por inducción:

① $x_{n+1} = 4x_n - x_n^2 \wedge x_0 = 4\mathrm{Sen}^2\theta$

Implica $x_{n+1} = 4\mathrm{Sen}^2(2^{n+1}\theta)$ $\theta \in [0, \pi/2]$

Caso base: $n=0$

$x_1 = 4x_0 - x_0^2$

$x_1 = 4 \cdot 4\mathrm{Sen}^2\theta - 16\mathrm{Sen}^4\theta.$

$= 16\mathrm{Sen}^2\theta - 16\mathrm{Sen}^4\theta$

$= 16\mathrm{Sen}^2\theta(1 - \mathrm{Sen}^2\theta)$

$= 16\mathrm{Sen}^2\theta\cos^2\theta.$

$= 4 \cdot 4\mathrm{Sen}^2\theta\cos^2\theta$

$= 4 \cdot (2\mathrm{Sen}\theta\cos\theta)^2,$

$= 4 \cdot (\mathrm{Sen}(2\theta))^2$

$= 4 \cdot \mathrm{Sen}^2(2^{0+1}\theta)$ ✓

Paso inductivo

Supongase que $x_{n+1} = 4x_n - x_n^2$

que $x_n = 4\mathrm{Sen}^2(2^n\theta)$

$x_{n+1} = 4x_n - x_n^2$

$x_{n+1} = 4(4\mathrm{Sen}^2(2^n\theta)) - (4\mathrm{Sen}^2(2^n\theta))^2$

$= 16\mathrm{Sen}^2(2^n\theta) - 16\mathrm{Sen}^4(2^n\theta)$

$= 16\mathrm{Sen}^2(2^n\theta)(1 - \mathrm{Sen}^2(2^n\theta))$

$= 16\mathrm{Sen}^2(2^n\theta)(\cos^2(2^n\theta))$

$= 4 \cdot (2 \cdot \mathrm{Sen}(2^n\theta) \cdot \cos(2^n\theta))^2$

$= 4(\mathrm{Sen}(2 \cdot (2^n\theta)))^2$

Por lo tanto, $4(\operatorname{sen}(2^{n+1}\theta))^2$

—— 11 —— 11 —— n ——

Probar que si $x_{n+1} = 4x_n - 4x_n^2$

y $x_0 = \operatorname{sen}^2\theta$

Implica que $x_{n+1} = \operatorname{sen}^2(2^{n+1}\theta)$

$\theta \in [0, \pi/2]$

• Demostración por inducción

Caso base: $n=0$

$x_1 = 4x_0 - 4x_0^2$

$x_1 = 4 \cdot \operatorname{sen}^2\theta - 4\operatorname{sen}^4\theta$

$x_1 = 4\operatorname{sen}^2\theta(1 - \operatorname{sen}^2\theta)$

$x_1 = 4\operatorname{sen}^2\theta \cos^2\theta$

$x_1 = (2 \cdot \operatorname{sen}\theta \cdot \cos\theta)^2$

$x_1 = (\operatorname{sen}(2\theta))^2$

Se puede escribir como

$x = \operatorname{sen}^2(2^n\theta)$

Supongase que $x_{n+1} = 4x_n - 4x_n^2$

$x_n = \operatorname{sen}^2(2^n\theta)$

$x_{n+1} = 4x_n - 4x_n^2$

$x_{n+1} = 4(\operatorname{sen}^2(2^n\theta)) - 4\operatorname{sen}^4(2^n\theta)$

$x_{n+1} = 4 \cdot \operatorname{sen}^2(2^n\theta))(1 - \operatorname{sen}^2(2^n\theta))$

$x_{n+1} = 4\operatorname{sen}^2(2^n\theta)(\cos^2(2^n\theta))$

$x_{n+1} = (2\operatorname{sen}(2^n\theta)\cos(2^n\theta))^2$ ✓

Considerese el sistema

(5) $A\vec{x} = b$

Donde A es una matriz triangular inferior

$$\begin{bmatrix} A_{11} & 0 & 0 & \cdots & \\ A_{21} & A_{22} & 0 & \cdots & \\ A_{31} & A_{32} & A_{33} & 0 & \cdots \\ \vdots & & & & \\ A_{n1} & A_{n2} & \cdots & & A_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

→ (1) $A_{11}x_1 = b_1$

(2) $A_{21}x_1 + A_{22}x_2 = b_2$

(3) $A_{31}x_1 + A_{32}x_2 + A_{33}x_3 = b_3$

⋮

(n) $A_{n1}x_1 + A_{n2}x_2 + \cdots + A_{nn}x_n = b_n$

(1) $x_1 = \frac{b_1}{A_{11}}$

(2) $A_{21}\frac{x_1}{A_{11}} + A_{22}x_2 = b_2$

$$x_2 = \frac{b_2 - A_2 x_1}{A_{22}}$$

(3) $x_3 = \frac{b_3 - A_{31}x_1 - A_{32}x_2}{A_{33}}$

$$x_3 = \frac{b_3 - (A_{31}x_1 + A_{32}x_2)}{A_{33}}$$

$$x_3 = \frac{b_3 - \sum_{j=1}^{2} A_{3j}\, x_j}{A_{33}}$$

$$x_n = \frac{b_n - A_{n1}x_1 - A_{n2}x_2 - \cdots - A_{nn-1}x_{n-1}}{A_{nn}}$$

$$x_n = \frac{b_n - (A_{n1}x_1 + A_{n2}x_2 + \cdots + A_{nn-1}x_{n-1})}{A_{nn}}$$

$$x_n = \frac{b_n - \sum_{j=1}^{n-1} A_{nj}\, x_j}{A_{nn}}$$

Entonces $n=i$ Cambio de variable

$$\rightarrow x_i = \frac{b_i - \sum_{j=1}^{i-1} A_{ij}\, x_j}{A_{ii}}$$