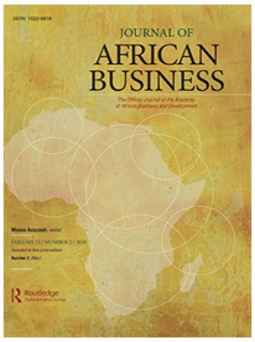

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Behavioural Portfolio Selection and Optimisation: Equities versus Cryptocurrencies

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ABSTRACT

This paper investigates if real investors other than rational investors could add value to their investment portfolios considering their mentality and psychology. The universe of assets constitutes 21 cryptocurrencies (37 international equities) and covers, respectively, the period from August 1, 2016, to March 31, 2018 (January 7, 2002, to March 23, 2018). The cumulative prospect theory and variant specifications were utilized to validate and compare the classification and selection of assets driven by some decision theories. The results of optimization analysis of all the formulated portfolios constituting assets from both markets showed that portfolios constituting assets with lower cumulative prospect theory values outperformed their counterpart with higher cumulative prospect theory values. The superiority of the cumulative prospect theory was established as an empirically corroborated theory of decision-making with rich psychological content. The findings of this paper are crucial for finance practitioners as they showcase an intuitive and coherent manner to guide fund managers, investors and other economic agents in their investment practices.

KEYWORDS

Equity; cryptocurrency; optimisation; portfolio; investor; behavioural asset selection

1. Introduction

Portfolio selection and optimization problem primarily focus on the allocation of assets with the prime aim of simultaneously maximizing returns while minimizing risk, given a time horizon. Markowitz in 1952 proposed a normative theory of decision-making known as the Mean-Variance model (hereafter, M-V) as a guiding principle in solving the portfolio optimization problem in achieving investors' investment objectives in financial markets. The theory till date remains a critical subject in theory and practice of finance and has a venerable history. Over the years, it has been comprehensively studied and experienced rapid development geared towards finding an optimum and efficient way of solving the portfolio optimization problem.

The M-V model provided a simple solution to the trade-off between the mean and the variance of a portfolio. The mean (variance) of a portfolio describes the portfolio's expected return (expected risk) respectively. One fundamental principle behind

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the M-V framework is to strike a balance between the expected return and the expected risk. Markowitz's model initialized the modern era of portfolio risk management and subsequently won the Nobel Prize in 1990 for his ground-breaking work that has revolutionized investment practice. The M-V model has long been widely adopted and applied both in academia and in industry.

According to Markowitz, the optimization problem involves solving two separate problems; namely, asset selection and portfolio optimization. This implies that the first phase of solving a portfolio optimization problem is a selection of market assets with the right M-V statistics to deliver higher returns in the future. Similarly, the second phase finds the right proportions of these market assets that could guarantee the attainment of a certain level of returns. Thus, the Markowitz approach seeks to search for assets within a universe that has the potentials to offer higher returns at a lower risk. The M-V portfolio selection over the years have witnessed significant advancement.

Within the Markowitz framework, investors are more concerned with two basic statistics of the return distribution. The two statistics, the mean and the standard deviation represent the first and the second moments of the returns distribution, respectively. These statistics remain key in solving the portfolio optimization problem. However, several scholars in the behavioral research argue strongly that many psychological factors greatly influence investors' decisions (see, for example, Chew & MacCrimmon, 1979; Kahneman & Tversky, 1979; Tversky & Kahneman, 1992; Bell, 1982; Quiggin, 1982; Chew, 1989; Schmeidler, 1989; Gul, 1991; Luce, 1991; Wakker & Tversky, 1993; Barberis et al., 1998; Rieger & Wang, 2006, 2012; Kontek & Lewandowski, 2013) which lead them to making sub-optimum decisions. They opine that apart from the gains derived from studying the statistical characteristics of returns distribution (i.e. return and risk), investors' mentality and psychology also affects their choices in real life. Thus, investors' behavior is usually influenced by their personality, psychological state, risk preference, and environmental elements, among other factors.

Markowitz's portfolio construction technique has ruled the field of investment for many years. Even though the technique has undergone massive revision, its underpinning assumptions about the mental state of investors have not changed. Under the Markowitz's framework, the following assumptions are operationalized; i) financial markets are efficient, and ii) investors are "fully" rational and "globally" risk-averse. Thus, investors pursue only one investment objective in financial markets which is to maximize returns while minimizing risk. More so, they are indifferent in the region of gains and losses.

Classical economics dictates that investors' mental state remains the same even when they are making gains or experiencing losses. This particular claim has been severely contested by many behavioral research scientists about the market state and investors' mental state. They argue that in the real world, investors depart significantly from these normative principles of investing. They further claim that investors in real life exhibit many mental biases that restrain them from making optimum investment decisions as rational economic agents in financial markets. Thus, investors apart from being cautious of the importance of the return distributions characteristics are equally influenced by many mental biases in their investments decision-making process.

Results of many practical experiments conducted by behavioral scientists have shown that assumptions about the absolute rationality of investors do not hold in real-life situations (Camerer, 1998). Key models in standard economics governed by the rational expectations assumption include the expected (subjective) utility theory, capital asset pricing model, efficient market hypothesis, modern portfolio theory among others. However, several studies (see, for example, Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) supported by empirical evidence contradict the rationality hypothesis underpinning many of these models where economic agents' decisions play a central role. Undoubtedly, the empirical findings by behavioral research scientists have several implications for many models in standard economics.

The behavioral economics literature has put forward many models as an alternative to standard economics decision theories specifically to resolve the disagreement between behavioral economics and neoclassical economics. In the behavioral contest, the following alternative models have been proposed (see, for example, fuzzy logic (Zadeh, 1965), elimination by aspects (Tversky, 1972), prospect theory (Kahneman & Tversky, 1979), rank-dependent utility (Quiggin, 1982), cumulative prospect theory (Tversky & Kahneman, 1992), and fast and frugal heuristics (Gigerenzer & Todd, 1999) among others to account for investors' mentality and psychology.

Even though several of these models exist in the economics literature, the cumulative prospective theory (hereafter, CPT) proposed by Tversky and Kahneman (1992) remains the most promising decision theory. The CPT model has a rich empirical content with more descriptive validity. However, the CPT model specification had not yet been perused and quantified fully in the financial economics literature especially about asset selection and portfolio optimization in emerging markets.

Since the inception of the CPT model, the model's parameters have been re-examined by many other scholars (see, for example, Andersen, Harrison, & Rutström, 2006; Donkers, Melenberg, and Van Soest (2001); Harrison & Rutström, 2009; Tu, 2005; Harrison and Rutström, 2009; Fehr-Duda, De Gennaro, and Schubert, 2006, Abdellaoui (2000); Abdellaoui, Vossmann, & Weber, 2005) as well as the functional form of the PWF (see, Goldstein & Einhorn, 1987; Prelec, 1998). Empirical findings of these studies either confirm the parameters estimated by Tversky and Kahneman (1992) or deviate slightly. The PWF specification in the original CPT model has only one parameter. Similarly, other alternative one- (two-) parameter PWF specifications have been proposed by Prelec (1998) (Goldstein & Einhorn, 1987). According to Stott (2006), several studies seem to favor the two-parameter PWF specification. However, no specific function has been pronounced as superior.

The application of the CPT model has significant implications in the selection of assets from a universe for portfolio optimization analysis. The CPT numerical value captures the product of a utility commonly referred to as value function (hereafter, VF) and a non-linear probability weighting function (hereafter, PWF). The PWF component of the CPT model's specification dictates the degree of investors' mental biases in making real-life choices. Primarily, it captures both the "elevation" and "curvature" of the S-shaped (non-linear) PWF which governs the degree of attractiveness of gambling and describes the degree of discrimination concerning changes in probabilities.

Asset selection for portfolio optimization analysis is governed and driven essentially by the rational expectation hypothesis. The rationality assumption, according to the behavioral decision researchers, believes that it is too rigid and restrictive. Many documented empirical evidence shows that real people in real-life show varied mental biases. To relax the rationality assumption and adequately capture investors' mental biases in solving portfolio optimization problem, the CPT models have been adopted and implemented in this paper with variant VF/PWF parameter estimates and PWF functional forms.

This paper, therefore, seeks to investigate if investors could add value to their portfolio by incorporating common human mental biases by investing in assets with behavioral underpinnings. The paper further attempts to find the most suitable decision-making model in selecting market asset by implementing the CPT model in optimizing behavioral-driven portfolios. The paper is a modest attempt to fill this void and contribute to the growing literature on the optimization of behavioral assets.

In choosing the market assets, the CPT model with different VF/PWF functional forms and corresponding estimates will be explored to guide the selection process from a universe of assets in the cryptocurrency and international equity markets. The CPT model is expected to provide a comprehensive, realistic and a more scientific approach in capturing investors' true behavior in real life. The superiority of the CPT model in the selection process of market assets is verified. This paper differs from previous studies in solving the portfolio optimization problem through the comprehensiveness of the empirical evaluation of the CPT model's suitability for understanding investors' decision-making process in the selection of market assets.

The rest of the paper is organized as follows. Section 2 presents the data and the methodology, while Section 3 reports the empirical results. Section 4 offers some conclusions.

2. Methodology

In this section, three PWFs proposed, respectively, by Tversky and Kahneman (1992), Goldstein and Einhorn (1987), and Prelec (1998) based on the CPT model were adopted to pre-select market assets to form portfolios for further optimization analysis. The multivariate Student-t Copula-GARCH construction approach is employed to describe the tail dependence between assets in the various portfolios. The M-V criterion is adopted as an optimization technique to evaluate the constructed portfolios. Other useful but related concepts that are necessary to understand CPT selection are as well explained.

2.1. Cumulative prospect theory

The CPT model developed by Tversky and Kahneman (1992) is primarily a combination of two models: i) the original PT (Kahneman & Tversky, 1979) and ii) the RDU (Quiggin, 1982, 2012). A brief review of CPT model is provided below to guide the selection of behavioral stocks. Evaluation of lotteries in prospect theory involves two key phases of analysis; namely, the editing phase and the evaluation phase. While in the editing phase, a reference point which differentiates the gains from the losses is established and hence easily recorded as such. On the other hand, the evaluation phase primarily utilizes a VF (v (.)) and a PWF (π (.)).

In a typical real choice context, choosing the RP is important for the CPT model, and it can often be set in several ways. For example, the current stock price may be taken as the reference point. Fixing the RP is an open research question, and investors set this RP differently. However, they gain experience with repeated risky choices under similar conditions (De Palma et al., 2008).

A prospect f is represented as a sequence of pairs(x_j, p_j), where x_j is the jthoutcome and p_j the associated objective probability. Investors are assumed to evaluate prospects in terms of gains and losses with respect to some RP, and their preferences are modeled jointly with a VF and a PWF. Given any lottery with three prospects: a) x with probability p; b) y with probability q; and c) the status quo with probability $1 - p - q$, then CPT value of the lottery is given by:

$$CPT = \pi(p)\, v(x) + \pi(q)\, v(y), \tag{1}$$

where π (.) is the decision weight (PWF), and v (.) is the value function (VF). Any lottery with more than two prospects is thus represented as

$$(x_j, p_j) : (x_{-j}, p_{-j}), (x_{-j+1}, p_{-j+1}), \ldots, (x_0, p_0), \ldots, (x_{k-1}, p_{k-1}), (x_k, p_k) \tag{2}$$

where (x_t, p_t); $t = -j; -j + 1; ; ; ; ; -1; 0; 1; ; : : k - 1; k$; means that the gambler wins$x_t$with probability p_t. Thus, the sum of all probabilities is equal to one (i.e. $\sum p_t = 1$). Let x_t define the outcome of the lottery and x_0 denote some numerical boundary that differentiates losses (i.e.$t < 0$; $x_t < x_0$) from gains (i.e. $t > 0$; $x_t > x_0$) called the reference point (i.e.$t = 0$; $x_t = x_0$). This constant numerical value depends solely on the investor's preference and varies from one investor to another. The CPT value for any given lottery with more than two prospects is thus given as:

$$CPT = \pi(p_{-j})\, v(x_{-j}) + \pi(p_{-j+1})\, v(x_{-j+1}) + \ldots + \pi(p_{k-1})\, v(x_{k-1}) + \ldots + \pi(p_k)\, v(x_k)$$

In a typical VF, the gains are assumed to be concave, and the losses are assumed to be convex, with a turning point at the RP which differentiates between gains and losses. A greater slope ratio in losses than in gains is assumed to capture loss aversion, that is, people tend to be more sensitive to decreases in their wealth than to increases (see, Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Figure 1 presents the CPT value function.

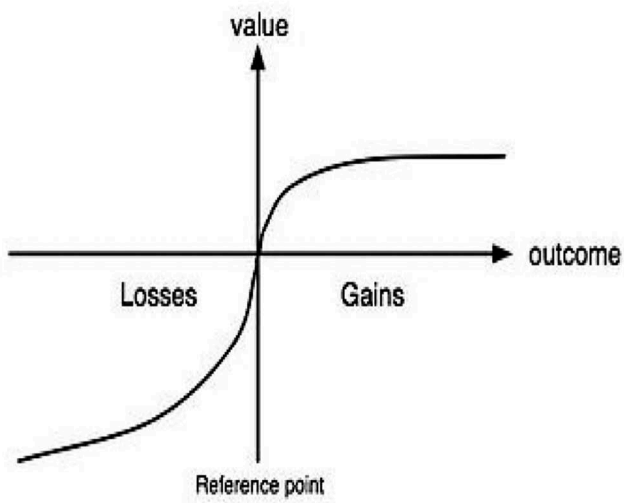

Figure 1. CPT Model Value Function.

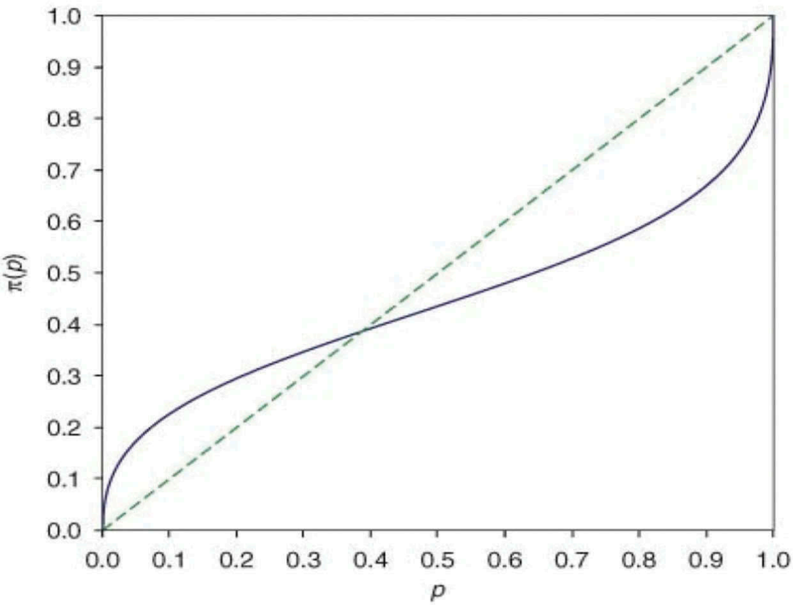

Figure 2. CPT Model Probability Weighting Function.

Likewise, Figure 2 shows the PWF, an inverse S-shaped curve. The overweighting of small probabilities indicates that investors are prone to risk seeking if they are offered low-probability but high-reward lotteries. However, under a high-probability, low-reward offering, decision-makers become risk aversion (Tversky & Kahneman, 1992).

In the CPT framework, Tversky and Kahneman (1992) employed cumulative decision weights instead of the separable decision weights used in PT model. Unlike the PT model, the CPT model was intended to accommodate any number of prospects and applied the cumulative function separately to gains and losses. Suppose a prospect f is represented as follows:

$$\begin{aligned} v(f) &= v(f^-) + v(f^+) \\ v(f^-) &= \sum_{i=-m}^{0} \pi_i^- \, v(x_i) \\ v(f^+) &= \sum_{i=0}^{n} \pi_i^+ \, v(x_i) \end{aligned} \tag{3}$$

where:

$v(f^-)$: is the value of the prospect losses.
$v(f^+)$: is the value of the prospect gains.
$\pi^-(f^-) = (\pi^-_{-m}, \ldots, \pi^-_0)$: are the decision weights of the losses, and
$\pi^+(f^+) = (\pi^+_0, \ldots, \pi^+_n)$: are the decision weights of the gains.

Positive subscripts are used to denote positive prospects (gains), negative subscripts are used to denote negative prospects (losses), and a zero subscript is used to indicate a neutral prospect (RP).

The following notations further define the decision weights:

$$\pi_{-m}^{-} = w^{-}(p_{-m})$$

$$\pi_{n}^{+} = w^{+}(p_{n})$$

$$\pi_{i}^{-} = w^{-}(p_{-m} \cup \ldots \cup p_{i}) - w^{-}(p_{-m} \cup \ldots \cup p_{i-1}), \ -m \leq i \leq 0 \quad (4)$$

$$\pi_{i}^{+} = w^{+}(p_{i} \cup \ldots \cup p_{n}) - w^{+}(p_{i+n} \cup \ldots \cup p_{i}), \ 0 \leq i \leq n-1$$

where$w^{-}$and w^{+}strictly increasing functions from the unit interval into itself satisfying $w^{-}(0) = w^{+}(0) = 0$ and $w^{-}(1) = w^{+}(1) = 1$.

Tversky and Kahneman (1992) proposed the following value function (i.e. a two-part power form) and weighting function which fit the CPT model assumptions.

$$v(x) = \begin{cases} x^{\alpha}, & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta}, & \text{if } x < 0 \end{cases} \quad (5)$$

In accordance with the principle of diminishing sensitivity, the VF is concave for gains ($\alpha \leq 1$) and convex for losses ($\beta \leq 1$). Thus, it is steeper for losses than for gains ($\lambda \geq 1$) per the principle of loss aversion. In the value function, the parameter $\lambda \geq 1$ describes and governs the degree of loss aversion and $\alpha = \beta = 1$ represents the case of pure loss aversion. Eq. (6) below represents the PWF for the prospect of a variety of gains and losses, respectively:

$$w^{-}(p) = \frac{p^{\delta-}}{\left[p^{\delta-} + (1 + p^{\delta-})\right]^{1/\delta-}} \text{ and } w^{+}(p) = \frac{p^{\delta+}}{\left[p^{\delta+} + (1 + p^{\delta+})\right]^{1/\delta+}} \quad (6)$$

The parametric values of δ define the curvature of the PWF and the point at which it crosses the 45-degree line. Decreasing δ causes the weighting function to become more curved and to cross the 45-degree line farther to the right.

Tversky and Kahneman (1992) estimated the values of these parameters and found that the best fits of the model are provided by $\alpha = \beta = 0.88$, $\lambda = 2.25$, $\delta^{-} = 0.69$, and $\delta^{+} = 0.61$. Findings of other empirical studies (Abdellaoui et al., 2008; Abdellaoui, 2000; Abdellaoui, Bleichrodt, & Paraschiv, 2007; Abdellaoui et al., 2005; Andersen et al., 2006; Fehr-Duda et al., 2006; Fennema & Van Assen, 1998; Harrison & Rutström, 2009; Tu, 2005) also confirm similar estimates of the CPT model specification. On the other hand, Wu and Gonzalez (1996), Camerer and Ho (1994) and Senbil and Kitamura (2004) also tested the fit of the CPT model and obtained values of$0.26 \leq \alpha \leq 0.31$.

Both the VF and the PWF play a vital role in the CPT model. Apart from the CPT model's PWF proposed by Tversky and Kahneman (1992), several other PWFs have since been proposed in the literature. Similarly, alternative PWFs that consider prospects in the region of gains or losses with a single or double parameter(s) are equally considered in the CPT model. One of the most commonly used PWF proposed before the CPT model is the linear-in-log-odds specification, introduced by Goldstein and Einhorn (1987), and given by:

Table 1. Empirical Estimates of Value Function.

Functional Form	Estimates			Properties				
Utility	α	β	λ	E	T	I	N	Author(s)
Power:*	0.88	0.88	2.25	md	c	n	25	**TK (92)**: Tversky and Kahneman (1992)
	0.89	0.92	–	md	c	y	40	**TK (92)**: Abdellaoui (2000)
	0.81	0.8	1.07	ml	c	y	90	**TK (92)**: Andersen et al. (2006)
	0.71	0.72	1.38	ml	c	y	158	**TK (92)**: Harrison and Rutström (2009)
	0.89	0.92	–	md	c	y	40	**GE (87)**: Abdellaoui (2000)
	0.91	0.96	–	md	c	n	41	**GE (87)**: Abdellaoui et al. (2005)
	1.01	1.05	–	md	c	y	181	**GE (87)**: Fehr-Duda et al. (2006)
	0.61	0.61	–	ml	b	n	2593	**Prelec-1**: Donkers et al. (2001)
	0.68	0.74	3.2	ml	b	n	1743	**Prelec-1**: Tu (2005)

*Note: Adopted names and notations do not form a convention, and are used for convenience. ± denote gains/losses.
* Estimates in Table 1 adopted from Booij et al. (2010)
* The utility function is specified on the complete real axis, where λ represents the loss aversion coefficient. The displayed utility function assumes α > 0 and β > 0, which is mostly found empirically. The function has a different specification for other parameter values Wakker (2008).
** Properties: E (estimator): mean; median (md); maximum likelihood (ml); T (task): choice; matching; both; I (incentives): yes (random lottery incentive scheme/ Becker, DeGroot, and Marschak (1964) procedure); *no* (fixed or no payment).

Table 2. Empirical Estimates of Different Probability Weighting Functional Forms.

Functional Form	Estimates				Properties				
Probability Weights	δ^+	γ^+	δ^-	γ^-	E	T	I	N	Author(s)
TK (92):		0.61	–	0.69		c	n	25	Tversky and Kahneman (1992)
		0.60	–	0.70		c	y	40	Abdellaoui (2000)
		0.76	–	0.76		c	y	90	Andersen et al. (2006)
		0.91	–	0.91		c	y	158	Harrison and Rutström (2009)
GE (87):	0.65	0.60	0.84	0.65		c	y	40	Abdellaoui (2000)
	0.98	0.83	1.35	0.84		c	n	41	Abdellaoui et al. (2005)
	0.87	0.51	1.07	0.53		c	y	181	Fehr-Duda et al. (2006)
Prelec-I:	–	0.413	–	0.413		b	n	2593	Donkers et al. (2001)
	–	1.00	–	0.77		b	n	1743	Tu (2005)

*Note: Adopted names and notations do not form a convention, and are used for convenience. ± denote gains/losses.
* Estimates in Table 2 adopted from Booij et al. (2010)
* The utility function is specified on the complete real axis, where λ represents the loss aversion coefficient. The displayed utility function assumes α > 0 and β > 0, which is mostly found empirically. The function has a different specification for other parameter values Wakker (2008).
** Properties: E (estimator): mean; median (md); maximum likelihood (ml); T (task): choice; matching; both; I (incentives): yes (random lottery incentive scheme/ Becker et al. (1964) procedure); no (fixed or no payment).

$$w^-(p) = \frac{\delta^- p^{\gamma-}}{\delta^- p^{\gamma-} + (1-p)^{\gamma-}} \text{ and } w^+(p) = \frac{\delta^+ p^{\gamma+}}{\delta^+ p^{\gamma+} + (1-p)^{\gamma+}} \quad (7)$$

Goldstein and Einhorn (1987) model specification, unlike the CPT model, has two parameters which govern both the curvature and the elevation of the PWF and has an inverted-S shape when $0 < \gamma < 1$.

On the other hand, Prelec (1998) proposed a one parameter PWF which accounts for prospects of a variety of gains and losses. The PWF specification introduced by Prelec (1998) is given as:

$$w^-(p) = \exp\left[-(-\ln p)^{\gamma-}\right] \, and w^+(p) = \exp\left[-(-\ln p)^{\gamma+}\right] \quad (8)$$

Tables 1 and 2 below present the various parameter estimates both the value and the probability weighting functions used in this study.

3. Empirical analysis and discussion of results

This section discusses in detail the empirical findings of the behavioral asset selection, portfolio and the portfolio optimization of behavioral assets.

3.1. Data

The data used in this paper include time series of daily closing prices of cryptocurrencies (international equities) publicly sourced from https://coinmarketcap.com/coins/ (Bloomberg database). The universe of cryptocurrencies (international equities) constitutes 21 (37) assets. The price data series for both markets covers the period from August 1, 2016, to March 31, 2018 (January 7, 2002, to March 23, 2018), resulting in a sample of 608 (3917) observations. Daily returns (r_t) for the constituent assets in the respective markets are calculated as follows: $r(t) = \ln(p_t/p_{t-1}) \times 100$, where$p_t$ is the value of the index at time t.

3.2 Preliminary results

The results are twofold. The first part shows the classification and selection of assets using the various decision theories under the cumulative prospect theory specification. The second part involves the mean-variance optimization of the classified and formulated portfolios based on the various decision theories.

In Table 3, we provide a detailed summary of parameter estimates of the VF (utility) proposed by Andersen et al. (2006), Harrison and Rutström (2009), Donkers et al. (2001), Tu (2005), Harrison and Rutström (2009), Fehr-Duda et al. (2006), Abdellaoui (2000), Abdellaoui et al. (2005). The universe of assets is made up of 21 (37) assets within the cryptocurrency (international equity) markets.

Table 3. Value Function (Utility).

	Value Function (Utility)			
Description	Authors	α	β	λ
Power Functional Form (Utility)	Tversky and Kahneman (1992)	0.88	0.88	2.25
	Abdellaoui (2000)	0.89	0.92	1.00
	Donkers et al. (2001)	0.61	0.61	1.00
	Tu (2005)	0.68	0.74	3.20
	Andersen et al. (2006)	0.81	0.80	1.07
	Fehr-Duda et al. (2006)	1.01	1.05	1.00
	Abdellaoui et al. (2007)	0.72	0.73	2.54
	Abdellaoui et al. (2008)	0.86	1.06	2.61
	Harrison and Rutström (2009)	0.71	0.72	1.38
Adjustment*	Abdellaoui et al. (2005)*	0.82	0.90	2.05

Note: This table displays the parameter estimates of the VF as proposed by different scholars.
* Estimates in Table 3 adopted from Booij et al. (2010)
* Arithmetic mean of Abdellaoui (2000, 2007 and 2008)parameters

Table 4. One-Parameter Probability Weighting Function.

	One-Parameters Probability Weighting Function		
Description	Authors	γ^{+}	γ^{-}
TK (1992)	Tversky and Kahneman (1992)	0.61	0.69
	Abdellaoui (2000)	0.60	0.70
	Andersen et al. (2006)	0.76	0.76
	Harrison and Rutström (2009)	0.91	0.91
Prelec I (1998)	Donkers et al. (2001)	0.413	0.413
	Tu (2005)	1.000	0.77

Note: This table displays the parameter estimates of the one-parameter PWF based on Tversky and Kahneman (1992) and Prelec (1998) functional forms.
* Estimates in Table 4adopted from Booij et al. (2010)

Table 5. Two-Parameter Probability Weighting Function.

	Two-Parameter Probability Weighting Function				
Description	Authors	δ^{+}	γ^{+}	δ^{-}	γ^{-}
Goldstein and Einhorn (1987)	Abdellaoui (2000)	0.65	0.60	0.84	0.65
	Abdellaoui et al. (2005)	0.98	0.83	1.35	0.84
	Fehr-Duda et al. (2006)	0.87	0.51	1.07	0.53

Note: This table displays the parameter estimates of the two-parameter PWF based on Goldstein and Einhorn (1987) functional form.
* Estimates in Table 5 adopted from Booij et al. (2010)

In the tradition M-V portfolio analysis, portfolios are principally selected based on the first two moments of the return distribution; namely, the mean (denoting return) and the standard deviation (denoting risk). In this paper, assets were first classified and pre-selected based on the CPT model with different VF and PWF parameter estimates which adequately capture real investors' mental biases.

Likewise, Tables 4 and 5 present the various parameter estimates of the PWF (non-linear probabilities) of the CPT model proposed, respectively, by Tversky and Kahneman (1992), Goldstein and Einhorn (1987) and Prelec (1998).

While Table 3 shows the various parameter estimates of the value function, Tables 4 and 5, on the other hand, show the one- and two-parameter estimates of the PWF proposed by different researchers.

3.2.1. Behavioral classification of cryptocurrencies and international equity indices

The portfolio classification principle adopted in this study is primarily based on the CPT model proposed by Tversky and Kahneman (1992). The PWF proposed by Tversky and Kahneman (1992) and Prelec (1998) has only one parameter while that of Goldstein and Einhorn (1987) has two parameters. Thus, five (three) different one-(two-) parameter PWF with different parameter estimates guided the classification and selection of assets in forming the various portfolios. Each of the PWF within the CPT specification was independently used in pre-classifying the assets.

Table 6 (Table 7) displays results of classified asset in the cryptocurrency (international equity) markets ranked by their CPT values. The one-parameter PWF with different parameter estimates proposed by the following: 1). Tversky and Kahneman (1992): Tversky and Kahneman (1992) represented as KT; Andersen et al. (2006) represented as Anderson; Harrison & Rutström (2009) represented as Harrison), and

Table 6. One-Parameter PWF and CPT Values of Cryptocurrencies.

TK	Asset	Anderson	Asset	Tu	Asset	Harrison	Asset	Donkers	Asset	Abd2000	Asset
ASSETS WITH HIGHER CPT VALUES											
0.5958	**DGB**	0.0561	**XVG**	0.6818	**DGB**	0.6703	**DGB**	0.7200	**DGB**	0.5919	**DGB**
0.5655	**SBD**	0.0402	**PIVX**	0.6519	**SBD**	0.6397	**SBD**	0.6927	**SBD**	0.5617	**SBD**
0.5418	**XVG**	0.0356	**DGB**	0.6341	**XVG**	0.6223	**XVG**	0.6774	**XVG**	0.5375	**XVG**
0.4871	**XEM**	0.0349	**XLM**	0.5839	**XEM**	0.5703	**XEM**	0.6264	**XEM**	0.4830	**XEM**
0.4558	**XRP**	0.0342	**SC**	0.5491	**XRP**	0.5363	**XRP**	0.5970	**XRP**	0.4519	**XRP**
0.3874	**XLM**	0.0323	**BTS**	0.4850	**XLM**	0.4721	**XLM**	0.5341	**XLM**	0.3836	**XLM**
0.3498	**REP**	0.0323	**DCR**	0.4532	**REP**	0.4388	**REP**	0.4981	**REP**	0.3459	**REP**
0.3430	**XMR**	0.0306	**ISK**	0.4479	**XMR**	0.4320	**XMR**	0.4934	**XMR**	0.3387	**XMR**
ASSETS WITH LOWER CPT VALUES											
0.2636	**DOGE**	0.0285	**WAVES**	0.3634	**ISK**	0.3477	**DOGE**	0.4115	**DOGE**	0.2596	**DOGE**
0.2608	**ISK**	0.0266	**DOGE**	0.3631	**DOGE**	0.3477	**ISK**	0.4085	**ISK**	0.2567	**ISK**
0.2546	**DASH**	0.0263	**ETC**	0.3554	**DASH**	0.3397	**DASH**	0.4010	**DASH**	0.2506	**DASH**
0.2404	**ETC**	0.0253	**DASH**	0.3394	**ETC**	0.3239	**ETC**	0.3859	**ETC**	0.2363	**ETC**
0.2234	**WAVES**	0.0244	**SBD**	0.3243	**WAVES**	0.3085	**WAVES**	0.3671	**WAVES**	0.2195	**WAVES**
0.2028	**FCT**	0.0237	**LTC**	0.3002	**FCT**	0.2854	**FCT**	0.3426	**FCT**	0.1993	**FCT**
0.1920	**ETH**	0.0237	**ETH**	0.2872	**ETH**	0.2717	**ETH**	0.3309	**ETH**	0.1882	**ETH**
0.1742	**BTC**	0.0181	**BTC**	0.2680	**BTC**	0.2522	**BTC**	0.3094	**BTC**	0.1707	**BTC**

Note: This table displays the universe of selected assets based Tversky and Kahneman (1992) and Prelec (1998) one-parameter functional forms with different parameter estimates. (*) indicates stocks constituting the various portfolios.

Table 7. One-Parameter PWF and CPT Values of International Equity Indices.

TK	Asset	Anderson	Asset	Tu	Asset	Harrison	Asset	Donkers	Asset	Abd2000	Asset
ASSETS WITH HIGHER CPT VALUES											
0.1558	**MICEX**	0.0109	**BOVESPA**	0.2430	**MICEX**	0.2276	**MICEX**	0.2880	**MICEX**	0.1524	**MICEX**
0.1253	**KOSPI**	0.0109	**BIST**	0.2057	**R. RTS**	0.1912	**R. RTS**	0.2481	**R. RTS**	0.1223	**KOSPI**
0.1252	**R. RTS**	0.0102	**MICEX**	0.2054	**KOSPI**	0.1911	**KOSPI**	0.2468	**KOSPI**	0.1222	**R. RTS**
0.1196	**BOVESPA**	0.0095	**R. RTS**	0.1989	**BOVESPA**	0.1848	**BOVESPA**	0.2403	**BOVESPA**	0.1167	**BOVESPA**
0.1159	**BUDAPEST**	0.0095	**BUDAPEST**	0.1938	**BUDAPEST**	0.1797	**BUDAPEST**	0.2352	**BUDAPEST**	0.1129	**BUDAPEST**
0.1128	**NIFTY 500**	0.0089	**FTSE JSE**	0.1903	**NIFTY 500**	0.1761	**NIFTY 500**	0.2302	**NIFTY 500**	0.1099	**NIFTY 500**
0.1124	**COLOMBO**	0.0084	**OMXS**	0.1877	**COLOMBO**	0.1740	**COLOMBO**	0.2284	**COLOMBO**	0.1096	**COLOMBO**
0.1012	**BIST**	0.0083	**KOSPI**	0.1752	**BIST**	0.1617	**BIST**	0.2152	**BIST**	0.0985	**BIST**
ASSETS WITH LOWER CPT VALUES											
0.0725	**S&P E/W**	0.0068	**IDX COMP**	0.1344	**S&P E/W**	0.1227	**S&P E/W**	0.1699	**S&P E/W**	0.0703	**S&P E/W**
0.0689	**SHANG**	0.0068	**NASDAQ**	0.1295	**SHANG**	0.1182	**SHANGHAI**	0.1643	**SHANG**	0.0669	**SHANGHAI**
0.0684	**D. JONES**	0.0068	**FTSE 100**	0.1287	**D. JONES**	0.1172	**D. JONES**	0.1632	**D. JONES**	0.0663	**D. JONES**
0.0677	**BEL 20**	0.0065	**S&P E/W**	0.1280	**BEL 20**	0.1165	**BEL 20**	0.1627	**BEL 20**	0.0656	**BEL 20**
0.0627	**RUSSELL**	0.0064	**SMI**	0.1210	**RUSSELL**	0.1099	**RUSSELL**	0.1545	**RUSSELL**	0.0607	**RUSSELL**
0.0622	**TASI**	0.0060	**S&P COM**	0.1196	**TASI**	0.1086	**TASI**	0.1536	**TASI**	0.0602	**TASI**
0.0575	**TAIEX**	0.0059	**D. JONES**	0.1133	**TAIEX**	0.1025	**TAIEX**	0.1453	**TAIEX**	0.0556	**TAIEX**
0.0573	**TA 35**	0.0058	**COLOMBO**	0.1132	**TA 35**	0.1024	**TA 35**	0.1450	**TA 35**	0.0553	**TA 35**

Note: This table displays the universe of selected assets based Tversky and Kahneman (1992) and Prelec (1998) one-parameter functional forms with different parameter estimates. (*) indicates stocks constituting the various portfolios.

Table 8. Two-Parameter PWF and CPT Values of Cryptocurrencies.

Fehr-Duda	Asset	Abd2000	Asset	Abd2005	Asset
ASSETS WITH HIGHER CPT VALUES					
0.5456	**DGB**	0.5864	**DGB**	0.6186	**DGB**
0.5153	**SBD**	0.5575	**SBD**	0.5881	**SBD**
0.4884	**XVG**	0.5284	**XVG**	0.5648	**XVG**
0.4332	**XEM**	0.4785	**XEM**	0.5130	**XEM**
0.4022	**XRP**	0.4467	**XRP**	0.4798	**XRP**
0.3338	**XLM**	0.3777	**XLM**	0.4127	**XLM**
0.2961	**REP**	0.3420	**REP**	0.3780	**REP**
0.2892	**XMR**	0.3345	**XMR**	0.3705	**XMR**
ASSETS WITH LOWER CPT VALUES					
0.2140	**DOGE**	0.2555	**DOGE**	0.2885	**DOGE**
0.2109	**ISK**	0.2527	**ISK**	0.2874	**ISK**
0.2054	**DASH**	0.2472	**DASH**	0.2807	**DASH**
0.1921	**ETC**	0.2326	**ETC**	0.2655	**ETC**
0.1763	**WAVES**	0.2161	**WAVES**	0.2497	**WAVES**
0.1581	**FCT**	0.1957	**FCT**	0.2283	**FCT**
0.1482	**ETH**	0.1852	**ETH**	0.2159	**ETH**
0.1326	**BTC**	0.1686	**BTC**	0.1980	**BTC**

Note: This table displays the universe of assets based Goldstein and Einhorn (1987) two-parameter functional form with different parameter estimation. (*) indicates stocks constituting the various portfolios.

Table 9. Two-Parameter PWF and CPT Values of International Equity Indices.

Fehr-Duda	Asset	Abd2000	Asset	Abd2005	Asset
ASSETS WITH HIGHER CPT VALUES					
0.1171	MICEX	0.1509	MICEX	0.1771	MICEX
0.0912	KOSPI	0.1213	KOSPI	0.1449	KOSPI
0.0911	R. RTS	0.1209	R. RTS	0.1446	R. RTS
0.0863	BOVESPA	0.1153	BOVESPA	0.1388	BOVESPA
0.0833	BUDAPEST	0.1116	BUDAPEST	0.1345	BUDAPEST
0.0807	NIFTY 500	0.1089	NIFTY 500	0.1315	NIFTY 500
0.0806	COLOMBO	0.1087	COLOMBO	0.1305	COLOMBO
0.0713	BIST	0.0971	BIST	0.1186	BIST
0.0486	S&P E/W	0.0694	S&P 500 E/W	0.0867	S&P E/W
0.0459	SHANG	0.0658	SHANG	0.0828	SHANG
0.0455	D. JONES	0.0655	D. JONES	0.0822	D. JONES
0.0449	BEL 20	0.0646	BEL 20	0.0814	BEL 20
0.0411	RUSSELL	0.0598	RUSSELL	0.0759	RUSSELL
0.0408	TASI	0.0592	TASI	0.0750	TASI
0.0372	TAIEX	0.0547	TAIEX	0.0701	TAIEX
0.0370	TA 35	0.0545	TA 35	0.0699	TA 35

Note: This table displays the universe of assets based Goldstein and Einhorn (1987) two-parameter functional form with different parameter estimation. (*) indicates stocks constituting the various portfolios.

2). Prelec (1998): Donkers et al. (2001) represented as Donkers; and Tu, (2005) represented as Tu guided the ranking of the assets. Classified assets with an estimated higher and lower CPT values were selected for further optimization analysis. Out of the classified assets, eight separate assets with extremely higher and lower CPT values were selected to form the various portfolios.

Similarly, Table 8 (Table 9) presents classified assets in both markets based on two-parameter PWF with different parameter estimates proposed by Goldstein and Einhorn (1987): Fehr-Duda et al. (2006) represented as Fehr-Duda; Abdellaoui (2000) as Abd 2000, and Abdellaoui et al. (2005) as Abd 2005.

The CPT values of all assets were mainly meant for classification purposes only drawing cognitive intuition into the selection process. Thus, the various PWF under the

CPT specifications were strictly adopted to classify the universe of assets for further optimization analysis using the M-V criterion.

Next, the study proceeds to formulate the various portfolios constituting both markets considering 1- and 2-parameter PWF. For each proposed VF/PWF estimates under the CPT specification, two separate portfolios emerge. Both the first and second portfolios constitute eight separate assets each with extremely higher and lower CPT values.

3.2.2. Portfolio formation

The various VF and PWF parameter estimates were used to guide the formulation of all portfolios comprising eight assets each. In all, eight different VF/PWF parameter estimates based on the CPT specification were used. The first (second) sets of portfolios constitute asset class with higher (lower) CPT values. Thus, for each specified VF/PWF, two separate portfolios emerge. The first portfolio constitutes asset class within the top eight ranked assets and the second portfolio from the bottom eight ranked assets. This implies that, for eight different VF/PWF, 16 different portfolios are expected, eight portfolios apart constituting assets with a higher and a lower CPT values.

However, several portfolios based on these VF/PWF share the same set of assets (see, Tables 6 to 9). Hence, the number of portfolios as earlier envisaged is greatly reduced. The formulated portfolios based on the CPT model with the various VF/PWF parameter estimates are explicitly described and summarised below in both markets.

Portfolio Set A: Cryptocurrency Market

(1) **Portfolio (1H)**: Classified and selected cryptocurrencies with higher CPT values
Assets: *DGB, SBD, XVG, XEM XRP, XLM, REP*, and *XMR*.
VF/PWF specification used: Tversky and Kahneman (1992), Abdellaoui (2000), Donkers et al. (2001), Abdellaoui et al. (2005), Tu (2005), Fehr-Duda et al. (2006), and Harrison and Rutström (2009)

(2) **Portfolio (1L)**: Classified and selected cryptocurrencies with lower CPT values
Assets: *DOGE, ISK, DASH, ETC, WAVES, FCT, ETH*, and *BTC*.
VF/PWF specification used: Tversky and Kahneman (1992), Tu (2005), Harrison and Rutström (2009), Donkers et al. (2001), Fehr-Duda et al. (2006), Abdellaoui (2000), and Abdellaoui et al. (2005).

(3) **Portfolio (2H)**: Classified and selected cryptocurrencies with higher CPT values
Assets: *XVG, PIVX, DGB, XLM, SC, BTS, DCR*, and *ISK*.
VF/PWF specification used: Andersen et al. (2006)

(4) **Portfolio (2L)**: Classified and selected cryptocurrencies with lower CPT values
Assets: *WAVES, DOGE, ETC, DASH, SBD, LTC, ETH*, and *BTC*.
VF/PWF specification used: Andersen et al. (2006)

Portfolio Set B: International Equity Market.

(1) **Portfolio (1H*)**: Classified and selected international equities with higher CPT values
Assets: *Russian MICEX, KOSPI, Russia RTS, BOVESPA, BUDAPEST, NIFTY 500*, Colombo, and BIST
VF/PWF specification used: Tversky and Kahneman (1992), Abdellaoui (2000),

Donkers et al. (2001), Abdellaoui et al. (2005), Tu (2005), Fehr-Duda et al. (2006), and Harrison and Rutström (2009).

(2) **Portfolio (1L*)**: Classified and selected international equities with lower CPT values
Assets: *S&P 500 E/W, Shanghai ALL Share, DOW JONES Share, BEL 20, RUSSELL 2000, TASI, Taiwan SE W/I*, and *Tel Aviv TA 35*
VF/PWF specification used: Tversky and Kahneman (1992), Tu (2005), Harrison and Rutström (2009), Donkers et al. (2001), Fehr-Duda et al. (2006), Abdellaoui (2000), and Abdellaoui et al. (2005).

(3) **Portfolio (2H*)**: Classified and selected international equities with higher CPT values
Assets: *BOVESPA, BIST, Russian MICEX, Russia RTS, BUDAPEST, FTSE JSE, OMXS*, and *KOSPI*.
VF/PWF specification used: Andersen et al. (2006)

(4) **Portfolio (2L*)**: Classified and selected international equities with lower CPT values
Assets: *IDX Comp, NASDAQ Comp, FTSE 100, S&P 500 E/W, SMI, S&P 500 Comp, Dow Jones Comp*, and *Colombo*
VF/PWF specification used: Andersen et al. (2006)

From the above formulated-portfolios, instead of constructing 16 portfolios as earlier predicted, we only had eight portfolios. From the empirical point of view, this primarily suggests that some of the VF/PWF implemented in this study resulted in similar conclusions. Thus, the empirical content of most of the VF/PWF compliments and supports the findings of Tversky and Kahneman (1992).

3.2.3. Descriptive statistics of classified and selected assets

A cursory inspection of the descriptive statistics in both markets shows evidence of leptokurtic distribution across the selected and classified assets. The results further show a rejection of the normality hypothesis captured by the Jarque–Bera test at the 1% level. With only two (one) exceptions (i.e. COLOMBO, SMI (BTC)) which showed positive (negative) skewness in the international equity (cryptocurrency) markets.

Apart from the few exceptions, the majority of the selected and classified assets showed positive (negative) skewness in the cryptocurrency (international equity) markets as evident, respectively, in Tables 10 and 11. Skewness of return distribution plays a significant role in the decision-making process of many investors and reflects the price of the financial asset. Thus, the probability of loss (gain) increases (decreases) with negative (positive) skewness. A tranche of studies in the literature suggests that the increase of investors' heterogeneity could lead to a negative skew distribution of the return series, for there existed short-sale constraints (see, for example, Chen, Hong, & Stein, 2001; Hong & Stein, 2003).

Evidentially, the characteristics of the return distribution of the entire assets conform to the stylized facts governing financial assets. Among other things, stylized facts stipulate that return distribution deviates from normal distribution.

3.2.4. Portfolio optimization

We proceed to evaluate the optimization result of all the formulated portfolios. The M-V criterion is adopted as optimization technique to evaluate the eight portfolios. The portfolio return and conditional Value-at-Risk (hereafter, CVaR) of the efficient

Table 10. Descriptive Statistics of Cryptocurrencies.

	1H						1L				
Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats	Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats
DGB	0.2812	5.0403	2.3381	23.5353	11,236.96*	**DOGE**	0.1794	3.4729	0.5569	11.0522	1673.97*
SBD	0.0492	3.9748	3.4335	48.7453	54,207.92*	**ISK**	0.2383	3.8077	0.2360	6.8328	377.81*
XVG	0.5405	8.5876	0.5705	7.1041	459.69*	**DASH**	0.2502	2.9678	0.8591	8.7085	900.31*
XEM	0.2506	4.0704	2.2428	23.5898	11,249.50*	**ETC**	0.1487	3.3971	0.1700	8.2243	694.35*
XRP	0.3178	4.0481	2.3298	22.2951	9981.63*	**WAVES**	0.2176	3.3738	0.1683	5.1871	124.05*
XLM	0.3286	4.6324	1.8042	13.3479	3042.55*	**FCT**	0.1679	3.6332	0.0102	4.9563	96.97*
REP	0.0823	3.9644	0.3262	9.8125	1186.52*	**ETH**	0.2504	2.7578	0.1602	6.9843	404.76*
XMR	0.3257	3.6553	1.3241	11.7409	2113.22*	**BTC**	0.1717	2.0347	−0.2232	8.1480	676.42*
	2H						2L				
Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats	Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats
XVG	0.5405	8.5876	0.5705	7.1041	459.69*	**WAVES**	0.2176	3.3738	0.1683	5.1871	124.05*
PIVX	0.5092	4.8693	0.8447	5.9128	287.24*	**DOGE**	0.1794	3.4729	0.5569	11.0522	1673.97*
DGB	0.2812	5.0403	2.3381	23.5353	11,236.96*	**ETC**	0.1487	3.3971	0.1700	8.2243	694.35*
XLM	0.3286	4.6324	1.8042	13.3479	3042.55*	**DASH**	0.2502	2.9678	0.8591	8.7085	900.31*
SC	0.2022	4.4386	0.8771	7.0059	484.49*	**SBD**	0.0492	3.9748	3.4335	48.7453	54,207.92*
BTS	0.2349	4.1570	0.7230	7.8963	660.30*	**LTC**	0.2429	2.9941	1.3240	13.8855	3179.37*
DCR	0.2184	3.9731	0.9043	5.8900	294.46*	**ETH**	0.2504	2.7578	0.1602	6.9843	404.76*
ISK	0.2383	3.8077	0.2360	6.8328	377.81*	**BTC**	0.1717	2.0347	−0.2232	8.1480	676.42*

Note: As shown in Table 4.1, the kurtosis of each asset price returns is larger than 3, and the skewness is either less than or greater than 0, which indicates that all series have fat tails and leptokurtosis. From the JB statistics, we know that they do not follow normal distribution. So it is effective to fit the series using a suitable GARCH model. An asterisk (*) indicates a rejection of the null hypothesis at the 1% level.

Table 11. Descriptive Statistics of International Equities.

	1H*						1L*				
Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats	Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats
MICEX	0.0169	1.0066	−0.2012	16.3979	29,322.77	**S&P 500 E/P**	0.0130	0.5732	−0.3469	11.721	12,491.46
KOSPI	0.0042	0.8167	−0.9961	16.7611	31,554.05	**SHANGHAI**	0.0090	0.7220	−0.4259	8.1210	4398.56
R. RTS	0.0142	0.9157	−0.3957	13.5947	18,421.81	**DOW JONES**	0.0108	0.4996	−0.1980	10.8167	9997.84
BOVESPA	0.0158	1.0562	−0.4977	9.9407	8023.97	**BEL 20**	0.0082	0.6262	−0.2003	8.9194	5744.94
BUDAPEST	0.0181	0.8842	−0.1562	10.8572	10,091.61	**RUSSEL**	0.0120	0.6573	−0.4225	8.8017	5610.15
NIFTY 500	0.0239	0.7102	−0.3563	12.0306	13,392.93	**TASI**	0.0136	0.6732	−1.1671	13.9752	20,548.41
COLOMBO	0.0240	0.5205	0.2262	33.113	148,029.3	**TAIEX**	0.0093	0.5882	−0.2630	5.9069	1424.31
BIST	0.0132	1.0522	−0.3496	8.4601	4945.40	**TA 35**	0.0160	0.5902	−0.3601	6.5001	2084.13
	2H*						2L*				
Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats	Assets	Mean	Std. Dev	Skewness	Kurtosis	JB Stats
BOVESPA	0.0158	1.0562	−0.4977	9.9407	8023.97	**IDX COMP**	0.0155	0.5907	−0.1170	9.0305	5944.36
BIST	0.0132	1.0522	−0.3496	8.4601	4945.40	**NASDAQ**	0.0155	0.5907	−0.1170	9.0305	5944.36
MICEX	0.0169	1.0066	−0.2012	16.3979	29,322.77	**FTSE 100**	0.0035	0.6067	−0.1710	13.0809	16,605.14
R. RTS	0.0142	0.9157	−0.3957	13.5947	18,421.81	**S&P 500 E/W**	0.0130	0.5732	−0.3469	11.7210	12,491.46
BUDAPEST	0.0181	0.8842	−0.1562	10.8572	10,091.61	**SMI**	0.0091	0.5448	0.2632	13.1544	16,873.81
FSTE JSE	0.0164	0.8055	−0.2780	8.2371	4526.87	**S&P 500 COM**	0.0095	0.5253	−0.2328	13.1602	16,883.51
OMXS	0.0096	0.7697	−0.0762	9.5168	6935.05	**DOW JONES**	0.0108	0.4996	−0.1980	10.8167	9997.84
KOSPI	0.0042	0.8167	−0.9961	16.7611	31,554.05	**COLOMBO**	0.0240	0.5205	0.2262	33.1130	148,029.30

Note: As shown in Table 4.1, the kurtosis of each asset price returns is larger than 3, and the skewness is either less than or greater than 0, which indicates that all series have fat tails and leptokurtosis. From the JB statistics, we know that they do not follow normal distribution. So it is effective to fit the series using a suitable GARCH model. An asterisk (*) indicates a rejection of the null hypothesis at the 1% level.

Table 12. Optimization Results of Cryptocurrencies.

Portfolio Classification		Optimisation Results							
1H	**Ticker**	**DGB**	**SBD**	**XVG**	**XEM**	**XRP**	**XLM**	**REP**	**XMR**
	Weight (%)	11.95	0.00	1.13	26.44	33.45	2.39	13.46	11.19
	Return				0.8336				
	CVaR				−0.1015				
1L	**Ticker**	**DOGE**	**ISK**	**DASH**	**ETC**	**WAVES**	**FCT**	**ETH**	**BTC**
	Weight (%)	0.00	26.94	7.33	19.31	6.54	6.50	4.50	28.88
	Return				1.0190				
	CVaR				0.1718				
2H	**Ticker**	**XVG**	**PIVX**	**DGB**	**XLM**	**SC**	**NTS**	**DCR**	**ISK**
	Weight (%)	0.00	19.36	6.86	23.68	8.34	13.77	10.06	17.92
	Return				0.9661				
	CVaR				0.0541				
2L	**Ticker**	**WAVES**	**DOGE**	**ETC**	**DASH**	**SBD**	**LTC**	**ETH**	**BTC**
	Weight (%)	9.13	10.64	6.84	21.83	6.39	16.43	8.84	19.90
	Return				0.9216				
	CVaR				0.0291				

Note: This table presents optimization results of portfolios made up of cryptocurrencies and benchmark for portfolio performance analysis (i.e. return and CVaR).

portfolio described by the M-V criterion are chosen as a benchmark to evaluate all the formulated portfolios. Specifically, the efficient portfolio's return (CVaR) statistics which represent the portfolio reward (risk) are the only two metrics for assessing the performance of all portfolios.

The M-V analysis was implemented on the four formulated portfolios containing assets which were selected and classified by the CPT model in both markets. Tables 12 and 13 present the results of the optimization analysis of portfolios constituting, respectively, assets from the cryptocurrency and the international equity markets. We seek to validate if investors could add value to their investment either by investing in assets classified as having higher/lower CPT values.

Cryptocurrency market

Table 12 displays the optimization results of four portfolios constituting assets from the cryptocurrency market only. Primarily, two pairs of portfolios are formulated based on two different VF/PWF specifications, and each portfolio is made up of assets with either a higher or a lower CPT value (see, the portfolio classifications in Table 11). Portfolio 1H (1L) and portfolio 2H (2L) represent portfolios made up of asset with higher (lower) CPT values.

Considering portfolios 1H (1L), while portfolio 1H recorded a return (risk) of 0.8336 (−0.1015), portfolio 1L, on the other hand, recorded a return of 1.0190 with a corresponding risk of 0.1718. Clearly, it is evident that portfolio 1L recorded a higher return in comparison with portfolio 1H. Likewise, the risk associated with portfolio 1H renders it unattractive to investors against that of portfolio 1L. This makes portfolio 1L more attractive to investors than portfolio 1H considering the return (risk) statistics. This implies that investing in portfolio 1L is more profitable than portfolio 1H considering the portfolio performance benchmarks.

All portfolios were highly diversified and constituted seven assets instead of the initial eight selected and classified assets. Thus, the diversification of the assets was not exhaustive. From Table 12, assets that received the highest (lowest) investment weights in portfolio 1H were recorded as XRP: 33.45% (XVG: 1.13%). In a similar vein, portfolio 1L, recorded BTC: 28.88% (ETH: 4.50%) as assets with highest (lowest)

Table 13. Optimization Results of International Equity Indices.

Portfolio Classification		Optimisation Results							
1H*	**Ticker**	**MICEX**	**KOSPI**	**RTS**	**BOVESPA**	**BUDAPEST**	**NIFTY**	**COLUMBO**	**BIST**
	Weight (%)	12.48	12.65	2.21	22.65	14.73	15.74	9.09	10.45
	Return					2.0619			
	CVaR					−0.0592			
1L*	**Ticker**	**S&P E/W**	**SHANGHAI**	**D. JONES**	**BEL 20**	**RUSSELL**	**TASI**	**TAIEX**	**TA 35**
	Weight (%)	3.97	5.60	7.12	13.50	23.39	11.24	28.81	6.37
	Return					2.2694			
	CVaR					0.1337			
2H*	**Ticker**	**BOVESPA**	**BIST**	**MICEX**	**RTS**	**BUDAPEST**	**FTSE_JSE**	**OMXS**	**KOSPI**
	Weight (%)	12.95	12.67	17.93	20.35	8.17	18.17	8.58	1.19
	Return					2.0333			
	CVaR					−0.0379			
2L*	**Ticker**	**IDX**	**NASDAQ**	**FTSE100**	**S&P E/W**	**SMI**	**S&P COMP**	**D. JONES**	**COLOMBO**
	Weight (%)	23.75	53.57	0.00	0.00	7.84	0.00	3.61	11.23
	Return					2.0833			
	CVaR					0.5349			

Note: This table presents the optimization results of portfolios made up of international equities and benchmark for portfolio performance analysis (i.e. return and CVaR).

investment weights. Assets in portfolio 1H/1L were selected and classified based on estimates from seven different researchers including the Tversky and Kahneman (1992).

Comparatively, results of portfolio 2H (2L) show counter evidence compared to portfolio 1H (1L). The portfolio return and its corresponding risk projected portfolio 2H as having the potential to outperform portfolio 2L. This contradicts the findings of portfolio 1H and 1L. For example, while the portfolio return (risk) of portfolio 2H were recorded as 0.9661 (0.0541), on the other hand, portfolio 2L was estimated to be 0.9216 (0.0291). The return of portfolio 2H is higher than 2L. Similarly, the risk associated with the two portfolios were found be positive. However, portfolio risk of 2H was greater than that of portfolio 2L. Thus, portfolio 2H is expected to be more attractive to investors than portfolio 2L based on the portfolio risk (return) statistics.

With respect to diversification, while portfolio 2H was found to be fairly diversified constituting seven out of the eight assets, portfolio 2L constitutes the entire assets class and exhaustive compared to portfolio 2H. Out of the seven (eight) assets of portfolio 2H (2L), the following assets XLM: 23.68%, PIVX: 19.36% (DASH: 21.83%, BTC: 19.30%) were found to have received the highest investments weights. Same portfolio sets unlike portfolio 1H (1L) also favored portfolios constituting assets with higher CPT values as having the potential in adding value to investors' portfolio.

Findings of preliminary empirical optimization analysis of these portfolio sets are inconclusive. Even though it is evident in Table 12 that investors' portfolio choice was found to be 1L and 2H, however, comparing portfolios 1L and 2H, portfolio 2L outperforms portfolio 2H based on the performance benchmark adopted. The results so far suggest that there is a greater investment potentials in investing in portfolio 1L than portfolio 2H. This implies that portfolios made up of assets classified as having lower CPT values have a greater potential in adding value to investors' portfolio.

For purposes of robustness, we proceed to conduct similar portfolio analysis adopting data from the international equity market. The significant differences between the cryptocurrencies and the international equities have to do with the trading days and currency type. While the cryptocurrencies are traded seven days per week with virtual/digital currencies, the international equities are traded five days per week with conventional/traditional currencies.

International equity markets

From Table 13, it is evident that portfolio 1L* outperforms portfolio 1H* considering the portfolio evaluation benchmarks. While portfolio 1L* recorded a return (risk) of 2.2694 (0.1337), portfolio 1H* on the other hand, recorded return (risk) of 2.0619 (−0.0592). Thus, investors are expected to be attracted to portfolio 1L* compared to 1H*. This is primarily because, portfolio 1L* has the highest portfolio return and a corresponding positive CVaR.

Both portfolios 1H* (1L*) were exhaustively diversified with BOVESPA: 22.65% and RTS: 2.21% (TAIEX: 28.81% and S&P E/W: 3.97%) representing assets with the highest (lowest) investment weights. This result is consistent with the findings of the previous results using assets in the cryptocurrency market.

On the other front, it is evident in Table 13 that portfolio 2L* is less diversified compared to portfolio 2H* which is well diversified. While portfolio 2H* constituted the entire eight assets, portfolio 2L* is made up of only five assets out of a possible eight assets. Thus, to achieve a portfolio return of 2.0833 and a corresponding risk of 0.5349, portfolio 2L* requires only five assets out of a total of eight assets. In this portfolio, the following assets, NASDAQ and IDX recorded the highest investment weights of 53.57% and 23.75%, respectively.

Similarly, portfolio 2H* recorded a portfolio return of 2.0333 with a corresponding risk of −0.0379. Unlike portfolio 2L*, portfolio 2H* included the entire eight assets to achieve this target return. Out of the eight assets in portfolio 2H*, RTS: 20.35% and FTSE-JSE: 18.17% received the highest investment weights. Based on the selected benchmark for evaluating the portfolio performances, portfolio 2L* emerged the investors choice.

The results reported in this paper have important implications from a portfolio management perspective. The paper's findings contribute to the literature by examining and providing original and empirical evidence on asset selection and portfolio optimization under behavioral setting. Thus, the paper provides insights by proposing a new empirical framework for pre-selecting assets for further optimization analysis.

4. Conclusions

This paper examined and attempted to validate if investors could add value to their investment holdings by investing in behaviorally driven assets in the cryptocurrency and international equity markets.

Preliminary empirical results relating to the behavioral asset classification show that six out of seven adopted VF/PWF affirmed the empirical results of the CPT model proposed by Tversky and Kahneman (1992). These six VF/PWF were independently put forward by the following researchers: Abdellaoui (2000), Donkers et al. (2001), Abdellaoui et al. (2005), Tu (2005), Fehr-Duda et al. (2006), and Harrison and Rutström (2009). These included both the 1- and the 2-parameter VF/PWF. Similarly, the only remaining VF/PWF, a 1-parameter function which was inconsistent with the results of the Tversky and Kahneman (1992) was introduced by Andersen et al. (2006).

Overall, the optimization results obtained in analyzing all formulated portfolios in both markets revealed the likelihood of investors to add value to their investment holdings by investing in assets with extremely lower CPT values. Also, the results showed that the VF/PWF specification introduced by Tversky and Kahneman (1992) is ideal to deliver such outcomes compared to alternative specifications proposed by Prelec (1998) and Goldstein and Einhorn (1987). Similarly, the alternative estimates of the VF/PWF specifications (see, Abdellaoui, 2000; Donkers et al., 2001; Abdellaoui et al., 2005; Tu, 2005; Fehr-Duda et al., 2006;; Harrison & Rutström, 2009) proposed by Goldstein and Einhorn (1987), Tversky and Kahneman (1992), and Prelec (1998) equally support the Tversky and Kahneman (1992) results.

Indeed, the CPT model has been established in this study as a gold standard in decision theory specifically in classifying market assets for optimization purposes. Also, the findings in this study are in contrast with a study by Stott (2006) who opined that several studies seem to favor the two-parameter PWF specification. However, according to Stott (2006), none of the PWF has been pronounced as superior.

Empirical findings in this study will be of interest to both academics and investment practitioners. On one hand, we add new insights to the sparse literature on Behavioural Economics and offer a complementary explanation to the asset selection and portfolio optimization in a market environment dominated by real and normal investors. More so, we show that investing in assets with behavioral foundation could play a significant role in the management of portfolio risk especially assets with lower CPT values. Specifically, our findings raise several questions which deserve further empirical research.

Disclosure statement

No potential conflict of interest was reported by the author.

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