MATH112 TOC

> MATH112 - 2024-01-09
>
> #notes #math112 #math #calc
>
> ## General Plan for semester
>
> - Doing vectors now, instead of later, so that we're actually prepped for physics
>   - Also gonna be working with coordinate systems
>   - Wrap back around to integration
>   - Integration by parts
>   - Apparently, lots of people don't like sequences
>
> ---
>
> Holy moly, we're actually doin stuff now
>
> $s(t) = \sin(2\pi t), t \geq 0$
> velocity
> $v(t) = \frac{d}{dt}\sin(2\pi t)2\pi, t \geq 0$
>
> - Normal convention is that positive velocity is towards the right
> - Let's say we have an object moving on a plane over time
> - If we take the derivative of a pos vector, we get a velocity vector!

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>
> ## General Goals Today
>
> - Into to vectors
>   - Geometry of vectors
>   - (Triangles)
>   - **Trig Calls.**
>   - Bracket notation for vectors

Alright, so we're back to our object wiggly woogling its way along

- We had our position vector, $\vec{s}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$
  - I and J are our unit position vectors
- So if we want a velocity vector, in that case we're going to have to take derivative of the pos vector
- $\frac{d}{dt}(x(t)\hat{\imath} + y(t)\hat{\jmath})$
  - Rewrite that shit
  - $\frac{d}{dt}x(t)\hat{\imath} + \frac{d}{dt}y(t)\hat{\jmath}$
- If you have a vector valued function and want to take derivatives, you just take em separately
- $$\vec{V}(t) = V_x(t)\hat{\imath} + V_y(t)\hat{\jmath}$$
- If have velocity and we want to get back around to pos, gonna have to integrate that
  - To do that, integrate the x portion and add it to the y portion
- If both components are positive, we're going northeast (towards quad 1)
- So let's say we just have a vector $V = V_x\hat{\imath} + V_y\hat{\jmath}$
  - Rember to draw vectors with an arrow on the end
  - Tail is the end, tip is the other side
  - Magnitude is actually written as $|\vec{V}|$
    - $\sqrt{Vx^2 + Vy^2}$

Bracket Notation

$< a, b >$ = $a\hat{\imath} + b\hat{\jmath}$

$< a, 0 >$

- A is just the length, if it's positive we're a straight line to the right, if it's negative we're in a straight line to the left
- This is known as the x component
  $< 0, b >$
- In this case, b is just the length, so if it's positive we goin up, if it's negative we're going down

Triangle Representation

- Triangle representation, you draw the components tip to tail
  - Start at the original tail, go to the last tip, you get $< a, b >$
- Parallellogram is the same thing, except the tails are touching and you send your vector off into the aether

Vector Addition

$< a, b > + < c, d > \quad = \quad < a + c, b + d >$

- Go tip to tail

Scalar Multiplication

- Scaling a vector by some number
- Direction stays the same, but magnitude is *scaled* by the *scalar*
- In general, if there's a constant hanging out in front of the vector, you "distribute" to all the components
-

Aaaaand now we peace out of vectors

Parametric Equations

- Parametrics represent an x-y curve in terms of t
- We need to know how to sketch a parametric (parametrized) equation
- We'll learn parameter elimination
  - Sometimes you can kill the parameter and just get back to like, $y = f(x)$
- Orientation of a curve
- Some special cases, like
  - Lines
  - Circles

If we're given a function like $y = x^2$, we know how to plot that

- If we're given like, a goldfish lookin function, we can try to parameterize that
  - Usually we're going to use t

    - Let x = cos(t)
    - and let y = sin9t)
    - and t is between $[0, 2\pi]$
- Went around counterclockwise
-

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$x = f(t)$
$y = g(t)$
$t(-\infty, \infty)$
Gives us x and y coordinates

- If an eye diagram actually has distinct points, yippee, you can probably get data out
    - (Eye is open)
- If you really kinda can't tell and get crazy stuff when parametrically plotting, you probably can't
    - The eye is closing

Parametrics

- Parameter elimination
    -
- Orientation
- Lines n Circles
- Alright, back to yesterday
    - When we have $x = \cos(t), y = \sin(t), t[0, 2\pi]$, we end up with a unit circle counterclockwise
    - Well, y'know what, what if $x = \cos(-t), y = \sin(-t), t[0, 2\pi]$
    - Remember, $\cos(-t)$ is the exact same damn thing as $\cos(t)$
    - $\sin(-t) = -\sin(t)$
    - Now we'll end up with a clockwise drawn circle
- Almost certainly going to get problems about restricting the domain of the parameter

Now les talk about LINES

- $y = 3x + 1$
  - We're chilling in slope intercept form
  - Turning it into a parametric equation
    - Technique we're going to use works for any $y = f(x)$
    - Let $x = t$
    - $y = 3t + 1$
  - Going back the other way
    - Given $x = 2 + 2t$
    - $y = 7 + 6t$
    - And that $t(-\infty, \infty)$
    - $t = \frac{x}{2} - 1$
    - $y = 7 + 6(\frac{x}{2} - 1)$
    - $y = 7 - 6 + 3x$
    - $y = 3x + 1$
    - And just like that, we've successfully murdered the t
  - Parameterization is not unique
    - There is no one right answer, but that's also kinda fun
  - Note, for parameter elimination:
    - Find the domain of x
    - We have $x = \cos(t)$ and $y = \sin^2(t)$
    - Anddd let's say $t[0, 2\pi]$
      - To kill t, we could try to set $t \arccos(x)$, which will work, but it does take a fair amount of work
      - We could also just use the pythagorean identity, and get $y = 1 - x^2$
- Lines in general
  - Line in point slope
    - $y - y_0 = m(x - x_0)$
      - This line will go through the point (x_0, y_0)
    - Let $x = x_0 + \Delta xt$
    - $y = y_0 + \Delta yt$
    - $t\epsilon(-\infty, \infty)$
- Find a segment from Point $(3, 4)$ to $(-1, -3)$
  - And we want to parameterize this
  - We've got our parameter t, and $t\epsilon[0, 1]$
  - Let $x = 3t + (1 - t)(-1)$
    - When t is $= 0$, $x = -1$, but when $t = 1, x = 3$, so now we've successfully got our x covered

- Let $y = 4t(1-t)(-3)$, and now y will equal -3 when $t=0$, and $y=-4$ when $t=1$
- Circles
  - $x^2 + y^2 = 1$
    - This is our friendly neighborhood unit circle, with a radius of 1 and centered at the origin
  - The generic circle equation, centered at x_0, y_0, would be $(x-x_0)^2 + (y-y_0)^2 = r^2$
  - Let x = $x_0 + r\cos(t)$
  - $y = y_0 + r\sin(t)$
  -

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Polar Coordinates!

- Given any point on the plane, you can represent it with cartesian coordinates (x_0, y_0)
- Polar coordinates are represented with (r, θ), where r is the radius and θ is the angle
- $(r, \theta) = (2, \frac{\pi}{4})$
  - Alright, so to get to this point, we go up to the angle $\frac{\pi}{4}$, then we go out a distance of 2
- Anytime you want to convert from polar to cartesian, $x = r\cos\theta, y = r\sin\theta$
- $\frac{y}{x} = \tan(\theta)$
- Polar coordinates are not unique!
  - You can wrap 2π around and be fine, or you can have a negative radius
  - $(r, \theta) = (-r, \theta + \pi)$
- Alright, going from cartesian to polar is noticeably more fun
  - Radius would just be $\sqrt{x_0^2 + y_0^2}$
  - Figure out quadrant from cartesian coordinates
  - Find θ_0 in the right quadrant such that $\tan(\theta_0) = \frac{y_0}{x_0}$

Polar Equations!

$$r = f(\theta)$$

- Circles centered at the origin are easy squeezy!

- They would just be like, $r = 6$, which makes a circle of radius 6 at the origin
- Spirals!
  - $r = \frac{1}{2\pi}\theta$
    - For theta $\geq$ 0
- Cardioid
  - $r = 1 + \sin\theta$
- Alright, now how do we get around to converting polar equations to cartesian?
  - If you see $r\cos\theta$, replace it with x
  - If you see $r\sin\theta$, replace it with y
  - Recognize lines
    - $r = \frac{c}{a\cos\theta + b\sin\theta}$ is the same thing as $ax + by = c$
  - Recognize circles
    - $r = 2a\cos\theta + 2b\sin\theta$
      - This is a circle $r = \sqrt{a^2 + b^2}$
      - center is at (a, b)
  - If we had something like $r = \sin\theta$, that's actually a circle, with $a = 0$ and $b = \frac{1}{2}$
- Example
  - $r = \frac{1}{2\cos\theta + 3\sin\theta}$
  - c = 1
  - a= 2
  - b=3
  - $2x + 3y = 1$ is what we could splice out of that
  - So to get x
    - $x = r\cos\theta = \frac{\cos\theta}{2\cos\theta + 3\sin\theta}$
    - $y = r\sin\theta = \frac{\sin\theta}{2\cos\theta + 3\sin\theta}$
    - $2x + 3y = \frac{2\cos\theta + 3\sin\theta}{2\cos\theta + 3\sin\theta}$

Integrate the two parts

$$\int_1^4 t^{\frac{-1}{2}} = 2\sqrt{t}, 4 - 2 = 2$$

$$\int_1^4 t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} = \frac{2(4)^{\frac{2}{3}}}{3} - \frac{2(1)^{\frac{3}{2}}}{3} = \frac{14}{3}$$

$$\text{Final answer of } \vec{r}' = 2\hat{i} + \frac{14}{3}\hat{j}$$

V-v-vectors!

- The main types of vector problems are planes and boats with vectors
  - Balanced net forces
- x tends to go down to left, y down to right, z straight up
- The point (4,3,5)
- Distance between two points
  - Big shock, it's basically the same, but you just add in the z component
- Midpoint between two points is just averaging them
-

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Reading Assignment

13.1
Applications of vectors, pages 811-813
13.2
Example 7, p.832

Alright apparently 6c is fun

- If you're given a problem, feel free to parametrize $y = t$ instead

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realzies notes

- Distance is the $\sqrt{}$ of all the differences between components squared
- So now let's define a 2d circle rq
  - $(x - x_0)^2 + (y - y_0)^2 = r^2$
    - Radius r, centered at (x_0, y_0)
    - If it's less than or equal to, it's a disk
      - $\geq$ is everything outside the circle
- a 3d circle is known as... a sphere! 🤯
  - $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$
    - Astoundingly, it still has radius r, and is now centered at (x_0, y_0, z_0)

- $\leq$ includes everything inside of it, and is called a ball

Example

- Find the sphere centered at the point $(1, 2, 0)$ that passes through the point $(3, 4, 5)$
  - We need our radius, which is really just finding the distance between the two points it passes through $r^2 = \sqrt{(3-1)^2 + (4-2)^2 + (5-0)^2} = r^2 = \sqrt{4+4+25}|||r^2 = 33$
  - $r = \sqrt{33}$
- Equation for the sphere is now $(x-1)^2 + (y-2)^2 + (z)^2 = 33$

Alright, that was fun with coords, back to vectors

- Vectors, again, have magnitude and direction
  - Let's say we have the point $P = (x_0, y_0)$
    - and $Q = (x_1, y_1)$
  - And we want $\vec{PQ}$
    - This is the vector with tail at P and tip at Q
  - So $\vec{PQ}$ would just be $< x_1 - x_0, y_1 - y_0 >$
- Vectors do not give *a single damn* about position
  - You can toss a vector anywhere in space
  - $P = (3, 4), Q = (2, 7)$
  - $S = (2, -3), T = (1, 0)$
  - Find the vector $\vec{PQ}$
    - This would end up as like, $\vec{PQ} =< -1, 3 >$
  - Find the vector $\vec{ST}$
    - Get like, $< -1, 3 >$
  - Oh hey, $\vec{PQ} = \vec{ST}$
  - Even though we got here different ways and they're vaguely different points in space, they're actually like, the same vector
- It's convention if you have the vector $< a, b >$, that this vector would be from the origin
  - Convention to call that a position vector
  - Rounded brackets are for points
-

Magnitude of a Vector

- Common notations
  - $| < a, b > | = || < a, b > ||$
    - Textbook tends to use single bar like it's an absolute value
    - These are both equal to $\sqrt{a^2 + b^2}$
      - Mildly shockingly, this works no matter how many dimensions your vectors have, which is a little surprising
- Scalar multiplication
  - $c < a, b >$, where c is a scalar and $< a, b >$ is our vector
    - Scalar just means that we scale each component by it
    - so resultant would just be $< ca, cb >$
    - if we want the magnitude of $|c < a, b > |$
      - $| < ca, cb > | = \sqrt{ca^2 + cb^2}$
        - Could also just be $\sqrt{c^2(a^2 + b^2)}$
        - We could drag that out $|c|\sqrt{a^2 + b^2}$
          - Abs is to force positive of the c, since any magnitude has to be positive
          - You're just scaling the mag, same as the rest of the vector! It's not hard!

Examples?

- if $|c| > 1$ then magnitude increases
- if $|c| < 1$ then we're going to shrink it
- if just straight up $c > 0$, then we keep the same direction
- if $c < 0$, we're looking like a family business based out of waco (flipping)
- if $c = 0$, then we end up with the vector $< 0, 0 >$, that's just a point, it's kinda boring
  - it is technically still a vector though

Vector Addition

- $< a, b, c > + < c, e, d > \ \ = < a + c, b + e, c + d >$
  - This works the *exact same* for subtraction
    - Let's say that first vector was $\vec{u}$ and the second was the $\vec{v}$
- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ commutative property goes WILD
- This is the algebraic approach to adding vectors together
- There is also the geometric point of view for vectors
  - This is what we in the business call "tip to tail"

        - The final resultantant is the original tail to final tip
    - Parallelogram also exists but like. bRuh.
- Easiest way to think about subtraction is like a scalar of -1
- Oh boy, one final definition
    - $\vec{v}$ is a unit vector is $|\vec{v}| = 1$
        - $\hat{\imath} =< 1, 0, 0 >$
        - $\hat{\jmath} =< 0, 1, 0 >$
        - $\hat{k} =< 0, 0, 1 >$
        - These are all referred to as the standard or coordinate unit vectors
    - If I have some vector $\vec{v}$, I can make a unit vector $\vec{u}$ in the same direction ad $\vec{v}$
        - Let $\vec{u}$ = $\frac{1}{|\vec{v}|} * \vec{v}$
        -

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- Cute n' quirky little aside - $\vec{0}$ is parallel to all vectors, since $\vec{a} = c\vec{b}$
- Quote from industry - best way not to have typos is to not type

readings

- 827 first page
- 832-833
    - work n' forces

dot products (yippeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeee)

- Defined algebraically and geometrically
- Projection of some vector $\vec{u}$ onto some other vector $\vec{v}$
- Dot product can be used to calculate work
    - Or decomposing forces

algebra way

- $\vec{u} =< u_1, u_2, u_3 >$
- $\vec{v} =< v_1, v_2, v_3 >$
- $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$
  - Dot product spits out a scalar
- $\vec{v} \cdot \vec{u}$ is the same thing as $\vec{u} \cdot \vec{v}$
  - Dot products are commutative
    - as previously stated, commutative property goes WILD
- $\vec{u} \cdot \vec{u} = u_1^2 + u_2^2 + u_3^2$
  - This is the same thing as $|\vec{u}|^2$

Example

$< -3, 2, 1 > \cdot < 2, 0, 1 >$

- $-3 * 2 + 2 * 0 + 1 * 1 = -6 + 0 + 1 = -5$

geometric way

- Law of cosines (affectionately referred to as Pythagorean's Theorem on steroids)
  - $a^2 + b^2 - 2ab \cos\theta = c^2$
    - You can toss in any number for θ
- Let's say I have a vector $\vec{v}$ and a vector $\vec{u}$ and I want to connect their heads together
- Hey - ho, law of cosines kicks in
- a^2 is just the magnitude of $\vec{u}$ squared, + $|\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta = |\vec{u} - \vec{v}|^2$
  - Let's do some quirky little aglebra
  - $|\vec{u} - \vec{v}|^2 = (\vec{u} - \vec{v}) * (\vec{u} - \vec{v})$
  - $\vec{u} \cdot \vec{u} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$
  - $|\vec{u}|^2 - 2\vec{u} \cdot \vec{v} + |\vec{v}|^2$
  - $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$
    - $\theta\, \epsilon [0, \pi]$
  - Alternate forms
    - $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}}$ where $\epsilon[-1, 1]$
    - $\theta = \cos^{-1}(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|})$

example

$\vec{u} =< -3, 2, 1 >$
$\vec{v} =< 2, 0, 1 >$
Now to find the angle between these vectors
So the dot product here would just be -5
And then we need to find the mag
$|\vec{u}| = \sqrt{9+4+1} = \sqrt{14}$
$|\vec{v}| = \sqrt{4+1} = \sqrt{5}$

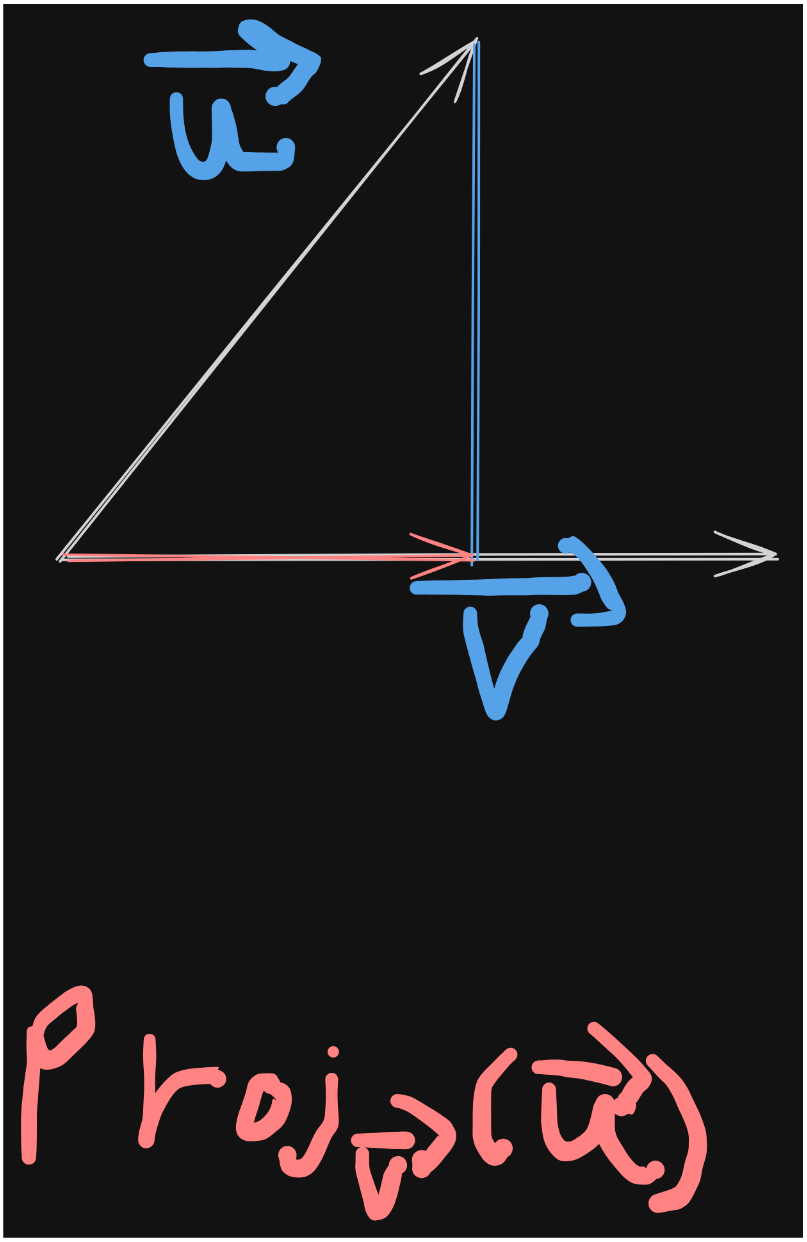
$\theta = \cos^{-} 1(\frac{-5}{\sqrt{14*5}})$
$\cos^{-1}(\frac{-5}{\sqrt(70)})$
Roundabout 2.21 radians, or 126.7 degrees
Sure, let's go with that

examples

- Find all vectors of the form $< 0, a, b >$ that are orthogonal to $< 4, 8, -2 >$
    - Orthogonal - same thing as perpendicular / normal
- The angle θ between the two vectors
- $< 0, a, b > \cdot < 4, 8, -2 >= 0 = \cos(\frac{\pi}{2})$
- $(0)(4) + (a)(8) + (b)(-2)$
- $8a - 2b = 0$
    - This is just algebra
    - $b = 4a$
- Any vector of the form $< 0, a, 4a$ is going to be orthogonal to $< 4, 8, -2 >$, and its dot product will be... 0
    - for any $\mathbb{R}$ (real number)
- Last topic of the day is

orthogonal projections

- Projection is like a "shadow



This is the (orthogonal) projection of $\vec{u}$ onto $\vec{v}$
$\text{proj}_{\vec{v}}(\vec{u}) = |\vec{u}| \cos\theta * \frac{\vec{v}}{|\vec{v}|}$
This is just getting the unit vector for $\vec{v}$ multiplied by some trig fun to get the angl

$\text{Proj}_{\vec{v}}(\vec{u}) = |\vec{u}| * \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} * \frac{\vec{v}}{|\vec{v}|}$

MATH112 - 2024-01-23

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Today is $\vec{u} \times \vec{v}$

- (cross product)
  - Both algebraically and geometrically

- Right Hand Rule
- Determinants are going to pop up
  - Conditions for parallel / orthogonal vectors
- Mayhaps area of a parallelogram
- Uses
  - Torque is cute n quirky
  - So are magnetic forces

- Bar Stool math
  - If you have three non colinear points, you can determine a plane
- Let's take some friendly neighborhood vectors, $\vec{u}$ and $\vec{v}$ that are not equal to each other and are not parallel
  - Alright, let's say that $\vec{u} = \hat{i}$ and that $\vec{v} = \hat{j}$
    - These two together create the $x - y$ plane
  - Ok smart guy, let's say we have $\hat{j} \& \hat{k}$
    - That forms the y-z plane
- Given some plane, find vectors orthogonal to the plane
  - Anything on the $y - z$ plane that's orthogonal would be $\pm\hat{k}$
- Given the plane from $\vec{u}, \vec{v}$, find $\vec{w}$ that is $\perp$ to $\vec{u}$ and $\perp$ to $\vec{v}$
  - $\vec{w} = \vec{u} \times \vec{v}$
    - $\vec{u} =< u_1, u_2, u_3 >$
    - $\vec{v} =< v_1, v_2, v_3 >$
    - $\vec{w} =< u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 >$
    - 23 32 31 13 12 21
  - Hey obligatory question, are we commutative?
    - Ehh, we're close
      - $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$
      - What we in the business call "anti-commutative"
  - Assorted other properties (left as exercises to the reader to prove)
    - $(a\vec{u}) \times (b\vec{v}) = (ab)(\vec{u} \times \vec{v})$ Associative
    - $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$ Distributive
    - $\vec{u} \times \vec{u}$ spits out the zero vector
      - I mean, that one makes sense without a proof
- Show that $\vec{u} \perp \vec{w}$
  - $\vec{a} * \vec{b} = |\vec{a}||\vec{b}| \cos(\theta) = 0$

    - $\vec{u} \cdot \vec{w} = u_1(u_2v_3 - u_3v_2) + u_2(u_3v_1 - u_1v_3) + u_3(u_1v_2 - u_2v_1)$
        - Okie dokie expansion time
        - $= u_1u_2v_3 - u_1u_3v_2 + u_2u_3v_1 - u_1u_2v_3 + u_1u_3v_2 - u_2u_3v_1$
            - wtf did I just write
        - $0 = 0$ skip skip hooray
- this does indeed imply that $\vec{u} \perp \vec{w}$
- Let $\vec{u} =< 1, 0, 0 >= \hat{\imath}$
- Let $\vec{v} =< 0, 1, 0 >= \hat{\jmath}$
- $\vec{u} \times \vec{v} =< 0 * 0 - 0 * 1, 0 * 0 - 1 * 0, 1 - 0 >$
    - $u_2v_3 - u_3v_2$
    - $u_3v_1 - u_1v_3$
    - $u_1v_2 - u_2v_1$
- We end up with $< 0, 0, 1 >= \hat{k}$

Right Hand Rule

- Curl your fingers from the first term to the second (ie, $\vec{u} \times \vec{v}$), and thumb will point in the direction of the cross product

Determinants

- Let's use determinants to calculate $\vec{u} \times \vec{v}$
- Let's say we have an n by n set of numbers
- $$\begin{matrix} a & b \\ c & d \end{matrix}$$
- Determinant is $ad - bc$
- $$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$$

  ```
  - Determinant would be $$a |\begin{matrix}
  ```

  e & f \ h & i
  \end{matrix} | - b |\begin{matrix}
  d & f \ g & i
  \end{matrix}| + c |\begin{matrix}

d & e \ g & h
\end{matrix}| $$

Which is the same thing as $a(ei - fh) - b(di - fg) + c(dh - eg)$

$$\vec{u} \times \vec{v} = |\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{matrix}| \$\$\$\$ \hat{\imath} |\begin{matrix} u_2 & u_3 \\ v_2 & v_3 \end{matrix}| - \hat{\jmath} |\begin{matrix} u_1 & u_3 \\ v_1 & v_3 \end{matrix}| + \hat{k} |\begin{matrix} u_1 & u_2 \\ v_1 & v_3 \end{matrix}|$$

Magnitude of a cross product is the $\sin\theta$ instead of the $\cos\theta$ that it was for dot products

The cross product of two colinear vectors is always going to be a big ol

0

The dot product though is $\pm|\vec{u}||\vec{v}|$

If they're perpendicular though,

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- You could break force vectors into magnitude * $< \cos\theta, \sin\theta >$

today

- In the past, we've parametrized curves in 2d space
  - So let's get to parametrizing curves in 3d space as vector valued functions (VVF)
- Find domain of parameter
- Find VVF of a line ℓ
- Relationship between $\ell_1$and ℓ_2
- Collision?

movin around

- okie dokies, so we have some object moving in space
- represent pos as a function of t
- $(x(t), y(t), z(t))$

    - For instance, it'd start at $(x(t_0), y(t_0), z(t_0))$
    - and move on to t_1, etc, etc
- Oh hey, if we're going in some direction $\vec{r}$, then t is just a scalar

example

$\vec{r}(t) = < \sqrt{t+2}, \sqrt{2-t} >$
Find domain of t
Uh, we have a problem if we try and $\sqrt{}$ a negative number, so our domain ends up just being $-2 \leq t \leq 2$

moving back on

- Any parametric can be expressed as a VVF
- $x = t$
- $y = t^2$
- Domain is any real number $\mathbb{R}$
- Range is a vector of the form $< t, t^2 >$ for any $\mathbb{R}$
- Vector is pointing from tail at the origin to all the tips at all the points on a curve

lines in 3d

- A line could be two distinct points
    - $p = (x_0, y_0, z_0)$, q = (x_1, y_1, z_1)
    - Hey ho, line
- You could also have a point, ie, $p = (x_0, y_0, z_0)$
    - and a ~~slope~~ direction $\vec{v} = < a, b, c >$
    - $\vec{r}(t) = < x_0, y_0, z_0 > + t < a, b, c >$
- Spoilers: these are basically just the same thing, you just produce a direction vector by doing $\vec{PQ} = < x_1 - x_0, y_1 - y_0, z_1 - z_0 >$

example

$p = (-3, 5, 8)$, $q = (4, 2, -1)$
okie, let's say that we're using p as our base
$\vec{PQ} = < 4 - -3, 2 - 5, -1 - 8 > = < 7, -3, -9 >$

Oh hey now we just have a dir
$\vec{r}(t) =< -3, 5, 8 > +t < 7, -3, -9 >$

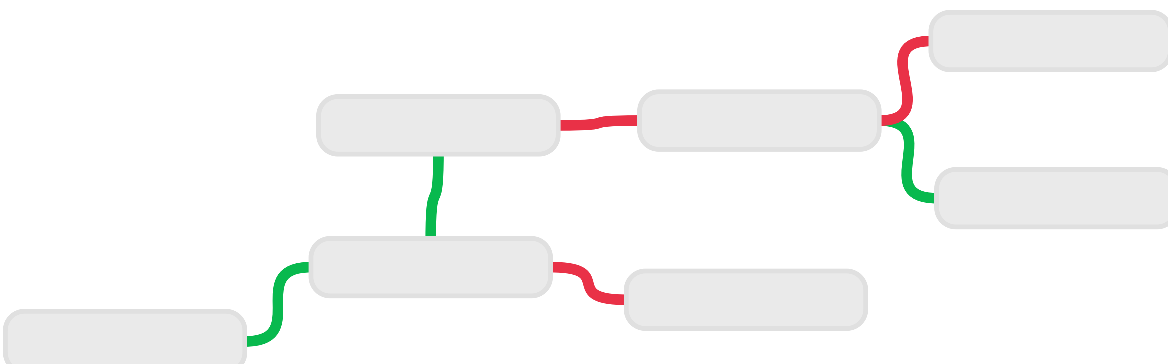
Find where this line crosses the $x - z$ plane

- So we gotta find where y = 0
- $t * -3 = 5$
- $t = \frac{5}{3}$
- Now that we know t, we could go ahead and plug it in
- $< -3 + 7 * \frac{5}{3}, 0, 8 - 9(\frac{5}{3}) >$

relationships between ℓ_1 and ℓ_2

they're in a relationship??? 😳😳😳😳

This is NOT going to render right
MATH112 Line Flowchart



$$\ell_1 = \vec{r}(t) =< t, 1 + t, 1 + 2t >$$

$$\ell_2 = \vec{r}_2(t) =< 2 + t, 4t, 3 + 4t >$$

- Alright, let's dice out some directions
  - Gotta find where things change with our parameter, t
  - $\vec{v}_1 =< 1, 1, 2 >$, it's just whatever scalar is sittin on t
  - $\vec{v}_2 =< 1, 4, 4 >$
  - Hey uh, those are NOT the same
    - Not even if you try *really* hard to scale them, there is no way of gettin there
- Alright, so we are NOT parallel, now to check for a common point

- Can I find t and s such that $\vec{r}_1(t) = \vec{r}_2(s)$
    - $< t, 1+t, 1+2t >^? =^? < 2+s, 4s, 3+4s >$
    - Equation one is look at the x components
        - $t = 2 + s$
    - y
        - $1 + t = 4s$
    - z
        - $1 + 2t = 3 + 4s$
    - uhhh, doing some math
        - $1 + 2 + s = 4s$
        - $s = 1$
        - which then makes it so $t = 3$
        - $7 = 7$ mmm, yes
        -

MATH112 - 2024-01-26

#notes #math112 #math #calc

"Algebra errors are like mold, they grow in cramped dark spaces"

notes

- so we have some line like $x = 5t + 4$, $y = 3t - 1$, $z = t + 1$
    - So you can solve for t and get all those
- you'll more likely be given like $\frac{x-4}{5} = \frac{y+1}{3} = z - 1 = t$
    - And then solve for t in all of those, and suddenly you have a parametrized function

Torque

- you torque me right round baby right round like a fulcrum baby right round right round
    - i'm going insane.
- fulcrum is some center, and then you have a lever arm (conventionally a wrench, in most of the problems we're going to have)
    - You'll have some force $\vec{F}$ being applied to the end of the wrench
        - Moments are momenting
- Put the tails together (at your fulcrum), and then your torque $\vec{\tau} = \vec{r} \times \vec{F}$

- Obligatory reminder for cross products that θ is between 0 and π
- Lots of problems ask for magntitude
    - Which would just be $|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$
        - That $\sin\theta$ does make sense, because your mag can't exactly be negative

alright back to lines

- Let's say $l_1 =< -1, 4t, 2-2t >$
- $l_2 =< 2t, 1, 1+t >$
    1. Let's find directions
        1. $\vec{V}_1 =< 0, 4, -2 >$
        2. $\vec{V}_2 =< 2, 0, 1 >$
        3. Not possible to make em parallel (womp womp)
    2. Direction is no dice, do they share a common point?
       1.x $-1 = 2s$
       2. $4t = 1$
       3. z $2-2t = 1+s$
       4. $s = -0.5$, $t = 0.25$
       5. $2-(0.25)2 = 1-0.5$
       6. $1.5 = 0.5$
       1. Those are NOT the same.
    3. no intersections, we're skew

planes

- equation for planes (not the kind that fly)
- Representation of planes
    - Can be shown by a point and a normal direction
- Calculate a plane from three points
- Intersections
    - intersections of a line and a plane
    - Planes and planes and planes and planes
    - Angle between planes
- Backing up to 2 dimensional
    - $3x+2y=1$
    - $3x+2y=4$
    - Parallel, but with different intercepts

- Big shocker, we can do it in 3dim as well
  - $ax + by + cz = d$
    - While the similar equation was a line in 2d, it's a plane here in 3d
  - Previously on calc 2
    - We said that two vectors $\vec{u}$ and $\vec{v}$, they *generally* represent a plane
    - Put tails together, connect the heads, get a triangle, yadayada
    - The cross product $\vec{n} = \vec{u} \times \vec{v}$
      - $\vec{n} \perp \vec{u}$ & $\vec{n} \perp \vec{v}$
- The direction of a plane is the normal vector (what direction is perpendicular to it)

example

- Normal vector is $\vec{n} =< a, b, c >$
  - If I pick some point Q in the plane, and there's some other point P, I make $\vec{PQ}$, I know that $\vec{n} \perp \vec{PQ}$
  - Rember we do have a 0 dot product from two orthogonal vectors
  - $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
  - $ax + by + cz = ax_0 + by_0 + cz_0$
    - That other half is just going to be some number d, oh hey, we're back to $ax + by + cz = d$
    - and the abc is the normal vector
  - $\vec{n} \cdot =< x, y, z >= \vec{n} \cdot < x_0, y_0, z_0 >$

examplier example (with numbers)

- Find the plane through the point $p = (-1, -6, -4)$ w/ direction $\vec{n} =< -5, 2, -2 >$
- We can just deadass use the equation
  - $< -5, 2, -2 > \cdot < x, y, z >=< -5, 2, -2 > \cdot < -1, -6, -4 >$
  - $-5x + 2y - 2z = -5(-1) + 2(-6) + (-2)(-4)$
  - $-5x + 2y - 2z = 1$

the less ideal case example

- Find the plane through
  - $p = (-1, 2, 1)$
  - $q = (0, -3, 2)$
  - $R = (1, 1, -4)$

- $\vec{pq} =< 1, -5, 1 >$
- $\vec{pr} =< 2, -1, -5 >$
- $\vec{pq} \times \vec{pr} = \vec{n} =$

$$\begin{matrix} \hat\imath & \hat\jmath & \hat k \\ 1 & -5 & 1 \\ 2 & -1 & -5 \end{matrix}$$

```
$$\hat\imath(25-(-1)), -\hat\jmath (-5-2), \hat k(-1-(-10))$$
$$26\hat\imath \ + 7\hat\jmath \ +9\hat k=\vec n$$
```

MATH112 - 2024-01-29

#notes #math112 #math #calc

previously on

- plane is $ax + by + cd = d$
- Angle between two intersecting planes
  - When planes intersect, their intersection is a line
  - Convention is to choose the smaller angle, so generally $\theta[0, \frac{\pi}{2}]$
- do remember that $\vec{n_1} \cdot \vec{n_1} = |\vec{n_1}||\vec{n_2}| \cos\theta$
  - $\theta = \cos^{-1}(\frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|})$
- If the directions are the same (or a scalar multiple), then the planes are parallel
- If the direction vectors are orthogonal, then they'll be orthogonal
  - $\theta = \frac{\pi}{2}$

example

- find the plane through the point $P = (3, 0, 8)$, I want this plane to be parallel to $2x + 5y + 8z = 17$
  - We need a point (P) and a direction $< 2, 5, 8 >$, from the coefficients of the previous
- $< 2, 5, 8 > \cdot < x, y, z >=< 2, 5, 8 > \cdot < 3, 0, 8 >$
- $2x + 5y + 8z = (6 + 0 + 64) = 70$

other example

Given plane 1 = $2x+5y-3z=0$ and plane 2 = $-x+5y+2z=8$
Find a plane orthogonal to plane 1 and plane 2
$\vec{n_1} =< 2, 5, -3 >$
$\vec{n_2} =< -1, 5, 2 >$
Find $\vec{n_3}$
Let $\vec{n} = \times$
not even going to tex this matrix
i j k
2 5 -3
-1 5 2

$10 - -15 = 25\hat{\imath} + 4 - 3 = 1\hat{\jmath} + 10 + 5 = 15\hat{k}$
$25x - y + 15z = d$ where d is any value

intersections

- line and a plane
  - either it lives in the plane or it punches through in exactly one point
    - $x+y+z=14$, $\vec{r}(t) =< 1, 1, 0 > +t < 0, 2, 4 >$
      - $x = 1 + 0(t)$
      - $y = 1 + 2t$
      - $z = 0 + 4t$
      - Sub allat into the plane formula
      - $1 + 1 + 2t + 4t = 14$
      - $2 + 6t = 14$
      - $t = 2$
    - $x = 1$, $y = 5$, $z = 8$
    - That's out intersection point
- plane and a plane (oh no)
  1. They could just be identical
     1. When those intersect, they overlap completely
        1. $2x + 3y + z = 6$
        2. $4x + 6z + 2z = 12$
        3. Smells like scalar up in here.
     2. Parallel planes
        1. Those are just where the point (number) at the end is not the same

3. Intersecting planes
    1. These spit out a line, which is cute
    2. Ex
        1. p =$x + 2y + z = 5$
        2. p2 = $2x + y - z = 7$
        3. Yeah there is not a chance in hell to scale the two of em, so they're definitely intersecting
        4. $< 1, 2, 1 >$
        5. $< 2, 1, -1 >$
2. The direction of the intersection line is given by $\vec{v} = \vec{n_1} \times \vec{n_2}$
    1. alright, so back to our cross shit, direction ends up as $\vec{v} =< -3, 3, -3 >$
    2. Picking any x (go with 0. it makes it easiest)
    3. Plane 1 = $0 + 2y + z = 5$
    4. Plane 2 = $0 + y - z = 7$
    1. this smells like algebra, two equations, two unknowns
    2. $y = 7 + z$, $2(7 + z) + z = 5$, $14 + 3z = 5$, $z = -3$, $y = 4$
    1. So our line ends up as $\vec{r}(t) =< 0, 4, -3 > +t < -3, 3, 3 >$

14.1

- Get more practice with vector valued functions
- match some function $\vec{r}(t)$ with assorted sketches
- Intersections of VVF's
- Multivariate limits and continuity
    - we can apply all that fun stuff from calc to what we did
    - spoiler: you just do them all separately and off you pop
- for the preclass, we had $\vec{r}(t) =< 4\cos(t), \sin(t), \frac{t}{2\pi} >$
    - in order to do this, we projected
    - turns out if we want to, say, project onto the $x - y$ plane
        - we just.... set z to 0
        - $< 4\cos(t), \sin(t), 0 >$
        - same if we want to project it onto literally any other view, just.. ignore it
-

MATH112 - 2024-01-30

#notes #math112 #math #calc

- In three dimensions, you have octants, instead of quadrants
  - quirky
- $x^2 + y^2 = z^2$ spits out a double cone
- www.geogebra.org

notesises

- Find $\vec{r}(t) = ?$
  - for a circle centered at 0,2,1 with a radius of 2 that is parallel to the x-z plane
  - For that whole parallel to the x-z and centered bit, we need $y = 2$
    - y doesn't change when we're hanging out on the other plane
- $\vec{r}(t) =< \cos(t), 2, \sin(t) >$
  - for that radius, scale the terms (that change) $< 2\cos(t), 2, 2\sin(t) >$
  - $< 2\cos(t), 2, 2\sin(t) + 1 >$
- Find the intersection of $y = \frac{1}{2}$ and $x^2 + y^2 + z^2 = 1$
  - $y = \frac{1}{2}$ is actually a plane
  - Just sub that in, $x^2 + (\frac{1}{2})^2 + z^2 = 1$
    - $x^2 + z^2 = \frac{3}{4}$
  - Aight vector valued
    - $\vec{r}(t) =< \frac{\sqrt{3}}{2}\sin(t), \frac{1}{2}, \frac{\sqrt{3}}{2}\cos(t) >$
- Find the intersection of $x^2 + y^2 + z^2 = 1$ and $y = z$
  - $x^2 + 2z^2 = 1$
    - hey dummy remember that they don't scale the same if they're an ellipse
- Writing a VVF
  - $\vec{r}(t) =< \cos(t), \sqrt{2}\sin(t), \sqrt{2}\sin(t)$
- $x^2 + y^2 = z^2$, $y = z^2$
  - $x^2 + (z^2)^2 = z^2$
  - $x^2 + z^4 = z^2$
  - $x^2 = z^2 - z^4$
  - $x = \pm\sqrt{z^2 - z^4}$
- Let $z = t$, and let $y = t^2$, which makes $x = \pm\sqrt{t^2 - t^4}$
- That $\pm$ really makes it so we have two VVFs
  - The $+$ is the intersection of the upper cone, and the $-$ is the lower cone
  - The domain on that is just t is between $[-1, 1]$

calc of VVFs

$$\lim_{t \to a} < f(t), g(t), h(t) >= \lim_{t \to a} \vec{r}(t)$$

Which is the same thing as

$$< \lim_{x \to a} f(t), \lim_{x \to a} g(t), \lim_{x \to a} h(t) >$$

This overall limit exists if each limit exists

$$\lim_{t \to 0} < 1 + t^3, te^{-t}, \frac{\sin(t)}{t}$$

which is

$$< 1, 0, \lim_{t \to 0} \frac{\sin(t)}{t} >$$

use a quick trip to the hospital

$$< 1, 0, 1 >$$

Do anotha one

$$\lim_{t \to \infty} < \cos(\pi t), \sin(\pi t), e^{-t} >$$

- Well, the z term goes to 0
- $\lim_{t \to \infty} \sin(\pi t) DNE$ because it just oscillates the whole time
  - round nround
- So this whole thing does not exist. womp womp.
- Quick reminder that to be continuous, we need to have the limits from both sides be the same, and equal the value at that point
- $\vec{r}(t)$ is continuous at $t = a$ if $f(t), g(t), h(t)$ are all continuous at the point
  - So if any of the x/y/z components are not continuous, we toss the whole thing out. zoinkers.

4.2 is derivatives n fun stuff

- Do a derivative of VVF
  - You do the derivative of each component
- Two new derivative rules
  - One for dots, one for cross products
- Tangent lines were calc 1, tangent vectors are what we're doing now

- Integrals are also just component by component

$\vec{r}'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$

MATH112 - 2024-01-31

#notes #math112 #math #calc

$\vec{r}(t) =< f(t), g(t), h(t) >$

- you're really just doing component wise derivatives
  - $\vec{r}'(t) =< f'(t, g'(t), h'(t)) >$
    - this is the derivative (duh)
    - also known as the tangent vector
    - if (t) is time and this is position, then we get velocity
    - $|r'(t)|$ is the magnitude of the derivative vector, also known as SPEEEEEEED
      - you could also divide the vector by this, which gives you the unit direction vector
  - $\vec{r}''(t) =< f''(t), g''(t), h''(t)$
  - r''', etc, etc

example

- find the line tangent to $\vec{r}(t) =< \ln(t), \sqrt{2t+1}, 4 >$
  - when t=4
  - so we'll get some point, which is $\vec{r}(4) =< \ln(4), 3, 4 >$
  - aaaand now we need a direction
    - $\vec{r}'(t) =< \frac{1}{t}, \frac{1}{2}(2t+1)^{\frac{-1}{2}}2, 0 >$
      - and we want that at 4
        - $< \frac{1}{4}, \frac{1}{3}, 0$
  - $\ell(t) =< \ln(4), 3, 4 > +t < \frac{1}{4}, \frac{1}{3}, 0 >$
  - $\ell(t) =< \ln(4)\frac{+t}{4}, 3 + \frac{t}{3}, 4 >$

new derivative rules kicking it around

dot product

$$\frac{d}{dt}(\vec{u}(t)\cdot\vec{v}(t)) = \vec{u}(t)\cdot\left(\frac{d}{dt}\vec{v}(t)\right) + \left(\frac{d}{dt}\vec{u}(t)\right)\cdot\vec{v}(t)$$

cross product

$$\frac{d}{dt} = (\vec{u}(t)\times\vec{v}(t)) = \vec{u}(t)\times\left(\frac{d}{dt}\vec{v}(t)\right) + \left(\frac{d}{dt}\vec{u}(t)\right)\times\vec{v}(t)$$

integrals

indefinite integrals

$\int \vec{r}(t)dt =< \int f(t)dt, \int g(t)dt, \int h(t)dt >$
$=< F(t)+c_1, G(t)+c_2, H(t)+c_3 >$

definite integrals

$$\int_a^b \vec{r}(t)dt =< \int_a^b f(t)dt, \int_a^b g(t)dt, \int_a^b h(t)dt >$$

14.3

- applying the 14.2 stuff (that we just did) to motion
- velocity, acceleration, gravity, yadayayayada

deribatibs

- let's say that the path is $\vec{r}(t) =< t, e^t, te^t >$
  - so the velocity, $\vec{v}(t) =< 1, e^t, te^t + e^t >$
    - so the acceleration, $\vec{a}(t) =< 0, e^t, te^t + e^t + e^t >$
      - e^t does not sound like a real term anymore

integrals

- Given $\vec{a}(t) =< \cos(t), 2\sin(t) >$
- We have that $\vec{v}(0) =< 0, 1 >$
- and that $\vec{r}(0) =< 1, 0 >$

- math time
  - $\vec{v}(t) =< \sin(t), -2\cos(t) + 3 >$
    - alright so now we need to find r
    - $\vec{r}(t) =< -\cos(t), -2\sin(t) + 3t > + \vec{c}$
      - $< 1, 0 >=< -1, 0 > + \vec{c}$
      - $\vec{c} =< 2, 0 >$
    - so the pos function, fully written out, would be $\vec{r}(t) =< -\cos(t) + 2, -2\sin(t) + 3t >$

example (trajectory problem with gravity)

- you'll be given some initial position, (x_o, y_o) and we'll probably get some v_0 and some α launch angle
  - Determine the range and max height
- $\vec{v}_0 = |v_0| < \cos(\alpha), \sin(\alpha)$
- $\vec{a}(t) =< 0, -g >$
  - g is a constant, $32\frac{ft}{s^2}$, or more commonly $9.8\frac{m}{s^2}$
- so $\vec{v}(t) = \int \vec{a}(t)dt >$
  - so we have $< 0, -gt > + \vec{c}$, so $\vec{c}$ is just going to be $< u_o, v_o >$
  - $\vec{v}(t) =< u_o, -gt + v_o >$
    - alrighty now we need to integrate (again)
    - $< u_o t, \frac{-gt^2}{2} + v_o t > + \vec{c}$
    - so $\vec{c}$ is just going to be $< x_o, y_o >$
    - end function is that $\vec{r}(t) =< u_o t + x_o, \frac{-gt^2}{2} + v_o t + y_o >$
    - and you can replace things with sin and cos as they should be
    -
  -

MATH112 - 2024-02-02

#notes #math112 #math #calc

"One of the quiz problems this weekend is graded wrong by Pearson
- Domain of $< \sqrt{8+t}, \sqrt{8-t} >$ is graded as $|t| >= 8$
- should be $\leq$ 8, for the record

- Last time on calc, we were doing trajectories
  - Given an initial posistion of $< 0, 0 >$ and a $|V_o| = 150\frac{m}{s}$

- $\alpha = 30^\circ = \frac{\pi}{6}$
- Find the Range
  - we wanna find when $y(t) = 0$
    - $\frac{-9.8t^2}{2} + V_o t = 0$
      - that's 0 when t is 0, yipee skippee
      - $t(-4.9t + V_o) = 0$
      - $t = \frac{V_0}{4.9}$
      - $t = \frac{150\sin(\frac{\pi}{6})}{4.9}$
        - that spits out 15.3 seconds
- max height occurs at half that, 7.65 seconds
  - you could either do this when y velocity is 0, or half the time
- "don't trick yourself into thinking in only one dimension"
- If the magnitude of your path is some constant R, you're on a circle (or a sphere, if you're chilling in 3d)
  - if $\vec{r}(t)$ is such that its magnitude is constant, then $\vec{r}(t) \cdot \vec{v}(t) = 0$

6.2

previous on calc 1

- we have some function $f(x) \geq 0$ on $x\epsilon[a, b]$
  - we want the area under the curve, we do $\int_a^b f(x)dx =$ area
- now in calc 2 land
  - $f(x) \geq g(x)$ on $x\epsilon[a, b]$
    - Our integral here is going to be $\int_a^b (f(x) - g(x))dx$

example

$f(x) = \frac{x}{x^2+1}$, $g(x) = \frac{x}{5}$

To find the "area between curves," make sure you pick the function that is on "top" to be $f(x)$

- :raised_eyebrow:

So we have $\int_{-a}^{0} \left(\frac{x}{5} - \frac{x}{x^2+1}\right)dx + \int_0^a \left(\frac{x}{x^2+1} \frac{-x}{5}\right)$
a is 2 and -2, respectively

MATH112 - 2024-02-05

#notes #math112 #math #calc

- If shit don't work out vertically, why not just flip it?
  - You can integrate over y, and that works out more or less the same
  - You integrate the right over the left instead of the top - bottom
    - I mean you could always just swap x and y, but it's waaaaaay easier to just flip it
- $$V \approx \sum_{k=1}^{n} A(x_k^*)\Delta x$$
  - A is the 2 dimensional area of some given cross section, and Δx is the width of the slice
  - So, you end up doing the limit, which just gets a nice integral of the area
- So, the area if we had a semicircle would be
  - $$\int_{-1}^{1} \left(\frac{(1-x^2)^2 * \pi}{2} \right) dx$$
  - $$\int_{-1}^{1} \left(\frac{\pi * r^2}{2} \right) dx$$

MATH112 - 2024-02-06

#notes #math112 #math #calc

volume (of pyramids / cones)

- They start off with some base shape
  - They proceed to get smaller and smaller up to a point
    - For instance, if you were to slice parallel to the base, you would just get smaller and smaller squares
- Truncated cone is basically a loft (or a cone with the top sliced off)
- Size of shape changes linearly as you go up
- For square bases, you need one side, for circles, you need the radius, etc

step by step (yippeee)

1. Find length of side s_{base}
2. Find length of side s_{top} (which could very well be 0, in a lot of cases)
3. Evaluate $s(y) = \frac{s_{top} - s_{base}}{h} y + s_{base}$
4. Find $A(y)$
   1. For a square, $A(y) = (s(y))^2 R$
   2. For a circle, $A(y) = \pi(s(y))^2$
5. Integrate, $\int_0^h A(y) dy$

example

- Some square pyramid, 6m by 6m, with a height of 10m
- Following our steps
  - $s_{base} = 6$
  - $s_{top} = 0$
  - $s(y) = \frac{-6}{10} y + 6$
    - $\frac{-3}{5} y + 6$
  - $A(y) = (\frac{-3}{5} y + 6)^2$
  - $V = \int_0^{10} (\frac{-3}{5} y + 6)^2 dy$
    - Spits out like, $120m^3$

today, find volumes by revolution

~~show those bourgeoise bastards~~

- Slices are disks
- oooor, slices that are washers (hole in the middle)
- Alright, so i have some curve $f(x) \geq 0 \text{ on } x[a, b]$
  - You integrate, find the area under the curve
    - Revolve that shape about the x axis, and wham bam, shape
- The y of $f(x)$ becomes your radius, so $A(x) = \pi(f(x))^2$
- So then we just $\int_a^b \pi(f(x))^2 dx$

example

- $f(x) = 4 - x^2, x \epsilon [-2, 2]$
- Revolve about the x axis

- $V = \int_{-2}^{2} \pi(4-x^2)^2 dx$
- $V = \int_{-2}^{2}(16x-8)$
  512/15 pi unit^3

Say $f(y) \geq 0$ for $y\epsilon[a,b]$
it's the same as about the x axis, $V = \int_a^b \pi(f(y))^2 dy$

$V = \int_0^4 (\pi)(\frac{1}{2}y)^2 dy$

Alright, for making donuts between two functions
It's basically outer - inner, which works out to be

$$\int_a^b \pi((f(x))^2 - (g(x))^2)dx$$

- If you're revolving around a weird line, just do inner vs outer

MATH112 - 2024-02-07

#notes #math112 #math #calc

We're going to make more revolute solids, but now we're going to start shelling em

- holy shit, war is hell

Revolving some rectangle about the same axis, that would make some cylinder as part of the objecect

So we have a can with no top, and no bottom, surface area would just be $2\pi rh$
Let's say that the thickness of the can's wall is Δx

$V \approx 2\pi rh\Delta x$
that approximately is because the radius changes

We could have some $f(x)$ that goes from a to b, some $g(x)$ that goes from a to b as well
Revolve ourselves about the y axis

$$V \int_a^b 2\pi x * (f(x) + g(x))$$

hershey kith

$f(x) = 1 - (x-1)^3$

$g(x) = 0$

$x \epsilon [0, 2]$

$v \int_0^2 2\pi x(1-(x-1)^3)dx$

We're actually going to integrate this one

$u = x - 1$

$\frac{du}{dx} = 1$, $du = dx$

$\int_0^2 2\pi x(1-(u)^3)$

alternative, x = u+1

$2\pi \int_{-1}^{1}(u+1)(1-u^3)du$

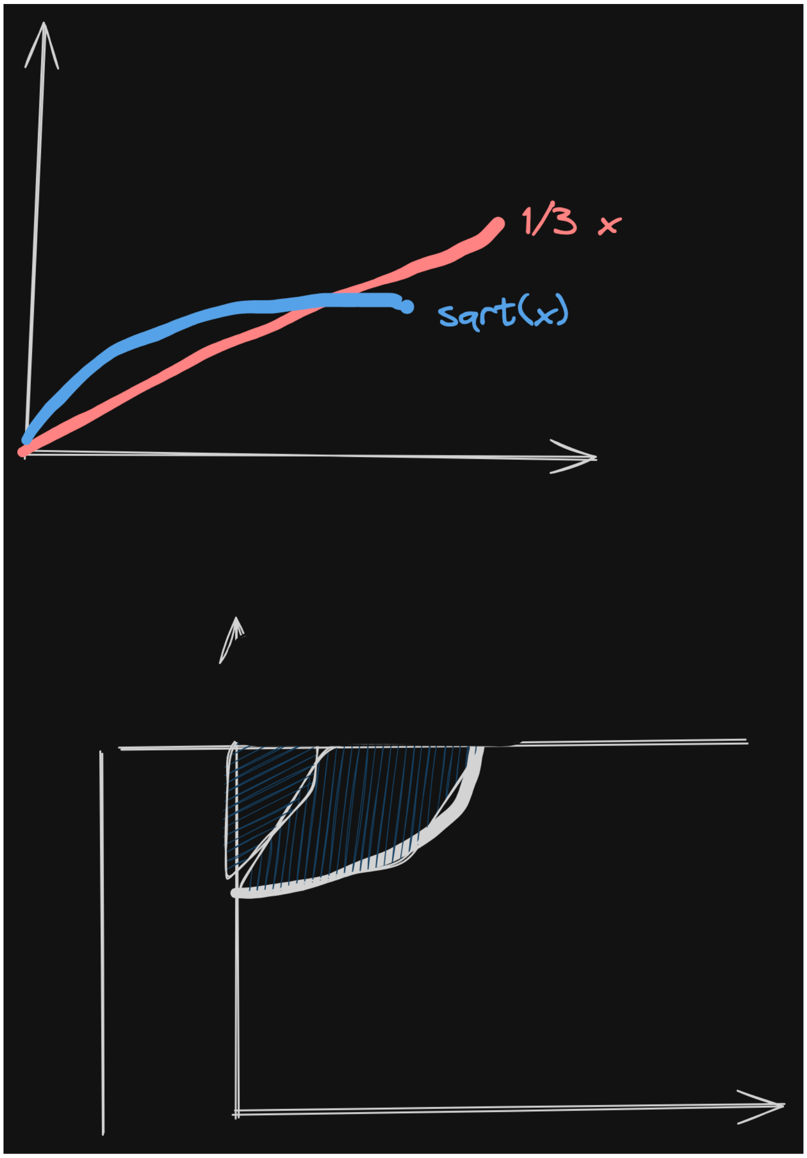
$2\pi \int_{-1} 1(u - u^4 + 1 - u^3)du$

$2\pi \left(\frac{u^2}{2} \frac{-u^5}{5} + u - \frac{u^4}{4}\right)$

....

$\frac{16}{5}\pi$ units^3

example... but again



These intersect at $(0, 0)$ and $(9, 3)$, and we're revolving around the y axis
Limits of integration are going to be 0 and 9

$$\int_0^9 2\pi x(\sqrt{x} - \frac{1}{3}x)$$

$$2\pi \int_0^9 \left(x^{\frac{3}{2}} - \frac{1}{3}x^2\right) dx$$

$$2\pi * \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{\frac{1}{3}x^3}{3}$$

....
that works out to like, $\frac{162}{5}\pi$

---

hey-ho, do it again, rotate around $x = -1$

Region;
$y = 6$
$y = x^2 + 2$
$x \geq 0$

So our integral setup would be $V = \int_0^6 2\pi(x+1)(6-(x^2+2))dx$

$$2\pi \int_0^6 (x+1)(4-x^2)dx$$

$$2\pi \int_0^6 (4x - x^3 + 4 - x^2)$$

Integrate that, get some garbage about

$$2\pi * (2x^2 - \frac{1}{4}x^4 + 4x - \frac{x^3}{3})$$

$$2\pi * (72 - \frac{1296}{4} + 24 - 72)$$

---

go backwards

$$V = \int_0^3 2\pi(3-x)(3-x)dx$$

Alright, so the first $(3-x)$ is the radius, the second is the height

MATH112 - 2024-02-13

#notes #math112 #math #calc

what's up with today

- all kinds of fun applications of integrals
  - mass/density
  - work integrals
    - springs
    - lifting things
    - Pumping water

starting off with mass

- Start with density $\rho(x) = \frac{mass}{length}$ at posistion x
    - Have some rod, where one end is 0, and going around the length is x
    - Mass of chunk k $\approx \rho(x_k) * \Delta x$
- Suppose $\rho(x) = \rho$ for all x
    - That's a uniform rod, so then the mass is just $\rho * \Delta x$
- Generally, mass = $\sum_{k=1}^{n} \rho(x_k)\Delta x$
    - Now, calc wants a limit, so $\lim_{n\to\infty} \sum_{k=1}^{n} \rho(x_k)\Delta x = \int_a^b \rho(x)dx$
- Given a 2 meter bar, with a density equal to $\rho(x) = 1 + x^2$ $\frac{kg}{m}$, find mass of bar
    - $\int_0^2 (1 + x^2)dx$
        - $1x + \frac{x^3}{3} from 0^2$
        - $2 + \frac{8}{3}$ = $\frac{14}{3}kg$

work

- Generally, $work = force * distance$ (yeah, I didn't make it \text, sue me)
- $w = F * d$

Common units

Metric

- Force, N- Newtons
- Distance, meter
- Work is N*M, also known as a Joule

Imperial

- Force, pounds
- Distance, feet
- Work is just ft lb

Calculus allows for force to be $F(x)$

$\lim_{n\to\infty} F(x_k) * \Delta x$, you sum all those up, yadayadayada, you get an integral.
$\int_a^b F(x)dx$

first up on the chopping block is springs

Hooke's Law says that for a spring, $F(x) = kx$
- where k is just our quirky silly little spring constant
- which is conventionally in like Joules

- spring starts at some distant equilibrium point called 0

example

40n is required to stretch a spring 20cm beyond equilibrium
40N is required to stretch a spring 0.2m

- Find the spring constant
  - 200 N*M
- So the force equation would be $200x$
- Find the work required to stretch from 20cm to 40cm
  - $\int_{0.2}^{0.4}(200x)dx$
  - $200\int_{0.2}^{0.4}(x)dx$
  - $100x^2$ from 0.2 to 0.4
    - That's some number alright ($16 - 4 = 12$J)
    -

example but again

find work to compress a spring from 50cm to 80cm
$W = \int_{-0.5}^{-0.8}(200x)dx$
$W = 100x^2$ from -0.5 to -0.8

- which will just come out to 39J

lifting problems

Gravity g

- we have some 70lb object
- (we generally use y)
  - if we're up at 60ft, calculate the work required to lift the bucket and the rope
  - There's generally two parts, getting the work of the bucket and the work of the rope
    - That bucket's super easy, it's constant, and is just going to be $70 * 60 = 4200ftlb$
  - Say the rope has density $\rho = 0.5\frac{lbs}{ft}$
  -

MATH112 - 2024-02-14

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves about the given lines.

y equals 25 minus x squaredy=25−x2,nbsp yequals=2525,nbsp xequals=55;

revolve about the line y equals 25#notes #math112 #math #calc

homework problem

So if we have some hemispherical tank (basically #6 on the worksheet)

- If we take some horizontal cross section, we have some circular cross section, and we need to find the area
  - So the area is $A(y) = \pi r^2$
  - $r^2 = 8^2 - (8-y)^2$
  - $A(y) = (\pi)(8^2 - (8-y)^2)$
    - So the integral would be $\int_0^8 \pi\rho * g(8^2 - (8-y)^2)(8+2-y)dy$

"basic" integration

u sub

$$I = \int x^{\frac{1}{2}} \cos(x^{\frac{3}{2}})dx$$

$$u = x^{\frac{3}{2}}$$

$$du = \frac{3}{2}x^{\frac{1}{2}}dx$$

Oh hop skiddly bop, we've got that floating around

$$\int \cos(u)\frac{2}{3}du$$

$$= \frac{2}{3}\sin\left(x^{\frac{3}{2}}\right) + C$$

Splitting Fractions

$$I = \int \frac{x+2}{x^2+4}dx$$

$$= \int \frac{x}{x^2+4}dx + \int \frac{2}{x^2+4}dx$$

$$= I_1 + I_2$$

$$\text{For } I_1, u = x^2+4, du = 2xdx$$

$$I_1 = \int \frac{1}{2}\frac{\frac{1}{2}du}{u} = \frac{1}{2}\ln|u| + C, \frac{1}{2}\ln|x^2+4| + C$$

$$I_2 = 2\int \frac{dx}{x^2+4} = \int \frac{\frac{1}{4}dx}{\frac{x^2}{4}+1}$$

$$\text{For } I_2, u = \frac{x}{2}, u^2 = \frac{x^2}{4}, du = \frac{1}{2}dx$$

$$\frac{1}{2}\int \frac{2du}{u^2+1} = \int \frac{du}{u^2+1} = \tan^{-1}(u) + C$$

$$\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\frac{1}{2}\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C$$

Completing the Square

$$(x-1)^2 = x^2 - 2x + 1$$

$$I = \int \frac{dx}{x^2-2x+10}$$

$$\int \frac{dx}{x^2-2x+10+1-1}$$

$$\int \frac{dx}{(x-1)^2+9} * \frac{\frac{1}{9}}{\frac{1}{9}}$$

$$\int \frac{\frac{1}{9}dx}{\frac{(x-1)^2}{9}+1}$$

$$u = \frac{x-1}{3}, u^2 = \frac{(x-1)^2}{9}, du = \frac{1}{3}dx$$

$$\frac{1}{9}\int \frac{3du}{u^2+1}$$

$$\frac{1}{3}\int \frac{du}{u^2+1}, = \frac{1}{3}\tan^{-1}(u)+C$$

$$\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right)+C$$

rational division

- we're basically doing long division

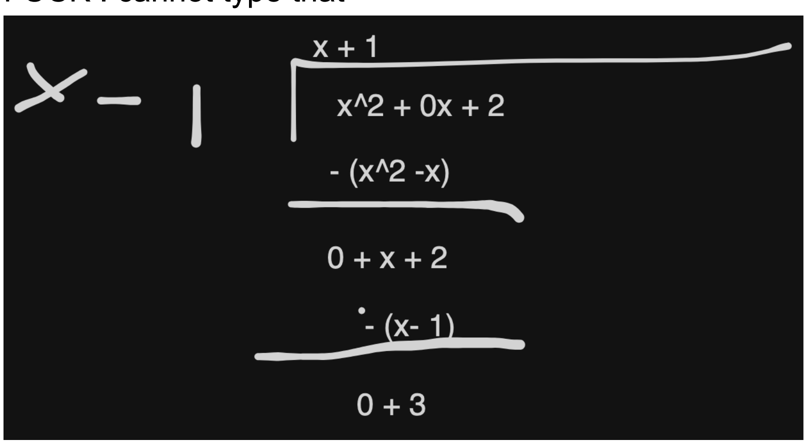
$$I = \int \frac{x^2+2}{x-1}$$

aside: this could be done with u-sub

- you sub, you call it a day, yippeee skippy

long division time

FUCK I cannot type that



$$\int (x+1+\frac{3}{x-1})dx$$

$$= \frac{x^2}{2}+x+3\ln(|x-1|)$$

some hints for worksheet

1. split fraction
2. u-sub (denominator)
3. u sub (argument of $\sqrt{}$)
4. same usub as 3 but tricksy
5. You can do long division for that
6. uses a technique called "multiply by 1".
    1. I will give you ONE guess how that works

MATH112 - 2024-02-21

#notes #math112 #math #calc

Integration by Parts

$$\int udv = uv - \int vdu$$

Steps
0. Uh, try an easier way. This shit is a pain.

1. $\int vdu$ must be easier than what you started with ($\int udv$)
2. You must be able to integrate dv
3. How to pick u
    1. LIPET
        1. Logarithms
        2. Inverse Trig
        3. Powers
        4. Exponentials
        5. Trig

example time

$$I = \int x^2 e^{3x} dx$$

$$u = x^2, dv = e^{3x}dx, v = \frac{1}{3}e^{3x}, du = 2xdx$$

$$I = (x^2)\left(\frac{1}{3}e^{3x}\right) - \int \frac{1}{3}e^{3x}2xdx$$

$$I = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}dx$$

$$I_1 = \int xe^{3x}dx$$

$$u = x, du = dx, dv = e^{3x}dx, v = \frac{1}{3}e^{3x}$$

$$I_1 = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx$$

$$I_1 = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

$$I = \frac{1}{3}x^2e^{3x} - \frac{2}{3}(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}) + C$$

$$I = \int \ln(x)dx$$

$$u = \ln(x), du = \frac{1}{x}, dv = 1, v = x$$

$$\int \ln(x)dx = \ln(x)x + \int \frac{x}{x}dx$$

$$\int \ln(x)dx = \ln(x)x - \int 1$$

$$\int \ln(x)dx = \ln(x)x - x$$

$$I = \int \sin^{-1}(x)dx$$

$$u = \sin^{-1}(x), dv = dx, du = \frac{1}{\sqrt{1-x^2}}dx, v = x$$

$$I = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}dx$$

$$I_1 = \text{that bullshit up there}, u = 1 - x^2, du = -2xdx$$

$$\int \frac{-\frac{1}{2}du}{\sqrt{u}}, -\frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{-1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = -(1-x^2)^{\frac{1}{2}}$$

$$I = x\sin^{-1}(x) + (1-x^2)^{\frac{1}{2}} + C$$

$$I = \int \tan^{-1} dx$$

$$u = \tan^{-1}(x), dv = dx, v = x, du = \frac{1}{x^2+1}$$

$$I = x\tan^{-1}(x) - \int \frac{x}{x^2+1}dx$$

$$I_1 = \int \frac{x}{x^2+1}dx$$

$$u = x^2+1, du = 2x$$

$$I_1 = \int \frac{\frac{1}{2}du}{u}, = \frac{1}{2}\int \frac{du}{u}$$

$$I_1 = \frac{1}{2}\ln(|u|) + C$$

$$I_1 = \frac{1}{2}\ln(|x^2+1|) + C$$

$$I = x\tan^{-1}(x) - \frac{1}{2}\ln(|x^2+1|) + C$$

$$I = \int e^x \sin(2x)dx$$

$$u = e^x, dv = \sin(2x)dx, du = e^x dx, v = -\frac{1}{2}\cos(2x)$$

$$I = \frac{-1}{2}e^x\cos(2x) + \frac{1}{2}\int e^x\cos(2x)dx$$

$$I_1 = \int e^x\cos(2x)dx$$

$$u = e^x, dv = \cos(2x)dx, du = e^x dx, v = \frac{1}{2}\sin(2x)$$

$$I_1 = \frac{1}{2}e^x\sin(2x) - \frac{1}{2}\int e^x\sin(2x)$$

$$I = \frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x) - \frac{1}{4}I$$

$$1.25(I) = \frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x)$$

$$I = \frac{\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x)}{\frac{5}{4}}$$

MATH112 - 2024-02-27

#notes #math112 #math #calc

Powers of trig functions, integrations

First up are sines and cosines

$$I = \int \sin^5(x)\cos^4(x)dx$$

$$I = \int (\sin(x))^5(\cos(x))^4 dx$$

$$\int \sin^4(x)\cos^4(x)\sin(x)dx$$

$$I = \int (1-\cos^2(x))^2\cos^4(x)\sin(x)dx$$

$$u = \cos(x), du = -\sin(x)$$

$$I = -1 * \int (1-u^2)^2 u^4 du$$

Expand all that garbage out

$$-1\int (u^4 - 2u^6 + u^8)du$$

$$-\frac{u^5}{5} - \frac{2u^7}{7} - \frac{u^9}{9} + C$$

$$-\frac{\cos^5(x)}{5} + \frac{2}{7}\cos^7(x) - \frac{1}{9}\cos^9(x) + C$$

$$\int \sin^m(x)\cos^n(x)dx$$

general steps

Case (1): Powers are such that m is positive and odd, and $n \epsilon \mathbb{R}$
Case (2): Powers are such that n is positive and odd, and $m \epsilon \mathbb{R}$
Case (3): Powers are such that m and n are positive and even

case 1 or 2

1. Separate one term from an odd power
    1. $I = \int \sin^3(x) \cos^3(x) dx$
        1. oh hey, this is Case 1 *and* Case 2
            1. Ain't that neat.
        2. $I = \int \sin^2(x) \cos^3(x) \sin(x) dx$
            1. $u = \cos(x),\ du = \sin(x)$
            2. $\int (1 - \cos^2(x)) \cos^3(x) \sin(x) dx$
            3.
        3. $I = \int \sin^3(x) \cos^2(x) \cos(x) dx$
2. Convert the remaining even power to the other trig function
    1. $I = \int \sin^3(x)(1 - \sin^2(x)) \cos(x) dx$
3. u-sub? Sure!
    1. $u = \sin(x), du = \cos(x)$
    2. $I = \int u^3(1 - u^2) du$
        1. $I = \int u^3 - u^5$
4. Expand + Integrate
    1. $I = \frac{u^4}{4} - \frac{u^6}{6} + C$
    2. Aaaand get back out of u land
    3. $I = \frac{1}{4} \sin^4(x) - \frac{1}{6} \sin^6(x) + C$

So, fun fact

$$I = \int \cos^3(x) dx$$

This is a quirky little case 2 where the power on $\sin(x)$ is 0

time for....

case 3

$$I = \int \sin^4(x) \cos^2(x) dx$$

$$I = \int (\sin^2(x))^2 \cos^2 dx$$

$$I = \int \left(\frac{1 - \cos(2x)}{2}\right)^2 \left(\frac{1 + \cos(2x)}{2}\right) dx$$

$$I = \frac{1}{8}\int(1 - 2\cos(2x) + \cos^2(2x))(1 + \cos(2x))dx$$

$$I = \frac{1}{8}\int(1 - \cos(2x) - \cos^2(2x) + \cos^3(2x))dx$$

$$I = \frac{1}{8}\int\left(1 - \cos(2x) - \left(\frac{1 + \cos(4x)}{2}\right) + \text{more bullshit with case 2}\right)dx$$

This shit is a *pain in the ass*

- official notes say "tedious" but let's be honest. pain.
- Luckily, there's an easier way
- We've got Reduction Formulas
  - $$\cos^n(x) = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int\cos^{n-2}(x)dx$$
  - Yeah uh, don't memorize that, but get with the idea

just when you thought the water was safe

$$\int\tan^m(x)\sec^n(x)dx$$

MATH112 - 2024-02-28

#notes #math112 #math #calc

today is fun with tangents and secants

- quick review of what we *should* already know
  - $\int\sec^2(x)dx = \tan(x) + C$
  - $\int\sec(x)\tan(x)dx = \sec(x) + C$
  - Activity Sheet 8.1
    - $\int\tan(x)dx$
      - $-1|\cos(x)| + C$
    - $\int\sec(x)dx$
      - $\ln(\sec(x) + \tan(x)) + C$
- $$\int\tan^m(x)\sec^n(x)dx$$

- Just like yesterday, we're going to have case 1, case 2, and the pain in the ass :(

cases for $\int \tan^m(x)\sec^n(x)dx$

Case 1

- n $(\sec)$ is positive even, m is $\mathbb{R}$

Case 2

- m is positive odd
- n is $\mathbb{R}$

Case 3 ☹

- m is positive even
- n is positive odd
- this is a pain

---

brief aside

$$\tan^2(x) + 1 = \sec^2(x)$$

$$I = \int \tan^3(x)\sec^4(x)dx$$

- hey ho, this is actually both case 1 and case 2, but we're going to treat it as a case 1 for examples sake

case 1

step one!

- separate out a $\sec^2(x)$

$$I = \int \tan^3(x) \sec^2(x) \sec^2(x) dx$$

step two!

- get back to tangent from our $\sec^2(x)$

$$I = \int \tan^3(x)(\tan^2(x) + 1)(\sec^2(x))dx$$

step three!

- u-sub, yippeee

$$u = \tan(x), du = \sec^2(x)dx$$

$$I = \int u^3(u^2 + 1)du$$

$$I = \int (u^5 + u^3)du$$

step four!

- expand and integrate, have fun

$$\frac{1}{6}u^6 + \frac{1}{4}u^4 + C$$

$$\frac{1}{6}\tan^6(x) + \frac{1}{4}\tan^4(x) + C$$

---

case 2

$$I = \int \tan^3(x) \sec^3(x) dx$$

step 1

- rip out a $\sec(x) \tan(x)$

$$I = \int \tan^2(x)\sec^2(x)\sec(x)\tan(x)dx$$

step 2

- replace

$$I = \int ((\sec^2(x) - 1)\sec^2(x))\sec(x)\tan(x)dx$$

step 3

- u sub, hipee

$$u = \sec(x), du = \sec(x)\tan(x)dx$$

$$I = \int (u^2 - 1)u^2 du$$

step 4

- expand, solve, blah blah blah

$$I = \int (u^4 - u^2)du$$

$$I = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

Replace the u, and there you go

$$\frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C$$

trig sub

usable if like

$$f(x) \text{ has } \sqrt{a^2 - x^2}, \sqrt{x^2 + a^2}, \sqrt{x^2 - a^2}$$

Some generally important things
Half angle formulas

$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}, \cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

Also have double angle

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

In a given right triangle with θ over to the right,

$$\sin(\theta) = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos(\theta) = \frac{x}{\sqrt{x^2+y^2}}$$

$$\tan(\theta) = \frac{y}{x}$$

case 1

$$\sqrt{a^2-x^2}$$

take a> 0, given that we're squaring it

trig sub

$$x = a\sin(\theta)$$

$$dx = a\cos(\theta)d\theta$$

$$I = \int \frac{dx}{16-x^2}$$

In this case, a=4

$x = 4\sin(\theta), dx = 4\cos(\theta)d\theta$

$$I = \int \frac{4\cos(\theta)}{\sqrt{16-(4\sin(\theta))^2}}$$

$$I = \int \frac{4\cos(\theta)}{\sqrt{16-16\sin^2(\theta)}}$$

$$\sqrt{16(1-\sin^2(\theta))}$$

$$4\sqrt{\cos^2(\theta)}$$

$$4\cos(\theta)$$

$$I = \int \frac{4\cos(\theta)d\theta}{4\cos(\theta)}$$

$$I = \theta + C$$

$$I = \sin^{-1}(\frac{x}{4}) + C$$

---

example with triangle

$$I = \int x^3\sqrt{4-x^2}dx$$

$$x = 2\sin(\theta)$$

$$dx = 2\cos(\theta)d\theta$$

$$\theta = \arcsin(\frac{x}{2})$$

$$I = \int (8\sin^3\theta)\sqrt{4-4\sin^2\theta}(2\cos(\theta)d\theta)$$

$$I = \int (8\sin^3\theta)(2\cos(\theta))(2\cos(\theta)d\theta)$$

$$I = 32\int \sin^3(\theta)\cos^2(\theta)d\theta$$

$$I = 32\int \sin^2\theta\cos^2\theta\sin\theta d\theta$$

$$I = 32\int (1-\cos^2(\theta))\cos^2\theta\sin(\theta)\mathrm{d}\theta$$

$$u = \cos(\theta), du = -\sin(\theta)d\theta$$

$$I = 32\int (1-u^2)u^2(-1)du$$

$$I = -32\int u^2 - u^4du$$

$$I = -32(\frac{1}{3}u^3 - \frac{1}{5}u^5) + C$$

$$I = -32(\frac{1}{3}\cos^3(\theta) - \frac{1}{5}\cos^5(\theta)) + C$$

$$I = -32(\frac{1}{3}(\frac{\sqrt{4-x^2}}{2})^3 - \frac{1}{5}(\frac{\sqrt{4-x^2}}{2})^5) + C$$

MATH112 - 2024-03-04

#notes #math112 #math #calc

previously on

- have some shenanigans like $\sqrt{a^2 - x^2}$, we let $x = a\sin(\theta)$

today

- some shenanigans like $\sqrt{x^2 + a^2}$
- Let $x = a\tan(\theta)$
- $dx = a\sec^2(\theta)d\theta$

example

$$I = \int \frac{dx}{(x^2+9)^2}$$

$$I = \frac{dx}{(\sqrt{x^2+9})^4}$$

$$x = 3\tan\theta, \theta = \tan^{-1}(\frac{x}{3})$$

$$dx = 3\sec^2(\theta)d\theta$$

$$\int \frac{3\sec^2(\theta)d\theta}{(9\tan^2(\theta)+9)^2}$$

Factor out a 9, get a +1

$$= \int \frac{3\sec^2(\theta)}{81\sec^4(\theta)}d\theta$$

$$\frac{1}{27}\int \frac{1}{\sec^2(\theta)}d\theta$$

$$I = \frac{1}{27}\int \cos^2(\theta)d\theta$$

pythagorean squared bullshit woo!

$$I = \frac{1}{27}\int \frac{1\cos(2\theta)}{2}$$

$$I = \frac{1}{54}(\theta + \frac{1}{2}\sin(2\theta)) + C$$

$$I = \frac{1}{54}(\tan^{-1}(\frac{x}{3}) + \frac{1}{2}\sin(2\theta)) + C$$

$$I = \frac{1}{54}(\tan^{-1}(\frac{x}{3}) + \frac{1}{2}2\sin\theta\cos\theta) + C$$

double angle bullshit WOO!

$$\frac{1}{54}(\tan^{-1}(\frac{x}{3}) + \frac{1}{2}\frac{x}{\sqrt{x^2+9}} * \frac{3}{\sqrt{x^2+9}}) + C$$

- this might be off by a factor of two. i don't really care.

---

more bullshit (case 3)

$$\sqrt{x^2 - a^2}$$

$$\text{Let } x = a\sec\theta$$

$$\theta = \sec^{-1}(\frac{x}{a})$$

$$dx = a\sec(\theta)\tan(\theta)d\theta$$

$$I = \int \frac{x^3}{(x^2-9)}$$

$$\text{Let } x = 3\sec(\theta)$$

$$dx = 3\sec(\theta)\tan(\theta)d\theta$$

$$I = \int \frac{\sec^3(\theta)}{(9\sec^2(\theta) - 9)^{\frac{3}{2}}}$$

$$I = \int \frac{(3\sec(\theta))^3}{9(\sec^2(\theta) - 1)}dx$$

$$I = \int \frac{27\sec^3(\theta) * 3\sec(\theta)\tan(\theta)}{27\tan^3(\theta)}d\theta$$

$$I = 3\int \sec^4(\theta)\tan^{-2}(\theta)d\theta$$

$$I = 3\int (\tan^2(\theta) + 1)\tan^{-2}\sec^2(\theta)d\theta$$

$$u = \tan(\theta), du = \sec^2(\theta)d\theta$$

$$I = 3\int (u^2+1)u^{-2}du$$

$$3(u - u^{-1}) + C$$

$$3(\tan(\theta)) - \cot(\theta) + C$$

$$3(\frac{\sqrt{x^2-9}}{3} - \frac{3}{\sqrt{x^2-9}}) + C$$

$$\sqrt{x^2-9} - \frac{9}{\sqrt{x^2-9}} + C$$

$$\frac{5x-2}{x^2-x}$$

$$\frac{2}{x} + \frac{3}{x-1} = \frac{5x-2}{x^2-x}$$

Start out with some rational $f(x)$, defined as $\frac{P(x)}{Q(x)}$

Case 1 is the easiest. this is my thirty-fourth reason why.

- Proper rational function with linear factors
- Proper: Degree of $P(x) < Q(x)$ (largest power)
- Linear factor: $(x - a_1)(x - a_2)....$ blah blah blah
- Ideally, we want to rewrite as $P\frac{P(x)}{Q(x)}$ as constant over factors

example

$$\frac{P(x)}{Q(x)} = \frac{3x-1}{x^2-1}$$

$$\frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

we're unfortunately gonna need a common denominator, assorted other fractions

$$\frac{3(x-1)}{(x-1)(x+1)} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

$$3x - 1 = A(x+1) + B(x-1)$$

$$3x - 1 = Ax + A + Bx - B$$

$$3x = Ax + Bx$$

$$3 = A + B$$

$$A = 3 - B$$

$$A - B = -1$$

$$3 - B - B = -1, 3 - 2B = -1$$

$$-1 + 2B = 3$$

$$4 = 2B, B = 2, A = 1$$

$$\frac{(x+1) + 2(x-1)}{(x-1)(x+1)} = \frac{x + 2x + 1 - 2}{(x-1)(x+1)} = \frac{3x - 1}{(x-1)(x+1)}$$

MATH112 - 2024-03-05

#notes #math112 #math #calc

quick worksheet aside

4a.

$$\int (2x)^2 \cos(3x) dx$$

- thiiiis is gonna need integration by parts
  - $$u = (2x)^2$$
  - we'll need a gross repeated integration by parts

example

$$\int x^3 e^{3x} dx$$

$$u = x^3, dv = e^{3x}$$

$$du = 3x^2 dx, v = \frac{1}{3} e^{3x}$$

$$x^3 * \frac{1}{3} e^{3x} - \int 3x^2 \frac{1}{3} e^{3x} dx$$

u	dv
x^3	e^{3x}

u	dv
$3x^2$	$\frac{1}{3}e^{3x}$
$6x$	$\frac{1}{9}e^{3x}$
6	$\frac{1}{27}e^{3x}$
0	$\frac{1}{81}e^{3x}$

$$I = +\left(x^3 * \frac{1}{3}e^{3x}\right) - (3x^2 * \frac{1}{9}e^{3x}) + (6x * \frac{1}{27}e^{3x}) - (6 * \frac{1}{81}e^{3x})$$

There are two end cases - either you have to take a derivative of 0, or I shows up again

---

today's fresh torture

$$\frac{3x-1}{(x+1)(x-1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

- solving it the way we did yesterday is generally known as matching coefficients
- This method is always gonna work, no if ands or buts about it, and it all makes sense

Method 2: Convenient Values of x

$$\frac{3x-1}{(x-1)(x+1)}(x-1) = A + \frac{B(1-1)}{1+1}$$

$$A = \frac{3(1)-1}{(1+1)}$$

$$\frac{(3x-1)}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$$

$$\frac{(3x-1)}{(x-1)\ \cancel{(x+1)}}\cancel{(x+1)} = \frac{A(x+1)}{(x-1)} + \frac{B}{\cancel{(x+1)}}\cancel{(x+1)}$$

$$\frac{3(-1)-1}{-1-1} = 0 + B$$

$$B = 2$$

What happens if

$$\frac{P(x)}{Q(x)}$$

is not proper?

If the degree of p is the same or greater than that of the denominator

Say we have

$$\frac{x^3-1}{x^2-x}$$

- We're uh, we're sent to long division hell.

$$x+1$$

$$x^2-x)\overline{x^3+0x^2+0x-1}$$

$$-(x^3-x^2)$$

$$x^2+0x-(x^2-x)$$

$$+x-1$$

  is our remainder

$$\frac{x^3-1}{x^2-x}=x+1+\frac{x-1}{x^2-x}$$

Oh hey, now we're proper, so we can actually do partial fractions (isn't that neat)

case 2 (the suffering never ends)

repeated linear factors

Now our denominator $Q(x)$, can be represented by $(x-a_1)^{M_1}(x-a_2)^{M_2}....(x-a_n)^{M_n}$

Example: $Q(x)=x^2(x-1)$

$$Q=(x-0)^2(x-1)^1$$

$$a_1=0, m_1=2$$

$$a_2=1, M_2=1$$

$$\text{For each (x-a)}^M \text{ we will use } (\frac{A_1}{(x-a)^1})+(\frac{A_2}{(x-a)^2})+....(\frac{A_M}{(x-a)^M})$$

$$\frac{P(x)}{Q(x)} = \frac{x^2 - x + 3}{x^2(x-1)} = \frac{A_1}{(x-0)^1} + \frac{A_2}{(x-0)^2} + \frac{B}{x-1}$$

$$B = \frac{x^2 - x + 3}{x^2}, = \frac{1 - 1 + 3}{1^2} = 3$$

- Here we multiplied everything by $(x-1)$, the linear factor associated with B

$$A_2 = \frac{x^2 - x + 3}{x - 1}|_0, A_2 = -3$$

- Multiplied everything by x^2
- go home and find A_1
-

MATH112 - 2024-03-06

#notes #math112 #math #calc

- If you're in a situation where you have a transcendental * a transcendental, pretty good odds you end up in some cyclical bullshit
- Exam statistics
    > - Median of 72, maxium of 97, mean of 73.25, Standard Deviation of 14.36

$$\int 12 \cos^4(3x) dx$$

$$12 \int \cos^2(3x) \cos^2(3x)$$

$$12 \int \frac{1 + \cos(6x)}{2} * \frac{1 + \cos(6x)}{2}$$

$$12 \int \frac{1 + 2\cos(6x) + \cos^2(6x)}{4}$$

$$3 \int 1 + 2\cos(6x) + \cos^2(6x)$$

$$12 \int \frac{1 + 2\cos(6x) + \cos^2(6x)}{4}$$

$$3 \int 1 + 2\cos(6x) + \frac{1 + \cos(12x)}{2}$$

In a situation where you end up with irreducible quadratic factors (ie, x^2+1) has no real roots

- For irreducible x^2+bx+c,

$$\frac{Ax+b}{x^2+bx+c}$$

$$I = \int \frac{3x^2-4x+5}{(x-1)(x^2+1)} dx$$

- this is proper, we sure cannot factor the base
-

$$\frac{A}{x-1} + \frac{Bx+c}{x^2+1} dx$$

$$A = \frac{3x^2-4x+5}{x^2+1}$$

$$A = \frac{3-4+5}{1+1} = 2$$

- for B and C we're..... gonna need to do more work

$$\frac{3x^2-4x+5}{(x-1)(x^2+1)} = \frac{2}{x-1} + \frac{Bx+c}{x^2+1}$$

Let x = 0, because that lets us find C by killing B (naur)

$$\frac{0-0+5}{-1(1)} = \frac{2}{-1} + \frac{C}{1}$$

$$-5+2 = C$$

$$C = -3$$

$$\frac{3x^2-4x+5}{(x-1)(x^2+1)} = \frac{2}{x-1} + \frac{-3x+C}{x^2+1}$$

Let x = -1

$$\frac{3+4+5}{(-2)(2)} = \frac{12}{-4} = -1$$

look, don't question it, you actually set it equal to the other sdie

---

Repeated irreducible Quadratics

$$\int \frac{dx}{x(x^2+1)^3} = \int \frac{A}{x} + \frac{B_1x+C_1}{x^2+1} + \frac{B_2x+C_2}{(x^2+1)^2} + \frac{B_3x+C_3}{(x^2+1)^3}$$

---

worksheet 2.7 # 2

$$\frac{1}{x^4-1} = \frac{1}{(x^2+1)(x^2-1)} = \frac{1}{(x^2+1)(x+1)(x-1)}$$

$$\frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+d}{(x^2+1)}$$

Briggs 8.9 - Improper integrals

type 1

- Happen when at least one end of the limit of integration is unbounded

$$\int_0^\infty e^{-2x}dx$$

The function is getting rather close to 0, as we go towards infinity

type 2

- happens when the integrand is unbounded

$$\int_0^1 \frac{1}{x}dx$$

- that's fine, as it goes past 1

FTOC

$$\int_a^b f(x)dx = F(b) - F(a)$$

- where F is the antiderivative of f
- that's wonderful. toss that shit out. does not apply to improper integrals

type 1 (but again)

$$\text{Definition} \int_a^\infty f(x)dx = \lim_{n\to\infty} \int_a^n f(x)dx$$

- Hey, if we use n as a finite upper limit, we can evaluate *that* using the FTOC
  - so yeah, do that, evaluate with FTOC
- aaand then take the limit
- we kinda need $f(x)$ to be continuous on $x \geq a$, just as a note
- If the limit exists (spits a number out)
  - theeen, the improper integral converges on some value
- If the limit does *not* exist
  - then the improper integral diverges

example

$$\int_0^\infty e^{-2x}dx = \lim_{n\to\infty} \int_0^n e^{-2x}dx = \lim_{n\to\infty} \frac{-1}{2}e^{-2x}|_0^n$$

$$= \lim_{n\to\infty} (-\frac{1}{2}e^{-2n} + \frac{1}{2}e^0)$$

$$= \lim_{x\to y}(-\frac{1}{2} * \frac{1}{e^{2n}} + \frac{1}{2})$$

$$= \frac{1}{2}$$

example 2

$$\int_1^\infty \frac{1}{x}dx$$

$$= \lim_{n\to\infty} \int_1^n \frac{1}{x}dx = \lim_{n\to\infty} \ln(x)|_1^n$$

$$= \lim_{n\to\infty} \ln(n) = \infty$$

MATH112 - 2024-03-08

#notes #math112 #math #calc

time for type 2 improper integrals

- just as a reminder, type 1 was

$$\int_0^\infty f(x)dx = \lim_{n\to\infty}\int_0^n f(x)dx$$

- WE can show that

$$\int_1^p \frac{1}{x^p}dx = \left(\frac{1}{p-1}\text{if p} \geq 1\right)$$

- Diverges if $p \leq 1$

Gabriel's horn shenanigans

$$\int_1^\infty \pi(\frac{1}{x})^2 dx = \pi\int_1^\infty \frac{1}{x^2}dx = \pi * (\frac{1}{1}) = \pi$$

---

other versions of type 1

$$\text{Defn: } \int_{-\infty}^b = \lim_{n\to -\infty}\int_n^b f(x)dx$$

$$\text{Definition: } \int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x)dx + \int_a^\infty f(x)dx$$

You can pick any number you please for a

example

$$\int_{-\infty}^\infty xe^{-x^2}dx = \int_{-\infty}^0 xe^{-x^2} + \int_0^\infty xe^{-x^2}dx$$

First one is I_1, second one is I_2

$$I_2 = \int_0^\infty xe^{-x^2}dx$$

$$u = x^2, du = 2xdx$$

$$\int \frac{1}{2}e^{-u}du = \frac{1}{2}\int e^{-u}du$$

$$= \frac{1}{2}(-1)e^{-u} + C$$

$$I_2 = -\frac{1}{2}e^{-x^2} + C$$

$$\lim_{n\to\infty}\int_0^n xe^{-x^2}dx$$

$$\lim_{n\to\infty} -\frac{1}{2}e^{-x^2}\Big|_{(0)}^{n}$$

$$\lim_{n\to\infty}(-\frac{1}{2}e^{-n^2} + \frac{1}{2})$$

$$= \frac{1}{2}$$

$$I_1 = \int_{-\infty}^{0} xe^{-x^2}dx = \lim_{n\to\infty}\int_n^0 xe^{-x^2}dx$$

$$\lim_{n\to\infty} -\frac{1}{2}e^{-x^2}\Big|_{n}^{0}$$

$$\lim_{n\to\infty} = (\frac{-1}{2} + \frac{1}{2}e^{-n^2})$$

$$\lim_{n\to\infty} = -\frac{1}{2}$$

$$I = -\frac{1}{2} + \frac{1}{2} = 0$$

$$\int_{-\infty}^{\infty} xdx$$

$$\int_{-\infty}^{\infty} xdx = \int_{-\infty}^{0} xdx + \int_0^{\infty} xdx$$

$$= \lim_{n\to-\infty}\frac{x^2}{2}\Big|_n^0 + \lim_{n\to\infty}\frac{x^2}{2}\Big|_0^n$$

$$\lim_{n\to-\infty}(-\frac{n^2}{2}) + \lim_{n\to\infty}\frac{n^2}{2}$$

$$-\infty + \infty = \text{ Womp womp}$$

This one diverges. Awesome!

$$\int_0^{\infty}\cos(x)dx = \lim_{n\to\infty}\int_0^n \cos(x)dx$$

$$= \lim_{n\to\infty} \sin(x)\Big|_0^n$$

$$= \lim_{n\to\infty} \sin(n) = \text{ DNE, oscillates}$$

This one diverges

---

ok now it's *actually* time for type 2 improper

$$\int_a^b f(x)dx$$

where $f(x)$ is unbounded as x goes to b^-

Pick some variable of t between a and b, and then we're going to move t infinitely close to b

$$\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$$

example time

$$\int_0^1 (1-x)^{-\frac{1}{2}} dx$$

$$-2(1-x)^{1/2} + C$$

$$\lim_{t\to 1^-} \sqrt{(-2(1-x))}\Big|_0^t$$

$$\lim_{t\to 1^-} (-2(1-t)^{1/2} + 2)$$

$$0 + 2 = 2$$

is the final answer for area under the curve

It can also blow up on the other end, you'd do the limit as $t \to a^+$

If it blows up in the middle, just split the difference and integrate on both sides

MATH112 - 2024-03-11

#notes #math112 #math #calc

sequences and series.

be not afraid.

just kidding. be afraid.

- A sequence is an ordered list of numbers

$$\{a_{1,}a_{2,}a_{3,}\dots\} = \{a_n\}_{n=1}^{\infty}$$

  Couple ways to define a sequence
- Explicity
  - The sequence

$$\{\frac{1}{n}\} = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots.\}$$

  - The sequence
  - $$\{-1^n * 2^n\} = \{-2, 4, -8 \dots.\}$$
- Define by recurrence
  - Given intial values
  - then we define new terms by a formula that depends on past terms

Initial value $a_1 = 1$

$$a_n = \frac{1}{2}a_{n-1}$$

$$a_1 = 1, a_2 = \frac{1}{2}, a_3 = \frac{1}{4}$$

You could explicitly write that as

$$a_n = \frac{1}{2^{n-1}}$$

$$a_1 = 1$$

$$a_2 = 1$$

$$a_n = a_{n-2} + a_{n-1}$$

$$a_1 = 1, a_2 = 1, a_2 = 2, a_3 = 3, a_4 = 5, 8, 13, 21, 34, 55, 89, 144$$

That's a Fibonacci just hangin out there.
(explicit would just be times 1.618181)

Explicit =

$$a_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1+\sqrt{5}}{2})^{-n}}{\sqrt{5}}$$

Important question for a sequence is, does it converge?

$$\{a_n\} = \{\frac{n+1}{n}\} = \{\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}\}$$

We could do a plot, with the "x" actually being the n axis along discrete integer values
Those are called stem plots!

$$\{a_n\} = \{\cos(n\pi)\}$$

Is the same as

$$\{(-1)^n\}$$

For some reason, people like to write these things with cosines and sines
Start with some series $\{a_{1,}a_{2,}a_{3,}\}$
Let $s_1 = a_1$
and

$$s_2 = a_1 + a_2$$

$$s_3 = a_1 + a_2 + a_3$$

and so on and so forth

These are what we call partial sums - we're summing together terms from the a series, but not all of em at once

Recursive definitions are a thing that exist and make sense

$$\{s_{1,}s_{2,}s_{3,}\dots\} = \{s_n\}$$

This is called a series, a sum of some other thing

The limit of the series

$$L = \lim_{n\to\infty} = \lim_{n\to\infty}\sum_{k=1}^{n} a_k = \sum_{n=1}^{\infty} S_n$$

Is L a number?
If so, this thing converges (ie, does it converge)
It could diverge to some infinity, or it could diverge by just oscillating

$$\text{Find} \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)}$$

Conjecture is that

$$s_n = \frac{n}{n=1} = \lim_{n \to \infty} s_n = \lim_{n \to \infty} \frac{n}{n+1} = 1$$

MATH112 - 2024-03-12

#notes #math112 #math #calc

10.2 sequences

- Find $\lim_{n \to \infty}$ an $\{a_n\}$
- geometric sequences
- growth rates

Given the sequence $\{a_n\}$, does it converge (or have a $\lim_{n \to \infty}$ of $\{a_n\}$)

function matching

- Let's say we have our sequence

$$\{a_n\} = \{\frac{3n^3}{n^3+1}\}$$

- Consider letting $f(x) = \frac{3x^3}{x^3+1}$, domain can't be -1
- note that $f(n) = a_n$ for n = $1, 2, 3...$
- Find the limit

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} f(n) = \lim_{x \to \infty} f(x) = \lim_{n \to \infty} \frac{3x^3}{x^3+1}$$

- Yeah that's three.
  > - you can do it using ye olde divide same (highest) power of x
- Alternatively, hopital that shit

$$= \frac{9x^2}{3x^2}$$

aaaaand that's three.

theorem

- Suppose $f(x)$ is such that $f(n) = a_n$ for $n = 1, 2, 3, \ldots \ldots$
- If $\lim_{x \to \infty} f(x) = L$, then the limit $\lim_{n \to \infty} a_n = L$
- L can be a number, and this theorem actually still works if $L = +\infty \text{ or } -\infty$

example

$$\{a_n\} = \{\frac{e^n}{n^2}\}$$

$$\text{Let } f(x) = \frac{e^x}{x^2}$$

aaand that'll work out with the theorem

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$$

Hopital that shit

$$\lim_{n \to \infty} \frac{e^x}{2x}$$

This... doesn't help. Do it again!

$$\lim_{x \to \infty} \frac{e^x}{2} = \infty$$

By the theorem, this means that the $\lim_{n \to \infty} a_n = \infty$

Matching function theorem

$$\{a_n\} = \{\frac{\arctan(n)}{n}\}$$

$$f(x) = \frac{\arctan(x)}{x} = \frac{\frac{\pi}{2}}{\infty} = 0$$

$$\lim_{n \to \infty} a_n = 0$$

$$\{a_n\} = \{\sin(\pi n\} = \{0, 0, 0, 0\}$$

$$\lim_{n\to\infty} \sin(\pi n) = 0$$

Try using the theorem, dne by oscillation

$$\{a_n\} = \{3\left(-\frac{1}{2}\right)^n\}$$

has a limit equal to 0, does some bouncy shenanigans
Let

$$f(x) = 3(\frac{-1}{2})^x$$

Geometric Sequence

$$\{ar^n\} = \{ar, ar^2, ar^3, \ldots..\}$$

r and a are both nonzero, if they are zero it gets pretty darn boring
r is known as the common ratio

Theorem

$$\lim_{n\to\infty} ar^n =$$

0 if $|r| < 1$
Situation like $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \}$
a if $r = 1$
DNE if $r = -1$
Also DNE if $|r| > 1$

Say we have the sequence

$$\{a_n\} = \{\frac{e^n + (-3)^n}{5^n}\} = \frac{e^n}{5^n} + \frac{(-3)^n}{5n}$$
$$= \left(\frac{-3}{5}\right)^n + \left(\frac{e}{5}\right)^n$$

Just need to look at our theorem

$$= (\frac{-3}{5})^n + (0)$$

aaand the other term is also going to 0

$$\lim_{n \to \infty} a_n = 0 + 0 = 0$$

growth rates

- How fast sequences grow relative to each other
- This only well and truly matters with divergent functions

Logs: $a_n = (\ln(n))^2 = \ln^2(n)$
Power: $a_n = n^4$
Power x Log: $a_n = n^3 \ln^4(n)$
Exponentials: 6^n
Factorial: $n!$
n to the n: $a_n = n^n$

$$\{a_n\}, \{b_n\}$$

$$\lim_{n \to \infty} \frac{a_n}{b_n} = 0$$

if b_n is growing faster, then the limit will be 0
Equivalent:

$$\lim_{n \to \infty} \frac{b_n}{a_n} = \inf$$

$$\{a_n\} << \{b_n\}$$

theorem

For any p, q, r, s > 0 and b > 1, then the following holds

$$\{\ln^q(n)\} << \{n^p\} << \{n^p \ln^r(n)\} << \{n^{p+s}\} << \{b^n\} << n! << \{n^n\}$$

MATH112 - 2024-03-13

#notes #math112 #math #calc

previously on

$$\{a_n\} \{b_n\}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0 = \{a_n\} << \{b_n\}$$

theorem

For any p, q, r, s > 0 and b > 1, then the following holds

$$\{\ln^q(n)\} << \{n^p\} << \{n^p \ln^r(n)\} << \{n^{p+s}\} << \{b^n\} << \{n!\} << \{n^n\}$$

$$\{3n\}_{n=1}^{\infty}, \{6n\}_{n=1}^{\infty}$$
$$\{3n\} \not\ll \{6n\}$$

$$\{n^2\} << \{1.1^n\}$$

---

briggs 10.3

- infinite series
  - geometric series
    - (last time we did geometric sequences)
- An infinite series is a sum of the form $\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 \ldots$
- Upper limit is unbounded, usually we have a lower bound of $k = 1$ or $k = 0$
- Sometimes we go elsewhere, but like, not normally
  - hasn't seen a negative start, but that is legal

Question, does $\sum_{k=1}^{\infty} a_k$ converge or diverge

- we generally want to know what it converges to

Another definition: Partial sums

$$s_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + \ldots. + a_n$$

s_n generally means you're summing the first n terms of your sequence

- If you start at 0, you go to $n - 1$

Another another definition:

We say that $\sum k=1^{\infty} a_k$ converges to L if $\lim_{n \to \infty} \sum k=1^n = L$

$$\sum_{k=0}^{\infty (\frac{1}{2})^k =} (1) + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots\ldots$$

$$s_1 = 1, s_2 = \frac{3}{2}, s_3 = \frac{7}{4}, s_4 = \frac{15}{8},$$

We can rewrite as

$$\frac{2^2-1}{2^2}, \frac{2^3-1}{2^2}, \frac{2^4-1}{2^3}$$

$$s_n = \frac{2^n-1}{2^{n-1}}$$

$$\sum_{k=0}^{\infty} s_n = \lim_{n \to \infty} \frac{2^n-1}{2^{n-1}}$$

There's a bunch of tricks but algebra is easiest

$$\lim_{n \to \infty} \frac{2 - \frac{1}{2^{n-1}}}{1} = 2$$

Generalizing this, we get...

$$Find \sum_{k=0}^{\infty} ar^k$$

$$s_n = ar^0, ar^1, ar^2, \ldots. + ar^{n+1}$$

$$rsn = ar^1 + ar^2 + ar^3 + \ldots\ldots + ar^n$$

$$s_n - rs_n = ar_0 - ar^n$$

$$s_n(1-r) = ar^0 - ar^n$$

$$s_n = \frac{ar^0 - ar^n}{1-r}$$

Valid if $r \neq 1$

$$\lim_{n \to \infty} \frac{ar^0 - ar^n}{1-r}$$

If, for example, $r = \frac{1}{2}$, n would head to 0

If it's greater than 1, it blows up off to ∞

So that limit would be equal to $\frac{a}{1-r}$ if $|r| < 1$

Diverges if $|r| \geq 1$

Find $5 - \frac{5}{4} + \frac{5}{16} - \frac{5}{64}$

$$\sum_{k=0}^{\infty} ar^k$$

$$a = 5, r = \frac{1}{4}$$

$$\frac{5}{\frac{5}{4}} = 4$$

MATH112 - 2024-03-25

#notes #math112 #math #calc

We have some geometric series as k goes to infinity of ar^k

- Which converges upon some value $\frac{a}{1-r}$ if $|r| < 1$
- For literally any other situation it diverges

$$\sum_{k=3}^{\infty} 3(\frac{3}{4})^k$$

$$\text{Let } l = k - 3$$

Which sets

$$k = l + 3$$

$$s = \sum_{l=0}^{\infty} 3(\frac{3}{4})^{l+3}$$

Works as

$$3 * \left(\left(\frac{3}{4}\right)^l * (\frac{3}{4})^3\right)$$

$$a = \frac{81}{64}, r = \frac{3}{4}$$

$$\sum_{l=0}^{\infty} \frac{81}{64}(\frac{3}{4})^l$$

$$S = \frac{\frac{81}{64}}{\frac{1}{4}}$$

$$S = \frac{81}{64} * 4 = \frac{81}{16}$$

Alternatively,

$$s = \sum_{k=0}^{\infty} 3(\frac{3}{4})^k - \text{all the terms from 0 to 2}$$

When $k = 1$, it just ends up being

$$\frac{ar}{r-1}$$

---

telescoping series

$$s = \sum_{k=1}^{\infty} (\cos(\frac{1}{k}) - \cos(\frac{1}{k+1}))$$

So the partial sum at any given point is just

$$\cos(1) - \cos(\frac{1}{n+1})$$

So our limit is just

$$\cos(1) - \cos(0), \text{or} \cos(1) - 1$$

$$s = \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

MATH112 - 2024-03-26

#notes #math112 #math #calc

So, what's been going on with geometric series

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}, if|r| < 1$$

$$\sum_{l=1}^{\infty} ar^{l-1} = \frac{a}{1-r} if|r| < 1$$

---

does $\sum_{k=1}^{\infty} a_k$ converge?

- We've been doing a bunch of nice sequences that have nice formulas, aaand now we're going into the jungle.
  - At least they've got fun and games.
- We're going to focus on $a_k > 0$

got some tests

- The divergence test
  - Warm up:
    - If object A is a dog, then it is a mammal.
    - If object A is not a mammal, then it is not a dog.
    - They're the same statement! $\equiv$
    - If object A is a mammal, then inconclusive if dog.
  - The DT
    - If $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k\to\infty} a_k = 0$
    - If $\lim_{k\to\infty} a_k \neq 0$, then $\sum_{k=1}^{\infty} a_k$ diverges
    - If $\lim_{k\to\infty} a_k = 0$, then we have absolutely no idea whether or not it converges
    - **Example**
      - $s = \sum_{k=1}^{\infty} \cos(\frac{1}{k})$
      - So the $\lim_{k\to\infty} \cos(\frac{1}{k}) = \cos(0) = 1$
      - So, this diverges
    - Slightly different
      - $\sum_{k=1}^{\infty} \sin(1/k)$
      - $\lim_{k\to\infty} \sin(\frac{1}{k}) \sin(0) = 0$
        - That... might diverge! We dunno!
        -
- The integral test

and more kinds of series!

- harmonic
- p-series

---

misc other examples

Classic Series

Harmonic Series

$$\sum_{k=1}^{\infty} \frac{1}{k}$$

$$\lim_{k \to \infty} \frac{1}{k} = 0$$

2 Series

$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\lim_{k \to \infty} \frac{1}{k^2} = 0$$

- Actually just a special case of the p series
- Divergence test tells us all of jack squat

I need something stronger!

- To test my series, of course. Not to drink...
- Let $f(x)$ be continous on $x \epsilon [1, \infty]$
  - Such that $f(x) > 0$
  - $f(x)$ is decreasing
    - Ie, $x_2 > x_1$, then $f(x_2) < f(x_1)$
  - Let $a_k = f(k)$ for $k = 1, 2, 3 \ldots$
- then
  - if $\int_1^\infty f(x)dx$ converges, then $\sum\limits_{k=1}^{\infty} a_k$ converges
  - If that integral instead diverges then the series also diverges.

- We know (from right rectangle) $\sum_{k=2}^{\infty} \leq \int_1^\infty f(x)dx \leq \sum_{k=1}^{\infty} a_k$

Applying this to our classic series

$$\sum_{k=1}^{\infty} \frac{1}{k}$$

$$f(x) = \frac{1}{x}$$

$$\int_1^\infty \frac{1}{x}dx = \ln(x)\Big|_1^\infty = \text{ diverges!}$$

This implies, from the integral test, that this also diverges.

Applying this to the 2 series

$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$f(x) = \frac{1}{x^2}$$

$$\int_1^\infty \frac{1}{x^2}dx = \int_1^\infty x^{-2}dx = -x^{-1}\Big|_1^\infty$$

$$= 0 - -1 = 1$$

This integral converges, which implies that

$$\sum_{k=1}^{\infty} \frac{1}{k^2} \text{ converges.... to something}$$

That apparently converges to

$$\frac{\pi^2}{6}$$

Thanks Euler, I guess

$$s = \sum_{k=1}^{\infty} \frac{1}{k^p} \text{where } p > 0$$

p = 1 is the harmonic series $\frac{1}{k}$
p = 2 is the 2 series $\frac{1}{k^2}$

So $f(x) = \frac{1}{x^p}$

$$\int_1^\infty \frac{1}{x^p} dx =$$

$$\int \frac{1}{x} dx \text{ if } p = 1$$

$$\int_1^\infty x^{-p} dx \text{ if } p \neq 1$$

$$= \frac{x^{-p+1}}{-p+1}\Big|_1^\infty$$

$$\sum_{k=1}^\infty \frac{1}{k^p} \text{ diverges if } 0 < p \leq 1, \text{ and converges if } p > 1$$

MATH112 - 2024-03-27

#notes #math112 #math #calc

Briggs 10.5

comparison tests

Given $\sum a_k$ and $\sum b_k$ series w both greater then zero

- If $a_k \leq b_k$ and $\sum b_k$ converges, then $\sum a_k$ converges
- If $b_k \leq a_k$ and $\sum b_k$ diverges, then $\sum a_k$ diverges

Given $\sum a_k, \sum_{k=1}^\infty \frac{1}{\sqrt{k}+2^k}$
Intuition for a large k, $\frac{1}{\sqrt{k}+2^k} \approx \frac{1}{2^k}$
Let $b_k = \frac{1}{2^k}$

$$\sum_{k=1}^\infty \frac{1}{2^k} = \sum_{k=1}^\infty (\frac{1}{2})^k$$

that all converges somewhere

lemme give you a word of warning

- if you pick the wrong b_k, your result could be inconclusive

Example

$$\sum_{k=1}^{\infty} \frac{k^3}{2k^4 - 1}$$

Intuition, vibe check a large k

$$\frac{k^3}{2k^4} \approx \frac{k^3}{2k^4} = \frac{1}{2k}$$

Sure, let's go let $b_k = \frac{1}{2k}$

$$\frac{1}{2} \sum^{\infty k=1} \frac{1}{k} = diverges!$$

MATH112 - 2024-04-01

#notes #math112 #math #calc

Limit Comparison Tests

- Good for p-lookin series
- Given $\sum a_k$ and $\sum b_k$ when they're both greater than 0, $\lim_{n \to \infty} L = \lim_{n \to \infty} \frac{a_k}{b_k}$
- If it's between 0 and infinity, then they either both converge or both diverge
- If the limit is 0 and $\sum b_k$ converges, then $\sum a_k$ converges
- If the limit is infinity and $\sum b_k diverges$ then a_k diverges
- L=0 and b_k diverges inconclusive
- L = inf and b_k converges is inconclusive

---

briggs 10.6, alternating series

- defining them, doing the alternating series test
- The Electric Koolaide Acid Test
- Remainder Estimation
- Absolute convergence vs conditional convergence
- \

alternating series

- starting with a harmonic series

- Diverges $\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots.$
- Alternating Harmonic Series, $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$
- $\{a_k\}, a_k > 0$
- $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$
- You can just kinda smack the alternating term on front

AST: Alternating Series Test

- Given some sequence $\{a_k\}$ w/
  1. $a_k > 0$
  2. $\lim_{n\to\infty} a_k = 0$
  3. a_k is decreasing for $k > N$ where N is some finite integer
- Then that'll converge
- Recall
  - $\{a_k\}$ w/ $a_k > 0$
  - $\lim_{n\to\infty} a_k = 0$
- Classic example
  - $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$
  1. That's greater than 0
  2. Limit goes to 0
  3. $\frac{1}{k+1} < \frac{1}{k}$
  4. AST tells us that this converges! sick!

why in the world does this shit work

- $S = 100 - 90 + 80 - 70 + 60 - 50.\ldots.$
- Partial sums, it basically keeps bouncing around till it eventually makes its way to the limit

example

$$\sum_{k=2}^{\infty} (-1)^k \frac{\ln(k)}{k}$$

1. That's positive!
2. Limit is heading down to 0 (k is growing faster than $\ln(k)$)
3. Uhhh, derivative!
    1. $$f(x) = \frac{\ln(x)}{x}, f'(x) = \frac{x * \frac{1}{x} - \ln(x) * 1}{x^2}$$
    2. $$1 - \ln(x) < 0, 1 < \ln(x)$$
    3. If k is larger than 3, we're gonna be negative

example (reprise)

$$S = \sum_{k=1}^{\infty} (-1)^k k^{-4} 2^k$$

$$a_k = \frac{2^k}{k^4}$$

1. Positive!
2. Limit is NOT 0.
    1. AST does not apply. Womp womp
3.

MATH112 - 2024-04-02

#notes #math112 #math #calc

previously on

- we were doing alternating series

$$\sum_{k=1}^{\infty} (-1)^{k+1} a_k$$

All of that garbage before a_k is just the alternating series definition

- Those converge when $a_k > 0$
- $\lim_{n \to \infty} = 0$
- Decreasing

today

remainder estimation theorem

- Suppose that I have the series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$
- Define remainder $R_k = S - S_k$
  - Where S is the final limit of converge and S_k is the kth partial sum
  - Then $|R_k| = |S - S_k| \leq a_{k+1}$

example

$$S = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k}$$

Find # of terms required to estimate S within 10^{-6}

$$|R_k| = |S - S_k| \leq a_{k+1} < 10^{-6}$$

$$\frac{1}{k+1} < 10^{-6}$$

$$k \geq 10^{6}$$

That's, uh, a million terms (that's fun)

quickly converging example

$$S = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k!}$$

$$|R_k| = |S - s_k| \leq \frac{1}{(k+1)!} < 10^{-6}$$

Yeah, that checks out for nine

If $\sum |a_k|$ converges, (it doesn't matter what terms converge to), we say $\sum a_k$ converges absolutely

So just as an example,

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k^2} = \sum_{k=1}^{\infty}|(-1)^{k+1}\frac{1}{k^2}| = \sum_{k=1}^{\infty}1/k^2$$

that's a series and therefore converges absolutely

If $\sum\limits_{k=1}^{\infty} |a_k|$ diverges and $\sum\limits_{k=1}^{\infty} a_k$ converges, then we say $\sum a_k$ converges conditionally

example

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$$

$$\sum_{k=1}^{\infty} |(-1)^{k+1} \frac{1}{k}| = \sum_{k=1}^{\infty} \frac{1}{k}$$

that's a conditional convergence!
Signs of the terms affect convergence

review problems

(1) Use tabular integration to solve $\int x^3 \sin(x) dx$

u	dv

2. Use u-sub to evaluate $\int_0^{\pi/3} \sin(x) \ln(\cos(x))$
    1. Use IBP to evaluate $\int_0^{\frac{\pi}{3}} \sin(x) \ln(\cos(x)) dx$
3. Evaluate $\int \sin^2(\theta) \cos^5(\theta) d\theta$
4. Evaluate $\int 10 \tan^9(x) \sec^2 x dx$
5. $\int \frac{\sqrt{9-x^2}}{x} dx$
6. $\int_0^6 \frac{z^2}{(z^2+36)^2} dz$
7. $\int \frac{dx}{\sqrt{x^2-81}}$
8.

MATH112 - 2024-04-03

#notes #math112 #math #calc

today: brigg's 10.7

- Not on exam 2
- Ratio Test
    - Works well with series that have exponentials and factorials

- Given the infinite series $\sum a_k$, (we know nothing about alternating, negative, positive, whatever), let $r = \lim\limits_{k\to\infty} |\frac{a_{k+1}}{a_k}|$
  - If $r < 1$, then the series converges absolutely, so $\sum a_k$ converges
  - If $r > 1$, including ∞, then $\sum a_k$ diverges
  - If $r = 1$, inconclusive

example

$$\lim_{n\to\infty} \frac{2k}{k!} = 0 \text{ womp womp}$$

$$\lim_{n\to\infty} \frac{\frac{2^{k+1}}{(k+1)!}}{\frac{2^k}{k!}}$$

$$\lim_{n\to\infty} \frac{2^{k+1}k!}{(k+1)!2^k}$$

$$\lim_{n\to\infty} \frac{2}{k+1} = 0, \text{ series converges absolutely since r<1}$$

$$\lim_{n\to\infty} \frac{a_{k+1}}{a_k} = r$$

Suppose r is just some number
As we get a whole ways out, $a_{k+1} \approx ra_k$
And then you just keep multiplying by r

$$r^2 a_k \approx ra_{k+2}$$

And you can just keep smacking powers on
Hey, that tail is starting to look like a geometric series
aaaand that's why the ratio test works. The tail becomes a geometric series, which we know how to handle.

MATH112 - 2024-04-05

#notes #math112 #math #calc

root test

- Given $\sum a_k$, let $\rho = \lim\limits_{n\to\infty} |a_k|^{1/k}$

- If $\rho < 1$, then $\sum a_k$ converges absolutely
- If $\rho > 1$, then $\sum a_k$ diverges
- If $\rho = 1$, then we're inconclusive

example

$$\sum_{k=1}^{\infty} \left(\frac{3}{4}\right)^k$$

$$\rho = \lim_{n\to\infty} \left(\left(\frac{3}{4}\right)^k\right)^{\frac{1}{k}}$$

$$\rho = \lim_{n\to\infty} \frac{3}{4} = \frac{3}{4}$$

Which is less than one, so that sure looks like it converges absolutely

trickier example

$$\sum_{k=1}^{\infty} (1 + \frac{2}{k})^{k^2}$$

Power depends on k, so good canidate to get rooty with it

$$\rho = \lim_{n\to\infty} \left(\left(1 + \frac{2}{k}\right)^{k^2}\right)^{\frac{1}{k}}$$

$$\lim_{n\to\infty} (1 + \frac{2}{k})^k = 1^\infty$$

That's a problem

$$\ln\rho = \lim_{n\to\infty} \ln\left(\left(1 + \frac{2}{k}\right)^k\right)$$

$$\ln(\rho) = \lim_{n\to\infty} k\ln(1 + \frac{2}{k}) = \infty * 0$$

That's a problem, again

$$\ln\rho = \lim_{n\to\infty} \frac{\ln(1 + \frac{2}{k})}{\frac{1}{k}} = \frac{0}{0}$$

That's a problem again

$$\frac{\frac{1}{1+\frac{2}{k}} * \frac{-2}{k^2}}{\frac{-1}{k^2}}$$

$$\frac{\frac{-2}{1}}{-1}$$

$$\ln(\rho) = 2$$

$$\rho = e^2$$

P-like series are absolutely going to come out to be one

$$\int \frac{\sqrt{9-x^2}}{x} dx$$

$$x = 3\sin(\theta)$$

You can get a triangle, hypotenuse is three,you end up with $\sqrt{9-x^2}$ for the missing side

$$dx = 3\cos(\theta)d\theta$$

$$\int \frac{\sqrt{9-9\sin^2\theta}}{3\sin(\theta)}(3\cos(\theta)d\theta)$$

$$\int 3\frac{\cos\theta}{\sin(\theta)} 3\cos\theta d\theta$$

$$\int \cos^2\theta * (\sin(\theta))^{-1} d\theta$$

$$3\int \cos^2\theta(\sin\theta)^{-2}\sin(\theta)d\theta$$

$$u = \cos\theta, du = -\sin\theta d\theta$$

$$3\int u^2 \frac{1}{1-u^2}(-du)$$

$$3\int \frac{u^2}{u^2-1}du$$

$$\int 1 + \frac{1}{u^2-1}$$

$$\int (1 + \frac{1}{(u+1)(u-1)})du$$

$$\frac{A}{u+1} + \frac{B}{u-1}$$

$$A = \frac{1}{2}, B = \frac{-1}{2}$$

$$I =$$

ooooooor, less annoying way

$$3\int(1-\sin^2\theta)\frac{1}{\sin\theta}d\theta$$

$$3\int(\frac{1}{\sin\theta}-\sin\theta)d\theta$$

$$3\int(\csc\theta-\sin\theta)d\theta$$

$$\int\frac{z^2}{(z^2+36)^2}$$

$$z = 6\tan(\theta)$$

$$\int\frac{36\tan^2\theta}{(36\tan^2\theta+36)^2}(6\sec^2\theta d\theta)$$

$$(36(\tan^2\theta)+1))^2\$\$\$\$\int\frac{36\tan^2\theta}{(36\sec^2\theta)^2}\$\$\$\$\frac{36*6}{36^2}\int\frac{\tan^2\theta\sec^2\theta}{\sec^4\theta}d\theta\$\$\$\$\frac{1}{6}\int\frac{\sin^2\theta}{\cos^2\theta}*\cos^2\theta =$$

MATH112 - 2024-04-08

#notes #math112 #math #calc

Find Σ notation for

$$s = 3e^{-1}+3e^{-2}+3^{-3}....$$

$$\sum_{k=1}^{\infty}3e^{-k}$$

$$S = \frac{1}{16}+\frac{3}{64}+\frac{4}{256}+\frac{5}{1024}$$

$$\cancel{\sum_{k=0}^{\infty}\frac{3^k}{2^{2k+2}}}$$

$$\sum_{k=1}^{\infty}\frac{3^{k-1}}{2^{2k+2}}$$

$$s = \frac{-1}{2}+\frac{4}{4}-\frac{9}{8}+\frac{16}{16}-\frac{25}{32}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{2^k}$$

$$\sum_{n=1}^{\infty} \frac{1}{2+3^n}$$

At extrema, this is basically

$$\frac{1}{3^n}, = 3^{-n}$$

so using direct comparison
Which is a geometric series with $r < 1$, so converges

$$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$$

$$\lim_{n\to\infty} \frac{2^n}{n^3} = \infty, \text{diverges by divergence test}$$

$$\text{(with growth rates)}$$

$$\sum_{n=1}^{\infty} \frac{1}{(n^2+2n)^{\frac{1}{3}}}$$

MATH112 - 2024-04-09

#notes #math112 #math #calc

Reminder of ratio n root tests:

- You don't need a b_k, you just have some series $\sum a_k$
- For ratio
  - Let r = $\lim_{n\to\infty} |\frac{a_{k+1}}{a_k}|$
  - If $r < 1$, converges (absolutely)
  - if $r > 1$ diverges
- For root
  - let $p = \lim_{n\to\infty} |a_k|^{\frac{1}{k}}$
  - if $p < 1$ conv
  - If $p > 1$ diverge

- if $p = 1$ inconclusive

ratio practice

$$\sum_{n=1}^{\infty} \frac{n^2}{e^{n^2}}$$

$$\lim_{n \to \infty} \frac{(n+1)^2}{e^{n+1^2}}$$

$$\lim_{n \to \infty} \frac{n^2+2n+1}{e^{n+1^2}} \cdot \frac{e^{n^2}}{n^2}$$

Growth rates,

$$e^{n^2} >> n^2 + 2n + 1, so \lim_{n \to \infty} = 0$$

Converges

$$\sum_{n=1}^{\infty} \frac{(-1)^n n!}{(2n-1)!}$$

Alternating series test here?

$$\lim_{n \to \infty} \frac{n!}{(2n-1)!} = 0, \text{growth rates}$$

Positive, yes

$$\frac{n!}{(2n-1)!} >> \frac{(n+1)!}{(2n)!}$$

Decreasing, yes

So, by alternating series it converges

$$\lim_{n \to \infty} \frac{\frac{(-1)^{n+1}(n+1)!}{(2n)!}}{\frac{(-1)^n n!}{(2n-1)!}}$$

$$\lim_{n \to \infty} \cdot \frac{(-1)^{n+1}(n+1)!}{(2n)!} \cdot \frac{(2n-1)!}{(-1)^n n!}$$

$$\lim_{n\to\infty} \frac{(-1)^{n+1}(n+1)!}{(-1)^n n!} \cdot \frac{(2n-1)!}{(2n+1)\,!}$$

$$n = 4, \frac{7!}{(8)!} = \frac{1}{8}$$

$$\frac{(7)!}{(9!)} = \frac{7*6*5*4*3*2*1}{9*8*7*6*5*4*3*2*1} = \frac{1}{9*8}$$

$$= \frac{1}{(2n)(2n+1)}$$

$$2nd\ term\ goes\ to\ \frac{1}{2n}$$

$$\lim_{n\to\infty} \frac{(-1)^{n+1}}{(-1)^n} \cdot \frac{(n+1)!}{n!} * \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{(-1)^n(-1)^1}{(-1)^n} * \frac{(n+1)!}{n!} * \frac{1}{n}$$

$$\lim_{n\to\infty} *(n+1) * \frac{1}{2n}$$

$$\lim_{n\to\infty} \frac{n+1}{n}$$

= 0, converges

$$\sum_{n=1}^{\infty} \frac{e^n}{n!}$$

$$\lim_{n\to\infty} \frac{\frac{e^{n+1}}{(n+1)!}}{\frac{e^n}{n!}}$$

$$\lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n}$$

$$\lim_{n\to\infty} e \cdot \frac{1}{n+1}$$

$$= 0 \text{ converges}$$

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$$

$$\lim_{n\to\infty}\left(\left(1-\frac{1}{n}\right)^{n^2}\right)^{\frac{1}{n}}$$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{\frac{n^2}{n}}$$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n}$$

$$\lim_{n\to\infty}=1^{\infty}$$

$$\ln(l)=\lim_{n\to\infty}n\ln(\frac{1-1}{n})=\infty*0=:($$

$$\lim_{n\to\infty}\frac{(\ln(1-\frac{1}{n}))}{\frac{1}{n}}$$

$$\lim_{n\to\infty}\frac{\frac{1}{1-\frac{1}{n}}*\frac{1}{n^2}}{-1\frac{1}{n^2}}$$

$$\lim_{n\to\infty}-\frac{1}{\frac{1-1}{n}}=\ln(\rho)=-1$$

$$\rho=e^{-1}$$

MATH112 - 2024-04-15

#notes #math112 #math #calc

- Linear approximation
    - Given $f(x)=e^{1-x}$
    - Approximate this with a linear function centered at $x=1$
        - SO the slope, $f'(x)=-e^{1-x}$
        - $f'(1)=-e^{0}=-1$
        -
- Quadratic Approximation
- Polynomial Approximation
    - Taylor polynomials
- point

$$(1,f(1))=(1,1)$$

$$y-f(1)=f'(1)(x-1)$$

$$L(x)=2-x\approx e^{1-x}$$

Most accurate for x near 1

Lettuce approximate $e^{\frac{1}{4}}$
Evaluate L at $\frac{3}{4}$, so then we get L = $\frac{5}{4}$, which has an error of 0.034025

Now we're up to

quadratic approximation

(we're generally going to x^n, eventually to approximate at $x=a$

$$P_0(x)=f(a)$$

$$P_1(x)=f(a)+f'(a)(x-a)$$

$$P_2(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2$$

$$P_2(a)=f(a)+0+0$$

$$P_2''(x)=\frac{f''(a)2}{2}=f''(a)$$

$f(x)=e^{1-x}$	$f(1)=1$
$f'(x)=-e^{1-x}$	$f'(1)=-1$
$f''(x)=e^{1-x}$	$f''(1)=1$

$$P_2(x)=1+(-1)(x-1)+\frac{1}{2}(x-1)^2$$

$$P_2(x)=\frac{5}{2}-2x+\frac{1}{2}x^2$$

$$e^{\frac{1}{4}=\frac{}{5}}2-2\left(\frac{3}{4}\right)+\frac{1}{2}\left(\frac{3}{4}\right)^2$$

Taylor Polynomials

Given $f(x)$ with derivatives

- these are all going to be things that we can keep taking derivatives of
  - $f'(x) = f^1(x)$
  - We're going to keep using power notation, because we're throwing a lot of primes on
  - Base function is $f^{(0)}(x)$
  - $0! = 1$ lol, bozo, L, does that shit make sense? no. eat shit.
- Polynomial centered at $x = a$
- $P_n(x) = f(a) + f^{(1)}(a)(x-a) + f^{(2)}\frac{a}{2!}(x-a)^2 + \dots \frac{f^{(n)(a)}}{n!}(x-a)^n$

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$f^{(0)} = \cos(x)$	$f(\pi/2) = 0$
$f^{(1)} = -\sin(x)$	$f'(\pi/2) = -1$
$f^{(2)} = -\cos(x)$	$f''(\pi/2) = 0$
$f^{(3)} = \sin(x)$	$f'''(\pi/2) = 1$

$$P_3 = f\frac{\pi}{2} + f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) + f''\frac{\frac{\pi}{2}}{2!}(\frac{x-\pi}{2})^2 + \frac{f'''(\pi/2)}{3!}(x-\pi/2)^3$$

$$P_3 = -1(x-\pi/2) + 1/6(x-\pi/2)^3$$

MATH112 - 2024-04-16

#notes #math112 #math #calc

k	k!	$f^{(k)}(x)$	$f^{(k)}(\pi/2)$	$\frac{1}{k!}f^{(k)}(\pi/2)$
0	1	$f^{(0)} = \cos(x)$	$f(\pi/2) = 0$	0
1	1	$f^{(1)} = -\sin(x)$	$f'(\pi/2) = -1$	-1
2	2	$f^{(2)} = -\cos(x)$	$f''(\pi/2) = 0$	0
3	6	$f^{(3)} = \sin(x)$	$f'''(\pi/2) = 1$	$1/6$
4	24	$f^{(4)} = \cos(x)$	$f''''(\pi/2) = 0$	0
5	120	$f^{(5)} = -\sin(x)$	$f'''''(\pi/2) = -1$	-1 / 120
6	720	$f^{(6)} = -\cos(x)$	$f''''''(\pi/2) = 0$	0

How accurate is our approximation?

- Remainder

$$R_n(x) = f(x) - P_n(x)$$

- Which is just actual - approximation

Theorem: Taylor's Inequality

$$|R_n(x)| = |f(x) - P_n(x)| \leq M \frac{|x-a|^{n+1}}{(n+1)!}$$

$$\text{where M is any number such that } |f'^{(n+1)}(c)| \leq M \text{ for all c between a and x inclusive}$$

$$\text{where a is the center value}$$

Note: Tricky to prove, so we're just going to use it

Apply to our problem from earlier, estimating the $\cos(3\pi/4)$

$$|R_5(x)| = |\cos(x) - P_5(x)| \leq M \frac{|x-\pi/2|^6}{6!}$$

where

$$|f^{(6)}(x)| \leq M$$

for x between a = $\frac{\pi}{2}$ and $x = \frac{3\pi}{4}$

$$|-\cos(x)| \leq M$$

$$|-\cos(x)| \leq M$$

$$|\cos(x)| \leq M$$

$$M_{ax} = \frac{\sqrt{2}}{2}$$

$$|R_5(x)| \leq \frac{\sqrt{2}}{2} \frac{|\frac{3\pi}{4} - \frac{\pi}{2}|^6}{6!} = \frac{\sqrt{2}}{2} \frac{(\pi/4)^6}{6!}$$

2

Find $p_2(x)$ for $f(x) = \sqrt{x}$, centered at x = 4

k	**k!**	$f^{(k)}(x)$	$f^{(k)}(4)$	$\frac{1}{k!}f^{(k)}(4)$
0	1	$\sqrt{x}$	2	2
1	1	$\frac{1}{2\sqrt{x}}$	$\frac{1}{4}$	$\frac{1}{4}$
2	2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{32}$	$-\frac{1}{64}$
3	6	$\frac{3}{8}x^{-5/2}$	$\frac{3}{256}$	$\frac{1}{512}$

$$p_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$$

Approximate $\sqrt{4.2}$
Minor problem, $P_2(x)$ is centered at $x = 4$
Evaluate $p_2(4.2)$

$$p_2(4.2) = 2 + \frac{1}{4}(4.2-4) - \frac{1}{64}()$$

MATH112 - 2024-04-22

#notes #math112 #math #calc

Given that

$$f(x) = \sum c_k(x-a)^k$$

If the series converges for x= b, then it converges for all x such that

$$|x-a| < |b|$$

where a is the center

If it diverges for $x = d$, then it diverges for all x such that $|(x-a) > |d|$

Time to start representing various $f(x)$ as power series

Start with some base series that we know, and then build off of it
That base series is

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

Use algebra tools and calculus tools to build new power series

Given $f(x) = \sum_{k=0}^{\infty} d_k x^k$

both w/ IOC = I

$$f(x) \pm g(x) = \sum_{k=0}^{\infty} (c_k \pm d_k) x^k$$

IOC = I

multiply by x^m

m is an integer, and $m + k \geq 0$ for all $c_k \neq 0$

$$x^m f(x) = x^m \sum_{k=0}^{\infty} c_k x^{k+m}$$

$$= \sum c_k x^{k+m}$$

composistion

Let $h(x) = bx^m$

$f(h(x))$

$$\sum_{k=0}^{\infty} c_k (h(x))^k$$

$$= \sum_{k=0}^{\infty} c_k b^k x^{mk}$$

IOC: All x such that $h(x)$ is in I (the original IOC)

$$f(x) = \frac{1}{1 + x^4} = \frac{1}{1 - (-x^4)}$$

$$h(x) = (-1)x^4$$

Use composition

$$f(x) = \sum_{k=0}^{\infty} = (-x^4)^k$$

$$\sum_{k=0}^{\infty} (-1)^k (x^{4k})$$

Find $|-x^4| < 1$

$$|x| < 1^{\frac{1}{4}}$$

$$IOC = (-1, 1)$$

$$f(x) = \frac{x^5}{1+2x}$$

$$f(x) = x^5 \ * \ \frac{1}{1-(-2x)}$$

$$h(x) = -2x$$

$$x^5 \sum_{k=0}^{\infty} (-2x)^k$$

$$= x^5 \sum_{k=0}^{\infty} (-2)^k x^k$$

$$\sum -2^k x^{k+5}$$

$$IOC : |-2x| < 1 => 2|x| < 1 => |x| < \frac{1}{2} => (\frac{-1}{2}, \frac{1}{2})$$

the calculus tools

- Calculus
- Derivative
  - $$f(x) = \sum_{k=0}^{\infty} c_k x^k = c_0 + c_1 x$$
  - $$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 \ldots.$$
  - $$f'(x) = 0 + c_1 + c_2 2(x-a) + c_3 3(x-a)^2$$
  - $$f'(x) = \sum_{k=1}^{\infty} c_k k(x-a)^{k-1}$$
  - With the same ROC = R
  - We could also reindex $\ell = k - 1$

- $$k = \ell + 1$$
- $$\sum_{\ell=0}^{\infty} c_{\ell+1}(\ell+1)(x-a)^{\ell}$$

Example

Previously, we had

$$f(x) = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

$$IOC = (-\infty, \infty)$$

$$f(x) = \frac{1}{0!}x^0 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2$$

$$f'(x) = 0 + 1 + \frac{2}{2!}x + \frac{3}{3!}x^2 + \ldots.$$

$$f'(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3$$

$$f(x) = e^x$$

MATH112 - 2024-04-23

#notes #math112 #math #calc

Last time, we had our starting series

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

Which had an IOC of

$$(-1, 1)$$

Derivative Tool

$$f(x) = \sum_{k=0}^{\infty} c_k (x-a)^k$$

We went on and derived that, and got

$$f'(x) = \sum_{k=1}^{\infty} c_k k(x-a)^{k-1}$$

---

example (similar to worksheet)

Find the power series of $g(x) = \frac{1}{(1-x)^2}$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}(1-x)^{-1} = -(1-x)^{-2} = \frac{-1}{(1-x)^2}$$

$$g(x) = -f'(x)$$

$$g(x) = -\frac{d}{dx}\sum_{k=0}^{\infty}$$

$$g(x) = -\sum_{k=1}^{\infty} kx^{k-1}$$

$$f(x), ROC = 1$$

$$ROC \text{ is in fact, still } 1$$

Integration

$$f(x) = \sum_{k=0}^{\infty} c_k(x-a)^k$$

$$\int f(x)dx = \int \sum_{k=0}^{\infty} c_k(x-a)^k$$

$$\int f(x)dx = \sum_{k=0}^{\infty} c_k \int (x-a)^k dx$$

$$c_0 \int (1) + c_1 \int (x-a) + c_2 \int (x-a)^2$$

$$c_0 x + \frac{c_1(x-a)^2}{2} + \frac{c_2(x-a)^3}{3} \ldots\ldots\ldots + C$$

$$= c_0 x - c_0 a + c_0 a$$

Toss that $c_0 a$ in the constant

$$= \sum_{k=0}^{\infty} \frac{c_k(x-a)^{k+1}}{k+1}$$

ROC still.... doesn't change!

example

Find the power series of

$$g(x) = \frac{1}{1+x}$$

$$g(x) = \frac{1}{1-(-x)}$$

$$h(x) = -x$$

$$\sum_{k=0}^{\infty}(-x)^k$$

$$\sum_{k=0}^{\infty}(-1)^k x^k$$

$$\int \frac{1}{1+x}dx = \ln(1+x) + C$$

$$\int \sum_{k=0}^{\infty}(-1)^k x^k dx = \sum_{k=0}^{\infty}(-1)^k \frac{x^{k+1}}{k+1} + C$$

Usually we "lock teh constant"

$$\ln(1+0) = \sum_{k=0}^{\infty}(-1)^k \frac{0^{k+1}}{k+1} + C$$

$$C = 0$$

$$\ln(1+x) = \sum_{k=0}^{\infty}(-1)^k \frac{x^{k+1}}{k+1}$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\ldots\ldots$$

Let's go test an endpoint, for shits and gigs

$x = 1$

$$\ln(1+1) = \sum_{k=0}^{\infty} \frac{(-1)^k 1}{k+1}$$

$$g(x) = \tan^{-1}(x)$$

$$g'(x) = \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$g'(x) = \frac{1}{1-(-x^2)}$$

$$\sum_{k=0}^{\infty}(-x^2)^k$$

$$\sum_{k=0}^{\infty}(-1)^k(x^2)^k$$

$$\sum_{k=0}^{\infty}(-1)^k x^{2k}$$

$$\tan^{-1}(x) = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1} + C$$

lockdown constant

$$\tan^{-1}(0) = 0 = 0 + C$$

$$\tan^{-1}(1) = \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$$

Taylor Series

Theorem: If $f(x)$ has a power series representation, then

$$f(x) = \sum_{k=0}^{\infty} f^{(k)}\frac{a}{k!}(x-a)^k$$

- unique
- When $a = 0$, it is also called a MacLauren series
-

MATH112 - 2024-04-24

#notes #math112 #math #calc

$$\frac{x}{1-x} = x * \frac{1}{1-x} = x(1+x+x^2)$$

---

just to reiterate

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

A is the center
Power series are unique
Find ioc

$$f(x) = e^x$$

$$f^{(0)} = e^x$$

$$f^{(1)} = e^x$$

and so on and so forth
Center at $x = 0$, which is also a maclauren series

$$f^{(0)}(0) = 1$$

$$f^{(1)}(0) = 1$$

yadayadayadayadayada

$$= \sum_{k=0}^{\infty} \frac{1}{k!}(x-0)^k$$

IOC for $e^x = (-\infty, \infty)$
We also might get something like approximate e^1 with five terms of the taylor series

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

Plugging in $x = 1$

$$1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$$

Alright, we're still doing e^x, but now we're going to center at like, four

So they're all going to be e^4

$$e^x = \sum_{k=0}^{\infty} \frac{e^4(4)}{k!}(x-a)^k$$

Find IOC

Let $L = \lim_{k\to\infty}$

$$L = \lim_{k\to\infty} \frac{e^4(x-4)^{k+1}}{(k+1)!} * \frac{k!}{e^4(x-4)^k}$$

$$\lim_{n\to\infty} \frac{(x-4)}{k+1}$$

$$|(x-4)| \lim_{k\to\infty} \frac{1}{k+1} = 0 < 1$$

Use five terms to estimate e^1

$$e^x \approx e^4 + e^4(x-4) + \frac{e^4(x-4)^2}{2!} + \frac{e^4(x-4)^3}{3!} + \frac{e^4(x-4)^4}{4!}$$

MATH112 - 2024-04-26

#notes #math112 #math #calc

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

Find Taylor series for different functions

Starting with $f(x) = \frac{1}{x}$, centered at $x = 3$

$f^{(k)}$	**Evaluate at $x = 3$**
x^{-1}	
$-x^{-2}$	
$2x^{-3}$	
$-6x^{-4}$	
$f^{(k)} = (-1)^k k! x^{-k-1}$	$(-1)^k$

$$\frac{1}{x} = \sum_{k=0}^{\infty} \frac{(-1)^k k! 3^{-k-1}}{k!}(x-3)^k$$

$$\frac{1}{x} = \sum_{k=0}^{\infty} (-1)^k 3^{-k-1}(x-3)^k = a_k$$

Ok, generally we need to find the IOC
So.... let's get to ratio testing

$$L = \lim_{k\to\infty} \frac{(x-3)^{k+1}}{3^{k+2}} \cdot \frac{3^{k+1}}{(x-3)^k}$$

$$L = \lim_{n\to\infty} \frac{(x-3)}{3} = \frac{|x-3|}{3} < 1$$

$$|x-3| < 3$$

$$-3 < x-3 < 3$$

$$0 < x < 6$$

So IOC is from 0 to 6, not including the endpoints $(0, 6)$
we need to go test the endpoints separately to figure out what's going on

testing the endpoints

$$\sum_{k=0}^{\infty} (-1)^k \frac{(0-3)^k}{3^{k+1}}$$

$$\sum \frac{3^k}{3^{k+11}}$$

$$\sum_{k=0}^{\infty} \frac{1}{3}$$

6 diverges by oscillation

Find MacLauren series of $f(x) = \sin(x)$

k	$f^{(k)}$	$f^{(k)}(0)$
0	$\sin(x)$	0
1	$\cos(x)$	+1
2	$-\sin(x)$	0
3	$-\cos(x)$	-1
4	$\sin(x)$	0
k		

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k 1}{(2k+1)!} x^{2k+1}$$

$$IOC = (-\infty, \infty)$$

$$ROC = \infty$$

What is the $f(x)$ for

$$\sum_{k=0}^{\infty} \frac{(-1)^k 1}{(2k+1)!}(3x)^{2k+1}$$

That's literally just $\sin(3x)$

Find T.S. of $\cos(x)$ centered at 0
(MacLauren series)

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k 1}{(2k+1)!} x^{2k+1}$$

$$\frac{d}{dx}\sin(x) = \frac{d}{dx}\sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k+1)!} x^{2k} + 1$$

$$\cos(x) = \sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k+1)!}\frac{d}{dx} x^{2k+1}$$

$$\cos(x) = \sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k+1)!}(2k+1)x^{2k}$$

$$\sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k)!} x^{2k}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

Hey that's quirky, cos is even and sine is odd

$$ROC = \infty \ => IOC = (-\infty, \infty)$$

We will have the taylor series for $e^{(x),} \sin(x), \cos(x), \frac{1}{1-x}, \ln(1+x)$

Applying TS

Approximate

$$\int_0^1 e^{-x^2} dx$$

to within 0.001 of its true value

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k}$$

$$\int_0^1 e^{-x^2} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k} dx$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_0^1 x^{2k} dx$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^{2k+1}}{2k+1}\Big|_0^1$$

$$[x - \frac{x^3}{3} + \frac{x^5}{2!5} - \frac{x^7}{3!7}]$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42}$$

Use AST to show converge

$$|R_k| < |a_{k+1}|$$

$$a_k = \frac{1}{(k!)(2k+1)} < 0.001$$

MATH112 - 2024-04-29

#notes #math112 #math #calc

previously on

$$\int_0^1 e^{-x^2} = \int_0^1 \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_0^1 x^{2k} dx$$

$$I = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^{2k+1}}{2k+1}$$

$$P = (3, 31)$$

$$Q = (2, -1, 0)$$

$$R = (-1, -3, 1)$$

$$Ax + By + Cz = D$$

$$PQ = (-1, -4, -1)$$

$$PR = (-4, -6, 0)$$

Cross product o those fellers, and you should end up with

$$x = 2 + s$$

$$y = 21 + 7s$$

$$z = 15 + 4s$$

bloop

$$x = -5 + 2t$$

$$y = -18 + 9t$$

$$z = -11 + 7t$$

---

one

Circle with radius $r = 2$
Center at $(-1, 2)$
CCW

Ok, so if we have

$$x = \cos(\theta)$$

and

$$y = \sin(\theta)$$

that's just our unit circle, but let's do some modifications

$$x = 2\cos(\theta), y = 2\sin(\theta)$$

that gets our radius of two
Aaaand then we just want to shift it over

$$x = 2\cos(\theta) - 1, y = 2\sin(\theta) + 2$$

$$\theta \epsilon [0, 2\pi]$$

$$x = e^t + 1$$

$$y = e^{3t} - 2$$

$$x - 1 = e^t$$

$$\ln(x-1) = t$$

$$y = e^{3\ln(x-1)} - 2$$

$$y = e^{\ln(x-1)^3} - 2$$

$$y = (x-1)^3 - 2$$

MATH112 - 2024-04-30

#notes #math112 #math #calc

$$u =< 3, 5, 0 >$$

$$v =< 0, 3, -6 >$$

Find the projection of u onto v

$$\text{proj}_v(u) = c \cdot \vec{v}$$

$$\text{proj}_v(u) = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\vec{v}$$

$$\vec{u} \cdot \vec{v} = (0) + 15 + 0$$

$$|v| = \sqrt{9 + 36} = \sqrt{45}^2 = 45$$

$$\text{proj}_v(u) = \frac{15}{45}\vec{v} = \frac{1}{3}\vec{v}$$

$$\text{proj}_v(u) =< 0, 1, -2 >$$

$$\int \frac{x}{\sqrt{x-5}} dx$$

$$u = x - 5, du = dx$$

$$\int \frac{x}{\sqrt{u}} du = \int \frac{u+5}{\sqrt{u}} du$$

$$\int \frac{u}{\sqrt{u}} + \frac{5}{\sqrt{u}}$$

$$\int \frac{u}{\sqrt{u}} + 5\int \frac{1}{\sqrt{u}}$$

$$\int \frac{u}{\sqrt{u}} + 5\int u^{\frac{-1}{2}}$$

$$\int \frac{u}{\sqrt{u}} + 5(\frac{1}{2}u^{\frac{1}{2}})$$

$$\int u^{\frac{1}{2}} + 5u^{\frac{-1}{2}}du$$

$$\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + 5\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

and y'know, plug u back in

$$\int (x)(x-5)^{\frac{1}{2}}dx$$

$$u = x$$

$$dv = (x-5)^{\frac{-1}{2}}dx$$

u	dv
x	$(x-5)^{-1/2}$
dx	$-2(x-5)^{1/2}$
0	$3(x-5)^{3/2}$

$$(2x)(x-5)^{\frac{1}{2}} - \frac{4}{3}(x-5)^{\frac{3}{2}+}C$$

$$(2x)(x-5)^{\frac{1}{2}} - 3(x-5)^{\frac{3}{2}}$$

$$\sum_{k=0}^{\infty}(-1)^k\frac{3^{2k}x^{2k}}{(2k)!}$$

$$\cos(x) = \sum_{k=0}^{\infty}\frac{(-1)^k x^{2k}}{(2k)!}$$

$$\cos(3x) = \sum_{k=0}^{\infty} \frac{(-1)^k (3x)^{2k}}{(2k!)}$$

$$\ln(1-x) = \sum_{k=0}^{\infty} (-1)^{k+1} \frac{(-x)^k}{k}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(-1)^k x^k}{k}$$

$$p1 : x + y - z = 1$$

$$p2 : 2x - 3y + 4z$$

There's a roundabout solving method, or

$$< 1, 1, -1 > \times < 2, -3, 4 >$$

$$< 1, 1, -1$$

$$< 2, -3, 4 >$$

$$=< (4) - 3, -(4+2), (-3-2) >$$

$$< 1, -6, -5 >$$

Try something like $x = 0$

$$y - z = 1$$

$$-3y + 4z = 5$$

$$y = 1 + z$$

$$-3(1+z) + 4z = 5$$

$$-3 - 3z + 4z = 5$$

$$-3 + z = 5$$

$$z = 8$$

$$y = 9$$

$$< 0, 9, 8 >$$

$$x = 0 + 1t$$

$$y = 9 - 6t$$

$$z = 8 - 5t$$

MATH112 - 2024-05-01

#notes #math112 #math #calc

converges absolutely or conditionally?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^{\frac{1}{2}}}$$

AST says we converge, but conditional or absolute?

$$\sum_{n=1}^{\infty} \frac{1}{1+n^{\frac{1}{2}}}$$

$$\int_{-1}^{1} e^{\frac{-x^2}{2}}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{\frac{-x^2}{2}} = \sum_{k=0}^{\infty} \frac{(\frac{-x^2}{2})^k}{k!} = \sum_{k=0}^{\infty} \frac{(-1)^k(\frac{1}{2})^k x^{2k}}{k!}$$

$$\int_{-1}^{1} e^{\frac{-x^2}{2}} dx = \int_{-1}^{1} \sum_{k=0}^{\infty} (-1)^k a_k x^{2k} dx$$

$$\sum_{k=0}^{\infty} \int_{-1}^{1} (-1)^k a_k x^{2k}$$

$$= \sum_{k=0}^{\infty} (-1)^k a_k \int_{-1}^{1} x^{2k} = \sum_{k=0}^{\infty} (-1)^k a_k \frac{x^{2k+1}}{2k+1}$$

$$\sum_{k=0}^{\infty} (-1)^k a_k \frac{1}{2k+1} - \sum_{k=0}^{\infty} (-1)^k a_k \frac{-1}{2k+1}$$

$$2\sum_{k=0}^{\infty} (-1)^k a_k \frac{1}{2k+1}$$

$$2\sum_{k=0}^{\infty} (-1)^k \frac{(\frac{1}{2})^k}{k!} \cdot \frac{1}{2k+1}$$

$$\text{Use alternating series remainder}$$

$$k_5 = -0.000047, \text{so use first 5 terms (k=0 -> k=4)}$$

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find t.s of $\sin(x)$ centered at $x = 0$

$$\sin(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}(x-0)^k$$

k	$f^k(x)$	$f^k(0)$
0	$\sin(x)$	0
1	$\cos(x)$	1
2	$-\sin(x)$	0
3	$-\cos(x)$	-1
4	$\sin(x)$	0

$$\sin(x) = \frac{f^{(0)}(0)}{0!}x^0 + f^{(1)}\frac{0}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2$$

$$= 0 + \frac{1}{1!}x^1 + 0 - \frac{1}{3!}x^3 + 0\ldots\ldots.$$

$$= x - \frac{x^3}{6} + \ldots\ldots$$