HOCHSCHULE MITTWEIDA
University of Applied Sciences

SIMULATION AND VISUALIZATION

Authors
Md Kamruzzaman
Bishwajit Karmaker
Aleda Islam Mou

Infectious Disease Modelling

A COVID-19 epidemic model with latency period using SEIRU and SEIRUδ Models[13]

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Faculty of **Applied Computer Sciences and Bio-sciences**

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Author:
Md Kamruzzaman
Bishwajit Karmaker
Aleda Islam Mou

Study Programme:
Applied Mathematics for Network and Data Sciences

Seminar Group:
MA-19w2-M

First Referee:
Schneider Kristan

Second Referee:
Gideon Ngwa

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Abstract

To be capable of transmitting the COVID-19 infection, the infected person needs or takes a certain amount of time to be capable of spreading the infection to other person. This time duration is known as the exposed or latency period and we have developed two types of differential equations to illustrate this period. The first model deals with the infected persons in the exposed class before the transmission is possible and the second model incorporates a time delay in infected person before the transmission is possible. We have applied both models to the COVID-19 epidemic in China and estimated the epidemiological parameters in the models, such as transmission rate and basic reproduction number using the data of reported cases. Through this procedure we will evaluate the role of the exposed or latency period in the dynamics of a COVID-19 epidemic. In this paper, we will examine the latency period of COVID-19 infection, that is, the period of time in which newly infected individuals are asymptotic and non-infectious.

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1 Introduction

Corona virus disease 2019 (COVID-19) is an infectious disease emerging in China in December 2019 which has widely spread throughout the whole world and WHO declared it to be a public health emergency of international concern on 30 January,2020. On 11 March, 2020, the confirmed cases of infection was more than 118,000 cases reported globally in 114 countries with having the 90 percent of cases in just four countries(Two of these in China and the republic of Korea)and WHO declared it to be a pandemic, the first one caused by a Corona virus. As of now, on 13 August 2020, there are 20,162,474 of confirmed cases where 737,417 confirmed death worldwide among 216 countries for this Corona virus[1]. Human-to-human transmission can occur via droplets or contaminated surfaces and materials[2] and elderly and people with chronic diseases are considered as a high risk population[3]. It is estimated that about 80 percent of infections only lead to mild or moderate symptoms[4,5,6] and most commonly shown symptoms are cough fever and rhinitis[7]. In an effort to decelerate the spread of this infection, drastic measures have been taken by many countries to minimize social contact even by closing the schools and forbidding social gatherings and in so many places, s total system was introduced to get shut down. We will examine the latency period of COVID-19 infection, that is, the period of time in which newly infected individuals are asymptotic and non-infectious. We will develop two mathematical models to study the impact of the latency period. One is a O.D.E.(Ordinary Differential Equation) model with an exposed class of infected individuals, who are not yet infectious. And, the other one is a D.D.E(Delay Differential Equation) model, with a time delay in newly infected individuals, before they become infectious. The D.D.E. model can be derived from a continuous age of infection model, which can be reduced to a system of D.D.E. Asymptotic infectious cases are not usually reported to medical authorities, and reported infectious cases are typically only a fraction of the total number of the symptotic infectious individuals. In this paper, we will evaluate the number of asymptotic infectious cases and unreported infectious cases, as well as the number of reported infectious cases, for the COVID epidemic in Italy. One of our objectives is to understand how these measures, such as isolation, quarantine, and public closings, reduce the final size of the epidemic. We examine how the latency period, tied contact tracing and to a 14-day medical observation or quarantine period for exposed persons, mitigates the final size of the epidemic.

The illustration of latency period given below:

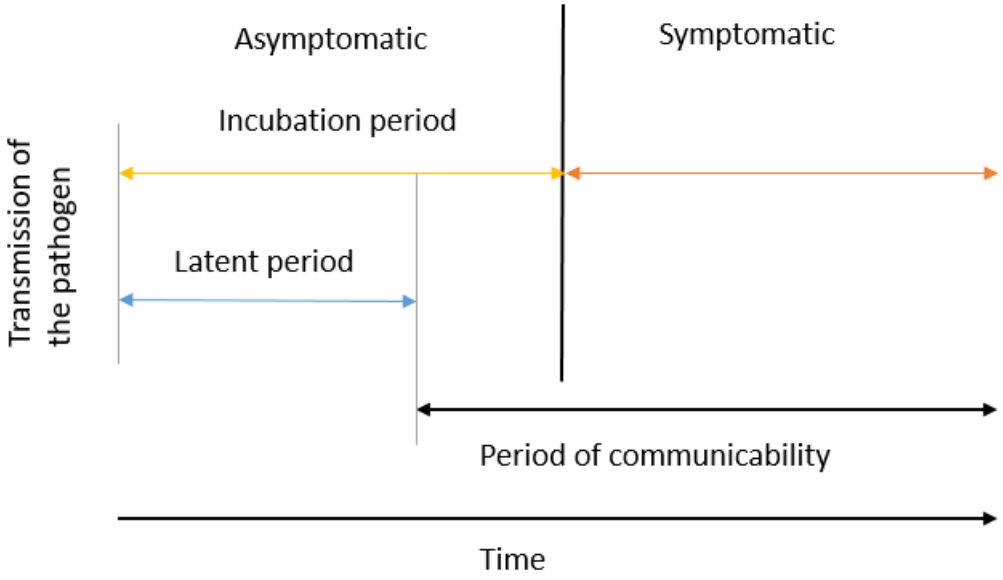

Figure 1.1: Key time periods of COVID-19 infection.

The latent or exposed period : before symptoms and transmissibility,

The incubation period : before symptoms appear,

The symptomatic period : May overlay the asymptomatic period.

2 Methodology of the Models

2.1 SEIRU Model

This model has a partition E in the structure of ODE that correlates with exposed infected individuals who are not yet infectious. We assign this model SEIRU model:

$$\begin{aligned} S'(t) &= -\tau(t)S(t)[I(t)+U(t) \\ E'(t) &= \tau(t)S(t)[I(t)+U(t)-\alpha E(t) \\ I'(t) &= \alpha E(t)-vI(t) \\ R'(t) &= v_1 I(t)-\eta R(t) \\ U'(t) &= v_2 I(t)-\eta U(t) \end{aligned} \tag{2.1}$$

t_0 : starting date of the epidemic with t$\leq t_0$ is time in days
S(t) : number of individuals open for infection at time t
E(t) : number of asymptomatic noninfectious individuals at time t
I(t) : number of asymptomatic but infectious individuals at time t
R(t) : number of reported symptomatic infectious individuals at time t
U(t) : number of unreported symptomatic infectious individuals at time t

This system is amplified by initial data.

$$S(t_0)=S_0>0, E(t_0)=E_0>0, I(t_0)=I(t_0)>0, U(t_0)=U_0>0, R(t_0)=R_0=0 \tag{2.2}$$

The time of exposure changes exponentially , the flow of exposed class is defined as -αE(t), the average value of the exposure time can be 6h, 12h, 1day, 2 days, 3 days, etc... which is defined by 1/α.

In the model,

I(t) equation : Asymptomatic infectious class.
R(t) equation : Reported symptomatic infectious individuals, symptomatic infectious individuals are break down to R(t) equation.
U(t) equation : Unreported symptomatic infectious individuals.
vI(t) : The flow of individuals depart from the class I.

We refer f as fraction of reported and (1-f) as unreported,
thus v_1 = fv and v_2 = (1-f)v.

Time dependent variable τ(t) is the transmission rate. Cumulative number of reported cases grows more or less exponentially at the beginning of the epidemic. At that time . is a constant value τ_0. After (date) to deal with the epidemic, government measures

isolation, quarantine, public closings, that impacted directly to the transmission of new cases but useful measures were complex, we use τ as time dependent exponentially decreasing transmission rate to deal with these effects after initial exponential increasing phase. We derive the formula for the exponential decreasing phase by fitting procedure to the data.

$$\tau(t) = \tau_0, \quad 0 \leq t \leq N$$
$$\tau(t) = \tau_0 \, exp\,(-\eta \times (t-N)),\ N < t. \tag{2.3}$$

Day N : day when public measures take effect
η : rate at which they take effect

Parameters and initial conditions are given in table 3.1. A schematic diagram is given below:

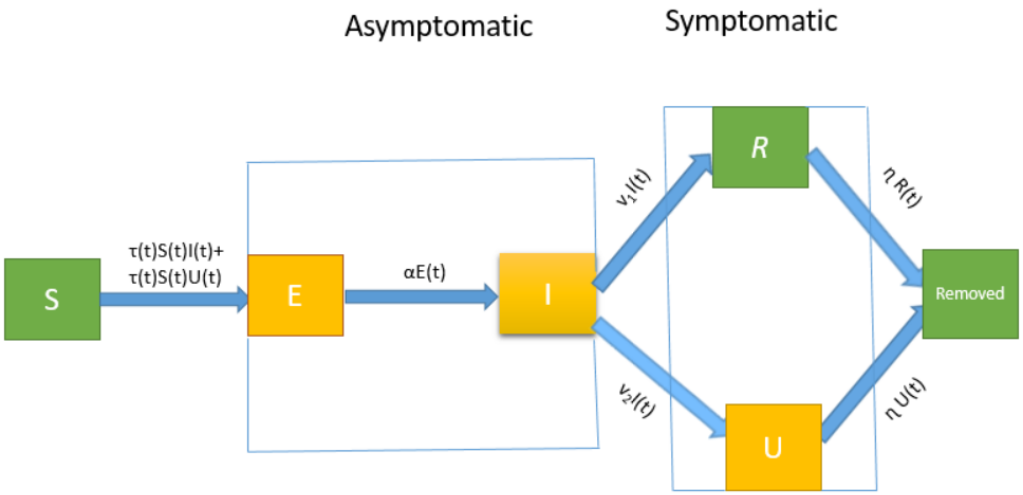

Figure 2.1: Flow chart for the model SEIRU

2.2 SEIRUδ Model

This model contains a time delay δ in the I(t) equation in the structure of DDE which has the latency period. This model is defined as SEIRUδ model:

$$\begin{aligned} S'(t) &= -\tau(t)S(t)[I(t)+U(t)] \\ I'(t) &= \tau(t-\delta)S(t-\delta)[I(t-\delta)+U(t-\delta)] - vI(t) \\ R'(t) &= v_1 I(t) - \eta R(t) \\ U'(t) &= v_2 I(t) - \eta U(t) \end{aligned} \tag{2.4}$$

This system is amplified by initial data.

$S(t_0+\theta) = S_0(\theta) > 0, I(t_0+\theta) = I_0(\theta) > 0, U(t_0+\theta) = U_0(\theta) > 0, \forall \theta \in [-\delta, 0],$ *and* $R(t_0) = 0$ (2.5)

Length of exposure is constant in this model and equal to δ.

The exposed class, E(t) = $\int_{t-\delta}^{t} \tau(\sigma)S(\sigma)[I(\sigma)+U(\sigma)]d\sigma$ (2.6)

alternative the can use the differential equation,

E'(t) = $\tau(t)S(t)[I(t)+U(t)] - \tau(t-\delta)S(t-\delta)[I(t-\delta)+U(t-\delta)]$ (2.7)

Parameters and initial conditions are given in table 2.1. A schematic diagram of the model is given below:

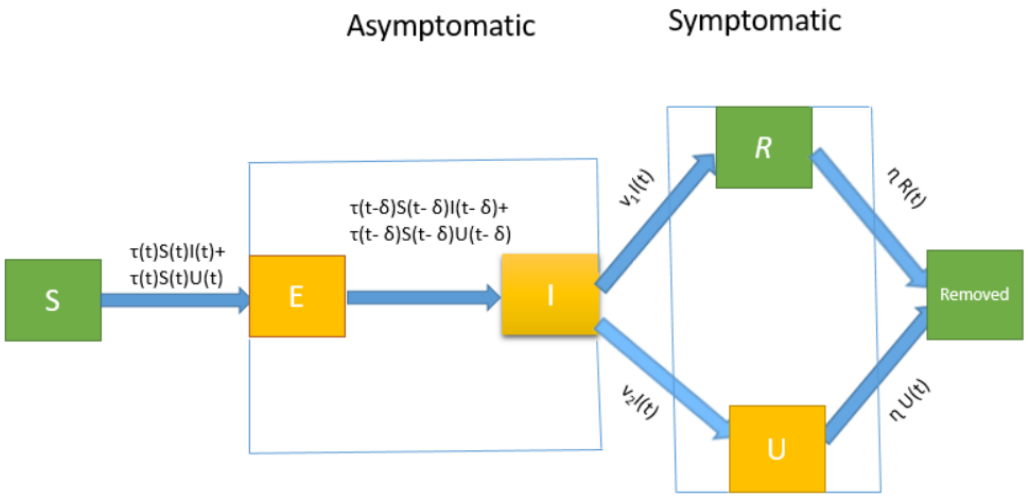

Figure 2.2: Flow chart for the model SEIRUδ

3 Parameters and initial conditions estimation

t_0	Time at which epidemic started	fitted
S_0	Number of susceptible at time t_0	fixed
E_0	Number of asymptomatic and noninfectious at time t_0	fitted
I_0	Number of asymptomatic but infectious at time t_0	fitted
U_0	Number of unreported symptomatic infectious at time t_0	fitted
.	Transmission rate	fitted
N	First day of the public interventions	fitted
μ	Intensity of the public interventions	fitted
$1/\alpha$	average duration of the exposed noninfectious period	fitted
$1/v$	Average time during which asymptotic infections are asymptomatic	fixed
f	Fraction of asymptotic infectious that become reported symptomatic infectious	fixed
$v_1 = fv$	Rate at which asymptomatic infectious become reported Symptomatic	fitted
$v_2 = (1-f)v$	Rate at which asymptomatic infectious become unreported Symptomatic	fitted
$1/\eta$	Average time symptomatic infectious have symptoms	fixed
τ	Transmission rate	fitted
δ	Latency period	fitted

Table 3.1: Parameters and initial conditions of the models

The parameters $\tau, v, v_1, v_2, \eta, \alpha, \delta$ that are used in the models along with the initial conditions $S(t_0), E(t_0), I(t_0), U(t_0)$ and starting time t_0 aren't fixed. Our goal is to calculate these by using reported symptomatic infectious cases data with respect to a certain period of time.
To find out the number of unreported asymptomatic infectious cases, we consider that the cumulative reported symptomatic infectious cases at time t contain a constant fraction f of the total number of symptomatic infectious cases at time t. Then the cumulative number of reported symptomatic infectious cases at time t,

$$CR(t) = v_1 \int_{t_0}^{t} I(s)ds \tag{3.1}$$

We assume the following formula for CR(t) in terms of early exponentially increasing phase:

$$CR(t) = x_1 exp(x_2 t) - x_3 \tag{3.2}$$

We assume that the reported cases data are exponentially increasing which takes place before any public intervention. Therefore we assume, $\tau(t) = \tau_0$ for both SEIRU and SEIRUδ models.

We use the method shown in section 3.1 to evaluate using the reported cases data. By using the following formula we can estimate the starting time t_0 for the models:

$CR(t_0) = 0 \Longleftrightarrow x_1 exp(x_2 t_0) - x_3 = 0 \Longrightarrow t_0 = \frac{1}{x_2}(ln(x_3) - ln(x_1))$.

We fix S_0 = 60,461,826 which is the total population of Italy. We assume that the change in S(t) is small and transmission rate $\tau(t) \equiv \tau_0$ is constant during exponentially growing phase.
We fix v, η, f, α. and calculate τ_0 from x_1, x_2, x_3 for each of the models SEIRU and SEIRUδ.

3.1 Parameters and initial conditions estimation for SEIRU model from the number of reported cases

We fix f, v, η, α

Step-1: as $f, \alpha, \eta, and v$ are fixed, we know that,
v_1 = fv and v_2 = (1-f) v

Step-2: by using equation (3.1) and (3.2) we obtain

$$CR'(t) = v_1 I(t) \Longleftrightarrow x_1 x_2 exp(x_2 t) = v_1 I(t) \tag{3.3}$$

and

$$\frac{exp(x_2 t)}{exp(x_2 t_0)} = \frac{I(t)}{I(t_0)}$$

therefore

$$I(t) = I_0 exp(x_2(t - t_0)) \tag{3.4}$$

Moreover by using (3.4) at t = t_0

$$I_0 = \frac{x_1 x_2 exp(x_2 t_0)}{fv} = \frac{x_3 x_2}{fv} \tag{3.5}$$

Step-3: In order to evaluate the parameters of the model we replace S(t) by $S_0 = 60.48e6$ in the right-hand side of (2.1) (which is equivalent to neglecting the variation of susceptibles due to the epidemic, which is consistent with the fact that t$\longrightarrow$CR(t) grows exponentially). Therefore, it remains to estimate τ_0, E_0, and U_0 in the following system:

$$\begin{aligned} E'(t) &= \tau(t_0) S_0(t)[I(t) + U(t)] - \alpha E(t) \\ I'(t) &= \alpha E(t) - vI(t) \\ U'(t) &= v_2 I(t) - \eta U(t) \end{aligned} \tag{3.6}$$

By using the second equation we obtain

$$E(t) = \frac{1}{\alpha}[I'(t) + vI(t)]$$

and therefore by using (3.4) we must have

$$I(t) = I_0 exp(x_2(t-t_0)) \; and \; E(t) = E_0 exp(x_2(t-t_0))$$

Then, by using the first equation we obtain

$$U(t) = \frac{1}{\tau S_0}[E'(t) + \alpha E(t)] - I(t) \; and \; then \; U(t) = U_0 exp(x_2(t-t_0)$$

By substituting these expression into (3.6), we obtain

$$\begin{aligned} x_2 E_0 &= \tau_0 S_0[I_0 + U_0] - \alpha E_0 \\ x_2 I_0 &= \alpha E_0 - v I_0 \\ x_2 U_0 &= v_2 I_0 - \eta U_0 \end{aligned} \tag{3.7}$$

Remark 3.1:The value x_2 is become the dominant eigenvalue of the linearized equation (3.7) we fix τ_0 in that way, and (E_0, I_0, U_0) is the positive eigen vector associated to this dominant eigenvalue x_2. Thus, we apply implicity the Perron-Frobenius theorem. Moreover, the exponentially growing solution (E(t), I(t), U(t)) that we consider (which is starting very close to (0, 0, 0)) follows the direction of the positive eigen vector associated with the dominant eigenvalue x_2

From the second and the third equation of (3.7) we obtain

$$E_0 = \frac{x_2 + v}{\alpha} I_0, \; U_0 = \frac{v_2}{x_2 + \eta} I_0$$

and by substituting these expressions into the first equation of (3.7) we obtain

$$\tau_0 = \frac{(x_2 + \alpha)E_0}{S_0[I_0 + U_0]} = \frac{(x_2 + v)(x_2 + \alpha)(x_2 + \eta)}{\alpha S_0(x_2 + \eta + v_2)} \tag{3.8}$$

We fix the fraction f = 0.1 for the reported symptomatic infectious cases which concludes that 10% of infectious cases are reported.

We assume,
The average time, 1/v is 5 days or 7 days when the patients are asymptomatic infectious.
The average time, $1/\eta$ is 7 days when a patient is symptomatic infectious.
We obtain,

$$v_1 = fv = 0.1/5\ (or\ 0.1/7)\ and\ v_2 = (1-f)v = 0.9/5\ (or\ 0.9/7) \quad (3.9)$$

We obtain basic reproduction number R_0 from section 6.1,

$$R_0 = \frac{(x_2+v)(x_2+\alpha)(x_2+\eta)}{\alpha v(x_2+\eta+v_2)}(1+\frac{(1-f)v}{\eta})$$

3.2 Parameters and initial conditions estimation for SEIRUδ model from the number of reported cases

Step 1: We have

$$v_1 = fv\ and\ v_2 = (1-f)v$$

Step 2: By using equation (3.2) we obtain

$$CR'(t) = v_1 I(t) \Longleftrightarrow x_1 x_2 exp(x_2 t) = v_1 I(t) \quad (3.10)$$

and

$$\frac{exp(x_2 t)}{exp(x_2 t_0)} = \frac{It}{It_0}$$

and therefore

$$I(t) = I(t_0) exp(x_2(t-t_0)) \quad (3.11)$$

Moreover, by using (3.10) at t = t_0,

$$I(t_0) = \frac{x_1 x_2 exp(x_2 t_0)}{fv} = \frac{x_3 x_2}{fv}, U(t_0) = \frac{v_2}{x_2+\eta} I_0 \quad (3.12)$$

Step 3: To evaluate to the parameters of model SEIRUδ, here we replace S(t) by S_0 = 60.48e6$\times 10^9$ in the right-hand side of (2.4) (which is equivalent to neglecting the variation of susceptibles due to the epidemic, and is consistent with the fact that t$\longrightarrow$CR(t) grows exponentially). Therefore it remains to estimate τ_0 and η in the following system:

I'(t) = $\tau S_0[I(t-\delta)+U(t-\delta)] - vI(t)$
$U'(t) = v_2 I(t) - \eta U(t)$ (3.13)

By using the first equation we obtain

$$U(t) = \frac{1}{\tau S_0}[I'(t) + vI(t)] - I(t)$$

and therefore by using (3.11) we must have

$$I(t) = I(t_0)exp(X_2(t-t_0)) and U(t) = U(t_0)exp(x_2(t-t_0))$$

so by substituting these expressions into (3.13) we obtain

$$\begin{aligned} x_2 I(t_0) &= \tau S_0[I(t_0) + U(0) \\ x_2 U(t_0) &= v_2 I(t_0) - \eta U(t_0) \end{aligned} \tag{3.14}$$

Remark 3.2: Here we fix τ_0 in such a way that the value x_2 becomes the dominant eigenvalue of the linearized equation (3.14) and (I$(t_0), U(t_0)$) is the positive eigen vector associated to this dominant eigenvalue x_2. Thus, we apply implicitly the Perron-Frobenius theorem. Moreover the exponentially growing solution (I$(t), U(t)$) that we consider (which is starting very close to (0,0)) follows the direction of the positive eigen vector associated with the dominant eigenvalue x_2.
By dividing the first equation of (3.14) by $I(t_0)$ we obtain

$$x_2 = \tau S_0[1 + \frac{U(t_0)}{I(t_0)}]e^{-x_2\delta} - v$$

and hence

$$\frac{U(t_0)}{I(t_0)} = \frac{(x_2 + v)}{\tau S_0} e^{x_2\delta} - 1 \tag{3.15}$$

By using the second equation of (3.14) we obtain

$$\frac{U(t_0)}{I(t_0)} = \frac{(v_2)}{\eta + x_2} \tag{3.16}$$

By using (3.15) and (3.16) we obtain

$$\tau = \frac{(x_2 + v)}{S_0} e^{-x_2\delta} \frac{\eta + x_2}{v_2 + \eta + x_2} \tag{3.17}$$

By using (3.12) we compute

$$U(t_0) = \frac{v_2}{\eta + x_2} I(t_0) = \frac{(1-f)v}{\eta + x_2} I(t_0) \tag{3.18}$$

The values of f, v, and η are equivalent to SEIRU model. From section (6.1) we obtain,

$$S(t_0+\theta) = S_0(\theta) = S_0,\ \theta \in [-\delta, 0] \tag{3.19}$$

$$I(t_0+\theta) = I_0(\theta) = \frac{x_3 x_2}{fv} e^{x_2\theta},\ \theta \in [-\delta, 0] \tag{3.20}$$

$$U(t_0+\theta) = U_0(\theta) = \frac{(1-f)v}{\eta + x_2} I_0(\theta),\ \theta \in [-\delta, 0] \tag{3.21}$$

$$\tau_0 = \frac{x_2 + v}{S_0} \frac{\eta + x_2}{v_2 + \eta + x_2} e^{x_2\delta} \tag{3.22}$$

From section (6.2) we obtain basic reproduction number R_0 for the model SEIRUδ

$$R_0 = \frac{\tau_0 S_0}{v}(1 + \frac{v_2}{\eta}) = \frac{x_2 + v}{v} \frac{\eta + x_2}{v_2 + \eta + x_2} e^{x_2\delta}(1 + \frac{(1-f)v}{\eta})$$

4 SEIRU and SEIRUδ models with data

Cummulative reported case data from January 28 to August 12 reported for Italy from[12].
The data corresponds to cumulative reported cases confirmed by testing.

Reported data : 0,0,0,3,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,14,62,53,97,93,78,250,238,
240,561,347,466,587,769,778,1247,1492,1797,977,2313,2651,2547,3497,2823,4000,3526,
4207,5322,5986,6557,5560,4789,5249,5210,6153,5959,5974,5217,4050,4053,4782,4668,
4585,4805,4316,3599,3039,3836,4204,3951,4694,4092,3153,2972,2667,3786,3493,3491,
3047,2256,2729,3370,2646,3021,2357,2324,1739,2091,2086,1872,1965,1900,1389,1221,
1075,1444,1401,1327,1083,802,744,1402,888,992,789,875,675,451,813,665,642,652,669,
531,300,397,584,593,516,416,355,178,318,321,177,518,270,197,280,283,202,379,163,346,
338,301,210,328,331,148,264,224,221,113,577,296,255,175,174,126,142,182,201,223,235,
192,208,137,193,214,276,188,234,169,114,162,230,231,249,218,190,12,280,306,252,274,254,
168,202,288,382,379,295,238,159,190,384,401,552,347,463,259,412.

Cumulative Reported data :
3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,17,79,132,229,322,400,650,888,1128,1689,2036,2502,
3089,3858,4636,5883,7375,9172,10149,12462,15113,17660,21157,23980,27980,31506,35713,
41035,47021,53578,59138,63927,69176,74386,80539,86498,92472,97689,101739,105792,
110574,115242,119827,124632,128948,132547,135586,139422,143626,147577,152271,156363,
159516,162488,165155,168941,172434,175925,178972,181228,183957,187327,189973,192994,
195351,197675,199414,201505,203591,205463,207428,209328,210717,211938,213013,214457,
215858,217185,218268,219070,219814,221216,222104,223096,223885,224760,225435,225886,
226699,227364,228006,228658,229327,229558,23015,230555,231139,231732,232248,232664,
233019,233197,233515,233836,234013,234531,234801,234998,235278,235561,235763,236142,
236305,236651,236989,237290,237500, 237828,238159,238011,238275,238499,238720,238833,
239410,239706,239961,240136, 240310,240436,240578,240760,240961,241184,241419,241611,
241819,241956,242149, 242363,242639,242827,243061,243230,243344,243506,24336,243967,
244216,244434,244624, 244752,245032,245338,245590,245864,246118,246286,246488,246776,
247158,247537,247832,248070,248229, 248419,248803,249204,249756,250103,250566,250825,
251237.

We use,
$x_1 = 0.2254,\ x_2 = 0.3762,\ x_3 = 1, \eta = 1/7,\ N = 45,\ S_0 = 60.48e6,\ f = 0.1,$
$v = 1/5[fig(4.1,4.3)]\ and\ 1/7[fig(4.2,4.4)]$
$\mu = 0.1276(a), 0.142(b), 0.166(c), 0.25(d)[fig4.1];\ 0.1539(a), 0.169(b), 0.198(c), 0.3(d)[fig4.2]$
$\mu = 0.1273(a), 0.1432(b), 0.177(c), 0.373(d)[fig4.3];\ 0.1515(a), 0.17(b), 0.2093(c), 0.454(d)[fig4.4]$
$\alpha = 1/6(a), 1/12(b), 1/24(c), 1/72(d)[Fig(4.1,4.2)], \delta = 1/4(a), 1/2(b), 1(c), 3(d)[Fig(4.3,4.4)].$

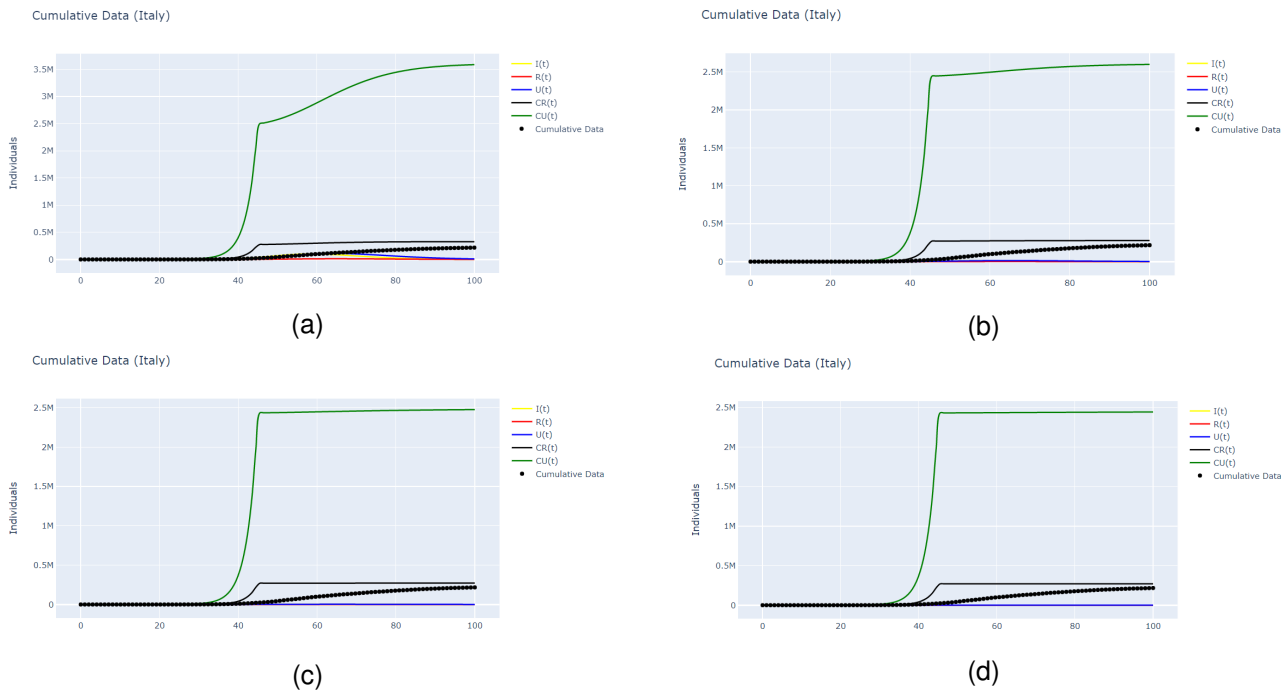

Figure 4.1: Cumulative epidemic curve(I) for SEIRU model

t→CR(t) (black solid line) : Reported Cumulative symptomatic infectious individuals.
t→CU(t) (green solid line) : Unreported Cumulative symptomatic infectious individuals.
t→R(t) (red solid line) : Reported symptomatic infectious individuals.
t→U(t) (blue solid line) : Unreported symptomatic infectious individuals.
t→I(t) (yellow solid line) : Asymptomatic infectious individuals.

The red dots are the data of the reported cumulative confirmed cases for Italy.

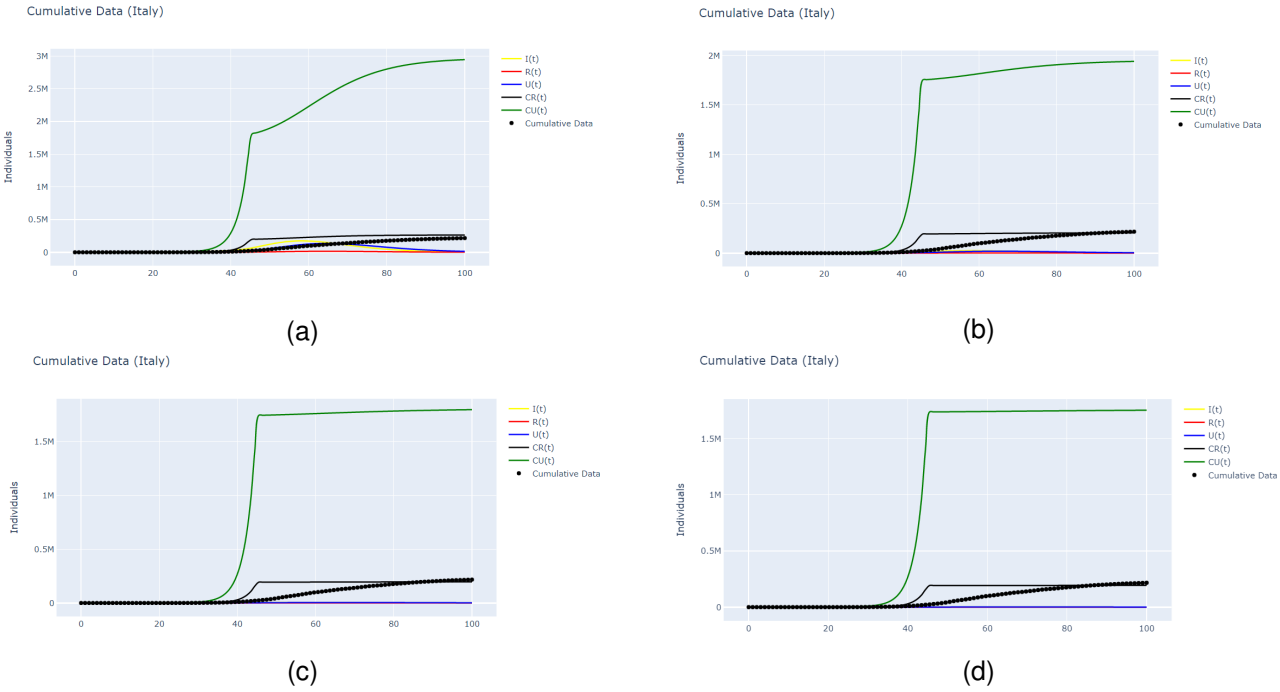

Figure 4.2: Cumulative epidemic curve(II) for SEIRU model

Figure	Final size Reported	Final size Unreported	Turning point of R(t), U(t)	Turning point of I(t)
Fig. 4.1(a)	326.8K	3.5M	day 64.6	day 65.6
Fig. 4.1(b)	278.2K	2.5M	day 64.6	day 68.6
Fig. 4.1(c)	272K	2.4M	day 64.6	NA
Fig. 4.1(d)	217.1K	2.4M	day 63.6	NA
Fig. 4.2(a)	316.1K	2.9M	day 64.6	day 58.5
Fig. 4.2(b)	217.1K	1.9M	day 65.6	day 58.5
Fig. 4.2(c)	196.2K	1.7M	day 65.6	day 58.5
Fig. 4.2(d)	193.8	1.7M	day 65.6	day 62.6

Table 4.1: Predicted turning point and final size of the ODE model SEIRU

Figure	Final size Reported	Final size Unreported	Turning point of R(t), U(t)	Turning point of I(t)
Fig. 4.3(a)	269.6K	2.4M	day 45.4	NA
Fig. 4.3(b)	269.6K	2.4M	day 45.4	NA
Fig. 4.3(c)	269.6K	2.4M	day 45.4	NA
Fig. 4.3(d)	269.6K	2.4M	day 45.4	NA
Fig. 4.4(a)	192.6K	1.7M	day 43.4	NA
Fig. 4.4(b)	192.6K	1.7M	day 43.4	NA
Fig. 4.4(c)	192.6K	1.7M	day 43.4	NA
Fig. 4.4(d)	192.6K	1.7M	day 43.4	NA

Table 4.2: Predicted turning point and final size of the ODE model SEIRU

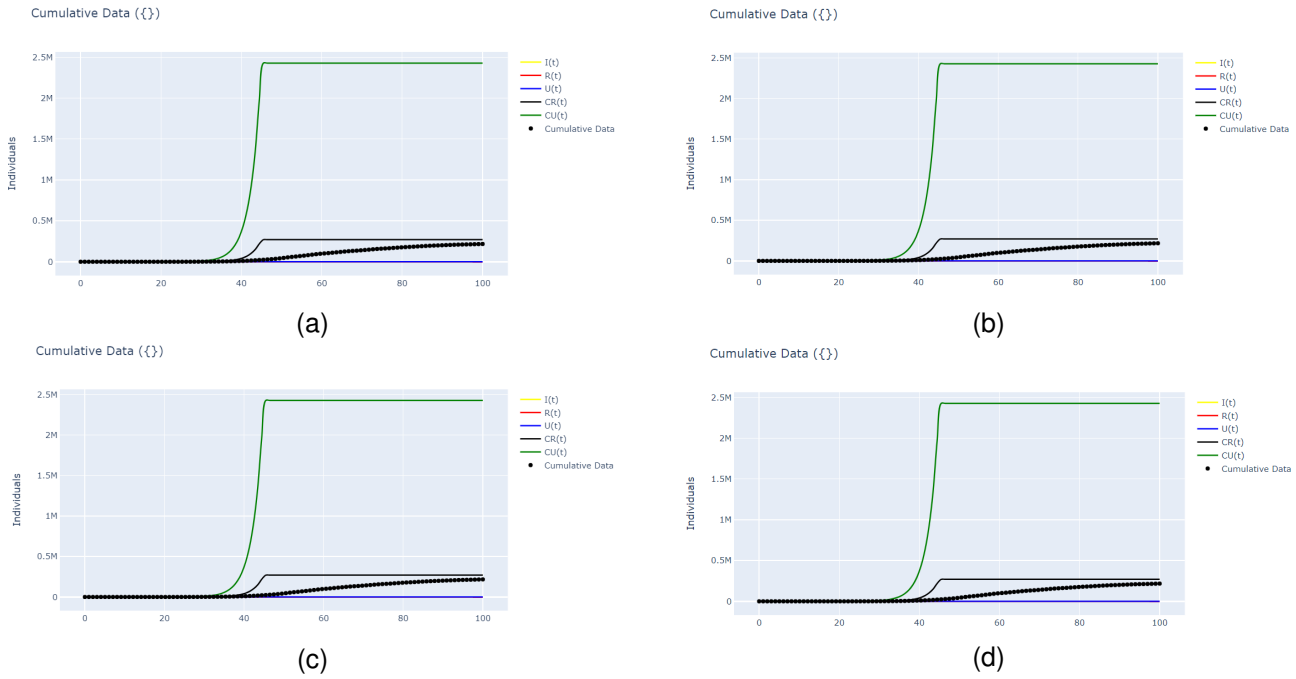

Figure 4.3: Cumulative epidemic curve(I) for SEIRUδ model

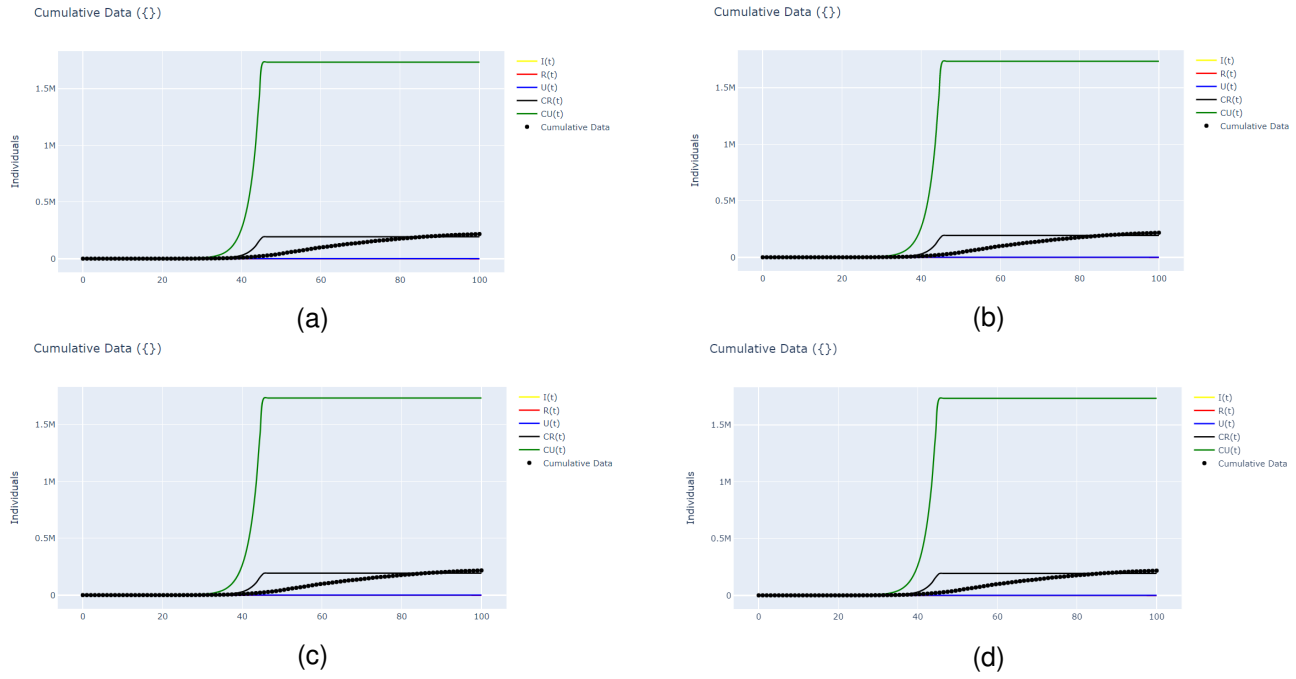

Figure 4.4: Cumulative epidemic curve(II) for SEIRUδ model

5 Predictions and Discussion

SEIRU MODEL: The value of μ is selected is such a way that the simulations align with the cumulative reported case data. In this way we can predict the future values of the epidemic from early cumulative reported case data. We see that simulations of Fig 4.1 with v = 1/5 and Fig 4.2 with v = 1/7 are almost same.

SEIRUδ MODEL: The value of μ is selected is such a way that the simulations align with the cumulative reported case data. In this way we can predict the future values of the epidemic from early cumulative reported case data. We see that simulations of Fig 4.1 with v = 1/5, the simulation of δ = 1/4 is almost same as $\delta = 1/2$ but simulations for δ=1 and δ=3 do not agree with the data. Thus we can conclude that δ can not be greater than 5 days. In Fig 4.4 we used v = 1/7 , simulations for δ=1/4,1/2,1 are almost the same but for $\delta = 3$ the simulation does not agree with the data thus δ can not be greater than 7 days.

We get to know the following important facts:

- Why public restrictions are important and what are the effects on the epidemic
- To interprete the number of reported cases, both reported and unreported cases are important.
- Asymptomatic infectious cases are also important in the desease transmission.

From table 4.1 we conclude that Fig 4.2(d) is best fit for SEIRU model which means v = 1/7 is better than v = 1/5.
From table 4.2 we conclude that Fig 4.2(c) is best fit for SEIRU model which means v = 1/7 is better than v = 1/5.
So, for both models v = 1/7 is better.

6 Calculation of basic reproduction number R_0 of the models

6.1 R_0 calculation for SEIRU

Here we apply result in Diekmann, Heesterbeek and Mertz [8] and Van den Driesssche and Watmough [9]. The linearized equation of the infectious part of the system is given by

$$\begin{aligned} E'(t) &= \tau S_0[I(t)+U(t)] - \alpha E(t) \\ U'(t) &= v_2 I(t) - \eta U(t) \\ I'(t) &= \alpha E(t) - vI(t) \end{aligned} \tag{6.1}$$

The corresponding matrix is

$$\text{A} = \begin{bmatrix} -\alpha & \tau s_0 & \tau s_0 \\ 0 & -\eta & v_2 \\ \alpha & 0 & -v \end{bmatrix}$$

and the matrix A can be rewritten as
A = V - S, where

$$\text{V} = \begin{bmatrix} 0 & \tau s_0 & \tau s_0 \\ 0 & 0 & v_2 \\ \alpha & 0 & 0 \end{bmatrix} \; and \; S = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & v \end{bmatrix}$$

Therefore, the next generation matrix is

$$\text{VS}^{-1} = \begin{bmatrix} 0 & \frac{\tau S_0}{\eta} & \frac{\tau S_0}{v} \\ 0 & 0 & \frac{v_2}{v} \\ 1 & 0 & 0 \end{bmatrix}$$

and we obtain that

$$R_0 = \frac{\tau S_0}{v}(1+\frac{v_2}{\eta}) \tag{6.2}$$

By using (3.8) we obtain

$$R_0 = \frac{(x_2+v)(x_2+\alpha)(x_2+\eta)}{\alpha S_0(x_2+\eta+v_2)} \frac{S_0}{v}(1+\frac{(v_2}{\eta})$$

and by using v_2 = (1 - f) v we obtain

$$R_0 = \frac{(x_2+v)(x_2+\alpha)(x_2+\eta)}{\alpha v(x_2+\eta+v_2)}(1+\frac{(1-f)v}{\eta}) \tag{6.3}$$

6.2 R_0 caculation for SEIRUδ model

Computation of the basic reproductive number R_0 of model SEIRUδ

The linearized equation of the infectious part of the system is given by

$$\begin{aligned} I'(t) &= \tau S_0[I(t-\delta)+U(t-\delta)] - VI(t), \\ U'(t) &= v_2 I(t) - \eta U(t). \end{aligned} \tag{6.4}$$

We apply the results in Thieme [10] to the linear operator
A:D(A) $\subset$ X $\rightarrow Xwhere$

$X = R^2 \times C([-\delta,0],R^2)$

$$\text{A}\begin{pmatrix} 0_R \\ 0_R \\ I \\ U \end{pmatrix} = \begin{pmatrix} -I'(0)+\tau S_0[I(-\delta)+U(-\delta)]-vI(0) \\ -U'(0)+v_2 I(0)-\eta U(0) \\ I' \\ U' \end{pmatrix}$$

with
D(A) = ${0_R}^2 \times C^1([-\delta,0],R^2)$

We split A into

$$\text{C}\begin{pmatrix} 0_R \\ 0_R \\ I \\ U \end{pmatrix} = \begin{pmatrix} \tau S_0[I(-\delta)+U(-\delta)] \\ v_2 I(0) \\ 0_C \\ 0_C \end{pmatrix}$$

$$\text{B}\begin{pmatrix} 0_R \\ 0_R \\ I \\ U \end{pmatrix} = \begin{pmatrix} -I'(0)-vI(0) \\ -U'(0)-\eta U(0) \\ I' \\ U' \end{pmatrix}$$

By using Theorem 3.5 in [11] we obtain that -B is invertible and

$$(-B)^{-1}\begin{pmatrix}\alpha\\ \beta\\ I\\ U\end{pmatrix} = \begin{pmatrix}0_R\\ 0_R\\ \tilde{I}\\ \tilde{U}\end{pmatrix}$$

where

$\tilde{I}(\theta) = v^{-1}[\alpha + I(0)] + \int_\theta^0 I(\sigma)d\sigma$

$\tilde{U}(\theta) = \eta^{-1}[\beta + U(0)] + \int_\theta^0 U(\sigma)d\sigma$

Thus we can compute

$$\mathrm{C}(-B)^{-1}\begin{pmatrix}\alpha\\ \beta\\ I\\ U\end{pmatrix}$$

and since the range of C is contained into $\mathrm{R}^2 \times {0_C}^2$ it is sufficient to compute

$$\mathrm{C}(-B)^{-1}\begin{pmatrix}\alpha\\ \beta\\ 0_C\\ 0_C\end{pmatrix} = \begin{pmatrix}\tau S_0[\frac{\alpha}{v} + \frac{\beta}{\eta}]\\ \frac{v_2}{v}\alpha\\ 0_C\\ 0_C\end{pmatrix}$$

Therefore, the next generation matrix is

$$\mathrm{V}S^{-1} = \begin{pmatrix}\frac{\tau S_0}{v} & \frac{\tau S_0}{\eta}\\ \frac{v_2}{v} & 0\end{pmatrix}$$

which is Leslie matrix, and the basic reproductive number R_0 is

$$R_0 = \frac{\tau S_0}{v}(1 + \frac{v_2}{\eta}) \tag{6.5}$$

By using (3.17) and $v_2 = (1-f)v$, we obtain

$$R_0 = \frac{x_2 + v}{v}\frac{\eta + x_2}{v_2 + \eta + x_2}e^{x_2\delta}(1 + \frac{(1-f)v}{\eta}) \tag{6.6}$$

.1 Acknowledgement and Group Contribution

First we sat together and read through our given article. Most of the things were new, so it was not easy for us. First we tried to understand the article with the help of other papers and internet materials. Then we divided works among ourselves. Unfortunately we were unable to find out any limitation of given models, we don't have any proposal of possbile extension of the models. Task sharing among group members is given below :

Bishwajit Karmaker (52073) : Abstract, Introduction, table 3.1 in latex and coding.

Aleda Islam Mou (52077) : Method to estimate the parameters and initial conditions, basic reproduction number calculations for both SEIRU and $SEIRU\delta$ models(3.1,3.2,6.1,6.2) in latex.

Md Kamruzzaman (52087) : Rest of the work in latex, modification and finalization.

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Erklärung

Hiermit erkläre ich, dass ich meine Arbeit selbstständig verfasst, keine anderen als die angegebenen Quellen und Hilfsmittel benutzt und die Arbeit noch nicht anderweitig für Prüfungszwecke vorgelegt habe.

Stellen, die wörtlich oder sinngemäß aus Quellen entnommen wurden, sind als solche kenntlich gemacht.

Mittweida, 22 August 2020