Finite impulse response filtering

Dr. Ing. Rodrigo Gonzalez

rodralez@frm.utn.edu.ar

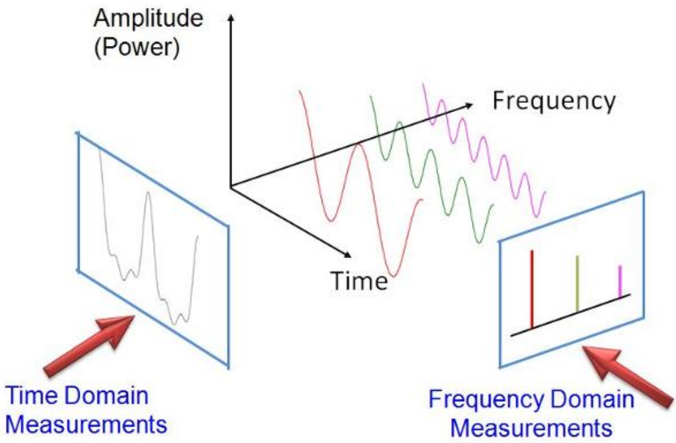
Técnicas Digitales III

Universidad Tecnológica Nacional,
Facultad Regional Mendoza.

Resumen

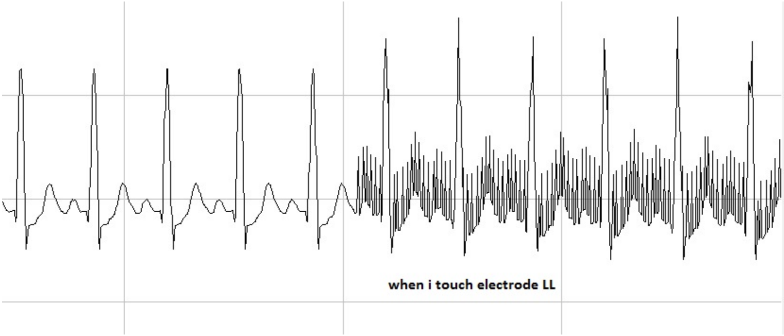
Filtering in different domains

- Filtering in **time domain** (signal restoration, smoothing, denoising).
- Filtering in **frequency domain** (signal separation).



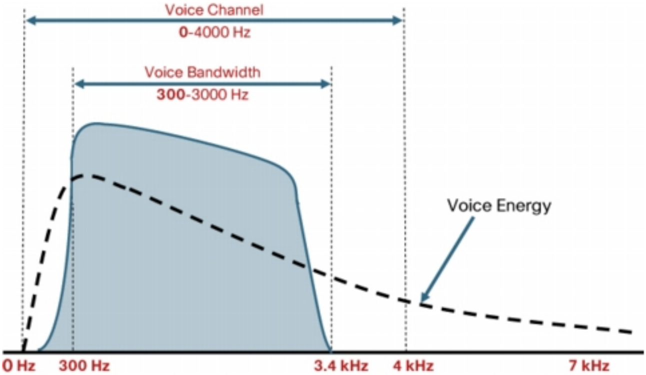
Information in time domain

- Information is contained in amplitude and time of the signal.
- Each sample contains information that is interpretable without reference to any other sample.
- The **step response** describes how information represented in the time domain is being modified by the system.
- Examples: electrocardiography (ECG) signal, accelerometer, gyroscope...



Information in frequency domain

- The information is contained in the relationship between many points in the signal.
- The frequency response shows how information represented in the frequency domain is being changed.
- Example: telephone voice channel, equalizer...



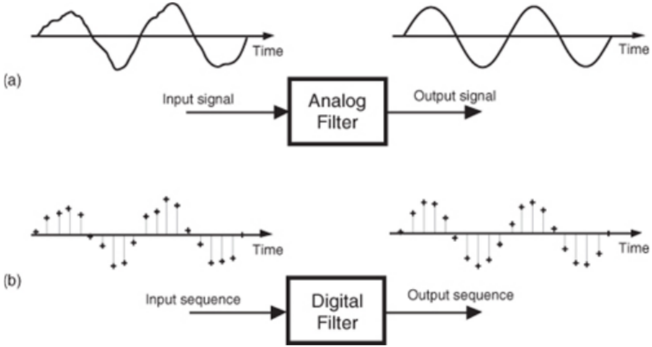
Classification of discrete filters

Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed Filters Equiripple Minimax	Bilinear z-transform

FIR filtering structure

Figure 5-1 Filters: (a) an analog filter with a noisy tone input and a reduced-noise tone output; (b) the digital equivalent of the analog filter.

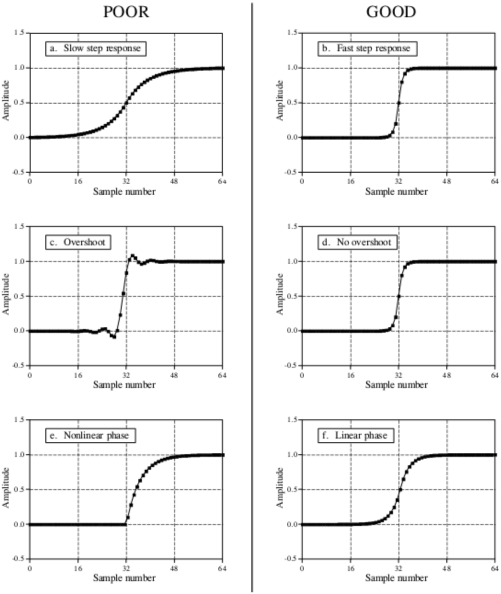


Time domain parameters, step response

- Risetime (between 10%∼90% amplitude).
- Overshoot.
- Linear phase.

It is not possible to optimize a filter for both domains.

Good performance in the time domain results in poor performance in the frequency domain, and vice versa.



Moving average filter

- The moving average filter is a convolution of the input signal with a rectangular pulse having an area of one.
- *Local average*.
- There is a delay of N/2 samples between input and output.

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1}\delta[n-k]\,, \tag{1}$$

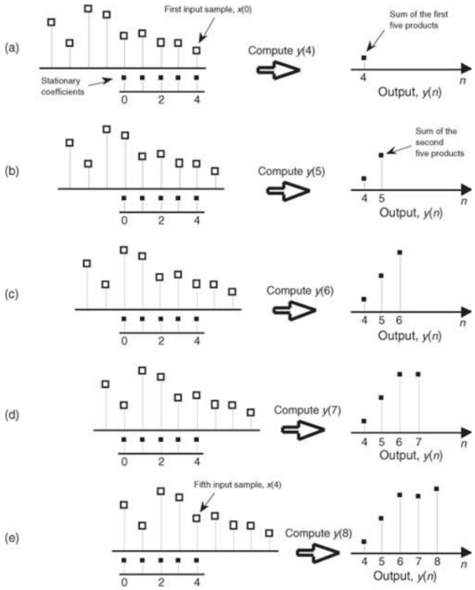
$$y[n] = x[n] * h[n] = \frac{1}{N}\sum_{k=0}^{N-1}x[n-k]\,, \tag{2}$$

$$N = \frac{\sigma_{in}^2}{\sigma_{out}^2}\,, \tag{3}$$

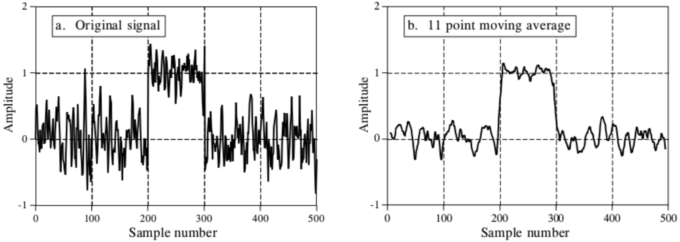
$$SNR = 10log_{10}(N)\,. \tag{4}$$

- It can be seen that the moving average filter is a FIR filter. Why?

Moving average filter, example



- MA reduces random white noise while trying to keep the sharpest step response.



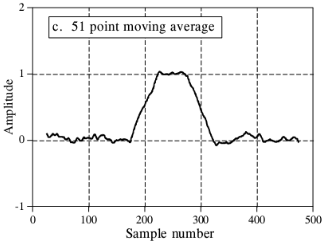
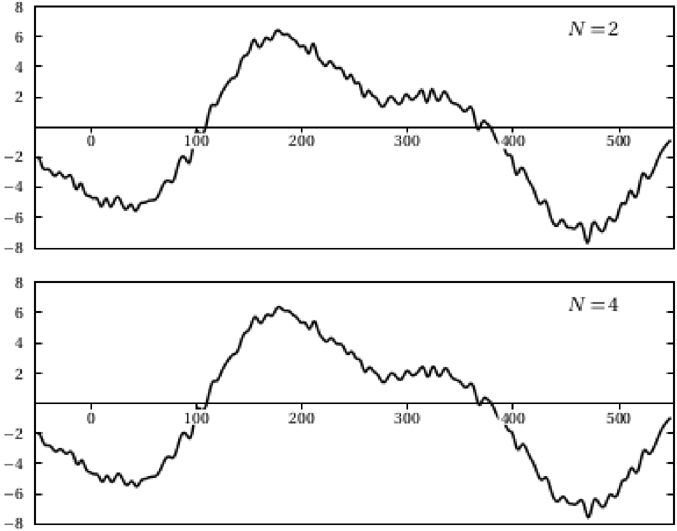


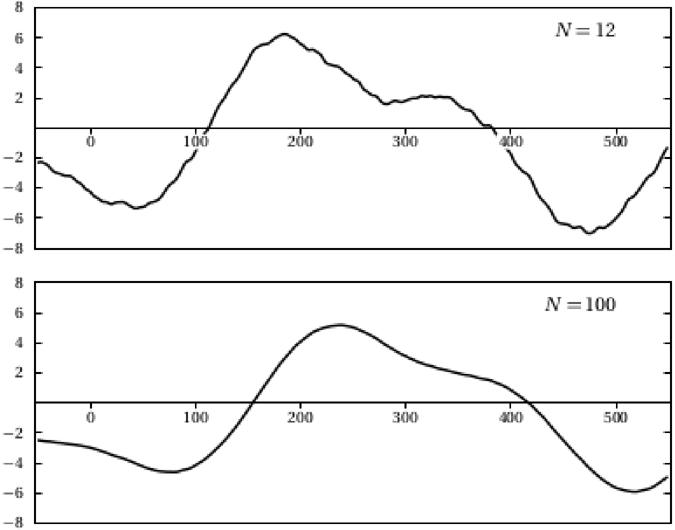
FIGURE 15-1
Example of a moving average filter. In (a), a rectangular pulse is buried in random noise. In (b) and (c), this signal is filtered with 11 and 51 point moving average filters, respectively. As the number of points in the filter increases, the noise becomes lower; however, the edges becoming less sharp. The moving average filter is the *optimal* solution for this problem, providing the lowest noise possible for a given edge sharpness.

Noise Reduction

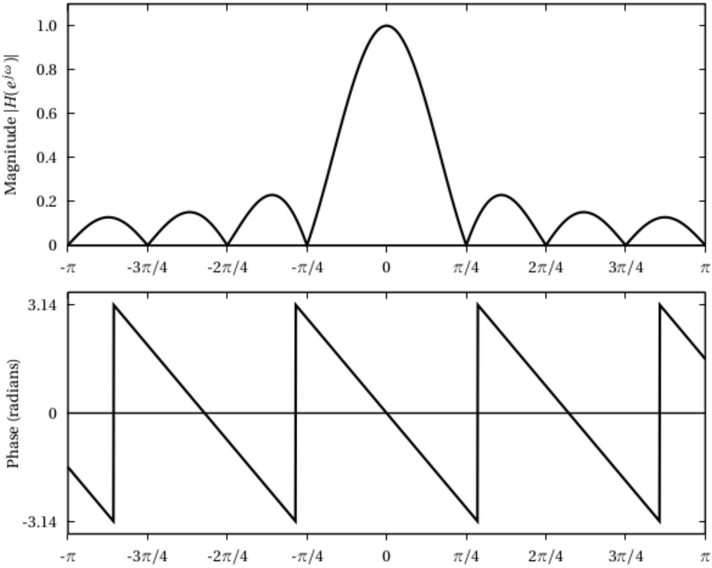


Noise Reduction

- Note how the signal is delayed as N grows.



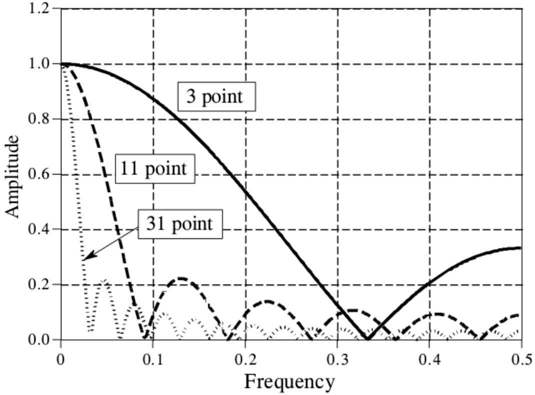
Frequency Response



Frequency Response

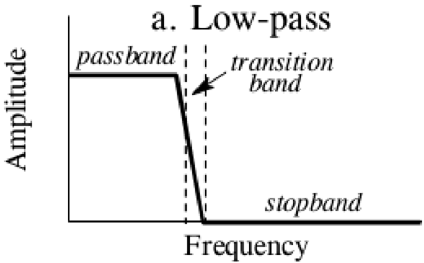
- The moving average filter is a bad low-pass filter.
- In short, the moving average is a good *smoothing filter* (the action in the time domain), but a bad low-pass filter (the action in the frequency domain).

$$H[e^{j\omega}] = \frac{1}{N}\frac{\sin(\omega N/2)}{\sin(\omega/2)} e^{-j\frac{N-1}{2}\omega}\,. \tag{5}$$

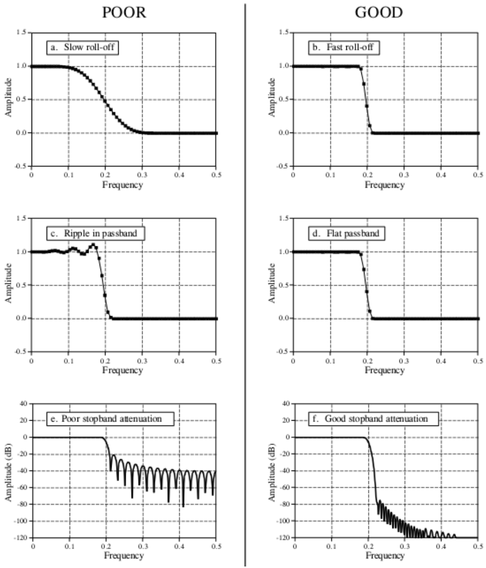


Frequency domain parameters

- Passband.
- Stopband.
- Transition band (fast roll-off).
- Passband ripple.
- Stopband ripple.



Frequency response parameters, cont'd



Strategy of filtering by windowing

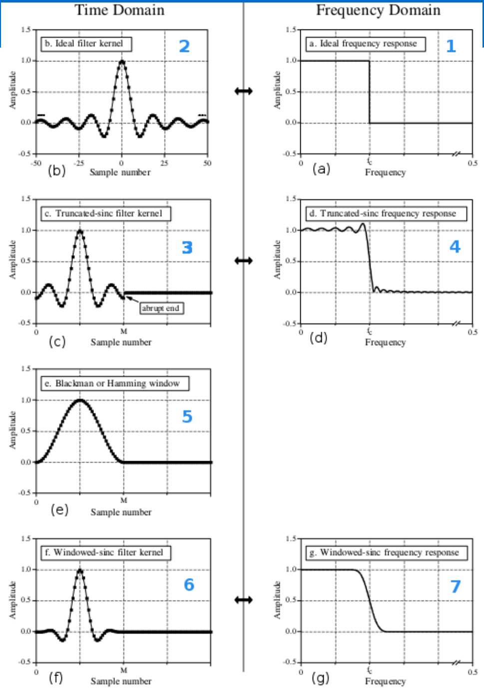
- Taking the Inverse Fourier Transform of an ideal frequency response (1) produces an ideal sinc filter kernel (2, impulse response).

$$hs[i] = \frac{\sin(2\pi f_C i)}{i\pi}$$

- Truncated-sinc (3) produces the Gibbs phenomenon in frequency response (4), no matter how long M is made.

$$h[i] = hs[i] \cdot w[i]$$

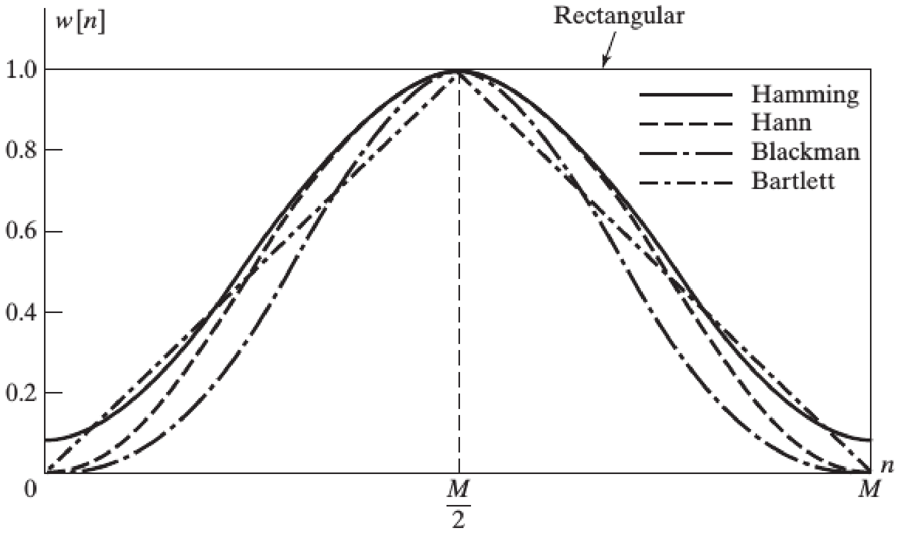
- Multiplying the truncated-sinc (3) by the Blackman window (5) results in the windowed-sinc filter kernel (6) with frequency response (7).



Windows

Name of window function $w(n)$	Mathematical definition
Rectangular	1
Hanning	$0.5 - 0.5\cos\left[\frac{2\pi n}{N-1}\right]$
Hamming	$0.54 - 0.46\cos\left[\frac{2\pi n}{N-1}\right]$
Blackman	$0.42 - 0.5\cos\left[\frac{2\pi n}{N-1}\right] + 0.08\cos\left[\frac{2\pi n}{N-1}\right]$
Kaiser	$\dfrac{I_0\left[\beta\sqrt{1-\left(\frac{\lvert 2n-N+1\rvert}{N-1}\right)^2}\right]}{-I_o(\beta)}$ Where, $I_0(x) = \sum_{k=0}^{\infty}\left(\frac{x^k}{2^k k!}\right)^2$

Windows in time domain



Windows in frequency domain

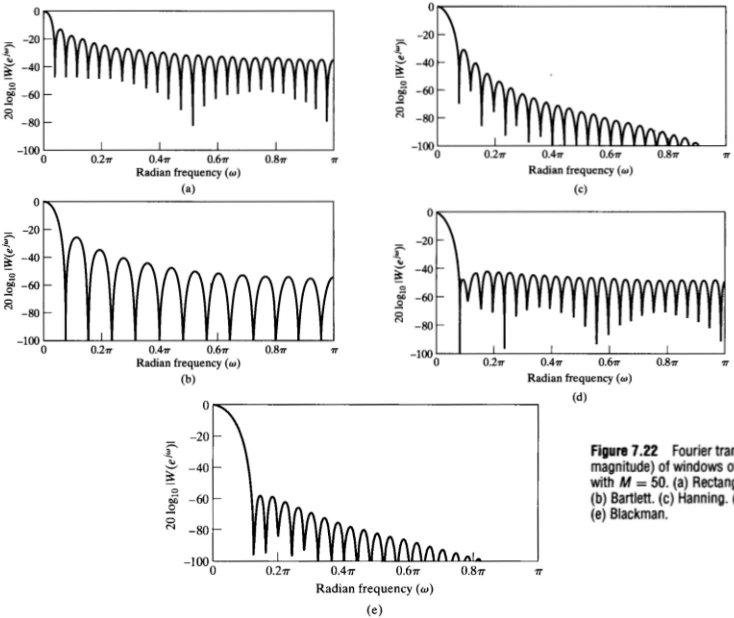
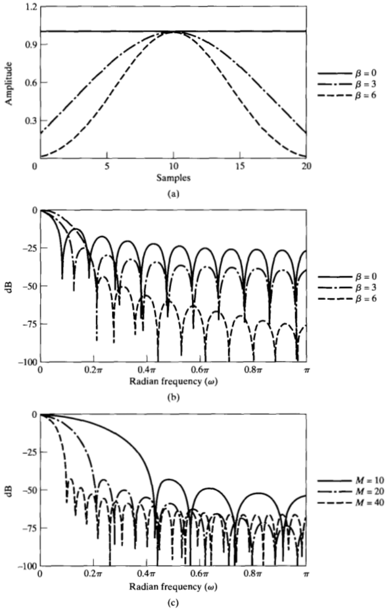


Figure 7.22 Fourier transforms (log magnitude) of windows of Figure 7.21. with $M = 50$. (a) Rectangular. (b) Bartlett. (c) Hanning. (d) Hamming. (e) Blackman.

Kaiser window filter

- The Kaiser window has two parameters:
  - Lenght, M+1.
  - Shape parameter, β.
- Trade-off between side-lobe amplitude and main-lobe width.

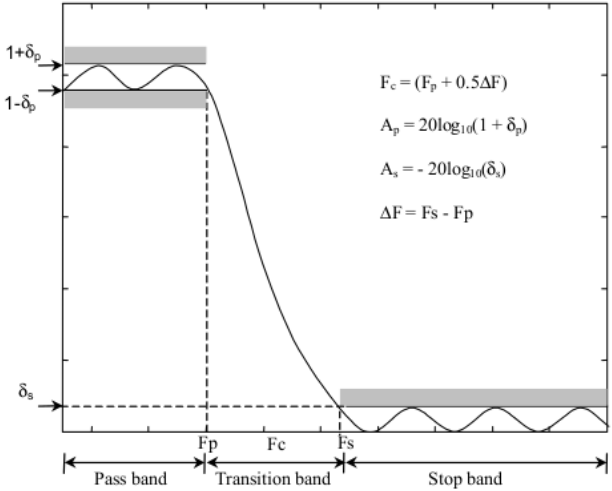


Kaiser window filter, cont'd

TABLE 7.2 COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hann	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

FIR filter design



Name of window function $w[n]$	Transition width ΔF in (Hz), (normalised)	Pass-band ripple A_p in (dB)	Ripple δ_p, δ_s	Side-lobe level in (dB)	Stop-band attenuation A_s in (dB)
Rectangular	0.9/N	0.741	0.089	-13	21
Hanning	3.1/N	0.0546	0.063	-31	44
Hamming	3.3/N	0.0194	0.0022	-41	53
Blackman	5.5/N	0.0017	0.000196	-57	74
Kaiser β=4.54	2.93/N	0.0274			50
β=5.65	3.63/N	0.00867			60
β=6.76	4.32/N	0.00275			70
β=8.96	5.71/N	0.000275			90

Example of FIR design

A FIR low-pass filter is required to have the following specifications:

1. Pass-band edge frequency $f_p = 2$ kHz
2. Transition band $\Delta f = 200$ Hz
3. Pass-band ripple $A_p = 0.1$dB
4. Minimum stop-band attenuation $A_s = 50$dB
5. Sampling frequency of $f_s = 10$ kHz

Pass-band ripple, $A_p = 20\log_{10}(1 + \delta_p)$

$$\delta_p = \log_{10}^{-1}\left[\frac{0.1}{20}\right] - 1 = 0.0116$$

Minimum stop-band attenuation $A_s = - 20\log_{10}(\delta_s)$

$$\delta_s = \log_{10}^{-1}\left[\frac{-50}{20}\right] = 0.00316$$

The normalised pass-band edge frequency

$$F_p = f_p / f_s = \frac{2x10^3}{10x10^3} = 0.2$$

The normalised transition width

$$\Delta F = \Delta f / f_s = \frac{200}{10x10^3} = 0.02$$

$$N = \frac{3.3}{0.02} = 165$$

$$H(z) = b_0 + b_1 z^{-1} + \ldots + b_{M-1} z^{M-1}$$

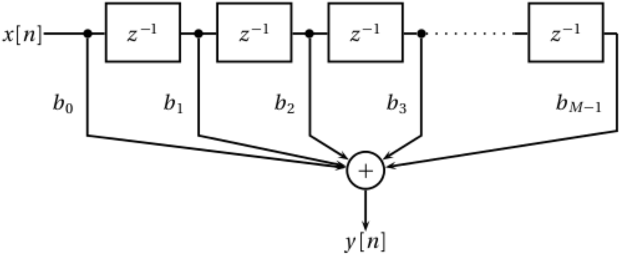


Figure 7.22 Direct FIR implementation.

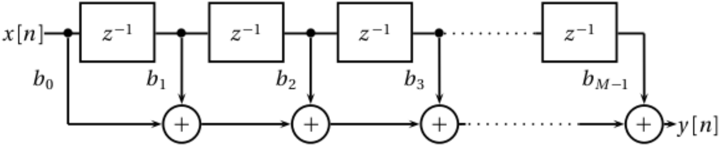


Figure 7.23 Transversal FIR implementation.

Bibliography

1 Alan V. Oppenheim and Ronald W. Schafer. *Discrete-time signal processing, 3rd Ed*. Prentice Hall. 2010. Sections 7.5 and 7.6.

2 Paolo Prandoni and Martin Vetterli. Signal processing for communications. Taylor and Francis Group, LLC. 2008. Sections 5.2, 5.3.1, 7.2.1, 7.4.1, and 7.1.1.

3 Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapters 14, 15, and 16. www.dspguide.com

4 Oliver Hinton. Digital Signal Processing Resources for EEE305 Course. Chapter 4. www.staff.ncl.ac.uk/oliver.hinton/eee305/