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RESEARCH PERSPECTIVES IN VEHICLE ROUTING AND SCHEDULING

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Abstract—The problem of vehicle routing and scheduling has been a popular research subject, yet much of the research has focused on developing methods that are accurate and computationally efficient but that are generally of limited scope. This article classifies these methodologies and appraises each class. Perhaps more important, the article identifies those restrictions and extensions that should be incorporated into a generalized vehicle routing and scheduling methodology and, therefore, points the direction for future research.

Although substantial research has already been carried out in the areas of vehicle routing and scheduling (VRS), much work remains. Such research is needed for several reasons. First, transportation management is a vital activity within most firms. The typical firm spends 10% of its sales dollar, and from one third to two thirds of its logistics budget on transportation (see LaLonde and Zinszer, 1976; and Stewart, 1965). Second, the new environment of relaxed regulation has meant that more firms are taking advantage of the expanding opportunities offered by private fleet ownership. Finally, the expanding presence of the computer in business operations is leading management to seek computational approaches that have sufficient realism to represent their routing problems while offering improved efficiency in the route design and scheduling of the truck fleets. This article compares various solution methodologies to the problem in order to show the strengths and weaknesses of each, and then gives suggestions about the directions of future research.

OVERVIEW OF THE ROUTING AND SCHEDULING PROBLEM

The basic vehicle routing and scheduling problem (VRS) is shown in Fig. 1. A company has a homogeneous fleet of vehicles that operates around a single depot. Requests for either pickup or delivery service, but not both, must be completely satisfied. The problem involves four decisions: (a) the size of the fleet of vehicles, (b) the assignment of customers (stops) to a vehicle, (c) the sequence, or order, that a vehicle travels to the customers assigned to it* and (d) the actual time schedule that the vehicle follows while completing its route, or tour.

Many techniques have been utilized to solve this basic problem. Unfortunately, no one technique has incorporated all, or even most, of the practical options or restrictions confronting a company. One can complain that the basic VRS problem can be adequately solved by existing techniques, but many real-life restrictions and extensions have not been treated, especially within a single model. Consequently, analysts have had to either custom develop their own solution techniques with the needed modifications or be satisfied with existing methods of limited scope.

The Appendix identifies the most frequently occurring extensions to the VRS problem. Probably no one company is faced with all of these extensions, but any given company will no doubt experience some of them. There are many other special extensions that a company's trucking system could incur. For instance, REVCO D. S., a large drug store chain, makes weekly deliveries of merchandise from a warehouse to its retail stores in the region. One special problem that it experiences is having a few stores whose weekly demands seasonally exceed the capacity of the delivery trucks. Store deliveries may need to be divided between two trucks during the peak seasons. One truck routed to one or more stops is usually assumed in the basic problem. Although the Appendix cannot deal with all of the possible extensions, we feel that the essential real-life restrictions that traffic managers are forced to consider when solving the VRS problem are represented.

METHODOLOGIES

Several excellent articles discussing various approaches to the vehicle scheduling and routing problem have recently been published (see Mole, 1979; Magnanti, 1981; Bodin and Golden, 1981; and Bodin, Golden, Assad and Ball, 1983). Figure 2 displays a general classification scheme summarizing these approaches. Various heuristics, optimization algorithms and interactive approaches have all been used to solve the VRS problem.

* Generally called the traveling salesman problem. A solution procedure must find the shortest route or tour for a vehicle that originates at the depot and visits only once all customers assigned to the vehicle.

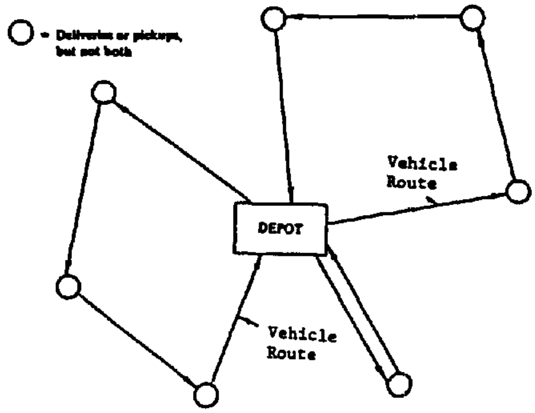

Fig. 1. The basic vehicle routing and scheduling problem.

The circular nature of the diagram highlights the fact that combinations of these three approaches have also been used. These four general approaches and examples of each are outlined in the following section. A brief discussion is offered of their abilities to deal with the extensions in the Appendix. Any general deficiencies with the approaches are noted as well.

Heuristic approaches

Due to the complex combinatorial nature of the VRS problem, heuristic techniques dominate the solution procedures. Four methodologies in this category are utilized. Briefly consider each of these.

Cluster-first, route-second procedures. First, group or cluster, customers and then determine feasible routes for each group as a second step. A good example of this is the method of Gillette and Miller (1974), which uses polar coordinates to develop vehicle routes.

It begins with a random starting point and then "sweeps"† around the depot to determine the stops on the route and finishes with a good traveling salesman heuristic to sequence the stops. Many of the extensions in the Appendix can be addressed by this method, however the procedure has two drawbacks. First, vehicles with different capacities (see Item 7 in the Appendix) cannot be easily incorporated due to the nature of the sweep procedure. Second, as the number of customers per route increases, the computation time increases dramatically.

Route-first, cluster-second procedures. These methods build a large (usually infeasible) route first. Then the large route is partitioned into a number of smaller feasible routes. Ball, Golden, Assad and Bodin (1983) developed two methodologies that deal directly with Items 1, 6, 8 and 14 in the Appendix. The procedures did not perform well for small problems.

Savings, or insertion, procedures. These methods build a solution (possibly an infeasible one) by calculating the savings generated by a new routing configuration, or determine the least expensive insertion of a customer into a route. Some of earliest VRS heuristics were based upon this logic, and many variations have been studied. The most widely known of these is the Clarke and Wright (1964) heuristic. The procedure begins with one stop per route, and combines a stop with another route if the combined route will "cost" less than the previously uncombined routes. This method theoretically can accommodate all of the extensions in the Appendix and has already been used on VRS problems involving many of the extensions (see Ballou and Chowdhury, 1980). However, the method has been labeled as "myopic" and has a tendency of having routes that can cross themselves, which is usually suboptimal.

Exchange, or improvement, procedures. These methods maintain feasibility in the solution while modifying the routes step by step to reduce costs. Russell (1977) has modified the "*r*-opt" stop exchange heuristic, see Lin and Kernighan (1973),‡ for the traveling salesman problem. Although this procedure in theory can give "optimal" solutions, long computation times are often required to do so. It is unclear as to how many of the extensions in the Appendix can be dealt with by this procedure, but the time window restrictions were included in the original study.

Exact, or optimal, approaches

Exact, or optimal solution procedures are always desirable. Unfortunately, such procedures have often been too computationally complex for even the fastest of computers to solve realistically sized VRS problems. Recent developments, particularly in the mathematical program-

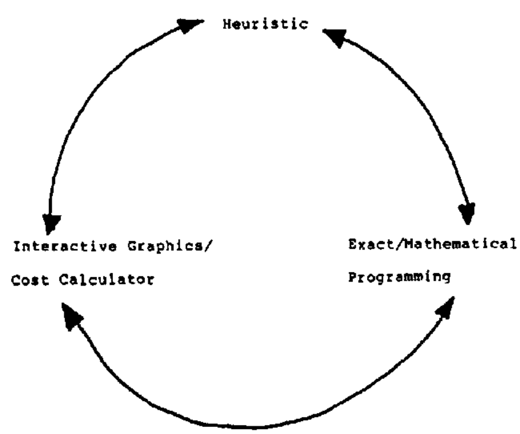

Fig. 2. Solution strategies for the vehicle routing and scheduling problem.

† The procedure generates a solution using each customer's polar coordinates as the starting position of an imaginary axis. Then the axis sweeps clockwise or counterclockwise around the depot, assigning customers to a route until some criteria is violated, that is capacity or distance limits. Then the sweep continues, building new routes until all customers have been assigned, and using a traveling salesman heuristic whenever a customer is added to a route.

‡ The *r*-opt stop exchange heuristic starts with a feasible solution and examines savings from switching the position of 1, or 2 . . . , or *r* customers.

ming area, have shown some promise. Christofides, Mingozzi and Toth (1981a,b) have derived two different branch and bound algorithms. The first uses dynamic programming to obtain effective lower bounds, whereas the second uses an improved Lagrangian relaxation procedure to optimally solve small problems. The main difficulty with all exact procedures is the huge number of constraints and variables needed to represent even the basic VRS problem and their adverse effect on computation time and computer storage space (see Magnanti, 1981).

Interactive approaches

The third solution strategy is the most simplistic, but in some respects it may be the most powerful of the strategies. Interactive methods involve either a simulation, or cost calculator approach, or some type of graphics capability to aid the decision maker. The cost calculator approach simply costs out a solution generated by the dispatcher so that he can quickly determine the effectiveness of his routes. Graphics give visual aids so that routes can be generated initially, redesigned or finally verified as satisfactory. Both methods are merely aids to the decision maker, who still must use some intuitive methods to solve the problem. Doll (1980) has developed a procedure that simply uses the idea of teardrop-shaped routes to quickly generate solutions. If both interactive methods are computerized and used by an experienced dispatcher, good, fast solutions are possible. Also, any of the extensions in the Appendix can be incorporated, although it is doubtful that all could be effectively managed simultaneously by a dispatcher. The obvious drawback to this approach is the skill and ability of the decision maker, particularly as the problem size and complexity increases.

Combination approaches

Combinations of the three strategies in Fig. 2 have proven very popular. Several studies combining mathematical programming and heuristic techniques have performed well (see Stewart, 1981; and Foster and Ryan, 1976). The Fisher and Jaikumar (1981) approach involves a "sweep" procedure similar to Gillette and Miller's method. The sweep generates an estimate of the cost of an optimal traveling salesman tour involving each customer, a "seed" customer and the depot for each vehicle in the problem. These costs are then utilized to solve a generalized assignment problem using Lagrangian relaxation, which assigns customers to vehicles without violating their capacity. The vehicle stop sequencing is then solved. If a vehicle has fewer than 10 stops per vehicle, a 3-opt stop exchange heuristic is implemented. Because this procedure is two-stage , a globally optimum solution is not guaranteed, but very good solutions can be expected. Once again, because the procedure clusters first, multiple vehicle types and distance or time limits are difficult to implement.

Cullen, Jarvis and Ratliff (1981) have coupled mathematical programming with a dispatcher to achieve an interactive optimization procedure. The dispatcher can generate clusters that are then improved upon by a generalized assignment algorithm. An iterative set partitioning procedure§ is then used to generate near-optimal solutions. The number of extensions that this procedure can incorporate is unclear, but because clustering is involved, several of the extensions (Items 5, 6 and 7 in the Appendix) are likely to be difficult to treat effectively. Comparative test results have not been published.

Cullen, Jarvis and Ratliff (1981) discuss several methods involving graphics, cost calculators and heuristics to solve the VRS problem. The authors tested an interactive heuristic procedure that iteratively calculated "prices" to aid the dispatcher in locating improved delivery solutions. For the basic VRS problem, near-optimal results for three test problems were achieved. All of the extensions in the Appendix are attainable using these types of procedures. Their main difficulty would be to consistently ensure very good or near optimal solutions.

When analyzing VRS solution methodologies, two performance criteria are apparent: (a) generalizability, or the ability to efficiently incorporate the extensions in the Appendix, and (b) accuracy of the solution, or nearness to an optimal solution. Criterion 1 measures the usefulness, or likelihood of implementation, of a methodology when applied in an actual company setting. Criterion 2 represents the possible cost savings due to decreased vehicle operating expenses, better customer service, and improved fleet investment.

Another important consideration is the intended area of planning in which the model will be used. In *strategic* planning, the models can be used to determine fleet investment and mix, design a fixed-route system, develop delivery costs for use in depot location analyses and determine the level of customer service. The models do not have to be extremely fast computationally or limited by storage considerations because mainframe or minicomputers are available at most companies for this level of planning. At the *tactical* planning level, models are used daily to develop routes and delivery schedules. In this case, very fast computation speed is essential, particularly if a micro- (or even a mini-) computer is involved. Computer storage space may also be limited, even on a mainframe, if the system is heavily utilized. Therefore, an interactive, graphics-capable methodology is definitely desirable. The graphics can help the dispatcher to quickly analyze routes in order to either accept them or to suggest improvements.

DIRECTIONS FOR FUTURE RESEARCH

When analyzing the directions of future research on VRS problems, three questions arise:

1. Of the presently available techniques or ideas, which seems most promising in terms of accuracy and generalizability, or alternatively, which technique is best suited to deal with each problem structure?

§ The procedure generates columns of routes using integer programming logic. By enumerating all possible routes the problem becomes one of selecting a set of columns such that every row is represented in exactly one column and the sum of "costs" of the columns selected is the smallest possible.

2. What are the most likely developments to occur in the future?, and

3. Which areas should be expanded upon or developed further?

The previous section has dealt somewhat with Question 1. Any of the combination approaches could give excellent results in the future. The mathematical programming-based heuristics should be well suited to strategic planning, whereas the interactive approaches would be best to use in a daily decision-making environment. Furthermore, the interactive approaches are in their infancy and further improvements seem highly likely.

In answer to Question 2, the most likely development is that researchers will continue to develop specialized procedures to deal with a few of the extensions in the Appendix, but not all simultaneously. Software companies probably will offer several procedures, or modules, and clients will choose the most appropriate method for their own environment. Optimizing methods will probably not be practical for problems of extended scope for a long time.

Several areas will likely be better developed. Interactive procedures should and will be expanded and refined. The study of artificial intelligence could ultimately prove beneficial to these interactive methods. One step in that direction would involve attempting to discern from experienced dispatchers what information they use and how they use that information to generate routes and schedules.

A standard series of realistically complex test problems should be developed, similar to what has been done by researchers in the forecasting area. A detailed mapping of problem extensions and the best techniques to solve them should be developed. Geometrical approaches to designing the routes should also be studied further.

Obviously, much work remains to be done in the area of vehicle routing and scheduling. Our suggestion is that future work should be directed toward generalized methods that incorporate the restrictions and extensions suggested in the Appendix and not so much on methods that treat only a few of them. This means heuristic and graphical methods that have the desired scope, rather than the exact approaches, are most likely to emerge in the short run.

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APPENDIX

Realistic extensions to the basic vehicle routing and scheduling problem

1. *For-hire carrier.* Examine the cost trade-off of using common or contract carriage in substitution for private carriage. For-hire carriage may be especially attractive for use with infrequently visited customers, isolated customers far from any other customers or LTL shipments. If the private fleet is not large enough to make all deliveries, then a decision must be made as to which subset of customers is serviced by the private fleet.

2. *Pickups and deliveries on the same route.* Better vehicle utilization is achieved by picking up cargo bound for the depot. Pickups can be made after all of the deliveries, or, if space permits, during the delivery legs of the route.

3. *Intermediate pickups and deliveries.* An intermediate pickup and delivery (IPUD) is one where neither the origin point nor the destination point is the domicile. For-hire carriage costs must be compared to the added cost of inserting an IPUD into a route.

4. *Multiple depots.* Large distribution networks involve multiple depots, and sometimes allow vehicles to be domiciled at more than one depot. The assignment of customers to depots and the number of vehicles per depot are the main considerations.

5. *Time windows.* Due to customer workforce scheduling requirements or municipal regulations designed to reduce traffic congestion, some stops may only be able to be served between certain hours of the day, or only on certain days of the week.

6. *Driving time or distance limits.* Government or union regulations concerning the number of driving hours per day can limit the length of time that is allowed on a route.

7. *Multiple types of vehicles.* Companies commonly have fleets composed of multiple types of vehicles. The vehicles can be differentiated on many different factors such as weight limits, specialized equipment or function. Designing routes that efficiently utilize various types of vehicles is a difficult task.

8. *Capacity limits.* The assignment of customers to a vehicle must ensure that the vehicle's capacity limits for either weight, cube or product mix are not violated. This is especially true for compartmentalized vehicles.

9. *Layovers.* By having an overnight stop on its route, a vehicle will be able to service additional customers. Extra driver costs are incurred, but vehicle utilization increases and another vehicle will not be needed to serve these customers. Daytime layovers may also be necessary in order to satisfy customer time windows.

10. *Loading and unloading times.* Consideration should be given to the time required to load or unload cargo at customers' facilities when the vehicle routes are designed.

11. *Forced stop sequencing.* Company policy, vehicle loading considerations or scheduling preferences may make the sequencing of certain customers on the route desirable.

12. *Multiple routes per vehicle per day.* A route may be of short enough duration to allow a vehicle to return to the depot upon completion and embark on one or more other routes the same day.

13. *Demand variability.* In many cases the size of a customer's order will vary over time. This is due to both seasonal patterns and random fluctuations. Both factors must be monitored very carefully for each route to ensure that vehicle capacity is not exceeded.

14. *Weekly scheduling.* If a vehicle can be domiciled at more than one depot or if layovers are used, the daily routings must be derived such that good vehicle utilization is achieved throughout the week.

15. *Multiple vehicle speeds.* Different vehicle speeds should be permitted on different legs of the route, as might be experienced between rural and urban movements. Additionally, for urban driving in particular, the vehicle speed will vary depending on the time of day.

16. *Estimation of driving distances.* Most VRS models require the driving distances from each customer to all other customers and the depot. Few companies maintain accurate records of these distances. Consequently the models must estimate them. Coordinate systems such as zip code, polar, or longitude and latitude can be used to represent the service territory. Adjustments to the coordinate system distances to account for rural or urban movement can then be used to estimate the actual driving distances. Ideally an exact mapping of the road network involved should be used, particularly for urban driving.

17. *Sleeper teams.* When long routes are involved, vehicles with two drivers can be used. The drivers alternate and more efficient vehicle utilization is possible.

18. *Relays of partial or full loads.* Relays occur when one tractor hauls a load over part of the route and one or more other tractors haul the load over the rest of route. The time that the load has to wait between being "dropped" by one tractor and "hooked" by another can vary.

19. *Continuous flow movement.* For some full truckload operations, when one pickup and delivery is completed, the driven deadheads to another pickup without returning to the depot. In fact, the depot may only be visited for maintenance activities. The decision process involves determining the assignment of vehicles to pickup and delivery pairs over some time frame.