MATH111 TOC

MATH111 - 2023-08-21

#math #math111 #notes

Office hours are (for now) Monday Wednesday afternoons, 1-2 and 1-3

- Limits for the first 4 weeks
- Derivatives for 5
- Applications of Derivatives for ~4 weeks
- Integrals are the last 3 weeks of class
    - Calc 2 is lots and lots of integrals
- 3 tests + final
- In class things are participation
    - Regular online homework
        - Thank goodness it's only 5 per day
- Access code comes with an ebook
- HW Mon/Tues/Fri

For Next Time

- Syllabus and stuff
- Browse Canvas
- Complete the Welcome survey by monday 8/28
- Connect to MyLab Math
- Tuesday is going to be about **LIMITS!**
-

MATH111 - 2023-08-22

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#Limits

Given a posistion function s(t), compute average velocity
Same thing, instantaneous velocity

Recognize slope of a secant line
Slope of a tangent line as instantaneous

Suppose we know position, find instantaneous velocity at a single point

Tossed Rock in the Air

s(t) = -16t^2 +96t

Try average velocity between t=1 and t=2

- Gets an average velocity of 48ft /s

Year 1/Semester 1/MATH111/Concepts/Limit as delta T approaches 0 = 64 for the rock

As the interval shortens, average velocity becomes a better approximation of instantaneous velocity

Average velocity is the slope of the Year 1/Semester 1/MATH111/Secant Line between two points

Slope of a Year 1/Semester 1/MATH111/Tangent Line is the same thing as instantaneous velocity! Wow!

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#Limits

Goals

- Look at limits, graphically, numerically, get our heads around it

Ex. Let f(x) = sqrt(x) -1 ÷ x-1

Q1: What is the domain of f(x)?

[0, 1) U (1, +inf).])

- Because of the sqrt, x has to be greater than or equal to 0
- Because of that (x-1), x=1 is going to be a problem

Q2: What is f(1)

- Undefined, persona non grata, don't talk about it

Q3: What happens to f(x) as x approaches 1?

- (Approaches but never equals)
- Wow, those outputs make it look like we're approaching 0.5 (1/2)

Notation is notating

The value of f(a) means NOTHING to the limit

Squeeze your fingers on both side of the graph - if they come together, you've got yourself a limit! Wowee.

Examples

Ex1.
Evaluate the following
f(3) = 5
limx->3 f(x) = 2
f(1) = 4
lim x->1 f(x) = 4
Ex2.
Evaluate the following
g(4) = 2
lim x-> 4 g(x) = no limit.... DNE

For a limit to exit it must converge, if it don't, it don't exist

Notation is notating, gotta find a good way to input that

One sided Limits

Examples but again

- Ex. 1:
  - lim x->3+ f(x) = 2
  - lim x->3- f(x) = 2
  - Wowzies, they're the same, it's almost like we have a real limit
  - lim x-> 3 f(x) = 2
- Ex. 2:

- lim x->4+ g(x) = 5
- lim x->4- g(x) = 2

Asymptotes tend to need one sided limits, but we'll do a lot of two sided

Theorem: Relationship between one-sided and two-sided limits

- Assume *f* is defined for all x near *a* except possibly at *a*. Then lim f(x) x->a = L if and only if the x->a+ is equal to x->a-
- A limit must be a singular number for it to exist

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#trig

- YES! WE ONLY NEED TO MEMORIZE PYTHAGOREAN IDENTITIES! YIPPEEEEE!

Trig Stuff (Review)

- They're all ratios
- Mines (calculus) expectation is to know the value of all 6 when evaluated at integer multiples pi/6, pi/4, pi/3, pi/2 radians

Special Right Triangles

45-45-90

- Legs are both √2 /2
  - Which is also 1/ √2
- Hypotenuse is 1

30-60-90

- √3 / 2 is opposite pi/3
- 1/2 is oposite pi/6
- (I have it memorized as 1, 2, radical 3)

Year 1/Semester 1/MATH111/Unit Circle Stuff

- (x,y) is (cos(θ), sin(θ))
- Positive angle is counterclockwise
- Sin = y/r
- Cos = x/r
- Tan = y/x

Reciprocal Identities are on the memorize list

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#trig

Year 1/Semester 1/MATH111/Unit Circle

latex example of sqrt(2) /2

$$\frac{\sqrt{2}}{2}$$

$$\sqrt{2}/2$$

Work in book for finding other identities with

$$cot(\theta) = -\frac{5}{12}$$

Identities

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Double/Half-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\sin^2\theta \ = \frac{1-\cos 2\theta}{2}$$

Example

Solve for theta

$$4\sin^2\theta + 4\sin\theta + 1 = 0$$

factors down to

$$(2\sin\theta+1)(2\sin\theta+1) = 0$$
$$2\sin\theta + 1 = 0$$
$$2\sin\theta \ = -1$$
$$\sin\theta \ = -1/2$$
$$\theta = \sin^{-1}(-1/2)$$

or

$$\frac{7\pi}{6} or \frac{11\pi}{6}$$

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#Limits

Big Goal: Evaluating Limits Algebraically

Quick Review:

f(a) means plug in a, get a number

lim x-> a f(x) is when we're very *close* to a

Some limits are easy!

Elementary

- f is (generally) elementary when its not piecewise defined and x is in the domain
- Example:
- $$\lim x \to \pi \cos(x) = \cos(\pi) = -1$$
- $$\lim x \to 2 \frac{3x}{\sqrt{4x+1}-1} = \frac{3(2)}{\sqrt{8+1}-1} = \frac{6}{3+1}$$

Theorem 2.3: Limit Laws

- They justify why you can just plug in a value when its not piecewise find limit laws image

Ex

$$\lim x \to 2 \frac{x^2-4}{x-2}$$

Try x = 2

$$\frac{2^2-4}{2-2} = \frac{0}{0}$$

Try factoring/cancel

$$\lim x \to 2 \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{(x-2)}$$

Do a little canceling of (x-2)

$$\lim x \to 2 \ (x+2)$$

2+2 = **4** <--- that's our answer!

Plugging in two directly doesn't really work (hole)

When you get

$$\frac{0}{0}$$

the answer is indeterminate, you need to actually do some work to find

The graphs of

$$\frac{x^2-4}{x-2} \ \ and \ y = x+2$$

are identical for ALL values of x EXCEPT for x = 2, which works for our limit because the limit doesn't care what's going on actually at the value

Example

$$\frac{x^3+x}{3x^2-4x}$$

factor/simplify some jazz

$$\frac{x^2+1}{3x-4}$$

oh hey, you can plug 0 into that

$$\frac{0^2+1}{3(0)-4}=-\frac{1}{4}$$

$$\lim_{x\to y}$$

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#Limits

$$\lim_{x\to 0}\frac{x^2+x}{3x^2-4x}$$

Whatcha do when the limit comes for you?

- Non-piecewise function where f(x) is not actually in the domain, you've got a couple strategies
  - Factor/cancel
  - Combining + simplifying fractions
  - Multiplying & Dividing by the conjugate

Ex

$$\lim_{x\to 0}\frac{\frac{1}{5+x}-\frac{1}{5}}{x}$$

Solo x in the denominator, icky, gross, indeterminate

Get a common denominator for them thar fractions in the numerator

$$\lim_{x\to 0} \frac{\frac{5}{(5+x)(5)} - \frac{5+x}{(5+x)(5)}}{x}$$

$$\lim_{x\to 0} \frac{\frac{-x}{(5+x)*5}}{x}$$

$$\lim_{x\to 0} \frac{-x}{(5+x)(5)} * \frac{1}{x}$$

$$\lim_{x\to 0} \frac{-1}{(5+x)*5} = \frac{-1}{(5+0)*5} = -\frac{1}{25}$$

New Strategy!

Multiply by the conjugate

- Can be helpful with limits that involve a square root
- What in the world is a conjugate?
  - Change the sign between two terms
  - Ex

$$x + y \to x - y$$

$$y - a^2 \to y + a^2$$

- Nice algebra happens when you multiply conjugates
- Ex

$$(\sqrt{x} + b)(\sqrt{x} - b)$$

- Foil that sh*t

$$(\sqrt{x})(\sqrt{x}) - b\sqrt{x} + b\sqrt{x} - b * b$$

$$x - b^2$$

- Ex 2

$$\lim_{x\to 1} \frac{\sqrt{10x-9} - 1}{x - 1}$$

  - Sadly plugging in 1 yields no goodies
  - Try: multiplying by conjugate (which would be)

$$\sqrt{10x-9} + 1$$

$$10x - 9 + \sqrt{10x-9} - \sqrt{10x-9-1}$$

$$10x - 10$$

- that was an aside, back to the problem itself

$$\lim_{x\to 1}\left(\frac{\sqrt{10x-9}-1}{x-1}\right)\left(\frac{\sqrt{10x-9}+1}{\sqrt{10x-9}+1}\right)$$

Pro Tip: Only multiply the conjugate terms

$$\lim_{x\to 1}\frac{10x-9-1}{(x-1)(\sqrt{10x-9}+1)}$$

$$\lim_{x\to 1}\frac{10x-10}{(x-1)(\sqrt{10x-9}+1)}$$

factor that sh*t

$$\lim_{x\to 1}\frac{(x-1)(10)}{(x-1)(\sqrt{10x-9}+1)}$$

Cancel!

$$\lim_{x\to 1}\frac{10}{\sqrt{10x-9}+1}$$

Plug n chug, see whatcha get

$$\frac{10}{2} = 5$$

Spooky warning for next time:

$$\frac{10}{y}$$

MATH111 - 2023-09-01

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Lil Bit of 2.3 (Computing piecewise algebraically)

- Super secret technique: use one sided limits

- Obligatory warning, you're (ALMOST) never going to be able to plug in f(a)
  - So just like, skip over it
- It is going to definitively not exist in most cases
- To get
- $$\lim_{x\to 2}, \lim_{x\to y^-}, \lim_{x\to y^+}$$

$$\lim_{x\to 2^+} = \lim_{x\to 2^+} (x^2 - 5)$$

- So now that its isolated, just go ahead and plug 2 in, 2^2 -5 = -1

  $$\lim_{x\to 2^-} = \lim_{x\to 2^-} (7 - 2x)$$

  So plug 2 in to that, 7 -4 = 3
  Yippee! It works!
- Those don't match though, so limit as x-> 2 Does Not Exist (DNE)

Infinite Limits {bum bum bum}

- What does it mean if the limit as x goes to a is infinity
- $$\lim_{x\to a} f(x) = \infty$$
- Function grows without bounds - the output gets really rather quite large
- Strictly speaking, an infinite limit Does Not Exist
- Two kinds of infinite limits
- Increases without bound as it gets close to *a*

  $$\lim_{x\to a} f(x) = \infty$$

- Decreases without bound as it gets close to *a* (very very big negative number)

  $$\lim_{x\to a} f(x) = -\infty$$

- Couple limits for tangent
- $$\lim_{x\to \frac{\pi}{2}^-} = \infty$$
- $$\lim_{x\to \frac{\pi}{2}^-} = -\infty$$

- $\lim_{x \to \frac{\pi}{2}} =$ DNE

Expectation for Limit Answers

- Write ∞ and $-\infty$ when possible, as they convey more info than just saying DNE
- Ex
  - Consider the graph of $y = 1/x^2$
  - $\lim_{x \to 0^+} 1/x^2$ = ∞
  - $\lim_{x \to 0^-} 1/x^2$ = ∞
  - $\lim_{x \to 0} 1/x^2$ = ∞

Definition

- The line $x = a$ is a vertical asymptote for a function f if any of the following limit statements are true
  - The function in question gets unbounded near a, in any way shape or form
    - $\lim_{x \to a} f(x) = \infty$
    - $\lim_{x \to a^+} f(x) = \infty$
    - $\lim_{x \to a^-} f(x) = \infty$
    - $\lim_{x \to a} f(x) = -\infty$
    - $\lim_{x \to a^+} f(x) = -\infty$
    - $\lim_{x \to a^-} f(x) = -\infty$
- Yip
- Ex
  - $f(x) = \frac{|x|}{x}$
  - Does $f(x)$ have a vertical asymptote at 0?
    - Nope! There are no values at $f(x) = 0$, but that does not mean it's a vertical asymptote, because none of them are infinite!
    - $\lim_{x \to 0^+} f(x) = 1$
    - $\lim_{x \to 0^-} f(x) = -1$
    - $\lim_{x \to 0^+} f(x) =$ DNE
    - DNE is not a guarantee that we have an asymptote
- $\frac{\neq 0}{0}$ is NOT an indeterminate form for a limit

- This type of limit will always be DNE, but we'll have to do some more work to figure out what's going on

$$\lim_{x \to 1^+} \frac{x+5}{1-x}$$

- Plug in one, we get $\frac{6}{0}$, which is slight uh oh, but now we know the limit DNE
- Can we tell if it\s ∞ or $-\infty$? Yeah! We just gotta do work (*shudder*)
- $\lim\limits_{x \to 1^+} \frac{x+5}{1-x} = -\infty$
  - We did it!
  - Think about what happens when you're getting very close - 1-1.00000001 is a very small negative number, and $\frac{6}{-0.00000000001}$ gets to basically $-\infty$
- $\lim\limits_{x \to 1^-} \frac{x+5}{1-x} = \infty$
  - Do the same thing for the other side, we get postive 0.0000000001, and 6/that is a very beeeg number($+\infty$)
  -

MATH111 - 2023-09-05

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Limits at Infinity

$$\lim_{x \to \infty} f(x) = L$$

means the outputs of $f(x)$ get arbitrarily close to L as the inputs x grow without bound

$$\lim_{x \to \infty} f(x) = M$$

means the outputs of $f(x)$ get arbitrarily close to M as the inputs x decrease without bound\

Definition: Horizontal Asymptote

- Those limits we mentioned earlier? L and M are horizontal asymptotes $y = L$ and $y = M$
- There is, at most, 2 horizontal asymptotes

Couple practice problems

$$\lim_{x \to \infty} \frac{3}{x^2} = 0$$

3 over a very big number is a very small number, which means we have a horizontal asymptote at y= 0

$$\lim_{x \to -\infty} \frac{3}{x^2} = 0$$

3 over a very big positive number is still 0 (not that it being positive matters)

Any number over ∞ is 0

Yet more practice problems

$$\lim_{x \to \infty} -2x^3 = -\infty$$

Look, there is NOTHIN stopping that infinity. It is GOIN (down).

$$\lim_{x \to -\infty} -2x^3 = \infty$$

Look, there is NOTHIN stopping that infinity. It is GOIN (up).

Technique for Limits to Infinity (with rational functions)

Example

$$\lim_{x \to \infty} \frac{3x+2}{x^2-1} = \frac{\infty}{\infty} = indeterminate =$$

Highest power in the denominator is x^2

$$\lim_{x \to \infty} \frac{3x+2}{x^2+1} = \lim_{x \to \infty} \frac{\frac{1}{x^2}}{\frac{1}{x^2}} * \frac{3x+2}{x^2-1} = \lim_{x \to \infty} \frac{\frac{3x}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \lim_{x \to \infty} \frac{\frac{3}{x}+\frac{2}{x^2}}{1-\frac{1}{x^2}} = \lim_{x \to \infty} \frac{0+0}{1-0} = \lim_{x \to \infty} \frac{0}{1} = 0$$

$y = 0$ is a horizontal asymptote for this function

$$\lim_{x \to -\infty} \frac{3x+2}{x^2-1} = \lim_{x \to -\infty} \frac{\frac{3}{x}+\frac{2}{x^2}}{1-\frac{1}{x^2}} = \frac{0+0}{1-0} = 0$$

Oh hey, it's the same (always!)

Now that we've done that icky, gross method, back to shortcuts

- If the power of x in the denominator is bigger than the numerator, asymptote is going to be 0

- If the degree of x is samesies, then the limit is going to be the coefficents divided, and the horizontal asymptote is also a/b
- If the numerator is greater than the denominator, then the limit is either ∞ or $-\infty$, and f has no horizontal asymptotes

Example but again

$$\lim_{x\to\infty} \frac{40x^4 + 4x^2 - 1}{10x^4 + 8x^2 + 1}$$

oh hey, the powers of the numerator and the denominator are samesies, so we're just looking at $\frac{40}{10}$, which is $\lim_{x\to\infty} 4 = 4$
because this is a rational function, it's going to be *samesies*, soooo, 4 too!

- checks out graphically, I'm too lazy to put that in here
-

MATH111 - 2023-09-06

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- Horizontal asymptotes are limits to infinity
- Vertical asymptotes are to a number, where the answer is infinity

Example

- Find all horizontal and vertical asymptotes for f and justify by showing the appropriate limit

ok other example rq

Find horizontal asymptotes, look at what happens at infinity

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

$\sqrt{x^2}$ is the same thing as $|x|$

do to this

$$\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2(1+\frac{1}{x^2})}}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2}+\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{x}{|x|\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{x}{x\sqrt{\frac{1}{x^2}}}$$

$$\lim_{x\to\infty}\frac{1}{\sqrt{1+0}}=1$$

The real example

$$f(x)=\frac{x^3-5x^2+6x}{x^4-4x^2}$$

Since f is rational, we only have 1 horizontal asymptote
Only look at $\lim_{x\to\infty} f(x)$

$$=\lim_{x\to\infty}\frac{x^3}{x^4}=\lim_{x\to\infty}\frac{1}{x}=0$$

Horizontal asymptote at $y=0$
Vertical asymptote need some work, so we gotta figure out what the hell a is first
Potential vertical asymptotes when the denominator is 0

$$x^4-4x^2=(x^2)(x^2-4)=0$$

So we have roots at $\pm$ 2 and 0
Now, the sticky wicket is that this does NOT mean we have vertical asymptotes, we gotta go check
Lets get to checking them

$$\lim_{x\to 2^+}\frac{x^3-5x^2+6x}{(x^2)(x-2)(x+2)}=\lim_{x\to 2^+}\frac{(x)(x^2-5x+6)}{x^2(x-2)(x+2)}=\lim_{x\to 2^+}\frac{(x)(x-2)(x-3)}{x^2(x-2)(x+2)}\lim_{x\to 2^+}\frac{(x)(x-3)}{x^2(x+2)}=$$

$$\lim_{x\to 2^+}-\frac{1}{8}$$

Oh hey, that's not ∞, (or -∞), so that's just a hole. Womp womp.

$$\lim_{x\to -2^+} f(x)=\frac{(x-3)}{x(x+2)}=\frac{-5}{0}$$

Which tells us the limit Does Not Exist, which isn't much good when you're trying to figure out an asymptote
Now, the real wicket is it $+\infty$ or $-\infty$

$$\lim_{x\to -2^+}\frac{x-3}{(x)(x+2)}=\frac{-5}{+smol(-)}=\frac{-}{-}=+$$

Sooooo, $\lim_{x\to -2^+} f(x)=+\infty$

MATH111 - 2023-09-08

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Log Rules

1. $$\ln(xy) = \ln(x) + \ln(y)$$
2. $$\ln(\frac{x}{y}) = \ln(x) - \ln(y)$$
3. $$\ln(x^r) = r * \ln(x)$$
4. $$e^{\ln(x)} = x$$
5. $$\ln(e^x) = x$$
6.

Notes

- y=0 is a horizontal asymptote for e^x
- . e = 2.717 *ish*, it keeps going
- e^{-x} is basically a reflection of e^x, it's the same as $\frac{1}{e^x}$

Practice stuff

$$\lim_{x\to\infty}(4e^{-2x} + 3x^{-1}) = \lim_{x\to\infty}\frac{4}{3^{2x}} + \frac{3}{x} = 0 + 0 = \lim_{x\to\infty} 0 = 0$$

$$\lim_{x\to-\infty}(4e^{-2x} + 3x^{-1}) = \lim_{x\to\infty}(\infty)$$

$$\lim_{x\to-\infty}\frac{5}{2+3^x} = \lim_{x\to-\infty}\frac{5}{2+0} = \lim_{x\to-\infty}\frac{5}{2} = \frac{5}{2}$$

- Logs and exponentials are inverses! Yippee.
- $\ln$ is really $\log_{10} e$
  ln(x)
- Domain (0, ∞)

- Range$(-\infty, \infty)$
- $\lim_{x \to \infty} \ln(x) = \infty$
- $\lim_{x \to 0^+} \ln x = -\infty$
-

MATH111 - 2023-09-11

#notes #math111 #math #calc

- Talking about continuity
- A function f is continuous at *a* if $\lim_{x \to a} f(x) = f(a)$
  - We can't have any holes, any breaks, any *shenanigans* going on at $f(a)$
- We're generally not going to be asked about what's going on at endpoints
- Why the *heck* do we care?
  - They're nice for calc, most calc theorems assume f is continuous

Continuity Checklist

1. $f(a)$ is defined
   1. (a number)
2. $\lim_{x \to a} f(x)$ exists
   1. (a number)
3. $\lim_{x \to a} f(x) = f(a)$
   1. those aforementioned numbers have to be samesies
4. Well, we kinda lied
   1. As long as we're not piecewise, it'll be continuous for everything in the domain

$$\frac{(x-1)(x-2)}{(x+1)(x-1)} = \frac{(x-2)}{(x+1)}$$

Checking some continuity jazz

1. $f(2) = 3$
   1. Yippeeeeeeeee
2. $\frac{(x+2)(x-2)}{(x-2)} = (x+2) = 4$ womp womp, doesn't match
3. $3 \neq 4$

1. Yipppppeeeeeee, definitely not continuous

checking more continuity jazz

- For $a = 3$
  1. Passes, $f(a)$ is *reaaaaalll*
  2. Limit DNE, the two one sided limits don't match
  3. DNE cannot equal a real number
- For $a = 5$
  1. $f(a)$ does not exist (is undefined)
  2. $\lim_{x \to 5}$ is infinity, which, not a number
  3. Undefined $\neq +\infty$
  4.
- Limits with continuous functions are eaaaaasy, just plug in
-

MATH111 - 2023-09-12

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Limit Review

- Plug in, as long as you don't get $\frac{0}{0}$ you're good - move on
- If you dooooo
  - Factoring/canceling
  - Multiply by conjugate
  - etc etc
-

MATH111 - 2023-09-13

#notes #math111 #math #calc

3.1 Definition of the [Year 1/Semester 1/MATH111/Concepts/Derivative]

- Quick reminder
  - Year 1/Semester 1/MATH111/Tangent Line is the instantaneous rate of change
  - Use secant lines to get an approximation, then take a limit to get the exact value
- $m_{\tan} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$
- x in this case is just some random given value
- The function we're taking the limit of is actually the Year 1/Semester 1/MATH111/Secant Line

Quick Example

Let $f(x) = x^2 - 8x$ Find the slope of the tangent line to f at x = 3

- Alright, let's get to pluggin and chuggin
  $m_{\tan} \lim_{x \to 3} \frac{f(x)-f(3)}{x-3}$
  = $\lim_{x \to 3} \frac{x^2-8x-(-15)}{x-3}$
  =$\lim_{x \to 3} \frac{x^2-8x+15}{x-3}$
  =$\lim_{x \to 3} \frac{(x-3)(x-5)}{(x-3)}$
  =$\lim_{x \to 3}(x-5)$
  =$\lim_{x \to 3} -2$
  -2, yipeeeeeee
  -2 is the slope of the function at $x = 3$

Quickly, scramble!

- We're reordering the function so we don't have to think so hard
- $m_{\sec} = \frac{f(a+h)-f(a)}{h}$
- $m_{\tan} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Quick Example (Reprise)

- Let $f(x) = x^2 - 8x$ and compete the following
- $f(3) = -15$
- $f(3+h) = (3+h)^2 - 24 - 8h$
  - I am, for the record, waaaaay too lazy to foil that sh*t
- $f(a) = a^2 - 8(a)$

- $f(a+h) = (a+h)^2 - 8(a+h)$
- Find the slope of the tangent line to f at $x = 3$
- $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h}$
- $\lim_{h \to 0} \frac{(3+h)^2-8(3+h)-(-15)}{h}$
- $\lim_{h \to 0} \frac{9+6h+h^2-24-8h+15}{h}$
- $\lim_{h \to 0} \frac{h^2-2h}{h} = \lim_{h \to 0} \frac{h-2}{1}$
- $\lim_{h \to 0} -2$

Derivative Time.

- Hey, we're ready for derivative time
- $f\backslash(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$
    - If that limit ($f\backslash(a)$) exists, we say that f is differentiable at $x = a$

Quick Example (Reprise (Reprise))

- Point (3,$f(3)$) = (3,-15)
- Slope = $f\backslash(3) = -2)$
- $y - -15 = -2(x-3)$
- $y + 15 = -2x + 6$
- $y = -2x - 9$
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MATH111 - 2023-09-15

#notes #math111 #math #calc

- The derivative of f at a, denoted $f\backslash(a)$
- Is given by $f\backslash(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$
    - $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$
    - (It's slope of a tangent line, aka instantaneous rate of change)
- f(6) = 5
- $\frac{\text{Rise}}{\text{Run}} = -2$

Let $f(x) = \sqrt{x+1}$

Use the limit definition of the Year 1/Semester 1/MATH111/Concepts/Derivative to find $f'(2)$

$\lim_{h \to 0} \frac{\sqrt{2+h+1} - \sqrt{3}}{h}$
$\lim_{h \to 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$
That *stinks* of multiply by the conjugate

$$\lim_{h \to 0} \frac{\sqrt{3+h} - \sqrt{3}}{h} * \frac{\sqrt{3+h} + \sqrt{3}}{\sqrt{3+h} + \sqrt{3}} = \lim_{h \to 0} \frac{3+h-3}{(\sqrt{3+h} + \sqrt{3}) * h} = \lim_{h \to 0} \frac{1}{\sqrt{3+h} + \sqrt{3}} = \lim_{h \to 0} \frac{1}{2\sqrt{3}}$$

The derivative of f is the function $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Example
let $f(x) = x^2 + 2x + 1$
$f(x) = (x+1)(x+1)$

$f(x+h) = (x+h+1)(x+h+1)$

$f'(x) = \lim_{h \to 0} \frac{(x+h+1)(x+h+1) - (x+1)(x+1)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h}$$

$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$
$\lim_{h \to 0} 2x + h + 2$
$= 2x + 2$

MATH111 - 2023-09-18

#notes #math111 #math #calc
Year 1/Semester 1/MATH111/Concepts/Derivative

$f'(x) = \lim_{h \to 0} \frac{(f(x+h) - f(x))}{h}$
obligatory writeout of point slope form $y - y_1 = m(x - x_1)$

$\frac{3}{5}$ is answer for the reflection

$f'(x) = 10$
$2x + 2 = 10$
$x = 4$
If we were looking for horizontal...
then slope

$f'(x) = 0$
$2x + 2 = 10$
$x = -1$

Differentiable if it's "locally linear," that meaning you can draw a non-vertical tangent line

If there's a sharp corner, there's two possible tangnet lines, no good

Vertical tangent line means that it's undefined

if it's non-continuous, we have nowhere to put a tangent line.

If you're differentiable, than you're for sure continuous - but if you're continuous, you're not necessarily differentiable

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Not continuous at -2, 2

Not differentiable at -2, 0, 2

MATH111 - 2023-09-19

#notes #math111 #math #calc

we did some matching stuff

- had the epiphany about degrees of functions, that's so much fun

Notion aside

$f'(x) = \frac{dy}{dx}$
$\frac{dy}{dx}$ is Leibniz notation, and we tend to use it as an operator
$(10)'$ means take the derivative of what's inside the parenthesis

Alright time to learn shortcuts

Constant Rule

- If $f(x) = c$ then $f'(x) = 0$
- If it's just a constant, then the slope of it 0 everywhere, wow, awesome

Power Rule

- If $f(x) = x^n$ then $f'(x) = nx^{n-1}$
- So if it's just $f(x) = x$, then it comes out that $f'(x) = 1$ (neat)
- $\frac{d}{dx}(x^{10}) = 10x^9$
- Remember that $\frac{1}{x^2}$ is the same thing as x^{-2}
- So the derivative of $\frac{1}{x^2}$ is $-2x^{-3}$

Constant Multiple Rule

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

So the derivative of $10x^{-1} = 10 * (-1)x^{-2}$

Sum (and Difference) Rule

$$\frac{d}{dx}(f(x) + g(x)) = (f'(x) = g'(x))$$

Differentiating two different functions that are being added(or subtracted) ? Eh, just do em independently and then stick em together

Strictly speaking, this is following limit rules; A limit of a sum = the sum of the limits

Example

Find $f'(x)$ for $6\sqrt{x} + \frac{x^4}{2} + \frac{8}{x^11} + 3x$

We gotta do some rewriting

$$f(x) = 6 * x^{\frac{1}{2}} + \frac{1}{2}x^4 + 8x^{-11} + 3x$$

So let's get derivin'

$$3x^{-0.5} + 2x^3 - 88x^{-12} + 3$$

MATH111 - 2023-09-25

#notes #math111 #math #calc

- Quick warning
  - Power rule is only good for x to a power
- product and quotient rules are a bit of a pain - remember that you can do algebra to get around doing them

Other fun fact

$$\frac{d}{dx}(e^x) = e^x$$

- There's some limit bs as to why, I'm half tempted to do the fanangling

okie dokie time for the real notes

Product Rule

$$\frac{d}{dx}(f(x) * g(x)) = f(x)g'(x) + g(x)f'(x)$$

Shorthand

$$(fg)' = fg' + gf'$$

it is definitely NOT

$$(fg)' \neq f'g'$$

Example: Find $h'(x)$ = $h(x) = 6x^3(2x^2 - 1)$

- $f'(x) = 18x^2$
- $g'(x) = 4x$
- $h'(x) = (6x^3)(4x) + (2x^2 - 1)(18x^2)$
- $h'(x) = 60x^4 - 18x^2$

ex (again) Find $f'(x) = 2x^2e^x$

$g'(x) = 4x$
$h'(x) = e^x$
$f'(x) = (4x)(e^x) + (2x^2)(e^x)$
$f'(x) = (e^x)(4x + 2x^2)$

ex(three) Find $h'(x)(x^2 + 1)(3x^{-5} + x^{-1})$

$f'(x) = 2x$
$g'(x) = -15x^{-6} - x^{-2}$
$h'(x) = (2x)(3x^{-5} + x^{-1}) + (x^2 + 1)(15x^{-6} - x^{-2})$
$h'(x) = 6x^{-4} + 2x + 15x^{-4} - 1 + 15x^{-6} - x^{-2}$

Why does the product rule work?

Because Deb said so.

MATH111 - 2023-09-26

#notes #math111 #math #calc

Quotient Rule

$$\frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

shorter version

$$(\frac{f}{g})' = \frac{g * f' - f * g'}{g^2}$$

$$\frac{Hi}{Lo} = \frac{Lo * DHi - Hi * DLo}{Lo^2}$$

- Why does the quotient rule work?
- We could prove it using the limit definition of the deriative
  - oooooooooooooor we can wait until we know the chain rule
    - you don't even really need the quotient rule, but the chain rule can do it instead (or product, combined shenanigans of both)

Ex

let $y = \frac{x^4+3x}{10-x}$

$$lo' = 0 - 1$$

$$hi' = 4x^3 + 3$$

$$\frac{(10-x)(4x^3+3) - (x^4+3x)(-1)}{(10-x)^2}$$

you coooould simplify slightly

$$\frac{dy}{dx} \frac{(10-x)(4x^3+3) + (x^4+3x)}{(10-x)^2}$$

we aa

Find $f'(x)$ of $f(x) = \frac{x^2}{4}$

quotient time
$lo' = 0$
$hi' = 2x$

$$\frac{(4)(2x) - (x^2)(0)}{4^2} = \frac{(4)(2x)}{4^2} = \frac{2x}{4} = \frac{x}{2}$$

other way time

$$0.25 * x^2 = 0.5x = \frac{x}{2}$$

Derivatives of Trig Functions

$\frac{d}{dx}\sin(x) = \cos(x)$
$\frac{d}{dx}\cos(x) = -\sin(x)$

They're the basic ones, you can plug everything else in

$\frac{d}{dx}\tan(x) = \sec^2 x$
$\frac{d}{dx}(\sec(x)) = \sec x \tan x$
$\frac{d}{dx}cot(x) = -\csc^2 x$
$\frac{d}{dx}\csc x = -\csc x \cot x$

Memory tip: sst

secx secx tanx
cscx cscx cotx

secx-> secx <- tanx

cscx-> -cscx <- cotx

Show that $\frac{d}{dx}(\tan x) = sec^2 x$

$$\frac{\sin}{\cos}' = \frac{(\cos)(\sin x)' - (\sin x)(\cos x)'}{\cos^2 x} = \frac{\cos x * \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

MATH111 - 2023-09-27

#notes #math111 #math #calc

ayo physics jumpscare

- derivative of position is velocity
- derivative of velocity is acceleration

Moving horizontally along a line (this is not what was in the worksheet)

$$f(t) = 2t^3 - 21t^2 + 60t$$

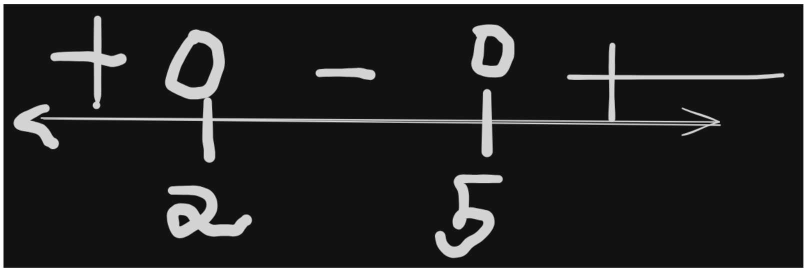
$$v(t) = 6t^2 - 42t + 60 \ = (6)(t^2 - 7t + 10) = (6)(t-5)(t-2)$$

$$a(t) = 12t - 42$$

1. Displacement?
    1. just plug that in
        1. so the answer is 77-0, 77 ft
2. Total distance traveled
    1. 52+27 +52 = 131ft (feet are cringe, I miss meters)
    2.

MATH111 - 2023-09-29

#notes #math111 #math #calc

- We're still dealing with local position function $f(t) = 2t^3 - 21t^2 + 60t$
- Differentiate to get velocity, we get $v(t) = 6t^2 - 42t + 60$
  - so lettuce figure out where it's 0
  - (6)(t-2)(t-5)
  - f(t) =0 at t = 2, 5
  - 

  - We gotta plug in values for each section
  - $v(0) = (6)(-2)(-5) = 60$
    - Everywhere left of 2 is gonna be positive
  - $v(3) = (6)(1)(-2) = -12$
    - Everywhere between two and five is going to be negative
  - $v(6) = (6)(4)(1)$
    - That's always gonna be positive
- Where velocity is positive we goin right, where velocity is negative we goin left
- t is moving right on the interval $(-\infty, 2) \cup (5, \infty)$
  - and to the left on the interval $(2, 5)$

- Acceleration is the ROC of velocity with respect to time
- An object is "speeding up" when its speed increases
  - An object will speed up when v(t) is > 0 and v'(t) > 0
  - v(t) is <0 and v'(t) < 0
- An object will speed up when velocity and acceleration have the same sign. An object will slow down when velocity and acceleration ave opposite signs

---

So, differentiate velocity, get $a(t) = 12t - 42$
$t = 0$ at $\frac{42}{12} = \frac{7}{2} = 3.5$
So if we were to draw the [Year 1/Semester 1/MATH111/Sign Chart], we got two halves
$a(0) = 12(0) - 42$ negative, yippeeeeeeeeeeeeeeeeeee
$a(4) = 12(4) - 42$ positive, hip hip hooray

MATH111 - 2023-10-02

#notes #math111 #math #calc

Chain Rule Objectives

- State the chain rule:

$$(f(g(x)))' = f'(g(x)) * g'(x)$$

- Apply the chain rule to compute derivatives
- Apply for powers to find derivatives, if n is any non-zero real number, then

$$\frac{d}{dx}((g(x))^n) = n(g(x))^{n-1}g'(x)$$

- Chain rule tells us how to differentiate a "composition" of functions
- So let's do an example

ex

$h'(x)$ for $h(x) = (3x^7 + 1)^2$

$f'(x) = 6x^7 + 2$
$g'(x) = 21x^6$
$f'(x) * g'(x) = (6x^7 + 2)(21x^6)$
$h'(x) = 126x^{13} + 42x^6$

Generalization Calls

- So, chain rule is quirky, and we can differentiate whatever the hell we want using the old rules

Chain rule for powers

$$\frac{d}{dx}[(g(x))^n] = n(g(x))^{n-1}g'(x)$$

Chain rule for exponentials

$$\frac{d}{dx}[e^{g(x)}] = e^{g(x)} * g'(x)$$

Examples (but again)

Let $y = \sqrt{x^4 - 7x}$

$y' = \frac{1}{2}(x^4 - 7x)^{\frac{-1}{2}} * (x^4 - 7x)''$
$y' = \frac{1}{2}(x^4 - 7x)^{\frac{-1}{2}} * (4x^3 - 7)$

Let $f(x) = 3e^{4\sqrt{x}}$

$f'(x) = 3e^{4\sqrt{x}} * 2x^{\frac{-1}{2}}$

Use the chain rule to find $(e^{ax})'$

- where A is just some number
- $f'(x) = e^{ax} * a$
-

MATH111 - 2023-10-03

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- Super important with the chain rule is figuring out what the "inside" of the function is

Mark it! Integral jumpscare!

MATH111 - 2023-10-04

#notes #math111 #math #calc

Implicit Differentiation

- Say what the hell that means
- Calculate $\frac{dy}{dx}$ for an implicitly defined function
- use it to do shenanigans

Solve $x^2 + y^2 = 25$ for some shenanigans

$y = \pm\sqrt{25 - x^2}$
The plus is the top half of the circle, the - is the bottom half of the circle

use $y = \sqrt{25 - x^2}$
to find $\frac{dy}{dx}$

$y = (25 - x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(25 - x^2)^{\frac{-1}{2}} - 2x$

$y' = \frac{-x}{\sqrt{25 - x^2}}$

$\Big|_a^b$

Maybe we can't actually solve for y, and we can't get the explicit differentiation

Using the equation $x^2 + y^2 = 25$

1. Differentiate both sides with respect to x

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$
$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^{2)} = \frac{d}{dx}(25)$$

Those ones with just x are *awesome*, we can do whatever we want with those ones on the other hand, when we see a y around, we have a problem

- We gotta ponder that $y = f(x)$
- Which, in that case, we need to conjure up the chain rule

gots to find
$\frac{d}{dx}(y^2)$
which is the same thing as

$$\frac{d}{dx}((f(x))^2) = 2(f(x)) * f'(x) = 2y * \frac{dy}{dx}$$

so hip hopping back up

$$2x + 2y * \frac{dy}{dx} = 0$$

2. Someone (if they're cringe) might ask us to solve for dy/dx

$$2x + 2y\frac{dy}{dx} = 0 \;\; = \;\; 2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

3. Plug in at a point (x, y) to find the slope at that point

$$\frac{dy}{dx}\Big|_{(3,4)} = \frac{-3}{4}$$

If we want to find where it's horizontal, dy/dx = 0, and you can just solve for that

Find the equation of the tangent line to $2x^3 + 2y^3 = 9xy$ at the point (2,1)

We gonna need to implicit this lil fella

$$\frac{d}{dx}(2x^3) + \frac{d}{dx}(2y^3) = \frac{d}{dx}(9xy)$$

- That first one is easy
  - Second one is a wonderful chain rule problem
  - Third one is an icky gross product rule trying to sneak its way into being a reasonable function with friends
  party time

$$6x^2 + 2 * 3y^2 * \frac{dy}{dx} = 9x\frac{dy}{dx} + 9y$$

that algebra is awful and I'm going to represent $\frac{dy}{dx}$ with a d just to make my life easier

$$6y^2d - 9xd = 9y - 6x^2$$

$$\frac{dy}{dx}(6y^2 - 9x) = 9y - 6x^2$$

$$\frac{dy}{dx} = \frac{9y - 6x^2}{6y^2 - 9x}$$

So now we have a somewhat reasonable equation, so let's go plug in $(2, 1)$

$$\frac{dy}{dx}\Big|_{(2,1)} \frac{9(1) - 6(2)^2}{6(1)^2 - 9(2)} = \frac{9 - 24}{6 - 18} = \frac{-15}{-12} = \frac{5}{4}$$

$$y - 1 = 1.25(x - 2)$$

MATH111 - 2023-10-06

#notes #math111 #math #calc

Quick Derivative Log Rules

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln |x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b(x)) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(\log_b |x|) =$$

We can prove $\frac{d}{dx}\ln(x) = \frac{1}{x}$
$y = \ln(x)$
And we want $\frac{dy}{dx}$
so $e^y = x$
$\frac{d}{dx}(e^{y)} = \frac{d}{dx}x$

$$e^y * \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$e^y = x, \text{sooooooo } \frac{dy}{dx} = \frac{1}{x}$$

MATH111 - 2023-10-09

#notes #math111 #math #calc

Quick Derivative Log Rules (Copy pasted from Friday)

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln |x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b(x)) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(\log_b |x|) = \frac{1}{x \ln b}$$

Proof for non- e log base

$$\log_b x = \frac{\ln x}{\ln b}$$

Derivative of that is just

$$\frac{1}{\ln b} * \ln x$$

Log base three of x differentiated would just be 1/ln3 * 1/x = 1/xln3

- i was way too lazy to LaTeX that

If we were to toss the chain rule in (ie $\ln(g(x))$) you would just get $\frac{1}{g} * g' *$

Find the derivative of $x \ln x$

$$x * \ln(x)' + \ln x * x' *$$

$$1 + \ln x$$

Find the derivative of ln (cos^2x)

$$\frac{1}{\cos^2} * (\cos^2 x)' = \frac{1}{\cos^2} * 2\cos x * -\sin x$$

Exponential Log Rules

$$\frac{d}{dx} b^x = (\ln b) * b^x$$

$$\frac{d}{dx}(b^{g(x)}) = (\ln b) * b^{g(x)} * g'(x)$$

Logarithmic Differentiation

helps us find the derivative for things like $\frac{1}{\cos^2} \frac{d}{dx} f(x)^{g(x)}$

To find the derivative of $f(x) = x^x$
$ln(f(x)) = ln(x^x)$

$$\ln(f(x)) = x \ln(x)$$

$$\frac{1}{f(x)} * f'(x) = x(\ln x)' + (\ln x)(x)'$$

$$\frac{f'(x)}{f(x)} = x * \frac{1}{x} + \ln(x)$$

$$f'(x) = (1 + \ln(x)) * x^x$$

$$f'(x) = x^x + x^x \ln(x)$$

MATH111 - 2023-10-10

#notes #math111 #math #calc

inverse trig (hip hip yipee)

- you're going to need to have some things memorized, because that's just kind of how it goes
- remember, strictly speaking, inverse functions "undo" one another (domain and range are reversed)
- sine woefully fails the horizontal line test
  - this isn't really important info, it's just funny to say
- If you restrict the domain, badaboom badabing, you can properly take the invrese
  - Restrict the domain of sine to $-\frac{\pi}{2}, \frac{\pi}{2}$
- $\sin^{-1}$, $\arcsin$, and inverse sine all mean the same thing
- Arcsin's range is the aforementioned -pi/2 to pi/2, domain is -1 to 1
-

$\sin^{-1}(1) = \frac{\pi}{2}$

Inverse cosine's range is from (0,π)
Domain is from -1 to 1 (inclusive)

Domain of tangent is -pi/2 to pi/2

So the domain of arctan is $-\infty, \infty$
Range is $\frac{\pi}{2}$ to $-\frac{\pi}{2}$

Using a right triangle to simplify expressions

cot (cos^1(x/4))

Let theta = cos^-1 (x/4) so cos(theta) = x/4

MATH111 - 2023-10-11

#notes #math111 #math #calc

Derivatives of Inverse Trig Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$

Derivatives of their inverse are the same, just toss a negative sign in there

hey, why is the derivative of inverse sin the way that it is?

$y = \sin^{-1}x$
Let's get differentiatin'
problem: ain't got no idea how to differentiate sin^-1 (if we're trying to prove it)

$siny = x$

$$\frac{d}{dx}(sin\ y) = \frac{d}{dx}x$$

$$\frac{d}{dx}(sin\ y) = 1$$

$$cos(y) * \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{cos(y)}$$

sin(y) = x
sin(y) = x/1

Derivatives of inverse trig functions (with the chain rule)

$$\frac{d}{dx}(sin^{-1}(g(x)) = \frac{1}{\sqrt{1-(g(x))^2}}) * g'$$

differentiate
$f(x) = 10tan^{-1}(e^{4x})$

$$f'(x) = 10 * (\frac{1}{1 + (e^{4x})^2}) * (e^{4x})'$$

$$f'(x) = 10 * (\frac{1}{1 + e^{8x}}) * e^{4x} * (4x)'$$

$$f'(x) = \frac{40e^{4x}}{1 + e^{8x}}$$

Maxima & Minima

Absolute Maxima and Minima

- Absolute extrema are the tippity top or the bitty bottom
- Absolute maximum is a y value that occurs *at* $f(x)$

Extreme Value Theorem

- A continuous function f that occurs on a closed interval has an absolute max and min on the interval
- This is the EVT - MVT is something that'll come back to bite us later
- If there's a hole, it can't be a max/min, because it needs to be a hole

Local Maxima and Minima

- Local max and min are the biggest/ smallest point around em
- Basically, if you chop your function up, you're the biggest in whatever interval you chop it up into
-
- you might find extrema where the derivative is not defined and where the derivative is = 0
    > - those are gonna be "critical points"

MATH111 - 2023-10-13

#notes #math111 #math #calc

More extrema stuff

Critical Numbers / Critical Points

- a value c is a critical number or critical point for a function f if
    1. c is in the domain of f
    2. $f'(c) = 0$ or $f'(c)$ does not exist
- WARNING: critical numbers are potential extrema, but this is not a guarantee

Remember, continuous function on a closed interval will have absolute extrema at either an endpoint or a critical point

Procedure

1. Compute $f'(x)$
2. Find all critical points c for $f'(x)$ in the interval $[a, b]$
3. Evaluate f at all the critical points from step 2, and at the two endpoints
4. The largest value from step 3 is the absolute max and the smallest value is the absolute min

Example

Let $f(x) = x^3 - 3x^2 + 1$ on the interval $[-1, 4]$
$f'(x) = 3x^2 - 6x$
Factor that sh*t
$(3x)(x - 2)$
We'll have 0's at $x = 0, 2$
We also need to find where it's undefined, but we can't really make this undefined
0 and 2 are both in our domain, so hip hip

Let's go shove these in our original function! yippeee

x	f(x)
-1	-3
4	17
0	1
2	-3

|

If we change the interval of our function from $[-1, 1]$ we gotta toss out 2 as a critical value

-3 becomes our absolute min
1 at 0 becomes our absolute max

$f(x) = x - x^{\frac{2}{3}}$
$f'(x) = 1 - \frac{2}{3}x^{\frac{-1}{3}}$
find where that derivative is 0 or not defined
$f'(x) = 1 - \frac{2}{3} * \frac{1}{x^{\frac{1}{3}}}$

$$\frac{3x^{\frac{1}{3}} - 2}{3x^{\frac{1}{3}}}$$

We can isolate some terms, do some shenanigans,
$1 = \frac{2}{3x^{\frac{1}{3}}}$
$x^{\frac{1}{3}} = \frac{2}{3}$
This is the cube root of x, which is neat

$$\frac{2}{3}^{3} = x = \frac{8}{27}$$

It's in -1 to 1, so yippee, lettuce keep it

x	f(x)
0	0
8/27	-4/27
-1	-2
1	0

Absolute max is 0, absolute min is -2

MATH111 - 2023-10-18

#notes #math111 #math #calc

We gots to talk about the mean value theorem

- We just did the EVT, but now we're doing the mean value theorem
- Used a whole bunch in proofs for theorems for a whoooole bunch of calc

- You could go a whole lot of potential speeds, but you'll have to pass through your mean at some point
- Somewhere, there will be a place where your average rate of change is the same as your instantaneous rate of change
- If f is continuous on $[a, b]$, and differentiable on (a, b), then there is at least one value c where $a < c < b$ such that the average rate of change of f over $[a, b]$ is equal to the instantaneous rate of change at c. That is,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- That fraction stuff is the average rate of change, $f'(c)$ is the instantaneous velocity at c
- Theorem states there is **at least one** value c - there could be more!
- minor problem with the MVT - it's simply an existence theorem, it doesn't do us any good in finding c
- Average rate of change is easy though, we can do that

---

ex

for a function $f(x) = \frac{x^2}{4} + 1$ on the interval $[-2, 4]$
We should be continuous and differentiable, because... we are (polynomial)

Let's find the endpoints!
At $x = -2, y = 2$
At $x = 4, y = 5$
Oookie, so we gotta find the average rate of change
$\frac{5-2}{4+2}$
$\frac{3}{6} = f'(c)$

$f'(x) = 0.5x$
$c = 1$

MATH111 - 2023-10-20

#notes #math111 #math #calc

- Today we're doing stuff with the first derivative
    - Tuesday is the second derivative test, shocking

How we can use the first derivative to test to identify local maxima and minima

- We can use the first derivative to tell where we're increasing or decreasing
  - If the function is increasing, the slope of the tangent line should be positive
- This checks out if we think about it, but a careful poof will use the Year 1/Semester 1/MATH111/Mean Value Theorem
  - MVT is useful because it can be used to algebraically connect the function to its derivative

First Derivative Test

- If c is a critical point for f, if f is continuous and differentiable (except *maybe* not at the critical point)
  - Quick aside: Critical points are where $f'(c) = 0$ or $f'(c) = ND$, and c is in the domain of f
- If $f'(c)$ changes from positive to negative at $x = c$, then we have a local max
- If $f'(c)$ changes from negative to positive, then we have a local min
- If it doesn't change, jack shit is happening there
-

Example

$f(x) = 2x^3 + 3x^2 - 12x + 11$
$f'(x) = 6x^2 + 6x - 12$
$f'(x) = (6)(x^2 + x - 2)$
$f'(x) = (6)(x - 1)(x + 2)$
We've got 0's at $x = -2, 1$
Hey uh, are those critical points?

- yeah goober, they're in the domain

(6)(-8)(-11)
We are testing in f', not f
Positive when <-2

$(6)(2)(-1)$
Zoinks, that's negative

(6)(4)(1)
That's all positive

So $x = -2$ is a local max, $x = 1$ is a local min
Plug in, the local max is 31, the local min is 4, and they occur at -2 and 1 respectively

Example 2

$f(x) = 2 + x^{-2}$
$f'(x) = \frac{-2}{x^3}$
So funny story, this derivative will never be 0
But where doesn't it exist? x^3=0, which happens at x=0
Is this a critcal point?
Naurr, because $f(0)$ is also problematic

Alright,we can make a sign chart for this

from $-\infty, 0$ and $0, \infty$
$\frac{-2}{-1} = 2$
That's positive
We're pawsitive to the right

$f'(2) = -2/2^3 = \frac{-2}{8}$ But that's negative!
So that'll look like a local max, but... it doesn't exist, so it's lame ahh hell

We have *no* local extrema

MATH111 - 2023-10-23

#notes #math111 #math #calc

Exam Stuff

Derivative Rules

Topics/Concepts:

- Implicit differentiation
  - Find a formula for dy/dx, or find a slope
- Extreme Value Theorem

        - Find an absolute max and min
- Rate of change problems
    - Find where something is moving left/right, speeding up, slowing down
- Mean Value Theorem
    - Rolle's Theorem exists
        - Just the mean value theorem, where $f(b) = f(a)$
        - so the overall slope is jack didley squat
        -

Inverse Trig

- Inverse trig stuff, $\sin^{-1}(1)$ and other triangle problem shenanigans
- Simplify something using a triangle $(\cos(\sin^{-1} \frac{x}{2})$

$f(1) = 0$
$f(-1) = 0$

$\frac{0}{2} = 0$

MATH111 - 2023-10-24

#notes #math111 #math #calc

- Second derivative helps determine "concavity" of a function

Year 1/Semester 1/MATH111/Concepts/Concavity

Concave Up

- It bends up like a cup
-
    - If the derivative is increasing on an interval then the function is concave up
    - NOTE: This does not say the derivative is always positive, it says it's going up
        - So we need the slope of our derivative
        - f''

Concave Down

- Bend's down like a frown

- Derivative is *decreasing* on the interval
- This means that f'' is negative

Inflection Point

- If something changes [Year 1/Semester 1/MATH111/Concepts/Concavity] at some point (from down to up or up to down), then that point is an inflection point
- This is some place - it has coordinates, and it's there

Example

- Give an x interval for concavity
- $f(x) = 5x^4 - 20x^3 + 10$
- $f'(x) = 20x^3 - 60x^2$
- $f''(x) = 60x^2 - 120x$
- $f''(x) = (60)(x)(x-2)$
- Concave up on the interval $[-\infty, 0), (2, \infty)$
- Concave down on the interval $(0, 2)$
- Inflection points are
    - $(0, f(0)) = (0, 10)$
    - $(2, f(2)) = (2, -70)$

- To be an inflection point you need to be on the graph

Second Derivative Test

- Tells us about local extrema
- If $f''(c) > 0$ then f has a local minimum at c
- If $f''(c) < 0$ then f has a local maximum at c
- If it's 0 we got a problem and have no idea what's going on there (indeterminate)
-

MATH111 - 2023-10-25

#notes #math111 #math #calc

Optimization

Extra Theorem

- One local Extremum implies absolute extremum
  - Suppose f is continuous on an interval I that contains exactly one critical number c
    - If the local max occurs at c, then that is the absolute max
    - If the local min occurs at c, then that is the absolute min

How to Optimization

1. Find a function of a single variable to be optimized
   1. That sounds easy but is actually kind of difficult
2. Determine the relevant domain for the function you are optimizing
3. Justify that you have found the abs. max or min
4. Answer the question, goober

Example

- A rancher has 400 feet of fencing to construct a rectangular yard, one side of the fence is against a barn and needs no fencing. What are the dimensions of the fence that encloses the largest area?
- GOAL: Maximize area
  - A = xy
  $2y + x = 400$
  $y = \frac{400-x}{2}$
  $y = 200 - \frac{x}{2}$

$f(x) = (x)(200 - \frac{x}{2})$
$f(x) = 200x - \frac{x^2}{2}$
Hey, single variate function

2. Relevant Domain
   1. X is a length, so it needs to be positive, and it can't be 0 because that's wonky for area
   2. Y is also a length, and has the same problem

Justification

- Extreme Value Theorem
  - We had a nice continuous function
- 1st Derivative Test
- 2nd Derivative Test
  - Uh, if we have a choice, do that
-

MATH111 - 2023-10-30

#notes #math111 #math #calc

Optimization Stuff

Some obligatory geometry review (don't forget your formulae, dummy)

Right Circular Cylinder

$r =$radius, $h =$ height
Volume: $V = \pi r^2 h$
Surface area = $S = 2\pi rh + 2\pi r^2$

Rectangular Solid

l = length, $w =$width, $h =$height
Volume = lwh
Surface area $S = 2lw + 2lh + 2wh$

MATH111 - 2023-10-31

#notes #math111 #math #calc

- Linear approximation (and Differentials)

$L(x) = f'(a)(x - a) + f(a)$

The tangent line of a differentiable function at a point is a pretty good approximation of what's going on at that function

example $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{\sqrt{x}} * \frac{1}{2}$
We're smackin a tangent line at 4
Point is $(4, f(4)) = (4, 2)$
Slope is just plugging in to the derivative = $\frac{1}{4}$
equation $y - 2 = \frac{1}{4}(x - 4)$
$y = \frac{x}{4} - 1 + 2$
$y = \frac{x}{4} + 1$
Alright that's our linear approximation, now let's get approximatin'
$y = \frac{4.1}{4} + 1 = 1.025 + 1 = 2.025$

Taylor Polynomials sound horrifying... anyways!

$L(x) \approx f(x)$ when $f(x)$ is near $a(= 4)$

Let $f(x) = \ln(x + 3)$

Tangent line approximation at $a = -2$ is $L(x) = x + 2$

MATH111 - 2023-11-01

#notes #math111 #math #calc

L'Hôpital's Rule

- Apply L'Hôpital's rule to indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$
- Indeterminate forms meant that we needed to do more work
- Some important notes
  - Quotient rule can bug off
  - We can use it multiple times in a row
  - Can ONLY apply to 0/0 and inf/inf

Ex

$\lim_{x \to \infty} \frac{x+1}{x^2-5} = \frac{\infty}{\infty}$ womp womp, it's indeterminate

- Before we went to the hospital, we would probably try to factor something, or barter with the largest powers of x, or some other shenanigans
    - old way has us going to 0
- Using the hospital means we divide the derivatives of top and bottom
- so we end up with $\lim_{x \to \infty} \frac{1}{2x} = 0$

do anotha one
$\lim_{x \to 0} \frac{\sin 0}{0}$
ok that's 0/0, hospital time
$\lim_{x \to 0} \frac{\cos 0}{2} = \frac{1}{1} = 1$

- this works with more tangent line shenanigans
  - y'know, math went from real exact to real "yeah, close enough" real fuckin quick
  $\lim_{x \to 0} \frac{x-2}{x^2+4} = \frac{-2}{4} = \frac{-1}{2}$
  yep that shit worked
  if you were to lop some itals, you'd get $\frac{1}{2x}$, plugging in 0 is $\frac{1}{0}$ which is DNE which is a proooooblem

more examples
$\lim_{x \to 0} \frac{\sec x - 1}{x^2}$ =$\frac{0}{0}$ indeterminate, hospital that shit

$\lim_{x \to 0} LHR \frac{-\sec x \tan x}{2x} = \frac{0}{0}$
$\lim_{x \to 0} = \frac{\sec x * \sec x^2 + \tan x * \sec x \tan x}{2}$
$\lim_{x \to 0} \frac{\sec x^3 + \sec x \tan x^2}{2}$
$\lim_{x \to 0} \frac{1+0}{2} = \frac{1}{2}$

---

yet more examples
$\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$ this is definitely indeterminate

MATH111 - 2023-11-03

#notes #math111 #math #calc

more L'Hôpital's shenanigans

- Today we're looking at other indeterminate forms (0*∞) and $\infty - \infty$

- $0 * 0 = 0, \infty * \infty = \infty, 10 * \infty = \infty, 10 * 0 = 0$
- $0 * \infty = ?????$ is something of a conundrum because we gotta figure out which one wins
- Soooo, we need to rewrite
  - If we have $0 * \infty$

ex

$\lim_{x \to 0^+} x \ln x$
We have ourselves a $0 * \infty$

- that shit is *in*determinate
- $\lim_{x \to 0^+} \frac{x}{\frac{1}{\ln x}}$
- or $\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}$
  - $\frac{\frac{1}{x}}{-x^{-2}}$
  - $\frac{\frac{1}{x}}{\frac{-1}{x^2}}$
  - $\lim_{x \to 0^+} (\frac{1}{x})(\frac{-x^2}{1}) = \lim_{x \to 0^+} (-x)$
  - And that's just 0, hip hip hooray

ex 2:

MATH111 - 2023-11-07

#notes #math111 #math #calc

$\int f(x)dx$

- Means find the family of antiderivatives
- Remember to put a +c on the end, which is to represent that you could add any constant you damn well please, and the derivative wouldn't change
- So $\int \cos x dx = \sin x + c$
- Integrate $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Power Rule (for integrals)

- $x^p dx = \frac{1}{p+1} x^{p+1} + C$
- So if we wanted to do $\int x^5 dx = \frac{1}{5+1} x^{5+1} + C = \frac{x^6}{6} + C$

- Do algebra first! It makes things easier
- $\int x^{-2}dx = \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$

Constant Multiple and Sum Rules

- If you have a constant multiple and you want to integrate, the constant multiple just hangs around! It's that easy!
- If you want to integrate the sum of two things, you can integrate them separately, then stick em together
- $\int 3x^5dx - \int 4 * x^{\frac{-1}{2}} + \int 2dx$
- $3\int x^5dx - 4\int x^{\frac{1}{2}} + 2\int dx$
- $\frac{\frac{3*x^6}{6} - 4*x^{\frac{3}{2}}}{\frac{3}{2}} + 2x + c$

$\frac{3x^2-5x^8}{x^4}$

\frac{3x^{2}}{x^{4}} \frac{-5x^{8}}}{{x^{4}}}}

$\frac{3*1}{x} - x^5 + c$

Integrals of Trigonometric functions

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc X + C$$

$$\int (\frac{\sin\theta}{2} + \frac{3}{\sin^2\theta})d\theta = \int (\frac{1}{2}\sin\theta + 3 * \csc^2\theta)d\theta$$

$$(\frac{1}{\sin^2\theta} - 3\cot\theta + C$$

Integrals of Inverse Trig & Logs n Shit

$$\int e^x dx = e^x + C$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} =$$

$$\int (\frac{1}{2x} + \frac{e^x}{2} + \frac{2}{1+x^2})$$

$$\int \frac{1}{2} * \frac{1}{x} + \frac{1}{2}e^x + 2 * \frac{1}{1+x^2}$$

$$\int \frac{1}{2} * \ln|x| + \frac{1}{2}e^x$$

How do I do the integral of $\int b^x dx$

$$\int b^x dx = \frac{1}{\ln b} * b^x + C$$

NOTE

All of these integral formulas are specific to ***X***
Make sure the x in the integrand matches the x in the dx

You can differentiate just about anything, but there are a lot of things that you just can't integrate

MATH111 - 2023-11-08

#notes #math111 #math #calc

- An equation involving an unknown function and its derivatives is called a differential equation

examples:

ex 1:

- Solve the following initial value problem

$$f'(t) = 7t(t^6 - \frac{1}{7}); f(1) = 2$$

  look the answer is just $f(t) = \frac{7}{8}t^8 - \frac{1}{2}t^2 + \frac{13}{8}$, I did it on the whiteboard and do NOT feel like typing it out here.

---

ex 2

$$x''(t) = t - \cos(t); x'(0) = 2, x(0) = -2$$

- $$x'(t) = \int (t - \cos t)dt$$

$$x'(t) = \frac{t^2}{2} - \sin t + C = x'(t) = 0 = \frac{0}{2} - 0 + 2$$

  $x'(t) = \frac{t^2}{2} - \sin t + 2$

$$x(t) = \frac{t^3}{6} + \cos t + 2t + C$$

$$x(0) = -2 || \frac{0}{6} + 1 + C$$

  $C = -3$

Real Notes

- Develop the idea of area under a curve, and later in the chapter we'll show the connection
  - So if derivative is slope, that means the opposite of the derivative is the area
- So uh, rectangle area isn't *awful*
- So we're going to use rectangles to approximate the area
  - More rectangles give a better approximation
  - Take a limit if we want the exact area (let the number of rectangles $n \to \infty$
- Consider $f(x) = x^2 + 1$ on the interval [1,3]
  - Find the exact area under the curve

- Let's divide that whole [1,3] into 2 intervals of equal width
- So let's slap some rectangles from [1,2] and [2,3], each of length one
    - So now we need the height, and we're going to use the rightmost endpoint
        - So for the one from 1 $\rightarrow$ 2, we're going to. use the endpoint at height 5 (x=2)
        - For the one that ends at 3, we use $f(x)$, which is height 10
        - So our rectangles are 1x5 and 1x10
        - Grand total area of 15
        - R_2 = 15
- So good news, R_4 = 12.75 (which is better)

MATH111 - 2023-11-10

#notes #math111 #math #calc

We're still looking at the function $f(x) = x^2 + 1$

- So what if we use *four* rectangles with right endpoints to get the height?
    - So to get four rectangles on our interval [1,3] we need to split into four equal intervals
    - Go halfsies, then go halfsies again
    - So the width of each rectangle is going to be $\frac{1}{2}$, we use the notation $\Delta x = \frac{1}{2}$
    - $\Delta x = \frac{b-a}{n}$
        - With a being the left hand side of the interval, b being the right hand side, and n being the number of rectangles
    - We're also going to need the height of the rectangles, which means given that we're using right endpoints, we go to the edge on teh right
    - We need f at 3/2, 2, 5/2, and 3
        - 13/4, 5, 29/4, 10
    - R_4 is just the sum of the areas of the four rectangles
- Fun fact, when we're taking the limit, it really does *not* matter how we define the height

---

Limit Definition

- To get the area exactly, fun fact, you can actually use whichever one you please
    - We're just going to do $\lim\limits_{x \to \infty}$ for right sums

- So we know that $\Delta x = \frac{b-a}{n}$
  - And that's the length of each subinterval (or the width of the rectangle, depending on how you want to phrase it)
- x_k = $a + k * \Delta x$
- $\sum_{K=1}$

MATH111 - 2023-11-13

#notes #math111 #math #calc

4. n=6, $\frac{5-2}{6} = 0.5 = \Delta x$
   $f(2) * \Delta x = (2)^2 - 1 * 0.5 = 1.5$
   $f(2.5) * \Delta x = (2.5)^2 - 1 * 0.5 = 2.625$
   $f(3) * \Delta x = (3)^2 - 1 * 0.5 = 4$
   $f(3.5) * \Delta x = (3.5)^2 - 1) * 0.5 = 5.625$
   $f(4) * \Delta x = (4)^2 - 1) * 0.5 = 7.5$
   $f(4.5) * \Delta x = (4.5)^2 - 1) * 0.5 = 9.625$
   Riemann Sum $= 1.5 + 2.625 + 4 + 5.625 + 7.5 + 9.625 = 30.875$

Right Riemann Sum

$$f(2.5) * \Delta x + f(3) * \Delta x + f(3.5) * \Delta x + f(4) * \Delta x + f(4.5) * \Delta x + f(5) * \Delta x$$
$$= 2.625 + 4 + 5.625 + 7.5 + 9.625 + 12 = 41.375$$

$\Delta x = 0.5$
L_4= $\Delta x * f(0) + \Delta x * f(0.5) + \Delta x * f(1) + \Delta x * f(1.5)$
L_4= $10 + 9 + 8 + 7 = 34$

$\Delta x = 0.5$
R_4= $\Delta x * f(0.5) + \Delta x * f(1) + \Delta x * f(1.5) + \Delta x * f(2)$
R_4=$9 + 8 + 7 + 6 = 30$

Break area under curve into triangle + rectangle
Triangle = base of 2, height of $20 - f(6) = 8$
Triangle area $= \frac{1}{2} b * h = 1 * 8 = 8$
Rectangle area below triangle = $2 * f(6) = 24$
Add em up, $24 + 8 = 32$ = exact area under the curve.

$$y'(\theta) = \frac{\sqrt{2}\cos^3\theta + 1}{\cos^2\theta}$$

$$y'(\theta) = \frac{\sqrt{2}\,\cancel{cos^3}\theta}{\cancel{\cos^2\theta}} + \frac{1}{\cos^2\theta} = \sqrt{2}\cos\theta + \sec^2\theta$$

$$\int y'(\theta) = \sqrt{2}\sin\theta + \tan\theta + C$$

$$= -12e^{\frac{-t}{6}}$$

$$\int \frac{x}{2x^2} + \frac{4x^5}{2x^2} dx = \int \frac{1}{2x} + 2x^3 dx = \int \frac{1}{2} * \frac{1}{x} + 2x^3 dx$$

$$= \frac{1}{2}\ln x + \frac{2}{4}x^4 + C = \frac{\ln|x| + x^4}{2} + C$$

$$\int 4 * \frac{1}{1+x^2} + 5 * \sec^2 x * dx = 4\tan^{-1} x + 5\tan x + C$$

MATH111 - 2023-11-27

#notes #math111 #math #calc

I really should've wrote down Fundamental Theorem of Calculus

- State the Fundamental Theorem
- Part 2
- $\int_{lower}^{upper}$

- Consider $f(x) = x^2 + 1$ on the interval [1,3]
- Find the exact area under the curve, $\int_1^3 (x^2+1)dx$ by evaluating the $\lim_{n\to\infty} R_n$
    - It's going to be a summation, $f(a + k * \Delta x) * \Delta x$
    - a is the lower, b is the upper limit
    - $\Delta x = \frac{b-a}{n}$ = $\frac{2}{n}$
    - $f(1 + k * \frac{2}{n})$
    - So now we plug that in to the original function
    - $(1 + k + \frac{2}{n}^2) + 1$
    - So this is the height of the k^{th} rectangle
    - $\sum[1 + k * \frac{2}{n}]^2 + 1) * \frac{2}{n} *$
    - Alright now we need to use the known summation formula to kill the summation
    - Foiling, $(1 + \frac{2k*2}{n} + k^2 * \frac{4}{n^2} + 1)[\frac{2}{n}]$
    - $\frac{4}{n} + k * \frac{8}{n^2} + \frac{8k^2}{n^3}$
    - We can split the summation up to be by term, also, summing a constant is just constant * n
    - So now we kill the summation, it comes out to $\lim_{x\to\infty}[\frac{4}{n} * n + \frac{8}{n^2} * n\frac{n+1}{2} + \frac{8}{n^3} *)\frac{n(n+1(2n+1)}{6}]$
    - $\lim_{x\to\infty}[4 + 4 * \frac{n^2+n}{n^2} + \frac{4}{3} * \frac{2n^3+3n^2+n}{n^3}]$
    - Continue simplifying the shit out of this, $\lim_{x\to\infty} 8\frac{+8}{3} = \frac{32}{3}$
- That shit was awful.

- Fundamental theorem, with that ugly-ass limit, could just be done by doing the antiderivative at the two ends
- $\int_1^3 (x^2+1)dx = \frac{x^3}{3} + x$ eval from 1 to 3
- $\left(\frac{3^3}{3} + 3\right) - (\frac{1}{3} + 1)$
- $9 + 3\frac{-1}{3} - 1 = \frac{32}{3}$

it's top - bottom, rember that

$sin^{-1}x$ we're doing that shit from 0 to (1/2)

- $\sin^{-1}(1/2) - \sin^{-}1(0)$
- $\frac{\pi}{6} - 0 = \frac{\pi}{6}$

$\frac{1}{2} * \frac{1}{4}e^{4x}$ evaluate from 0 to ln(2)
$\frac{1}{8}e^{4\ln 2} - \frac{1}{8}e^{0}$

MATH111 - 2023-11-29

#notes #math111 #math #calc

u sub day 2

- use u sub to evaluate $\int \frac{\cos\frac{3}{x}}{x^2} dx$
  - We're gonna say that $u = \frac{3}{x}$
    - $u = 3x^{-1}$
    - $du = -3x^{-2}dx$
    - Hey, so we don't see that, but what we doooo see is $-3 * \frac{dx}{x^2}$
    - If you're off by a constant, that's all okie dokie, you can just substitude
    - $\int \cos{(u)} * \frac{du}{-3}$
    - yeah, I can do that
    - $\frac{-1}{3} * \sin{(u)} + C$
    - $\frac{-1}{3}\sin{(\frac{3}{x})} + C$

- ok actual u-sub day two
- If you can't find u, probably go check the denominator
- Remember to do ln() shenanigans, it's probably wild
- Plug the endpoints of a definite integral
  - $\frac{1}{3}du$

MATH111 - 2023-12-04

#notes #math111 #math #calc

- Alright, fundamental theorem says that you do the antiderivative and evaluate at the two ends
- $F'(x) = F(b) - F(a)$
  - if you integrate a rate, you get a change in amout
- Example:
  - The growth rate of cats is given by the function $P'(t) = 1 + 3\sin(2\pi t)$

- Initially there is one cat
- So the answer is P(0) and the change in number of cats is the definite integral from 0 to 5

$$\int_0^5 (1 + 3\sin(2\pi t))dt$$

$$u = 2\pi * t$$

du = 2pi *dt* = $\frac{du}{2\pi} = dt$
t = 0 $u = 2\pi * 0 = 0$
$t = 5u = 2\pi * 5 = 10\pi$

$$\int_0^{10\pi} (1 + 3\sin(u))\frac{du}{2\pi} = \frac{1}{2\pi}\int_0^{10\pi} (u - 3\cos u)$$

evaluate from 0 to 10pi

$$\frac{1}{2\pi}(10\pi - 3\cos(10\pi)) - \frac{1}{2\pi}(0 - 3\cos(0))$$

$$\frac{1}{2\pi}(10\pi - 3 + 3) = 5 + 1 = 6 \text{ cats!}$$