

GEOMETRIC URN.

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DEFINING THE EXPECTED VALUE OF A GEOMETRIC DISTRIBUTION

$$\rightarrow E[X] = \frac{1}{p}$$

• GIVEN DATA

• URN CONTAINS SOME RED BALLS AND BLACK BALLS

• EXPECTED VALUE OF DRAWS TO OBTAIN RED = 20

↓

$$E[X] = 20, \text{ solving for } p \rightarrow 20 = \frac{1}{p}$$

$$\therefore p = \frac{1}{20}$$

$$p = 0.05$$

↓

USING PROBABILITY OF RED TO FIND # OF RED AND BLACK BALLS

$$\text{RED} = 100 \cdot p \Rightarrow 100 \cdot 0.05$$

$$\text{RED} = 5$$

↓

$$\text{BLACK} = \text{TOTAL} - \text{RED}$$

$$\text{BLACK} = 100 - 5$$

$$\text{BLACK} = 95$$

• WE WOULD EXPECT THE URN TO HAVE:

5 RED BALLS AND 95 BLACK BALLS.

DRAGON DICE

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GAME :

PICK A NUMBER (1, 2, 3, 4, 5, 6) = GUESS
ROLL 3 SIX SIDED DICE

$\left\{ \begin{array}{l} \text{IF 1 DIE} == \text{GUESS} + 1 \\ \text{IF 2 DICE} == \text{GUESS} + 2 \\ \text{IF 3 DICE} == \text{GUESS} + 3 \end{array} \right\}$ SUCCESS

IF 0 DICE == GUESS - 1 \rightarrow LOSS

$E[X]$ = WEIGHTED SUM OF THE PROBABILITIES OF THE 4 OUTCOMES



• EVENT 1 \rightarrow GUESS == ALL THREE DICE

$$P(\text{EVENT 1}) = 1/216 \text{ or } (1/6 \cdot 1/6 \cdot 1/6)$$

• EVENT 2 \rightarrow GUESS == TWO DICE

$$P(\text{EVENT 2}) = ((1 \cdot 1 \cdot 5) \cdot (1 \cdot 5 \cdot 1) \cdot (5 \cdot 1 \cdot 1)) / 216 \text{ or } 15/216$$

• EVENT 3 \rightarrow GUESS \neq ~~ONE~~ ^{ALL} DICE

$$P(\text{EVENT 3}) = (5 \cdot 5 \cdot 5) / 216 \rightarrow 5^3 / 216 = 125/216$$

• EVENT 4 \rightarrow ~~GUESS == ALL THREE DICE~~ GUESS == ONE DIE

$$P(\text{EVENT 4}) = 1 - P(\text{EVENT 1}) - P(\text{EVENT 2}) - P(\text{EVENT 3})$$

$$P(\text{EVENT 4}) = 1 - (1/216) - (15/216) - (125/216) = 75/216$$

$$\therefore E[X] = 3 \cdot P(\text{EVENT 1}) + 2 \cdot P(\text{EVENT 2}) + P(\text{EVENT 4}) - P(\text{EVENT 3})$$

$$E[X] = 3(1/216) + 2(15/216) + (75/216) - (125/216)$$

$$E[X] = -17$$

HERMIONE'S ~~ONLY~~ EXPECTED OUTCOME $\rightarrow -17$.

NEWTON - PEPYS PROBLEM

BINOMIAL DISTRIBUTION

$$B(n, k) = \binom{n}{k} (1-p)^{n-k} \cdot p^k$$

P(AT LEAST 1 6 APPEARS): # OF DICE = 6

$$\rightarrow = 1 - b(6, 0)$$

$$= 1 - 0.33 \Rightarrow \underline{0.665102}$$

P(AT LEAST 2 SIXES APPEAR): # OF DICE 12

$$= 1 - b(12, 0) - b(12, 1)$$

$$= 1 - 0.112157 - 0.269126$$

$$= 1 - 0.381283$$

$$= \underline{0.618667}$$

P(AT LEAST 3 SIXES APPEAR) # OF DICE 18

$$= 1 - b(18, 0) - b(18, 1) - b(18, 2)$$

$$= 1 - 0.0307561 - 0.135220 - 0.229874$$

$$= \underline{0.597346}$$

EVENT WITH HIGHER PROBABILITY OF SUCCESS IS:

{ OBTAINING AT LEAST ONE SIX WHEN SIX INDEPENDENT
DICE ARE TOSSED

→ SUCCESS RATE $\approx 66.5\%$ WHICH PROVES NEWTON WAS CORRECT!