# Multi Final Review

watch for complete square ↗

## Ch 13

$|a\cdot(b\times c)|$ = volume of paralleliped

$|a\times b|$ = area of parallelogram

Spherical: $(\rho,\theta,\varphi)\rightarrow r=\rho\sin\varphi$

plane: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$\vec{a}\cdot\vec{b}=ab\cos\theta$ $\qquad a\times b=ab\sin\theta$

Direction Cosines: $\frac{\vec{a}\cdot\hat{i}}{a\,i}=\frac{a_1}{a}=\cos\alpha$

Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ $\qquad$ Cone $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

Elliptic Paraboloid $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ $\qquad$ Hyperboloid(1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Hyperbolic Paraboloid $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ $\qquad$ Hyperboloid(2) $=-1$

Scalar projection: $\left(\frac{\vec{a}\cdot\vec{b}}{|a|}\right)\left(\frac{\vec{a}}{|a|}\right)$

$\vec{n}\cdot\vec{r}=\vec{n}\cdot\vec{r}_0\rightarrow$ in a plane

for skew, d is ∥ $(V_1\times V_2)$

don't intersect, not ∥

## Ch 14

normal plane: N+B

osculating plane: T+N

$L=\int_a^b|r'(t)|$

$\hat{T}=\frac{\vec{v}}{v}$

$\hat{N}=\vec{a}_\perp/a_\perp$ or $\hat{B}\times\hat{T}=\frac{T'(t)}{|T'(t)|}$

✓ $\vec{a}_\perp=\vec{a}-\vec{a}_\parallel$

✓ $\vec{a}_\parallel=\frac{\vec{v}\cdot\vec{a}}{v}\left(\frac{\vec{v}}{v}\right)$

$\hat{B}=\hat{T}\times\hat{N}$ or $\frac{\vec{v}\times\vec{a}}{|\vec{v}\times\vec{a}|}$

$r'(t)=\lim_{h\to 0}\frac{r(t+h)-r(t)}{h}$

smooth if $r'(t)\neq 0$

$s=\int_a^t|r'(u)|\,du$ $\qquad \frac{ds}{dt}=|r'(t)|$

$K=\frac{|\vec{v}\times\vec{a}|}{v^3}$ or $\frac{|\vec{a}_\perp|}{v^2}$ = 1/radius of curvature $=\left|\frac{d\hat{T}}{ds}\right|=\frac{|\hat{T}'(t)|}{|\vec{r}'(t)|}$

→ plane: $\frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$

$\vec{a}=v'\hat{T}+Kv^2\hat{N}$

$a_T=\frac{\vec{v}\cdot\vec{a}}{v}\hat{T}$

$a_N=\frac{|\vec{v}\times\vec{a}|}{|v|}\hat{N}$

## Ch 15

$f_x(a,b)=\lim_{h\to 0}\frac{f(a+h,b)-f(a,b)}{h}$

Finding limits: if it's not going to work, find 2 paths

$dz=f_x\,dx+f_y\,dy\rightarrow z-z_0=f_x(x-x_0)+f_y(y-y_0)\rightarrow 0=f_x(x)+f_y(y)-f_z(z)$

Critical point: $\nabla f=0$ or f isn't differentiable

1st you need to find what to minimize/maximize ← 2nd Derivative Test: $H=\begin{bmatrix}f_{xx}&f_{xy}\\f_{yx}&f_{yy}\end{bmatrix}$ if det $H>0$ + $f_{xx}>0\rightarrow$ min

det $H>0$ + $f_{xx}<0\rightarrow$ max

det $H<0\rightarrow$ saddle

det $H=0\rightarrow$ FAIL

$\frac{dy}{dx}=\frac{-F_x}{F_y}$

Tangent to level surface: normal $=\nabla F(x_0,y_0,z_0)$

$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$

Lagrange: $\nabla f(x,y,z)=\lambda\nabla g(x,y,z)$ $\qquad g(x,y,z)=K$

solve these

## Ch 16

$\langle x \rangle = \dfrac{\iint_D x\rho \, dA}{\iint_D \rho \, dA}$ moment about $x$-axis is $\langle y \rangle$

$\iint_D \rho \, dA \rightarrow = m$

$I = \int r^2 dm$ radius of gyration: $mR^2 = I$

$dS = |d\vec{r}_x \times d\vec{r}_y| = \sqrt{1 + f_x^2 + f_y^2}\, dx\, dy$

$= \sqrt{1 + f_r^2 + \frac{1}{r^2} f_\theta^2}\, r\, dr\, d\theta$

$dV = dx\,dy\,dz = r\,dr\,d\theta\,dz = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$ $\rightarrow u = -\cos\varphi$

inverse: $\frac{1}{\det}$ | negate, swap | Joint Density $P(X)$ is between $A + B$ and $P(Y)$ is between $C + d$

$= \int_a^b \int_c^d f(x,y)\, dx\, dy$

average time 10 min $\rightarrow \frac{1}{10} e^{-x/10}$ * swapping integrals

Jacobians: $dx\,dy = \left| \dfrac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$ or $\left| \dfrac{\partial(x,y)}{\partial(u,v)} \right| = \left| \dfrac{\partial(u,v)}{\partial(x,y)} \right|^{-1}$

$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$

## Ch 17

Fundamental Thm of Line Integrals: $\int_C \nabla f = f(P_b) - f(P_a)$

Green's Thm: $\oint_C \vec{F} \cdot d\vec{r} = \iint_D \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \, dA$ (counterclockwise)

$A = \oint_C x\, dy$ curl $= \nabla \times \vec{F}$

if $d\vec{r}$ is normal, find it Stokes Thm: $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\, \vec{F} \cdot d\vec{S}$ div $= \nabla \cdot \vec{F}$

flux Divergence Thm: $\oiint_{SS} \vec{F} \cdot d\vec{S} = \iiint_V \text{div}\vec{F}\, dV$ curl$(\nabla f) = 0$

$V = \oiint_S x\, \hat{i} \cdot d\vec{S}$ $\nabla \cdot (\nabla \times \vec{F}) = 0$

$\int_C f(x,y)\, ds \rightarrow \dfrac{ds}{dt} = \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}$

you need to parameterize $x + y$

$\int_C f(x,y)\, d\vec{x} = \int_a^b f(x(t), y(t))\, x'(t)\, dt$

$\int_C \vec{F} \cdot d\vec{r} = \vec{F} \cdot \vec{r}'(t)\, dt$

iff conservative, $P_y = Q_x$

To find $f$ from $\vec{F}$, integrate each

Surface Area =

solenoidal: $\nabla \cdot \vec{G} = 0$