Portfolio management strategies of cryptocurrencies

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Abstract: This study explores the portfolio management of cryptocurrencies by assessing the out-of-sample performance of selected portfolio strategies in the literature. Using daily data from 500 randomly selected cryptocurrencies with monthly and weekly revision, the scaled and stable mean-variance-entropic (MVE) value-at-risk portfolios outperform other portfolio strategies closely followed by 1/*N* portfolios. The mean Sharpe ratio with transaction costs of both MVE and 1/*N* was higher than that of benchmark, Coinbase index. Indeed, diversification across cryptocurrencies does improve investment results and mitigates risk exposure. The findings of this research are crucial for practitioners as they showcase a coherent manner to aid fund managers and investors in their investment practices.

Keywords: cryptocurrencies; portfolio strategy; diversification; risk.

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1 Introduction

Ever since the seminal work by Harry Markowitz in 1952 on mean-variance (MV) portfolio, researchers have assessed whether this approach of optimal diversification outperforms other portfolio optimisation methods. Kan and Zhou (2007) proved that MV has a problem of estimation risk in the parameters of the model, whereas DeMiguel et al. (2009) demonstrated that using estimated means and variances performs worse than a naively diversified portfolio strategy. Atta Mills et al. (2016) found out that other portfolio strategies, when compared to MV, have a better out-of-sample performance with high Sharpe ratios and relatively low turnover rates. The authors add to the debate by examining the performance of MV and other portfolio strategies in cryptocurrency markets.

The domain of money and investment is revolutionising before our eyes. Digitised assets and state-of-the-art financial conduits, securities and systems are designing innovative paradigms for monetary transactions and creating alternative channels of capital. During the tech boom of the 90's, there were attempts at digital currency creation with virtual currencies like Beenz, E-gold, Flooz and DigiCash budding on the market but unavoidably failing because of fraud, cybercrime and systemic problems among others (Cermak, 2017).

Cryptocurrencies are decentralised digital currencies that use encryption to authenticate transactions. Nakamoto (2008) released his research work describing Bitcoin. In January 2009, Nakamoto (2008) released the open-source client that propelled Bitcoin. In 2019, Bitcoin is the most commonly known and used cryptocurrency with a market capitalisation of $153,743,076,304 according to http://www.coinmarketcap.com on May 29, 2019.

During the past few years, cryptocurrency markets have considerably evolved. Amid vast public interest (Ciaian et al., 2016), trading cryptocurrencies has increased due to cryptocurrencies' unmatched price increases providing users with the possibility of achieving extremely high rewards in the shortest possible time while posing legitimate and supervisory disputes to central authorities (Fry and Cheah, 2016). As a medium of exchange, cryptocurrencies seem appealing due to the anonymity it for provides its users, low transaction costs because of the absence of intermediaries and the acceptance of Bitcoin payments by retailers. Thus, cryptocurrency users could trade goods and services, illegal ones inclusive (Foley et al., 2019). Nonetheless, apart from the prospect in using cryptocurrencies to fund illegal undertakings, cryptocurrencies are principally useful for speculation instead of being used as a conventional means of exchange (Blau, 2018).

Researchers have described cryptocurrencies as financial assets rather than currencies (Baur et al., 2018; Yermack, 2015) due to their variability (Köchling et al., 2019), susceptibility to speculative bubbles (Fry, 2018) and fat-tail behaviour (Huynh, 2019; Osterrieder et al., 2017), long memory (Phillip et al., 2018) among other characteristics. Inspired by the huge cryptocurrency price volatility (Katsiampa et al., 2019) as compared to traditional financial assets such as stocks (Bouri et al., 2017; Dyhrberg, 2016), the risks cryptocurrency users and traders face and the apparent interdependencies within cryptocurrency markets, researchers have studied volatility dynamics of cryptocurrencies and other statistical properties (Katsiampa, 2019).

All in all, the findings of previous studies, high volatility, leverage effects, susceptibility to speculative bubbles as well as growing efficiencies in the price dynamics of individual cryptocurrencies (Nadarajah and Chu, 2017; Tiwari et al., 2018), provide enough plausible reasons to consider diversified and optimal cryptocurrency investments as a way of minimising risk exposure in cryptocurrency markets. Brauneis and Mestel (2019) examine the benefits from developing cryptocurrency portfolios in a MV framework while Corbet et al. (2018) address diversification benefits of other cryptocurrencies other than Bitcoin since Bitcoin was considered as a hedge to other financial asset classes when constructing optimal portfolios. Bouri et al. (2018) found that Bitcoin could be employed as an effective diversification asset and, in some time horizons, also exhibit safe-haven and hedge characteristics.

Furthermore, studies on cryptocurrencies prove the potential likelihood of diversification in this budding market for investors and decision (Liu, 2019). A good proportion of cryptocurrencies have large market capitalisation and they usually have lower transaction costs because of the absence of intermediaries (Kim, 2017), which implies sufficient liquidity. There are also cryptocurrencies that the dynamics are comparatively isolated to others (Corbet et al., 2018), which provides diversification rewards. The number of cryptocurrencies is still on the surge and therefore, the cryptocurrency market has a role to play in diversification and portfolio optimisation.

Motivated by the viable reasons to consider diversified cryptocurrency investments to alleviate risk exposure, this study aims to close the research gap by comparing the out-of-sample performance of portfolio strategies using performance evaluation criteria to determine which optimisation model performs the best for cryptocurrency only investments. The authors also study the portfolio performance by setting different rebalancing periods to reflect a real-world scenario. The authors extend the studies of (Brauneis and Mestel, 2019; Liu, 2019; Platanakis et al., 2018) by considering other portfolio strategies in literature. Authors' choice of portfolio strategies is based on an extensive literature review.

This paper continues with the following content. Section 2 provides an overview of selected portfolio studies in the literature. Section 3 describes the data and approach, Section 4 presents the evaluation criteria, and Section 5 describes results and discussion. Concluding remarks is described in Section 6.

2 Selected portfolio strategies in literature

In finance, a portfolio is a pool of assets such as stocks, cash equivalents and now cryptocurrencies among other financial assets. The primary concern in asset allocation is constructing an optimal portfolio and it aims to find a risk-reward trade-off. The modern portfolio theory or MV portfolio strategy by Markowitz (1952) pursues optimal allocation of assets among a basket of financial investment and determines optimality based on risk and expected returns trade-off. Mathematically, Markowitz MV approach is a quadratic optimisation problem of the form:

$$\min \; w^T Q w$$

$$\text{subject to } \mu^T w \geq R \tag{1}$$
$$e^T w = 1$$
$$w \in \mathbb{R}^+,$$

where μ is a vector of mean returns, the variance-covariance matrix of portfolio return Q, e is a vector of ones and R is specified return level. This study imposes non-short selling constraint and normalisation of portfolio weights w. Minimum variance (MinV) follows the form of MV model but without the first constraint, i.e., expected returns constraint.

The MV model is criticised because it utilises returns of assets as deterministic denoted by a single point estimate thus, mean returns which contribute to the MV model being sensitive to the estimation of the returns, i.e., estimation risk. Brodie et al. (2009) studied the use of norms in tackling estimation risk. They proposed l_1-penalised version of the MV model. This optimisation problem is of the form:

$$\min \; w^T Q w$$
$$\text{subject to } \mu^T w \geq R \tag{2}$$
$$e^T w = 1$$
$$\| w \|_1 = \Xi^2$$
$$w \in \mathbb{R}^+,$$

where $||w||_1 = \sum_{k=1}^{N} || w_k ||$ and Ξ is a threshold value. Filomena and Lejeune (2012) proposed a probabilistic version of the MV model to tackle estimation risk. They also considered transaction costs to rebalance or revise the portfolio, which is another critique of the MV model. The chance constrained MV model with proportional transaction cost is of a stochastic optimisation form:

$$\min w^T Q w \tag{3}$$

subject to

$$\mathbb{P}\left(\sum_{k=1}^{N} \left(\varsigma_k w_k - c^b w_k^b - c^s w_k^s\right) \geq R\right) \geq 1 - \epsilon \tag{4}$$
$$\sum_{k=1}^{N} w_k + c^b \sum_{k=1}^{N} w_k^b + c^s \sum_{k=1}^{N} w_k^s = 1, \tag{5}$$
$$w_k = w_k^0 + w_k^b - w_k^s, \quad \forall k \tag{6}$$
$$e^T w = 1, \tag{7}$$
$$w, w^b, w^s \in \mathbb{R}_N^+, \tag{8}$$

where N is the number of assets, ς_k, $k = 1, \ldots, N$ is the random return of asset k, w_k^b is purchases (portion used) of asset k, w_k^s is sales (portion obtained) of asset k, w^0 is the portion of capital allocated to asset k and ϵ represents confidence interval. The transaction costs associated with selling and purchasing an asset is c^s and c^b, respectively. The above problem minimises risk of the portfolio and subjected to a set of constraints. Constraint (4) is a chance constraint that requires portfolio return to exceed the specified level of return R with a probability of $1 - \epsilon$. Constraint (5) is a budget constraint and constraint (6) is the balance constraint.

Various research works have been done to obtain a better measure of risk as variance used in the MV model penalises negative and positive deviations from mean return equally. Alternate risk measures for portfolio strategies have been studied and can be categorised into dispersion measures and safety-first measures. The idea of a coherent risk measure was proposed by Artzner et al. (1999) as a tool for determining the characteristics of a good risk measure. This led to the formulation of conditional value-at-risk (CVaR), which is a coherent risk measure and provides a better measure of downside risk than value-at-risk (Rockafellar and Uryasev, 2000).

Roman et al. (2007) studied mean-variance-CVaR (MVC) portfolio strategy and concluded that MVC model does not expel MV and mean-CVaR (MC) models but rather find a balance between attaining large CVaR and large variance from portfolios deduced from each respective model. Roman et al. (2007) achieved a better model using three indexes in comparison to MV or MC by solving:

$$\begin{aligned}
&\min w^T Q w \\
&\text{subject to } \mu^T w \geq R \\
&\mathrm{CVaR}_{1-\epsilon}(w) \leq L \\
&e^T w = 1 \\
&w \in \mathbb{R}^+,
\end{aligned} \tag{9}$$

where L is the specified level of risk. In solving the above problem, please refer to Rockafellar and Uryasev (2000) for an equivalent function of CVaR.

One problem with CVaR is that it dependent on tail of the distribution thus, not smooth. Ahmadi-Javid (2012) proved that intractable stochastic programming problems with CVaR are efficiently obtained with entropic value-at-risk (EVaR). In the light of this, Atta Mills et al. (2017) proposed a scaled and stable portfolio optimisation model based on variance and EVaR. Their model utilised squared l_2-norm constraint to mitigate estimation risk and pursued rescaling by capital available after revision of a portfolio. The scaled and stable MV-EVaR (MVE) model is of the fractional programming form

$$\begin{aligned}
&\min \frac{w^T Q w + \mathrm{EVaR}_{(1-\varepsilon)}(w)}{(e^T x)^2} \\
&\text{subject to } \mu^T w \geq R, \\
&\sum_{k=1}^{N} w_k + c^b \sum_{k=1}^{N} w_k^b + c^s \sum_{k=1}^{N} w_k^s = 1 \\
&w_k = w_k^0 + w_k^b - w_k^s, \quad \forall \kappa \\
&w_k^b \cdot w_k^s = 0, \\
&\|x\|_2^2 \leq \Xi^2 \\
&e^T w = 1 \\
&w, w^b, w^s \in \mathbb{R}_N^+,
\end{aligned} \tag{10}$$

where $||w||_2^2 = \sum_{k=1}^{N} w_k^2$. Returns are assumed to follow a Gaussian distribution, i.e., $X \sim \mathcal{N}(\mu, \sigma)$. Therefore, $\text{EVAR}_{1-\epsilon} = \mu + \sqrt{2ln\frac{1}{\epsilon}}\sigma$.

In the case of the naively diversified portfolio, a portfolio weight of $1/N$ is assigned to each asset in the portfolio and use $1/N$ with revision as in the case of DeMiguel et al. (2009). Thus, $w_k = \frac{1}{N}, \forall k$.

3 Data and approach

This section presents a practical application of portfolio optimisation models selected from an extensive literature review and fund managers' opinion to establish optimal portfolios and examine out-of-sample performance using cryptocurrencies traded on several exchanges worldwide like BitMEX, DigiFinex, among others. Historical daily log returns derived from closing prices of 500 randomly selected cryptocurrencies traded worldwide over the period 1 January 2016 to 30 April 2019 (1,216 trading days) were extracted from http://www.coingecko.com. Risk-free assets or any other risky assets are not considered as part of the portfolio implying that at any time, all capital funds are allocated to only cryptocurrencies. The selection criterion of a random sampling of cryptocurrencies is based on cryptocurrencies traded within the evaluation time horizon.

This paper also utilises Coinbase index daily cryptocurrency price data to provide a benchmark test of the results over the same time horizon even though 32% of 500 randomly selected cryptocurrencies are constituents of Coinbase index, which is weighted by market capitalisation. In an attempt to seek optimal cryptocurrency portfolios, we rely on selected portfolio strategies in Table 1.

Table 1 Overview of selected cryptocurrency portfolio strategies

Portfolio strategy	*Notation*
Naively diversified (DeMiguel et al., 2009)	$1/N$
Mean-variance (Markowitz, 1952)	MV
Mean-variance-CVaR (Roman et al., 2007)	MVC
l_1-penalised mean-variance (Brodie et al., 2009)	l_1-MV
Scaled and stable mean-variance-EVaR (Atta Mills et al., 2017)	MVE
Minimum variance	MinV
Stochastic mean-variance (Filomena and Lejeune, 2012)	SMV

The authors employ rolling estimation window of three months for parameter estimations. To construct the optimal portfolios, this paper considers the lower bound of mean return as required return level. This research normalises initial positions for cryptocurrencies by considering equal initial endowments thus, $w_k^0 = \frac{1}{N}, k = 1, \ldots, N$. In the real world, constructing a new portfolio or rebalancing an existing portfolio compels cost to be incurred and must be introduced in practical applications. Proportional transaction costs are incorporated into the framework of portfolio strategies understudy with an implicit assumption that transaction costs are paid at the beginning of the period. Portfolio revision or rebalancing is assumed to occur monthly (i.e., b = 30) since in practice they lead to reasonably low transaction costs. This study assumes proportional transaction costs of 25 basis points.

4 Evaluation criteria

The annualised out-of-sample Sharpe ratio with transaction cost (SRt) is reported. Sharpe ratio is the measure of average return earned in excess of the risk-free rate to a unit of volatility. Sharpe ratio is mathematically defined as $\text{Sharpe ratio} = \frac{(\rho_p - \rho_f)}{\sigma_p}$, where ρ_p and ρ_f are portfolio return and return from three-month US Treasury Bill rate. σ_p is the standard deviation of portfolio returns. This can help investors understand the return of cryptocurrency investment compared to its risk by showing how much additional return on cryptocurrency investment an investor earns by taking additional risk. A portfolio with a higher Sharpe ratio is regarded as superior relative to its counterparts.

In terms of portfolio risk (variance), risk reduction was used and this is defined as the measure of portfolio risk from a selected portfolio strategy to that from the MV model with transaction costs (Li, 2014). A portfolio with lower risk is regarded as superior relative to its counterparts. In the presence of norm-constrained portfolios, portfolio weights may be sparse leading to undiversified portfolios. This paper reports the mean active components, i.e., cryptocurrencies with non-zero portfolio weights over time. A sparse portfolio has few non-zero weights leading to low transaction costs.

Another way to evaluate portfolios is to use portfolio turnover (Li, 2014). Portfolio turnover measures the change in the cryptocurrency portfolio over a defined time. It measures the frequency with which cryptocurrencies are traded. This paper defines portfolio turnover as $\text{Turnover} = \frac{\sum_{t=T}^{\tilde{T}-1} \sum_{k=1}^{N} (|w_{k,t+1} - w_{k,t+1}^{-}|)}{\tilde{T} - T - 1}$, where $w_{k,t+1}$ is the portfolio weight for cryptocurrency k at $t + 1$, $w_{k,t+1}^{-}$ represents portfolio weight before rebalancing at time $t + 1$ and $\tilde{T} - T - 1$ is a representation of the length of non-zero constituents in total portfolio return. Higher values of turnover rates imply an actively managed portfolio.

5 Results and discussion

Descriptive statistics of historical daily log returns show that the cryptocurrency dataset has relatively high average returns and high risk. Not surprisingly, there exist asymmetric and leptokurtic return distributions. Also, there was a substantiation of non-normality, autocorrelation and heteroscedastic returns. 92% of all pairwise correlations between the randomly selected 500 cryptocurrencies fall within the interval –0.08 to 0.50, signifying a substantial degree of diversification effects. Most cryptocurrencies returns are weakly correlated even though they have common characteristic features. However, most cryptocurrencies are moderately correlated with Bitcoin as can be seen in Pearson correlation coefficients for different pairs of five cryptocurrencies (Table 2). It is important to note that 67% of 499 cryptocurrencies have a positive correlation with Bitcoin.

Table 2 Pearson product-moment correlation coefficient of five cryptocurrencies

	Bitcoin	*Bytecoin*	*Emercoin*	*Monero*	*Rubycoin*
Bitcoin	1.0000	0.3309	0.1380	0.4921	0.3600
Bytecoin	0.3309	1.0000	0.0717	0.2154	0.1193
Emercoin	0.1380	0.0717	1.0000	0.0996	0.0909
Monero	0.49208	0.2154	0.0996	1.0000	0.2149
Rubycoin	0.3600	0.1193	0.0909	0.2149	1.0000

Table 3 Summary statistics of five cryptocurrencies

	Bitcoin	*Bytecoin*	*Emercoin*	*Monero*	*Rubycoin*
Mean	0.0028	0.0021	0.0010	0.0039	–0.0002
Std. dev.	0.1379	0.0400	0.2340	0.0718	0.1199
Skewness	4.8416	–0.1824	0.5016	1.1307	1.4516
Kurtosis	97.4617	9.1995	441.7116	12.5298	35.4095
Jarque-Bera	45,6850.1	1,954.027	9,751,754	4,860.428	53,646.15
Prob.	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4 Out-of-sample performance of cryptocurrency portfolio strategies

Portfolio strategy	*SRt*	*Risk reduction*	*Sparsity*	*Turnover*
$1/N$	0.1524	0.0621	500.0000	0.0451
MV	0.1308	1.0000	500.0000	17.1241
MVC	0.1402	0.1801	500.0000	12.2231
l_1-MV	0.1392	0.3234	260.3215	12.0132
MVE	0.1641	0.0502	208.2420	5.2319
MinV	0.1311	0.0491	500.0000	7.2313
SMV	0.1337	0.3141	500.0000	16.0324

Summary statistics for returns of five selected cryptocurrencies are reported in Table 3 and return dynamics in Figure 1. The daily mean closing log-returns are positive for all cryptocurrencies except for Rubycoin. It is not particularly surprising that Bitcoin has a superior risk-return pattern, as it is the best performing cryptocurrency in May 2019. All five cryptocurrencies are leptokurtic as a result of high kurtosis, with Emercoin exhibiting the highest excess kurtosis. While Bitcoin returns are negatively skewed implying that it has a longer left tail, the opposite result holds for others. Jarque-Bera test confirms the evidence of non-normality for five cryptocurrencies returns series. The sample results confirm that of the descriptive statistics of randomly selected 500 cryptocurrencies.

Tables 4 and 5 show the annualised out-of-sample portfolio performance and computational results of some additional comparison metrics. Investors can examine risk-adjusted returns in exchange for the risk they bear by using the Sharpe ratio. The higher the Sharpe ratio, the more rewards an investor accumulates per unit of risk. Mean-variance-entropic (MVE) and naively diversified portfolios outperform other portfolios with MVE having the highest ratio. In terms of risk reductions (variance), scaled and stable MV-EVaR has a higher resolution to risk, therefore, has a lower risk than other portfolio strategies except for minimum variance. As the number of cryptocurrencies increase, MVE, MinV and 1/*N* attain lower risk than other portfolio strategies.

Figure 1 Return dynamics of five cryptocurrencies (see online version for colours)

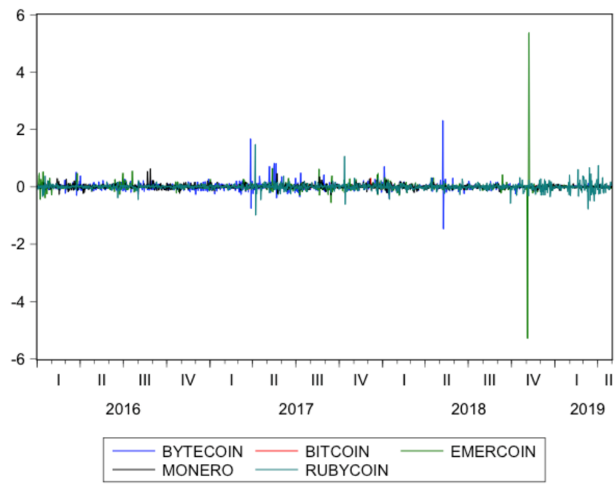

Table 5 Comparison of portfolios based on mean and standard deviation

	1/N	*MV*	*MVC*	*l_1-MV*	*MVE*	*MinV*	*SMV*
Mean							
b = 30	0.0069	0.0053	0.0064	0.0059	0.0074	0.0050	0.0062
b = 7	0.0071	0.0054	0.0066	0.0064	0.0078	0.0055	0.0063
Std. dev.							
b = 30	0.0458	0.0386	0.0410	0.0391	0.0434	0.0331	0.0393
b = 7	0.0464	0.0391	0.0413	0.0394	0.0455	0.0342	0.0399

Portfolio turnover assesses the frequency with which cryptocurrencies in a portfolio are traded. Turnover rate is preferably small. By convention, 1/*N* portfolios have the lowest turnover. Among all other actively managed portfolios, MVE and MinV have the lowest turnover rates. The authors noticed that large turnovers and reasonable transaction costs reduce the economic gains of some cryptocurrency portfolios. The highest turnover portfolio is from MV analysis leading to the smallest Sharpe ratio. About 40% of cryptocurrencies are selected by MVE and l_1-MV leading to a sparse portfolio that optimises the allocated funds by focusing on cryptocurrencies that result in diversification.

Table 5 reports net mean daily returns with corresponding standard deviations (std. dev.) for the selected portfolio strategies with different holding time horizons (*b* = 30) and (*b* = 7). In comparison to benchmark Coinbase index (Mean = 0.005821 and Std. dev. = 0.00361), most portfolios have higher rewards and higher risk.

From Table 5, an increment in transaction cost as a result of frequent revision with a portfolio strategy leads to a higher return but at the expense of slightly higher risk. MVE portfolios lead the way with highest returns notwithstanding different holding time horizons. Table 4 shows that portfolios MVE and 1/*N* seem to have better return-risk measure than other portfolios. The Sharpe ratios with transaction costs exhibited in Table 5 substantiates this point. MVE and 1/*N* portfolios outperform other portfolio strategies. The mean Sharpe ratio with transaction costs of both MVE (0.1641) and 1/*N* (0.1524) is higher than the Coinbase index Sharpe ratio of 0.1518. These results indicate that portfolio strategies MVE and 1/*N* have empirical backing of being a better choice in constructing cryptocurrency portfolios with MVE being the best. In comparison to individual results for each cryptocurrency (sample in Table 3), the Sharpe ratio increased, inferring that diversification across cryptocurrencies does improve investment results and mitigates risk exposure.

6 Conclusions

This paper contributes to the studies on cryptocurrencies and portfolio management by evaluating the performance of selected portfolio strategies of 500 randomly selected cryptocurrencies. This study concludes that scaled and stable MV-EVaR and 1/*N* portfolios outperform other portfolios. The mean Sharpe ratio with transaction costs of both MVE and 1/*N* was higher than that of Coinbase index. The results hold for different holding periods for transaction costs. Diversification across cryptocurrencies does improve investment results and mitigates risk exposure.

The portfolio strategies used in this paper considers proportional transaction costs, whereas actual transaction cost models may differ. An assumption to consider in the future is the problem of portfolio revision with transaction costs which are paid at the beginning of the planning horizon. The strategies used in this paper may be further investigated with the suggestions given above.

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