Filtros digitales
Conceptos fundamentales

Ing. Rodrigo González
rodralez@frm.utn.edu.ar

Técnicas Digitales III

Universidad Tecnológica Nacional,
Facultad Regional Mendoza.

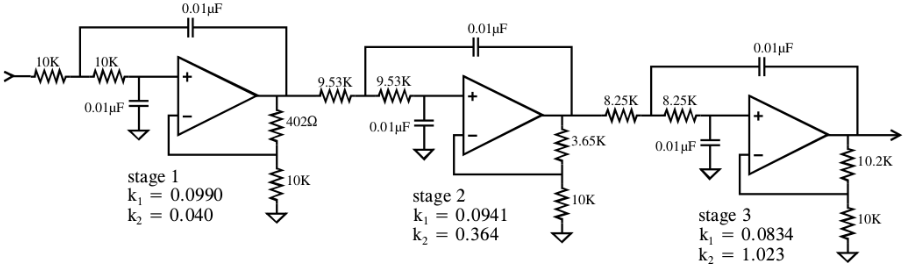
Resumen

1. Introduction to Digital Filters
   - Filter Basics
   - How Information is Represented in Signals
   - Time Domain Parameters
   - Frequency Domain Parameters
   - High-Pass, Band-Pass and Band-Reject Filters
   - Filter classification
2. Moving Average Filters
   - Implementation by Convolution
   - Noise Reduction vs. Step Response
   - Frequency Response
   - Relatives of the Moving Average Filter
3. Windowed-Sinc Filters
   - Strategy of the Windowed-Sinc
   - Designing the Filter

Analog vs digital

Which is better?

- Analog filters are cheap, fast, and have a large dynamic range in both amplitude and frequency.
- Digital filters, in comparison, are vastly superior in the level of performance that can be achieved.
- E.g.: FIR gain of 1 +/- 0.0002 from DC to 1K Hz, and a gain of less than 0.0002 for frequencies above 1.001K Hz.

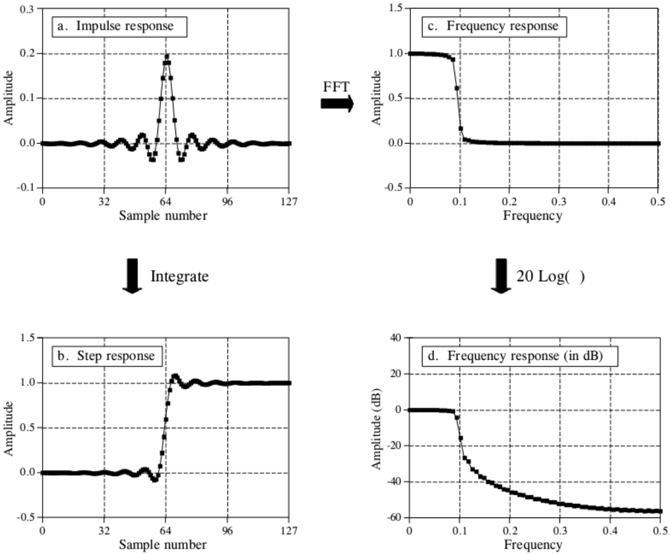


Types of digital filters

Filters have two uses:

- Signal restoration (time domain).
- Signal separation (frequency domain).

Filter parameters



Classification

Information represented in the time domain

- Describes when something occurs and what the amplitude of the occurrence is.
- Each sample contains information that is interpretable without reference to any other sample.
- The step response describes how information represented in the time domain is being modified by the system.
- Example: EKG signal.

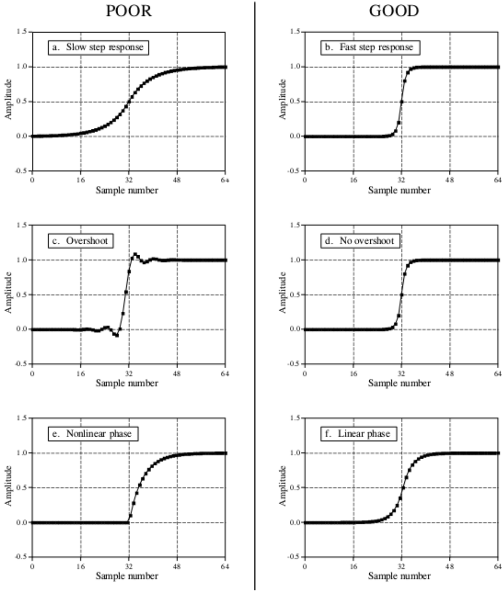
Information represented in the frequency domain

- The information is contained in the relationship between many points in the signal.
- Each sample contains information that is interpretable without reference to any other sample.
- The frequency response shows how information represented in the frequency domain is being changed.
- Example: digital filter for a hearing aid.

It is not possible to optimize a filter for both applications. Good performance in the time domain results in poor performance in the frequency domain, and vice versa.

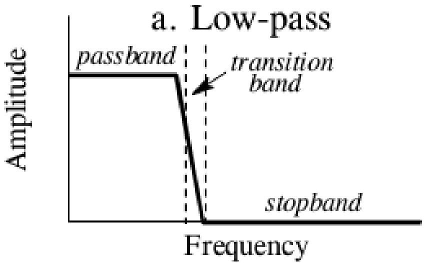
Step response parameters

- Risetime (between 10%∼90% amplitude).
- Overshoot.
- Linear phase.

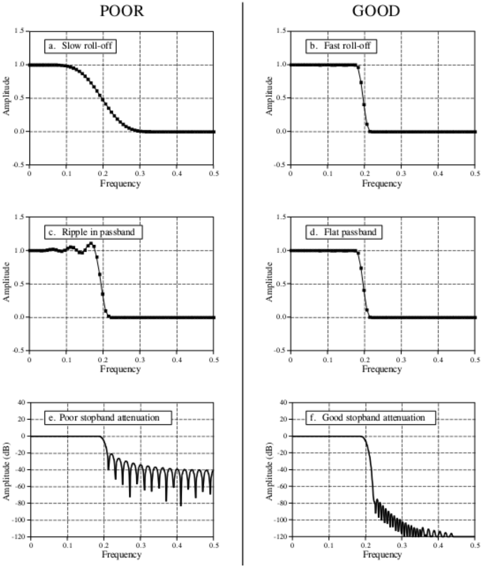


Frequency response parameters

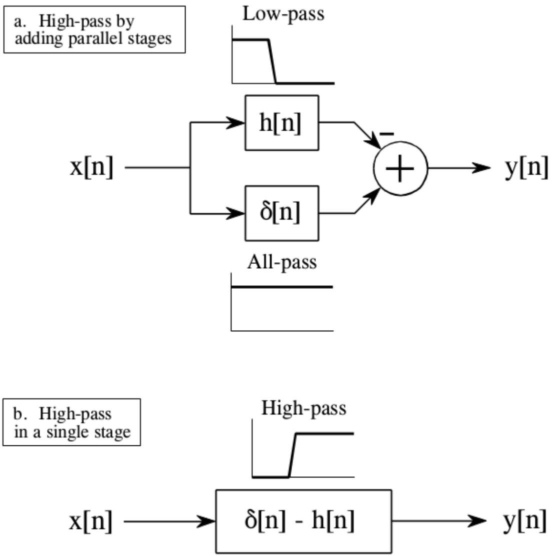
- Passband.
- Stopband.
- Transition band (fast roll-off).
- Passband ripple.



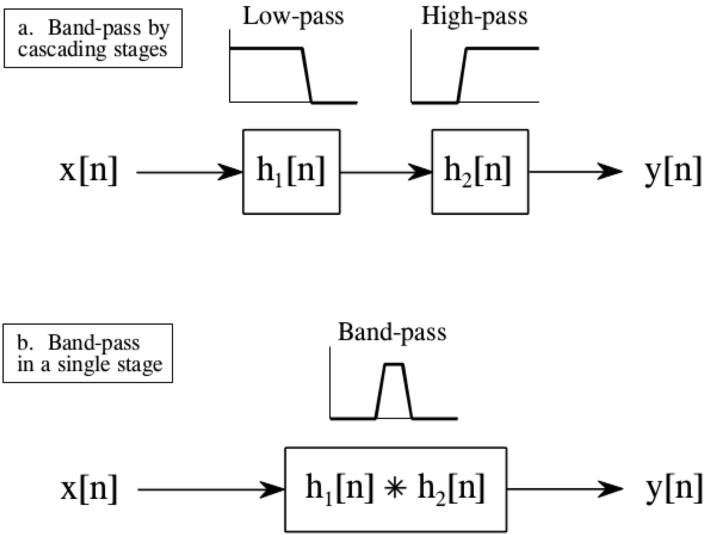
Frequency response parameters, cont'd



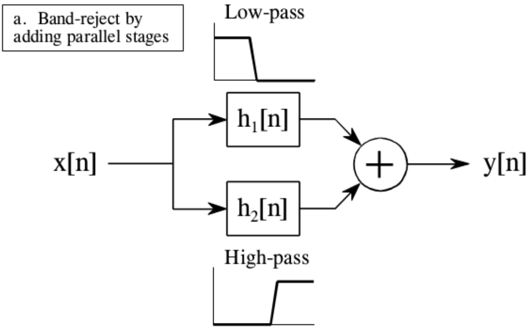
High-Pass Filter

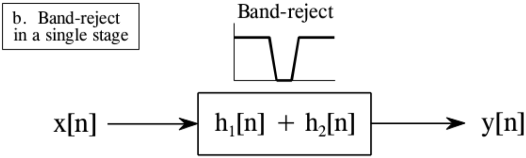


Band-Pass Filter



Band-Reject Filter





Convolution and Recursion filters

- Convolution (FIR): convolution between the input signal and the digital filter's impulse response (filter kernel).
- Recursion (IIR): convolution between the output signal and the recursion coefficients.
- FIR can have far better performance than IIR, but need more computation time.

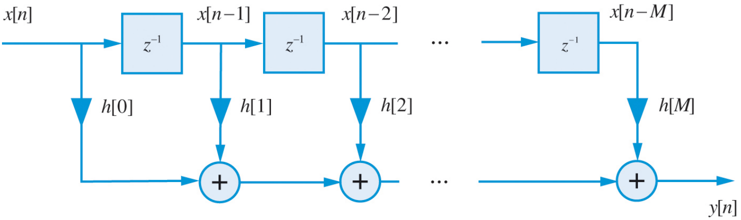


Figure : Block diagram representation of an FIR system.

Filter classification

FILTER USED FOR:	FILTER IMPLEMENTED BY: Convolution *Finite Impulse Response (FIR)*	FILTER IMPLEMENTED BY: Recursion *Infinite Impulse Response (IIR)*
Time Domain *(smoothing, DC removal)*	Moving average (Ch. 15)	Single pole (Ch. 19)
Frequency Domain *(separating frequencies)*	Windowed-sinc (Ch. 16)	Chebyshev (Ch. 20)
Custom *(Deconvolution)*	FIR custom (Ch. 17)	Iterative design (Ch. 26)

Implementation by Convolution

The moving average filter is a convolution of the input signal with a rectangular pulse having an area of one.

One side averaging,

$$y[i] = \frac{1}{M} \sum_{j=0}^{M-1} x[i+j]$$

Symmetrical averaging,

$$y[i] = \frac{1}{M} \sum_{j=-(M-1)/2}^{(M-1)/2} x[i+j], \quad \text{M odd}$$

- It reduce random white noise while keeping the sharpest step response.
- The moving average filter is the *optimal* solution for this problem, providing the lowest noise possible for a given edge sharpness.
- $SNR = 10log_{10}(\sqrt{M})$

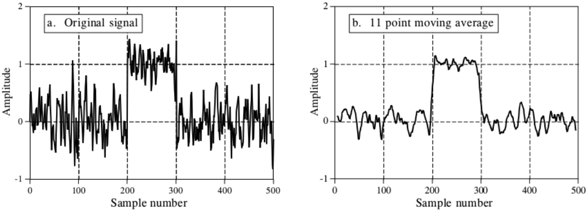
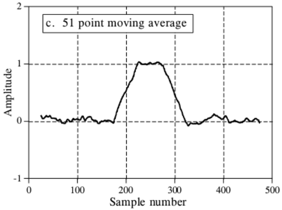
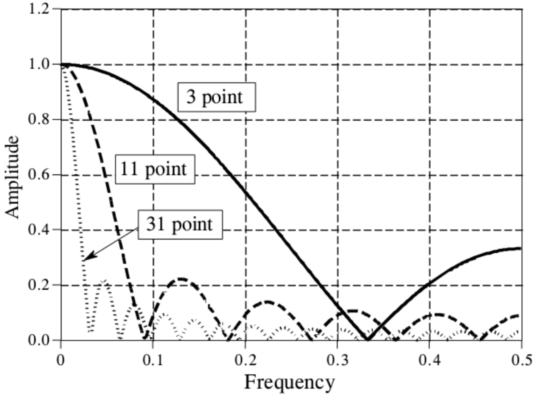


FIGURE 15-1
Example of a moving average filter. In (a), a rectangular pulse is buried in random noise. In (b) and (c), this signal is filtered with 11 and 51 point moving average filters, respectively. As the number of points in the filter increases, the noise becomes lower; however, the edges becoming less sharp. The moving average filter is the *optimal* solution for this problem, providing the lowest noise possible for a given edge sharpness.

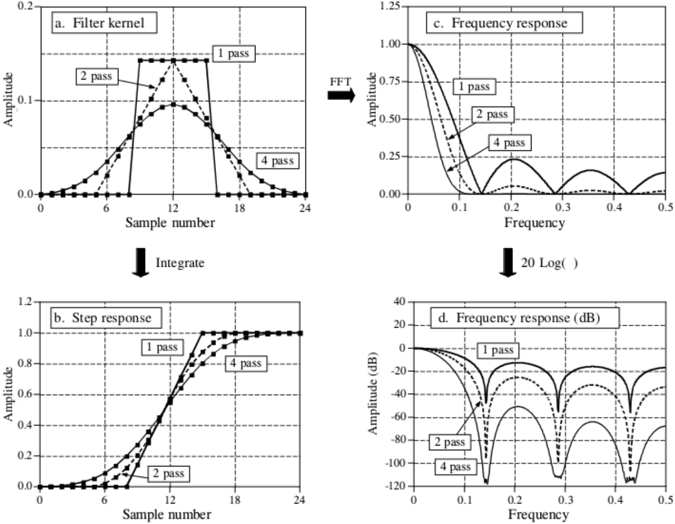


- The moving average filter cannot separate one band of frequencies from another.
- In short, the moving average is an exceptionally good *smoothing filter* (the action in the time domain), but an exceptionally bad low-pass filter (the action in the frequency domain).

$$H[f] = \frac{\sin(\pi f M)}{M \sin(\pi f)}$$

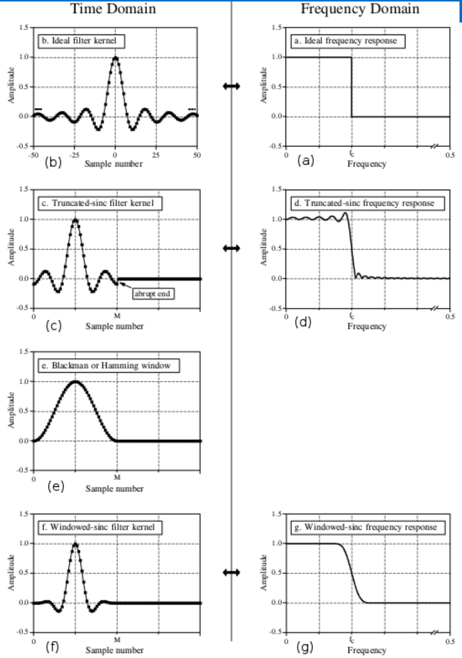


- Blackman window.
- Gaussian window.
- Better stopband attenuation. Step responses are *smooth* curves.



Strategy of the Windowed-Sinc

- Taking the Inverse Fourier Transform of this ideal frequency response (a) produces the ideal filter kernel (b, impulse response).
$$h[i] = \frac{\sin(2\pi f_C i)}{i\pi}$$
- Truncated-sinc (c) and frequency response (d). Discontinuity is significant no matter how long M is made.
- Multiplying the truncated-sinc (c) by the Blackman window (e) results in the windowed-sinc filter kernel (f) with frequency response (g).



Characteristics of the Blackman and Hamming windows

- Hamming window has about a 20% faster roll-off than the Blackman.
- Blackman has a better stopband attenuation, -74dB (- 0.02%) vs -53dB (-0.2%).
- In general, the Blackman should be your first choice; a slow roll-off is easier to handle than poor stopband attenuation.
- Other windows: Bartlett, Hanning, rectangular.

- Blackman window
  $h[i] = 0.42-$
  $-0.5\ cos(2\pi i/M)+$
  $+0.08\ cos(2\pi i/M)$

- Hamming window
  $h[i] = 0.54-$
  $-0.46\ cos(2\pi i/M)$

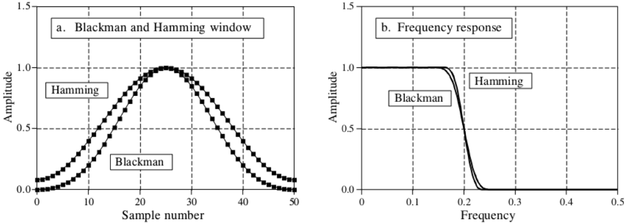
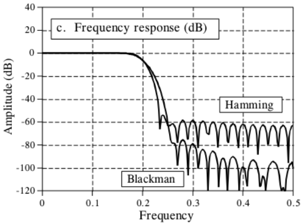


FIGURE 16-2
Characteristics of the Blackman and Hamming windows. The shapes of these two windows are shown in (a), and given by Eqs. 16-1 and 16-2. As shown in (b), the Hamming window results in about 20% faster roll-off than the Blackman window. However, the Blackman window has better stop-band attenuation (Blackman: 0.02%, Hamming: 0.2%), and a lower passband ripple (Blackman: 0.02% Hamming: 0.2%).

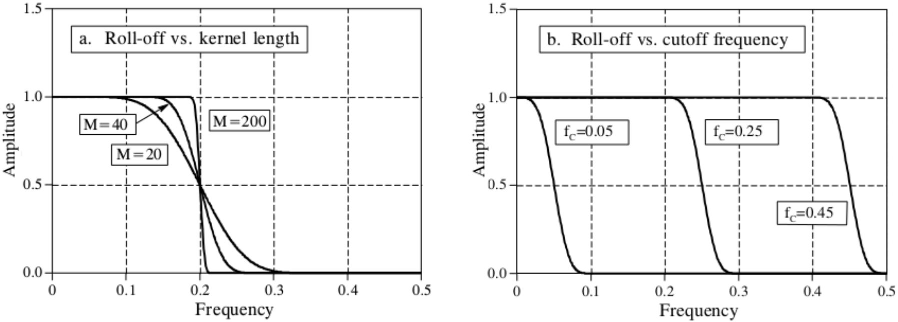


Frequency Response

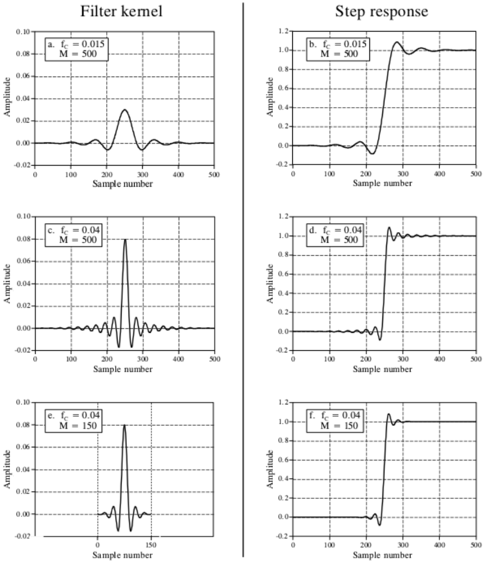
Parameters:

- Cutoff frequency, f_C.
- Sampling frequency, f_S.
- Length of the filter kernel, M.

$$M \approx \frac{4}{BW}, \quad 0 < BW < 0.5$$



Step Response



Bibliography

- Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapters 14, 15, and 16.