into an example



- Find V[x], M[x], and diagrams
- We're going to do an FBD of the whole thing. Because this is statics, and why not.



- Lookin at ye olde FBD to the left of the cut



- You generally have to make a new cut when your FBD changes
  - You could probably say that the cut is valid from $0 \leq x \leq 10$, which works in abstract math land but would get weird if you were actually dealing with supports.

$$\sum F_y = -V[x] - 100x + 500 = 0$$

$$V[x] = 500 - 100x$$

$$\sum M_{cut} = 0 \circlearrowleft_{+ccw}$$

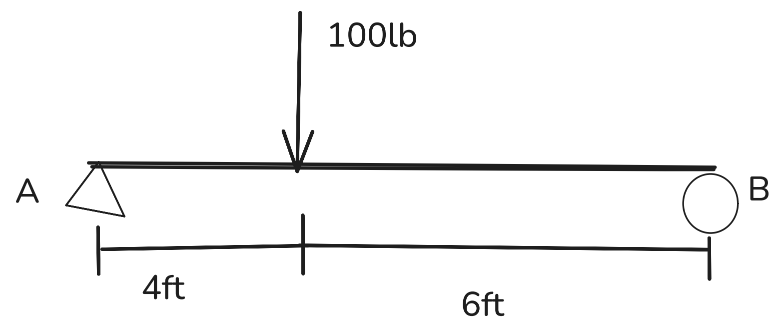
$$+M[x] + 100x * \frac{x}{2} - 500x = 0$$

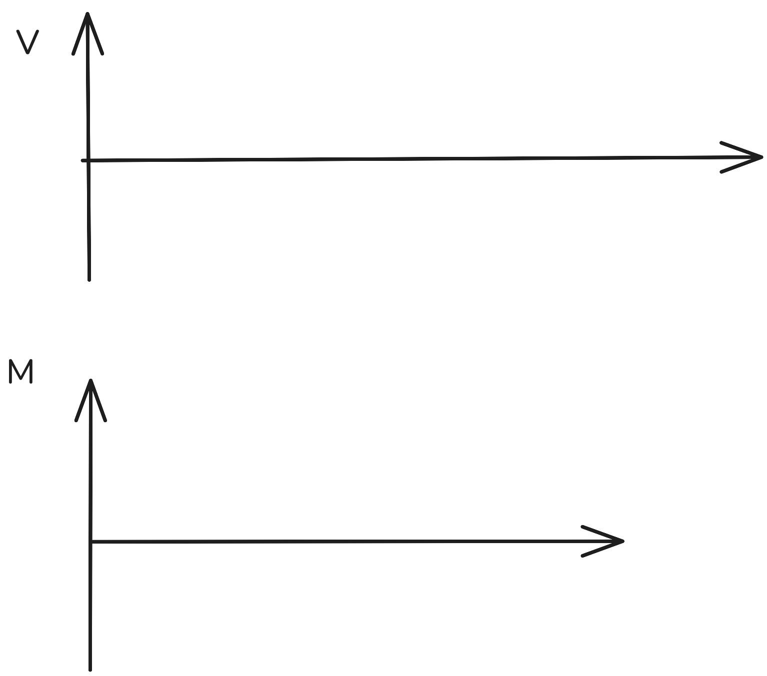
$$M[x] = 500x - \frac{100x^2}{2}$$

- Relationships between w, V, and M
  - Slope of $V = w$, $\frac{dV}{dx} = w$
  - $\Delta V = \int w dx$
  - Slope of $M = v$, $\frac{dm}{dx} = v$
  - $\Delta m = \int v dx$
- Doing some solving
  - $V[x] = 500 - 100x$
  - $500 - 100x = 0, x = 5ft$

- $M_{max} = M[5] = 500(5) - \frac{100*5^2}{2} = 1250ft - lb$
- You can also use Δm to find M_{max} which is $\Delta m = \int vdx$
- Area under shear is $(\frac{1}{2})(500)(5)$
    - Those triangles are just going to be ± 1250
- The graphical approach uses relationships with the shapes and such to draw V and M

New Example Time (also friday is going to be the same thing)





$$A_y = 600lb$$

$$B_y = 400lb$$

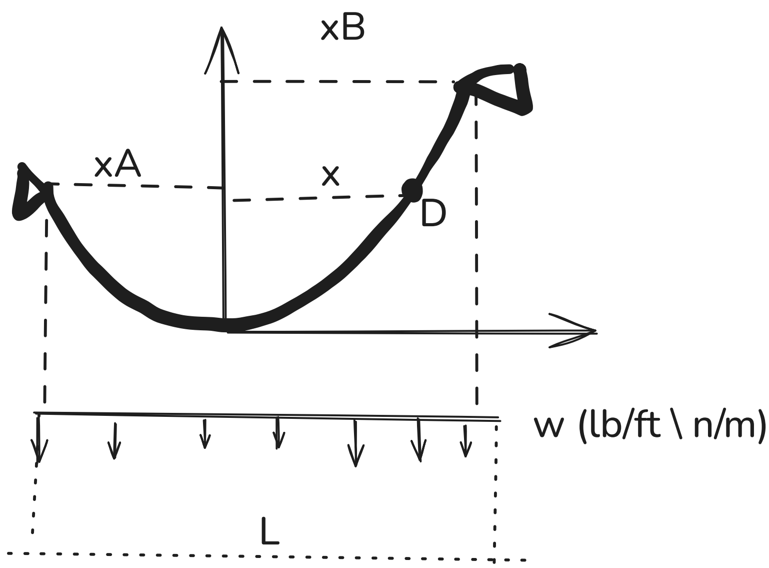
- Focus on starting/ending values, slopes, and $\Delta V, \Delta m$
-

CEEN241 - 2025-04-02

#notes #meche #ceen241

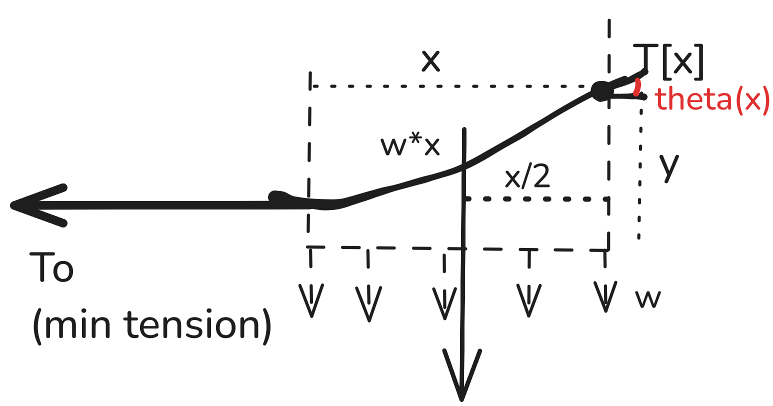
Cables Subject to Distributed Loads

- Lowest point has the least tension



- Origin of x-y goes at lowest point where slope $= 0$
- When supports are at the same elevation, origin is halfway between

FBD of CD



$$\sum M_D = 0 \, CCW$$

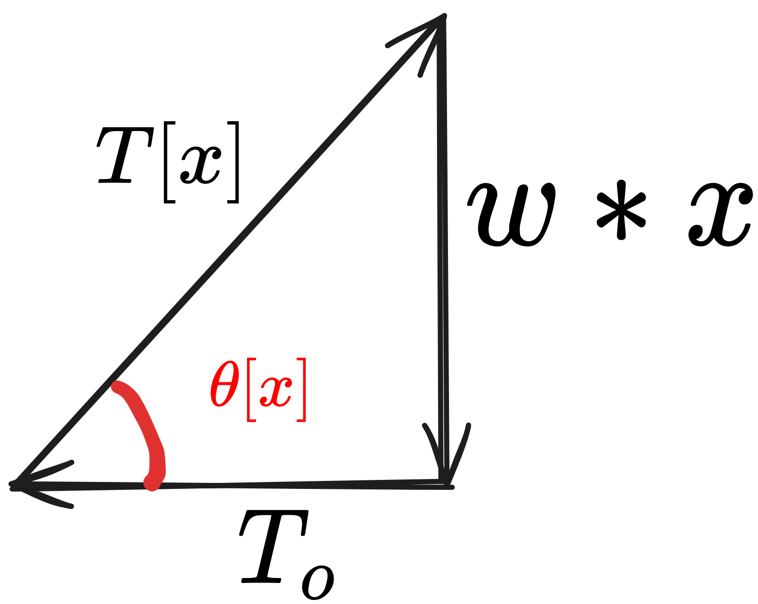
$$wx\left(\frac{x}{2}\right) - T_0 * y = 0$$

$$y = \frac{wx^2}{2T_0}$$

- And that's the actual slope/shape of the cable.

$$T_o = \frac{wx^2}{2y}$$

- We're going to "use a force triangle," whatever the hell that means
  - Force triangle implements $\sum F_x$ and $\sum F_y$ simultaneously.



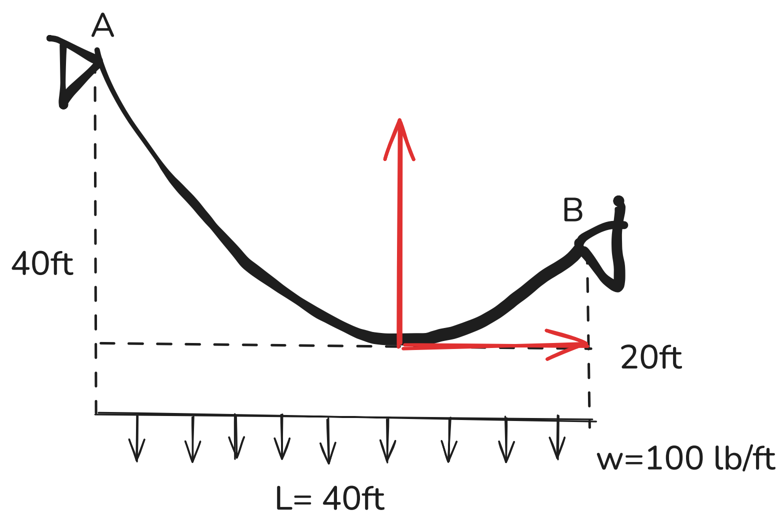
$$T[x] = \sqrt{T_o^2 + (wx)^2}$$

$$\theta[x] = \tan^{-1}\left(\frac{wx}{T_0}\right)$$

Fun aside, there are only really three other relevant possible FBDs

- The FBD of the entire damn system (I don't want to draw these because *I'm lazy*)
  - we can use $\sum F_x, \sum F_y, \sum M$, only problem is that's three equations and we have four unknowns
- FBDs to the left and right of the low point
  - They'd both have a T_o like we just drew
  - Problem is we don't know x_A or x_B to either side, but we do know we have resultant forces from our distributed loads
  - We still have $\sum F_x, \sum F_y, \sum M$, but also not a convenient three unknowns.

Example



- We want to find T_o, T_{max}
- We have two options: Do this with some FBD shenanigans, or just jump straight into the neat equations we derived

$$y = \frac{wx^2}{2T_o}$$

- We don't know much, but we can figure out where the low point is going to be.

$$y_A = \frac{wx_A^2}{2T_0}$$

$$y_B = \frac{wx_B^2}{2T_0}$$

$$w = 100, y_A = 40, y_B = 20$$

- This makes our unknowns x_A, x_B, T_0
  - That looks like two equations and three unknowns. Great news: $x_A + x_B = L = 40$
- To go about solving this thing, why not smack em together

$$\frac{40 = \frac{100x_A^2}{2T_0}}{20 = \frac{100x_B^2}{2T_0}} = 2 = \frac{x_A^2}{x_B^2}$$

$$x_B = 40 - x_A$$

$$2 = \frac{x_A^2}{(40 - x_A)^2}$$

$$2(40 - x_A)^2 - x_A^2 = 0$$

$$x_A = 136.5ft \text{ or } 23.43ft$$

- 136.5 is what we in the business call out of bounds, so $x_A = 23.43$ feet, meaning that $x_B = 16.57$ feet

$$T_o = \frac{wx_A^2}{2y_A} = \frac{100 * (23.43)^2}{80} = 686.08 \text{ lb}$$

- If we go out to solve for T_{max}, it's going to be at A since that has the steepest slope

$$T[x] = \sqrt{T_o^2 + (wx)^2}$$

$$T[23.43] = \sqrt{686.08^2 + (100(23.43))^2} = T_{max} = T_A = 2440 \text{ lb}$$

CEEN241 - 2025-04-07

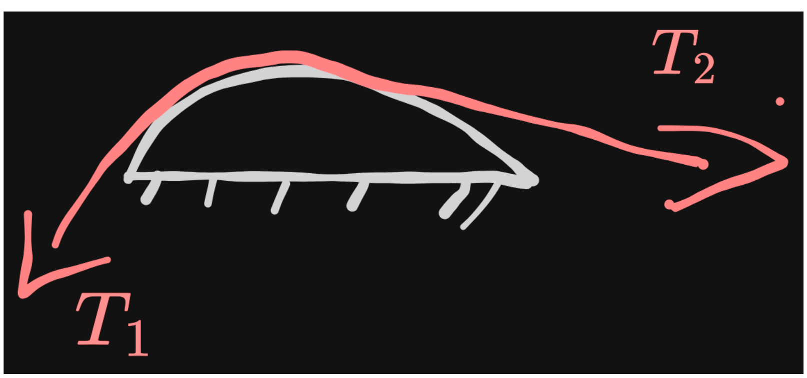
#notes #meche #ceen241

$$T_1 = T_2 e^{\mu\beta}$$

Get Friction-y with it (I totally didn't miss friction day 1)

- Friction goes opposite the direction of impending motion, as you would expect

Friction on Flat Belts

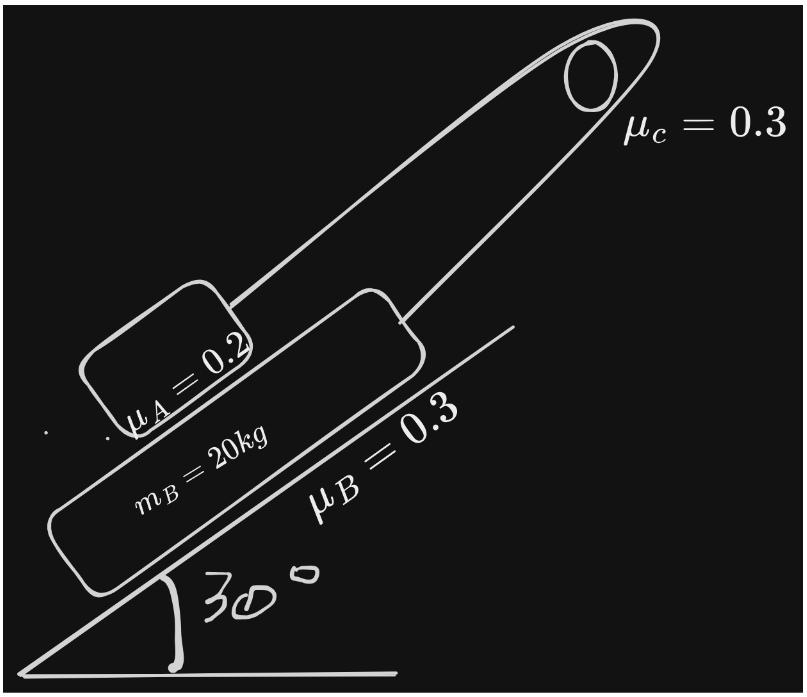
- 

$$T_1 = T_2 e^{\mu\beta}$$

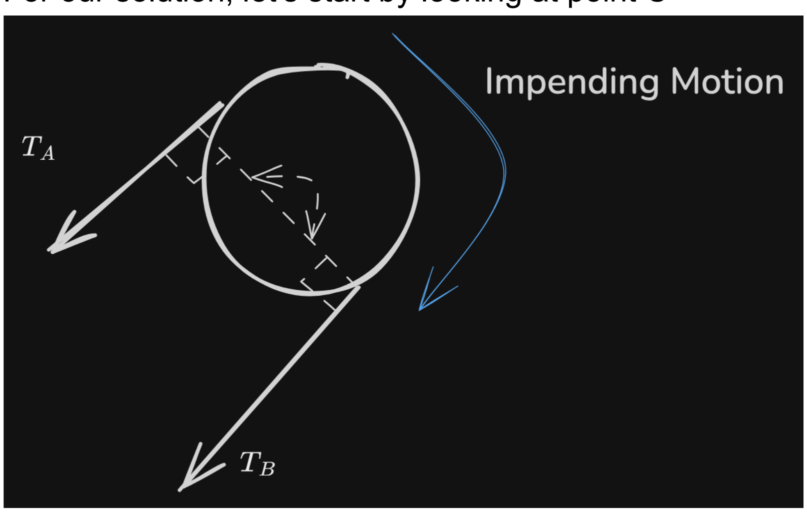
- In this case, $e = 2.718$, Euler's Number
- $\mu =$ coefficient of friction, as we'd expect
- $\beta =$ angle of contact in radians (gross)
- If we're given θ in terms of relative to t_2 to a vertical, $\beta = 180 - \theta$ to the horizontal because it swaps around

  - again, β is in RADIANS. it normally just makes sense to work in degrees then swap over, but still.
- If you have an applied couple moment, impending motion is always to the opposite.
  - Okie, so we're looking at it and we have $T_1 = T_2 e^{\mu\beta}$
  - $\sum m_o = T_1 * r - T_2 * r - m = 0$

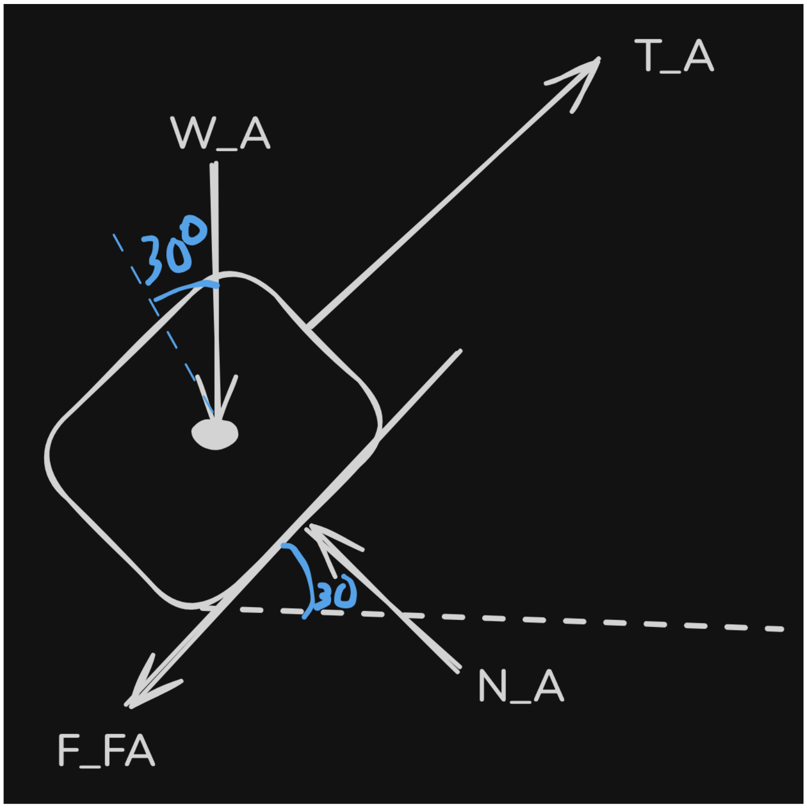
Class example



- For our solution, let's start by looking at point C



- Because of the impending motion being down the ramp, $T_B > T_A, T_B = T_A e^{\mu\beta}$



- Weight in the y direction is $W_A \cos(30)$, and weight in the x is $W_A \sin(30)$

$$\sum F_x = -W_A \sin(30) + T_A + \mu_A N_A = 0$$

$$\sum F_y = -w_A \cos(30) + N_A = 0$$

- You also draw an FBD of B, but I kinda don't wanna, so the equations that pop out are
- $F_{FB} = \mu_B N_B$

$$\sum F_x = -w_B \sin_3 0 + \mu_A N_A + \mu_B N_B + T_B = 0$$

$$\sum F_y = -N_A - w_B \cos(30) + N_B = 0$$

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Hydrostatic Fluid Pressure

- Pressure is based on the "unit weight" of water, γ
- For water, $\gamma = 62.4 \, \frac{lb}{ft^3}$
  - A $1' \times 1' \times 1'$ cube of water weighs 62.4 lb
    - Somewhere my brain knew that. D&D probably?

        - In SI units, base is kg
            - We start with a unit density of $\rho = 1000\ \frac{kg}{m^3}$
                - You can make that into a unit weight, which would just be $\rho * g$, or $9.81\ \frac{kN}{m^3}$
- How can we calculate the pressure at any given depth?
    - Hydrostatic fluid pressure is entirely based on how deep you are in the water
        - So let's say we're some h_A feet down, maybe, 10ft.
        - $P_A = \gamma * h_A = 62.4\ \frac{lb}{ft^3} * 10ft = 624 \frac{lb}{ft^2}$
            - For reference, that's about 4.33 PSI, which is a noticeably more reasonable sounding measurement.
    - If we were to work in SI
        - $h_A = 3.05m$
        - $P_A = \rho * \gamma * h_A = 1000 * 9.81 * 3.05 = 29.92 \frac{kN}{m^2}$

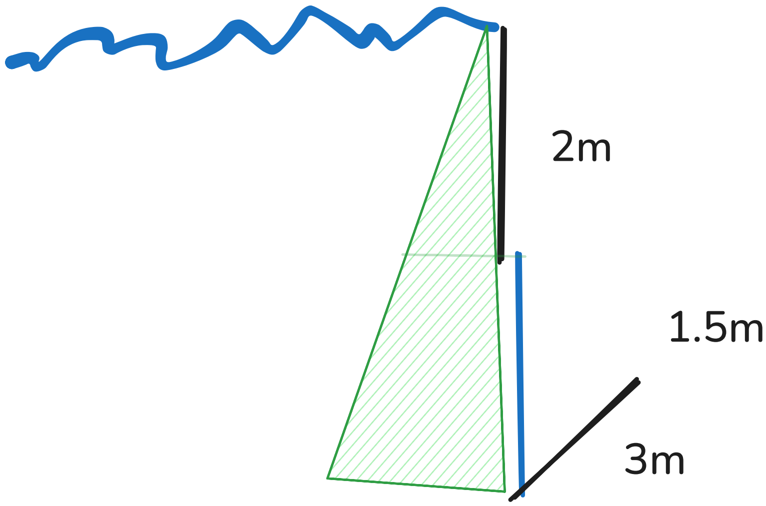
Normal Forces

- Hydrostatic fluid pressure is always normal to a surface

Pressure Distribution on a Vertical Wall

- We have some wall of a pool
    - Let's call $h = 10$ft for the pool
    - We end up with some triangular pressure distribution
        - Said force acts normal to the wall
        - Pressure at the bottom of the pool, A, is going to be $\gamma * h_A$
            - In this case, p_A is going to be 624 lb/ft^2
            - Now you just make it into a distributed load, $w = \text{Area of } p$
                - $w = \frac{1}{2} p_A * h_A = 3120 \frac{lb}{ft}$
    - $F_R = 3120 \frac{lb}{ft} * (75) = 234,000$ lb

Example 9.14 from the textbook



- We want to find total hydrostatic force on the gate, which I drew in blue, AB
  - $p_A = \rho * g * 2 = 19.62 \frac{kN}{m^2}$
  - $p_B = \rho * g * 5m = 49050 \frac{kN}{m^2}$
  - Now find the area of the p distribution
    - $w_R = \left(3m * 19.62 \frac{kN}{m^2}\right)$ and acts at 1.5m off the bottom of the gate
      - $w_R = 58.86 \frac{kN}{m}$
    - $w_T = (1.5 * (p_B - p_A)) = 44.145 \ \frac{kN}{m}$ acting at 1m off the bottom of the gate
  - Third and final thing to find is the total force
    - $F_1 = 1.5 * 58.86 = 88.29 kN$
    - $F_2 = 1.5 * w_T = 66.22 kN$
    - $\boxed{F_{total} = 154.51 kNB}$
-

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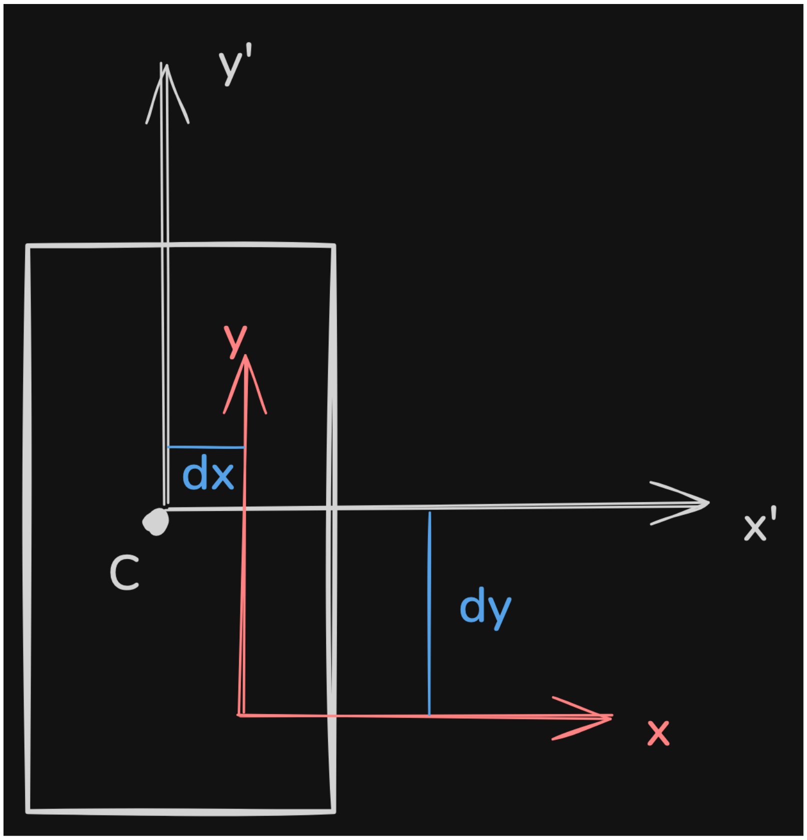
Not a word about the dates or missing content between this and the last note.

- Polar moment of inertia is for torsion, all you really need to know is that $J = Ix + Iy$, and that's literally all you care about.

Parallel Axis Theorem



- If we did the math, we'd find that $Ix' = \frac{1}{12}bh^3$
  - and that $Iy' = \frac{1}{12}b^3h$
- And we have symmetry, so $Ix'y' = 0$, which just means that one of them is the most and one of them is the least.

Equations for Parallel Axis Theorem

- These let us find equations for any axis that's parallel to our original, ie what's marked in red

$$\boxed{Ix = Ix' + A * d_y^2}$$

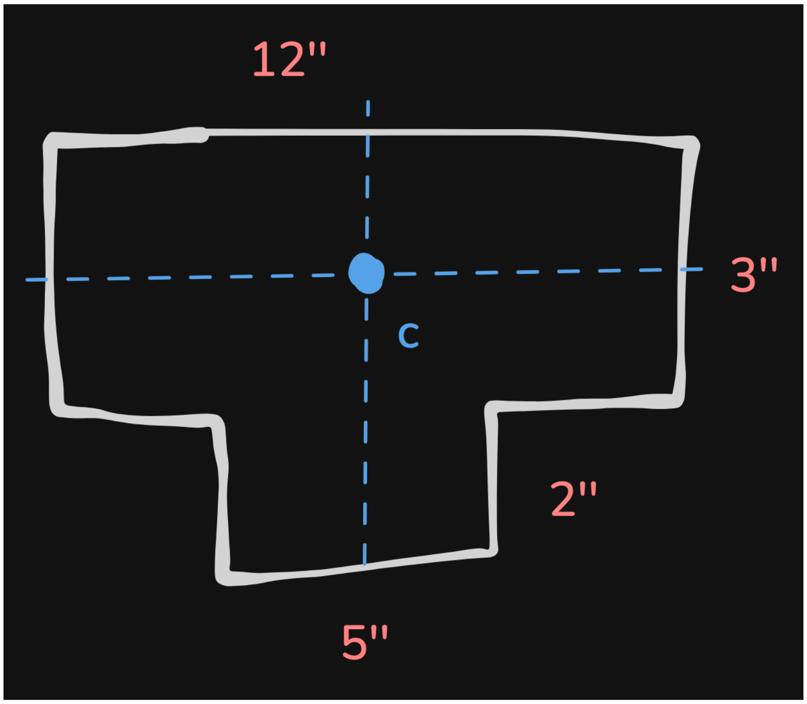
$$\boxed{Iy = Iy' + A * d_x^2}$$

$$\boxed{Ixy = Ix'y' + A * dx * dy}$$

Signs on dx and dy

- Based on the coordinate of the $x'y'$ origin in the $x - y$ system.

Example



- Deflection, δ, is an equation that I just barely missed writing down. Motherfucker.
- For our solution, we're just going to use the classic $I = I' + Ad^2$
- Before we can actually do anything, we do in fact need to find centroid.
  - Step 1a, draw x'_1, y', and $x - y$ axes
  - As well as $\tilde{x}, \tilde{y}$ for all yo segments
  - Gotta be measured from the same edge as $\bar{y}$ (or like $\bar{x}$)

$$\bar{y} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2}$$

$$A_1 = 5 * 2 = 10, A_2 = 3 * 12 = 36$$

$$\tilde{y}_1 = 4, \tilde{y}_2 = 1.5$$

- Plug it all in, you get that $\bar{y} = 2.04$ inches
- Go forth and use a table for the parallel axis theorem

Seg	b	h	A	$Ix' = \frac{1}{12}bh^3$	$Iy' = \frac{1}{12}bh^3$	dx	dy
1	5	2	10	3.33	20.83	0	-1.96
2	12	3	36	27	432	0	0.54

$$Ix_1 = Ix'_1 + A_1 d^2_{y_1} = 3.33 + 10(-1.96)^2 = 41.75$$

$$Ix_2 = Ix'_2 + A_2 d^2_{y_2} = 37.50$$

$$Ix = Ix_1 + Ix_2 = 79.24 \text{ in}^4$$

$$Ixy_1 = 0$$

$$I_y = Iy_1 + Iy_2$$

$$Ixy = 0$$

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Hands on #8

- Diameter of bucket is 10.5in
- Empty bucket is 1.77lb
-