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Cryptocurrency portfolio optimization with multivariate normal tempered stable processes and Foster-Hart risk☆

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ABSTRACT

We study portfolio optimization of four major cryptocurrencies. Our time series model is a generalized autoregressive conditional heteroscedasticity (GARCH) model with multivariate normal tempered stable (MNTS) distributed residuals used to capture the non-Gaussian cryptocurrency return dynamics. Based on the time series model, we optimize the portfolio in terms of Foster-Hart risk. Those sophisticated techniques are not yet documented in the context of cryptocurrency. Statistical tests suggest that the MNTS distributed GARCH model fits better with cryptocurrency returns than the competing GARCH-type models. We find that Foster-Hart optimization yields a more profitable portfolio with better risk-return balance than the prevailing approach.

1. Introduction

Cryptocurrency is an entirely new finanical asset, which rapidly increases market capitalization and is attracting growing attention from market participants. From an econometric point of view, it is of interest to explore which type of time series model accounts for highly volatile returns on and accurately forecasts risks associated with cryptocurrencies. Among expanding literature, Caporale and Zekokh (2019), Cerqueti et al. (2020), and Troster et al. (2019) utilize generalized autoregressive conditional heteroscedasticity (GARCH) models. They share the common conclusion that the normally distributed GARCH model is inadequate for describing cryptocurrency returns, and the introduction of non-Gaussian distribution substantially improves the fit of a GARCH-type model. Other approaches include the stochastic volatility model (Chaim and Laurini, 2018), and the generalized autoregressive score model (Troster et al., 2019). The high volatility of cryptocurrency also highlights its speculative nature. Brauneis and Mestel (2019) investigate the risk-return relationship of an optimized cryptocurrency portfolio based on the Markowitz mean-variance framework.

This paper studies portfolio optimization of cryptocurrencies by employing a union of sophisticated time series models and risk measures. Four major cryptocurrencies are selected as samples: Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), and XRP. Our time

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series model is the multivariate normal tempered stable (MNTS) distributed GARCH model. The MNTS distribution (Kim et al., 2012) has demonstrated excellent fit to joint dynamics of physical asset returns in a number of empirical studies (Anand et al., 2016; Anand et al., 2017; Bianchi and Tassinari, 2020; Kim et al., 2015; Kurosaki and Kim, 2013a; Kurosaki and Kim, 2013b; Kurosaki and Kim, 2019; and Shao et al., 2015). Our portfolio optimization strategy is based on Foster-Hart risk, which is highly sensitive to risky left tail events. Foster-Hart (FH, hereafter) risk was originally introduced in the field of game theory (Foster and Hart, 2009), and was subsequently applied to financial market risk management (Anand et al., 2016; Kurosaki and Kim, 2019; Leiss and Nax, 2018). These cutting-edge techniques have not yet been documented in the context of cryptocurrency. Statistical tests demonstrate that the MNTS distributed GARCH model has better explanatory power for cryptocurrency returns than the normally distributed GARCH model. Also, when in tandem with FH risk, we find that the model creates more profitable portfolios than the traditional mean-variance approach.

The rest of this paper is organized as follows. Section 2 briefly introduces our methodology. Section 3 describes the dataset. Section 4 outlines the empirical results of statistical tests. Section 5 conducts portfolio optimization and discusses performance. Section 6 summarizes our findings.

2. Methodology

We introduce the methodology implemented to achieve efficient portfolio optimization. See also Online Appendix B for supplementary explanations.

2.1. Non-Gaussian time series model

We utilize a GARCH-type model with autoregressive (AR) and moving average (MA) processes to describe the dynamics of cryptocurrency returns. After the most standard ARMA(1,1)-GARCH(1,1) filtering, we obtain independent and identically distributed (i.i.d.) standard residuals η_t with a mean of zero and unit variance for each cryptocurrency. To describe complicated interdependency among cryptocurrencies, we conduct multivariate modeling on η_t for each cryptocurrency jointly. We employ an i.i.d. standard MNTS as an assumptive distribution that η_t follows. We also hypothesize that η_t follows an i.i.d. standard normal and student t as competing models. Hereafter, we denote the ARMA(1,1)-GARCH(1,1) model with multivariate normal, student t, and NTS distributed standard residuals as AGNormal, AGT, and AGNTS, respectively.

In contrast to Lévy's stable distribution, a tempered stable distribution has finite variance (Rachev et al., 2011). This property is desirable when fit into standardized residuals. The MNTS is especially advantageous due to its flexibility with respect to a multivariate extension. Both the estimation of the MNTS from real data and the scenario generation based on the estimated MNTS are feasible without computational difficulty, even in considerably high dimensional settings. These features are critical in their application to portfolio optimization.

2.2. Risk measures

We introduce FH risk. Let a gamble be any bounded random variable g with a positive expected value and a positive probability of losses: $\mathbb{E}(g) > 0$, $\mathbb{P}(g < 0) > 0$. FH risk is the minimum reserve that an agent should initially possess to prevent itself from almost certainly going bankrupt, even after the infinite repetition of the gamble g. Foster and Hart (2009) demonstrate that, for a gamble g, irrespective of the utility function, FH risk $R(g)$ is the unique positive root of the following equation (1):

$$\mathbb{E}\left(\log\left[1+\frac{g}{R(g)}\right]\right) = 0. \tag{1}$$

The bankruptcy-proof property endows FH risk with extremely high sensitivity to negative events. By regarding investments in financial assets as a gamble, FH risk is expected to sense market downturn in a forward-looking manner.

We also utilize more popular risk measures, Value at Risk (VaR) and Average VaR (AVaR), in order to supplement FH risk.[1] Risk forecasting accuracy is an important aspect of time series models. Statistically backtesting VaR and AVaR is feasible as they are relatively simplistic, whereas no backtesting methodology has been established for FH risk. We backtest VaR using the Christoffersen's likelihood ratio (CLR) test, and AVaR with the Berkowitz's likelihood ratio (BLR) tail test and Acerbi and Szekely (AS) test. See Christoffersen (1998), Berkowitz (2001), and Acerbi and Szekely (2014), respectively.

3. Data and estimation

Our dataset contains daily logarithmic returns of cryptocurrency exchange spot rates in U.S. Dollars per unit from 08/31/2015 to 03/31/2020, resulting in 1674 observations for each cryptocurrency. In reference to Caporale and Zekokh (2019), we select the following four cryptocurrencies as samples: Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), and XRP. Although we are aware of a greater number of the representative cryptocurrencies, these four cryptocurrencies overwhelm the others in both trading volume and

[1] AVaR is also called Conditional VaR or Expected Shortfall.

data length (cf. Online Appendix C). The data source is CoinMarketCap.[2] Table 1 reports the descriptive statistics of our dataset. All cryptocurrencies have larger kurtosis than those of the normal distribution and show non-zero skewness, and this observation motivates us to apply the non-Gaussian model. To obtain the AGNTS model, we first estimate the univariate AGT model for each cryptocurrency and subsequently fit the standard MNTS distribution to the same residuals η_t. We estimate all models based on maximum likelihood estimation. We refer the readers to Kurosaki and Kim (2019) for details regarding these techniques.

Our analysis procedure is as follows. First, we arrange a moving window with a length of 500 days. The first window, ranging from 08/31/2015 to 01/12/2017, moves ahead day by day until 03/31/2020, which amounts to 1175 distinct windows. Subsequently, we iteratively estimate time series models from returns data within each window and forecast a one-day-ahead return distribution. Finally, we assess the resulting 1175 models for statistical performance, as well as the optimized portfolio profitability based on a forecasted return distribution.

4. Statistical tests

To assess the capability of our multivariate GARCH-type models to account for marginal return dynamics of each cryptocurrency, we examine the statistical performance of the 1175 iteratively-estimated models using in-sample and out-of-sample tests.

4.1. *In-sample test*

As an in-sample test, we investigate the fitting performance of standard residuals η_t of the univariate ARMA(1,1)-GARCH(1,1) model for the assumptive distributions (normal, student t, and NTS) using the Kolmogorov-Smirnov (KS) and the Anderson-Darling (AD) tests. While both tests are designed to assess the proposed distributions for goodness-of-fit, the AD test puts more emphasis on fitting at the tail. Under the reasonable postulation that our sample is sufficient in number, we can compute p-values for both tests.

Tables 2 and 3 report the number of rejections for the KS and AD tests out of 1175 iterated estimations for AGNormal, AGT, and AGNTS residuals, respectively. AGNormal is almost always rejected by both tests and thus significantly underperforms in comparison to AGT and AGNTS. In the KS test, AGNTS has a smaller number of rejections than AGT in three out of four cryptocurrencies at the 10% level. More clearly, in the AD test, AGNTS has a smaller number of rejections than AGT in four (three) out of four cryptocurrencies at the 5% (10%) level due to the excellent ability of AGNTS to track tail behavior. Overall, AGNTS is the most preferable model.

While out-of-sample tests require a considerably large number of observations, in-sample tests apply to a limited number of observations. Therefore, as a robustness check, we conduct additional in-sample tests for expanded twenty cryptocurrencies during the most recent sample period. The results also support the superiority of AGNTS (cf. Online Appendix C).

4.2. *Out-of-sample test*

As an out-of-sample test, we backtest VaR and AVaR. Each iteratively-estimated model forecasts one-day-ahead VaR and AVaR, constituting a time series of VaR and AVaR forecasts for 1175 days from 01/12/2017 to 03/31/2020.[3] In line with the Basel accord, we adopt the 99% confidence level for VaR and AVaR. Backtesting is achieved by comparing VaR and AVaR with actual returns every day. Out-of-sample testing is more important than in-sample testing since our research interest lies in portfolio risk forecasting and optimization. In order to clarify our results, we divide the sample period into three subperiods and conduct out-of-sample tests by subperiod. Specifically, let Periods 1, 2, and 3 cover 01/12/2017 to 03/31/2018, 04/01/2018 to 03/31/2019, and 04/01/2019 to 03/31/2020, respectively.[4]

Table 4 summarizes the p-values of out-of-sample tests. The CLR test with conditional coverage is for VaR forecasts, and the BLR tail and the AS tests are for AVaR forecasts.[5] First of all, AGNormal has lower p-values than AGT and AGNTS, especially in the BLR and AS tests. AGNTS passes the CLR tests during any period and with any cryptocurrency, including at the 10% level, whereas AGT fails in Periods 1 and 2 at the same level. Also, AGNTS always passes the BLR tests except for in Period 2 and in BTC. By contrast, AGT fails in Periods 1 and 2 at the 5% level. Finally, AGNTS fails the AS tests for at most one cryptocurrency in each subperiod at the 10% level. However, AGT fails for two cryptocurrencies in Periods 1 and 3 at the same level. Therefore, we conclude that AGNTS shows the best performance in out-of-sample tests more clearly than in-sample tests.

5. Portfolio optimization

We practice portfolio optimization with cryptocurrencies consisting of BTC, ETH, LTC, and XPR, in line with Kurosaki and Kim (2019). The portfolio risk and reward are forecasted through multivariate time series models. The optimization is carried out by minimizing the objective risk measure under the tradeoff with expected returns and transaction costs following the procedure detailed in Appendix A. We exploit FH risk as the objective risk measure to be minimized, as well as standard deviation (SD) and AVaR.

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[2] https://coinmarketcap.com/

[3] Following Kim et al. (2010), the VaR estimation with AGNTS relies on the discrete Fourier Transform.

[4] Notice that each subperiod includes some form of turmoil. The cryptocurrency boom and crash around the end of 2017 for Period 1, the crash following the release of "hardfork" of Bitcoin cash at the end of 2018 for Period 2, and the Covid-19 crisis at the beginning of 2020 for Period 3.

[5] The p-values of the AS test are computed from 10^4 sample statistics generated by time series models and for a left tail.

Table 1
Descriptive statistics of our dataset. The sample period is from 08/31/2015 to 03/31/2020.

Cryptocurrency	Number of observation	Mean	Max	Min	Standard deviation	Kurtosis	Skewness
BTC	1674	0.0020	0.2251	− 0.4647	0.0405	13.9441	− 0.9392
ETH	1674	0.0027	0.3028	− 0.5507	0.0624	7.0200	− 0.1820
LTC	1674	0.0016	0.5103	− 0.4490	0.0565	12.8712	0.8100
XRP	1674	0.0019	1.0274	− 0.6163	0.0689	43.1126	2.8848

Table 2
Number of rejections of Kolmogorov-Smirnov tests out of 1175 iterated estimations for GARCH residuals.

Significance level	1%			5%			10%		
Model	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
BTC	1009	92	107	1079	186	185	1163	303	259
ETH	1110	136	134	1173	337	383	1175	542	523
LTC	889	416	488	1167	532	577	1175	648	693
XRP	1174	683	650	1175	783	764	1175	812	791

Table 3
Number of rejections of Anderson-Darling tests out of 1175 iterated estimations for GARCH residuals.

Significance level	1%			5%			10%		
Model	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
BTC	1022	137	81	1175	537	155	1175	629	236
ETH	1142	106	103	1175	329	287	1175	503	523
LTC	1144	377	462	1175	544	543	1175	665	648
XRP	1175	557	729	1175	812	801	1175	835	821

Table 4
p-values of three out-of-sample tests.

Test	CLR			BLR			AS		
Model	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
Period 1 (01/12/2017 to 03/31/2018)									
BTC	0.0020	0.0174	0.7307	0.0003	0.0338	0.7824	0.0000	0.0186	0.6921
ETH	0.0450	0.6691	0.6691	0.0000	0.6304	0.8778	0.0000	0.5016	0.3827
LTC	0.4489	0.4489	0.4489	0.0009	0.7565	0.7650	0.5963	0.8748	0.8455
XRP	0.0450	0.2217	0.4153	0.0000	0.5008	0.5698	0.0000	0.0835	0.0510
Number of p-values less than 5%	3	1	0	4	1	0	3	1	0
Number of p-values less than 10%	3	1	0	4	1	0	3	2	1
Period 2 (04/01/2018 to 03/31/2019)									
BTC	0.0388	0.0990	0.2236	0.0000	0.1242	0.0546	0.0000	0.0014	0.0001
ETH	0.0137	0.4419	0.7271	0.0000	0.3352	0.8721	0.0000	0.1670	0.2896
LTC	0.4419	0.2236	0.4419	0.0872	0.0064	0.1640	0.0319	0.2083	0.2491
XRP	0.7271	0.3366	0.3366	0.5057	0.3174	0.5176	0.2599	0.9671	0.9657
Number of p-values less than 5%	2	0	0	2	1	0	3	1	1
Number of p-values less than 10%	2	1	0	3	1	1	3	1	1
Period 3 (04/01/2019 to 03/31/2020)									
BTC	0.0394	0.4449	0.7295	0.0000	0.4436	0.2497	0.0000	0.0177	0.0420
ETH	0.1002	0.7295	0.6733	0.0000	0.9374	0.8386	0.0000	0.3157	0.6138
LTC	0.1002	0.4449	0.7295	0.0000	0.9158	0.9124	0.0000	0.1602	0.2592
XRP	0.2258	0.2258	0.4449	0.0000	0.5051	0.7859	0.0000	0.0731	0.1274
Number of p-values less than 5%	1	0	0	4	0	0	4	1	1
Number of p-values less than 10%	1	0	0	4	0	0	4	2	1

Table 5
Performance of each optimized portfolio ($\lambda = 0$). Both long and short positions are allowed.

Portfolio	Cumulative return (a)	SD (b)	AVaR (c)	FH risk (d)	a/b	a/c	a/d
AGNormal forecasts							
Mean-SD	0.8662	0.0415	0.2293	1.2554	20.8968	3.7773	0.6900
Mean-AVaR	− 0.3812	0.0464	0.2766	0.4555	− 8.2220	− 1.3785	− 0.8371
Mean-FH	0.0677	0.0509	0.2761	22.5218	1.3288	0.2450	0.0030
AGT forecasts							
Mean-SD	0.7612	0.0427	0.1981	1.4200	17.8362	3.8424	0.5361
Mean-AVaR	1.4448	0.0465	0.2392	0.9514	31.0754	6.0402	1.5186
Mean-FH	1.7895	0.0673	0.3590	1.6465	26.6043	4.9845	1.0869
AGNTS forecasts							
Mean-SD	0.7612	0.0427	0.1981	1.4200	17.8362	3.8424	0.5361
Mean-AVaR	2.1916	0.0488	0.2347	0.6627	44.9341	9.3391	3.3072
Mean-FH	**2.5889**	0.0527	0.2518	0.7347	**49.0979**	**10.2808**	**3.5235**

Table 5 exhibits optimization results under the absence of transaction costs ($\lambda = 0$), through a combination of time series models and objective risk measures. When the portfolio is optimized with respect to SD, AVaR, and FH under the tradeoff against expected returns, we refer to the corresponding portfolio as mean-SD, mean-AVaR, and mean-FH portfolio, respectively. Column 2 reports the cumulative returns that each optimized portfolio accrues from 01/12/2017 to 03/31/2020.[6] We see that the mean-FH portfolio with AGNTS forecasts yields the largest profit, followed by the mean-AVaR portfolio with AGNTS forecasts. Columns 3 through 5 show the SD, AVaR, and FH risk of the optimized portfolio itself, which are computed from their historical returns. Columns 6 through 8 indicate the cumulative returns adjusted by each risk measure.[7] We find that the mean-FH portfolio with AGNTS forecasts shows the highest return-to-risk ratios of any risk measures. Therefore, we conclude that this combination not only achieves the largest cumulative returns but also generates the most ideal balance between risk and reward.

As a robustness check, we conduct optimization in the presence of transaction costs, where the corresponding cost ($\lambda \cdot |w_t - w_{t-1}|$) is deducted from daily returns. We also consider the restriction that a short position is prohibited, which is realistic for conservative investors. Table 6 reports the performances of the optimized portfolios along with the naïve $1/N$ (equally-weighted) portfolio for reference. Here, the portfolios are optimized based on AGNTS forecasts for different cost aversion ($C = 0.01, 0.1, 1$), as well as in cases without transaction costs ($\lambda = 0$), when a short position is included (Table 6(a)) and excluded (Table 6(b)).[8] For visualization, Fig. 1 shows how the cumulative returns of the optimized portfolio evolve temporarily over the investment period. We observe that the mean-FH portfolio accrues the largest profit irrespective of cost aversion parameters. Therefore, the superiority of the combination of AGNTS and FH risk still holds under more realistic situations.

Before concluding, we discuss the naïve $1/N$ portfolio. Brauneis and Mestel (2019) and Liu (2019) suggest its effectiveness in cryptocurrency portfolio construction. In line with the existing literature, we confirm the effectiveness of the naïve $1/N$ portfolio in the sense that it shows a better risk-return balance than the mean-SD and the mean-AVaR portfolios. However, in terms of cumulative returns as well as risk-return balance, we also find that in most cases only the mean-FH portfolio outperforms the $1/N$ portfolio. We conclude that the Markowitz framework can finally beat the naïve $1/N$ strategy, but only if such sophisticated techniques proposed in this paper are combined.

6. Concluding remarks

This paper studies the portfolio optimization of four major cryptocurrencies. Statistical analysis demonstrates that the introduction of MNTS distribution substantially enhances the explanatory power of the GARCH-type model for cryptocurrency return dynamics, especially in terms of risk forecasting. FH risk is an immeasurably sensitive predictor for market crashes, for example, in scenarios such as those caused by the Covid-19 pandemic. The combination of the MNTS distributed GARCH model and FH risk leads to desirable portfolio optimization concerning cumulative returns as well as risk-return balance. We first document the effectiveness of those sophisticated techniques in the context of cryptocurrency. It is of practical importance to explore whether this effectiveness holds when we expand cryptocurrencies; the additional in-sample tests imply that our techniques are still promising. As daily observed data are accumulated, we leave the comprehensive analysis based on the expanded sample cryptocurrencies for our future research.

[6] Since AGT and AGNTS share the same residuals, both models produce the same mean-SD portfolio. Also, note that the first revenue recognition takes place on the day after the first optimization, 01/13/2017.

[7] Note that the return-to-SD ratio in Column 6 is the well-known Sharpe ratio.

[8] Note that the naïve $1/N$ portfolio is free of transaction cost.

Table 6
Performance comparison: each optimized portfolio and the naïve $1/N$ portfolio ($N = 4$). The forecasts of future returns are created by AGNTS.

(a) Case I: Both long and short positions are allowed.

Portfolio	Cumulative return (a)	SD (b)	AVaR (c)	FH risk (d)	a/b	a/c	a/d
			$\lambda = 0$				
Mean-SD	0.7612	0.0427	0.1981	1.4200	17.8362	3.8424	0.5361
Mean-AVaR	2.1916	0.0488	0.2347	0.6627	44.9341	9.3391	3.3072
Mean-FH	**2.5889**	0.0527	0.2518	0.7347	49.0979	10.2808	3.5235
			$C = 0.01$				
Mean-SD	0.4584	0.0431	0.2132	2.4016	10.6382	2.1497	0.1909
Mean-AVaR	0.8907	0.0534	0.2976	1.9448	16.6758	2.9933	0.4580
Mean-FH	1.3654	0.0541	0.2889	1.3393	25.2296	4.7263	1.0195
			$C = 0.1$				
Mean-SD	0.6785	0.0441	0.2310	1.7234	15.3700	2.9378	0.3937
Mean-AVaR	1.4453	0.0510	0.2652	1.1138	28.3494	5.4497	1.2976
Mean-FH	**2.8348**	0.0523	0.2543	0.6838	**54.2364**	11.1471	**4.1457**
			$C = 1$				
Mean-SD	1.2317	0.0427	0.2135	0.9101	28.8192	5.7699	1.3534
Mean-AVaR	2.0403	0.0490	0.2348	0.7194	41.6500	8.6889	2.8359
Mean-FH	**3.0913**	0.0506	0.2334	0.5708	**61.1124**	13.2444	**5.4158**
Naïve $1/N$	2.5708	0.0496	0.1917	0.6218	51.8551	13.4138	4.1347

(b) Case II: Short positions are prohibited.

Portfolio	Cumulative return (a)	SD (b)	AVaR (c)	FH risk (d)	a/b	a/c	a/d
			$\lambda = 0$				
Mean-SD	3.0081	0.0620	0.3129	0.9044	48.5249	9.6138	3.3260
Mean-AVaR	3.1988	0.0625	0.3151	0.8572	51.1716	10.1523	3.7317
Mean-FH	**3.7819**	0.0633	0.2952	0.7536	**59.7812**	12.8134	**5.0184**
			$C = 0.01$				
Mean-SD	2.7898	0.0623	0.3188	0.9628	44.8108	8.7500	2.8976
Mean-AVaR	3.0361	0.0626	0.3151	0.8933	48.4957	9.6359	3.3989
Mean-FH	**3.8834**	0.0638	0.2952	0.7452	**60.8829**	13.1575	**5.2112**
			$C = 0.1$				
Mean-SD	2.8726	0.0618	0.3129	0.9321	46.4583	9.1809	3.0818
Mean-AVaR	3.1656	0.0625	0.3151	0.8639	50.6318	10.0468	3.6645
Mean-FH	**4.0395**	0.0635	0.2952	0.7252	**63.5682**	**13.6862**	**5.5700**
			$C = 1$				
Mean-SD	2.9810	0.0620	0.3129	0.9102	48.0867	9.5274	3.2750
Mean-AVaR	3.0378	0.0634	0.3333	0.9158	47.8948	9.1136	3.3172
Mean-FH	**3.8179**	0.0637	0.2951	0.7526	**59.9722**	12.9356	**5.0730**
Naïve $1/N$	2.5708	0.0496	0.1917	0.6218	51.8551	13.4138	4.1347

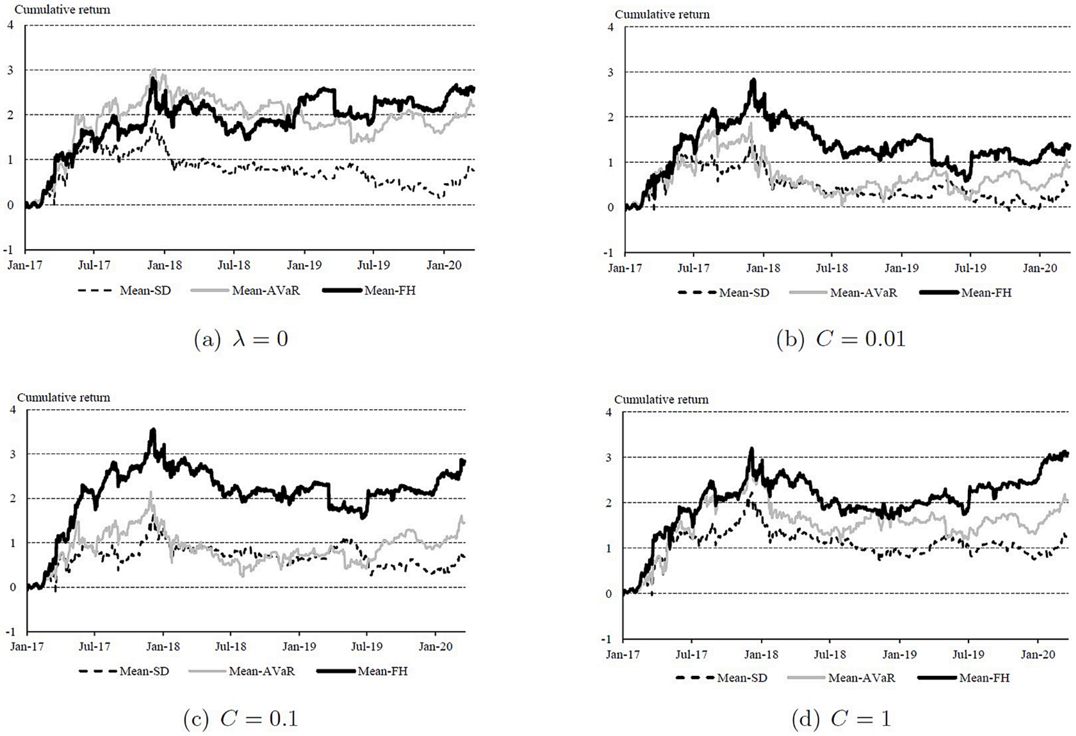
CRediT authorship contribution statement

Tetsuo Kurosaki: Conceptualization, Methodology, Software, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization, Project administration. **Young Shin Kim:** Conceptualization, Methodology, Software, Writing - review & editing.

Appendix A. Portfolio Optimization Procedure

The presented problem is to search for the optimized weight of each cryptocurrency i with expected return $\mu_{i,t}$ at time t, denoted by $w_{i,t}$ $(1 \le i \le 4)$, conditional on the information available up to time t, under the tradeoff between risk and reward. We identify this problem as minimizing the risk-to-reward ratio (disutility), where the risk and the reward are quantified by FH risk and expected returns, respectively. We supplementarily utilize one-day-ahead AVaR at the 99% confidence level and SD as risk measures for reference. We exclude VaR because of its non-convexity. Note that the optimization based on SD is equivalent to the classical Markowitz framework.

We impose some constraints on weights $w_{i,t}$. First, as a benchmark, we allow for both long and short positions up to the unit, while we consider the restriction that a short position is prohibited as a robustness check. Second, we maintain positive expected returns from



(a) $\lambda = 0$

(b) $C = 0.01$

(c) $C = 0.1$

(d) $C = 1$

Fig. 1. Time evolution of optimized portfolio's cumulative returns. The forecasts of future returns are created by AGNTS. Both long and short positions are allowed.

the portfolio. The second constraint stems from the principle of speculation as well as the necessary condition for FH risk to be defined. Additionally, we consider the cost of reallocating portfolio weights, which partly deprives the portfolio of cumulative returns. We also take investor cost aversion into account.

Consequently, our optimization problem is described using the following equation:

$$\min_{\substack{-1 \leq w_{i,t} \leq 1 \\ 0 \leq w_t^\top \mu_t}} \frac{\rho(w_t)}{w_t^\top \mu_t} + C\left[\frac{\lambda \cdot (w_t - w_{t-1})}{w_t^\top \mu_t}\right]^2, \tag{A.1}$$

where $w_t = (w_{1,t}, \cdots, w_{4,t})^\top$, $\mu_t = (\mu_{1,t}, \cdots, \mu_{4,t})^\top$, and $\rho(\cdot)$ is a risk measure of the portfolio. The second quadratic term of the change in weights corresponds to the transaction cost (e.g., Fabozzi et al., 2006). λ and C are the parameters for transaction cost charged per unit weight change and investor cost aversion relative to risk-to-reward ratio, respectively. We set λ as 10^{-7} when transaction costs are present.[9] We assume several distinct values for C as it depends on investors. Notice that our objective function (A.1) is set as homogeneous with respect to the size of w_t, as long as $\rho(\cdot)$ satisfies the homogeneity.

We, therefore, have the 1175 iteratively-estimated time series models from 01/12/2017 to 03/31/2020. Every day during this investment period, we generate one-day-ahead 10^4 scenarios for each cryptocurrency return by utilizing estimated AGNormal, AGT, and AGNTS models. Under these generated scenarios, we forecast risk and reward, thereby finding the optimal portfolio w_t to solve equation (A.1). The portfolio is optimized into w_t at the end of day t and creates profit or loss from the return at the end of day $t + 1$.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.frl.2021.102143.

[9] Since $\mu_{i,t}$ is typically on the order of 10^{-3} in our dataset, $\lambda = 10^{-7}$ suggests that transaction costs are roughly on the order of 1 bps to the portfolio expected returns.

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