CMS-380

1) PROBLEM 1 → WIZARD PEOPLE

- P(SHE'S A WITCH) = 0.75
- P(NOT RECEIVING A LETTER | SHE'S A WITCH) = 0.03
- P(NOT RECEIVING A LETTER | SHE'S NOT A WITCH) = 0.99

P(NOT RECEIVING A LETTER) = P(NRL | IS A WITCH) · P(IS A WITCH) + P(NRL | NOT A WITCH) · P(NOT A WITCH)

$P(NRL) = (0.03) \cdot (0.75) + (0.99) \cdot (0.25) =$

$P(NRL) = (0.0225) + (0.2475) = 0.27$

BAYE'S RULE:

A = SHE'S A WITCH

B = NOT RECEIVING A LETTER

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(B}$$

$$P(A \mid B) = \frac{(0.75) \cdot (0.03)}{P(NRL)}$$

$$P(A \mid B) = \frac{(0.75) \cdot (0.03)}{0.27} = \frac{0.0225}{0.27}$$

$$\underline{P(A \mid B) = 0.083}$$

2) PROBLEM 2 → CHOCOLATE FROGS

* 30 TOTAL UNSEEN CARDS

* WHAT IS THE EXPECTED NUMBER OF FROGS THAT NEED TO BE PURCHASED AND OPENED TO ~~BE/ONE~~ GET ALL UNIQUE CARDS

* FIRST CARD ALLWAYS UNIQUE 30/28 ~ 1.0714 TO FIND A THIRD UNIQUE CARD

$$E[X] = 1 + 30/29 + 30/28 + \dots + 30 \qquad k = 30$$

$$x_i = \frac{k}{k-i} \implies \sum_{i=0}^{29} x_i \implies \sum_{i=0}^{29} \left(\frac{k}{k-i}\right)$$

$$\sum_{i=0}^{29} \left(\frac{30}{30-i}\right) = 119.849623927\dots \simeq 120$$

* HERMIONE HAS TO BUY 120 CHOCOLATE FROGS TO GET ALL UNIQUE CARDS

3) PROBLEM 3 → HAT ~~%~~ PROBLEM 

* ALL EVIL STUDENTS GO INTO SLYTHERIN

* P(HUFFLE PUF) = 40%
* P(GRYFFINDOR) = 20%
* P(SLYTHERIN) = 20%
* P(RAVEN CLAW) = 20%

* 10% OF STUDENTS ARE EVIL

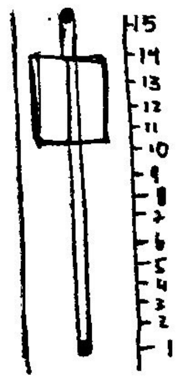
P(EVIL / SLYTHERIN)

* PROBABILITY OF A RANDOM STUDENT CHOSEN FOR SLYTHERIN

$$P(S) = P(N \mid S)\, P(N) + P(E \mid S) \cdot P(E)$$
$$= (0.2)(0.9) + (1) \cdot (0.1)$$
$$= 0.28$$

$$P(\text{EVIL} \mid \text{SLYTHERIN}) = E/S$$
$$= P(E \mid S) = (0.1)/(0.28) = \underline{0.35714286}$$

PROBLEM 4 => DUMBIEVATOR



- NUMBER OF TOTAL TRIPS = 28
  (TO TRAVEL DOWN AT FLOOR 13)
- STOPS ON THE WAY DOWN = 4
  - 1 @ floor 15
  - 2 @ floor 14
  - 1 @ floor 13

- P(ELEVATOR GOING DOWN)

↓ $P(ED) = \frac{\text{\# of stops going down}}{\text{\# of total stops}}$

* P(ELEVATOR GOING DOWN) = $1/7$

↳ $4/28 \Rightarrow 1/7$

PROBLEM 5 → URN WHILE YOU LEARN

* MAGIC URN CONTAINS: 10 BLACK BALLS, 5 RED BALLS

↳ URN DISCARDS ANOTHER BALL AFTER EACH DRAW

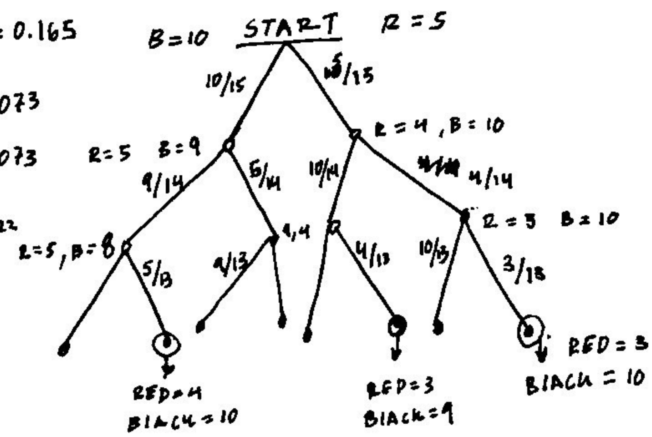
* P(SECOND BALL IS RED) ⟹ $P(x)$

↓ $P(9\ \text{black}, 4\ \text{red}) = \frac{10}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} = 0.165$

$P(9\ \text{black}, 3\ \text{red}) = \frac{10}{15} \cdot \frac{5}{14} \cdot \frac{4}{13} = 0.073$

$P(9\ \text{black}, 3\ \text{red}) = \frac{5}{15} \cdot \frac{10}{14} \cdot \frac{4}{13} = 0.073$

$P(10\ \text{black}, 2\ \text{red}) = \frac{5}{15} \cdot \frac{4}{14} \cdot \frac{3}{13} = 0.022$



* $P(x) = P(9,4) + P(9,3) + P(9,3) + P(10,2)$

* $P(x) = (0.165) + (0.073) + (0.073) + (0.022)$

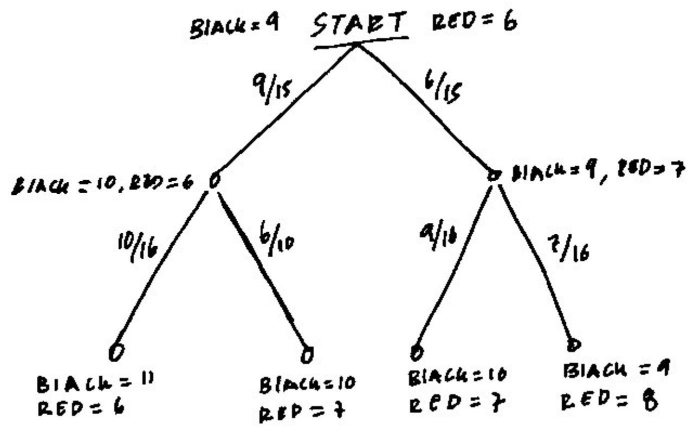
$P(x) = (0.238) + (0.095)$

$P(x) = 0.333$

PROBLEM: 6 $\longrightarrow$ PÓIAY'S URN

* URN ~~IN~~ INITIAL STATE $\rightarrow$ WAT IS THE EXPECTED VALUE OF BLACK AND RED BALLS?

- 9 BLACK
- 6 RED



- $P(11 \text{ BLACK}, 6 \text{ RED}) = \frac{9}{15} \cdot \frac{10}{16} = 0.375$
- $P(10 \text{ BLACK}, 7 \text{ RED}) = \frac{9}{15} \cdot \frac{6}{10} = 0.450$
- $P(10 \text{ BLACK}, 7 \text{ RED}) = \frac{6}{15} \cdot \frac{9}{16} = 0.450$
- $P(10 \text{ BLACK}, 8 \text{ RED}) = \frac{6}{15} \cdot \frac{7}{16} = 0.175$

$\hookrightarrow$ * THE SUM OF ALL PROBABILITIES HAVE TO EQUAL = 1

TOTAL $P = (0.375) + (0.450) + (0.450) + (0.175)$

$\hookrightarrow P = 1$

* CALCULATED EXPECTED VALUES FOR EACH COLOR:

$\downarrow$ BLACK BALLS $= \sum$ (NUM OF BLACK BALLS) $\cdot$ (IT'S PROBABILITY)

$\hookrightarrow = (11 \cdot 0.375) + (10 \cdot 0.450) + (9 \cdot \cancel{0.}175) = 10.2$

* BECAUSE THEY ARE BALLS THEY HAVE TO BE FULL NUMBERS

RED BALLS $= (6 \cdot 0.375) + (7 \cdot 0.450) + (8 \cdot 0.175) = 6.79$

$\therefore$ BLACK BALLS = 11, RED BALLS = 7

PROBLEM 7: ~~ARM~~ ARITHMACY

* THE DISCRETE UNIFORM DISTRIBUTION IS DEFINED OVER A FINITE SET OF INTEGER VALUES $\longrightarrow \{a, a+1, a+2 \ldots, b\}$

↳ EACH OF WHICH ARE EQUALLY LIKELY TO OBSERVED

* 6 SIDED DIE = $a = 1$, $b = 6$
* 20 SIDED DIE = $a = 1$, $b = 20$

o PROVE THAT THE EXPECTED VALUE OF A DISCRETE UNIFORM DISTRIBUTION WITH:

↳ $a = 1 \quad b = n \quad \longrightarrow P(a) = \frac{1}{n}$

• $E[x] = \frac{n+1}{2}$

↓ $E[x] = \sum_x x \cdot P(x)$

$\therefore E[x] = \sum_x x \cdot P(x) = 1 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n} + 3\frac{1}{n} \ldots + n\frac{1}{n}$

COMON MULTIPLE $\frac{1}{n}$ PULLED OUT $= \frac{1}{n}\underbrace{(1+2+3+4 \ldots n)}$

RE-WRITEN AS: $\sum_{k=1}^{n} \frac{1}{2}n(n+1)$

↓

$\sum_{k=1}^{n} k = \frac{1}{2} \cdot n(n+1)$, MULTIPLY BY $\frac{1}{n}$

↓ $\frac{1}{n} \cdot \left(\frac{1}{2}n(n+1)\right) = \frac{1}{\cancel{n}} \cdot \frac{1}{2}\cancel{n}(n+1) = \frac{1}{2}(n+1)$

$\therefore = \frac{n+1}{2}$ //

PROBLEM 8: BIRTH DAY ATTACK:

o WHAT IS THE PROBABILITY THAT NO TWO STUDENTS SHARE THE SAME BIRTH DAY

o TOTAL DAYS = 365 = X

o STUDENTS IN HERMONE'S YEAR = 40

$P(\text{NOT SHARING BIRTHDAY}) = \frac{X}{X} \cdot \frac{X-1}{X} \cdot \frac{X-2}{X} \cdots$ WHERE X = TOTAL DAYS IN THE YEAR

$P(\text{NOT SHARING BIRTHDAY}) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdots$

$P(\text{NOT SHARING BIRTHDAY}) = 0.1087682$