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Investing with cryptocurrencies – evaluating their potential for portfolio allocation strategies

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Cryptocurrencies (CCs) have risen rapidly in market capitalization over the past years. Despite striking volatility, their high average returns and low correlations have established CCs as alternative investment assets for portfolio and risk management. We investigate the benefits of adding CCs to well-diversified portfolios of conventional financial assets for different types of investors, including risk-averse, return-maximizing and diversification-seeking investors who may trade at different frequencies, namely, daily, weekly or monthly. We calculate out-of-sample performance and diversification benefits for the most popular portfolio-construction rules, including mean-variance optimization, risk-parity, and maximum-diversification strategies, as well as combined strategies. Our results demonstrate that CCs can improve the risk-return profile of portfolios, but their benefit depends on investor objectives. In particular, diversification strategies (maximizing the portfolio diversification index or equating risk contributions) draw appreciably on CCs and show, in line with spanning tests, CCs to be non-redundant extensions of the investment universe. However, when we introduce liquidity constraints via the LIBRO method to account for illiquidity of many CCs, out-of-sample performance drops considerably, while the diversification benefits persist. We conclude that the utility of CC investments strongly depends on investor characteristics.

Keywords: Cryptocurrency; CRIX; Investments; Portfolio management; Asset classes; Blockchain; Bitcoin; Altcoins; DLT

JEL Classification: C01, C59, G11

1. Introduction

Cryptocurrencies (CCs) have exhibited remarkable performance in the decade since Nakamoto (2008) invented the blockchain. Accompanied by huge inflows of capital into the market and strong swings in prices, CCs have gained strongly in market value. Accordingly, indices like CRIX (Trimborn

and Härdle 2018, thecrix.de) were introduced to capture the market evolution. Driven by these developments, cryptocurrency markets became increasingly attractive to investors, who have started to consider CCs as a novel class of alternative investments. However, the potential role of CCs in investors' portfolios is still not fully understood and thus remains controversial. Our paper studies this role from the perspective of an investor choosing his portfolio who is not restricted to CCs but considers including CCs to a well-diversified portfolio.

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Table 1. List and categorization of all asset allocation models we implement, including their abbreviations and references.

Model	Reference	Abbreviation
<i>Model-free strategies</i>		
Equally weighted	DeMiguel <i>et al.</i> (2009)	EW
<i>Risk-oriented strategies</i>		
Mean-Variance – min Var	Merton (1980)	MinVar
Mean-CVaR – min risk	Rockafellar and Uryasev (2000)	MinCVaR
Equal Risk Contribution (Risk-parity)	Maillard <i>et al.</i> (2010)	ERC
Maximum Diversification	Rudin and Morgan (2006)	MD
<i>Return-oriented strategies</i>		
Risk-Return – max return	Markowitz (1952)	RR-MaxRet
<i>Risk-Return-oriented strategies</i>		
Mean-Variance – max Sharpe	Jagannathan and Ma (2003)	MV-S
<i>Combination models</i>		
Naïve Combination	Schanbacher (2015)	CombNaïve
Weight Combination	Schanbacher (2014)	Comb

While we review the relevant literature extensively in section 2, it is well-established by numerous studies that CC investments exhibit remarkably high average realized returns by the standards of traditional financial assets[†] – and correspondingly high risk and uncertainty. Not only is price volatility high; also unfavorable properties obtrude, including frequent pricing bubbles (Fry and Cheah 2016, Hafner 2018, Chen and Hafner 2019, Núñez *et al.* 2019), accumulation of jumps (Scaillet *et al.* 2018), even evidence of price manipulation (Gandal *et al.* 2018).

At the same time, there is evidence of low correlations of CC returns with those of traditional financial assets and other CCs. Therefore, the high risk of CC positions may be compensated by appropriate returns as well as provide an opportunity to increase portfolio diversification. Results to that effect have been spearheaded by Brière *et al.* (2015) and Eisl *et al.* (2015), the first to include Bitcoin (BTC) in a portfolio of traditional assets, and subsequently bolstered by Elendner *et al.* (2018), who include a broad cross-section of CCs, Chuen *et al.* (2017), who instead add CRIX, and lately Platanakis and Urquhart (2019) and Akhtaruzzaman *et al.* (2019), who include Bitcoin in advanced portfolio optimization and find it enhances the risk-return profile.

Therefore, evidence exists that CCs can be beneficial to investors (Pele *et al.* 2020). However, taking the investor’s perspective, and acknowledging that prior studies have covered crucially important aspects of investing with CCs, some critical questions remain open for two reasons. First, the key result of a diversification benefit of CCs (or BTC) cannot be established unless a broad set of non-CC alternative investments is included. For simplicity, in this paper, we refer to all non-CC investments as ‘traditional assets’, including alternative investments like gold or real estate. We do this in order to focus on the potential of adding CCs to well-diversified

[†] We do not compare CCs to derivatives, as they clearly constitute underlyings—in fact, a common complaint, albeit ignoring their economic role, laments that, in the words of Alan Greenspan, ‘You have to really stretch your imagination to infer what the intrinsic value of Bitcoin is. I haven’t been able to do it’ (Kearns 2013). CC derivative markets, while growing, still remain quite nascent.

portfolios. Second, it remains unclear which strategies can actually be implemented in practice unless specific care is taken to address the frequently extremely dry liquidity in CC markets.

The importance of addressing liquidity concerns is pinpointed by Trimbom *et al.* (2019), who introduce LIquidity Bounded Risk-return Optimization (LIBRO) when considering a large sample of CCs added to a portfolio consisting of the S&P100, US bonds and commodities. Given the low liquidity of CCs as compared to traditional markets, LIBRO is designed to protect investors from an inability to trade a CC in necessary amounts due to low trading volume.

Against this background, we take the investor’s perspective and perform a large-scale comparative investment-strategy study including *both* a broad range of traditional assets together with a broad cross-section of CCs. Therein, we test the performance of an extensive set of common investment strategies and thus consider different types of investors, while we employ the LIBRO method to handle liquidity concerns. We consider risk-oriented, return-oriented, risk-return-oriented, and combined strategies; see table 1 for a full list of strategies under consideration. We estimate optimal portfolios for a sizable number of common objective functions and compare out-of-sample performance and diversification across all strategies, each based on three re-allocation frequencies, namely, daily, weekly and monthly. To the best of our knowledge, we thus present the broadest study on investing with CCs conducted so far.

Closest related to our paper are Akhtaruzzaman *et al.* (2019) and Platanakis and Urquhart (2019), both also studying the influence of CC investment on optimal portfolio composition. However, both include only Bitcoin,[‡] whereas we consider a broad cross-section of 52 distinct CCs. Moreover, both consider fewer traditional assets: industry portfolios (i.e. equity only) in the former paper, US equity and bond investments in the latter, plus commodities in a robustness test. In

[‡] Platanakis and Urquhart (2019) do run a robustness test replacing Bitcoin with CRIX, acknowledging the importance of altcoins. Naturally, diversification *across* CCs necessitates an optimization including their individual, distinct return series.

Table 2. List of traditional constituents of the investment universe.

Name	Asset class
EURO STOXX 50	Equity
S&P100	Equity
NIKKEI225	Equity
FTSE100	Equity
SSE (Shanghai Stock Exchange) index	Equity
MSCI ACWI COMMODITY PRODUCERS	Commodities
GOLD	Commodities
FTSE EPRA/NAREIT DEV REITS	Real Estate
EUR/USD	Fiat currency
GBP/USD	Fiat currency
CNY/USD	Fiat currency
YEN/USD	Fiat currency
Eurozone 10Y Gov Bonds	Fixed income
UK 10Y Gov Bonds	Fixed income
USA 10Y Treasuries	Fixed income
Japan 10Y Gov Bonds	Fixed income

Note that we term all these asset classes, including alternative assets, ‘traditional’ in order to contrast them with investments in cryptocurrencies (CCs). We obtain price series for all traditional investments from Bloomberg.

contrast, our set of traditional assets is critically broader: first, as CCs trade globally, our international approach includes equity returns for each of the five major economic areas (Europe, USA, Japan, UK, China), as well as region-specific bond returns. Second, we always include alternative investments, namely, gold, real estate, commodities, and the returns to FX trades between the five regions’ fiat currencies. Table 2 lists the traditional assets all our portfolios include. As we have pointed out, this emphatically goes beyond quantitatively extending prior studies: unless *both*, a broad cross-section of CCs *and* of traditional assets, are included, it remains impossible to determine the magnitude of diversification benefits, and more critically, also impossible to distinguish whether apparent benefits of CCs are indeed present, or if CCs merely proxy for alternative assets.

Moreover, we cover a longer time horizon and can thus include more than two years after peak CC prices; also, we consider more allocation strategies. Most importantly, since we take the investor’s perspective, we implement LIBRO and contrast portfolios with weights that observe the liquidity constraints with otherwise identical portfolios which do not: it turns out to critically affect performance for several popular trading strategies.

The research hypotheses which we test provide a number of insights about the advantages of considering CCs in portfolio optimization. Spanning tests show that some CCs can expand the efficient frontier even beyond that from our broad set of traditional assets. The performance of the optimized portfolios under consideration differs significantly from that of a standard mean-variance-optimal portfolio, as do performances across various investor profiles. We show that purely risk-minimizing investors choose to optimally forego CC investment; however, investors with higher target returns optimally include CCs. Diversification-seeking investors benefit most, even in terms of final cumulative wealth.

Our hypotheses tests also show that Sharpe Ratios are unaffected by the inclusion of CCs to most portfolios,

implying that additional risk from high CC volatility is already adequately compensated via higher expected returns and CC markets well integrated with financial markets. In terms of risk improvement, we also show how the diversification effect differs substantially depending on investor profile. Variation between respective measures is large, illustrating the need for a differentiated view of the effect of CCs for different types of investors. Testing whether differences in trading frequency affect portfolio returns, we find no statistically significant difference. Thus, rare (monthly) rebalancing is sufficient to benefit from CC investments (and reduces transaction costs).

Furthermore, we test the hypothesis if the efficient frontier is affected by including CCs, and it is: investors should not only focus on BTC, despite the fact that many CCs provide no longer statistically significant improvements in portfolio construction. Hence investors should focus on a selection of CCs. Thereby, they need to mind the low liquidity of several CCs, because we show that tackling it with the LIBRO approach leads to significantly different return distributions of portfolios which are strongly exposed to a singular CC or a plethora of illiquid CCs. Our results highlight how analyses that do not take liquidity into account will compute investment returns that are infeasible for any but the smallest personal portfolios.

The paper is organized as follows. First, section 2 reviews the related literature and section 3 develops our research hypotheses. Section 4 provides an overview of the asset-allocation models under consideration, with a focus on connections between them; therein section 4.2 explains the approach of model averaging across investment strategies. Section 5 reviews the LIBRO method. In section 6, we explain the methodology for comparing the performance of the models considered. Our dataset of portfolio components is described in section 7, and section 8 presents the results of our analyses of out-of-sample performance of all portfolio strategies with CCs and traditional assets. We conclude in section 9.

Code to produce the results of this paper is available via www.quantlet.de.

2. Literature review

Modern portfolio theory builds on the CAPM (Markowitz 1952, Sharpe 1964, Lintner 1965), both a theoretical equilibrium model and a directly applicable statistical approach. Yet, financial markets do not meet its assumptions, so it lacks empirical accuracy. Asset pricing and portfolio optimization address this lack in one of two ways.

The first we call the *financial-economics approach*: it follows Ross’s (1976) arbitrage-pricing theory[†] which keeps the linear structure and adds more factors to capture systematic patterns in returns. Popularized by Fama and French (1992, 1993), it was extended to factors for momentum (Jegadeesh and Titman 1993, Carhart 1997) or

[†] This approach puts the emphasis on the equilibrium model and is thus often preferred by theorists.

profitability and investment (Fama and French 2015). In principle, the approach renders portfolio optimization straightforward and unidimensional: a portfolio is better, the higher its alpha (the intercept after accounting for all factors' loadings). In practice, the choice of factors depends on the investment universe, and also for given asset classes controversy remains about factors (the 'zoo' of Cochrane 2011), how to choose them (Feng *et al.* 2020), even basic methodology (Novy-Marx 2014).

A strand of the literature on cryptocurrencies (CCs) is devoted to finding and using factors in CC markets (see Hubrich 2017, Elendner 2018, Sovbetov 2018, Liu *et al.* 2019, Shen *et al.* 2020); however, in this paper, we pursue the second approach.

We term it the *quantitative-finance approach*, due to its statistical nature. Its idea, in essence, says: if we can capture the (joint) return distribution (and its dynamics) of all investable assets (and parameters affecting them), then we can directly estimate portfolio weights to optimize the desired performance metric. Owing to the abundance of statistical techniques for the variety of modeling choices and investment objectives, this approach is most common in fund management.[†] However, the easy customization has precluded a standard, unique approach. A portfolio's optimal allocation thus depends crucially on three elements: the investment universe, the investment strategy, and the investment objective as defined by the metric of optimization.

Most fundamental is the determination of the *investment universe*. Our paper focuses on its role by analyzing it for extensive sets of common strategies and objective functions; concretely, on the potential of adding CCs. Historically, starting from stocks[‡] and a risk-free interest rate,[§] the diversification benefits to adding bonds (Liu 2016), foreign exchange (Kroencke *et al.* 2013, Barroso and Santa-Clara 2015, Ackermann *et al.* 2017), real estate (Benjamin *et al.* 2001, Addae-Dapaah and Loh 2005), and commodities (Belousova and Dorfleitner 2012) including gold (Hoang *et al.* 2015) have been established in the literature.[¶] We term *all* these assets 'traditional investments', and we include proxies for all of them in our benchmark portfolio. This breadth is key, as our goal is to investigate the effect of including *additionally* CCs. Only the broad traditional portfolio ensures we assess the diversification potential of CCs as investments: otherwise, CCs might merely substitute for other alternative investments.

Considering CCs as investments contains subtle irony, as Nakamoto (2008) pseudonymously introduced the blockchain as a technology to serve as money,^{||} not a profitable investment opportunity. Thus, initially doubt and debate

shrouded the economic role^{††} of CCs (Glaser *et al.* 2014, Yermack 2015, Böhme *et al.* 2015, Baur *et al.* 2018).^{‡‡}

However, after more than a decade of increasing demand, market capitalizations, and trading volumes for a multiplying number of CCs,^{§§} the recently flourishing academic literature converges to the consensus that CCs constitute investments, generally, and a distinct asset class in particular. This literature can be categorized along two dimensions: first, which CC investment is considered? Only Bitcoin, also a fistful of other highly visible CCs like Ethereum or Ripple, or a broad cross-section of tradable CCs?^{¶¶} Second, which portfolio allocations are considered? Only the CC(s), or also traditional markets? If the latter, only equity markets, or a broad range of traditional investments?

Regarding the first point, the literature started by investigating the properties of the Bitcoin price process (Kristoufek 2015, Chu *et al.* 2015, Cheah and Fry 2015, Urquhart 2017, Bariviera *et al.* 2017, Osterrieder 2017, Blau 2018, Liu and Tsyvinski 2018), establishing, in essence, the presence of all critical properties of equity returns, yet often up to an order of magnitude stronger: CCs exhibit exceptionally high mean returns, and likewise volatility and drawdowns; returns also feature extremely heavy tails and high heteroskedasticity. Correlations with common return series turn out extremely low to non-existent.

Given such low correlations, it is intuitive why including Bitcoin can enhance a portfolio of traditional assets.

Whereas the high riskiness leads to low portfolio weights for risk-averse investors, an inclusion is beneficial as it improves diversification.

At the same time, exploding interest in blockchain led to an explosion in the number of investable CCs.^{|||} This kickstarted investigations into the joint-return properties of a broad cross-section of CCs (Elendner *et al.* 2018, Wang and Vergne 2017, Brauneis and Mestel 2018, Zhang *et al.* 2018, Wei 2018), which confirmed the return characteristics of Bitcoin to be representative for the entire asset

[†] An additional benefit is how it links potential empirical shortcomings to insufficiently captured statistical properties, offering remedy via more refined methods.

[‡] Markowitz (1952).

[§] Sharpe (1964).

[¶] In fact, already Roll (1977) had stressed the 'market portfolio' ought to include *all* wealth. Naturally, his critique has led to innumerable suggestions for further asset classes that cannot all be part of our analysis, including private equity (Gompers *et al.* 2010), fine art (Mei and Moses 2002, Campbell 2008), or even fine wine (Fogarty 2010, Chu 2014).

^{||} More precisely, the intent was a protocol with the emphasis on the tokens' role as medium of exchange, not as stores of value.

^{††} Some debate centered on the question whether investments in CCs play an economic role similar to gold: See Dyhrberg (2016), Shahzad *et al.* (2019) for affirmative views, and Klein *et al.* (2018) for a dissenting one.

^{‡‡} Generally, mainstream economics has joined the research effort on CCs deplorably late; it is now catching up, see for instance Schilling and Uhlig (2019), Abadi and Brunnermeier (2019). Game-theoretic modeling has been more active, including Houy (2016), Dimitri (2017), Caginalp and Caginalp (2019), Bolt and van Oordt (2020).

^{§§} At the latest update of this writing, in 2021-Q1, the leading dedicated information platform coinmarketcap.com records more than 8000 CCs traded at more than 33,000 markets, totalling a market capitalization close to 1 trillion USD (almost two thirds of which are due to Bitcoin), with a 24-h trading volume surpassing 150 billion USD.

^{¶¶} Note that the commonly reported thousands of CCs include mostly such with extremely low liquidity: As of 2020-04-28, only 10 CCs exhibit daily trading volumes exceeding 1 billion USD; volume below 100 000 USD exists already among the top 200 CCs.

^{|||} Quick growth in the number of traded CCs was mostly driven by the free-software nature of Bitcoin, allowing forks, and to a lesser degree by development of new (sometimes blockchainless) CCs.

class;[†] yet generally so-called *altcoins* exhibit still higher risk and mean returns. (Even more extreme were returns of Initial Coin Offerings, ICOs, in particular during their peak in 2017—see, for instance, Adhami *et al.* (2018), Momtaz (2018, 2019a, 2019b). However, despite the important economic role of ICOs and STOs (Security Token Offerings) as novel channels of venture-capital investment, they are unsuitable for rules-based portfolio allocation, and hence fall outside the scope of our paper.)

A key finding is that correlations are low even among CCs, as long as they are no close substitutes or forks. This implies a potential diversification benefit from a broad basket of CCs (Chuen *et al.* 2017). Alessandretti *et al.* (2018), optimizing CCs-only portfolios with LSTMs and decision trees, also find enhanced return performance.

As one consequence, CC indices were developed: The CRIX (Trimborn and Härdle 2018) captures the broad CC market movement with a statistically optimized varying number of constituents; CCI30 (Rivin and Scevola 2018) is a simple, close analog to stock-market indices; F5 (Elendner 2018) is a momentum-factor-based, transaction-cost-optimized basis for an exchange-traded portfolio; C20 (Schwartzkopff *et al.* 2017) is an on-chain crypto-asset itself. VCRIX (Kim *et al.* 2019) is a volatility index for option pricing. A first paper on option pricing of cryptos is Hou *et al.* (2020).

The second key finding of cross-sectional analyses is that CCs beyond the most prominent exhibit considerably low liquidity. Portfolio calculations ignoring liquidity might suggest trades which are impossible without extreme price impact. Trimborn *et al.* (2019) introduce Liquidity Bounded Risk-return Optimization (LIBRO) to account for illiquidity in CC portfolio formation. Since our focus is to evaluate the potential of adding CCs to traditional portfolios, i.e. we take the investor's perspective, we provide results both without and with the inclusion of LIBRO constraints.

In summary, the literature on CCs so far has solidly established potential benefits of holding CCs in investment portfolios (foremost high returns and low correlations), as well as certain difficulties (critically low liquidity). Yet open questions remain; prime among those whether CCs 'only' proxy for alternative (non-CC) assets, or provide investment opportunities that cannot be realized without CC positions. We close this gap by evaluating a wide range of common asset-allocation models with and without CC positions.

3. Research questions and hypotheses development

In light of the present body of knowledge about CCs reviewed in the previous section, we now turn to flesh out the research hypotheses that tackle unanswered questions from the perspective of an individual investor who is contemplating the addition of CCs to his portfolio. In this quest, we put our

emphasis on a realistic setting insofar as we allow the investor to already own a well-diversified portfolio of traditional assets to which CCs would be a (likely minor) *addition*, and we also take into account the lack of *liquidity* in the markets of many but the most prominent coins.

These two conditions will be faced by any rational CC investor. However, investors differ significantly with regard to their risk profiles, investment targets, individual trading behaviors, and generally their diverse motives and preferences. Consequently, the implications and benefits from including CCs into financial portfolios may differ depending on the investor. Naturally, many investors strive to optimize their risk-return profile, thus a natural benchmark strategy is a mean-variance optimized portfolio. Consequently, a plethora of financial studies have evaluated strategies with such a target, as they are commonly applied in practice. Therefore, our first goal is to clarify if an investment in CCs is equally beneficial across different types of investors, or if strategies with different targets do not perform differently from what a mean-variance optimizing investor achieves:

Hypothesis 1. *When CCs are included, realized returns of optimal portfolios do not differ from a standard mean-variance optimal portfolio.*

In order to extend our analysis to risk-adjusted returns, we also run tests about risk-adjusted returns:

Hypothesis 2. *When CCs are included, the Sharpe ratios of optimal portfolios do not change; the same holds true for Certainty Equivalents.*

Moreover, diversification generally improves when adding more assets to a portfolio, although it also increases the costs and complexity of the portfolio. For CCs, further risks include (but are not limited to) the drying out of liquidity, or a delisting of a CC from exchanges. Thus, being exposed to a lower variety of CCs may be beneficial, too. Thus we study if investors with different profiles include the same number of CCs into their optimal portfolios, which amounts to the following hypothesis:

Hypothesis 3. *The diversification effect from CCs is indistinguishable across all strategies under consideration.*

A second aspect of investor behavior is at the center of the question: To which type of investor are CC investments most useful? Only professional traders who rebalance their portfolio frequently, or also less actively trading retail investors?

While CC markets exhibit a lot of short-term trading, it is important to know whether the beneficial properties that have been documented in the literature can only be taken advantage of by professional investors who are able to maintain a high trading frequency or whether CC positions are also useful to investors who may rebalance their portfolio as rarely as once a month. We address this question by formally testing if the return differential between trading frequencies for otherwise identical strategies differs significantly from zero:

Hypothesis 4. *Daily, weekly, and monthly rebalanced portfolios entail no different cumulated returns when compared on the lower frequency.*

[†] The reasons to consider CCs an asset class naturally go beyond the similarity of their return processes; the major reason is that their economic rationale differs decisively from all other asset classes, as they constitute the only means to provide real resources to decentralized apps.

Our third line of inquiry addresses the breadth of optimal CC holdings: Should investors focus on one particular coin (e.g. Bitcoin), a selected few, or rather consider the broad cross-section of CCs?

Our most direct approach to this question is to test which coins effectively extend the set of achievable risk/return combinations, formally:

Hypothesis 5. *The efficient frontier is significantly affected by including CCs in the investment universe.*

Our focus on real-world feasibility leads us to investigate the important question: Can these strategies be implemented in practice? In particular, are all CCs liquid enough for inclusion in an investment portfolio? If not, how can investors still profit from promising CCs with little trading volume without exposing their portfolio too much to illiquidity? Most importantly, how is performance affected by honoring such portfolio restrictions?

We address these questions by implementing the LIBRO methodology and comparing thus restricted portfolios with their counterparts which optimize identical targets but without such a liquidity restriction:

Hypothesis 6. *Returns of unrestricted and restricted (via LIBRO) optimal portfolios stem from the same distribution.*

At the heart of testing these hypotheses lie comparisons of different optimal portfolio strategies. We therefore calculate portfolio optimizations for a range of asset-allocation models, which we detail in the following section.

4. Asset-allocation models

Consider a matrix $X \in \mathbb{R}^{P \times N}$ of log returns of N assets for P days. In our comparative analysis we rely on a moving-window approach. Specifically, we choose an estimation window of length $K = 252$ days (corresponding to the number of trading days in a calendar year). We investigate the performance of strategies for three rebalancing frequencies k : monthly, with $k = 21$ days, weekly, with $k = 5$ days, and daily with $k = 1$ day.[†] For each rebalancing period t ($t = 1, \dots, T$, with T the number of moving windows, defined as $T = \frac{P-K}{k}$), starting on date $K + 1$, we use the data in the previous K days to estimate the parameters required to implement a particular strategy. These parameter estimates are then used to determine the relative portfolio weights w in the portfolio of risky assets. Based on these weights, we compute the strategy’s return in rebalancing period $t + 1$. This process is iterated by adding the k daily returns for the next period in the dataset and dropping the corresponding earliest returns, until the end of the dataset is reached. The outcome of this rolling-window approach is a series of $P - K$ daily out-of-sample returns generated by each of the portfolio strategies listed in table 1. To simplify notation, we omit the index t for moving window or rebalancing period.

The traditional evaluation literature (e.g. DeMiguel *et al.* 2009, Schanbacher 2014) considers an investor whose preferences are specified in terms of utility functions and fully described by the portfolio mean μ_P and variance σ_P .

However, Merton (1980) showed that a very long time series is required in order to receive accurate estimates of expected returns. Due to this high margin of error of expected-return estimates some authors, including Haugen and Baker (1991), Chopra and Ziemba (1993) and Chow *et al.* (2011), suggest to rely only on estimates of the covariance matrix as input of the optimization procedure. Thus, investors assume that all stocks have the same expected returns and under this strong assumption the optimal portfolio is the global minimum-variance portfolio. The minimum-variance portfolio strategy represents one of the so-called risk-based portfolios, i.e. the only input used is the estimate of the variance-covariance matrix. In this paper we consider the most popular ones: Maximum Diversification, Risk-Parity, Minimum Variance and Minimum CVaR portfolio. In section 4.1 we describe the individual strategies from the portfolio-choice literature that we consider. In addition to traditional approaches, we consider a decision maker with risk preferences specified in percentile terms, and portfolio construction based on higher moments of the portfolio return-distribution, such as skewness and kurtosis. Therefore, in our comparative study we distinguish three groups of strategies: return-oriented, risk-oriented (or risk-based, as in Clarke *et al.* (2013)), as well as a tangency portfolio with Maximum Sharpe Ratio (MV-S), which we categorize as a risk-return-oriented strategy.

Taking into account that the ranking of models changes over time, and motivated by the fact that in many fields a combination of models performs well (see, e.g. Clemen 1989, Avramov 2002), we also extend our analysis to include the combination of portfolio models based on a bootstrap approach inspired by Schanbacher (2014, 2015). The detailed methodology of combined portfolio models is discussed in section 4.2.

4.1. Common asset-allocation models

In this section, we review those models that we consider in the empirical analysis. We also discuss links between the strategies and give conditions under which they are equivalent. In general, when bringing the theoretical models to the data, we employ in-sample moments of return distributions as estimators of their theoretical counterparts; naturally, all evaluation then concerns out-of-sample performance. As subsequent prices provide new information about assets' returns, all estimates are updated before any rebalancing trades.

In all models, we rule out short selling, a standard assumption in the CC literature, given that—with the exception of bitcoin, for which futures are traded since December 2017—taking short positions on CCs is at the very least impractical, if not outright impossible.

4.1.1. Equally weighted portfolio. The most naïve portfolio strategy sets equal weights (EW) for all constituents: every asset gets a weight $w_i = 1/N$ for $i = 1, \dots, N$. If all

† We also test strategies on extending windows as in Trimborn *et al.* (2019); since the insights are similar, these results are not reported.

constituents have the same expected returns and covariances, the EW portfolio is mean-variance optimal. However, there is no need for assumptions or estimates regarding the distribution of the assets' returns to implement EW. Moreover, as DeMiguel *et al.* (2009) show, EW allocations can actually perform well, in particular in settings of high uncertainty, i.e. parameter instability—the model-free approach avoids over-fitting. This is also the reason why the F5 crypto strategy builds on an EW baseline benchmark.

4.1.2. Optimal mean-variance portfolio. Many portfolio managers still rely on Markowitz' risk-return or mean-variance (MV) rule, combining assets into an efficient portfolio offering a risk-adjusted target return (Härdle and Simar 2015). MV portfolios are optimal if the financial returns follow a normal distribution (which, generally, they do not), or if risk can be fully captured via volatility (which, generally, it cannot). Otherwise, MV serves as an approximation, which in favor of tractability and convenience accepts the drawbacks widely discussed in the literature: high portfolio concentration, i.e. high portfolio weights for a limited subset of the investment universe, and high sensitivity to small changes in parameter estimates of μ and σ , see Jorion (1985), Simaan (1997), Kan and Zhou (2007). In a Gaussian world, portfolio weights w are obtained by solving the following optimization problem:

$$\begin{aligned} \min_{w \in \mathbb{R}^p} \quad & \sigma_P^2(w) \stackrel{\text{def}}{=} w^\top \Sigma w \\ \text{s.t.} \quad & \mu_P(w) = r_T, \\ & w^\top 1_N = 1, \quad w_i \geq 0 \end{aligned} \quad (1)$$

where $\Sigma \stackrel{\text{def}}{=} E_{t-1}\{(X - \mu)(X - \mu)^\top\}$ and $\mu \stackrel{\text{def}}{=} E_{t-1}(X)$ are the sample covariance matrix and vector of mean returns respectively, $\mu_P(w) \stackrel{\text{def}}{=} w^\top \mu$, is the portfolio mean and r_T the target return, ranging from minimum return to maximum return to trace out an efficient frontier. E_{t-1} is the expectation operator conditional on the information set available at time $t - 1$.

We include three benchmark mean-variance portfolios in our analyses: first, the global minimum variance portfolio ('MinVar' in table 1); second, the tangency portfolio ('MV-S'), and third the portfolio with the highest in-sample return ('RR-MaxRet'). In our classification approach, MinVar is a risk-based decision rule, since it is the most averse to risk and accepts the lowest target portfolio return. At the opposite end of Markowitz' efficient frontier lies the return-orientated RR-MaxRet portfolio, accepting any risk to choose the (currently) highest possible reward. In between these two endpoints, the MV-S portfolio occupies middle-ground: it maximizes the Sharpe ratio (16), in this way involving both risk and return estimation for the portfolio construction. We characterize MV-S as a risk-return-based strategy.

4.1.3. Optimal conditional-Value-at-Risk portfolio. A strong limitation of Markowitz-based portfolio strategies lies in the assumption of Gaussian distributions of assets' log-returns. Absent those, for investors whose preferences are not fully

described by a quadratic utility function, variance or volatility is an insufficient risk measure, leading the MV strategy to give a non-optimal portfolio composition. Importantly, returns of CCs have even heavier tails as compared to those of equities, as detailed in Chuen *et al.* (2017) and Elendner *et al.* (2018). The descriptive statistics of our investment universe in figure A1 and table A3 in appendix A.2 again provide strong evidence of this heavy-tailed distributions for CCs. Therefore, we include a strategy that accounts for higher moments via Conditional Value at Risk (CVaR): we include a Mean-CVaR-optimized portfolio as in Rockafellar and Uryasev (2000), Krokmal *et al.* (2002).

For a given $\alpha < 0.05$ risk level, the CVaR-optimized portfolio weights w are derived as:

$$\begin{aligned} \min_{w \in \mathbb{R}^N} \quad & \text{CVaR}_\alpha(w), \text{ s.t. } \mu_P(w) = r_T, w^\top 1_p = 1, w_i \geq 0, \quad (2) \\ \text{CVaR}_\alpha(w) = & -\frac{1}{1 - \alpha} \int_{w^\top X \leq -\text{VaR}_\alpha(w)} w^\top X f(w^\top X | w) dw^\top X, \quad (3) \end{aligned}$$

with $\frac{\partial}{\partial w^\top X} F(w^\top X | w) = f(w^\top X | w)$ the probability density function of the portfolio returns with weights w . $\text{VaR}_\alpha(w)$ is the corresponding α -quantile of the cumulative distribution function, defining the loss to be expected in $(\alpha \cdot 100)\%$ of the times.

As for the MV portfolio, we construct the efficient frontier, from which we derive the portfolios to add to our analyses. As a risk-oriented strategy, we add the MinCVaR strategy, minimizing the risk in terms of CVaR. As far as a return-oriented strategy is concerned, given our methodology, the maximal expected return arises in the same way as in the maximum-return portfolio ('RR-MaxRet' in table 1), by investing in the riskiest asset only. Thus, we report this portfolio only as RR-MaxRet.

4.1.4. Risk-parity portfolio (with equal risk contribution, ERC). One traditional risk-based portfolio strategy is based on the concept of risk parity. The underlying idea is to set weights such that each asset has the same contribution to portfolio risk, see Qian (2006). Maillard *et al.* (2010) derive properties of such portfolios and rename them 'equal-risk-contribution' (ERC) instruments. The Euler decomposition of the portfolio volatility $\sigma_P(w) = \sqrt{w^\top \Sigma w}$ (Härdle and Simar 2015) allows to present it in the following form:

$$\sigma_P(w) \stackrel{\text{def}}{=} \sum_{i=1}^N \sigma_i(w) = \sum_{i=1}^N w_i \frac{\partial \sigma_P(w)}{\partial w_i}, \quad (4)$$

where $\frac{\partial \sigma_P(w)}{\partial w_i}$ is the marginal risk contribution and $\sigma_i(w) = w_i \frac{\partial \sigma_P(w)}{\partial w_i}$ is the risk contribution of the i th asset. So, to construct the ERC portfolio, we calibrate:

$$\sigma_i(w) = \frac{1}{N} \quad \forall i \quad (5)$$

The ERC portfolio can be compared to the EW portfolio: instead of allocating capital equally across all assets, the ERC

portfolio allocates the total risk equally across all assets. Consequently, if variances of log-returns were all equal, the ERC portfolio would become identical to EW portfolio. The ERC portfolio is also comparable to the MinVar portfolio, which focuses on parity of marginal contributions of all assets.

4.1.5. Maximum-diversification portfolio (based on the Portfolio Diversification Index, PDI). Originally, the Maximum Diversification portfolio (MD) uses an objective function introduced in Choueifaty and Coignard (2008) that maximizes the ratio of weighted average asset volatilities to portfolio volatility or diversification ratio as in Equation (22). In our study, instead of the diversification ratio we maximize the Portfolio Diversification Index (PDI) proposed by Rudin and Morgan (2006). It consists in assessing a Principal Component Analysis (PCA) on the weighted asset returns' covariance matrix, i.e. identifying orthogonal sources of variation. In its original form, PDI does not account for the actual portfolio weights, here we incorporate weighted returns. We optimize:

$$\max_{w \in \mathbb{R}^N} PDI_P(w), \quad \text{s.t. } w^\top 1_P = 1, \quad w_i \geq 0 \quad (6)$$

$$PDI_P(w) = 2 \sum_{i=1}^N i W_i - 1, \quad (7)$$

where $W_i = \frac{\lambda_i}{\sum_{i=1}^N \lambda_i}$ are the normalized covariance eigenvalues λ_i in decreasing order, i.e. the relative strengths. Thus, an 'ideally diversified' portfolio, i.e. when all assets are perfectly uncorrelated and $W_i = 1/N$ for all i , then $PDI = N$. On the contrary a $PDI \approx 1$ indicates diversification is effectively impossible. Thus, in case of perfectly uncorrelated assets the MD portfolio will be exactly the EW portfolio.

The PDI summarizes the diversification of a large number of assets with a single statistic and can compare the diversification across different portfolios or time periods.

4.2. Averaging of portfolio models

Additional to individual allocation models, we also consider combinations of models. After all, every individual model is subject to estimation risk; the idea of combining (or averaging) models in order to reduce such risk received attention in various areas, and particularly in forecasting (Avramov 2002). Traditional model-averaging methods use information criteria—like AIC or BIC—to identify relative shares of models. Across portfolio-allocation models the likelihood is unknown, however, therefore we calculate model shares with the loss function l , defined as

$$l(w) = w^\top \hat{\mu} - \frac{\gamma}{2} w^\top \hat{\Sigma} w. \quad (8)$$

The parameter γ reflects the investor's risk aversion, with γ being large (small) for a risk-averse (risk-seeking) investor. We use two approaches to construct combined strategies: Naïve averaging of the portfolio weights, as well as the combination method based on a bootstrap procedure described in

Schanbacher (2014). However, in order to account for possible time series dependencies at a daily frequency, we apply the stationary bootstrap algorithm of Politis and Romano (1994) with automatic block-length selection proposed by Politis and White (2004).

Consider a set of m asset-allocation models. The corresponding portfolio weights per model are given by $W = (w^1, \dots, w^m)$. Relative shares of (or beliefs in) individual models are $\pi = (\pi^1, \dots, \pi^m)$, such that $\pi^\top 1_m = 1$.

Then the asset weights for the combined portfolio are given by:

$$w^{comb} = \sum_{i=1}^m \pi^i w^i \quad (9)$$

The Naïve combination over all asset-allocation models just assigns equal shares, i.e. $\pi_t^i = \frac{1}{m}$ for all $i = 1, \dots, m$.

The alternative, more sophisticated approach is to set the share π_t^i equal to the probability that model i outperforms all other models. We apply a bootstrap method to estimate these probabilities. For every period t we generate a random sample (with replacement) of k returns using returns $X_{k(t-1)+1} \dots X_{k(t-1)+1+K}$, i.e. K -long returns vectors of the $t-1$ rolling window. We apply all m asset allocation models to these bootstrapped returns. The procedure is repeated B times. Let $s_{i,b} = 1$ if model i outperforms in terms of the loss function other models in the b th bootstrapped sample, otherwise $s_{i,b} = 0$. The probability of model i being best is then estimated as

$$\hat{\pi}_t^i = \frac{1}{B} \sum_{b=1}^B s_{i,b} \quad (10)$$

where $B = 100$ is our number of independent bootstrap samples, and $s_{i,b} = 1$ if model i is the best model in the b -th sample.

5. Liquidity constraints with the LIBRO framework

In this section, we review the LIBRO framework for portfolio formation, which prevents too high portfolio weights for low-liquidity assets, by introducing weight constraints in the portfolio optimization which depend on liquidity.

Liquidity, however, does not have a unique definition; different concepts and measures abound. Von Wyss (2004), Vayanos and Wang (2013) survey the extensive literature on liquidity measures in equity markets; the literature on CC liquidity is still scarce, with notable exceptions of Brauneis *et al.* (2020), Scharnowski (2020). Due to the highly fragmented market structure of CC exchanges (no dominant or central exchange is trading all assets), we employ Trading Volume (TV) as our proxy for liquidity. TV is also the basis for the widely used Amihud (2002) illiquidity measure and proved suitable for the LIBRO methodology. In principle, alternative measures like the bid-ask spread would also be applicable, as many exchanges report bid and ask prices; however, reliable order-book data aggregated across exchanges and for all CCs is lacking. TV, in contrast, is available for practically all CC markets, and aggregated without problems. For these reasons, we follow Trimbom *et al.* (2019) and employ TV as our

liquidity measure. TV is defined as

$$TV_{ij} = p_{ij} \cdot q_{ij}, \quad (11)$$

where p_{ij} is the closing price[†] of asset i at date j , and q_{ij} is the volume traded at date j of asset i . The liquidity of asset i in period t can then be measured with the sample median of trading volume,

$$TV_i = \frac{1}{2}(TV_{i,up} + TV_{i,lo}), \quad (12)$$

where $TV_{i,up} = TV_{i,\lceil \frac{t+1}{2} \rceil}$ and $TV_{i,lo} = TV_{i,\lfloor \frac{t+1}{2} \rfloor}$.

Define M as the total amount invested in all N assets, so that Mw_i denotes the market value held in asset i . Trimbom *et al.* (2019) formulate the constraint on the weight of asset i as

$$Mw_i \leq TV_i \cdot f_i, \quad (13)$$

where f_i captures the speed with which an investor intends to be able to clear the current position in asset i via multiples of TV. For example, $f_i = 0.5$ implies the position in asset i must not exceed 50% of median trading volume. It results in a boundary for the weight on asset i as

$$w_i \leq \frac{TV_i \cdot f_i}{M} = \hat{a}_i. \quad (14)$$

The beauty of this approach lies in its ease to include it into any portfolio optimization.

6. Evaluating the performance of portfolios

While section 4 presents the set of common asset-allocation models we implement, no unique metric exists to evaluate and compare them. In order to draw conclusions about the effect of adding CCs to broadly diversified portfolios, we pursue three dimensions: First, we calculate a range of widely used performance measures in section 6.1. Second, in section 6.2, we run direct tests for differences between strategies on a pair-wise basis. Third and finally, in section 6.3, we address the diversification effect of CCs directly by calculating three well-known measures of portfolio concentration.

6.1. Performance measures

To assess the performance of the investment strategies we consider as it develops over time, we employ the following five common performance criteria widely used in literature, as well as by practitioners. Performance measures are computed based on the time series of daily out-of-sample returns generated by each strategy.

[†] Technically, CC markets never close; the terminology ‘closing price’ is still used in reference to the last price of a day, where days are customary defined on UTC time.

First, we measure the *cumulative wealth* (CW) generated by each strategy i

$$W_{i,t+1} = W_{i,t} + \hat{w}_{i,t}^\top X_{t+1}, \quad (15)$$

starting with an initial portfolio wealth of $W_0 = \$1$. Cumulative wealth, while naturally of high interest as a measure of performance achieved over the period considered, is not sufficient to rank our allocation approaches. Therefore, we also compute two traditional measures of risk-adjusted returns: the Sharpe ratio, and the certainty equivalent. Moreover, we provide the Adjusted Sharpe Ratio (ASR) in order to address the MinCVaR strategy and the non-Gaussian nature of the return distributions.

The *Sharpe Ratio* (SR) of strategy i is defined as the sample mean of out-of-sample excess returns (over the risk-free rate), scaled by their respective standard deviation. This definition presumes an unambiguous risk-free rate, inexistent in the global context of CCs. Fortunately, our sample period is characterized by most of the global economy at or very close to the zero lower bound on interest rates; so we can sidestep the question by implicitly setting the riskless rate to 0 and defining

$$\widehat{SR}_i = \frac{\hat{\mu}_i}{\hat{\sigma}_i^2}. \quad (16)$$

The *Certainty Equivalent* (CEQ) captures, for an investor with a given risk aversion γ , the riskless return that said investor would consider of equal utility as the risky return under evaluation. For the case $\gamma = 1$, it is equivalent to the closed-form solution of Markowitz (1952) portfolio optimization problem in Equation (1).

$$\widehat{CEQ}_{i,\gamma} = \hat{\mu}_i - \frac{\gamma}{2} \hat{\sigma}_i^2 \quad (17)$$

While there is debate about the risk-aversion coefficient best describing investors going back to Mehra and Prescott (1985), we argue that current CC investors are unlikely to be characterized by extremely high risk aversion, and calculate the CEQ in the empirical part of our paper with a γ of 1. As can be noted, the CEQ corresponds to the loss function l defined in Equation (8).

The CEQ and in particular the SR are more suitable to assess strategies when assets exhibit normally distributed returns. To address this drawback, Pezier and White (2008) propose the *Adjusted Sharpe Ratio* (ASR). ASR explicitly incorporates skewness and kurtosis:

$$\widehat{ASR}_i = \widehat{SR}_i \left[1 + \left(\frac{S_i}{6} \right) \widehat{SR}_i - \left(\frac{K_i}{24} \right) \widehat{SR}_i^2 \right] \quad (18)$$

where SR_i denotes the Sharpe Ratio, S_i the skewness, and K_i the excess kurtosis of asset i . Thus, the ASR accounts for the fact that investors generally prefer positive skewness and negative excess kurtosis, as it contains a penalty factor for negative skewness and positive excess kurtosis.

To assess the impact of potential transaction costs associated with asset rebalancing, we also calculate two measures

for turnover. *Portfolio turnover* is computed to capture the amount of trade necessary on rebalancing dates as

$$TO_i = \frac{1}{T-K} \sum_{t=1}^{T-K} \sum_{j=1}^N |\hat{w}_{i,j,t+1} - \hat{w}_{i,j,t}| \quad (19)$$

where $w_{i,j,t}$ and $w_{i,j,t+1}$ are the weights assigned to asset j for periods t and $t+1$ and $\hat{w}_{i,j,t+}$ denotes its weight just before rebalancing at $t+1$. Thus, we account for the price change over the period, as one needs to execute trades in order to rebalance the portfolio towards the w_t target. High turnover will imply significant transaction costs; consequently, the lower turnover of a strategy, the better it performs.

Target turnover, the second turnover-related measure, captures the amount of change in target weights between two consecutive rebalancing dates as

$$TTO_i = \frac{1}{T-K} \sum_{t=1}^{T-K} \sum_{j=1}^N |\hat{w}_{i,j,t+1} - \hat{w}_{i,j,t}| \quad (20)$$

In contrast to Equation (19), here the difference between weights spans the time interval of one rebalancing period, instead of the (conceptually infinitesimal) duration of rebalancing trades. Therefore, the realized price paths of the assets affect the measure only insofar as they lead to different parameter estimates and thus a revision in target weights. The difference between the two turnover measures is best illustrated by considering the EW strategy: it may require high turnover to return to exactly equal weights per asset every rebalancing date; yet by definition it will never exhibit target turnover.

6.2. Testing for performance differences between strategies

To test hypothesis 1, we perform Kolmogorov–Smirnov tests for equal distributions of the returns for each of the risk-oriented, return-oriented, risk-and-return-oriented, or combined strategies with respect to the benchmark strategy of equally weighted (EW) assets.

To test hypothesis 2, whether strategies are significantly different from each other in risk-adjusted terms, we provide the p -values of pairwise tests. The common approach by Jobson and Korkie (1981) is widely used in the performance evaluation literature (e.g. also in DeMiguel et al. 2009). However, this test is not appropriate when returns have tails heavier than the normal distribution. Therefore, as a testing procedure we rely on the Ledoit and Wolf (2008) test with the use of robust inference methods. We test for difference of both CEQ and SR, and report results for the HAC (heteroskedasticity and autocorrelation) inference version. The procedure is described in appendix A.1.

6.3. Measuring diversification effects

To evaluate portfolio concentration and portfolio diversification effects, we calculate three measures:

- (a) the *Portfolio Diversification Index* (PDI) as introduced in Equation (7),

- (b) *Effective N* as introduced by Strongin et al. (2000), and
- (c) the *diversification ratio*.

Effective N is defined as

$$N_{eff}(w_t) = \frac{1}{\sum_{j=1}^N w_{j,t}^2} \quad (21)$$

with $j = 1, \dots, N$ indexing assets. Effective N varies from 1 in the case of maximal concentration, i.e. the portfolio entirely invested in a single asset, to N —its maximum achieved by an equally-weighted portfolio. The design of Effective N is related to other traditional concentration measures, e.g. the Herfindahl Index, the sum of squared market shares to measure the amount of competition. Effective N can be interpreted as the number of equally-weighted assets that would provide the same diversification benefits as the portfolio under consideration.

The *diversification ratio*, suggested by Choueifaty et al. (2011), measures the proportion of a portfolio's weighted average volatility to its overall volatility:

$$DR(w_t) = \frac{w_t^\top \sigma_t}{\sqrt{w_t^\top \Sigma_t w_t}} = \frac{w_t^\top \sigma_t}{\sigma_{P,t}(w_t)} \quad (22)$$

Thus, the diversification ratio has the form of the Sharpe Ratio in Equation (16), with the sum of weighted asset volatilities replacing the expected excess return. In case of perfectly correlated assets, the DR equals 1; in contrast, in a situation of 'ideal diversification,' i.e. perfectly uncorrelated assets, $DR = \sqrt{N}$. Hence, in our empirical study we report the results for DR^2 , for two reasons: First, to make it comparable to the other two metrics used, and second, because Choueifaty and Coignard (2008) demonstrate that for a universe of N independent risk factors, the portfolio that weighted each factor by its inverse volatility would have a DR^2 equal to N . Hence DR^2 can be viewed as a measure of the effective degrees of freedom within a given investment universe.

7. Data

For the empirical analysis, we collect daily price data on a sample of CCs and traditional financial assets (including alternative investments) over the period 2015-01-01 to 2019-12-31 (1304 daily log-returns). CC prices are provided by CoinGecko, data for traditional assets is acquired from Bloomberg. Many CCs were established only after January 2015 or ceased to trade prior to the period we study. Since investors who apply rules-based optimization techniques usually only consider assets with sufficient price histories, we require CCs to have a continuous return time-series over the period of our study in order to be included. By excluding coins that did not already circulate in January 2015, went extinct before December 2019, or have only patchy price series, we effectively focus on solid CCs, of interest to investors considering positions in this novel asset class.† We also sidestep

† We also run our entire analysis for a sample period extending until end of December 2017. For this shorter period, 55 CCs fulfill our

ICOs. Hence, our final data sample for portfolio construction includes 52 CCs next to 16 traditional assets. In order to cover three different reallocation frequencies (daily, weekly, monthly), we calculate with daily, weekly and monthly return series for all assets treated equally.

We employ a rolling-window approach for the portfolio construction. The initial portfolio weights are determined from estimations based on the first year (2015), after which we 'roll' through the dataset by estimating new portfolio weights at the reallocation frequency. Depending on the employed frequency approach, this adds one day, week, or month of data to the estimation set and leaves out the oldest day, week or month of data, in order to capture potentially time-varying parameters.[†]

To evaluate the performance of each of the strategies we consider, our research question studies the effects of including CCs *as an addition* to classical, well-diversified portfolios. Therefore, our investment universe always includes 16 traditional assets from 5 asset classes: equity, fixed-income, fiat currencies, commodities, and real estate. Since CCs are global in nature, our traditional assets cover the five main economic areas around the globe (Europe, USA, UK, Japan, China). In this way, the asset space is sufficiently broad to allow diversification without CCs, ensuring any relevance of CCs we find is genuine, and at the same time is still narrow enough to allow us to add each CC individually as an asset without leading to high-dimensionality issues in covariance estimation. The full list of traditional constituents of the investment universe is provided in table 2. Tables A3 and A2 in appendix A.2 report summary statistics of all constituents considered in our empirical study.

The main properties of our data correspond to the findings of the prior literature, e.g. Chuen *et al.* (2017): CCs outperform traditional asset classes in terms of average daily realized returns, their returns exhibit higher volatility, with means mostly positive while the medians are mostly negative, positive movements occur less frequently than negative ones, but with higher magnitudes (absolute values of minima and lower deciles are less than of maxima and higher deciles for the majority of CCs). Correlation analysis of the top five CCs by market capitalization with traditional asset classes shows the potential of CCs to increase diversification: As can be seen from table A1, correlation coefficients with none of the traditional assets exceed 0.1.

8. Empirical results

8.1. Hypotheses evaluation

In this section, we test the hypotheses formulated in section 3. To do so, we evaluate the out-of-sample performance of all the portfolio allocation strategies studied in terms of risk-adjusted performance, compare both realized and potential outcomes

as well as diversification benefits of each method, and clarify the role of liquidity and rebalancing frequency when adding CCs to investment portfolios.

Hypothesis 1. *When CCs are included, realized returns of optimal portfolios do not differ from a standard mean-variance optimal portfolio.*

First we examine cumulative wealth produced by the allocation strategies we study. Figures 1 and 2 display the dynamics of cumulative wealth for eight of the strategies considered, with and without enforcing liquidity constraints, respectively. As benchmarks we also plot S&P100, EW, MV-S and MinVar portfolios built only from traditional investment constituents (Traditional Assets, 'TrA'). The EW strategy is displayed separately in figure 3 and discussed subsequently. Table 3 summarizes all performance indicators.

The following conclusions can be drawn regarding final Cumulative Wealth (CW) over the entire period of our study when ignoring liquidity: despite CCs trading far below their historical peaks at the end of our time span, most portfolios with CCs generally outperform benchmark portfolios with only conventional constituents. However, the discrepancies across strategies are huge, and the worst-performing strategy RR-MaxRet, which invests always in the asset with the highest expected return (and thus most often in a CC), ends up with what can be called a catastrophic result: over the four years of our study, it loses 97% of its initial wealth by the end of 2019. Critically, the strategy did provide stellar results during the boom phase of 2017, exceeding a multiple of 20 times initial wealth at its peak. Yet clearly, historical returns were no long-term predictor of expected returns for the best-performing CCs, and the lack of diversification hurt this strategy badly.

On the other end of the spectrum, the highest result is achieved by MD, with an accumulated final wealth of 275%. This amounts to an annualized rate of return of just below 30% over a 4-year period in which the S&P100 lost 10%. Critically, this result is also achieved by investing in small CCs (and therefore also follows the boom-and-bust cycle to a comparable degree): the difference is driven by the very strong diversification the MD strategy pursues by design. It is therefore not surprising that ERC turns out the second-best strategy, with a +22% return over the period. Its construction successfully limits its exposure to the extremes during 2017/2018 to about an order of magnitude lower than MD.

Regarding the combined strategies, the naïve version is strongly susceptible to RR-MaxRet, while the bootstrapped version performs quite well.

Finally, the model-free EW strategy with CCs underperforms with a final loss of 13%, while equal weighting across only traditional assets achieves the best performance among the benchmark strategies. However, figure 3 shows how EW performance exhibits high variation over the time span, similar in nature to MaxRet and MD. The figure displays MD and EW separately, to elucidate two important points: first, how disproportionately the performance of small coins exceeded the gains of established CCs in the 2017 price explosion; second, how seriously calculated results of portfolio allocation rules can diverge from returns achievable by investors if lack of liquidity is not taken into account.

criteria, and with minor exceptions only for combined strategies, all our results remain qualitatively unchanged.

[†] As a robustness test, we also calculate with extending windows, where no historical data is dropped and only new observations added as they become observable. The results are qualitatively the same.

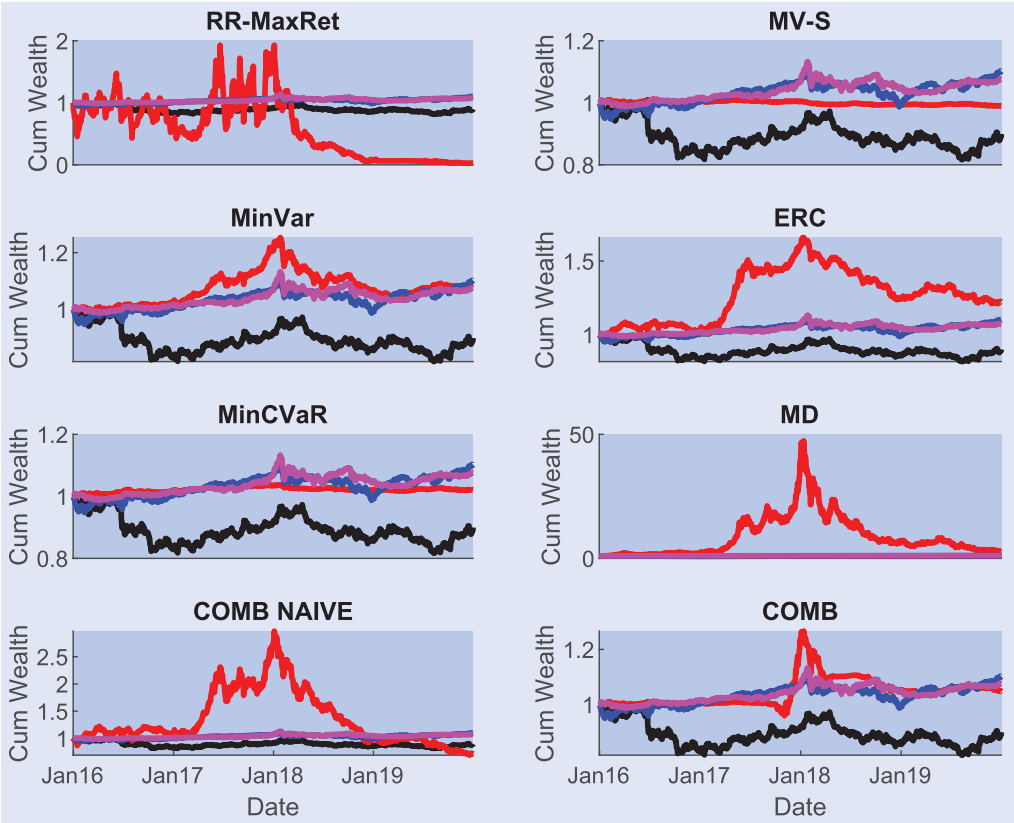



Figure 1. Performance in terms of cumulative wealth of portfolio strategies without liquidity constraints with monthly rebalancing ($l = 21$) over the period from 2016-01-01 to 2019-12-31 with the following color code: S&P100, **EW-TrA**, **RR-MaxRet-TrA** and the corresponding allocation strategy from table 1. ‘TrA’ denotes only traditional, i.e. non-CC assets are included. Note that the date axes are aligned, but the wealth axes are not, due to large dispersion in scales.

 CCPPerformance

Generally, LIBRO portfolios have mixed results in terms of cumulative wealth. Most importantly, MD underperforms when enforcing LIBRO constraints. Of course, this implies that the high performance of unconstrained optimization can only be reached for very small investment sums. For larger portfolios, when the liquidity constraints turn binding, performance need not necessarily suffer. By limiting the exposure to individual (and thus also small) coins, some strategies, including RR-MaxRet, are positively affected by LIBRO. When ignoring the liquidity risk, this strategy retained 3% of its initial value; with LIBRO it retains 59.1%. Also the combined strategy COMB, which provides a positive performance without LIBRO, further improves by 8.6% when protecting the portfolio from liquidity risk.

Next, we analyze risk-adjusted performance for all portfolios. While MD demonstrates superior absolute performance, ERC dominates in terms of risk-adjusted performance, in particular in terms of its (adjusted) Sharpe Ratio of 0.033. Importantly, turnover is much lower at 4.2 (unconstrained, constrained: 9.8), slightly below that of EW and above MV-S. Turnover and target turnover per strategy are reported in the last four columns in table 3, which show that the only strategy with appreciably lower trading is RR-MaxRet at 0.69 (constrained: 0.73), with the above-mentioned harsh result. This is expected, given that the strategy is by construction the most concentrated one, consisting of the one asset with the highest return (see also figures 4 and 5).

In testing hypothesis 1, the Kolmogorov–Smirnov test statistics in table 3 (in parentheses) clearly reject the hypothesis that the strategies under consideration entail returns no different than those of the benchmark mean-variance strategy: this holds true for all unconstrained portfolios reported in the first column as well as for strategies with liquidity constraints enforced (second column).[†]

Hypothesis 2. When CCs are included, the Sharpe ratios of optimal portfolios do not change; the same holds true for Certainty Equivalents.

Table 4 reports when the differences between strategies in terms of CEQ or SR are significant, based on tests described in appendix A.1. Although MD, COMB, and in particular MV-S have SR and CEQ higher than the EW strategy, tests do not support significance of this difference. In contrast, the ERC portfolio exhibits a higher SR and this difference is significant. The comparison of risk-adjusted metrics for MinVar and MinCVaR reveals that they differ significantly from each other—testament to the strong deviation of CC returns from the normal distribution. MinCVaR also differs significantly from the diversifying strategies MD and ERC. Thus, hypothesis 2 exhibits a more nuanced picture: while the hypothesis is rejected for the CEQ for the maximum-return strategy against

[†] These results are robust for all Mean-Variance portfolio versions used as a benchmark: MV-S TrA, MinVar, MV-S and RR-MaxRet.

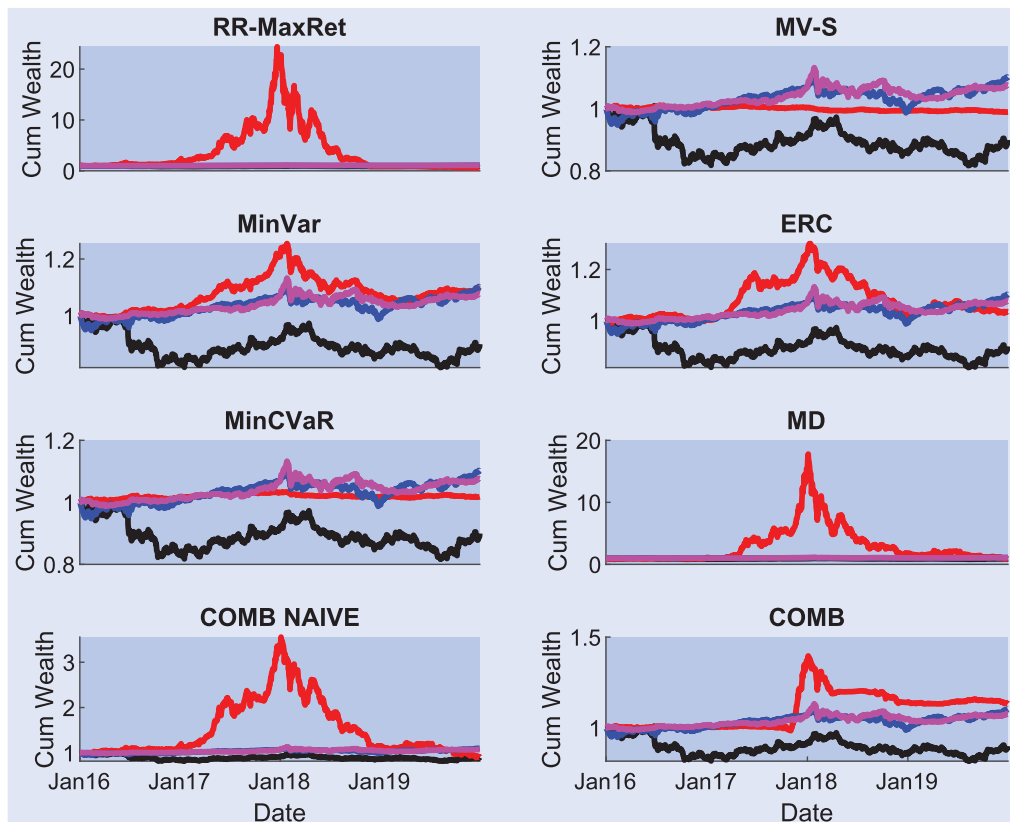


Figure 2. *Performance* in terms of cumulative wealth of portfolio strategies with liquidity constraints (based on LIBRO at the level of USD 10 mln) and monthly rebalancing ($l = 21$) over the period from 2016-01-01 to 2019-12-31 with the following color code: S&P100, EW-TrA, RR-MaxRet-TrA and the corresponding allocation strategy from table 1. 'TrA' denotes only traditional, i.e. non-CC assets are included. Note that the date axes are aligned, but the wealth axes are not, due to large dispersion in scales.

CCPPerformance

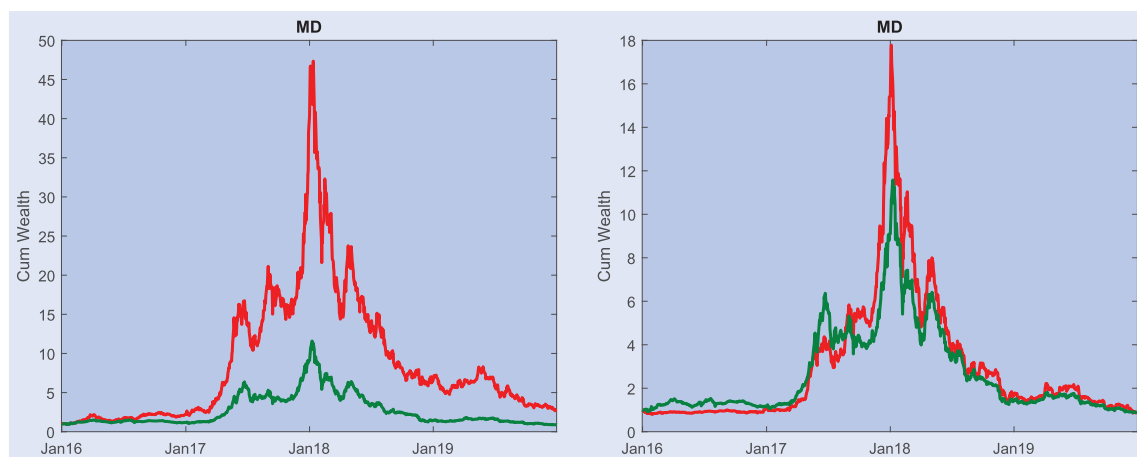


Figure 3. *Performance* in terms of cumulative wealth of portfolio strategies of the maximum-diversification strategy (MD) without (left panel) and with (right panel) liquidity constraints (based on LIBRO at the level of USD 10 mln), with monthly rebalancing ($l = 21$) over the period from 2016-01-01 to 2019-12-31. For reference, the equally-weighted EW strategy is displayed. Note that the date axes are aligned, but the wealth axes are not, due to large dispersion in scales.

CCPPerformance

all other strategies at the 1% level, for a sizable amount of pairs the hypothesis is accepted, both for SR as well as CEQ.

As a robustness check, we also conduct all analyses for weekly and daily portfolio rebalancings. Results are provided in appendix A.4, generally confirming the conclusions so far, and show that the qualitative results are robust with regard to the rebalancing frequency.

Hypothesis 3. *The diversification effect from CCs is indistinguishable across all strategies under consideration.*

Explicitly analyzing diversification characteristics of the allocation rules is important for two reasons: On the one hand, CCs are known from the literature for their diversifying properties; on the other hand, the most diversifying strategies MD and ERC performed best. First, we examine

Table 3. Performance measures for all investment strategies as well as benchmarks over the entire time period from 2016-01-01 to 2019-12-31, with monthly rebalancing ($l = 21$).

Allocation strategy	Portfolio performance measures: monthly rebalancing											
	CW		SR		ASR		CEQ		TO		TTO	
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln
<i>Benchmark strategies</i>												
S&P100	0.900	0.900	− 0.016	− 0.016	− 0.016	− 0.016	0.000	0.000	0.000	0.000	0.000	0.000
EW TrA	1.102	1.102	0.033	0.033	0.033	0.033	− 0.001	− 0.001	3.615	3.615	0.000	0.000
MV-S TrA	1.076	1.076	0.028	0.028	0.028	0.028	0.000	0.000	2.199	2.199	0.274	0.274
EW	0.877	0.877	− 0.003	− 0.003	− 0.003	− 0.003	− 0.001	− 0.001	4.345	4.345	0.000	0.000
<i>Risk-oriented strategies</i>												
MinVar	0.990 (0.27)***	0.990 (0.27)***	− 0.041	− 0.011	− 0.041	− 0.011	− 0.007	− 0.002	8.541	7.672	0.056	0.056
MinCVaR	1.021 (0.22)***	1.018 (0.23)***	0.026	0.023	0.026	0.023	0.000	0.000	14.884	8.093	0.112	0.114
ERC	1.224 (0.22)***	1.035 (0.12)***	0.033	0.009	0.033	0.009	0.000	0.000	4.193	9.840	0.058	0.064
MD	2.751 (0.43)***	0.858 (0.39)***	0.020	− 0.003	0.020	− 0.003	0.000	− 0.001	20.315	48.707	0.391	0.209
<i>Return-oriented strategies</i>												
RR-MaxRet	0.030 (0.41)***	0.591 (0.36)	− 0.015	− 0.016	− 0.015	− 0.016	0.000	0.000	0.687	0.731	0.687	0.479
<i>Risk-Return-oriented strategies</i>												
MV-S	1.090 (0.09)***	1.096 (0.07)***	0.024	0.028	0.024	0.027	0.000	0.000	4.021	8.591	0.291	0.290
<i>Combination of models</i>												
COMB NAÏVE	0.716 (0.35)***	0.908 (0.33)***	− 0.015	− 0.005	− 0.015	− 0.005	− 0.001	0.000	3.553	36.731	0.211	0.156
COMB	1.048 (0.14)***	1.134 (0.14)***	0.010	0.029	0.010	0.029	0.000	0.000	6.758	5.759	0.148	0.145

Notes: The performance measures are final cumulative wealth (CW), the Sharpe ratio (SR), the adjusted Sharpe ratio (ASR), the certainty equivalent (CEQ), and turnover. ‘TrA’ denotes only traditional, i.e. non-CC assets are included. Strategies are detailed in table 1. Highest results are highlighted in red. Kolmogorov–Smirnov test statistics for differences between the distribution of each return series from the MV-S TrA portfolio are shown in parentheses. Significance is indicated by * at the 0.1, ** at the 0.05, and *** at the 0.01 level.

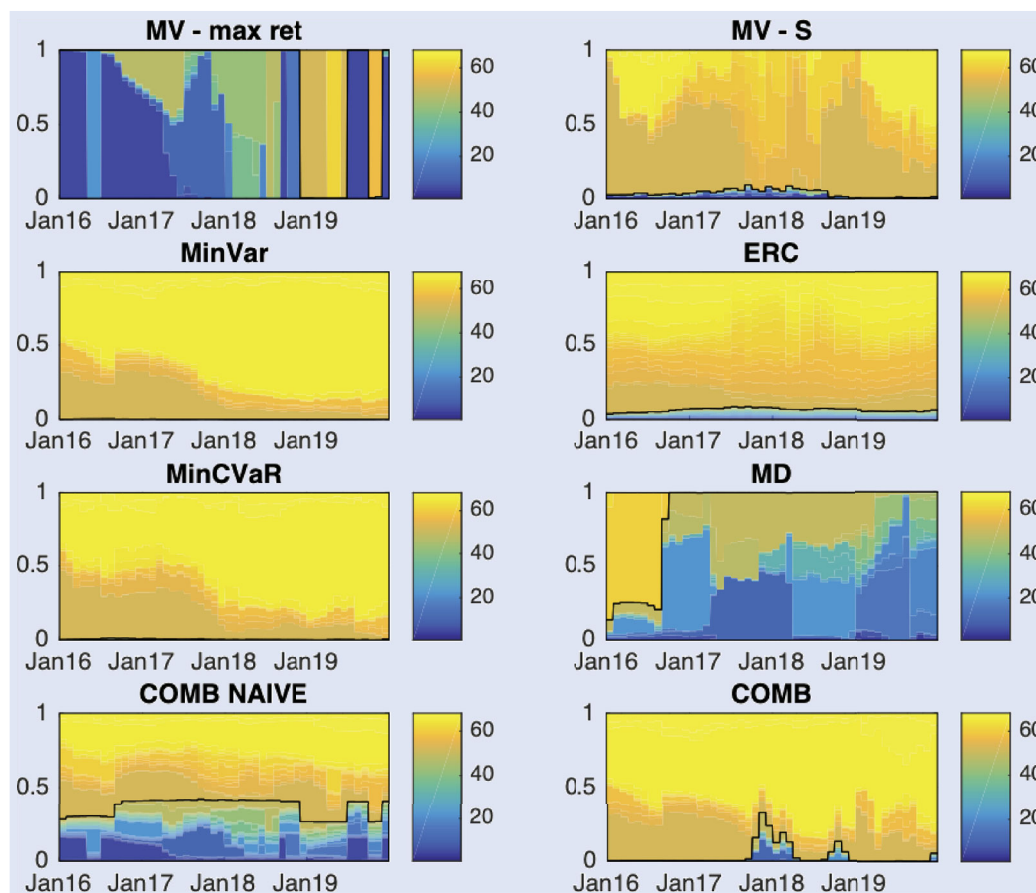


Figure 5. Evolution of the *portfolio composition* (i.e. relative weights) of all allocation strategies with a position limit of USD 10 mln (via the LIBRO approach) with monthly rebalancing over the period from 2016-01-01 to 2019-12-31: the black line separates conventional assets ('TrA,' upper yellow part of the spectrum) from cryptocurrencies (CCs, lower green-blue part of the spectrum).

CCPWeights

To shed more light on how these weights affect the performance of each strategy's portfolio, we also compare the risk structures for all strategies (plotted in figures A2 and A3 in appendix A.3). As expected, the volatility structure of CCs leads to disproportionate risk contributions relative to their capital weights: traditional assets affect changes in portfolio values to a visibly lower degree.

Table 5 reports results on our three chosen diversification metrics (detailed in section 6.3). Not surprisingly, no diversification benefits exist for the RR-MaxRet strategy—by definition it consists of only one asset at a time (unless LIBRO forces it into more than one asset). The range of values across diversification metrics emphasizes that diversification has different aspects and its quantification depends on the definition used. Consequently, different measures do not always provide identical conclusions about the diversification effects of CCs in portfolios. For instance, in terms of a DR^2 of 13.73 (13.44),[†] MinCVaR is characterized as most diversified strategy. Slightly lower measures pertain for MinVar and ERC portfolios with 12.02 (11.72) and 8.71 (9.41), respectively. The MD portfolio is a special case regarding this type of diversification, with DR^2 of 2.64 (2.07) and at the same time a PDI of 21.06 (21.01). Clearly PDI is the highest for

the MD portfolio because of its objective function, maximizing diversification via the number of independent sources of variation in the portfolio.

The ERC portfolio is characterized by the highest Effective N of 17.16 (13.61) by a large margin, also a typical result (see, e.g. Clarke *et al.* 2013) due to its nature: it includes all assets by definition. Apart from MaxRet, the lowest Effective N of 2.68 (2.69) arises for the MinVar portfolio, containing only traditional assets, showing that fewer than 3 uncorrelated equally-weighted stocks would provide the same diversification by this measure. All other individual strategies also exhibit Effective N ranging between 3 and 4. One more remarkable result concerns the combined portfolios' concentration: While COMB's Effective N lies in the range of individual strategies, COMB Naïve exceeds 10 both in constrained and unconstrained portfolios. In terms of DR^2 , the combined strategies rank inversely, reaching 3.43 (3.62) for COMB Naïve and 10.0 (9.44) for COMB; their PDIs are similar to those of the other risk-oriented portfolios MinVar, MinCVaR and ERC.

Note that with the exception of ERC, liquidity constraints do not strongly affect the diversification features of portfolios: all metrics display only minor changes. This result is due to the fact that LIBRO generally lowers the weight of constituents, but does not completely exclude them.

[†] Here and henceforth we provide the values of the performance metric for LIBRO portfolios in parentheses.

Table 5. Measures of diversification for all investment strategies and a benchmark, over the entire time period from 2016-01-01 to 2019-12-31, with monthly rebalancing ($l = 21$). DR^2 denotes the squared diversification ratio, PDI the portfolio diversification index; all three measures are detailed in section 6.3.

Allocation strategy	Portfolio diversification effects: monthly rebalancing					
	DR^2		Effective N		PDI	
	No const	10 mln	No const	10 mln	No const	10 mln
<i>Benchmark strategies</i>						
MV-S TrA	5.39	5.39	3.11	3.11	4.92	4.92
<i>Return-oriented strategies</i>						
RR-MaxRet	1.00	1.00	1.00	1.48	1.00	1.00
<i>Risk-oriented strategies</i>						
MinVar	12.02	11.72	2.68	2.69	20.60	20.60
ERC	8.71	9.41	17.16	13.61	20.62	20.62
MinCVaR	13.73	13.44	3.15	3.15	20.60	20.61
MD	2.64	2.07	3.99	3.17	21.06	21.01
<i>Risk-Return-oriented strategies</i>						
MV-S	8.03	7.48	3.26	3.35	20.62	20.62
<i>Combination of models</i>						
COMB NAÏVE	3.43	3.62	11.60	10.92	20.65	20.67
COMB	10.00	9.44	3.39	3.40	20.61	20.61

Notes: 'TrA' denotes only traditional, i.e. non-CC assets are included. Strategies are detailed in table 1. Results without liquidity constraints (columns 'No const') are contrasted with those when applying LIBRO with a threshold of USD 10 mln (column '10 mln'). Highest results are highlighted in red.

We particularly highlight the difference of diversification of the MV-S portfolios with and without CCs: diversification measured by DR^2 increases through the inclusion of CCs from 5.39 (5.39) to 8.03 (7.48), and PDI scales up even more strongly, from 4.92 (4.92) to 20.62 (20.62). Thus, the inclusion of CCs remarkably improves portfolio diversification, especially in terms of the distribution of principal portfolio variances, albeit investors with certain profiles do benefit more strongly. As such the hypothesis can be rejected.

Hypothesis 4. *Daily, weekly, and monthly rebalanced portfolios entail no different cumulated returns when compared on the lower frequency.*

To formally test hypothesis 4 comparing different rebalancing frequencies, we perform the following test: for a fixed strategy and two differing rebalancing frequencies, we cumulate the returns of the higher-frequency series to the lower frequency (incorporating all effects from the higher-frequency's strategy trading repeatedly within each longer interval) and compare these returns with the aligned returns of the lower-frequency strategy. For example, when comparing a daily to a monthly series, the end-of-month results of the daily rebalanced portfolio are contrasted with the coincident results of the strategy that trades only monthly; and similarly for all other combinations. Table 6 reports test statistics of Kolmogorov–Smirnov tests for equal distributions. The fact that we observe no statistics exceeding the significance threshold leads us to accept the hypothesis that it is not necessary to trade on a daily basis in order to reap the benefits of the return distribution from an optimal portfolio including CC assets as long as monthly rebalancing is possible.

Hypothesis 5. *The efficient frontier is significantly affected by including CCs in the investment universe.*

Figure 6 plots efficient frontiers for three groups of assets: only traditional assets, traditional assets & CCs without liquidity constraints, and traditional assets & liquid CCs, up to the constraint defined via the LIBRO approach with an investment sum of USD 10 mln.[†] The top row depicts frontiers from mean-variance optimization, the lower three panels are based on mean-CVaR-optimal allocations. All panels show frontiers built on a daily basis, evolving over time. The impression of how much CCs extend the frontier to the area of higher risk/higher return appears remarkable.

For both optimization rules, including CCs leads to a distinct extension of the frontiers: for low levels of risk, portfolios with CCs give a similar level of return as without them, but much higher expected returns can be sought when CCs are included. The second important observation is that mean-variance frontiers, in most cases, are shorter than mean-CVaR frontiers (the same level of returns has lower variance than CVaR), evidence of risk not being inadequately captured by variance, in line with expectations.

However, to formally test hypothesis 5, we conduct two mean-variance spanning tests on each of the 52 CCs: first, the corrected test of Huberman and Kandel (HK, 1987), second the step-down test by Kan and Zhou (2012).

Table 7 lists only CCs with at least one test rejecting the hypothesis that traditional assets span the frontier at the 10% level. Recall that our definition of traditional assets includes a broad set of alternative investments, all but CCs. The corrected HK test rejects spanning for 3 CCs. In contrast, the step-down test provides information on the source for spanning rejection: F_1 tests for spanning of tangency portfolios,

[†] Of course, in principle the efficient frontier is unique, thus identical for all allocation strategies. However, it depends on the risk measure (variance or CVaR in this paper), as well as on whether liquidity constraints are enforced (via LIBRO in this paper) or not.

Table 6. Kolmogorov–Smirnov test of difference between distributions of return series with various rebalancing frequencies. Significance is indicated by * at the 0.1, ** at the 0.05, and *** at the 0.01 level.

Allocation Strategy	Daily-Monthly		Weekly-Monthly		Daily-Weekly	
	No const	10 mln	No const	10 mln	No const	10 mln
RR-MaxRet	0.0211	0.0221	0.0350	0.0214	0.0204	0.0176
MV-S	0.0278	0.0221	0.0171	0.0167	0.0172	0.0166
MinVar	0.0173	0.0154	0.0150	0.0120	0.0110	0.0112
ERC	0.0106	0.0106	0.0087	0.0100	0.0084	0.0074
MinCVaR	0.0173	0.0230	0.0147	0.0205	0.0130	0.0144
MD	0.0182	0.0192	0.0278	0.0191	0.0172	0.0090

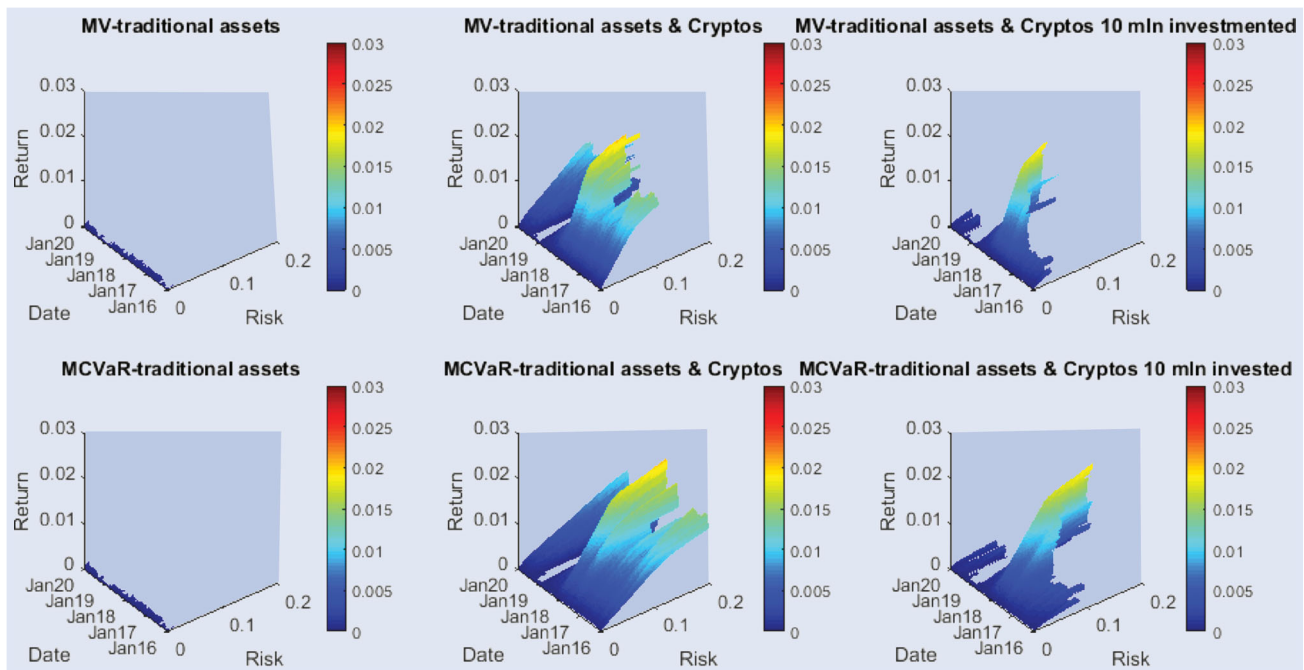


Figure 6. Efficient frontiers surfaces: the first column displays the frontiers for portfolios with only traditional assets (including alternative investments, but no cryptocurrencies, CCs) as constituents, the second column adds CCs without liquidity constraints, and the third column instead adds only CCs up to a liquidity constraint (via the LIBRO approach with an investment sum of USD 10 mln). The top row depicts frontiers from mean-variance optimization, the lower one from mean-CVaR optimization. All frontiers are built on a daily basis and plotted over the period from 2016-01-01 to 2019-12-31.

CCPEfficient_surface

whereas F_2 tests spanning for global minimum portfolios. From table 7, we see that the F_1 test rejects spanning for only 2 CCs, pointing out that tangency portfolios which include CCs are significantly different from the benchmark tangency portfolio, but also that the inclusion of the two years 2018–2019 has dramatically reduced that number from previously 27 CCs, which included Bitcoin (BTC), Ripple (XRP), Dash (DASH) and Litecoin (LTC). F_2 rejects spanning for 5 CCs for the entire time period, still including one of the coins with the highest market capitalization, XRP.

Thus, we conclude there still exists evidence that a MV-S portfolio can be improved by 7 out of 52 CCs, but that the integration of CC with financial markets has progressed markedly. Anecdotal evidence in line with this finding comes from the recent outbreak of the corona-virus pandemic, when initially CC markets moved for the first time with strong positive correlation together with financial markets, driven by institutional investors rebalancing in favor of cash holdings, before CCs resumed their diversifying role in subsequent weeks.

Also, there is little evidence that a MinVar portfolio can be improved. This result is supported by the dynamics of the portfolios' composition presented in figures 4 and 5 for unconstrained and LIBRO portfolios, respectively: MinVar portfolios in both cases are constructed entirely from traditional assets, whereas MV-S portfolios have a (varying) CC component throughout the whole investment period.

In sum, the results imply that investors should consider a broader selection of CCs, not only BTC. However, as of recently only a small fraction of CCs continue to improve the efficient frontier.

Hypothesis 6. *Returns of unrestricted and restricted (via LIBRO) optimal portfolios stem from the same distribution.*

Referring back to figure 6, we see that the LIBRO approach shortens the frontiers especially in the beginning of the investment period, because it limits the influence of turbulently growing CCs with low trading volumes. At the same time,

Table 7. Spanning tests for individual cryptocurrencies with respect to the efficient frontier constructed from all traditional investment assets, including alternative assets (see table 2 for a complete list; p -value in parentheses).

Cryptocurrency	F -Test	F -Test1	F -Test2
BCN	3.28 (0.04)	1.23 (0.27)	5.32 (0.02)
DOGE	1.73 (0.18)	0.01 (0.92)	3.46 (0.06)
EAC	1.70 (0.18)	0.09 (0.76)	3.32 (0.07)
NLG	2.79 (0.06)	4.31 (0.04)	1.26 (0.26)
PPC	3.19 (0.04)	0.61 (0.44)	5.78 (0.02)
XMG	1.86 (0.16)	3.44 (0.06)	0.28 (0.60)
XRP	1.88 (0.16)	0.83 (0.36)	2.93 (0.09)

Notes: F -Test refers to the corrected test of Huberman and Kandel (1987), F_1 and F_2 to step-down tests by Kan and Zhou (2012), testing for spanning of tangency portfolios and for global minimum portfolios, respectively. Only CCs for which at least one test rejects spanning at the 10% level are reported.

it is visible that starting roughly in January 2017, the difference between frontiers with (LIBRO) and without constraints all but vanishes—a change driven by the extreme growth of trading volumes and market capitalisations of the entire CC market during that boom period.

The CC market crash in early 2018 is also clearly visible as the frontiers collapse. At the trough, series of strongly negative returns amidst high volatility and evaporating liquidity lead to CCs playing close to no role in optimal portfolios. As the market consolidates, in 2019 CCs pick up their role again in extending the efficient frontier: however, until today the discrepancy between portfolios with and without concern for liquidity considerations remains pronounced. Consequently, the importance of limiting exposure to illiquid CCs remains high. Portfolio optimization without liquidity constraints may promise an attractive performance in theory which cannot be realized in the market.

For instance, the performance of unconstrained MD profits from unreasonably high weights on small and illiquid CCs. Table 5 illustrates this in terms of Effective N and PDI: MD reaches a very low Effective N of 3.99, although it includes only CCs, compare the weight composition in figures 4 and 5. PDI is clearly higher than for other strategies, in line with the objective of MD, and the PDI only shrinks marginally when incorporating LIBRO, whereas Effective N drops by about 1. This implies that the strategy focuses disproportionately on (a) singular CC(s), driving the high returns, which cannot be traded sufficiently for a portfolio of USD 10 mln. At the other end of the spectrum, the minimum-risk strategies focus on traditional assets with high trading volume, therefore they are little affected by LIBRO.

It is interesting to note that for the strategies with strong diversification, in particular MD and ERC, but also MV-S, enforcing the LIBRO constraints leads to *higher* turnover. This is of concern to investors, as it prompts higher

Table 8. Kolmogorov–Smirnov test of difference between distributions of return series without liquidity constraints and return series of LIBRO portfolios.

Allocation strategy	Rebalancing frequency		
	Monthly	Weekly	Daily
EW	0.0000	0.0000	0.0000
RR-MaxRet	0.1419***	0.1508***	0.1448***
MV-S	0.0345	0.0394	0.034
MinVar	0.0096	0.0096	0.0115
ERC	0.1179***	0.1134***	0.1141***
MinCVaR	0.0192	0.0221	0.0182
MD	0.0748***	0.0845***	0.0690**
COMB NAÏVE	0.0786***		
COMB	0.0182		

Note: Significance is indicated by * at the 0.1, ** at the 0.05, and *** at the 0.01 level.

transaction costs. At first sight this observation appears counterintuitive, as restricted weights could be expected to reduce trading needs (due to positions partially remaining at their binding limits). The puzzle is explained by the last two columns, reporting target turnover: clearly, changes in *target weights* are mitigated via the liquidity constraints, corresponding to intuition. At the same time, it is exactly small and illiquid CCs which exhibit the largest volatility, and thus prompt *larger* trades when at the next rebalancing date positions are brought back to target weights. Enforcing LIBRO constraints leads to positions in more (and prone to be less mature and less capitalized) CCs, triggering larger rebalancing needs in terms of portfolio turnover.

This leads to the question how significantly the LIBRO restrictions affect returns of the various strategies. To that end, we test hypothesis 6 by running Kolmogorov–Smirnov tests for equal distributions always pairing each unconstrained strategy with its LIBRO-constrained counterpart. The results are reported in table 8. The hypothesis that enforcing the restriction does not significantly affect the return distribution is accepted for risk-oriented strategies—those that invest minimally if at all in CCs—while it is strongly rejected for return-oriented or diversification-intensive strategies. This fits well with our intuition that LIBRO is relevant for strategies which suggest investment in many or smaller and thus illiquid CCs.

8.2. Interpretation of the results and discussion

In this section, we discuss the broader meaning of our results and stress their contribution in terms of practical insights for market participants considering CC investments. First, our findings make explicit the specific benefits for different groups of investors who are heterogeneous with regard to their objectives and rebalancing schemes. Second, we derive recommendations about the implementation feasibility of investment strategies, namely the reliance on historical price time series to build the portfolios, and additionally the effects of (i)liquidity of different coins. Finally, we summarize popular portfolio rules' characteristics, including their ability to manage risk and simultaneously take advantage of CCs-return properties, as well as their weight and risk compositions.

We started by asking: For whom is investing in the CC market valuable? Do risk-orientend, return-oriented, and diversification-seeking investors benefit similarly from CC investments? The efficient frontiers in figure 6 show that the main benefit of CCs accrue to investors who make use of their high-risk/high-expected-return character; investors with low risk tolerance benefit least. While it is not surprising that CCs constitute risky investments, figure 4 shows how risk-oriented strategies (minimizing variance or CVaR) hold almost always entirely traditional assets. At least a risk-return orientation is necessary for CCs to play a role in portfolios. At the other end of the spectrum, the extremely CC-affine MaxRet strategy, despite stellar performance during the boom phase of 2017, was all but wiped out by the end of our sample period (−97%).

By far the best performance was achieved by investors targeting strong (or even maximal) diversification. These strategies, ERC and MD, lead to sizeable exposures to a broader cross-section of CCs, while they limit the risks the EW strategy incurs.

So our general conclusion is that the usefulness of CCs in a portfolio strongly depends on the investor's objective. In particular investors targeting a well-diversified portfolio while willing to bear some risk are advised to consider CCs for their investments.

Our second focus was on implementation feasibility: On the one hand, we addressed whether the commonly high trading frequencies imply that only professional investors who rebalance their portfolios daily can profit from CCs, or if also less actively trading retail investors can.

Updating parameter estimates and trading portfolio positions back to their targets on a daily, weekly, or monthly basis does influence the performance of investors' portfolios. For instance, over our study period cumulative wealth for the MV-S strategy grows by 5% when readjusting the portfolio on a daily basis, by 12% with weekly, and 9% with monthly position changes (differences that may become more pronounced when transaction costs are deducted, as turnover is naturally higher at a higher rebalancing frequency).

However, these differences are not statistically significant; moreover, the overall picture stays unchanged across rebalancing frequencies: it is always diversification-seeking investors (ERC and MD) who outperform the other investment strategies. Therefore, our general conclusions about the effect of adding CCs into investment portfolios apply qualitatively in the same way to daily traders, weekly rebalancing and monthly reallocation (retail investors).

On the other hand, implementation feasibility means to consider limits of liquidity in CC markets. Bounds on CC weights by LIBRO (Trimborn *et al.* 2019) frequently bind, indicating that CCs are often not sufficiently liquid for investors with deeper pockets. Since the approach allows the inclusion of illiquid CCs up to restricted amounts, investors can still perform diversification strategies to quite a reasonable degree—strategies that rank among the most profitable. However, the impressive results by strategies with very broad CC exposure turn out not to be scalable. For instance, the MD strategy's excellent performance without liquidity constraints (+175%) turns out to vanish with LIBRO, when its final CW drops below initial wealth (−14%).

We would therefore urge all investors to always consider liquidity of any CC in a portfolio when they assess its calculated historical performance.

Regarding the question whether investors should focus on a single CC (if at all) or consider a broader cross-section, our findings clearly indicate that diversification also across CCs is beneficial. At the same time, investors could diversify too much. As table 5 shows, the MD strategy, which had the highest return, showcases an Effective N of only 3.26. ERC has much higher Effective N of 17.16, still it features considerably lower final cumulative wealth, at least in unconstrained optimization. Judged by PDI, MD is the most successful strategy, which of course is driven by the fact that the target-weight allocation of MD is derived precisely by maximizing PDI. However, this also indicates that including as many assets in the portfolio as possible is not necessary to adequately represent the covariance matrix, and not beneficial in terms of cumulative wealth. Figures 4 and 5 caution the interpretation of MD dominating in terms of accumulated returns. Both figures show that MD includes a broad range of CCs, whereas MinVar and MinCVaR—both with comparable Effective N and PDI—almost entirely exclude them, giving weight only to traditional assets. In this sense, we do find evidence that CCs can substitute for traditional assets in portfolio optimization.

Regarding the ERC strategy, while it reaches optimal diversification for the alternative metric of Effective N , it provides sizable gains in cumulative wealth and at the same adequately diversifies the portfolio. Figures 4 and 5 indicate that CCs and traditional assets are mixed in the portfolio, while the PDI is close to the one of MD, and DR^2 only second to the pure risk-oriented strategies MinVar and MinCVaR.

Therefore, including CCs to diversify the portfolio is beneficial to achieve high target returns and balancing traditional assets and CCs is advisable.

Even though CCs are highly volatile, the past pricing series are informative for portfolio allocation. As such, quantitative methodologies for portfolio allocation are applicable and one is not restricted to non-quantitative or model-free investment schemes. The model-free EW strategy is a special case: its performance in the middle of our time period was extraordinary; and so was its collapse when the 2017 price rally in CCs disintegrated. As with MaxRet, both parts are driven by the high weight of small CCs—these were precisely the ones that gained disproportionately in value during the price rally, and subsequently suffered the severest. Therefore, the EW strategy can exhibit phases of extraordinary returns but does not manage risk well. At the other end of the spectrum, however, strategies exclusively targeted at lowering risk at all cost do not benefit from CCs. This is, of course, unsurprising since lower risk must go at the expense of lesser expected return, most clearly visible in the efficient frontiers in figure 6.

We also confirm that the patterns of generally high means, high volatilities, excess kurtosis, and low correlations with traditional assets, documented frequently in prior research (see also our literature review in section 2), are also present in our sample (see the descriptive statistics in appendix A.2). Our contribution addresses the effect of including CCs in already broadly diversified portfolios: beyond the findings

already discussed, our central result is that the key conclusion of prior studies—that CCs are valuable additions to the investment universe—holds true even in our much broader investment universe.

While diversification strategies prove most promising, including only the top CCs foregoes diversification potential. Most importantly, returns of broad CC portfolios that are calculated without accounting for liquidity remain virtual: they cannot be realized by professional investors.

Finally, for certain types of investors, namely those highly risk averse, the benefits can prove too risky to pursue.

9. Conclusion

This study investigates cryptocurrencies (CCs) as new investment assets available to portfolio management. We investigate the benefit for different types of investors when they consider adding CCs to a well-diversified portfolio of conventional financial assets. We consider risk-averse, return-maximizing as well as diversification-seeking investors, each of whom trade at different frequencies, namely, daily, weekly or monthly.

We analyze the performance of common asset-allocation models based on historical prices and trading volumes of 52 cryptocurrencies, combined with 16 traditional assets. The rules-based investment methods cover a broad spectrum of investor objectives, from classical Markowitz optimization to recent strategies maximizing portfolio diversification. Along with pure allocation strategies we also include combined strategies from model averaging. The out-of-sample performance of portfolios is evaluated with a range of different measures, including cumulative wealth, risk-adjusted performance metrics, turnover, as well as the diversification effects produced by the optimal portfolios.

We find that due to the volatility structure of CCs, traditional risk-minimizing strategies, such as minimum-variance and minimum-CVaR, do not improve investment performance significantly. In contrast, approaches with high target returns, in particular, diversification-seeking portfolios, reach higher expected returns for investors via higher and broader cryptocurrency exposure. Regarding diversification benefits, we demonstrate a benefit beyond well-diversified, global portfolios of conventional assets without CCs. We also document how the rules we study have differing effects on portfolio diversification, depending on the concept of diversification and its respective quantification.

Finally, we show how constraints mitigating liquidity risks of cryptocurrencies (LIBRO) can significantly affect the outcome of strategies that rely on a larger cross-section of CCs. For portfolios as small as USD 10 mln, out-of-sample performance drops considerably. At the same time, the diversification benefits persist coherently across all frameworks.

Further extensions can be made along three main lines: first, more involved estimators of expected returns and the covariance matrix could be employed; second, more performance measures could be used to evaluate the outcomes of investment strategies; and third, additional portfolio-allocation

strategies could be included in the comparison. In particular, factor-based APT (arbitrage-pricing theory) models would constitute the complementary approach to the statistical-optimization techniques studied in this paper.

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Appendix

A.1. Test for difference of SR or CEQ between two strategies

We employ the test by Ledoit and Wolf (2008). Let $v = (\mu_i, \mu_j, \sigma_i, \sigma_j)$ denote the vector of the moments of two strategies i and j .

Then we can test for a difference of the strategies’ CEQs or SRs via the test statistics defined as the differences of those measures,

$$f_{CEQ}(v) = \mu_i - \frac{\gamma}{2}\sigma_i^2 - \mu_j + \frac{\gamma}{2}\sigma_j^2,$$

(A1)

or

$$f_{SR}(v) = \frac{\mu_i}{\sigma_i} - \frac{\mu_j}{\sigma_j},$$

(A2)

respectively.

Applying the delta method yields that if $\sqrt{T-M}(\hat{v} - v) \xrightarrow{d} N(0, \Psi)$, then

$$\sqrt{T-M}(\hat{f} - f) \xrightarrow{d} N(0, \nabla' f(v) \Psi \nabla f(v)),$$

(A3)

where ∇f stands for the derivative of f .

The standard error for such a test statistic \hat{f} then amounts to:

$$SE(\hat{f}) = \sqrt{\frac{\nabla' f(v) \Psi \nabla f(v)}{T-M}},$$

(A4)

so we require a consistent estimator $\hat{\Psi}$ for Ψ .

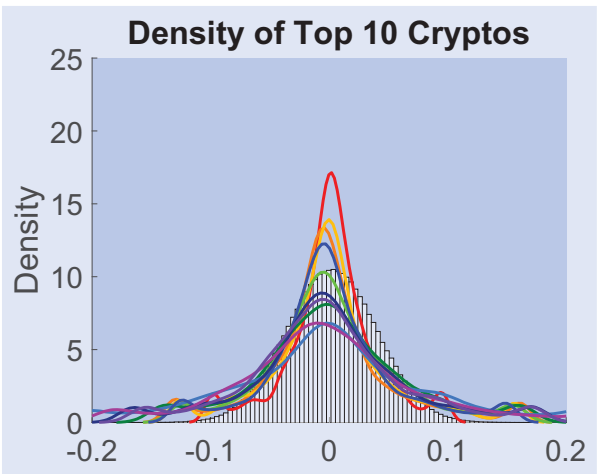


Figure A1. Density of daily log returns of the top 10 CCs (BTC, XRP, LTC, DASH, XMR, BCN, DOGE, BTS, DGB, NXT) against a normal distribution with same mean and variance as BTC. The time span covers our entire dataset and includes the estimation window, running from 2015-01-01 to 2019-12-31.

 CCPHistReturnsDensity

The standard method to provide such an estimator is to apply heteroskedasticity- and autocorrelation-robust kernel estimation to obtain the estimate

$$\Psi_{T-M} = \frac{T-M}{T-M-4} \sum_{j=-T+M+1}^{T-M-1} Ker\left(\frac{j}{S_{T-M}}\right) \hat{\Gamma}_{T-M}(j),$$

(A5)

where a kernel function $Ker(\cdot)$ and a bandwidth S_{T-M} need to be chosen.

Then a two-sided p -value for the hypothesis $H_0: f = 0$ is given as:

$$\hat{p} = 2\Phi\frac{|\hat{f}|}{SE(\hat{f})}.$$

(A6)

A.2. Descriptive statistics of portfolio components

For completeness, we present descriptive statistics both for our traditional assets as well as all 52 CCs in our sample. Table A1 shows that, as expected, correlations between CCs and traditional assets are low to non-existent, also in our sample. Tables A2 and A3 show univariate distributional properties of daily log returns on traditional

Table A1. Correlation coefficients of daily log returns of the top ten CCs with all conventional financial assets in our analysis (detailed in table 2) over the entire sample period from 2016-01-01 to 2019-12-31.

	BTC	XRP	LTC	DASH	XMR	BCN	DOGE	BTS	DGB	NXT
CNY	0.000	−0.050	−0.010	0.000	−0.020	−0.040	−0.030	−0.030	−0.030	0.000
FTSE REIT	0.000	0.020	−0.010	0.010	0.050	0.010	0.010	0.020	0.010	0.030
EUR	0.000	0.050	0.000	0.060	−0.030	0.020	0.030	0.000	0.010	0.030
GBP	0.010	0.030	−0.010	0.030	0.000	0.010	0.020	−0.010	0.040	0.010
GOLDS	0.020	0.020	0.020	−0.010	−0.010	0.020	0.050	0.020	0.030	0.020
JPY	−0.010	0.000	−0.020	0.060	0.030	0.020	0.000	0.020	0.020	0.010
MSCI CP	0.020	0.060	0.010	0.070	0.040	0.050	0.030	0.050	0.080	0.050
NIKKEI225	−0.020	0.030	0.000	0.020	0.000	0.020	0.000	0.020	0.010	0.050
SSE	−0.030	−0.040	−0.010	0.010	0.040	−0.010	−0.050	−0.020	0.030	−0.030
S&P 100	0.020	0.040	0.000	0.080	0.030	0.030	0.030	0.050	0.080	0.040
EURO STOXX 50	0.030	0.020	0.020	0.050	0.050	0.010	0.040	0.050	0.040	0.040
FTSE100	0.020	0.060	0.040	0.060	0.070	0.030	0.040	0.070	0.060	0.050
UK 10Y	0.010	0.020	0.000	−0.100	0.020	0.010	−0.010	0.000	0.000	0.030
Japan 10Y	−0.010	0.000	0.000	−0.030	−0.040	−0.020	0.000	−0.010	−0.060	0.010
USA 10Y	0.000	0.030	0.020	−0.060	−0.030	0.000	0.010	0.000	0.000	0.020
EURO 10Y	0.000	0.040	0.030	−0.080	0.030	0.000	−0.010	−0.020	0.010	0.050

Table A2. Descriptive statistics for daily log returns (in %) of all conventional assets in our baseline portfolio (detailed in table 2) over the entire sample period from 2016-01-01 to 2019-12-31.

Asset name	Max	P_{90}	Med	Mean	P_{10}	Min	SD
CNY	1.84	0.24	0.00	0.01	-0.23	-1.20	0.23
JPY	2.22	0.58	0.00	-0.01	-0.61	-3.78	0.53
EUR	3.02	0.59	0.00	-0.01	-0.63	-2.38	0.52
GBP	3.00	0.67	-0.02	-0.01	-0.65	-8.40	0.61
FTSE REIT	4.14	1.04	0.03	0.02	-1.08	-9.38	0.96
GOLDS	4.58	0.90	0.02	0.02	-0.84	-3.38	0.77
MSCI CP	2.67	0.78	0.04	0.02	-0.76	-4.88	0.69
NIKKEI225	7.43	1.24	0.01	0.02	-1.22	-8.25	1.19
SSE	5.60	1.47	0.01	-0.00	-1.33	-8.87	1.46
S&P100	4.84	0.97	0.03	0.03	-0.84	-4.18	0.83
EURO STOXX50	4.60	1.18	0.04	0.01	-1.20	-9.01	1.07
FTSE100	3.51	0.96	0.02	0.01	-0.95	-4.78	0.86
UK 10Y	2.00	0.30	0.00	0.00	-0.29	-1.43	0.28
Japan 10Y	0.74	0.09	0.00	-0.00	-0.08	-0.63	0.10
USA 10Y	1.29	0.30	0.00	0.00	-0.28	-1.55	0.27
EURO 10Y	0.85	0.22	0.00	-0.00	-0.21	-1.90	0.22

Note: P_{10} and P_{90} denote the first and ninth decile, respectively, 'Med' the median, and 'SD' standard deviation.

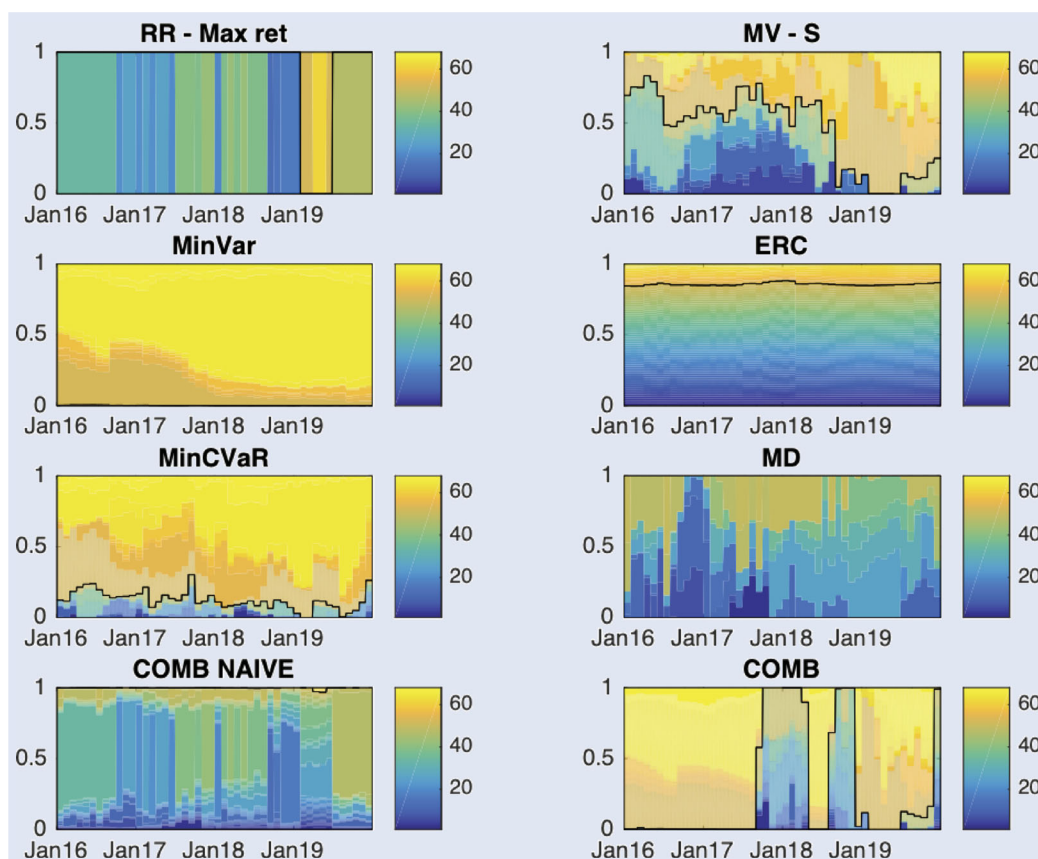


Figure A2. Evolution of *risk contributions* (i.e. fraction of portfolio value changes driven by each constituent) of all allocation strategies (without liquidity constraints) with monthly rebalancing over the period from 2016-01-01 to 2019-12-31: the black line separates conventional assets ('TrA,' upper yellow part of the spectrum) from cryptocurrencies (CCs, lower green-blue part of the spectrum).

CCPRisk_contribution

assets and CCs, respectively. The generally elevated magnitude for CCs is clear; figure A1 visually confirms the strong leptokurtic nature of CC returns.

A.3. Dynamics of risk contributions for portfolio strategies

The outcome of portfolio optimization can be viewed in two different ways: first, in terms of the weights the chosen strategy assigns to each asset; second, in terms of the risk each constituent contributes to

the portfolio. While flip sides of the same coin, with strongly divergent statistical properties across assets, as in our case, relative risk contributions can differ noticeably from relative portfolio shares. For instance, if a portfolio were to hold the same percentage of its value in UK bonds and in bitcoin, the changes in portfolio value over time driven by BTC will amount to a multiple of those stemming from the same-sized fixed-income position.

While we reported weights in figures 4 and 5 in the main text, for completeness we present the risk contributions as a function of time

Table A3. Descriptive statistics for daily log returns (in %) of all 52 CCs eligible for our portfolio strategies (detailed in table 1) over the entire sample period from 2016-01-01 to 2019-12-31.

CC	Max	P_{90}	Med	Mean	P_{10}	Min	SD
ABY	35.10	14.18	− 0.19	0.01	− 13.65	− 29.69	12.13
AUR	29.85	12.50	− 0.12	− 0.09	− 12.68	− 27.22	11.00
BCN	21.80	10.40	− 0.20	− 0.12	− 10.93	− 20.65	8.77
BLK	22.88	9.61	− 0.25	− 0.07	− 9.59	− 22.44	8.55
BTC	9.58	5.01	0.22	0.24	− 4.03	− 9.88	3.80
BTS	17.26	8.04	− 0.32	− 0.07	− 8.03	− 16.64	6.71
BURST	21.70	10.51	0.24	0.09	− 10.44	− 20.09	8.57
BYC	30.16	12.10	0.00	− 0.14	− 11.80	− 26.84	10.57
CANN	37.87	12.56	− 0.06	0.18	− 12.44	− 28.67	12.00
CURE	25.31	11.55	− 0.26	− 0.05	− 11.22	− 20.68	9.35
DASH	15.67	7.01	− 0.16	0.21	− 6.04	− 12.56	5.59
DGB	22.92	10.36	− 0.56	0.09	− 9.22	− 18.47	8.23
DGC	54.02	16.36	− 0.33	− 0.53	− 17.31	− 62.84	18.93
DMD	17.83	9.57	− 0.13	− 0.01	− 9.16	− 19.58	7.62
DOGE	14.99	6.75	− 0.25	0.05	− 5.62	− 12.58	5.34
EAC	41.30	12.35	− 0.07	− 0.04	− 13.06	− 37.18	13.11
EMC2	25.40	11.60	− 0.35	− 0.01	− 10.82	− 23.49	9.68
FTC	27.50	11.43	− 0.77	− 0.20	− 10.64	− 21.49	9.55
GRC	37.14	13.80	− 0.50	0.21	− 13.23	− 23.57	11.61
HUC	28.23	12.92	0.00	0.05	− 12.93	− 22.96	10.43
IOC	28.86	14.69	− 0.07	0.25	− 13.03	− 28.26	11.55
LTC	15.71	6.36	− 0.07	0.12	− 5.99	− 12.70	5.33
MAX	80.52	21.21	− 0.44	− 0.24	− 22.02	− 81.86	26.40
NAV	26.75	12.16	− 0.34	0.10	− 11.07	− 20.07	9.53
NEOS	28.82	12.37	0.00	− 0.22	− 12.43	− 25.50	10.36
NLG	18.09	8.89	− 0.20	0.04	− 8.53	− 14.55	6.91
NMC	16.64	7.34	− 0.20	− 0.07	− 7.27	− 16.26	6.27
NOTE	27.56	11.89	− 0.35	− 0.39	− 12.44	− 25.80	10.47
NVC	20.18	7.19	− 0.17	− 0.08	− 7.92	− 15.06	6.61
NXT	17.15	8.26	− 0.54	− 0.22	− 7.92	− 15.68	6.56
POT	20.16	9.77	− 0.07	− 0.03	− 10.39	− 19.77	8.13
PPC	16.94	6.98	− 0.19	− 0.07	− 7.47	− 15.07	6.26
QRK	37.18	11.97	− 0.38	− 0.05	− 12.22	− 30.88	12.15
RBY	25.58	12.34	0.00	0.10	− 12.33	− 28.14	10.41
RDD	32.46	14.26	− 0.05	0.08	− 13.67	− 28.14	11.90
SLR	25.15	11.21	− 0.23	0.01	− 11.01	− 21.65	9.41
START	29.14	14.48	− 0.66	− 0.16	− 12.99	− 26.97	11.36
SYS	24.83	10.61	− 0.14	0.20	− 10.24	− 19.16	8.72
UNO	22.07	11.13	− 0.01	0.16	− 9.47	− 23.61	8.79
VIA	25.06	10.68	− 0.00	− 0.01	− 11.35	− 20.53	9.22
VRC	33.85	14.29	− 0.51	0.04	− 13.28	− 28.46	11.86
VTC	27.29	10.98	− 0.41	0.03	− 10.74	− 19.54	9.09
WDC	29.58	11.66	0.00	− 0.42	− 12.39	− 33.05	11.24
XCN	59.83	20.82	− 0.42	− 0.16	− 22.36	− 51.64	20.06
XCP	22.41	10.77	− 0.48	− 0.19	− 10.48	− 21.09	8.84
XDN	24.72	11.73	− 0.29	− 0.13	− 12.12	− 22.17	9.58
XMG	31.69	12.70	− 0.18	0.24	− 11.33	− 23.38	10.54
XMR	16.29	8.54	− 0.05	0.26	− 7.22	− 14.12	6.36
XPM	21.99	9.61	− 0.25	− 0.20	− 10.58	− 19.71	8.45
XRP	16.18	6.70	− 0.39	− 0.06	− 6.05	− 13.14	5.52
XST	29.94	13.98	− 0.43	− 0.04	− 13.15	− 26.17	11.24
ZET	32.50	16.43	− 0.30	− 0.13	− 15.80	− 33.04	13.16

Note: P_{10} and P_{90} denote the first and ninth decile, respectively, ‘Med’ the median, and ‘SD’ standard deviation.

in figures A2 and A3 for portfolio optimizations without and with enforced liquidity constraints, respectively.

A.4. Results for daily and weekly rebalanced portfolios

While our main analysis maintained the industry standard of rebalancing on a monthly basis, we deem it important to also consider

higher trading frequencies in the CC market. We therefore report the performance results based on weekly rebalancing in table A4, as well as for daily reallocations in table A6.

Since diversification effects can also be affected by the rebalancing frequencies, tables A5 and A7 display the diversification measures for a weekly and daily frequency, respectively.

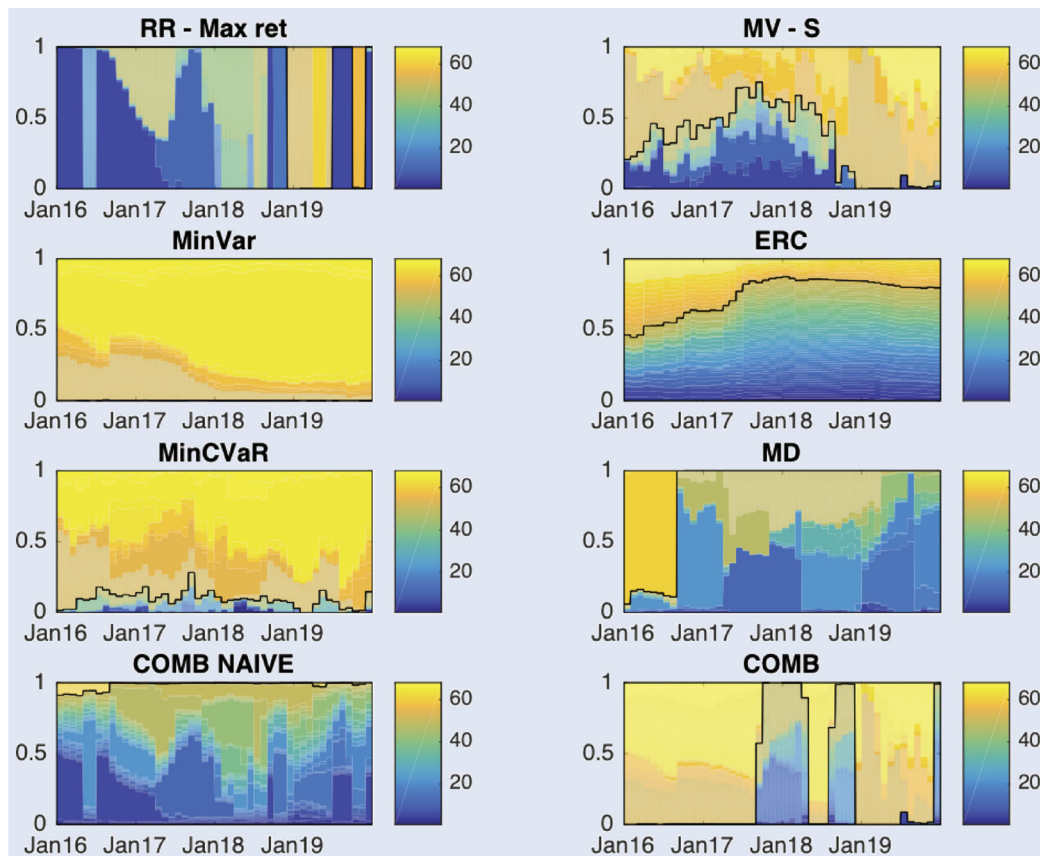


Figure A3. Evolution of *risk contributions* (i.e. fraction of portfolio value changes driven by each constituent) of all allocation strategies with a position limit of USD 10 mln (via the LIBRO approach) with monthly rebalancing over the period from 2016-01-01 to 2019-12-31: the black line separates conventional assets ('TrA,' upper yellow part of the spectrum) from cryptocurrencies (CCs, lower green-blue part of the spectrum).

CCPRisk_contribution

Table A4. Performance measures for all investment strategies as well as benchmarks over the entire time period from 2016-01-01 to 2019-12-31, with *weekly rebalancing* ($k = 5$).

Allocation Strategy	Portfolio		performance measures: weekly rebalancing								TO		TTO	
	CW		SR		ASR		CEQ							
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln		
<i>Benchmark strategies</i>														
S & P100	0.901	0.901	−0.016	−0.016	−0.016	−0.016	0.000	0.000	0.000	0.000	0.000	0.000		
EW TrA	1.103	1.103	0.033	0.033	0.033	0.033	−0.001	−0.001	9.557	9.557	0.000	0.000		
MV-S TrA	1.069	1.069	0.028	0.028	0.027	0.027	0.000	0.000	7.631	7.631	0.195	0.195		
EW	0.889	0.889	−0.003	−0.003	−0.003	−0.003	−0.001	−0.001	6.136	6.136	0.000	0.000		
<i>Risk-oriented strategies</i>														
MinVaR	0.99	0.99	−0.019	0.001	−0.019	0.001	−0.006	−0.001	15.370	11.352	0.035	0.035		
MinCVaR	1.014	1.014	0.017	0.017	0.017	0.017	0.000	0.000	32.224	8.301	0.087	0.082		
ERC	1.213	1.036	0.032	0.009	0.032	0.009	0.000	0.000	9.951	7.050	0.035	0.042		
MD	1.992	0.895	−0.002	−0.002	0.014	−0.002	0.000	−0.001	36.707	34.133	0.252	0.119		
<i>Return-oriented strategies</i>														
RR-MaxRet	0.151	2	1.074	−0.015	−0.016	−0.015	−0.016	0.000	0.000	0.454	2.952	0.454	0.278	
<i>Risk-Return-oriented strategies</i>														
MV-S	1.124	1.107	0.034	0.032	0.034	0.032	0.000	0.000	6.662	6.475	0.202	0.205		

Notes: The performance measures are final cumulative wealth (CW), the Sharpe ratio (SR), the adjusted Sharpe ratio (ASR), the certainty equivalent (CEQ), and turnover. 'TrA' denotes only traditional, i.e. non-CC assets are included. Strategies are detailed in table 1. Highest results are highlighted in red.

Table A5. Measures of diversification for all investment strategies and a benchmark, over the entire time period from 2016-01-01 to 2019-12-31 for *weekly rebalancing* ($k = 5$).

Allocation strategy	Portfolio diversification effects: weekly rebalancing					
	DR^2		Effective N		PDI	
	No const	10 mln	No const	10 mln	No const	10 mln
<i>Benchmark strategies</i>						
MV-S TrA	5.310	5.310	3.120	3.120	4.870	4.870
<i>Return-oriented strategies</i>						
RR-MaxRet	1.000	1.000	1.000	1.500	1.000	1.000
<i>Risk-oriented strategies</i>						
MinVar	12.030	11.730	2.670	2.680	20.520	20.520
ERC	8.710	9.400	17.200	13.600	20.540	20.530
MinCVaR	13.870	13.480	3.160	3.150	20.520	20.520
MD	2.650	2.060	4.000	3.170	20.970	20.920
<i>Risk-Return-oriented strategies</i>						
MV-S	8.010	7.430	3.320	3.390	20.540	20.540

Notes: DR^2 denotes the squared diversification ratio, PDI the portfolio diversification index; all three measures are detailed in section 6.3. 'TrA' denotes only traditional, i.e. non-CC assets are included. Strategies are detailed in table 1. Results without liquidity constraints (columns 'No const') are contrasted with those when applying LIBRO with a threshold of USD 10 mln (column '10 mln'). Highest results are highlighted in red.

Table A6. Performance measures for all investment strategies as well as benchmarks over the entire time period from 2016-01-01 to 2019-12-31, with *daily rebalancing* ($k = 1$).

Allocation strategy	Portfolio performance measures: daily rebalancing											
	CW		SR		ASR		CEQ		TO		TTO	
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln
<i>Benchmark strategies</i>												
S&P100	0.900	0.900	-0.016	-0.016	-0.016	-0.016	0.000	0.000	0.000	0.000	0.000	0.000
EW TrA	1.102	1.102	0.033	0.033	0.033	0.033	-0.001	-0.001	10.557	10.557	0.000	0.000
MV-S TrA	1.029	1.029	0.012	0.012	0.012	0.012	0.000	0.000	6.832	6.832	0.096	0.096
EW	0.877	0.877	-0.003	-0.003	-0.003	-0.003	-0.001	-0.001	19.400	19.400	0.000	0.000
<i>Risk-oriented strategies</i>												
MinVaR	0.985	0.985	-0.052	-0.011	-0.052	-0.011	-0.009	-0.002	19.930	18.083	0.013	0.013
MinCVaR	1.003	1.004	0.004	0.005	0.004	0.005	0.000	0.000	15.207	14.233	0.026	0.034
ERC	1.216	1.031	0.032	0.008	0.032	0.008	0.000	0.000	13.144	12.347	0.014	0.019
MD	2.149	0.967	0.016	-0.001	0.016	-0.001	0.000	-0.001	55.774	58.244	0.121	0.038
<i>Return-oriented strategies</i>												
RR-MaxRet	0.006	0.561	-0.022	-0.023	-0.022	-0.023	0.000	0.000	0.245	1.639	0.240	0.145
<i>Risk-Return-oriented strategies</i>												
MV-S	1.053	1.040	0.015	0.012	0.015	0.012	0.000	0.000	8.432	6.713	0.099	0.101

Notes: The performance measures are final cumulative wealth (CW), the Sharpe ratio (SR), the adjusted Sharpe ratio (ASR), the certainty equivalent (CEQ), and turnover. "TrA" denotes only traditional, i.e. non-CC assets are included. Strategies are detailed in Table 1. Highest results are highlighted in red.

Table A7. Measures of diversification for all investment strategies and a benchmark, over the entire time period from 2016-01-01 to 2019-12-31 for *daily rebalancing* ($k = 1$).

Allocation strategy	Portfolio diversification effects: daily rebalancing					
	DR^2		Effective N		PDI	
	No const	10 mln	No const	10 mln	No const	10 mln
<i>Benchmark strategies</i>						
MV-S TrA	5.340	5.340	3.130	3.130	4.870	4.870
<i>Return-oriented strategies</i>						
RR-MaxRet	1.000	1.000	1.000	1.500	1.000	1.000
<i>Risk-oriented strategies</i>						
MinVaR	12.060	11.750	2.670	2.680	20.490	20.490
ERC	8.720	9.400	17.220	13.620	20.510	20.510
MinCVaR	13.900	13.570	3.160	3.160	20.500	20.500
MD	2.650	2.060	4.010	3.180	20.950	20.900
<i>Risk-Return-oriented strategies</i>						
MV-S	8.030	7.460	3.310	3.400	20.510	20.510

Notes: DR^2 denotes the squared diversification ratio, PDI the portfolio diversification index; all three measures are detailed in section 6.3. 'TrA' denotes only traditional, i.e. non-CC assets are included. Strategies are detailed in table 1. Results without liquidity constraints (columns 'No const') are contrasted with those when applying LIBRO with a threshold of USD 10 mln (column '10 mln'). Highest results are highlighted in red.