Extending Defeasible Reasoning beyond Rational Closure Literature Review

Alec Lang
lngale007@myuct.ac.za
University of Cape Town
Cape Town, South Africa

ABSTRACT

Formal logic is used in artificial intelligence to represent and reason upon knowledge of the world. Classical reasoning is monotonic, meaning that the addition of new information will not lead to the withdrawal of previously drawn conclusions, even if they contradict each other, leading to an unsatisfiable knowledge base. Defeasible reasoning aims to fix this, with it being non monotonic. This means that new opposing information can result in previously drawn conclusions to be withdrawn from the knowledge base. Rational closure, the first and most conservative form of defeasible entailment relations introduced, is reviewed, with it providing a starting point from which will be expanded in the project. The goal is then to propose and implement algorithms for a predefined class of defeasible reasoning approaches that extend rational closure.

CCS CONCEPTS

• **Theory of computation** → **Automated reasoning**; • **Computing methodologies** → **Nonmonotonic, Rational closure**; **Propositional logic**.

KEYWORDS

Artificial Intelligence, Knowledge Representation and Reasoning, Defeasible reasoning, Logic-based reasoning

1 INTRODUCTION

Knowledge Representation and Reasoning (KRR) is a sub-field of artificial intelligence where knowledge can be represented in a structured way by using formal logic. The represented knowledge can then be reasoned on, whereby it is manipulated using a set of rules to infer new information[2]. The goal of KRR is to enable AI systems to make well informed judgements and solve complex problems, in turn allowing them to reason about the world as if it were a human. The aim of this literature review is to provide an overview into rational closure, but before that can take place we first need to familiarise ourselves with some of the fundamentals.

We will first look into propositional logic including its grammar and semantics, and then touch on the flaws of propositional reasoning. We will then investigate defeasible reasoning as well as the KLM approach to defeasible reasoning. Rational closure will then be explored along with the algorithms to compute it. Finally we can conclude in how we may possibly modify rational closure to formulate a better approach to defeasible entailment.

2 PROPOSITIONAL LOGIC

Propositional logic is a logical system that is used to reason about, and model knowledge of the world. The logic used to represent knowledge in this paper will be propositional logic. A proposition is a statement that can be attributed a truth value. An example of such could be "*Cape Town is a capital of South Africa*" (This proposition is true because Cape Town is a capital of South Africa, however not the only one!). In propositional logic, complex statements can be built by connecting basic statements with *logical operators*, with the truth of the new statement relying on the truth of the statements used in its construction[1]. Propositional logic was chosen to be used in this project as it is simple to grasp and work with, as well as being the underlying logic used by Kraus, Lehmann and Magidor to create the KLM framework, which will be looked at later in the review.

2.1 Grammar

The language of propositional logic is made up of variables known as *propositional atoms*, referred to as atoms from here on out. An atom is a statement, or a representation of a statement, eg; *bird*, *ostrich* and *robin* are all atoms. The set of all atoms is denoted as $\mathcal{P}$, and is finite. Atoms are often represented with meta variables, denoted with small Latin alphabet letters $\mathcal{P} = \{b, o, r, \ldots\}$ with the letter o representing ostrich in this case[8]. Each atom in the set is attributed a truth value of either true (T) or false (F). These atoms can be combined together using a set of boolean operators to create propositional formulas, denoted using lower case Greek letters $\{\alpha, \beta, \gamma, \ldots\}$, with the set of all formulas, ie; the propositional language, denoted by $\mathcal{L}$ and we refer to a finite set of formulas as a knowledge base, often denoted by $\mathcal{K}$.

Name	Type	Symbol
Negation	Unary	$\neg$
Conjunction	Binary	$\land$
Disjunction	Binary	$\lor$
Implication	Binary	$\rightarrow$
Equivalence	Binary	$\leftrightarrow$

Figure 1: Propositional logic boolean operators

Negation is a unary operator that requires a single operand, while all other operators are binary and act on two operands. The precedence of these operators is such that negation has the highest priority, with each operator thereafter in the table having lower and lower precedence[7]. Using these 5 boolean operators we are able to define all the other formulas.

We can recursively define the set of all propositional formulas $\mathcal{L}$ from our atoms and operators; for every $p \in \mathcal{P}, p \in \mathcal{L}$, and if $\alpha, \beta \in \mathcal{L}$, then $\{\neg\alpha, \alpha \wedge \beta, \alpha \vee \beta, \alpha \rightarrow \beta, \alpha \leftrightarrow \beta\}$
In addition, The constants $\top$ and $\bot$ are also in $\mathcal{L}$: $\top$, known as "top", represents a tautology (a statement that is always true), while $\bot$, known as "bottom" represents the opposite (a statement that is always false).

2.2 Semantics

2.2.1 ***Interpretations.*** The semantics of propositional logic involves the concept of *validity*, which refers to the property of a statement where the truth of the premises guarantees the truth of the conclusion. In propositional logic, a statement is valid if and only if its conclusion follows logically from its premises. The assignment of truth values to atoms is referred to as interpretations or worlds. Mathematically, this can be expressed as $u : \mathcal{P} \rightarrow \{T, F\}$, where $\mathcal{P}$ is the set of propositional atoms and $\{T, F\}$ represents the set of truth values. This can be shown by using truth tables.$\neg p$ can be expressed as $\overline{p}$, and this convention will be employed to indicate the valuations. The set of all possible valuations a formula can take is denoted with $\mathcal{U}$. When $\mathcal{P} = \{p, q\}, \mathcal{U}$ is expressed as $\{pq, p\overline{q}, \overline{p}q, \overline{pq}\}$ as this indicates all conceivable arrangements of truth values for the atoms in $\mathcal{P}$.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Figure 2: A truth table for p and q

2.2.2 ***Satisfaction.*** If a valuation assigns a truth value of true to a propositional atom, then the atom is said to be satisfied in that valuation. *Satisfaction* is denoted with the symbol $\Vdash$, such that for some valuation u, if it is the case that for some $p \in \mathcal{P}$, $u(p) = \text{T}$, then $u \Vdash p$, and if $u(p) = \text{F}$, then $u \nVdash p$[1]. This can be extended to any propositional formulae; $\alpha, \beta \in \mathcal{L}$, such that a valuation u can be said to satisfy any formula in $\mathcal{L}$ using the following criteria:

- $u \Vdash \neg\alpha$ if and only if $u \nVdash \alpha$
- $u \Vdash \alpha \wedge \beta$ if and only if $u \Vdash \alpha$ or $u \Vdash \beta$
- $u \Vdash \alpha \vee \beta$ if and only if either $u \Vdash \alpha$ or $u \Vdash \beta$
- $u \Vdash \alpha \rightarrow \beta$ if and only if $u \nVdash \alpha$ or $u \Vdash \beta$
- $u \Vdash \alpha \leftrightarrow \beta$ if and only if $u(\alpha) = u(\beta)$
- $u \Vdash \top$ for every $u \in \mathcal{U}$
- $u \nVdash \bot$ for every $u \in \mathcal{U}$

Let α be an element of the language $\mathcal{L}$, and define $\hat{\alpha}$ as the subset of $\mathcal{U}$ consisting of all valuations that satisfy α, $\hat{\alpha} = \{u \in \mathcal{U}$ $u \Vdash \alpha\}$. A valuation $u \in \hat{\alpha}$ is referred to as a model of α. Therefore, $\hat{\alpha}$ is a subset of $\mathcal{U}$ that includes only the valuations satisfying α. A formula α is considered a tautology if for all valuations $u \in \mathcal{U}$, $u \Vdash \alpha$. On the other hand, α is satisfiable if there exists at least one valuation $u \in \mathcal{U}$ such that $u \Vdash \alpha$. If no such valuation exists, α is unsatisfiable[1].

2.2.3 ***Logical Consequence.*** A logical consequence, known as an *entailment*, is an outcome that arises from a statement or set of statements. Entailment is denoted by the symbol $\models$. If we have two formulas α and β, both belonging to $\mathcal{L}$, we say that β is a logical consequence of α, denoted by $\alpha \models \beta$, when for every $u \in \mathcal{U}$ such that u satisfies α (i.e., $u \Vdash \alpha$), u also satisfies β (i.e., $u \Vdash \beta$). In other words, $\alpha \models \beta$ holds if and only if $\hat{\alpha} \subseteq \hat{\beta}$ ($\hat{\alpha}$ is a subset of $\hat{\beta}$).

Classical entailment can then be defined as a logical consequence from a set of statements: If we have a set of formulas $\mathcal{K}$ and a formula α, we say that α is entailed by $\mathcal{K}$, denoted by $\mathcal{K} \models \alpha$, when for every $u \in \mathcal{U}$ such that u satisfies $\mathcal{K}$ (i.e., $u \Vdash \mathcal{K}$), u also satisfies α (i.e., $u \Vdash \alpha$). In other words, $\mathcal{K}$ entails any α if every model of $\mathcal{K}$ is also a model of α[1].

2.2.4 ***Object and Meta Levels.*** In propositional logic, the *object-level* is comprised of the propositional formulas that represent knowledge in the language L, this being the boolean connectives $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \top, \bot\}$, as well as the propositional atoms. In contrast, the *meta-level* is anything that operates above the object-level. This includes entailment $\models$, satisfaction $\Vdash$ and the knowledge base $\mathcal{K}$. The object-level represents knowledge, with object-level formulas being evaluated using truth tables, where its meaning is defined in all cases. The meta-level deals with the statements about the knowledge, rather than the knowledge itself, with the truth of statements not being set in all cases[8].

2.2.5 ***Flaws with Propositional reasoning.*** Propositional reasoning follows the principle of *monotonicity*, which implies that the addition of new information might help draw new conclusions, but it will never lead to the retraction of a previously drawn conclusion[14]. However, this also means that if the new knowledge includes instances that contradict the existing rules, the entire knowledge base becomes unsatisfiable.

The following is an example of this. Say we have a knowledge base:

$$\mathcal{K} = \{o \rightarrow b, b \rightarrow f, o \rightarrow \neg f\}$$

Where o, b, f represent ostrich, bird and fly respectively, and the knowledge base states that ostriches are birds, birds fly and ostriches do not fly. Propositional reasoning will conclude from the statements $o \rightarrow b$ and $b \rightarrow f$ that because an ostrich is a bird and birds fly, then an ostrich also flies. But $o \rightarrow \neg f$ contradicts this conclusion, and because previously drawn conclusions cannot be withdrawn, we come to the contradiction that ostrich's both can and cannot fly. The only way to resolve this issue is to conclude that ostriches do not exist. Defeasible reasoning is a way to fix this issue.

3 DEFEASIBLE REASONING

Defeasible reasoning is *non-monotonic*, meaning conclusions are drawn based on incomplete or uncertain information, where the conclusions can be subjected to revision in the occurrence of new conflicting information[9, 13]. This allows for systems to reason in a way which is closer to humans. This literature review will focus on the KLM style of defeasible reasoning[10].

3.1 KLM framework for defeasible reasoning

In the KLM[10, 12] approach to defeasible reasoning, Kraus, Lehmann and Magidor extended propositional logic to include the defeasible implication symbol $\mid\!\sim$. Take the statement $\alpha \mid\!\sim \beta$. This equates to mean "α typically implies β", so if α is true, then β is probable to also be true. Using this approach lets look at our previous example again. If we swap implication symbol $\rightarrow$ with the typically implies symbol $\mid\!\sim$ in the statements $b \rightarrow f$ and $o \rightarrow \neg f$, we now have a knowledge base:

$$\mathcal{K} = \{o \rightarrow b, b \mid\!\sim f, o \mid\!\sim \neg f\}$$

From this we get that an ostrich is a bird, birds typically fly and ostrich's typically do not fly. Using this proposed symbol we are now able to reason with uncertain information. Unlike in propositional logic, in defeasible reasoning there is no set method to determine defeasible entailment (denoted $\mid\!\approx$). KLM[10, 12] proposed the following properties that had to be satisfied by a defeasible entailment relation, with any such form of defeasible entailment that satisfies these properties being known as $LM - Rational$:

(Reflexivity) $\quad \mathcal{K} \mid\!\approx \alpha \mid\!\sim \alpha$

(Left logical equivalence) $\quad \dfrac{\alpha \equiv \beta,\ \mathcal{K} \mid\!\approx \alpha \mid\!\sim \gamma}{\mathcal{K} \mid\!\approx \beta \mid\!\sim \gamma}$

(Right weakening) $\quad \dfrac{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \beta,\ \beta \models \gamma}{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \gamma}$

(And) $\quad \dfrac{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \beta,\ \mathcal{K} \mid\!\approx \alpha \mid\!\sim \gamma}{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \beta \wedge \gamma}$

(Or) $\quad \dfrac{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \gamma,\ \mathcal{K} \mid\!\approx \beta \mid\!\sim \gamma}{\mathcal{K} \mid\!\approx \alpha \vee \beta \mid\!\sim \gamma}$

(Cautious monotonicity) $\quad \dfrac{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \beta,\ \mathcal{K} \mid\!\approx \alpha \mid\!\sim \gamma}{\mathcal{K} \mid\!\approx \alpha \wedge \beta \mid\!\sim \gamma}$

(Rational monotonicity) $\quad \dfrac{\mathcal{K} \mid\!\approx \alpha \mid\!\sim \gamma,\ \mathcal{K} \not\mid\!\approx \alpha \mid\!\sim \neg\beta}{\mathcal{K} \mid\!\approx \alpha \wedge \beta \mid\!\sim \gamma}$

3.1.1 ***Preferential Interpretations.***

The semantics that were defined for $\mid\!\sim$ by KLM[10] and Lehmann and Magidor[12] is based off of the preferential semantics defined previously by Shoham[15–17]. Preferential semantics imposes an ordering over valuations. An ordering is such that a preferred valuation would be seen as more typical. This preference ordering is established by introducing a concept of "states", where each state is associated with a classical valuation. It's important to note that states are not the same as valuations, as there can be multiple states that correspond to the same valuation. Because of this, there could be an infinite number of states within a single interpretation. We will now take a look at a subset of preferential interpretations, named *ranked interpretations*.

3.1.2 ***Ranked Interpretations.***

The semantics of KLM-style rational defeasible implications use structures referred to as *ranked interpretations*[12]. These are structures that assign a rank to every possible valuation of a set of atoms $\mathcal{P}$, in order of typicality. These structures have ranks of typicality such that rank 0 has the most typical worlds, with the level of atypicality increasing all the way to some rank ∞, which would include the impossible worlds. No rank may be empty in a ranked interpretation. A more formal definition from Kaliski[8]: A ranked interpretation $\mathscr{R}$ is a function from $\mathscr{U}$ to $\mathscr{N} \cup \{\infty\}$, satisfying the following convexity property: for every $i \in \mathscr{N}$, if there exists a $u \in \mathscr{U}$ such that $\mathscr{R}(u) = i$, then there must be a $v \in \mathscr{U}$ such that $\mathscr{R}(v) = j$ with $0 \leq j < i$. Given a ranked interpretation $\mathscr{R}$ and $\alpha, \beta \in \mathcal{L}$, $\mathscr{R}$ satisfies the conditional $\alpha \mid\!\sim \beta$ (denoted $\mathscr{R} \Vdash \alpha \mid\!\sim \beta$) if in the most typical rank, where α holds, β also holds. $\mathscr{R}$ is said to be a ranked model of $\alpha \mid\!\sim \beta$[4].

To give a visual representation of a ranked interpretation structure we can show the following example: We have atoms $\mathcal{P} = \{o, b, f\}$ and a knowledge base $\mathcal{K} = \{o \rightarrow b, b \mid\!\sim f, o \mid\!\sim \neg f\}$. The ranked interpretation would look as follows[4].

∞	$o\overline{bf}\ \overline{ob}f$
2	obf
1	$\bar{o}b\bar{f}ob\bar{f}$
0	$\overline{obf}\ o\bar{b}f\ \bar{o}bf$

Figure 3: A ranked interpretation for $\mathcal{P} = \{o, b, f\}$

In this ranked interpretation the worlds in rank 0 ($\overline{obf}$, $o\bar{b}f$, $\bar{o}bf$) are the most typical, with the atypicality increasing as the ranks increase. $o\overline{bf}$ and $\overline{ob}f = \infty$ and are said to be impossible. Classical propositional statements can be expressed as defeasible implications (DIs): $\mathscr{R} \Vdash \alpha$ if and only if $\mathscr{R} \Vdash \neg\alpha \mid\!\sim \bot$[3, 4].

3.2 Rational Closure

Rational closure was the first non-monotonic entailment relation defined by Lehmann and Magidor[12] and is a form of defeasible entailment that satisfies LM-Rationality. It is a pattern of *prototypical reasoning*[11], and is the most conservative of all the defeasible entailment relations in the KLM framework. This means that members of a world only inherit the properties of that world if they are the most typical. To demonstrate this we can look at our previous example where we have a knowledge base with "ostriches are birds", "birds fly" and "ostriches do not fly". Prototypical reasoning would conclude that since ostriches do not fly, they are atypical birds and would not inherit any typical attributes of a typical bird since they do not follow the convention of a typical bird. Two approaches to computing rational closure will be defined. The first one, *minimal ranked entailment*, defines rational closure semantically using a unique ranked model of $\mathcal{K}$[8]. The second defines an algorithm to compute the rational closure of a defeasible knowledge base $\mathcal{K}$, first put forward by Lehmann and Magidor[12]

3.2.1 ***Minimal Ranked Entailment.***

Minimal ranked entailment involves defining a partial ordering of all ranked models of some knowledge base $\mathcal{K}$, with this denoted by $\preceq_{\mathcal{K}}$. A definition to impose an ordering $\preceq_{\mathcal{K}}$ on the typicality of the ranked interpretations in a knowledge base $\mathcal{K}$ was given by Casini et al.[4] and is as follows: $\mathcal{R}1 \preceq_{\mathcal{K}} \mathscr{R}2$ if for every $v \in \mathcal{U}$, $\mathcal{R}1(v) \leq \mathcal{R}2(v)$. Further Giordano et al. [6] showed that the partially ordered set $\langle \mathcal{R}, \preceq_{\mathcal{K}} \rangle$ has a minimal element, $\mathcal{R}^{\mathcal{K}}_{RC}$ for $\mathcal{K}$, with minimal interpretations being "pushed down"[8] as much as is possible. Minimal ranked entailment $\models\!\approx$ can be defined given a defeasible knowledge base $\mathcal{K}$ and the minimal ranked interpretation satisfying $\mathcal{K}$, $\mathcal{R}^{\mathcal{K}}_{RC}$: for any defeasible implication $\alpha \mid\sim \beta$, $\mathcal{K} \models\!\approx \alpha \mid\sim \beta$ iff $\mathcal{R}^{\mathcal{K}}_{RC} \Vdash \alpha \mid\sim \beta$.

3.2.2 ***Algorithmic approach.***

Before the algorithm can be looked at the *materialisation* of the knowledge base must first be defined: The material counterpart of a defeasible implication $\alpha \mid\sim \beta$ is the propositional formula $\alpha \rightarrow \beta$. Given a defeasible knowledge base $\mathcal{K}$, the material counterpart, denoted $\overrightarrow{\mathcal{K}}$, is the set of material counterparts, $\alpha \rightarrow \beta$, for every defeasible implication $\alpha \mid\sim \beta \in \mathcal{K}$[8]. A propositional statement α is then said to be exceptional with regards to a knowledge base $\mathcal{K}$ if and only if $\mathcal{K} \models \neg\alpha$ (i.e., α is false in all the most typical interpretations in every ranked model of $\mathcal{K}$)[4, 8].

3.2.3 ***BaseRank.***

The first step in calculating rational closure is the *BaseRank* algorithm. BaseRank is an algorithm that separates formulas in a knowledge base into ranks, based on how general they are, with the most general statements being in the lowest rank. Each propositional formula in $\mathcal{K}$ is mapped to the set of natural numbers and infinity: $\{0, 1, 2, 3, ...\} \cup \infty$ (Most typical in rank 0 and so forth). The input to the algorithm is a defeasible knowledge base with the output being a tuple of sets of classical implications that are the material counterparts to the defeasible implications in $\mathcal{K}$, corresponding to the sequence $\mathcal{E}^{\mathcal{K}}_n$ of exceptional subsets of $\mathcal{K}$[8].

An outline of the BaseRank algorithm:

- $\mathcal{K}$ is first separated into into classical components $\mathcal{K}_C$ and defeasible components $\mathcal{K}_D$.
- The material counterpart of the defeasible components $\overrightarrow{\mathcal{K}_D}$ is placed in $\mathcal{E}^{\mathcal{K}}_0$.
- For each propositional formula $\alpha \rightarrow \beta$ in $\mathcal{E}^{\mathcal{K}}_0$, check whether $\mathcal{E}^{\mathcal{K}}_0 \cup \mathcal{K}_C \Vdash \neg\alpha$.
- If this holds, then the antecedent (α) is *exceptional* with regards to the knowledge base, and all formulas in $\mathcal{E}^{\mathcal{K}}_0$ with α are moved to $\mathcal{E}^{\mathcal{K}}_1$.
- The most general rank $\mathcal{R}_0$ is assigned to the formulas $\mathcal{E}^{\mathcal{K}}_0$.
- This is repeated for the next subset/s $\mathcal{E}^{\mathcal{K}}_n$ until a subset is reached in which there are no formulas that need to be moved to a less general level.
- Finally the classical statements $\mathcal{K}_C$ are assigned to the least general rank $\mathcal{R}_\infty$.

3.2.4 ***RationalClosure.***

The second step in calculating the rational closure is the *RationalClosure* algorithm. RationalClosure determines if a statement is defeasibly entailed by the knowledge base. It takes as input a knowledge base $\mathcal{K}$ and a defeasible implication $\alpha \mid\sim \beta$, and returns true if and only if the implication $\alpha \mid\sim \beta$ is in the rational closure of $\mathcal{K}$ ($\mathcal{K} \models_{RC} \alpha \mid\sim \beta$)[5].

An outline of the RationalClosure algorithm:

- For a defeasible implication $\alpha \mid\sim \beta$, RationalClosure checks if the negation of the antecedent is entailed by the materialisation of the knowledge base, $\overrightarrow{\mathcal{K}} \models \neg\alpha$.
- If this holds then the antecedent α is an exceptional formula, and ranks of the statements are continually removed from the ranked knowledge base, starting with the most general rank ($\mathcal{R}_0$), until the algorithm finds a rank in which the antecedent is no longer exceptional. The knowledge base with the removed ranks is given as $\mathcal{K}'$
- If this does not hold then the antecedent α is not exceptional. The algorithm then checks if the materialisation of the query ($\alpha \rightarrow \beta$) is entailed by the statements in the current rank and outputs the result.
- If the knowledge base is empty then $\mathcal{K}$ does not entail the implication, $\mathcal{K} \not\models\!\approx \alpha \mid\sim \beta$

The following is an example of how the BaseRank and RationalClosure algorithm work.
Given the knowledge base $\mathcal{K} = \{o \rightarrow b, b \mid\sim f, o \mid\sim \neg f, b \mid\sim w\}$, BaseRank would work as follows:

- Separates $\mathcal{K}$ classical components $\mathcal{K}_C = \{o \rightarrow b\}$.
- Defeasible components $\mathcal{K}_D = \{b \mid\sim f, o \mid\sim \neg f, b \mid\sim w\}$ are materialised giving $\mathcal{K}_D = \{b \rightarrow f, o \rightarrow \neg f, b \rightarrow w\}$.
- $\mathcal{E}^{\mathcal{K}}_0 = \{b \rightarrow f, o \rightarrow \neg f, b \rightarrow w\}$.
- $\mathcal{E}^{\mathcal{K}}_1 = \{o \rightarrow \neg f\}$ since o is found to be an exceptional antecedent.
- $\mathcal{E}^{\mathcal{K}}_2$ is empty so we stop.
- $\mathcal{R}_\infty = \{o \rightarrow b\}$.

- $\mathcal{R}_1 = \{o \rightarrow \neg f\}$.
- $\mathcal{R}_0 = \{b \rightarrow f, b \rightarrow w\}$.

This leaves us with the following ranking of statements in the knowledge base:

0	$b \rightarrow f, b \rightarrow w$
1	$o \rightarrow \neg f$
∞	$o \rightarrow b$

Figure 4: Ranked statements

Now RationalClosure is used to determine if the query $o \mid\!\sim \neg f$ is entailed by a subset of $\mathcal{K}$:

- First we check if the negation of the antecedent in the query, $\neg o$, is entailed by $\overrightarrow{\mathcal{K}}$.
- It holds, therefore $\neg o$ is exceptional with regards to $\overrightarrow{\mathcal{K}}$ and the most general rank $\mathcal{R}_0$ is removed.

1	$o \rightarrow \neg f$
∞	$o \rightarrow b$

Figure 5: Updated Ranked statements

- We check again whether $\overrightarrow{\mathcal{K}} \models \neg o$ and we find that it does not hold. So there exists a world in $\overrightarrow{\mathcal{K}}$ that satisfies o.
- We now check if $\overrightarrow{\mathcal{K}} \models o \mid\!\sim \neg f$.
- $\overrightarrow{\mathcal{K}}$ does entail $o \mid\!\sim \neg f$. Therefore $\mathcal{K} \mid\!\approx o \mid\!\sim \neg f$ and True is returned from the algorithm.

3.3 Drawbacks of Rational Closure

The major drawback of rational closure is that it is too conservative, in being that it is a form of prototypical reasoning. Again what is meant by this is that that members of a world only inherit the properties of that world if they are the most typical, and the consequence of this is that rational closure will often throw out too much information that it could have used otherwise. There are other forms of defeasible entailment that builds on rational closure to help improve its flaws, one being *lexicographic closure*, introduced by Lehmann[11]. Lexicographic closure is a form of presumptive reasoning that distinguishes itself from prototypical reasoning in that it assumes that properties of a world apply to all of the members of said world, unless there is contradictory knowledge to suggest otherwise. In going forward with this project, improving on rational closure and implementing our own form of defeasible entailment, we can look to what has been done before as inspiration. Of such improvements that could be made, one could impose a further refinement on the worlds in a ranked interpretation, on a level by level basis.

4 CONCLUSIONS

In this review we established how propositional logic is used to model knowledge about the world. The concept of satisfaction was outlined as well as how new information can be implied from a knowledge base through entailment. We found how the monotonic nature of classical propositional reasoning gave results that are inadequate when dealing with new information being added to its knowledge base, thus making it unfit for concluding accurate implications in the real world, where there can often be a typicality to statements. We explored how the KLM approach for defeasible reasoning can make amends for the errors of propositional entailment, due to its non-monotonicity. The defeasible entailment relation that was then focused on in particular was rational closure. We then gave an overview into rational closure and the algorithms used to calculate it. Rational closure was shown to be an improvement on propositional entailment, but we concluded that it still had shortcomings in the fact that it throws out to much information that could otherwise be used to reason with.

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