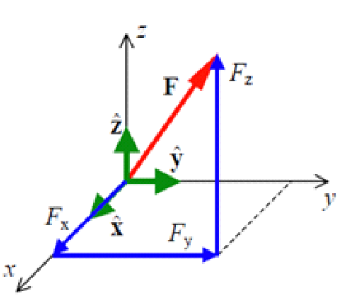
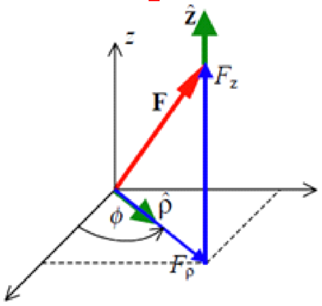
SISTEMAS DE COORDENADAS ORTOGONALES

Vector posición



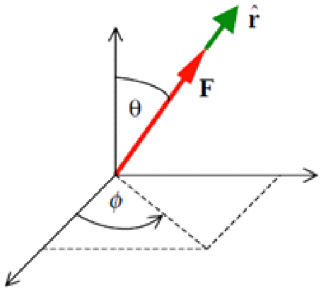
$\mathbf{F} = F_x\,\hat{\mathbf{x}} + F_y\,\hat{\mathbf{y}} + F_z\,\hat{\mathbf{z}}$

Cartesiano



$\mathbf{F} = F_\rho\,\hat{\rho} + F_z\,\hat{\mathbf{z}}$

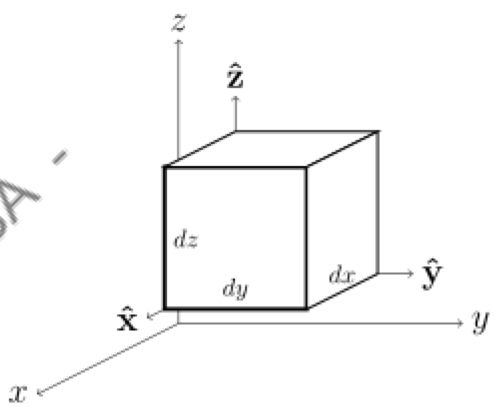
Cilíndrico

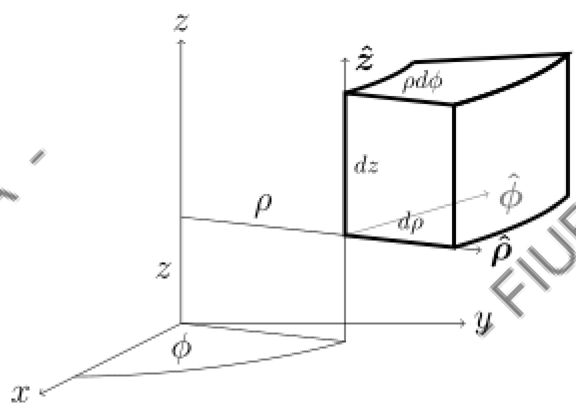


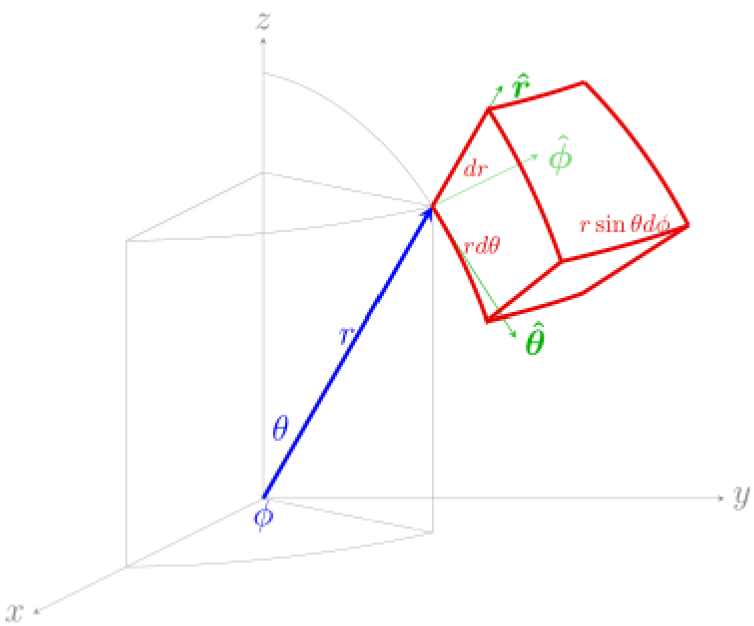
$\mathbf{F} = F_r\,\hat{\mathbf{r}}$

Esférico

Sistemas de coordenadas







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SUSTITUCIONES PARA TRANSFORMAR CAMPOS ESCALARES

	A coordenadas cartesianas	A coordenadas cilíndricas	A coordenadas esféricas
De coordenadas cartesianas	$x = x$ $y = y$ $z = z$	$x = \rho\, cos(\phi)$ $y = \rho\, sin(\phi)$ $z = z$	$x = r\, sin(\theta) cos(\phi)$ $y = r\, sin(\theta) sin(\phi)$ $z = r\, cos(\theta)$
De coordenadas cilíndricas	$\rho = \sqrt{x^2 + y^2}$ $\phi = tan^{-1}(y/x)$ $z = z$	$\rho = \rho$ $\phi = \phi$ $z = z$	$\rho = r\, sin(\theta)$ $\phi = \phi$ $z = r\, cos(\theta)$
De coordenadas esféricas	$r = \sqrt{x^2 + y^2 + z^2}$ $\phi = tan^{-1}(y/x)$ $\theta = cos^{-1}\left(z/\sqrt{x^2 + y^2 + z^2}\right)$	$r = \sqrt{\rho^2 + z^2}$ $\phi = \phi$ $\theta = tan^{-1}(\rho/z)$	$r = r$ $\phi = \phi$ $\theta = \theta$

DIFERENCIALES DE LONGITUD

Sistema de coordenadas	Coordenada que varía sobre la trayectoria	dl	$d\vec{l}$
Cartesianas	x y z	dx dy dz	$\hat{x}\, dx$ $\hat{y}\, dy$ $\hat{z}\, dz$
Cilíndricas	ρ ϕ z	$d\rho$ $\rho\, d\phi$ dz	$\hat{\rho}\, d\rho$ $\hat{\phi}\, \rho\, d\phi$ $\hat{z}\, dz$
Esféricas	r ϕ θ	dr $r\, sin(\theta) d\phi$ $r d\theta$	$\hat{r} dr$ $\hat{\phi}\, r\, sin(\theta) d\phi$ $\hat{\theta}\, r d\theta$

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DIFERENCIALES DE ÁREA

Sistema de coordenadas	Coordenada que se mantiene constante sobre la superficie	dS	$d\vec{S}$
Cartesianas	x y z	$dy\ dz$ $dx\ dz$ $dx\ dy$	$\hat{x}\ dy\ dz$ $\hat{y}\ dx\ dz$ $\hat{z}\ dx\ dy$
Cilíndricas	ρ ϕ z	$\rho\ d\phi\ dz$ $d\rho\ dz$ $\rho\ d\phi\ d\rho$	$\hat{\rho}\ \rho\ d\phi\ dz$ $\hat{\phi}\ d\rho\ dz$ $\hat{z}\rho\ d\phi\ d\rho$
Esféricas	r ϕ θ	$r^2 \sin(\theta)\, d\theta\ d\phi$ $r\ d\theta\ dr$ $r \sin(\theta)\, dr\ d\phi$	$\hat{r}\ r^2 \sin(\theta)\, d\theta\ d\phi$ $\hat{\phi}\ r\ d\theta\ dr$ $\hat{\theta} r \sin(\theta)\, dr\ d\phi$

DIFERENCIALES DE VOLUMEN

Sistema de coordenadas	Diferencial de volumen
Cartesiano	$dV = dx\ dy\ dz$
Cilíndrico	$dV = \rho\ d\rho\ d\phi\ dz$
Esférico	$dV = r^2 \sin(\theta)\, dr\ d\phi\ d\theta$

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TRANSFORMACIÓN DE VECTORES UNITARIOS (VERSORES)

	A coordenadas cartesianas	A coordenadas cilíndricas
De coordenadas cartesianas		$\hat{x} = \hat{\rho}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{\rho}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} \quad \hat{z}$
De coordenadas cilíndricas	$\hat{\rho} = \hat{x}\cos(\phi) + \hat{y}\sin(\phi)$ $\hat{\phi} = -\hat{x}\sin(\phi) + \hat{y}\cos(\phi)$ $\hat{z} = \hat{z}$	

	A coordenadas cartesianas	A coordenadas esféricas
De coordenadas cartesianas		$\hat{x} = \hat{r}\sin(\theta)\cos(\phi) + \hat{\theta}\cos(\theta)\cos(\phi)$ $-\hat{\phi}\sin(\phi)$ $\hat{y} = \hat{r}\sin(\theta)\sin(\phi) + \hat{\theta}\cos(\theta)\sin(\phi)$ $+\hat{\phi}\cos(\phi)$ $\hat{z} = \hat{r}\cos(\theta) - \hat{\theta}\sin(\theta)$
De coordenadas esféricas	$\hat{r} = \hat{x}\sin(\theta)\cos(\phi) + \hat{y}\sin(\theta)\sin(\phi)$ $+\hat{z}\cos(\theta)$ $\hat{\theta} = \hat{x}\cos(\theta)\cos(\phi) + \hat{y}\cos(\theta)\sin(\phi)$ $-\hat{z}\sin(\theta)$ $\hat{\phi} = -\hat{x}\sin(\phi) + \hat{y}\cos(\phi)$	

	A coordenadas cilíndricas	A coordenadas esféricas
De coordenadas cilíndricas		$\rho = \hat{r}\sin(\theta) + \hat{\theta}\cos(\theta)$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{r}\cos(\theta) - \hat{\theta}\sin(\theta)$
De coordenadas esféricas	$\hat{r} = \hat{\rho}\, sin(\theta) + \hat{z}\, cos(\theta)$ $\hat{\theta} = \hat{\rho}\, cos(\theta) - \hat{z}\, sin(\theta)$ $\hat{\phi} = \hat{\phi}$	

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Fórmulas del gradiente en distintos sistemas de coordenadas

Cartesianas:

$$\vec{\nabla} g(\vec{r}) = \left(\frac{\partial g(\vec{r})}{\partial x}, \frac{\partial g(\vec{r})}{\partial y}, \frac{\partial g(\vec{r})}{\partial z} \right) = \frac{\partial g(\vec{r})}{\partial x}\, \breve{e}_x + \frac{\partial g(\vec{r})}{\partial y}\, \breve{e}_y + \frac{\partial g(\vec{r})}{\partial z}\, \breve{e}_z$$

Cilíndricas:

$$\vec{\nabla} g(\vec{r}) = \frac{\partial g(\vec{r})}{\partial \rho}\, \breve{e}_\rho + \frac{1}{\rho} \frac{\partial g(\vec{r})}{\partial \phi}\, \breve{e}_\phi + \frac{\partial g(\vec{r})}{\partial z}\, \breve{e}_z$$

Esféricas:

$$\vec{\nabla} g(\vec{r}) = \frac{\partial g(\vec{r})}{\partial r}\, \breve{e}_r + \frac{1}{r\, sen\vartheta} \frac{\partial g(\vec{r})}{\partial \phi}\, \breve{e}_\phi + \frac{1}{r} \frac{\partial g(\vec{r})}{\partial \vartheta}\, \breve{e}_\vartheta$$

Fórmulas de la divergencia en distintos sistemas de coordenadas

Cartesianas:

$$\vec{\nabla} \bullet \vec{F}(\vec{r}) = \frac{\partial}{\partial x} F_x(\vec{r}) + \frac{\partial}{\partial y} F_y(\vec{r}) + \frac{\partial}{\partial z} F_z(\vec{r})$$

Cilíndricas:

$$\vec{\nabla} \bullet \vec{F}(\vec{r}) = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[\rho\, F_\rho(\vec{r}) \right] + \frac{1}{\rho} \frac{\partial}{\partial \phi} F_\phi(\vec{r}) + \frac{\partial}{\partial z} F_z(\vec{r})$$

Esféricas:

$$\vec{\nabla} \bullet \vec{F}(\vec{r}) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2\, F_r(\vec{r}) \right] + \frac{1}{r\, sen\vartheta} \frac{\partial}{\partial \phi} F_\phi(\vec{r}) + \frac{1}{r\, sen\vartheta} \frac{\partial}{\partial \vartheta} \left[sen\vartheta\, F_\vartheta(\vec{r}) \right]$$

Fórmulas del rotor en distintos sistemas de coordenadas

Cartesianas:

$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left\{ \left[\frac{\partial F_z(\vec{r})}{\partial y} - \frac{\partial F_y(\vec{r})}{\partial z} \right] \breve{e}_x + \left[\frac{\partial F_x(\vec{r})}{\partial z} - \frac{\partial F_z(\vec{r})}{\partial x} \right] \breve{e}_y + \left[\frac{\partial F_y(\vec{r})}{\partial x} - \frac{\partial F_x(\vec{r})}{\partial y} \right] \breve{e}_z \right\}$$

Cilíndricas:

$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left\{ \left[\frac{1}{\rho} \frac{\partial F_z(\vec{r})}{\partial \phi} - \frac{\partial F_\phi(\vec{r})}{\partial z} \right] \breve{e}_\rho + \left[\frac{\partial F_\rho(\vec{r})}{\partial z} - \frac{\partial F_z(\vec{r})}{\partial \rho} \right] \breve{e}_\phi + \frac{1}{\rho} \left[\frac{\partial \left[\rho\, F_\phi(\vec{r}) \right]}{\partial \rho} - \frac{\partial F_\rho(\vec{r})}{\partial \phi} \right] \breve{e}_z \right\}$$

Esféricas:

$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left\{ \frac{1}{r\, sen\vartheta} \left[\frac{\partial \left[F_\phi(\vec{r})\, sen\vartheta \right]}{\partial \vartheta} - \frac{\partial F_\vartheta(\vec{r})}{\partial \phi} \right] \breve{e}_r + \frac{1}{r} \left[\frac{\partial \left[r\, F_\vartheta(\vec{r}) \right]}{\partial r} - \frac{\partial F_r(\vec{r})}{\partial \vartheta} \right] \breve{e}_\phi + \right.$$

$$\left. + \frac{1}{r} \left[\frac{1}{sen\vartheta} \frac{\partial F_r(\vec{r})}{\partial \phi} - \frac{\partial \left[r\, F_\phi(\vec{r}) \right]}{\partial r} \right] \breve{e}_\vartheta \right\}$$