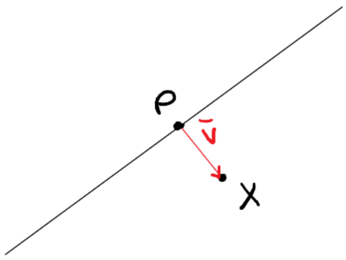
Exercise 1. Total least squares line fitting.

1.

COMPUTER VISION
EXERCISE ROUND 5

Exercise 1.

1.



We need to find a unit vector $\bar{v}$ perpendicular to the line

$$l \equiv ax + by = d \quad \Rightarrow \quad \bar{v} = \frac{1}{\sqrt{a^2+b^2}}(a,b) = (a,b)$$
$$a^2 + b^2 = 1$$

The distance d between P and X will be:

$$\left| \vec{PX} \cdot \vec{v} \right| = \begin{pmatrix} x_i - x_p \\ y_i - y_p \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \left| a(x_i - x_p) + b(y_i - y_p) \right.$$

Since P belongs to l: $ax_p + by_p = d$

$$\text{distance} = \left| ax_i + by_i - d \right|$$

2.

$\boxed{2.}$

$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0$$

$$a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} d = 0$$

$$a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i = nd$$

$$d = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} y_i$$

$$d = a\bar{x} + b\bar{y}$$

3.

$\boxed{3.}$

$$E = \sum_{i=1}^{n} (ax_i + by_i - a\bar{x} - b\bar{y})^2$$

$$E = \sum_{i=1}^{n} \left(a(x_i - \bar{x}) + b(y_i - \bar{y}) \right)^2$$

$$E = \left\| \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} \right\|^2 = (U \cdot N)^T (U \cdot N)$$

$$E = N\, U^T \cdot U \cdot N^T \quad \text{where } N = (a, b)$$

4.

$\boxed{4.}$

If we calculate de porcial $\frac{\partial E}{\partial N}$

Knowing $E = N U^T U N$ from 3)

We get:

$$\frac{\partial E}{\partial N} = 2\,(U^T U N)$$

The minum of $\| U(a,b)^T \|$ must satisfy

$$2(U^T U N) = 0$$

$$(U^T U) N = 0$$

subject to $\|N\|^2 = 1$.

The solution for this equation by definition is the eigenvector of $U^T U$ associated with the eigenvalue $\lambda = 0$

Applying Sylvester's criterion, we know that $U^T U$ is positive-semidefine.

Since the upper-left corner of $U^T U$ is positive

$$U^T U = \begin{pmatrix} \sum_{i=1}^{n} (x_i - \bar{x})^2 & \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{n} (y_i - \bar{y})^2 \end{pmatrix}$$

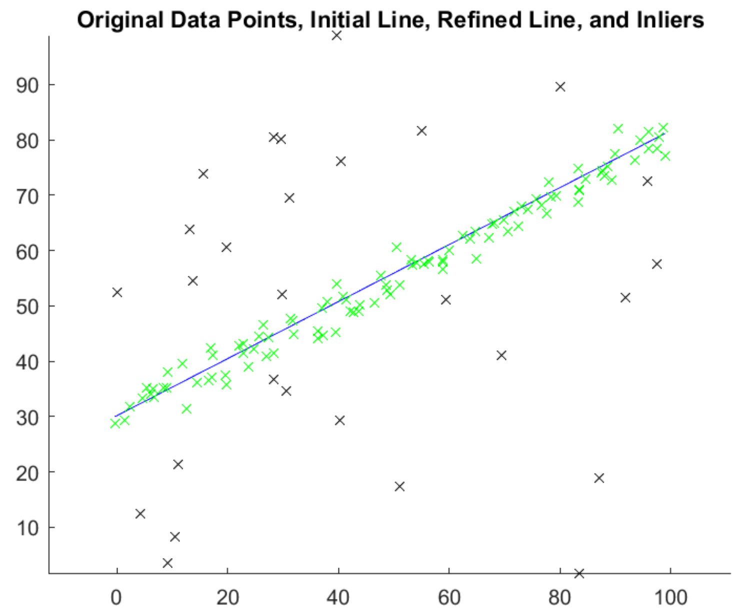
$\Rightarrow$ All the eigenvalues λ are greater or equal to 0 $(\lambda \geq 0)$, which means that $\lambda = 0$ is the smallest eigenvalue.

Once we know $N = (a, b)$ we can obtain $d = a\bar{x} + b\bar{y}$ using the expression from 2)

Exercise 2. Robust line fitting using RANSAC.

Result of applying RANSAC with N=88 and t=5 before least squares fitting.

$$0.49x - y = 30.78$$



Result after refitting the previous solution with least squares fitting.

$$-0.45x - y = 0.89$$