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Vehicle Routing Problems with Profits

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Abstract

This paper is a survey of the broad class of vehicle routing problems with profits. In routing problems with profits, a profit is associated with each customer and, contrary to what happens in the classical routing problems, the subset of customers to be served is chosen. The decision is made on the basis of an objective function that includes the collected profit and/or the travel cost. We review the literature on this class of problems focusing on the distinction between single vehicle problems and multiple vehicle problems.

Keywords: Routing problems with profits, Vehicle routing, Survey, Literature.

1 Introduction

The key characteristic of the class of vehicle routing problems with profits is that the set of customers to serve is not given, which is the case for the most classical vehicle routing problems. Therefore, two different decisions have to be simultaneously taken: which customers to serve and how to sequence them in one or several routes. In general, a profit is associated with each customer that makes such customer more or less attractive. Thus, any route or set of routes can be measured both in terms of length and in terms of profit. The two measures may either be combined in a single objective function, or one of them may be bounded in a constraint.

There are several classes of applications that can be modeled by means of a problem of this class:

- scheduling of the daily operations of a steel rolling mill (see, for example, Balas [11] and Balas [14]);
- design of tourist trips to maximize the value of the visited attractions in a limited period (see, for example, Vansteenwegen and Van Oudheusden [79]);

- identification of suppliers to visit to maximize the recovered claims with a limited number of auditors (see Ilhan et al. [46]);
- recruiting of athletes from high schools for a college team (see Butt and Cavalier [21])
- planning of the visits of a salesperson (see, for example, Ramesh and Brown [63]);
- routing of oil tankers to serve ships at different locations (see Golden et al. [43]);
- delivery of home heating fuel, where the urgency of a customer request for fuel is treated as a score (see Golden et al. [42]);
- reverse logistics problem of a firm that aims to collect used products from its dealers (see Aras et al. [2]);
- customers selection in less-than-truckload transportation (see Archetti et al. [7]).

The most basic problems of this class are those with only one tour and are often presented as variants of the Traveling Salesman Problem (TSP) (see Fischetti et al. [33]). The Orienteering Problem (OP) has been first studied in Tsiligirides [75] and Golden et al. [43]. The OP is also known with the names Selective Traveling Salesperson Problem (see, for example, Laporte and Martello [54], Gendreau et al. [37] and Thomadsen and Stidsen [72]), Maximum Collection Problem (see, for example, Kataoka and Morito [48] and Butt and Cavalier [21]) and Bank Robber Problem (see Awerbuch et al. [10]). The name is derived from the orienteering sport where each participant has to maximize the total collected prize associated with visited points while returning to the starting point within a given time limit. The other two basic routing problems with profits with one vehicle only are the Profitable Tour Problem (PTP) and the Prize Collecting Traveling Salesman Problem (PCTSP). In the literature, there is no consistent definition of these two problems. In this survey we will follow the definitions given in [31] and explain, case by case, the different contributions.

Among the routing problems with profits and multiple vehicles the only one that has been studied in depth is the Team Orienteering Problem (TOP). The TOP was introduced by Butt and Cavalier [21] with the name Multiple Tour Maximum Collection Problem. The first paper where the name TOP appeared is that of Chao et al. [24].

Two surveys appeared covering parts of the literature on routing with profits. The survey by Feillet et al. [31] is focused on the routing problems with profits with one vehicle only whereas the recent survey by Vansteenwegen et al. [76] covers the OP and the TOP.

Problem name	Objective	Constraints	Vehicles
Orienteering Problem (Selective Traveling Salesperson Problem, Maximum Collection Problem, Bank Robber Problem)	max profit	route duration	single
Profitable Tour Problem	max (profit - cost)	–	single
Prize Collecting Traveling Salesman Problem	min cost	route profit	single
Team Orienteering Problem (Multiple Tour Maximum Collection Problem)	max profit	route duration	multiple

Table I: Summary of routing problems with profits

In this chapter we will cover all the vehicle routing problems with profits. We will start with the problems with one vehicle (the OP in Section 2, PTP in Section 3, PCTSP in Section 4 and variants in Section 5). We will then present the TOP in Section 6 and all the other contributions on problems with multiple vehicles in Section 7.

Following [76] we formulate the problems on a directed graph. We will mention how the models change in the case the graph is undirected. Although different papers often use different notations, for the ease of presentation we have chosen a standard notation that we will use throughout the paper to present the various problems. A complete graph $G = (V, A)$ is given where $V = \{0, \dots, n\}$ is the set of vertices and A is the set of arcs. Vertices $1, \dots, n$ correspond to the customers while vertex 0 corresponds to the depot. For any set of vertices $S \subset V$, we define $\delta^+(S) = \{(i, j) : i \in S, j \notin S\}$. Two nonnegative values may be associated with each arc $(i, j) \in A$, a traveling cost c_{ij} and a traveling time t_{ij}. In most of the problems only one of these two values is relevant. A nonnegative profit p_i is associated with each customer i. The profit of each customer can be collected at most once. One or more vehicles are available to collect the profit of a subset of customers. Each vehicle route starts from and ends at the depot. While in some problems a time limit $T_{\max}$ is set on the time duration of a route, in some others a minimum value p_{min} on the profit to be collected is used.

In Table I we summarize the problems covered in this survey and their characteristics. In the first column we indicate the names we use for the problems and, in parentheses, the other names used in the literature. In the following columns, we show the objective and the kind of constraints.

2 The orienteering problem

In this section we introduce the Orienteering Problem and its mathematical programming formulation. In the following subsections we overview the exact, heuristic and approximation algorithms for the OP and its variants.

The traveling time t_{ij} is associated with each arc $(i,j) \in A$. One vehicle is available at the depot with maximum route duration $T_{\max}$. The OP is the problem of finding the vehicle route that maximizes the total collected profit while satisfying the maximum duration constraint on the route.

Let us introduce the problem variables:

- $y_i =$ binary variable equal to 1 if vertex $i \in V$ is visited by the vehicle route, and 0 otherwise,
- $x_{ij} =$ binary variable equal to 1 if arc $(i,j) \in A$ is traversed by the vehicle, and 0 otherwise.

We present now the mathematical programming formulation of the directed version of OP.

$$\max \ \sum_{i \in V \setminus \{0\}} p_i y_i \tag{1}$$

$$\sum_{j \in V} x_{ij} = y_i, \qquad \forall\, i \in V \tag{2}$$

$$\sum_{j \in V} x_{ji} = y_i, \qquad \forall\, i \in V \tag{3}$$

$$y_0 = 1, \tag{4}$$

$$\sum_{(i,j) \in \delta^+(S)} x_{ij} \geq y_h, \quad \forall\, S \subset V, 0 \in S, \forall h \in V \setminus S \tag{5}$$

$$\sum_{(i,j) \in A} t_{ij} x_{ij} \leq T_{\max}, \tag{6}$$

$$y_i \in \{0,1\}, \qquad \forall\, i \in V \tag{7}$$

$$x_{ij} \in \{0,1\}, \qquad \forall\, (i,j) \in A. \tag{8}$$

The objective function (1) maximizes the collected profit. Constraints (2) and (3) ensure that an arc enters and an arc leaves from each vertex which is visited, while (4) imposes to visit the depot. Subtours are eliminated through (5) while (6) is the maximum duration constraint on the route. Finally, (7) and (8) are variables definition.

In case of an undirected graph $G = (V,E)$, variables x_{ij} are only defined for $i < j$ and are binary variables for $i > 0$ while $x_{0j} \in \{0,1,2\}$ for $j = 1,...,n$. Let us define $\delta(S) = \{(i,j) : (i \in S, j \notin S) \text{ or } (i \notin S, j \in S)\}$.

The formulation changes as follows (OP-U):

$$\max \ \sum_{i\in V\setminus\{0\}} p_i y_i \tag{9}$$

$$\sum_{j\in V, i<j} x_{ij} + \sum_{j\in V, j<i} x_{ji} = 2y_i, \qquad \forall\, i \in V \tag{10}$$

$$y_0 = 1, \tag{11}$$

$$\sum_{(i,j)\in\delta(S), i<j} x_{ij} \geq 2y_h, \qquad \forall\, S \subset V, 0 \in S, \forall h \in V\setminus S \tag{12}$$

$$\sum_{(i,j)\in E, i<j} t_{ij} x_{ij} \leq T_{\max}, \tag{13}$$

$$y_i \in \{0,1\}, \qquad \forall\, i \in V \tag{14}$$

$$x_{ij} \in \{0,1\}, \qquad \forall\, (i,j) \in E, i<j, i \in V\setminus\{0\} \tag{15}$$

$$x_{0j} \in \{0,1,2\}, \qquad \forall\, j \in V\setminus\{0\}. \tag{16}$$

The meaning of the objective function and of the constraints is analogous to the case of a directed graph.

2.1 Algorithms

A scheme to obtain an upper bound on the optimal value of the OP is presented in Laporte and Martello [54] while an upper bound on the number of vertices visited in an optimal solution is given in Millar and Kiragu [58]. Instances with a number of vertices between 10 and 90 are solved to optimality. Instances with 10 vertices are solved in Millar and Kiragu [58] by solving the mathematical programming formulation with CPLEX. In Leifer and Rosenwein [55] valid inequalities are added to the formulation presented in [54]. An optimal algorithm using a Lagrangean relaxation within a branch-and-bound algorithm is proposed in Ramesh et al. [64], who solved instances with up to 150 vertices. A new relaxation to the OP is presented in Kataoka et al. [49] together with two methods to improve its lower bound. A branch-and-cut algorithm using several families of valid inequalities is presented in Fischetti et al. [32] and instances with up to 500 vertices are solved to optimality.

Several heuristics were proposed for the OP. The first heuristics are due to Tsiligirides [75] and Golden et al. [43]. A comparison of the first heuristics can be found in Golden et al. [43] and Keller [51]. A heuristic that combines previously algorithmic concepts, along with learning capabilities, is proposed in Golden et al. [44]. In Chao et al. [23] a new heuristic is proposed and compared with the 6 previously published ones:

two published in Tsiligirides [75] and the others published in Golden et al. [43], Golden et al. [44], Keller [51] and Ramesh and Brown [63]. The comparison is carried out on the 49 instances originally proposed in [75] and on additional ones. All these methods use classical local ascent schemes and tend to become trapped in local optima. A dynamic programming heuristic is presented in Hayes and Norman [45]. A neural network based heuristic is proposed in Wang et al. [80], whereas the first tabu search algorithm for the OP is presented in Gendreau et al. [38]. The instances tested had up to 300 vertices. An ant colony and a tabu search heuristics are proposed in Liang et al. [56].

The first constant-factor approximation algorithm for the OP can be found in Blum et al. [18]. Other approximation results are proposed in Awerbuch et al. [10]. The most recent approximation results both for the directed and the undirected case of the OP can be found in Chekuri et al. [25]. In Angelelli et al. [1] the undirected OP is studied together with the variant where a service time is associated with the visit of a customer. The complexity of classes of instances, corresponding to special structures of the underlying graph, is studied. Polynomial algorithms for the polynomially solvable cases and fully polynomial-time approximation schemes for some NP-hard cases are presented.

3 The profitable tour problem

In this section we present the Profitable Tour Problem and its mathematical programming formulation. In the following subsections we overview the solution algorithms for the PTP and its variants.

As in the OP, in the PTP only one vehicle is available. The traveling cost c_{ij} is associated with each arc $(i, j) \in A$. The PTP is the problem of finding the vehicle route that maximizes the difference between the total collected profit and the total traveling cost. No constraint is imposed on the vehicle route.

For the mathematical programming formulation of the PTP we use the same variables y_i and x_{ij} defined for the OP:

$$\max \ \textstyle\sum_{i \in V \setminus \{0\}} p_i y_i - \sum_{(i,j) \in A} c_{ij} x_{ij} \tag{17}$$

$$\textstyle\sum_{j \in V} x_{ij} = y_i, \qquad \forall\, i \in V \tag{18}$$

$$\textstyle\sum_{j \in V} x_{ji} = y_i, \qquad \forall\, i \in V \tag{19}$$

$$y_0 = 1, \tag{20}$$

$$\textstyle\sum_{(i,j) \in \delta^+(S)} x_{ij} \geq y_h, \qquad \forall\, S \subset V, 0 \in S, \forall h \in V \setminus S \tag{21}$$

$$y_i \in \{0,1\}, \qquad \forall\, i \in V \tag{22}$$

$$x_{ij} \in \{0,1\}, \qquad \forall\, (i,j) \in A. \tag{23}$$

The objective function (17) aims at maximizing the difference between the collected profit and the traveling cost. The constraints have the same meaning illustrated above for the OP.

Note that (17) can be written as:

$$\begin{aligned} \max \sum_{i \in V \setminus \{0\}} p_i y_i - \sum_{(i,j) \in A} c_{ij} x_{ij} = \\ -\min\Big(\sum_{(i,j) \in A} c_{ij} x_{ij} - \sum_{i \in V \setminus \{0\}} p_i y_i + \sum_{i \in V \setminus \{0\}} p_i\Big) = \\ -\min\Big(\sum_{(i,j) \in A} c_{ij} x_{ij} + \sum_{i \in V \setminus \{0\}} p_i (1 - y_i)\Big). \end{aligned} \tag{24}$$

Thus, maximizing the difference between the total collected profit and the total traveling cost is equivalent to minimizing the sum of the total traveling cost and the total uncollected profit.

In the case of an undirected graph, the changes needed in variables definitions and problem formulation are analogous to those presented for the OP.

3.1 Algorithms

To the best of our knowledge, neither specific exact approaches nor computational analysis of heuristic algorithms have been specifically proposed for the PTP. The problem, however, has received much attention in terms of approximation results, probably due to its simple structure.

An approximation algorithm for the undirected version of the PTP is proposed in Goemans and Bertsimas

[40]. Several approximation results appeared later in the literature under the assumption that the c_{ij} satisfy the triangle inequality (see Bienstock et al. [17], Goemans and Williamson [41], Goemans [39]).

An approximation algorithm for the directed version of the PTP was recently presented in Nguyen and Nguyen [62] and Nguyen [61]. In Angelelli et al. [1] the undirected PTP is studied together with the variant where a service time is associated with the visit of a customer. Some polynomially solvable classes of instances, corresponding to special structures of the underlying graph, are identified and studied.

4 The prize collecting traveling salesman problem

In this section we introduce the Prize Collecting Traveling Salesman Problem and its mathematical programming formulation. Then, we overview the algorithms proposed for the PCTSP and its variants.

We adopt the definition of PCTSP used in Feillet et al. [31] and in Vansteenwegen et al. [76].

One vehicle is available at the depot to collect a profit from visited customers. The total collected profit cannot be smaller than a threshold $p_{\min}$. The value c_{ij} represents the traveling cost associated with arc $(i,j) \in A$. The PCTSP is the problem of finding the vehicle route that minimizes the total traveling cost with the constraint that the total collected profit is at least $p_{\min}$.

To define the model of PCTSP we use the same variables y_i and x_{ij} defined for the OP. Thus, the mathematical programming formulation of the directed PCTSP is as follows:

$$\min \sum_{(i,j)\in A} c_{ij}x_{ij} \tag{25}$$

$$\sum_{j\in V} x_{ij} = y_i, \qquad \forall\, i \in V \tag{26}$$

$$\sum_{j\in V} x_{ji} = y_i, \qquad \forall\, i \in V \tag{27}$$

$$y_0 = 1, \tag{28}$$

$$\sum_{(i,j)\in\delta^+(S)} x_{ij} \geq y_h, \quad \forall\, S \subset V, 0 \in S, \forall h \in V \setminus S \tag{29}$$

$$\sum_{i\in V\setminus\{0\}} p_i y_i \geq p_{\min}, \tag{30}$$

$$y_i \in \{0,1\}, \qquad \forall\, i \in V \tag{31}$$

$$x_{ij} \in \{0,1\}, \qquad \forall\, (i,j) \in A. \tag{32}$$

The objective function (25) aims at minimizing the total traveling cost. Constraint (30) imposes to collect a profit not smaller than p_{min} while the remaining constraints have the same meaning as in the case of the OP and PTP. Also, for the case of an undirected graph, the changes needed in variables definitions and problem formulation are analogous to those presented for the OP.

In the paper which first introduced the name PCTSP, due to Balas [11], the objective function was the sum of the total traveling cost and a total penalty for the non visited customers. Denoting by γ_i the penalty associated with customer i, $i \in V \setminus \{0\}$, to distinguish this more general problem we call it PCTSP *with penalties*. The problem is formulated as:

$$\min \sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{i\in V\setminus\{0\}} \gamma_i(1-y_i) \tag{33}$$

with constraints (26)-(32).

The objective function (33) is equivalent to the maximization of the difference between the penalties (that can be interpreted now as profits) of the visited customers and the total distance traveled (see the transformation (24)). Then, the problem turns out to be a generalization of the PTP, obtained when $p_{\min} = 0$, and of the PCTSP, obtained when $\gamma_i = 0$, $\forall i$.

As in Balas [11], also in Dell'Amico et al. [29] two different values are associated with each customer i, a profit, used in constraint (30), and a penalty.

The online version of the PCTSP with penalties where customers are disclosed over time is studied in Ausiello et al. [9].

4.1 Algorithms

A polyhedral study of the PCTSP with penalties, and thus obviously also of the PCTSP, can be found in Balas [12]. Bounding procedures, based on different relaxations, are proposed for the same problem in Fischetti and Toth [34] and Dell'Amico et al. [29]. A branch-and-cut algorithm for the PCTSP is proposed in Bérubé et al. [15] with computational results on instances with more than 500 vertices.

In Balas [13] it is shown that if some additional constraints are imposed on the structure of the route, the PCTSP with penalties can be solved in polynomial time.

A first heuristic for the PCTSP with penalties is proposed in Dell'Amico et al. [28]. The Lagrangian heuristic, that starts from a lower bound to the problem and makes the solution feasible, is evaluated on randomly generated instances and real instances.

The first approximation algorithm for the undirected PCTSP with penalties is presented in Balas [11] and is improved in Awerbuch et al. [10] where an approximation result is also obtained for the undirected PCTSP. Both results are, in fact, derived from previous results for the minimum-weight k-tree problem.

In Angelelli et al. [1] the undirected PCTSP is studied together with the variant where a service time is associated with the visit of a customer. The complexity of classes of instances, corresponding to special structures of the underlying graph, is analyzed. Polynomial algorithms for the polynomially solvable cases and fully polynomial time approximation schemes for some NP-hard cases are presented.

5 Variants of single vehicle routing problems with profits

A variant of the OP, called Generalized Orienteering Problem (GOP), is considered in Wang et al. [81] and Silberholz and Golden [67] that differs from the OP mainly in the objective function that is a non-linear function of the profits collected from the visited vertices. The GOP is a generalized version of the OP in which each city is assigned a number of scores for different attributes and the overall function to optimize is a function of these attribute scores. The OP with time windows is studied in Kantor and Rosenwein [47] where a heuristic is proposed. An exact algorithm for the OP with time windows is proposed in Righini and Salani [65]. The algorithm is based on a bi-directional and bounded dynamic programming algorithm with state space relaxation. A penalty-based greedy heuristic and a branch-and-bound algorithm are proposed in Erkut and Zhang [30] for a variant of the OP where the profit is a time dependent, decreasing, linear function.

A more general problem than the OP, called the One-Period Bus Touring Problem, is studied in Deitch and Ladany [27] where an attractiveness, that we can also see as a profit, is associated also with edges. A transformation of the problem into the OP is presented together with a heuristic that is compared with one of the heuristics proposed in Tsiligirides [75]. A different generalization of the OP, called time-dependent orienteering, is studied in Fomin and Lingas [35], where an approximation algorithm is proposed. The OP with a given set of compulsory vertices is studied in Gendreau et al. [37]. A branch-and-cut algorithm based on several families of valid inequalities is proposed that can solve to optimality instances with up to 300 vertices.

The OP with stochastic profits is the problem of finding a tour that visits a subset of vertices within a pre-specified time limit and maximizes the probability of collecting more than a pre-specified target profit level. This variant of the OP is studied in Ilhan et al. [46].

In Vansteenwegen and Van Oudheusden [79] a class of problems, called Tourist Trip Design Problems, is introduced that is associated with the application area of creating a feasible plan for tourists in order to visit attractions within the available time span. The OP is the simplest problem of this class.

The only studied variant of the PTP is the Capacitated version of the PTP (CPTP) where each customer has a demand and the vehicle has a capacity. Thus, a solution has to satisfy the capacity of the vehicle. A branch-and-cut algorithm for the solution of the undirected version of the CPTP is proposed in Jepsen [59] where instances with up to 800 vertices are solved to optimality. In the Capacitated PCTSP with penalties the difference with respect to the PCTSP with penalties is that customers have a demand and the vehicle has a capacity. Thus, the route must satisfy the capacity constraint, besides guaranteeing that the minimum total profit p_{min} is collected. An iterated local search heuristic for the Capacitated PCTSP with penalties was recently proposed in Tang and Wang [71]. The heuristic is tested on real instances.

In Bérubé et al. [16] the bi-objective problem is studied where a profit is associated with each customer and the goal is to visit a subset of vertices while addressing the conflicting objectives: maximize the collected profit and minimize the traveling cost. The multi-objective version of the OP is studied in Schilde et al. [66].

6 The team orienteering problem

In this section we consider the multiple vehicle extension of the OP. It was first introduced in Butt and Cavalier [21], who called it the Multiple Tour Maximum Collection Problem, whereas the name Team Orienteering Problem (TOP) was coined by Chao et al. [24] to highlight the connection with the more widely studied single vehicle case. More precisely, the TOP calls for the determination of a set of at most K vehicle routes that maximize the total collected profit while satisfying a maximum duration constraint for each route. Vehicle flow models, extending those presented in Section 2, are proposed in the literature (see, e.g., Tang and Miller-Hooks [70]).

We present here a formulation for the directed case of the TOP that is an extension of that presented for the directed OP in Section 2 and uses the following decision variables:

- y_{ik} = binary variable equal to 1 if vertex $i \in V$ is visited by vehicle route $k = 1, \dots, K$, and 0 otherwise,
- x_{ijk} = binary variable equal to 1 if arc $(i, j) \in A$ is traversed by vehicle route $k = 1, \dots, K$, and 0 otherwise.

The mathematical programming formulation for the directed TOP is as follows:

$$\max \ \sum_{i\in V\setminus\{0\}} p_i \sum_{k=1}^{K} y_{ik} \tag{34}$$

$$\sum_{j\in V} x_{ijk} = y_{ik}, \qquad \forall\, i \in V, k = 1,\dots,K \tag{35}$$

$$\sum_{j\in V} x_{jik} = y_{ik}, \qquad \forall\, i \in V, k = 1,\dots,K \tag{36}$$

$$\sum_{k=1}^{K} y_{0k} \le K, \tag{37}$$

$$\sum_{k=1}^{K} y_{ik} \le 1, \qquad \forall\, i \in V \setminus \{0\} \tag{38}$$

$$\sum_{(i,j)\in\delta^+(S)} x_{ijk} \ge y_{hk}, \qquad \forall\, S \subset V, 0 \in S, \forall h \in V \setminus S, k = 1,\dots,K \tag{39}$$

$$\sum_{(i,j)\in A} t_{ij} x_{ijk} \le T_{\max}, \qquad k = 1,\dots,K \tag{40}$$

$$y_{ik} \in \{0,1\}, \qquad \forall\, i \in V, k = 1,\dots,K \tag{41}$$

$$x_{ijk} \in \{0,1\}, \qquad \forall\, (i,j) \in A, k = 1,\dots,K. \tag{42}$$

The objective function and most of the constraints are extensions to the multiple vehicle case of those presented for the OP. Constraint (37) limits the number of routes to be at most K, while constraints (38) impose that each customer is visited at most once. Finally, the constraints (39) impose that the solution is connected and (40) limit the maximum distance for each tour.

6.1 Algorithms

The first optimal solution procedure for the TOP is proposed by Butt and Ryan [22]. They start from a set partitioning formulation and their algorithm makes efficient use of both column generation and constraint branching. The algorithm is able to solve instances with up to 100 potential customers when routes include only a few customers each. More recently, a branch-and-price algorithm was presented by Boussier et al. [20] which, thanks to various acceleration procedures in the column generation step, is able to solve instances with up to 100 potential customers from the large set of benchmark instances proposed in Chao et al. [24].

The first heuristic proposed for the TOP is a simple construction algorithm introduced in Butt and Cavalier [21] and tested on small size instances with up to 15 vertices by comparing the results with a mathematical programming model similar to (34)-(42). A more sophisticated construction heuristic is given in Chao et al. [24] in which the initial solution is refined through customer moves and exchanges and various restart strategies. The resulting algorithm is tested on a set of 353 test instances with up to 102 customers and up to

4 vehicles.

More recently, several metaheuristics were applied to the TOP, starting from the tabu search algorithm introduced in Tang and Miller-Hooks [70] which is embedded in an adaptive memory procedure that alternates between small and large neighborhoods during the search and outperforms previous heuristics. Archetti et al. [8] propose two variants of a generalized tabu search algorithm and a variable neighborhood search algorithm. Ke et al. [50] use an ant colony optimization approach which uses four different methods to construct candidate solutions. Other metaheuristic paradigms are successfully applied to the TOP, such as guided local search (Vansteenwegen et al. [77]), path relinking (Souffriau et al. [68]) memetic algorithms (Bouly et al. [19]) and particle swarm optimization-based memetic algorithm (Dang et al. [26]), the latter being the current best-in-class.

7 Other routing problems with profits with multiple vehicles

In Archetti et al. [7] the capacitated version of the TOP, called Capacitated TOP (CTOP), is studied. In this problem a demand is associated with each customer and each vehicle has a maximum capacity. The objective is to maximize the total collected profit while satisfying the capacity and duration constraint for each route. In the same paper, the capacitated version of the PTP (CPTP) is studied. The authors propose a branch-and-price algorithm adapted from the one proposed by Boussier et al. [20] for the TOP. The algorithm is able to solve instances derived from classical CVRP instances and including up to 200 customers. Heuristic algorithms are also proposed for both problems adapted from the heuristics proposed in Archetti et al. [8] for the TOP. The same problems are studied in Archetti et al. [6] where a branch-and-price algorithm is proposed which improves the results obtained in [7]. Archetti et al. [5] studied a variant of the CTOP where a partial service to each customer is allowed and they propose a branch-and-price algorithm. Further variants of the CTOP are studied in Archetti et al. [3] and in Archetti et al. [4]. In [3] split deliveries are allowed but each customer has to be either entirely served or not served, while in [4] incomplete service is also allowed. A branch-and-price exact algorithm and heuristic algorithms are proposed for the solution of both problems.

The TOP variant with Time Windows (TOPTW) received considerable attention from the heuristic community in the last few years. Vansteenwegen et al. [78] were among the first to study this problem in the context of electronic tourist guides which assist tourists in planning their trip. They propose an iterated local search which quickly produces good solutions on large instances derived from VRPTW ones. Montemanni and Gambardella [60] developed an ant colony optimization approach for TOPTW which was later improved by Gambardella et al. [36]. Other metaheuristic approaches for the TOPTW were recently introduced and overall obtain very good average results on benchmark instances. Tricoire et al. [74] apply to the TOPTW a variable neighborhood search (VNS) developed for the multi-period variant of the problem. Labadie et al. [53] use a hybrid approach which combines a greedy randomized adaptive search procedure and an evolutionary search, whereas Labadie et al. [52] present a VNS and a hybrid approach which combines VNS with the granular search by Toth and Vigo [73]. Finally, Lin and Yu [57] propose two heuristics based on the simulated annealing paradigm.

8 Conclusions

In this chapter we reviewed the large family of variants of the Vehicle Routing Problem with profits. These problems are widely studied due to the practical relevance of the applications they may model and the scientific interest of their structure in which not all customers have to be served and are selected by considering a set of penalties or prizes.

We first of all provided a homogeneous description of the main problems of the family since in the literature some discrepancies in problem definitions and naming are encountered. Then for each problem and the known main variants we reviewed the most relevant results with particular attention to the computational testing of the proposed methods.

In this area most of the research has been devoted to the single vehicle case and to the so-called Orienteering variants and relatively little attention was given to multiple vehicle extensions. Therefore, there is still considerable room for valuable and systematic research in this field particularly for unified approaches capable of successfully tackling several variants. In addition, important characteristics arising in practical application deserve specific attention such as the multiple depots or customer clustering which are frequently encountered in distribution problems arising in small package shipping (see, for example, Stenger et al. [69]).

References

[1] E. Angelelli, C. Bazgan, Zs. Tuza, and M.G. Speranza. Complexity and approximation for traveling salesman problems with profits. Technical Report 352, Department Quantitative Methods, University

of Brescia, 2010.

[2] N. Aras, D. Aksen, and M.T. Tekin. Selective multi-depot vehicle routing problem with pricing. *Transportation Research Part C*, 19:866–884, 2011.

[3] C. Archetti, N. Bianchessi, A. Hertz, and M.G. Speranza. The split delivery capacitated team orienteering problem. *Networks*, (to appear).

[4] C. Archetti, N. Bianchessi, A. Hertz, and M.G. Speranza. Incomplete service and split deliveries in a routing problem with profits. Technical Report 356, Department Quantitative Methods, University of Brescia, 2011.

[5] C. Archetti, N. Bianchessi, and M.G. Speranza. The capacitated team orienteering problem with incomplete service. *Optimization Letters*, (to appear). doi: 10.1007/s11590-012-0559-4.

[6] C. Archetti, N. Bianchessi, and M.G. Speranza. Optimal solutions for routing problems with profits. *Discrete Applied Mathematics*, (to appear).

[7] C. Archetti, D. Feillet, A. Hertz, and M.G. Speranza. The capacitated team orienteering and profitable tour problems. *Journal of the Operational Research Society*, 60:831–842, 2009.

[8] C. Archetti, A. Hertz, and M.G. Speranza. Metaheuristics for the team orienteering problem. *Journal of Heuristics*, 13:49–76, 2007.

[9] G. Ausiello, V. Bonifaci, and L. Laura. The online prize-collecting traveling salesman problem. *Information Processing Letters*, 107:199–204, 2008.

[10] B. Awerbuch, Y. Azar, A. Blum, and S. Vempala. New approximation guarantees for minimum-weight k-trees and prize-collecting salesmen. *SIAM Journal on Computing*, 28:254–262, 1998.

[11] E. Balas. The prize collecting traveling salesman problem. *Networks*, 19:621–636, 1989.

[12] E. Balas. The prize collecting traveling salesman problem. II: Polyhedral results. *Networks*, 25:199–216, 1995.

[13] E. Balas. New classes of efficiently solvable generalized travelling salesman problems. *Annals of Operations Research*, 86:529–558, 1999.

[14] E. Balas. The prize collecting traveling salesman problem and its applications. In G. Gutin and A. Punnen, editors, *Traveling Salesman Problem and its variations*, pages 663–695. Kluwer Academic Publishers, Dordrecht, 2002.

[15] J.-F. Bérubé, M. Gendreau, and J.-Y. Potvin. A branch-and-cut algorithm for the undirected prize collecting traveling salesman problem. *Networks*, 54:56–67, 2009.

[16] J.-F. Bérubé, M. Gendreau, and J.-Y. Potvin. An exact ε-constraint method for bi-objective combinatorial optimization problems: Application to the travelling salesman problem with profits. *European Journal of Operational Research*, 194:39–50, 2009.

[17] D. Bienstock, M.X. Goemans, D. Simchi-Levi, and D. Williamson. A note on the prize collecting traveling salesman problem. *Mathematical Programming*, 59:413–420, 1993.

[18] A. Blum, S. Chawla, D.R. Karger, T. Lane, A. Meyerson, and M. Minkoff. Approximation algorithms for orienteering and discounted-reward TSP. *SIAM Journal on Computing*, 37:653–670, 2007.

[19] H. Bouly, D.-C. Dang, and A. Moukrim. A memetic algorithm for the team orienteering problem. *4OR*, 8:49–70, 2010.

[20] S. Boussier, D. Feillet, and M. Gendreau. An exact algorithm for the team orienteering problem. *4OR*, 5:211–230, 2007.

[21] S.E. Butt and T.M. Cavalier. A heuristic for the multiple tour maximum collection problem. *Computers and Operations Research*, 21:101–111, 1994.

[22] S.E. Butt and D.M. Ryan. An optimal solution procedure for the multiple tour maximum collection problem using column generation. *Computers and Operations Research*, 26:427–441, 1999.

[23] I-M. Chao, B.L. Golden, and E.A. Wasil. A fast and effective heuristic for the orienteering problem. *European Journal of Operational Research*, 88:475–489, 1996.

[24] I-M. Chao, B.L. Golden, and E.A. Wasil. The team orienteering problem. *European Journal of Operational Research*, 88:464–474, 1996.

[25] C. Chekuri, N. Korula, and M. Pál. Improved algorithms for orienteering and related problems. *ACM Transactions on Algorithms*, 8:661–670, 2012.

[26] D.-C. Dang, R.N. Guibadj, and A. Moukrim. A PSO-based memetic algorithm for the team orienteering problem. In C. Di Chio, A. Brabazon, G. Di Caro, R. Drechsler, M. Farooq, J. Grahl, G. Greenfield, C. Prins, J. Romero, G. Squillero, E. Tarantino, A. Tettamanzi, N. Urquhart, and A. Uyar, editors, *Applications of Evolutionary Computation, Lecture Notes in Computer Science*, pages 471–480. Springer, Berlin, 2011.

[27] R. Deitch and S.P. Ladany. The one-period bus routing problem: Solved by an effective heuristic for the orienteering tour problem and improvement algorithm. *European Journal of Operational Research*, 127:69–77, 2000.

[28] M. Dell'Amico, F. Maffioli, and A. Sciomachen. A lagrangian heuristic for the prize collecting travelling salesman problem. *Annals of Operations Research*, 81:289–306, 1998.

[29] M. Dell'Amico, F. Maffioli, and P. Väbrand. On prize-collecting tours and the asymmetric travelling salesman problem. *International Transactions in Operational Research*, 2:297–308, 1995.

[30] E. Erkut and J. Zhang. The maximum collection problem with time-dependent rewards. *Naval Research Logistics*, 43:749–763, 1996.

[31] D. Feillet, P. Dejax, and M. Gendreau. Travelling salesman problems with profits. *Transportation Science*, 39:188–205, 2005.

[32] M. Fischetti, J.J. Salazar González, and P. Toth. Solving the orienteering problem through branch-and-cut. *INFORMS Journal on Computing*, 10:133–148, 1998.

[33] M. Fischetti, J.J. Salazar González, and P. Toth. The generalized traveling salesman and orienteering problems. In G. Gutin and A. Punnen, editors, *Traveling Salesman Problem and its Variations*, pages 609–662. Kluwer Academic Publishers, Dordrecht, 2002.

[34] M. Fischetti and P. Toth. An additive approach for the optimal solution of the prize-collecting traveling salesman problem. In B.L. Golden and A.A. Assad, editors, *Vehicle Routing: Methods and Studies*, pages 319–343. North Holland, Amsterdam, 1988.

[35] F.V. Fomin and A. Lingas. Approximation algorithms for time-dependent orienteering. *Information Processing Letters*, 83:57–62, 2002.

[36] L.M. Gambardella, R. Montemanni, and D. Weyland. Coupling ant colony systems with strong local searches. *European Journal of Operational Research*, 220:831–843, 2012.

[37] M. Gendreau, G. Laporte, and F. Semet. A branch-and-cut algorithm for the undirected selective travelling salesman problem. *Networks*, 32:263–273, 1998.

[38] M. Gendreau, G. Laporte, and F. Semet. A tabu search heuristic for the undirected selective travelling salesman problem. *European Journal of Operational Research*, 106:539–545, 1998.

[39] M.X. Goemans. Combining approximation algorithms for the prize-collecting tsp. *Proceedings of CoRR*, 2009.

[40] M.X. Goemans and D.J. Bertsimas. On the parsimonious property of connectivity problems. *SODA '90 Proceedings of the first annual ACM-SIAM Symposimum on Discrete algorithms*, pages 388–396, 1990.

[41] M.X. Goemans and D.P. Williamson. A general approximation technique for constrained forest problems. *SIAM Journal on Computing*, 24:296–317, 1995.

[42] B.L. Golden, A.A. Assad, and R. Dahl. Analysis of a large scale vehicle routing problem with an inventory component. *Large Scale Systems*, 7:181–190, 1984.

[43] B.L. Golden, L. Levy, and R. Vohra. The orienteering problem. *Naval Research Logistics*, 34:307–318, 1987.

[44] B.L. Golden, Q. Wang, and L. Liu. A multifaceted heuristic for the orienteering problem. *Naval Research Logistics*, 35:359–366, 1988.

[45] M. Hayes and J.M. Norman. Dynamic programming in orienteering: Route choice and siting of controls. *Journal of the Operational Research Society*, 35:791–796, 1984.

[46] T. Ilhan, S.M.R. Iravani, and M.S. Daskin. The orienteering problem with stochastic profits. *IIE Transactions*, 40:406–421, 2008.

[47] M.G. Kantor and M.B. Rosenwein. The orienteering problem with time windows. *Journal of the Operational Research Society*, 43:629–635, 1992.

[48] S. Kataoka and S. Morito. Single constraint maximum collection problem. *Journal of the Operations Research Society of Japan*, 31:515–530, 1988.

[49] S. Kataoka, T. Yamada, and S. Morito. Minimum directed 1-subtree relaxation for score orienteering problem. *European Journal of Operational Research*, 104:139–153, 1998.

[50] L. Ke, C. Archetti, and Z. Feng. Ants can solve the team orienteering problem. *Computers and Industrial Engineering*, 54:648–665, 2008.

[51] C.P. Keller. Algorithms to solve the orienteering problem: A comparison. *European Journal of Operational Research*, 41:224–231, 1989.

[52] N. Labadie, R. Mansini, J. Melechovský, and R. Wolfler Calvo. The team orienteering problem with time windows: An LP-based granular variable neighborhood search. *European Journal of Operational Research*, 220:15–27, 2012.

[53] N. Labadie, J. Melechovský, and R. Wolfler Calvo. Hybridized evolutionary local search algorithm for the team orienteering problem with time windows. *Journal of Heuristics*, 17:729–753, 2011.

[54] G. Laporte and S. Martello. The selective travelling salesman problem. *Discrete Applied Mathematics*, 26:193–207, 1990.

[55] A.C. Leifer and M.B. Rosenwein. Strong linear programming relaxations for the orienteering problem. *European Journal of Operational Research*, 73:517–523, 1994.

[56] Y.-C. Liang, S. Kulturel-Konak, and A.E. Smith. Meta heuristics for the orienteering problem. *Proceedings of the 2002 Congress on Evolutionary Computation*, pages 384–389, 2002.

[57] S.-W. Lin and V.F. Yu. A simulated annealing heuristic for the team orienteering problem with time windows. *European Journal of Operational Research*, 217:94–107, 2012.

[58] H. H. Millar and M. Kiragu. A time-based formulation and upper bounding scheme for the selective traveling salesperson problem. *Journal of the Operational Research Society*, 48:511–518, 1997.

[59] Jepsen M.K. Branch-and-cut and branch-and-cut-and-price algorithms for solving vehicle routing problems. *PhD Thesis, Technical University of Denmark*, 2011.

[60] R. Montemanni and L.M. Gambardella. Ant colony system for team orienteering problems with time windows. *Foundations of Computing and Decision Sciences*, 34:287–306, 2009.

[61] V.H. Nguyen. A primal-dual approximation algorithm for the asymmetric prize-collecting TSP. In W. Wu and O. Daescu, editors, *Combinatorial Optimization and Applications, Lecture Notes in Computer Science*, pages 260–269. Springer, Berlin, 2010.

[62] V.H. Nguyen and T.T.T. Nguyen. Approximating the asymmetric profitable tour. *Electronic Notes in Discrete Mathematics*, 36:907–914, 2010.

[63] R. Ramesh and K.M. Brown. An efficient four-phase heuristic for the generalized orienteering problem. *Computers and Operations Research*, 18:151–165, 1991.

[64] R. Ramesh, Y.-S. Yoon, and M.H. Karwan. An optimal algorithm for the orienteering tour problem. *ORSA Journal on Computing*, 4:155–165, 1992.

[65] G. Righini and M. Salani. Decremental state space relaxation strategies and initialization heuristics for solving the orienteering problem with time windows with dynamic programming. *Computers and Operations Research*, 36:1191–1203, 2009.

[66] M. Schilde, K.F. Doerner, R.F. Hartl, and G. Kiechle. Metaheuristics for the bi-objective orienteering problem. *Swarm Intelligence*, 3:179–201, 2009.

[67] J. Silberholz and B. Golden. The effective application of a new approach to the generalized orienteering problem. *Journal of Heuristics*, 16:393–415, 2010.

[68] W. Souffriau, P. Vansteenwegen, G. Vanden Berghe, and D. Van Oudheusden. A path relinking approach for the team orienteering problem. *Computers and Operations Research*, 37:1853–1859, 2010.

[69] A. Stenger, D. Vigo, S. Enz, and M. Schwind. A variable neighborhood search algorithm for a vehicle routing problem arising in small package shipping. *Transportation Science*, (to appear), doi: 10.1287/trsc.1110.0396.

[70] H. Tang and E. Miller-Hooks. A tabu search heuristic for the team orienteering problem. *Computers and Operations Research*, 32:1379–1407, 2005.

[71] L. Tang and X. Wang. An iterated local search heuristic for the capacitated prize-collecting travelling salesman problem. *Journal of the Operational Research Society*, 59:590–599, 2008.

[72] T. Thomadsen and T. Stidsen. The quadratic selective travelling salesman problem. IMM-Technical Report-2003-17, Technical University of Denmark, 2003.

[73] P. Toth and D. Vigo. The granular tabu search and its application to the vehicle routing problem. *INFORMS Journal on Computing*, 15:333–346, 2003.

[74] F. Tricoire, M. Romauch, K.F. Doerner, and R.F. Hartl. Heuristics for the multi-period orienteering problem with multiple time windows. *Computers and Operations Research*, 37:351–367, 2010.

[75] T. Tsiligirides. Heuristic methods applied to orienteering. *Journal of the Operational Research Society*, 35:797–809, 1984.

[76] P. Vansteenwegen, W. Souffriau, and D. Van Oudheusden. The orienteering problem: A survey. *European Journal of Operational Research*, 209:1–10, 2011.

[77] P. Vansteenwegen, W. Souffriau, G. Vanden Berghe, and D. Van Oudheusden. A guided local search metaheuristic for the team orienteering problem. *European Journal of Operational Research*, 196:118–127, 2009.

[78] P. Vansteenwegen, W. Souffriau, G. Vanden Berghe, and D. Van Oudheusden. Iterated local search for the team orienteering problem with time windows. *Computers and Operations Research*, 36:3281–3290, 2009.

[79] P. Vansteenwegen and D. Van Oudheusden. The mobile tourist guide: An OR opportunity. *OR Insight*, 20:21–27, 2007.

[80] Q. Wang, X. Sun, B.L. Golden, and J. Jia. Using artificial neural networks to solve the orienteering problem. *Annals of Operations Research*, 61:111–120, 1995.

[81] X. Wang, B.L. Golden, and E.A. Wasil. Using a genetic algorithm to solve the generalized orienteering problem. In B.L. Golden, S. Raghavan, and E.A. Wasil, editors, *The Vehicle Routing Problem: Latest Advances and New Challenges*, pages 263–274. Springer, New York, 2008.