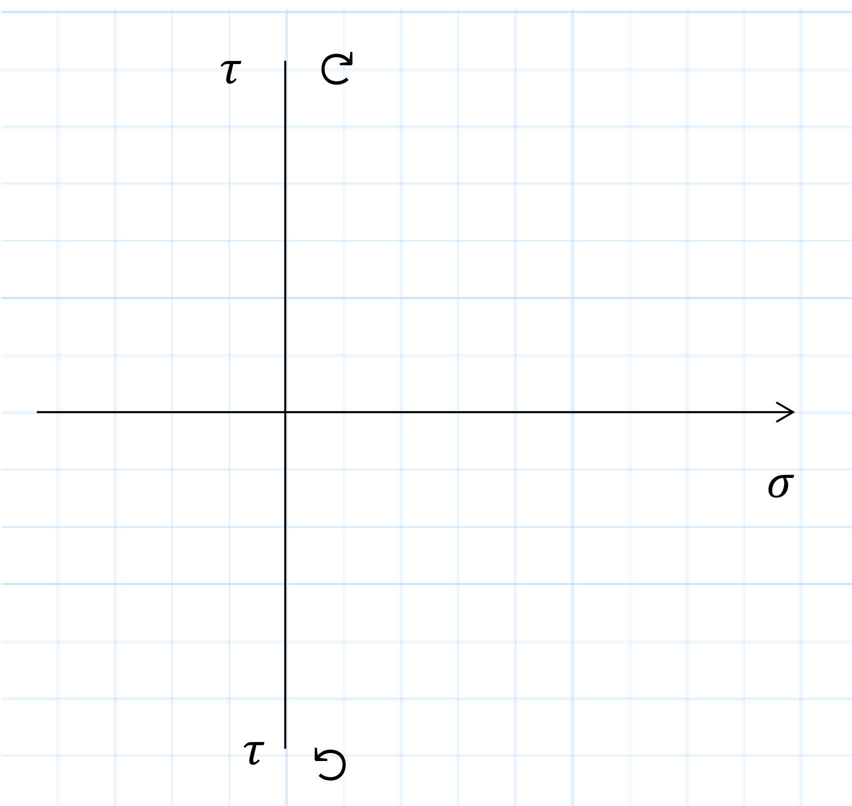
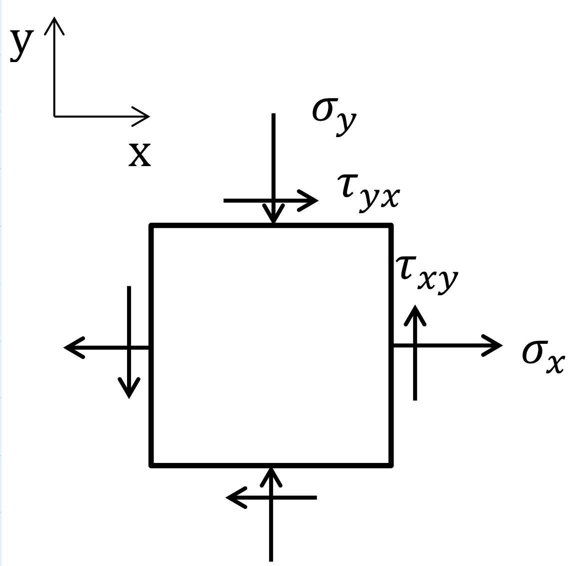
MEGN 212 Solid Mechanics

Plane Stress – Mohr's Circle for Stress, Principal Stresses and Maximum In-Plane Shear Stress

- The circle represents all combinations of normal and shear stresses (σ, τ) acting on any plane at a point, represented by a "state of stress" or "volume" element, in loaded structure.
- The normal and shear stresses vary cyclically with rotation of a reference (inclined) plane at a point.
- A point on the circle represents a plane on the stress element.
- The coordinates of the point are the stresses acting on the plane.
- The position of a point on the circle gives the relative position of the plane relative to another known plane.
- The ***maximum*** and ***minimum normal stresses*** are called ***Principal Stresses*** and occur on planes 90° apart on the ***physical system*** (volume element) or 180° apart on ***Mohr's Circle****.* [The planes of maximum/minimum normal stress are caned ***Principal Planes*** on the stress element.]
- The shear stress is always zero on the Principal Planes.
- Planes of ***maximum "in-plane" shear stress*** are 45° from the Principal Planes in the physical system; 90° from the Principal Planes on Mohr's Circle.
- The sum of the normal stresses acting on ***any*** two perpendicular planes is an invariant; i.e. $\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = constant$.
- The ***"in-plane" maximum shear stress*** equals one-half the difference of the ***Principal Stresses****.*
- The third Principal Plane is the *stress-free* plane where $\sigma = \tau = 0$ for a two-dimensional state of stress. (For our purposes, this is usually the z-plane.)





Procedure for Mohr's circle for stress:

1. Draw and label axes
2. Plot $X(\sigma_x, \tau_{xy})$ and $Y(\sigma_y, \tau_{yx})$
3. Connect X and Y with diameter and draw circle
4. Find center of the circle: $C = \frac{\sigma_x+\sigma_y}{2}$
5. Find radius: $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$
6. Find first principle stress: $\sigma_1 = C + R = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$
7. Find second principle stress: $\sigma_2 = C - R = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$
8. Find average normal stress $\sigma_{ave} = C = \frac{\sigma_x+\sigma_y}{2}$
9. Find max. in-plane shear stress $\tau_{\max in-plane} = R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$
10. Find $2\theta_p = \tan^{-1}\left(\frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}\right)$
11. Find $2\theta_s = \tan^{-1}\left(\frac{-(\sigma_x-\sigma_y)/2}{\tau_{xy}}\right)$