11.8 Exercises

1. What is a power series?

2. (a) What is the radius of convergence of a power series? How do you find it?
(b) What is the interval of convergence of a power series? How do you find it?

3–36 Find the radius of convergence and interval of convergence of the power series.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

4. $\displaystyle\sum_{n=1}^{\infty} (-1)^n n x^n$

5. $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}\, x^n$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{5^n} x^n$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{5^n}{n} x^n$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1} x^n$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

13. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

14. $\displaystyle\sum_{n=1}^{\infty} n^n x^n$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

16. $\displaystyle\sum_{n=1}^{\infty} 2^n n^2 x^n$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{\sqrt{n}} x^n$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n 5^n} x^n$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n(n^2+1)} x^n$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$

21. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n} (x-1)^n$

23. $\displaystyle\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln n}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$

27. $\displaystyle\sum_{n=4}^{\infty} \frac{\ln n}{n} x^n$

28. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} x^n$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{b^n} (x-a)^n, \quad b > 0$

30. $\displaystyle\sum_{n=2}^{\infty} \frac{b^n}{\ln n} (x-a)^n, \quad b > 0$

31. $\displaystyle\sum_{n=1}^{\infty} n!(2x-1)^n$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$

33. $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$

34. $\displaystyle\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

35. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\,x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$

---

37. If $\sum_{n=0}^{\infty} c_n 4^n$ is convergent, can we conclude that each of the following series is convergent?

(a) $\displaystyle\sum_{n=0}^{\infty} c_n(-2)^n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n(-4)^n$

38. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?

(a) $\displaystyle\sum_{n=0}^{\infty} c_n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n 8^n$

(c) $\displaystyle\sum_{n=0}^{\infty} c_n(-3)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

39. If k is a positive integer, find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$$

40. Let p and q be real numbers with $p < q$. Find a power series whose interval of convergence is

(a) (p, q) (b) $(p, q]$ (c) $[p, q)$ (d) $[p, q]$

41. Is it possible to find a power series whose interval of convergence is $[0, \infty)$? Explain.

42. Graph the first several partial sums $s_n(x)$ of the series $\sum_{n=0}^{\infty} x^n$, together with the sum function $f(x) = 1/(1-x)$, on a common screen. On what interval do these partial sums appear to be converging to $f(x)$?

43. Show that if $\lim_{n\to\infty} \sqrt[n]{|c_n|} = c$, where $c \neq 0$, then the radius of convergence of the power series $\sum c_n x^n$ is $R = 1/c$.

44. Suppose that the power series $\sum c_n(x-a)^n$ satisfies $c_n \neq 0$ for all n. Show that if $\lim_{n\to\infty} |c_n/c_{n+1}|$ exists, then it is equal to the radius of convergence of the power series.

45. Suppose the series $\sum c_n x^n$ has radius of convergence 2 and the series $\sum d_n x^n$ has radius of convergence 3. What is the radius of convergence of the series $\sum (c_n + d_n)x^n$?

46. Suppose that the radius of convergence of the power series $\sum c_n x^n$ is R. What is the radius of convergence of the power series $\sum c_n x^{2n}$?

15. D **17.** C **19.** C **21.** D **23.** D **25.** C
27. C **29.** f is neither positive nor decreasing.
31. $p > 1$ **33.** $p < -1$ **35.** $(1, \infty)$
37. (a) $\frac{9}{10}\pi^4$ (b) $\frac{1}{90}\pi^4 - \frac{17}{16}$
39. (a) 1.54977, error $\leqslant 0.1$ (b) 1.64522, error $\leqslant 0.005$
(c) 1.64522 compared to 1.64493 (d) $n > 1000$
41. 0.00145 **47.** $b < 1/e$

EXERCISES 11.4 ■ PAGE 764

1. (a) Nothing (b) C **5.** (c) **7.** C **9.** D
11. C **13.** D **15.** C **17.** C **19.** D
21. D **23.** C **25.** D **27.** C **29.** D
31. C **33.** C **35.** C **37.** D **39.** C
41. 0.1993, error $< 2.5 \times 10^{-5}$
43. 0.0739, error $< 6.4 \times 10^{-8}$
53. Yes **55.** (a) False (b) False (c) True

EXERCISES 11.5 ■ PAGE 772

Abbreviations: AC, absolutely convergent;
CC, conditionally convergent
1. (a) A series whose terms are alternately positive and negative (b) $0 < b_{n+1} \leqslant b_n$ and $\lim_{n\to\infty} b_n = 0$, where $b_n = |a_n|$ (c) $|R_n| \leqslant b_{n+1}$
3. D **5.** C **7.** D **9.** C **11.** C **13.** D
15. C **17.** C **19.** C
21. (a) The series $\Sigma\, a_n$ is absolutely convergent if $\Sigma\, |a_n|$ converges. (b) The series $\Sigma\, a_n$ is conditionally convergent if $\Sigma\, a_n$ converges but $\Sigma\, |a_n|$ diverges. (c) It converges absolutely.
23. CC **25.** CC **27.** AC **29.** AC **31.** CC
33. CC **35.** -0.5507 **37.** 5 **39.** 5
41. -0.4597 **43.** -0.1050
45. An underestimate **47.** p is not a negative integer.
49. $\{b_n\}$ is not decreasing. **53.** (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$; $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

EXERCISES 11.6 ■ PAGE 778

1. (a) D (b) C (c) May converge or diverge
3. AC **5.** D **7.** AC **9.** AC **11.** D
13. AC **15.** AC **17.** AC **19.** D **21.** AC
23. AC **25.** D **27.** CC **29.** AC **31.** D
33. AC **35.** D **37.** AC **39.** (a) and (d)
43. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521
(b) $n \geqslant 11$, 0.693109

EXERCISES 11.7 ■ PAGE 781

1. (a) C (b) C **3.** (a) C (b) D
5. (a) D (b) C **7.** (a) C (b) D
9. D **11.** CC **13.** D **15.** D **17.** C **19.** C
21. C **23.** C **25.** C **27.** C **29.** D **31.** D
33. D **35.** C **37.** C **39.** C **41.** D
43. C **45.** D **47.** C

EXERCISES 11.8 ■ PAGE 786

1. A series of the form $\Sigma_{n=0}^{\infty} c_n(x-a)^n$, where x is a variable and a and the c_n's are constants
3. $1, [-1, 1)$ **5.** $1, (-1, 1)$ **7.** $5, (-5, 5)$
9. $3, [-3, 3)$ **11.** $1, [-1, 1)$ **13.** $\infty, (-\infty, \infty)$
15. $4, [-4, 4]$ **17.** $\frac{1}{4}, \left(-\frac{1}{4}, \frac{1}{4}\right]$ **19.** $2, [-2, 2)$
21. $1, [1, 3]$ **23.** $2, [-4, 0)$ **25.** $\infty, (-\infty, \infty)$
27. $1, [-1, 1)$ **29.** $b, (a-b, a+b)$ **31.** $0, \left\{\frac{1}{2}\right\}$
33. $\frac{1}{5}, \left[\frac{3}{5}, 1\right]$ **35.** $\infty, (-\infty, \infty)$ **37.** (a) Yes (b) No
39. k^k **41.** No **45.** 2

EXERCISES 11.9 ■ PAGE 793

1. 10 **3.** $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n, (-1, 1)$ **5.** $\displaystyle\sum_{n=0}^{\infty} x^{2n}, (-1, 1)$

7. $\displaystyle 2\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, (-3, 3)$ **9.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2^{4n+4}}, (-2, 2)$

11. $\displaystyle -\frac{1}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n 3x^n}{2^{n+1}}, (-2, 2)$

13. $\displaystyle\sum_{n=0}^{\infty} \left(-1 - \frac{1}{3^{n+1}}\right) x^n, (-1, 1)$

15. (a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n (n+1) x^n, R = 1$

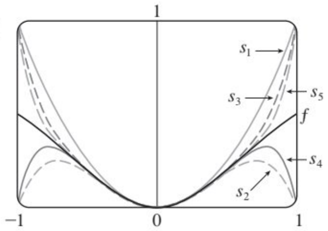
(b) $\displaystyle\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n (n+2)(n+1) x^n, R = 1$

(c) $\displaystyle\frac{1}{2}\sum_{n=2}^{\infty} (-1)^n n(n-1) x^n, R = 1$

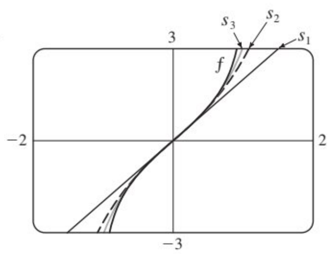
17. $\displaystyle\sum_{n=0}^{\infty} (-1)^n 4^n (n+1) x^{n+1}, R = \tfrac{1}{4}$

19. $\displaystyle\sum_{n=0}^{\infty} (2n+1) x^n, R = 1$ **21.** $\displaystyle\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, R = 5$

23. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n+2}, R = 1$



25. $\displaystyle\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}, R = 1$



27. $\displaystyle C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, R = 1$