Research practice II

Final report

Conceptualization of the method of building Nash's equilibrium price bidding strategies in an unregulated electricity market

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Abstract

This article presents a method for the development of mixed betting strategies for N players in an unregulated electricity market, implemented in *Maple* software and based on the seminal work of Von de Feh and Harbord in 1992. The built algorithm had an improvement in the process of solving the ordinary differential equations, to facilitate the construction of the Nash equilibrium between players, in addition to a data replication strategy that was performed in the results section, where the profiles of mixed strategies are presented by means of the accumulated probability curves. The aim of this project is to establish an improvement in the way energy is auctioned in these markets, in order to guarantee the balanced participation of all players.
Keywords: Ordinary differential equations, reverse auction, finite players, Maple, Nash equilibrium

1 Introduction

Energy has been for centuries the engine of many cities and the pillar of human progress in many areas, so knowing the mechanism in how the energy market is distributed and the entities that produce it is of great interest to me. The document is oriented to the non-regulated electricity market which is not monitored by any legal entity and therefore generates a competition factor among taxpayers. A generalized method was developed for the N energy generators with their respective characteristics that seeks that their daily participation is balanced among them and this is desired to be achieved with an algorithm in which systems of differential equations of first order are numerically solved.

As results obtained in this practice is the improvement of the algorithm and the proposed strategy of replication of data from a player to obtain balances of Nash, this strategy was implemented in the examples presented in the results section and each of these examples contains

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the accumulated probability curve that refers to the profile of mixed strategy of price betting for the next day. The document is divided by the literature review section that contains the contributions of different authors to the method of competition between players especially that of *Von de ferh and Harbord* that performs the method for two energy generators, then presents the mathematical basis and the formalization of the method that allows the verification of the process for N generators and finally there are the conclusions and future work that can be done with this project.

2 Problem definition

Due to the great importance of the energy market worldwide, a topic of interest was developed in the way in which the draw of those entities that will be able to enter this market is carried out, which has recently taken a strategy of non-supervision of the government with in order to carry out a negotiation process between the marketer and the consumer, this being an unregulated market where a reverse auction process is carried out to reverse which entities and how many units with their respective price will be used on the next day.

It is intended to implement a generalized method for mixed strategy profiles of Nash equilibrium price betting, using an unused algorithm. It is important to highlight that the method does not guarantee exact mathematical solutions because it uses numerical methods to solve certain consecutive systems of first-order ordinary differential equations with initial conditions that will describe the strategy for each player. For the application of this method we have the following elements to consider: N generators $g_1, ...g_N$, with daily production capacity $K_1, ...K_N \in N$, marginal production costs $c_1, ...c_N > 0$, and in which the demand is a discrete random variable d that takes values $1, 2, ..., \sum iK_i$ with $Pr(d = i) = \pi_i$.

The previous knowledge that should be considered for the implementation of this method are probability theory, game theory and auction theory; this being the mathematical elements that will be the support to the system of equations that will be generated for each strategy proposed. The main aim of the Nash's method is to reach a equilibrium for all the players; being this a great interest to branches as the economy, finance and any study that requires a fair way of choosing elements in a balanced way giving them the corresponding value, it should be noted that the method is designed to admit only the necessary elements that describe an unregulated electricity market mentioned above, all this being the basis of the research project.

For the purpose deepen the conceptual basis of the project, the term unregulated market will be developed, which refers to the non-regulation by some legal entity such as the government, so that the form of participation in this market is more competitive and accessible to the consumer so prices are not standardized.

The research practice 2 will focus on the same problem posed in the research practice 1 but with a conceptual and theoretical look not yet covered, which is necessary to support the strategy used previously concerning the replication of a player who is in balance within the game.

2.1 Electricity purchase mechanism

The mechanism used for the realization of the algorithm is based on the following structure, where it is assumed that the electricity generating system is formed by N company generators $g_1, ..., g_N$, that each company g_j has a single generating plant, which is capable of producing at most a certain positive integer number k_j of units per day, and which has a unit production cost $c_j \geq 0$. Let $k = \sum_{j=1}^{K} k_j$ the maximum amount of electricity that can be generated by the system in one day. It is assumed that the amount of electricity demanded by consumers in a d is a random variable d that takes whole values $1, 2, ..., K$, and that the probability that d takes the value i is π_i. In addition, certain market policies require that companies can only offer unit prices $p \in R$ in the interval $[0, \bar{p}]$ Each day (day t), each g_j secretly sends a number $p_j \in [0, \bar{p}]$ to a regulatory body, which expresses g_j's desire to produce the next day (day $t+1$) any amount of electricity equal to or less than its capacity k_j, provided it is paid to p_j or more per unit. At this point we emphasize that the generators decide their p_j with knowledge of the probability distribution of d, that is, of the π_i, but without knowing the particular value that d takes the day $t+1$. Once the regulator receives the unit prices offered $p_1, ..., p_N$, it chooses a ranking of these prices, that is, it chooses a tuple $(r_1, ..., r_N)$ with $\{r_1, ..., r_N\} = \{1, ..., N\}$ and such that $p_{r1} \leq pr_2 \leq ... \leq pr_N$. Note that this rank is not necessarily unique. Therefore the regulator considers all possible rankings of prices $p_1, ..., p_N$, and chooses one of them according to a uniform probability distribution. Once the regulator has chosen a rank r for the prices $p_1, ..., p_N$, and knows the particular value that d takes the day $t+1$, it considers the numbers $K_0 = 0$ and $K_j = \sum_{i=1}^{j} k_{ri}$ for $j = 1, ..., N$, and determines the value of j for which $K_{j-1} < d \leq K_j$. If we call this value j_s, then the regulator determines that each generator g_{r_j} with $j < j_s$ is bought all its capacity k_{r_j}, that the generator $g_{r_{js}}$ is bought $d - K_{j_s-1}$, that the rest of the generators are not bought anything, and that everyone is paid $p_{r_{js}}$ per unit. This $p_{r_{js}}$ value is called the *spot price*. This explains the use of the letter s in the note. According to this, the utility of each generator grj with $j < j_s$ will be $u_{r_j} = k_{rj}(p_{r_{js}} - c_{r_j})$, the utility of the generator $g_{r_{js}}$ will be $u_{r_{js}} = (d - K_{j_s-1})(p_{r_{js}} - c_{r_{js}})$, and the utility of the rest of the generators will be zero. As usual, let's assume that the companies want to maximize their utilities, and that they are rational agents. The whole situation can then be interpreted as the repetition of a classic non-cooperative game of three stages: first, companies chooses its prices in the interval$[0, \bar{p}]$; then nature chooses a demand value according to the probability distribution $Pr(d = i) = \pi_i, i = 1, ..., K$; and finally the regulator dispatches according to the rules already mentioned. Note that the last step involves a random selection that has the fin to resolve ties. The utilities of companies depend on the prices offered by all companies, the value taken by the demand, and the particular rank chosen by the regulator. The conditions of the game lead players to offer their prices in a random way, according to a profile of mixed strategies $F_1, ..., F_N$ that is a Nash equilibrium. A mixed strategy for offering prices is described by a probability distribution function F defined in the range $[0, \bar{p}]$, this referring to the fact that F is monotonous and not decreasing, continues on the right, where $F(0) = 0$ y $F(\bar{p}) = 1$: if $a, b \in [0, \bar{p}]$ with $a < b$, the probability of choosing a price $p \in (a, b]$, is $F(b) - F(a)$. A profile of mixed strategies $F_1, ..., F_N$ is a Nash equilibrium if for each $j = 1, ..., N$,

$$E_{f_1}, ..., F_j, ..., F_N[u_j] \geq E_{f_1}, ..., F_j^*, ..., F_N[u_j]$$

for all mixed strategies F_j^*. Here u_j denotes the utility of the generator g_j. The elements that the method needs to carry out the competition between players are: N, $k_1, ...k_N$, $c_1, ..., c_N, \pi_1, ..., \pi_k$, to propose profiles $F_1, ..., F_N$ that are Nash's equilibrium.

3 State of the art

Deregulated markets allow electricity providers to compete and sell electricity directly to consumers, this being the reduction or elimination of government power in a particular industry, generally promulgated to create more competition within the industry. The reason for this is that fewer regulations and simpler ones will lead to a higher level of competitiveness, therefore, greater productivity, more efficiency and lower prices in general Estay (2002), Due to this, so many investigations are carried out around these markets and the important role that all these mathematical theories can perform for the development of this one is evident, that as you spend time it takes more development force and has been nourished by increasingly complex models, deregulation gained momentum in the 1970's, influenced by research at the University of Chicago and the theories of Ludwig von Mises, Friedrich von Hayek, and Milton Friedman, among others Reddy (2014). After this quite a few authors have been interested in the development of models that can help to generate fair competences using the Nash balance as the main objective of these interactions between the elements of the model.

One of the clearest examples is the research that has been carried out throughout Europe for these unregulated markets where oligopoly markets (a situation where the market is dominated by a minority of large companies) are the clearest case of the energy markets in the UK in which only nash balances were sought in non-cooperative spot markets as a one-shot game. Since the bidding process is repeated daily and bids are published shortly after they are made, it is wait that not there would be any "Learning" problems in reaching these equilibria also Bolle (1990) applied the Klemperer-Meyer framework to the electricity spot market, modeling supply functions for three specifications of a bidding game Richard J. Green (1992). Normally the models found in the literature are implemented for a specific number of generators and with elements irrelevant to the model you want to develop as in the case of the model of Severin Borenstein & Stoft (2000) in which two geographically distributed electricity markets are used can be thought of as a first-level approximation to the market distinctions between northern and southern California. In this situation where each market is dominated by a single supplier, the benefits of increased transmission capacity manifest as greater competition, in some cases with less actual power flow. The mere threat of competitive entry that is provided by additional transmission capacity acts as a restraining influence on the dominant supplier in each market, causing each to produce nearer to competitive levels even though the threatened imports are not in fact realized.

Grossman (1981) and Hart (1985) studied supply function equilibria in the absence of uncertainty. It's consider this approach problematic for two reasons.This approach is considered problematic for two reasons. First, Nash equilibria in supply functions can support a huge multiplicity of results in the absence of uncertainty. Second, in the absence of uncertainty, the motivation for modeling firms as competing via supply functions is not compelling. Without uncertainty, a firm knows its equilibrium residual demand with certainty, and it therefore has a single profit-maximizing point, which it could achieve by choosing either a fixed price or a fixed quantity. It gains nothing from the ability to choose a more general supply function, so the use of these more general strategies would not be robust even to arbitrarily tiny costs of maintaining the greater flexibility they embody Paul D. Klemperer (1989).

The particular organization of the electricity spot market makes standard oligopoly models inadequate as analytical tools. It's propose instead to model this market as a sealed-bid, multiple-unit auction. In the first stage of the model, firms simultaneously submit offer prices at which they are willing to supply their (given) capacities. As in the UK industry, firms (generators) can submit different offer prices for each individual plant or generating set, i.e. firms offer step supply functions. Sets are then ranked according to their offer prices (i.e. a supply function is constructed). In the final stage, demand is realized and the system marginal price is determined by the intersection of demand and supply; that is, by the offer price of the marginal operating unit Nils-Henrik M. von der Fehr (1992).

Mentioned this, it is expected that the generalized model for an unregulated market to be considered in the investigation is required for a simple implementation to use and that Nash equilibrium can be achieved for all players.

4 Solution method / Methodology

In this practice we built an adaptation of the algorithm made in Maple which is a program oriented to the resolution of mathematical problems, capable of performing symbolic, algebraic and computational algebraic calculations. With this, the process of extension of each function was facilitated, and it is more evident the construction of each accumulated probability curve of the players.

Given the needs that arose in past research practice, it was thought that the process of improving the algorithm was fundamental to the experimentation and manipulation of this, so some changes were made in its structure and functions.

For the construction of the algorithm, the formula (1) was implemented, which generates an admissible profile for the players. The terminology and theorems necessary for the construction of the algorithm will be presented below:

4.1 Terminology

- The support of a mixed strategy F, is the smallest closed subset S of $[0, p]$ with the property that $\mu_F([0, p] - S) = 0$,this support refers to the spot price mentioned above in the electricity purchase mechanism.

- The profiles of mixed strategies are included under a theorem in which $F_1, ..., F_N$ is a balance of Nash if and only if for each $j = 1, ..., N$, the restriction to F_j support of the function $\Phi j = [0, p] \to R$ defined as $\Phi j(p) = E_{F_1}, ..., F_N[u_j - p_j = p]$, is a constant function.

- Let $F_1, ..., F_N$ be an admissible profiles $F_1, ..., F_N$ is a equilibrium of Nash if and only if for all $k = 1, ..., N$,if $j \geq k$ then $\Phi_j^{'}$ is zero in the interval (p_k^m, p_{k-1}^m) .

4.1.1 Theorem

Define the profile of mixed strategies $F_1, ..., F_N$ is admissible, if there are numbers $0 < p_N^m \leq p_{N-1}^m \leq ... \leq p_1^m < \bar{p}$, such that:

i For each j, $F_j(p) = 0$ for all $p \in [0, p_j^m]$.

ii There is at most one j such that $F_j(\bar{p}-) := \lim_{p \to \bar{p}^-} -F_j(p) < 1$.

ii If for each j it is define:

$$\tilde{F_j}(p) = \begin{cases} F_j(p) & \text{if } p \in [0, \bar{p}) \\ F_j(\bar{p}-) & \text{if } p = \bar{p} \end{cases}$$

4.1.2 Theorem

Let $F_1, ..., F_N$ be an admissible profile Then:

1) Each of the functions Φj is continuous the interval $[0, p]$.
2) For all $p \in [0, p) - p_i^m : i = 1, ..., N$, $\Phi_j^{'}(p)$ exists and is given by

$$\begin{aligned} \phi_j^{\square'}(p) = & \sum_{A \bigcup \{c\} \bigcup B = \{1,...N\} j \in A} - \left\{ k_j(p - c_j) \left(\sum_{i=1}^{k_c} \pi_{k_A+i} \right) \prod_{a \in A \ a \neq j} F_a(p) \right. \\ & \prod_{b \in B} (F_b(\bar{p}-) - F_b(p)) \left(1 + \sum_{n \in B} \frac{1 - F_n(\bar{p}-)}{F_n(\bar{p}-) - F_n(p)} \right) F^{'}c(p) \\ + & \\ & \sum_{A \bigcup \{j\} \bigcup B = \{1,...,N\} \square} \left(\sum_{i=1}^{Kj} i \pi_{k_A+i} \right) \frac{d}{dp} \left\{ (p - c_j) \prod_{a \in A \square} F_\alpha(p) \right. \\ & \prod_{b \in B} (F_b(\bar{p}-) - F_b(p)) \left(1 + \sum_{n \in B} \frac{1 - F_n(\bar{p}-)}{F_n(\bar{p}-) - F_n(p)} \right) \end{aligned} \tag{1}$$

where K_A denotes $\sum_{a \in A} K_a$ for each $A \subset \{1, ...N\}$.

4.1.3 Theorem

Let $F_1, ..., F_N$ is an admissible profile. is a equilibrium Nash's if and only if for everything $k = 1, ..., N$ if $j \geq k$ then $\phi_j^{'}$ is zero in the interval(P_k^m, P_{k-1}^m) where (P_0^m) is $(\bar{p})$

Each competitor has a $\phi_j^{\square'}(p)$ that makes reference to the admissible profiles with which it is expected to arrive at a strategy with which it is possible to play in equilibrium for all the players, the solutions of these differential equations of first order are in function of the drifts of the $F_1, ...F_N$ with which it was used the method *dsolve* that is already by effect in the software

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According to the author Von fehr when the case of When N = 2 there exists a unique mixed-strategy Nash equilibrium in which player i's strategy is given by: Play $p \in [p^m, \bar{p}]$ according to the probability distribution $F_i(p)$, where $F_i(p)$, $i = 1, 2$ are given by

$$p^m = \begin{cases} \frac{\bar{p}-c}{e} + c & where\ \pi = \frac{1}{2} \\ [\bar{p} - c][\frac{\pi}{1-\pi}]^{\frac{\pi}{1-2\pi}} + c & where\ \pi \neq \frac{1}{2} \end{cases} \tag{2}$$

Under the theorem exposed by the author it was decided to carry out a search for Nash equilibrium using the fact that the last two solutions must end in the same value with which replicated one of them significant changes in the value of KA are carried out.. With this theorem, tests were made for the values shown in the results section where this strategy was implemented.

Steps	***Description of the construction method***
Step 1	Make $i = 0$
Step 2	Mage $G_1, ..., G_N : \bar{p} \leq [0, 1]$ such that $G_1(\bar{p}) = \frac{n-i}{n}$, $G_2(\bar{p}) = ... = G_N(\bar{p})$
Step 3	Is intended to build a successful backward extension of $G_a, ...G_N$
Step 4	if the extension cannot be built and $ei < n$, increase $iin1$ and go to step 2. if the extension cannot be built $ei = n$, stop the process and declare that the process does not produce any results
Step 5	If the extension could be built, be it $\hat{G}_1, ...\hat{G}_N[b, p] \to [0, 1]$ and sea $q \in [b, p)$ as in the definition of successful extension.
Step 6	If $\hat{G}_1(q) = ... = \hat{G}_N(q) = 0$ then the process stops and if $F1$ is defined as the function that is worth zero in $[0, q]$, which matches $\hat{G}_1$ in $[q, \bar{p})$ and $F1(\bar{p}) = 1$; and F_j with $j = 2, .N$ as the function that is worth zero in $[0, q]$, and coincides with $\hat{G}_j$ in the interval $[q, \bar{q}]$, it turns out that $F_1, .F_N$ is a permissible profile which is a Nash equilibrium.
Step 7	If for some$j, \hat{G}_j(q) > 0$, be $G_1, .., G_N$ the restrictions of the functions $\hat{G}_1, ...\hat{G}_N$ to the interval $[q, p]$ and return to *Step 3*.

If this process does not produce a result, then it is done again but taking as $g1_1$ any other generator.

5 Results

In order to follow the same strategy of obtaining the equilibrium Nash's for various players, we continued to observe the technique of data replication by complying with *step 6* of the construction of the method and making small changes in the data to finally achieve this equilibrium. I will then present the replication process

- **Nash equilibrium for three players**

Two player values				
Elements	**Capacities**	**Cost**	**Initial values**	**Probabilities**
Values	(1, 1, 1)	(40, 40, 0)	(1, 1, 0.7886)	(0.2, 0.2,0.6)

Table 1: Data for three players, replicating the information of one

Three player values				
Elements	**Capacities**	**Cost**	**Initial values**	**Probabilities**
Values	(1,1,1)	(0, 40, 42)	(1,1,0.7886)	(0.2,0.2,0.6)

Table 2: Data for three players, with different information

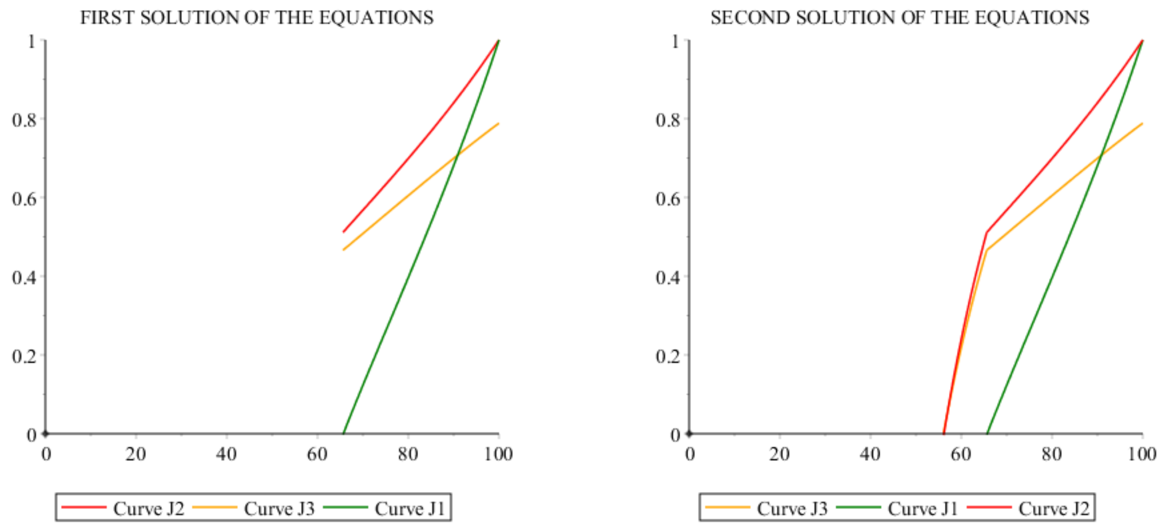

Figure 1: Cumulative probability curves for players

In *Table 1* is shown the use of replication as the player 1 and player 2 share the same data, by using the algorithm we get an equilibrium for only two players "supposedly" which is evidenced *step 6* of the construction of the method which indicates that if only two players are in the price draw will find a balance of Nash for the two where their strategy profiles will be equal in the interval and thus start from a balance to get another with more players. In the *Table 2* it is evident the variation of the data for which it is easier to find the balance because it would only be necessary to move the initial condition to obtain the balance between all, these techniques are supported with the theorems mentioned in the section of Terminology which encourage the construction of balances more simply

Let's remember that to carry out this procedure in the built algorithm it became somewhat tedious to find the new initial conditions, so speeding up this process was essential for the development of the cumulative probability curves of each player, so an extra function was performed which extensively solves the ordinary differential equations without the need for manual calculation that was done before.

The same procedure was performed again with the improved algorithm and the data replication strategy to continue obtaining the balance but for four players

Four player values				
Elements	**Capacities**	**Cost**	**Initial values**	**Probabilities**
Values	(1,1,1,1)	(0, 40,40, 41)	(1,1,1,0.7)	(0.2,0.2,0.3,0.3)

Table 3: Data for four players, replicating the information of one

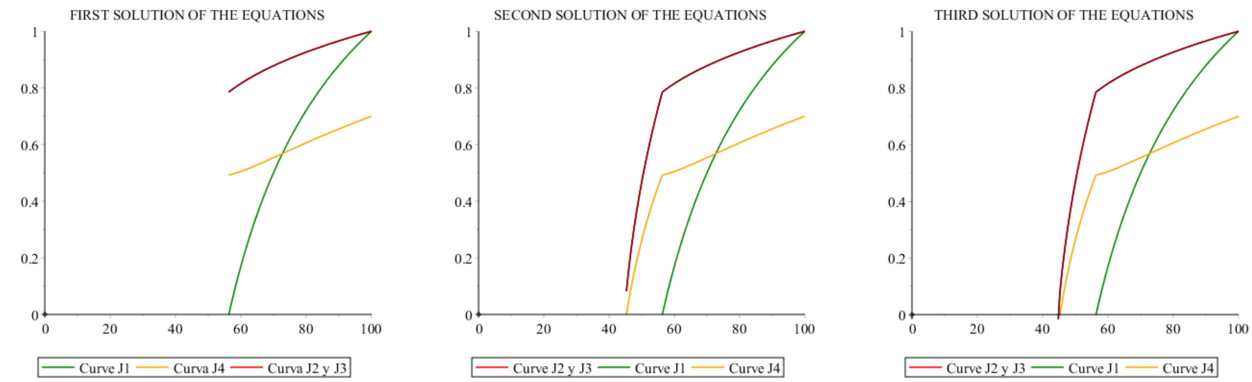

Figure 2: Cumulative probability curves for four players, replicating the information of one

For these four players we implemented the same replication technique in which the red curve that offers the profile of permissible strategies for the player $J1$ and $J2$ is equal because they share data with each other, we observe that the value of this curve in the interval $[0, \bar{p}]$ is equal to that of the curve of $J4$.

Four player values				
Elements	**Capacities**	**Cost**	**Initial values**	**Probabilities**
Values	(1,1,1,1)	(0, 40, 41, 42)	(1,1,1,0.75)	(0.2,0.2,0.3,0.3)

Table 4: Data for four players, with different information

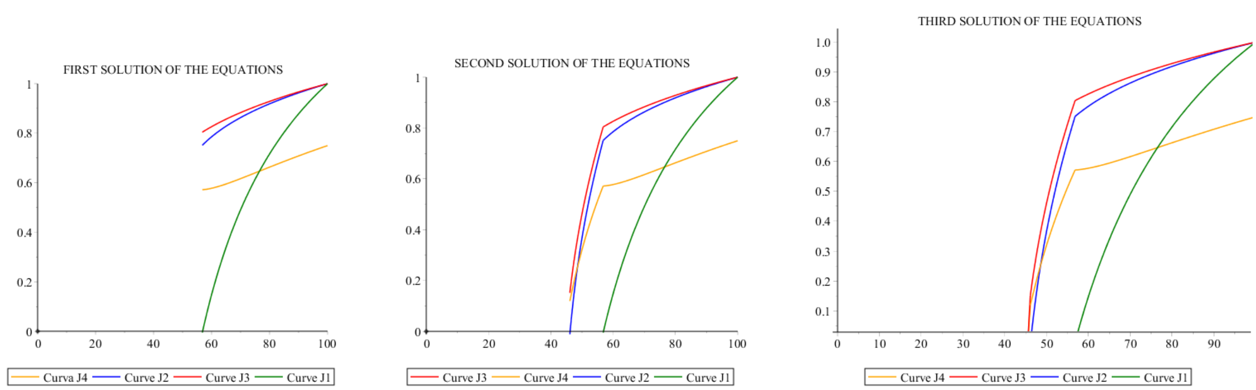

Figure 3: Cumulative probability curves for four players with different information

Note that in the *Table 4* there is a change in the data of each player and a curious case of this graph is that the initial value of player four had to be varied to achieve the balance of Nash for all players, note that this change is achieved through in *step 6* of the construction of the method. Which indicates that if the last two functions do not end at the same point in the interval, the initial value must be varied to obtain the equilibrium.

6 Conclusions and future research

The improvements made in the algorithm were very useful when conducting experiments which facilitated the search for balances Nash for several players, it is noteworthy that the strategy used replication is not yet proven its validity because it was not achieved to make enough experiments to verify the strategy, a future work for the project is to make a deepening in the concept of replication and generate a mathematical support that supports this.

One of the most important results was the generation of the extension concept mentioned in step 6 of the construction of the method that was essential in the speed of the process of construction of Nash balances, with this new advance that if it made generates a very great hand fan of future works where a simulation could be made to generate an analysis of the behavior of Nash for a group of players

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