Does Frequent Breathing Help When Exercising?

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Abstract

In this paper, we model the diffusion rate of Oxygen within lungs and compare the absorption capacity in two states, resting and exercising.

1 Model of Lung

Here, we model the lung as a simple membrane permeable to Oxygen:

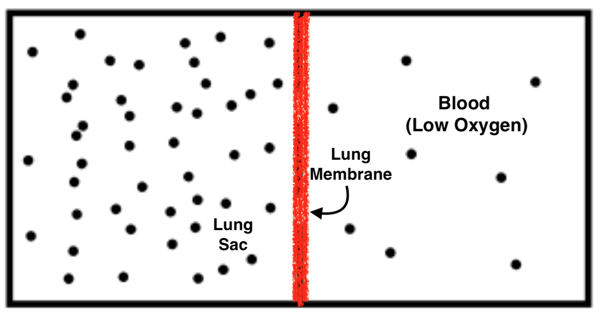

Figure 1: Diffusion

The oxygen is diffused from the high-concentrated lung sac into low-concentrated blood capillaries.
On the other hand, we model the breathing behavior with a two steps action; *Holding* after a fast inhaling, and *tranquillity* after a fast exhaling (Fig 2).

We ignore those fast rising and falling transitions in our simplified modeling. Consider α to be the percentage of time for holding or inhaling (*i.e.,* $\frac{\alpha}{1-\alpha}$ as the inhale to exhale ratio), N_L as the number of Oxygen molecules inside a full lung, and N_B as the whole number of Oxygen molecules inside the capillaries in direct contact with the sacs, before absorbing new molecules. We define $N = N_L + N_B$. The diffusion rate from one side with n molecules to the another, is (linearly)

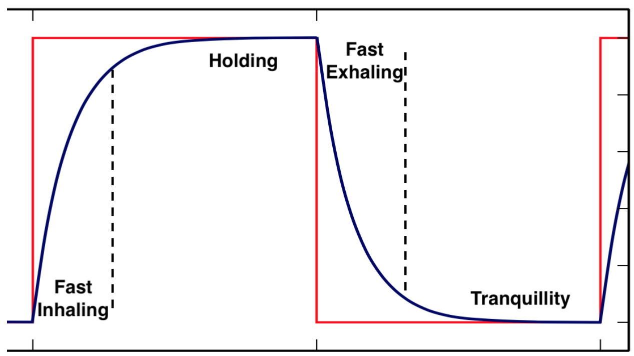

Figure 2: Breathing steps

related to the number of molecules in this side:

$$-\beta n$$

Besides, we assume that diffusion merely happens during the holding period, when the lungs have no interaction with outside. If $n(t)$ is the number of Oxygen molecules inside lungs, $N - n(t)$ will be the number of Oxygen molecules inside capillaries, and we will have the following equation:

$$\frac{dn}{dt} = -\beta n + \beta(N - n) = -2\beta(n - \frac{N}{2}) \tag{1}$$

and the answer is as follows ($n(0) = N_L$):

$$n(t) = \frac{N}{2} + (N_L - \frac{N}{2})e^{-2\beta t}. \tag{2}$$

After Δt seconds holding the breath, the total amount of Oxygen absorbed by the capillaries is as follows:

$$\Delta n = N_L - n(\Delta t) = (N_L - \frac{N}{2})(1 - e^{-2\beta \Delta t}). \tag{3}$$

2 Comparison Between Normal and Frequent Breathing

We compare a normal breathing with the period of T, and a fast breathing with the period of $\frac{T}{k}$ (breathing increased with a rate of k). In the first case, we absorb the following amount of Oxygen during T seconds ($\Delta t = \alpha T$):

$$\Delta n_1 = (N_L - \frac{N}{2})(1 - e^{-2\alpha\beta T}) \tag{4}$$

and for the second case, we absorb the following amount of Oxygen during T seconds:

$$\Delta n_2 = k(N_L - \frac{N}{2})(1 - e^{-2\alpha\beta\frac{T}{k}}). \tag{5}$$

To have a better comparison, we approximate the exponential term with $e^x \approx 1 + x + \frac{x^2}{2}$. So we get:

$$\Delta n_1 = (N_L - \frac{N}{2})(1 - e^{-2\alpha\beta T}) \approx (N_L - \frac{N}{2})(2\alpha\beta T - 2\alpha^2\beta^2 T^2) \tag{6}$$

$$\Delta n_2 = k(N_L - \frac{N}{2})(1 - e^{-2\alpha\beta\frac{T}{k}}) \approx (N_L - \frac{N}{2})(2\alpha\beta T - 2\alpha^2\beta^2\frac{T^2}{k}) \tag{7}$$

that shows Δn_2 is slightly greater than Δn_1, supporting the advantage for breathing faster during intense exercising!