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A Markov-switching COGARCH approach to cryptocurrency portfolio selection and optimization

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Abstract

Blockchain is a new technology slowly integrating our economy with cryptocurrencies such as Bitcoin and many more applications. Bitcoin and other versions of it (known as Altcoins) are traded everyday at various cryptocurrency exchanges and have drawn the interest of many investors. These new types of assets are characterized by wild swings in prices, and this can lead to large swings in profit and losses. To respond to these dynamics, cryptoinvestors need adequate tools to guide them through their choice of portfolio selection and optimization. Bitcoin returns have shown some form of regime change, suggesting that regime-switching models could more adequately capture the volatility dynamics. This paper presents a two-state Markov-switching COGARCH-R-vine (MSCOGARCH) model for cryptocurrency portfolio selection and compares the performance to the single-regime COGARCH-R-vine (COGARCH). The findings here are in line with the literature where MSCOGARCH outperforms the single-regime COGARCH with regard to the expected shortfall risk. The COGARCH specifications here capture the structural breaks and heavy tailness within each state of the Markov switching in order to achieve a minimal risk and a maximum return. The flexibility of R-vine copula allows adequate bivariate copula selection for each pair of cryptocurrencies to achieve suitable dependence structure through pair-copula construction architecture.

Keywords Long range dependence · Lévy processes · Differential evolution · R-vine copula · Portfolio optimization

JEL Classification C02 · G11 · G17

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1 Introduction

The increasing interest on cryptocurrencies was fueled by Bitcoin, the first successful implementation of a peer-to-peer network that uses strong cryptography to secure financial transactions. This success has led to the introduction of alternative versions of Bitcoin such as Ethereum, Ripple, Litecoin, Bitcoin Cash, Tether, to name the few [1]. Cryptocurrency exchanges, such as Binance, Bitfinex, Bittrex and many others, allow customers to trade cryptocurrencies for other assets, such as conventional fiat money, or to trade between digital currencies, thus the name cryptoassets. Selecting a basket (portfolio) of cryptoassets to invest in is a crucial task here, since these cryptoassets are characterized by extreme upswings and downswings in prices that can result in dependence shifts and portfolio gains or losses (see, e.g., Brunnermeier 2009; Moshirian 2011; Florackis et al. 2014; Bekiros et al. 2015). As more investors and institutions including banks, hedgefunds and even governments join the growing cryptocurrency community, it is of importance to find technical tools that are able to deal with such underlying wild swings, known as volatility, to profitably trade these cryptoassets.

The most commonly used models in the literature for modeling volatility are the generalized autoregressive conditional heteroskedasticity (GARCH) models. GARCH framework was introduced by Bollerslev (1986) by extending the autoregressive conditional heteroskedasticity (ARCH) specification of Engle (1982). From this development, various specifications of GARCH come into being such as the exponential GARCH (EGARCH) model of Nelson (1991), the threshold GARCH (TGARCH) model of Zakoian (1994), the GJRGARCH model of Glosten et al. (1993), the IGARCH (integrated GARCH) of Engle and Bollerslev (1986) and many others.

GARCH models have been applied to analyze and understand the dynamics of cryptocurrencies price movement. For example, Dyhrberg (2016a, b) used GARCH models to explore the Bitcoin capabilities as financial asset, and the asymmetric GARCH model to explore the Bitcoin hedging capabilities, respectively. Hansen and Lunde (2005) showed that a GARCH (1, 1) specification outperforms more sophisticated models in the case of exchange rates. Chu et al. (2017) fitted twelve GARCH models (GARCH (1,1), GJRGARCH (1,1), APARCH (1,1), SGARCH (1,1), EGARCH (1,1), IGARCH (1,1), CSGARCH (1,1), AVGARCH (1,1), NGARCH (1,1), TGARCH (1,1), NAGARCH (1,1) and ALL GARCH (1,1)) to each of the seven most popular cryptocurrencies and realized that the IGARCH (Integrated GARCH) of Engle and Bollerslev (1986), and the GJRGARCH of Glosten et al. (1993) models provide the best fits, in terms of modeling of the volatility in the largest and most popular cryptocurrencies. Cryptocurrencies data series display structural breaks due to various factors such as the *non-intrinsic value of cryptocurrencies*, as they mostly depend upon market sentiments; *lack of regulatory oversight*, since the cryptomarket is so free and unregulated that causes high market manipulation by some key players who hold a large volume of the overall cryptocurrency; and *herd mentality* (thousands investing and divesting in fear of losing money) (see Linuma (2018)).

[1] At the time of this writing, there are 2103 tradable cryptocurrencies according to Coinmarketcap (https://coinmarketcap.com).

Recent studies have shown that structural breaks result in biased estimates of GARCH models and poor volatility forecasts (see Bauwens et al. (2014)). To deal with this issue, Markov-switching GARCH (MSGARCH) models have been proposed, whose parameters can change over time according to a discrete latent (i.e., unobservable) variable. Ardia et al. (2018) have tested the presence of regime changes in the GARCH volatility dynamics of Bitcoin using MSGARCH models and found that Bitcoin log returns exhibit regime changes in their volatility dynamics, with MSGARCH models outperforming standard single-regime GARCH models. This is supported by Caporale and Zekokh (2019) who found that two-regime models yield better results in terms of prediction for both value-at-risk (VaR) and expected shortfall (ES) and that using single-regime GARCH models may yield incorrect VaR and ES predictions and hence result in ineffective risk management, portfolio optimization, pricing of derivative securities, etc. In this paper, we propose a two-state Markov-switching COGARCH (MSCOGARCH) model to effectively estimate the conditional value-at-risk (CVaR) of a cryptocurrency portfolio. Continuous GARCH (COGARCH) models were introduced by Klüppelberg et al. (2004) as a continuous-time analogue to the influential and successful discrete-time GARCH volatility model of Bollerslev (1986), where the time is continuous and the innovation follows a Lévy process. This approach intends to capture within each state excessive structural breaks/jumps. Gronwald (2014) reported that an autoregressive jump-intensity GARCH model fits the Bitcoin data better than a standard GARCH.

We are going to use here the COGARCH(1, 1) model driven by compound Poisson Lévy process which accounts not only for the volatility clustering (heteroskedasticity) and heavy-tailed distribution, but also for the jumps. While accounting for these features, as well as statistical dependence, it is important to select adequate optimization algorithm. Evolutionary algorithm such as differential evolution (DE) algorithm introduced by Storn and Price (1997) has shown remarkable performance on continuous numerical problems and optimizing portfolios under non-convex settings see [Ardia et al. (2011), Krink and Paterlini (2011), Krink et al. (2009), Maringer and Oyewumi (2007), Yollin (2009)].

Given the ability of Student's t-copula to capture the tail dependence structure, the t-copula differential evolution (tCDE) was introduced in Mba et al. (2018) and displays great performance as compared to the standard DE in portfolio optimization. However, t-copula allows symmetric tail dependence while financial returns in general (Cherubini et al. 2004) and cryptoassets returns in particular display asymmetric dependence and fat tail characteristics. Moreover, in higher dimension where there are different dependence structures between different pairs of random variables (financial returns), more than one copula is required to model all these various dependencies. An alternative approach is to use vine copula originally proposed by Joe (1994) and extended by Bedford and Cooke (2001, 2002) and Kurowicka and Cooke (2006) with the introduction of graphical models called **regular vines**. For portfolios of higher dimensions, Low et al. (2013) find that modeling asymmetries within the marginals and the dependence structure with the Clayton canonical vine copula consistently outperformed other models incorporating elliptical or symmetric dependence structures. In this study, we will use the regular vine copula which is flexible enough to identify suitable copula family for each pair of assets to adequately model the dependency

structure. The advantage of vine structure in higher-dimensional data is that they split the dependence of large sets of returns into pair of returns and easily employ various bivariate copulas as the building blocks to capture the dependence structure between these pairs of variables, hence the name **pair-copula construction** (PCC).
To obtain random processes with dependence structure (co-movement), jumps and heavy-tailed marginal distributions, we consider in this study the MSCOGARCH-R-vine copula model where the optimal portfolio is obtained using the heuristic, derivative-free search algorithm differential evolution, known for its global solution search ability.
The rest of the paper is organized as follows: Sect. 2 introduces the methodology, Sect. 3 discusses the empirical findings, and the last Sect. 4 concludes the work.

2 Methodology

The pioneering work of Markowitz (1952, 1959) on the mean-variance (MV) portfolio optimization procedure is the milestone of modern theory for optimal portfolio construction, asset allocation and investment diversification. In this procedure, investors respond to the uncertainty of an investment by selecting portfolios that maximize profit subject to achieving a specified level of calculated risk or, equivalently, minimize variance subject to obtaining a predetermined level of expected gain (Markowitz 1952, 1959, 1991; Merton 1972). One of the drawbacks of the Markowitz's approach is the use of sample variance as a measure of risk, the variance which is known to be symmetric may severely mislead in capturing the downside risk. For example, Frankfurter et al. (1971) find that the portfolio selected according to the Markowitz mean-variance criterion is likely not to be as effective as an equally weighted portfolio. Jorion (1985), Best and Grauer (1991) and Britten-Jones (1999) suggest the main difficulty concerns the extreme weights that often arise when constructing sample efficient portfolios that are extremely sensitive to changes in asset means. Another school suggests that the estimation of the correlation matrix plays an important role in this problem. Our paper complements the theoretical work of Markowitz by developing a new bias-corrected estimator to reliably capture the essence of portfolio selection.
Motivated, in particular by the availability of high-frequency data, classical diffusion limits have been used in a natural way to suggest continuous-time limits of discrete-time processes, including for the GARCH models. The best known of these is due to Nelson (1990) which is given by

$$\mathrm{d}Y_t = \sigma_t \mathrm{d}W_t^{(1)}, \quad t \geq 0 \tag{2.1}$$

where the volatility process σ_t satisfies

$$\mathrm{d}\sigma_t^2 = (\beta - \eta\sigma_t^2)\mathrm{d}t + \phi\sigma_t^2 \mathrm{d}W_t^{(2)}, \quad t > 0 \tag{2.2}$$

with $W^{(1)}$ and $W^{(2)}$ two independent Brownian motions and $\beta > 0$, $\eta \geq 0$ and $\phi \geq 0$ constants. This Nelson's diffusion limit of the GARCH process is driven by two independent sources of randomness, whereas the discrete-time GARCH process is driven only by a single white noise sequence. One of the features of the GARCH process is the idea that large innovations in the price process are almost immediately manifested as innovations in the volatility process, but this feedback mechanism is lost in models such as the Nelson continuous-time version. Corradi (2000) modified Nelson's method to obtain a diffusion limit depending only on a single Brownian motion—but then the equation for σ_t^2 degenerates to an ordinary differential equation. Using a Brownian bridge between discrete-time observations, Kallsen and Taqqu (1998) found a complete model driven by only one Brownian motion. But continuous-time limits found in such a way can be probabilistically and statistically different from their discrete-time progenitors. As shown by Wang (2002), parameter estimation in the discrete-time GARCH and the corresponding continuous-time limit stochastic volatility model may yield different estimates.

Klüppelberg et al. (2004) proposed a radically different approach to obtaining a continuous-time model. Their continuous-time GARCH model (COGARCH) is a direct analogue of the discrete-time GARCH, based on a single background driving Lévy process, and generalizes the essential features of the discrete-time GARCH process in a natural way.

2.1 COGARCH(1, 1) models

The COGARCH(1, 1) is the stochastic volatility model introduced by Klüppelberg et al. (2004) as the continuous counterpart of the following discrete-time model GARCH(1, 1):

$$y_t = \varepsilon_t \sigma_t, \quad \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2.3}$$

where α, β, ω are strictly positive.

The specification of the COGARCH model consists of two equations analogous to those of Eq. 2.3 with the single source of variation driven by the Lévy process $(L_t)_{t\geq 0}$. More precisely, the COGARCH process $(G_t)_{t\geq 0}$ is defined in terms of its stochastic differential equation

$$\mathrm{d}G_t = \sigma_{t-}\mathrm{d}L_t, \quad t \geq 0 \tag{2.4}$$

where the volatility process, σ_t, is given by the following stochastic differential equation

$$\mathrm{d}\sigma_t^2 = (\omega - \eta\sigma_{t-}^2)\mathrm{d}t + \alpha\sigma_{t-}^2 d[L, L]_t^d, \quad t > 0, \tag{2.5}$$

for constants $\omega > 0$, $\eta \geq 0$ and $\alpha \geq 0$. The quadratic variation process $[L, L]_t^d$ of L is given by

$$[L, L]_t^d := \sum_{0<s\leq t} (\Delta L_s)^2 \tag{2.6}$$

where $\Delta L_t = L_t - L_{t-}$ for $t \geq 0$ with $L_{0-} = 0$. An auxiliary Lévy process $X = (X_t)_{t\geq 0}$ defined by

$$X_t = \eta t - \sum_{0<s\leq t} \log(1 + \alpha(\Delta L_s)^2), \quad t \geq 0 \tag{2.7}$$

can be used to obtain the solution to the stochastic differential Eq. 2.5, as

$$\sigma_t^2 = \exp(-X_t)\left(\omega \int_0^t \exp(X_s)\mathrm{d}s + \sigma_0^2\right), \ t \geq 0, \tag{2.8}$$

This reveals that σ_t^2 follows a generalized Ornstein–Uhlenbeck process parameterized by (ω, η, α) and driven by the Lévy process L. Financial log returns are modeled by the increments of the process $G_{t,h} = G_{t+h} - G_t$. The Lévy process is the sole source of randomness, and when it jumps, both the price and the volatility jump at the same time.

2.2 Markov-switching COGARCH

Following Ardia et al. (2018), the general Markov-switching COGARCH specification is as follows:

$$\mathrm{d}G_t|(S_t = k, I_{t-1}) = \sigma_{k_{t-}}\mathrm{d}L_{k_t}, \quad t \geq 0 \tag{2.9}$$

where

$$\mathrm{d}\sigma_{k_t}^2 = \left(\omega_k - \eta_k \sigma_{k_{t-}}^2\right)\mathrm{d}t + \alpha_k \sigma_{k_{t-}}^2 d[L_k, L_k]_t^d, \quad t > 0 \tag{2.10}$$

for constants $\omega_k > 0$, $\eta_k \geq 0$ and $\alpha_k \geq 0$, and I_{t-1} being the information set available at time $t-1$. The state variable S_t evolves according to a first-order homogeneous Markov chain with transition probability matrix $\mathbf{P} = (p_{ij})_{i,j=1}^k$ where $p_{ij} = P[S_t = j|S_{t-1} = i]$. In this paper, we consider the two-state regime, i.e., $k \in \{1, 2\}$.

2.3 Regular vine copula

In terms of co-movement, the linear correlation which is often used to analyze the dependency has some drawbacks, as illustrated in Embrechts et al. (2003) and Rachev et al. (2005). For example, when the variance of the returns happens to be infinite, particularly with cryptoassets where extreme events are frequently observed, the linear correlation between these returns is undefined. Furthermore, due to the invariance of the linear correlation under nonlinear strictly increasing transformations, returns might be correlated, whereas prices are not or vice versa. Moreover, linear correlation measures only the degree of dependence but fails to clearly capture the structure of dependence. In order to overcome these limitations of the linear correlation, a more prevalent approach to model the dependence structure is to use the copula model introduced by Sklar (1959). Copulas have been used widely in quantitative finance to model and minimize tail risk and portfolio optimization applications (see, for example, Mba et al. (2018), Simo-Kengne et al. (2018), Low et al. (2013)). The advantage of using copula is that it allows a more general modeling of dependency and the dependency of extreme events called **tail dependence**.

The analysis of dependence structure between random variables has gained much attention in probability and statistics. To understand the nonlinear dependence structure between random variables, copula function was introduced. It provides a link among multivariate joint distributions and univariate marginal distributions. It is designed to provide an idiosyncratic description of the dependence structure between random variables, irrespective of their marginal distribution.

A n-dimensional copula C is a n-dimensional distribution function on $[0, 1]^n$, with standard uniformly distributed marginals $U(0, 1)$ on $[0, 1]$. Sklar's theorem (1959) states that every multivariate distribution F with margins $F_1, \ldots, F_n$ can be written as:

$$F(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)), \tag{2.11}$$

for some copula C, which is uniquely defined on $[0, 1]^n$ for distribution F with margins that are absolutely continuous. Conversely, any copula C may be used to join any collection of univariate distribution functions $F_1, \ldots, F_n$ using in Eq. 2.11 to create a multivariate distribution function F with margins $F_1, \ldots, F_n$. Given a n-dimensional random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)'$ with continuous and strictly increasing margins, the copula C of their joint distribution function may be extracted from Eq. 2.11 by evaluating:

$$C(u_1, u_2, \ldots, u_n) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots, F_n^{-1}(u_n)), \tag{2.12}$$

where F_i^{-1} are the quantile functions of the univariate marginal distribution.

From the recent developments have emerged several types of copulas from two families: elliptical (like Gaussian or Student's t copula) and Archimedean copula (like Clayton, Gumbel or Frank copula). When modeling the dependence structure between three or more random variables, multivariate copulas have been introduced but lack flexibility in higher dimension. This problem is overcome by vine copula models which use bivariate conditional copulas as building blocks, making these models flexible enough in catching the underlying dependence and tail dependence structure. In dimension d (i.e., d random variables), a multivariate density is constructed by $d(d-1)/2$ bivariate (conditional) copulas Bedford and Cooke (2001) as building blocks, thus the name pair-copula construction (PCC) given to this construction process. For example, let X_1, X_2 and X_3 be three random variables with distribution functions F_1, F_2 and F_3, respectively. The joint density can be decomposed as

$$f(x_1, x_2, x_3) = f_{3|12}(x_3|x_1, x_2) f_{2|1}(x_2|x_1) f_1(x_1) \tag{2.13}$$

where

$$\begin{aligned} f_{2|1}(x_2|x_1) &= c_{12}(F_1(x_1), F_2(x_2)) f_2(x_2) \\ f_{3|12}(x_3|x_1, x_2) &= c_{13;2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) f_{3|2}(x_3|x_2) \\ f_{3|2}(x_3|x_2) &= c_{23}(F_2(x_2), F_3(x_3)) f_3(x_3) \end{aligned} \tag{2.14}$$

with

$$F(x|\mathbf{v}) = \frac{\partial C_{x,v_j;\mathbf{v}_{-j}}\{F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})\}}{\partial F(v_j|\mathbf{v}_{-j})} \tag{2.15}$$

for every v_j of the vector $\mathbf{v}$ with $\mathbf{v}_{-j} = \mathbf{v}\ \{v_j\}$ in the general case.
Note that the construction is not unique. All the possible constructions are illustrated by a set of nested trees $T_i = (V_i, E_i)$ where V_i are the nodes and E_i the edges. This set of trees is called a *vine* (Bedford and Cooke 2001).
A nested set of trees is a regular vine if and only if the trees fulfill the following conditions (Bedford and Cooke 2001):

1. T_1 is a tree with nodes $V_1 = \{1, \ldots, d\}$ and edges E_1.
2. For $i \geq 2$, T_i is a tree with nodes $V_i = E_{i-1}$ and edges E_i.
3. If two nodes in T_{i+1} are joint by an edge, the corresponding edge in T_i must share a common node (proximity condition).

A PCC is called a regular vine (R-vine) copula if all marginal densities are uniform.

3 Results and analysis

The data used are composed of the daily returns (100 times the difference in logarithms of Crypt/USD exchange rates) of the top 10 cryptocurrencies by market capitalization from marketcoincap.[2] It spans the period October 01, 2017, to June 21, 2019, and consists of the following cryptoassets: Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), Bitcoin Cash (BCH), Stellar (XLM), Ripple (XRP), Binance Coin (BNB), Eos (EOS), Cardano (ADA), Tether (USDT). Figures 4 and 5 display the historical prices and the log returns of these cryptocurrencies.

These cryptocurrencies exhibit structural breaks (see Figs. 4, 5), and all prices appear to move in the same direction except for the stablecoin Tether which is paired with the US Dollar (see Table 1). (Except for Tether, the correlation coefficients between the remaining cryptocurrencies are greater or equal to 0.5.) This appears to be of concern in terms of portfolio diversification for a cryptoinvestor. That is why a proper dependence structure needs to be taken into account, coupled with structural breaks/jumps to adequately select an optimal portfolio. The R-vine copula and Lévy process in the COGARCH model account for these features, respectively. On the other hand, looking at the log returns (Figs. 4, 5), there is evidence of high-volatility clustering. Moreover, the skewness and kurtosis in Table 2 point to the leptokurtic skewed

[2] https://coinmarketcap.com/.

Table 1 Correlation coefficients

	btc.ret	eth.ret	ltc.ret	bch.ret	xlm.ret	xrp.ret	bnb.ret	eos.ret	ada.ret	usdt.ret
btc.ret	1.00000	0.68560	0.68330	0.53764	0.49395	0.46616	0.57171	0.58889	0.51235	0.12982
eth.ret	0.68560	1.00000	0.78370	0.65422	0.55880	0.63690	0.52895	0.68681	0.58929	0.08994
ltc.ret	0.68330	0.78366	1.00000	0.56827	0.48292	0.54552	0.50946	0.61377	0.52304	0.14446
bch.ret	0.53760	0.65422	0.56830	1.00000	0.40182	0.46353	0.37897	0.58199	0.40601	0.03593
xlm.ret	0.49390	0.55880	0.48290	0.40182	1.00000	0.57439	0.38911	0.49297	0.62118	0.04924
xrp.ret	0.46620	0.63690	0.54550	0.46353	0.57439	1.00000	0.36042	0.55085	0.62622	−0.00546
bnb.ret	0.57170	0.52895	0.50950	0.37897	0.38911	0.36042	1.00000	0.45475	0.40924	0.02835
eos.ret	0.58890	0.68681	0.61380	0.58199	0.49297	0.55085	0.45475	1.00000	0.50406	−0.06586
ada.ret	0.51230	0.58929	0.52300	0.40601	0.62118	0.62622	0.40924	0.50406	1.00000	−0.00298
usdt.ret	0.12980	0.08994	0.14450	0.03592	0.04924	−0.00546	0.02835	−0.06586	−0.00298	1.00000

Table 2 Kurtosis and Skewness

	BTC	ETH	XRP	BCH	LTC	EOS	BNB	USDT	XLM	ADA
Kurtosis	3.221352	2.307838	15.71108	5.926475	6.717989	3.608644	7.614789	13.39305	10.9872	22.35613
Skewness	0.08545709	−0.1502868	1.936578	0.692238	1.20586	0.6715525	0.8987349	0.5082862	1.586178	2.796865

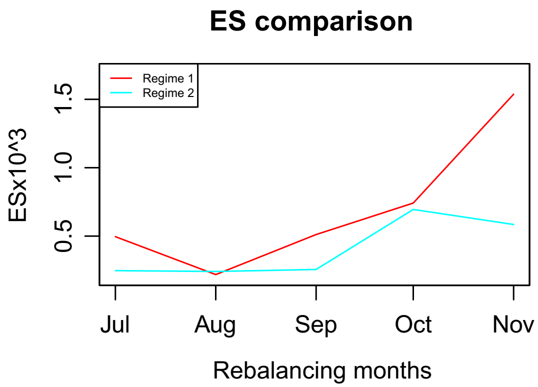
type of distribution for these returns, suggesting that large fluctuations are more likely on the fat tails. A Markov-switching COGARCH model, whose parameters can change over time according to a discrete latent (i.e., unobservable) variable, has the ability to deal with such characteristics.

3.1 Portfolio optimization

The optimization is achieved with the differential evolution algorithm implemented in the software R through the package DEoptim developed by Mullen et al. (2011). Each portfolio is re-balanced monthly over a 5-month period. To allow diversification and mitigate the risk, we impose the constraint that the weight ω_i for cryptoasset i should satisfy $0.2 \leq \omega_i \leq 0.6$. The expected shortfall (ES), the return and weights are displayed in the tables:
Regime 1: Tables 3 and 4; Regime 2: Tables 5 and 6; single regime: Tables 7 and 8. Regime 1 captures the high-volatility state, whereas Regime 2 captures the low-volatility state as depicted in Fig. 1. Allowing this transition between one state to another through Markov-switching COGARCH specifications results in a better estimation of risk (ES) in a portfolio, as illustrated in Fig. 2. Here, a single-regime optimal portfolio displays a high level of risk across the entire period as compared to Markov-switching ones. This is in line with the observations of Caporale and Zekokh (2019), who found that MSGARCH models outperform standard single-regime GARCH models in terms of VaR and ES prediction.

The Markov-switching framework illustrates great ability in terms of modeling cryptocurrencies volatility and portfolio risk, but very little or no influence at all on the return of a cryptoportfolio (see Fig. 3). Moreover, in terms of portfolio returns, the single-regime COGARCH model can peform as much as Regime 1 state of the MSCOGARCH model. As already advocated in the literature, Markov-switching frameworks have great ability to model volatility of cryptocurrencies. So, cryptoinvestors should consider using Markov-switching models, such as the MSCOGARCH proposed in this paper, for their investment decisions.

Fig. 1 Volatility level between Regime 1 and Regime 2



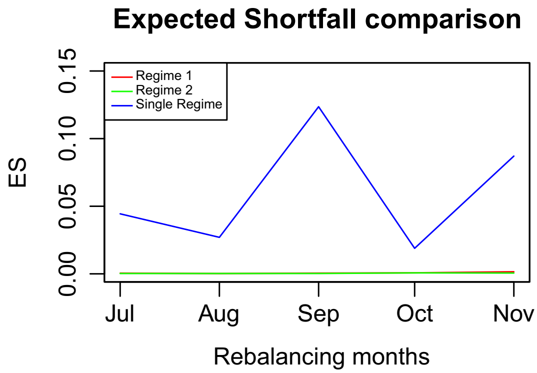


Fig. 2 Expected shortfall comparison between single regime COGARCH and MSCOGARCH

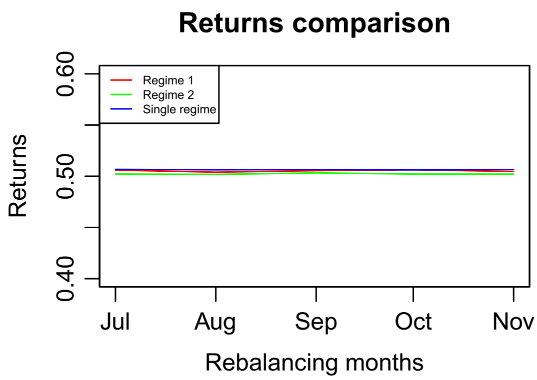


Fig. 3 Portfolio returns for the single regime COGARCH and MSCOGARCH

4 Conclusion

This paper presents a Markov-switching COGARCH-R-vine (MSCOGARCH) model for cryptocurrency portfolio selection and compares the performance to the single-regime COGARCH-R-vine (COGARCH). The findings here are in line with the literature with MSCOGARCH outperforming the single-regime COGARCH. The COGARCH specifications here capture the structural breaks and heavy tailness within each state of the Markov switching to achieve a minimal risk and a maximum return. The flexibility of R-vine copula allows adequate bivariate copula selection for each pair of cryptocurrencies to achieve suitable dependence structure through pair-copula construction architecture. This is very important for cryptocurrencies in terms of portfolio diversification, since the majority of them are strongly positively correlated. So, cryptoinvestors should consider using Markov-switching models, such as the MSCOGARCH-R-vine proposed in this paper, for their investment decisions.

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5 Appendix

See Tables 3, 4, 5, 6, 7, 8 and Figs. 4, 5.

Table 3 Regime 1 portfolio

Rebal periods	ES	Mean
P_1	0.00049573	0.50598
P_2	0.00021852	0.50394
P_3	0.00051131	0.50546
P_4	0.00074151	0.50627
P_5	0.00153713	0.50466

Table 4 Weights Regime 1

Rebal periods	DL.btc	DL.eth	DL.ltc	DL.bch	DL.xlm	DL.xrp	DL.bnb	DL.eos	DL.ada	DL.usdt
P_1	0.022	0.246	0.020	0.020	0.022	0.452	0.020	0.168	0.020	0.020
P_2	0.176	0.370	0.076	0.020	0.020	0.260	0.026	0.020	0.020	0.020
P_3	0.126	0.070	0.020	0.058	0.022	0.070	0.022	0.038	0.560	0.024
P_4	0.080	0.020	0.530	0.184	0.048	0.024	0.024	0.020	0.058	0.022
P_5	0.262	0.020	0.454	0.034	0.140	0.020	0.020	0.020	0.020	0.020

Table 5 MSCOGARCH Regime 2 (ES and mean)

Rebal. periods	ES	Mean
P_1	0.00024745	0.50214
P_2	0.00024130	0.50180
P_3	0.00025597	0.50325
P_4	0.00069471	0.50214
P_5	0.00058486	0.50205

Table 6 MSCOGARCH Regime2 weights

Rebal. periods	DL.btc	DL.eth	DL.ltc	DL.bch	DL.xlm	DL.xrp	DL.bnb	DL.eos	DL.ada	DL.usdt
P_1	0.43	0.02	0.26	0.04	0.08	0.02	0.02	0.09	0.02	0.03
P_2	0.02	0.02	0.03	0.02	0.03	0.02	0.08	0.36	0.41	0.02
P_3	0.37	0.02	0.02	0.03	0.02	0.08	0.02	0.06	0.37	0.02
P_4	0.60	0.02	0.05	0.02	0.12	0.02	0.09	0.02	0.04	0.03
P_5	0.03	0.09	0.05	0.11	0.02	0.04	0.02	0.13	0.50	0.02

Table 7 COGARCH single-regime (ES and mean)

Rebal. periods	ES	Mean
P_1	0.04437879	0.5067
P_2	0.02699725	0.5063
P_3	0.12357561	0.5065
P_4	0.01887074	0.5063
P_5	0.08707254	0.5065

Table 8 COGARCH single-regime weights

Rebal. periods	DL.btc	DL.eth	DL.ltc	DL.bch	DL.xlm	DL.xrp	DL.bnb	DL.eos	DL.ada	DL.usdt
P_1	0.03	0.37	0.10	0.05	0.03	0.02	0.02	0.10	0.27	0.02
P_2	0.20	0.36	0.02	0.26	0.02	0.02	0.07	0.02	0.02	0.02
P_3	0.21	0.35	0.14	0.04	0.02	0.02	0.04	0.09	0.08	0.02
P_4	0.17	0.33	0.03	0.03	0.03	0.04	0.02	0.03	0.31	0.02
P_5	0.06	0.58	0.02	0.02	0.03	0.07	0.02	0.03	0.12	0.06

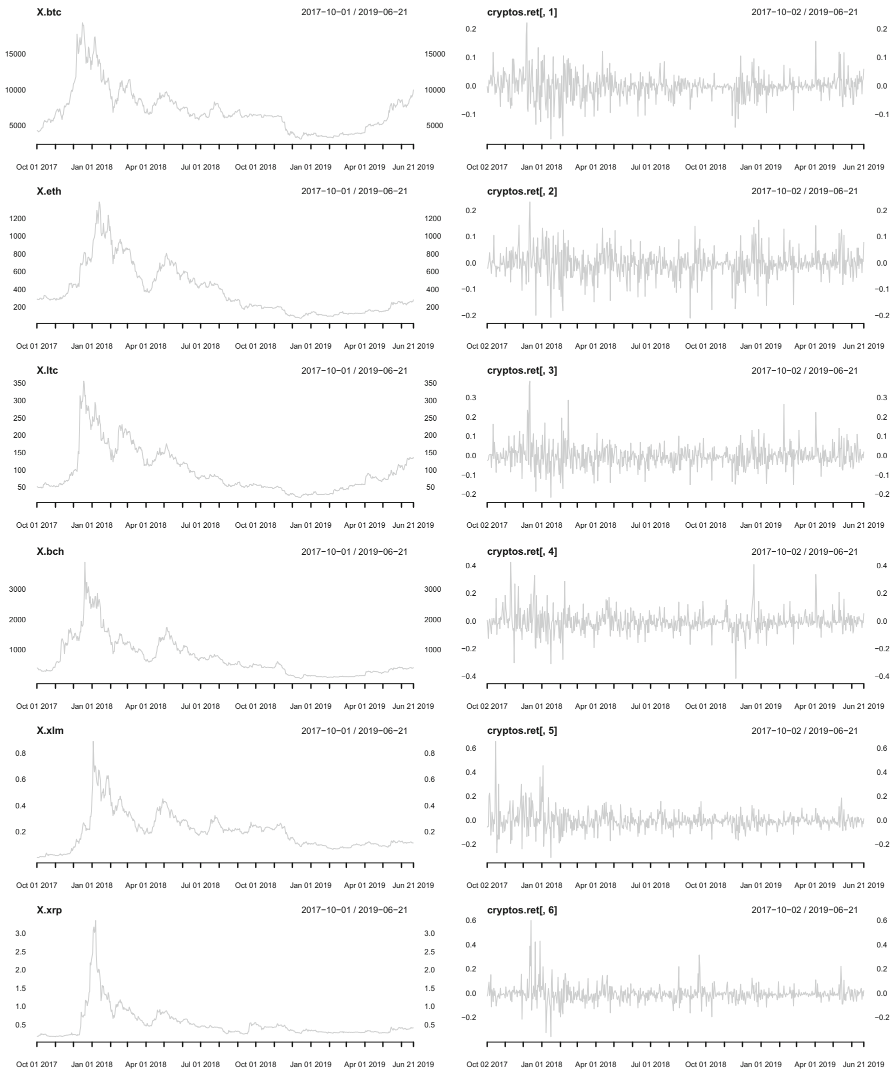

Fig. 4 Historical prices and returns

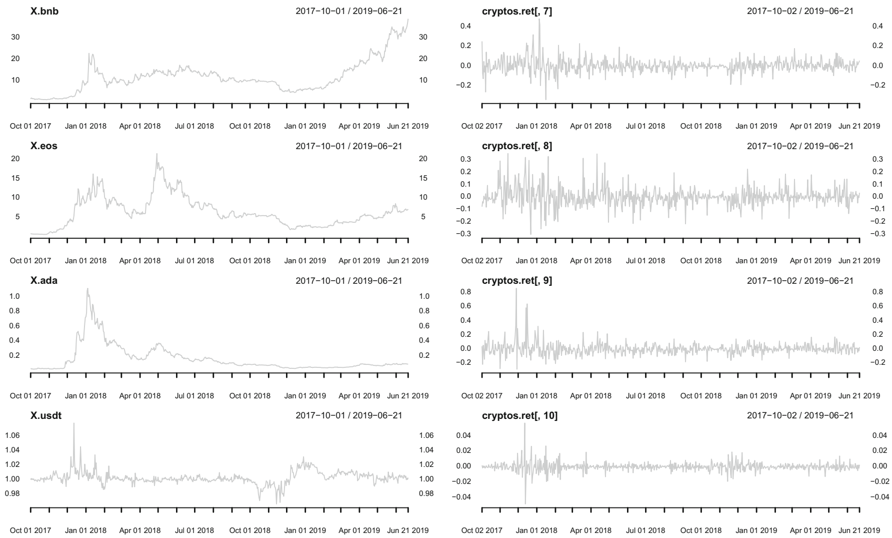

Fig. 5 Historical prices and returns

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