Dimostrazioni Preservazione F

16/11

Lemma 1

$$F^+ \subseteq G^+ \iff F \subseteq G$$

Dim:

$\Rightarrow$ Banale

Dimostro $\Leftarrow$

$$F \subseteq G \Rightarrow F^+ \subseteq G^+$$

(Applico Assiomi ARMSTRONG)

$f \in F^+ = F^A$

$\exists f_1 \dots f_n \in F$ t.c. $\{f_1, \dots, f_n\} \xrightarrow{A} f$

$\forall f_i \in F \subseteq G^+ \quad \exists g_i^1, \dots, g_i^{m_i} \in G \quad \{g_i^1, \dots, g_i^{m_i}\} \xrightarrow{A} f_i$

$$\left.\begin{matrix} \{g_1^1, \dots, g_1^{m_1}\} \xrightarrow{A} f_1 \\ \vdots \\ \{g_m^1, \dots, g_m^{m_m}\} \xrightarrow{A} f_m \end{matrix}\right\} \xrightarrow{A} f \in G^A = G^+$$

Teorema $Z_f = X_G^+$ Correttezza dell'Algoritmo per calcolo X_G^+ data F

①

Dim $Z_f \subseteq X_G^+$

x Induzione

- CASO BASE

$$Z_0 = X \subseteq X_G^+ \quad \checkmark$$

- Caso Induttivo

$$Z_i \subseteq X_G^+ \Rightarrow Z_{i+1} \subseteq X_G^+$$

$$A \in Z_{i+1} = Z_i \cup S_i$$

2 POSSIBILITÀ

1) $A \in Z_i \subseteq X_G^+ \ \checkmark$

2) $A \in S_i = \bigcup_{j=1}^{k} [(Z_i \cap R_j)_F^+ \cap R_j]$

$\Rightarrow \exists j \in \{0, .., k\}$ t.c. $A \in (Z_i \cap R_j)_F^+ \cap R_j$

$A \in R_j$

$A \in (Z_i \cap R_j)_F^+$

$$\underset{\underset{R_j}{\cap\!\!\!|}}{Z_i \cap R_j} \rightarrow \underset{\underset{R_j}{\cap\!\!\!|}}{A} \in F^A (= F^+)$$

$$\Pi_{R_j}(F) = \{X \rightarrow Y \in F^+ / XY \subseteq R_j\}$$

$Z_i \cap R_j \to A \in \pi_{R_j}(F) \subseteq G$

So che $Z_i \subseteq X_G^+$

$\Rightarrow Z_i \cap R_j \subseteq X_G^+$

$\Rightarrow X \to Z_i \cap R_j \in G^A (= G^+)$

$\Rightarrow X \to A \in G^A$

$\Rightarrow A \in X_G^+$ ✓ se dim. di $Z_{i+1} \in G^A$ allora ogni elt. ne appartiene

② Dim $X_G^+ \subseteq Z_f$ SOLO PER LA LODE

Osserva che:

X e Y 2 insiemi qualunque

e so che $X \subseteq Y$

$\Rightarrow X_G^+ \subseteq Y_G^+$

$\downarrow$

$X \subseteq Z_f$

$\Downarrow$

$X_G^+ \subseteq (Z_f)_G^+$ ora ne dimostro che $Z_f = (Z_f)_G^+$

$S = \{ A \mid \exists\, Y \to V \in G \mid Y \subseteq Z_f,\ A \in V \}$

$\exists\, y \in \{1, \dots, k\} \mid Y \to V \in \pi_{R_j}(F)$ $\quad A \in R_j$

$Y \to V \in F^+ = F^A \Rightarrow A \in (Y)_F^+ \Rightarrow A \in (Z_f \cap R_j)_F^+ \Big\} A \in$

$y \in \mathcal{Z}_f$

$$A \in (\mathcal{Z}_f \cap R_{\bar{j}})^+_F \cap R_{\bar{j}}$$

$$\cap$$

$$S_f$$

$\Rightarrow$ l'algoritmo della chiusura

SI BLOCCA SUBITO