Vari Esercizi di ripasso

Foglio ④ 5, 6, 7

Es 5

A	produce	40%	con	errore	2%
B	//	10%	//	//	3%
C	//	50%	//	//	4%

1) $P(\text{pezzo difettoso}) = P(A) \cdot P(A_{dif}) + P(B) \cdot P(B_{df}) + P(C) \cdot P(C_{df})$

$= 0{,}4 \cdot 0{,}02 + 0{,}1 \cdot 0{,}03 + 0{,}5 \cdot 0{,}04 = 0{,}031 = 3{,}1\%$

2)

- $P(A \mid df) = \dfrac{P(A) \cdot P(A_{df})}{P(df)} = \dfrac{0{,}4 \cdot 0{,}02}{0{,}031} =$
- $P(B \mid df) = \dfrac{P(B) \cdot P(B_{df})}{P(df)} = \dfrac{0{,}1 \cdot 0{,}03}{0{,}031}$
- $P(C \mid df) = \dfrac{P(C) \cdot P(C_{df})}{P(df)} = \dfrac{0{,}5 \cdot 0{,}04}{0{,}031}$

Es 6

Invio 0 con $p = 0{,}45$ e arriva correttamente con $p = 0{,}94$

Invio 1 // $p = 0{,}55$ // // // con $p = 0{,}91$

$P(\text{Ricevo } 1) = P(\text{invio } 1) \cdot P(1 \text{ corretto}) + P(\text{invio } 0) \cdot P(0 \text{ sbagliato})$

$= 0{,}55 \cdot 0{,}91 + 0{,}45 \cdot 0{,}06 = 0{,}5275$

$P(\text{ricevo } 0) = \text{simile}$

$$P(\text{invio } 1 \mid \text{ricevo } 1) = \frac{P(\text{invio } 1) \cdot P(\text{ricevo } 1 \mid \text{invio } 1)}{P(\text{ricevo } 1)} = \frac{0{,}55 \cdot 0{,}91}{0{,}5275}$$

$$P(\text{invio } 0 \mid \text{ricevo } 0) = \frac{P(\text{invio } 0) \cdot P(\text{ricevo } 0 \mid \text{invio } 0)}{P(\text{ricevo } 0)}$$

$$P(\text{errore}) = P(\text{invio } 1) \cdot P(\text{ricevo } 0 \mid \text{invio } 1) + P(\text{invio } 0) \cdot P(\text{ricevo } 1 \mid \text{invio } 0)$$

$$= 0{,}55 \cdot 0{,}05 + 0{,}45 \cdot 0{,}06 = 0{,}0765$$

Es 7

Armando gioca 10 partite scommettendo 1 euro

p vince $= \frac{18}{37}$ $\left(p \text{ perde} = \frac{19}{37}\right)$

1) $P(00001) = \left(\frac{19}{37}\right)^4 \cdot \frac{18}{37}$

2) $\displaystyle\sum_{i=2}^{10} \binom{10}{i} \left(\frac{18}{37}\right)^i \left(\frac{19}{37}\right)^{10-i}$

3) $P(\text{Armando capitale } 2€) = \binom{10}{6} p^6 (1-p)^4$

Foglio 5 1, 2, 3, 8

Es 1

$P(\text{omozigoti}) = p$

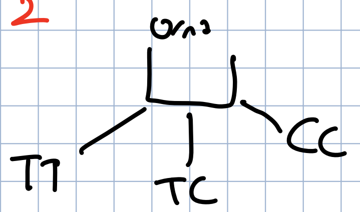
$P(\text{eterozigoti}) = (1-p)$ — stesso sesso 50% / sesso diverso 50%

$$P(\text{omo} \mid \text{stesso sesso}) = \frac{P(\text{omo}) \cdot P(\text{stesso sesso} \mid \text{omo})}{P(\text{stesso sesso})} = \frac{p \cdot 1}{\frac{1}{2} + \frac{1}{2}p}$$

$$P(\text{stesso sesso}) = P(\text{omo}) \cdot 1 + P(\text{etero}) \cdot \frac{1}{2} = p + (1-p) \cdot \frac{1}{2} = p + \frac{1}{2} - \frac{1}{2}p = \frac{1}{2} + \frac{1}{2}p$$

$$P(\text{sesso diverso}) = P(\text{omo}) \cdot 0 + P(\text{etero}) \cdot \frac{1}{2} = p \cdot 0 + (1-p) \cdot \frac{1}{2} = \frac{1}{2} - \frac{1}{2}p$$

Es 2



$$P(T) = P(TT) \cdot 1 + P(TC) \cdot \frac{1}{2} = \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{1}{2}$$

$$P(TC \mid T) = \frac{P(TC) \cdot P(T \mid TC)}{P(T)} = \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$$

$P(T \mid T_2)$

$$P(T_2) = P(TT) \cdot P(T) \cdot P(T) + P(TC) \cdot P(T) P(T)$$

$$\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} =$$

Es 3

Referendum, ciascuno vota con $p = \frac{1}{2}$ e vota SI con $p = \frac{1}{2}$

1) Un individuo vota SI

$P(\text{vota SI}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

2) $P(n° SI = k) = \sum_{i=k}^{n} \binom{n}{i} \left(\frac{1}{4}\right)^{i} \left(\frac{3}{4}\right)^{n-i}$

Es 8

Moneta Truccata

$P(T) = p$

$P(a \text{ teste prima di } b \text{ C}) = \sum_{i=a}^{a+b-1} \binom{i}{a} p^{a} (1-p)^{i-a}$

Foglio ③ 1, 2, 3

Es 1

9 studenti 3 compiti diversi

1) 9 studenti in 3 gruppi $\binom{9}{3}\binom{6}{3}\binom{3}{3}$

2) $3!$

3) $\dfrac{\binom{9}{3}\binom{6}{3}\binom{3}{3}}{3!}$

4) $\dfrac{\binom{9}{5}\binom{4}{2}\binom{2}{2}}{2!}$

5) 1

Es 2

Mazzo di carte napoletane, 10 carte a ciascun giocatore

1) $P(1B, 2B, 3B) = P(\text{napoli } B) = \dfrac{\binom{1}{1}\binom{1}{1}\binom{1}{1}\binom{37}{7}}{\binom{40}{10}}$

2) $P(\text{napoli } B \text{ e } C) = \dfrac{\binom{34}{4}}{\binom{40}{10}}$

3) $P(\text{Almeno 1 napoli}) = P($

Es 3

Problema di Calcetto

1) A(Calcetto sceglie tutti)

$\Omega = \{(\omega_1, \dots, \omega_4)\ \omega_i \in \{1, \dots, 30\} \wedge \omega_i \neq \omega_j\}$ $\quad |\Omega| = \binom{39}{4}$

$$P(A) = \frac{\binom{19}{4}}{\binom{39}{4}}$$

2)

$\Omega = \{\text{compiti diversi}\}$

$$|\Omega| = \binom{50}{5} \cdot \binom{30}{4} \cdot \binom{10}{1}$$

3 B(Calcetto sceglie tutti e 10 ...)

$$P(B) = \frac{\binom{20}{5} \cdot \binom{19}{4} \binom{5}{1}}{\binom{50}{5} \cdot \binom{30}{4} \cdot \binom{10}{1}}$$

4)

C(sceglie ESATTAMENTE 3, 2, 1)

$$P(C) = \frac{\binom{20}{3} \binom{30}{2} \cdot \binom{10}{2} \cdot \binom{20}{2} \cdot \binom{3}{1}}{\binom{50}{5} \cdot \binom{30}{4} \cdot \binom{10}{1}}$$

Foglio 2

Es 7

Aula I e II 50 studenti

Aula III 100 studenti

1) $\binom{200}{100}\binom{100}{50}\binom{50}{50} = \frac{200!}{100!\,100!} \cdot \frac{100!}{50!\,50!} \cdot \frac{50!}{50!} = \frac{200!}{100!\,50!\,50!}$

2)

$A_1 = \{A,B \text{ in } I,\ V \text{ in } II\} = \binom{197}{48} \cdot \binom{149}{45}\binom{100}{100}$

$A_2 = \{A,B \text{ in } I,\ V \text{ in } III\} = \binom{197}{48}\binom{149}{50}\binom{99}{99}$

$A_3 = \{A,B \text{ in } II\ \ V \text{ in } I\} = \binom{197}{49}\binom{148}{48}\binom{100}{100}$

$A_4 = \{A,B \text{ in } II,\ V \text{ in } III\} = \binom{197}{50}\binom{147}{48}\binom{99}{99}$

$A_5 = \{A,B \text{ in } III,\ V \text{ in } I\} = \binom{197}{49}\binom{148}{50}\binom{98}{98}$

$A_6 = \{A,B \text{ in } III,\ V \text{ in } II\} = \binom{197}{50}\binom{147}{49}\binom{98}{98}$

$P(A) = \sum_{i=1}^{6} \frac{A_i}{\frac{200!}{100!\,50!\,50!}}$

Foglio 6

Es 5

1) $P(\text{Alice supera esame}) = \sum_{i=7}^{10} \binom{10}{i} \left(\frac{1}{4}\right)^i \left(\frac{3}{4}\right)^{10-i}$

$P(X_{\text{corrette}}) = \binom{10}{x} p^x (1-p)^{10-x}$

2) $E(\text{Corrette}) = np = 10 \cdot \frac{1}{4} = \frac{10}{4}$

$E(\text{Sbagliate}) = 10 - \frac{10}{4}$

$E(X) = \underbrace{\frac{10}{4} \cdot 3}_{\text{risp. corretta}} \underbrace{-1\left(10 - \frac{10}{4}\right)}_{\text{risp. sbagliata}} = \frac{30}{4} - 10 + \frac{10}{4} = \frac{40}{4} - 10 = 0$

Foglio 7 1, 4, 5,

Es 1

$T = \{ n° \text{ lanci}$ $\qquad X = \begin{cases} 5 \\ 6 \end{cases}$

1) $P(T=3, X=5) = \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{4}{9} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{4}{27} \cdot \frac{1}{2} = \frac{4}{54} = \frac{2}{27}$

$P(X=5) = \frac{1}{2}$

$P(T) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

2) $P(T=t) = \left(\frac{2}{3}\right)^{t-1} \frac{1}{3}$

3) $P(X=i) = \frac{1}{2}$

Es 4

Circuito in serie $\qquad$ ciclo $\qquad$ $T \sim Geo(p)$

$P(T=J) = p \cdot \left[(1-p)^{J-1} \right]^{K}$ $K \to$ componenti del circuito

Circuito in parallelo si rompe se tutti si rompono.

$P(T=J) = \underbrace{\binom{J-1}{K-1} p^{K-1} (1-p)^{J-1-K-1}}_{K-1 \text{ componenti si rompono nei primi } J-1 \text{ cicli}} \cdot \underbrace{p \cdot (1-p)^{J-1}}_{\text{l'ultimo componente si rompe al ciclo } J}$

K componenti

Es 5

1) $P(X=1) = p(1-p) + (1-p)\,p$

$P(X=2) = p^2(1-p) + (1-p)^2\,p$

$\vdots$

$P(X=k) = p^k(1-p) + (1-p)^k\,p$

$$\mathbb{E}(X) = \sum_{n=1}^{\infty} n \cdot P(X=n)$$

esame già fatto

Foglio 8 1, 2, 3, 5, 7

Es 1

2% macchine campione di 100 persone $X \sim Bin(100, 2\%)$

P(almeno 3 macchine) = 1 − P(2 macchine)

$$P(X=2) = \sum_{k=0}^{2} e^{-2}\frac{(2)^k}{k!} = e^{-2}\cdot\frac{(2)^2}{1} + e^{-2}\cdot\frac{(2)^2}{2}$$

$$= e^{-2}\cdot 2 + e^{-2}\cdot\frac{4^2}{\cancel{2}} + e^{-2}\frac{(2)^0}{0!} =$$

$$= 2e^{-2} + 2e^{-2} + e^{-2} = 5\cdot e^{-2}$$

P(almeno 3 macchine) $= 1 - 5e^{-2}$

Es 2

$$\frac{e^{-\lambda p}(\lambda p)^k}{k!}$$

Es 3

Y = n° email Spam Z = email leggittimo

$$P(Y=k) = \frac{e^{-\lambda p}(\lambda p)^k}{k!}$$

$$P(Z=J) = \frac{e^{-\lambda(1-p)}(\lambda p)^J}{J!}$$

Es 5

7 Batterie 3 nuove, 2 usate sane, 2 difettose

me devo scegliere 3

X batterie nuove Y batterie usate funzionanti

1) Distribuzione Congiunta e Marginali

Parto dalla marginale

$$P(X=x) = \frac{\binom{3}{x}\binom{4}{3-x}}{\binom{7}{3}}$$

$$P(Y=y) = \frac{\binom{2}{y}\binom{5}{2-y}}{\binom{7}{3}}$$

Congiunta

$$P(X=x, Y=y) = \frac{\binom{3}{x}\binom{2}{y}\binom{2}{3-x-y}}{\binom{7}{3}}$$

3) $P(\text{[illegible]}) = P(X=3, Y=0) + P(X=2, Y=1) + P(X=1, Y=2)$

$$\frac{\binom{3}{3}}{\binom{7}{3}} + \frac{\binom{3}{2}\binom{2}{1}}{\binom{7}{3}} + \frac{\binom{3}{1}\binom{2}{2}}{\binom{7}{3}}$$

Foglio 10 2, 3, 4, 5

$$X \sim \text{Gauss}(\overset{\mu}{2}, \overset{\sigma^2}{25})$$

1) $P(|X-2| \geq 7)$

$$Z = \frac{X-(2)}{\sqrt{25}} = \frac{X-2}{5} \qquad X = 5Z+2$$

$$P(|5Z+2-2| \geq 7) = P(5Z \geq 7) + P(5Z < 7)$$

$$2P(Z \leq \tfrac{7}{5}) = 0{,}16$$

2) $P(0 < 5Z+2 < 7) = P(-\tfrac{2}{5} < Z < 1)$

$$= P(Z > -\tfrac{2}{5}) \cdot P(Z < 1) = P(Z \leq \tfrac{2}{5}) \cdot P(Z \leq 1) = 0{,}48$$

Es 3

$$X \sim \text{Gauss}(5, 0{,}25)$$

1) $P(X < 4) + P(X > 6)$

$$Z = \frac{X-(5)}{0{,}5} = \frac{X-5}{0{,}5} \Rightarrow X = \frac{1}{2}Z+5$$

$$P\left(\tfrac{1}{2}Z+5<4\right)+P\left(\tfrac{1}{2}Z+5>6\right) = P\left(\tfrac{1}{2}Z<-1\right)+P\left(\tfrac{1}{2}Z>1\right)$$

$$P(Z<-2)+P(Z>2) = P(Z>2)+(1-P(Z<2))$$

$$= 1-(P(Z<2)) + (1-P(Z<2))$$

$$= 2\cdot[1-(P(Z<2))] = 0{,}04 \Rightarrow 4\%$$

Es 4

$$B=\begin{cases} 1 & +2 \\ 0 & -2 \end{cases} \qquad A=\begin{cases} N=+2V \\ N=-2V \end{cases}$$

$X = \mu + Z$

se $X \geq 0{,}5 \Rightarrow B=1$

se $X < 0{,}5 \Rightarrow B=0$

1) A invia 0, B decodifica 1

$$P(X=-2+Z \geq 0{,}5) = P(Z \geq 2{,}5) = 1-P(Z \leq 2{,}5) = 0{,}0062$$

2) A invia 1, B decodifica 0

$$P(X=2+Z<0{,}5) = P(Z<-1{,}5) = P(Z \geq 1{,}5) = 1-P(Z<1{,}5) = 0{,}0668$$

$$P(A \text{ invia } 1) = P(A \text{ invia } 0) = \tfrac{1}{2}$$

$$P(B \text{ decodifica } 1) = \tfrac{1}{2}P(A \text{ invia } 0, B \text{ dec } 1) + \tfrac{1}{2}(A \text{ invia } 1, B \text{ deco } 1)$$

$$= 0{,}0031 + \tfrac{1}{2}\,0{,}9932 = 0{,}0031 + 0{,}4966 = 0{,}4997$$

$$\cong 50\%$$

$P(A_{mus\,1}, B_{dec\,1}) = P(X = 2+Z > 0{,}5) = P(Z > -1{,}5)$

$= P(Z < 1{,}5) = 0{,}9332$

4) $P(A_{mus\,1} | B_{dec\,1}) = \dfrac{P(A_{mus\,1}) \cdot P(B_{dec\,1} | A_{mus\,1})}{P(B_{dec\,1})} = \dfrac{\frac{1}{2} \cdot 0{,}9332}{0{,}4997} = 0{,}933$

FAI ES S