Inequality and Misallocation under Production Networks

Rojas-Bernal, Alejandro*

Vancouver School of Economics, The University of British Columbia

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Abstract

In this paper, I develop an aggregation theory for distorted production network economies with heterogeneous households and endogenous labor supply. I provide nonparametric formulas that capture the impact of changes in the income and consumption distributions on the aggregate and distributional propagation of microeconomic shocks. I demonstrate that macroeconomic theories that rely solely on an aggregate production function and ignore variations in the consumption distribution are biased when households have no symmetry in the centrality of their expenditure on the income distribution. I compare the outcomes of a decentralized economy with those of a constrained social planner and show that the distributions of income and consumption are inefficient when the dilution of the consumption expenditure that reaches labor revenue is not symmetric across workers. Finally, I estimate the first quantitative implementation for a production network environment in the United States with household heterogeneity. The results suggest that income distribution variations fostered growth before the Great Recession by increasing TFP by 8.2% and after the Great Recession hindered growth by reducing TFP by 7.5%. Additionally, the variations in the income distribution are responsible for 20% of the business cycle volatility, and microeconomic shocks and the production network play a significant role in explaining income and real consumption inequalities.

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*arojas01@mail.ubc.ca; la.rojas1560@gmail.com

1 Introduction

"While we often must focus on aggregates for macroeconomic policy, it is impossible to think coherently about national well-being while ignoring inequality and poverty, neither of which is visible in aggregate data. Indeed, and except in exceptional cases, macroeconomic aggregates themselves depend on distribution."

– Deaton (2016)

Modern economic production operates under a cobweb of linkages in which billions of independent agents interact through complex social systems. The decisions of these agents on how and what to produce, consume, and supply depend on their income, the price system, and access to markets for primary factors and goods, which are geographically and institutionally constrained. In its simplest form, this economic architecture portrays interdependent multilayered networks of consumption, supply of factors, and transfers, through which disaggregated flows of goods and money circulate.

The question of how microeconomic shocks propagate in complex social systems is foundational for any macroeconomic theory on aggregation. However, such a theory faces many challenges, as it needs to address how the position and dimension of each firm and household in the economy, and consequently the network structure, are essential to understand the propagation of a shock.

In this paper, I contribute to the theory of aggregation for complex social systems by building a neoclassic environment for production network economies with heterogenous households, distortions, and endogenous factor supply. My main objective is to explain the role of endogenous distributional variations in the aggregate and idiosyncratic impact of microeconomic shocks. The common idea behind the mechanisms that this paper introduces is that variations in how expenditure enters and flows in an economy are essential to understand the reallocation of factors across firms. The main theoretical contribution of this paper is to show that reallocation is neutral on aggregate or across households only under highly restrictive conditions. I prove this by identifying sufficient statistics that capture, under an agnostic nonparametric setting, the aggregate and idiosyncratic influence from any endogenous variation in the income distribution, the consumption expenditure distribution, and the recomposition of demand from firms and households. The main requirements for the non-technological channels of growth that my sufficient statistics capture require distortions, endogenous labor supply, and some degree of heterogeneity in households' preferences and firms' technologies.

Using my model, I estimate the first empirical implementation of a production network environment with heterogeneous households for the United States. The model explains Total Factor Productivity (TFP) growth and intermediate input markets are essential for this match. By itself, accounting for intermediate linkages boosts the R^2 of the model prediction on TFP from 4% to 55%. Over the first two decades of the XXI century, the technological component was the main source of growth in TFP. More precisely, the productivity shocks in the oil and gas extraction and computer and electronic product industries were the primary drivers of this growth. The secondary influence on TFP from distributional driven changes in the misallocation of workers was due to the opposing effects before and after the Great Recession. On the one hand, before the Great Recession, distributional effects reduced the misallocation of workers and fostered economic growth by increasing TFP by 8.2%. On the other hand, after the Great Recession, variations in the income distribution reduced TFP by 7.5%. These empirical results contribute to the literature by bringing a new distributional explanation for the stagnated growth environment that the United States faced after the Great Recession.

Theoretical Contribution

The theory of aggregation for multisector economies with input-output networks has relied on the Domar aggregation developed by Hulten (1978), which builds on the growth accounting work from Solow (1957) and Domar (1961). Hulten's theorem shows that in a perfectly competitive neoclassical representative-household economy with an inelastic labor supply, the sales distribution is a sufficient statistic for the first-order effect of firm-level technological shocks on TFP. Therefore, the network structure is not necessary to understand the aggregate effect of microeconomic shocks.

Hulten's theorem relies on symmetry between the downstream propagation of costs (from labor costs to final expenditure) and the upstream propagation of revenue (from final expenditure to labor income). Introducing non-wasteful distortions (e.g., taxation, tariffs, financial constraints, nominal rigidities, and market power) breaks this symmetry, changes the system of prices, and keeps workers and intermediate inputs away from their first-best allocation. For this reason, under distortions, the first-order variation of TFP also depends on a non-technological component that captures the influence of the endogenous reallocation of resources across firms (Baqaee & Farhi, 2020). Consequently, readily observable distributions are no longer sufficient to account for changes in TFP. The firms', workers', and households' location in the network and the network structure become necessary to understand the aggregate effects of microeconomic shocks.

In the absence of an elastic labor supply, I segment the influence of the reallocation of resources on TFP into effects coming from exogenous variations in distortions and endogenous changes in the income distribution. I show that for each worker, the value-added to labor income ratio, which I call distortion centrality, indicates the extent of their labor income dependence on firms that operate within heavily distorted supply chains. Aggregate misallocation worsens as labor income shifts toward workers with large distortion centralities. Consequently, an economy with distortions faces two counterintuitive second-best results. First, the allocation of resources improves as the income distribution becomes more distant from the value-added distribution. Second, the allocation of resources improves with profits as money flows are extracted "earlier" from the payment chains and freed from the distortions that factoral income faces. This decomposition coincides with the results from Baqaee & Farhi (2020), but the workers' distortion centralities and the second-best results are novel.

Including a standard labor-leisure tradeoff allows me to have idiosyncratic labor wedges that capture how distortions influence the labor supply from workers. I derive the first-order approximation for these labor wedges around a distorted equilibrium and show that distortions, the recomposition of demand, and the redistribution of expenditure across households are sources of variation. This result extends the first-order approximation for the labor wedge that Bigio & La'O (2020) derive for a representative household model with production networks, which holds only around the undistorted equilibrium, where distortions are its only source of variation.

From the first-order approximation for the labor wedges, I obtain a theory for the local variation of the income distribution. Using the local variations in the workers' labor income shares, I decompose the distributional effects on TFP in channels that capture the influence of demand recomposition and redistribution of expenditure across households. For each household and firm, there is a sufficient statistic for the average distortion centrality their income faces as it flows through the network and reaches labor compensation. These sufficient statistics convey that aggregate misallocation rises as expenditure shifts toward households or demand shifts toward firms with incomes that reach workers with high distortion centralities. The intuition is that these households and firms directly or indirectly demand labor through relatively efficient supply chains from workers essential in the production processes of heavily distorted sectors. Misallocation rises as workers move from heavily distorted sectors into relatively efficient supply chains. From here, I establish two neutrality results. First, demand recomposition and expenditure redistribution are neutral on TFP around the undistorted equilibrium. Second, expenditure redistribution is neutral on TFP if and only if households' sufficient statistics are symmetric. This symmetry is satisfied when households have the same consumption bundle, indicating that aggregate demand non-homotheticity is necessary but insufficient for redistribution to influence TFP. The second result establishes when an aggregate production function theory of aggregation can be blind to the redistribution of expenditure without introducing biases on TFP.

I show that the decentralized market solution amplifies the welfare loss from misallocation by centralizing households' decisions on a constrained social planer. The planer's solution requires symmetric distortion centralities, which implies a homogenous dilution of the value added by each worker. Workers do not internalize this symmetry condition when choosing their labor supply, creating an externality on aggregate welfare. The constraint social planer is akin to a representative household, with the added feature of choosing distributional allocations. Consequently, the representative household economy coincides with the heterogeneous household model only under the highly restrictive condition of symmetry in distortion centralities, and only then the distributional allocations are efficient from the perspective of the constrained planner. This result allows me to prove that for a representative household or a constrained planner environment, the distributional gains on TFP are equal to the contraction in the aggregate labor wedge. This relationship integrates into a single equation the two equilibrium objects that characterize the prototypical, neoclassic, representative firm, representative household economy à la Chari, Kehoe, & McGrattan (2007).

Household heterogeneity also allows me to consider the distributional effects of microeconomic shocks. For this reason, I introduce the positional terms of trade (PTT) as an object that captures idiosyncratic efficiency wedges. I use the term "positional" because they depend on the location of households across multiple networks. The changes in PTTs are a distributional decomposition of the variations in TFP.

Empirical Contribution

The model successfully explains the variations of TFP in the US, and intermediate input markets are essential for this match. By itself, accounting for input-output linkages boosts the R^2 of the model prediction from 5% to 50%.

From 1997 to 2020, the long-run growth of TFP was mainly attributable to technological shocks. Two industries carried the technological growth of TFP. Without productivity shocks in oil and gas extraction or the computer and electronic products industries, my model predicts that TFP would have grown 11.1% and 6.6% less, respectively. The reallocation of resources played a secondary role; TFP would have grown 2.5% more without changes in competitiveness and 2.8% more without labor misallocation coming from variations in the income distribution. Despite the secondary role of resource misallocation, some industries were important. In particular, TFP would have increased by 4.1% more without the higher profit margins in the credit intermediation industry.

Furthermore, roughly 60% of the TFP volatility was attributable to the reallocation of resources. Out of this, 40% corresponds to variations in the sectoral profit margins and the remaining 20% to changes in the income distribution. Productivity shocks explain the remaining 40% of the volatility. Productivity and distortion shocks in oil and gas extraction, chemical products, utilities, and the financial industries explain most TFP volatility. These results indicate the dichotomy in TFP between growth and volatility in the United States. The former required technological sources of variation, while the latter involved resource misallocation.

However, for specific business cycles, the misallocation of resources did play a significant role in growth. In particular, aggregate competitiveness and labor misallocation felt from 2002 to 2009, and increased from 2010 to 2020.

For the cycle before the Great Recession (2002 to 2009), without the reduction in aggregate competitiveness, TFP would have grown 19.3% more. The higher profit margins in the oil and gas extraction, utilities, housing, computer and electronics, telecommunications, and chemical products industries were the primary drivers of the decline in aggregate competitiveness. Without the reductions in competitiveness from these six industries, TFP would have increased by 14.3% more, and the oil and gas extraction industry would have explained by itself a 6.6% rise. The reduction in labor misallocation partially compensated for the declining competitiveness, and without these favorable changes in labor misallocation, TFP would have been 8.2% lower. The main drivers of this reduction in labor misallocation were the higher profit margins, particularly in the oil and gas extraction and the computer and electronic products industries, which extracted expenditure flows earlier from heavily distorted payment chains.

Higher labor misallocation partially washed off the positive influence of higher aggregate competitiveness after the Great Recession (2010 to 2020). Without the increase in aggregate competitiveness, TFP would have grown 9.8% less, and the primary drivers of this positive effect were the lower profit margins for the oil and gas extraction and the housing sectors, which respectively explain an increase of 6.3% and 3.1% in TFP. Without the rise in labor misallocation, TFP would have grown 7.5% more.

These aggregate variations hide a rich story of distributional effects captured by the variation of the positional terms of trade. According to the positional terms of trade, the last two decades, on the one hand, have been unfavorable for low-skill industrial workers with occupations that have a labor income heavily exposed to the printing, shoe, leather, and textile industries. On the other hand, the same shocks have benefited high-skill workers with occupations related to computer science and mathematics.

Related Literature

This article relates to the literature on disaggregated national accounts, production networks, heterogenous agents, growth accounting, and misallocation. The most foundational is the literature on disaggregated national accounts with heterogenous consumers and producers. The roots of this literature trace back to the work from Cantillon (1756) and Quesnay (1758), who considered that a successful system of macroeconomic accounts should build up from bilateral flows that add up to the national aggregates. These principles inspired the diagrams of circular flow developed by Lahn (1903), Foster (1922), Knight (1933), Meade & Stone (1941), and Kuznets (1946), and the measures of inter-industrial trade from Leontief (1928, 1986). These studies are the foundation for modern national accounts (Stone, 1961). However, the disaggregated transactions that these accounts collect are incomplete. For example, they capture no information about the flows between firms and households. For this reason, Andersen, Hansen, Huber, Johannesen, & Straub (2022) take a step forward in measuring these flows in Denmark, where accessible administrative data and information on credit card transactions from the largest retail bank allows them to estimate direct bilateral flows. My model defines new measures of bilateral centrality, including the direct and indirect channels connecting any two households or firms throughout the economy.

The production network literature builds on the canonical multisector models from Hulten (1978) and Long & Plosser (1983). The main emphasis of this literature has been on the propagation of sectoral productivity shocks (Foerster et al., 2011; Horvath, 1998, 2000; Dupor, 1999; Acemoglu et al., 2012, 2016; Carvalho et al., 2021). However, the same models have been used to study the propagation of sectoral distortions under specific (Basu, 1995; Ciccone, 2002; Yi, 2003; Jones, 2011; Asker et al., 2014) and generic (Jones, 2013; Baqaee, 2018; Liu, 2019; Baqaee & Farhi, 2020; Bigio & La'O, 2020) input-output structures. In a broader sense, the literature on production networks belongs to the broader attempt to map the aggregate effects from "granular" microeconomic shocks that follow the seminal work from Gabaix (2011). My model nests all of these environments and shocks as specific cases.

Within the extensive work on heterogenous agents, my article is related to the literature on asymmetries in marginal propensities to demand goods and labor. These publications show that static marginal propensities to consume can be heterogenous across regions, countries, sectors, or categories of goods (Clayton et al., 2018; Jaravel, 2019; Cravino et al., 2020; Argente & Lee, 2021; Huneeus et al., 2021). This argument is captured in production network environments with heterogeneous households by the models from Baqaee & Farhi (2019b, 2022) and Devereux et al. (2023). My model differs from Baqaee & Farhi (2019b) in taking distortions into account and from Baqaee & Farhi (2019b, 2022) in the inclusion of a microfounded labor-leisure tradeoff; however, relative to these two papers, the most crucial difference is that I represent the production network as separated substochastic matrices, which allows me to introduce new measures of bilateral centrality. Relative to the open economy environment with production networks and an endogenous labor supply from Devereux et al. (2023), my model generalizes its distributional implications from a Cobb-Douglas environment to a generic nonparametric specification. The treatment of the elastic labor supply borrows from the representative-household environment in Bigio & La'O (2020).

Finally, in the growth accounting literature opened by Solow (1957), and developed by Domar (1961); Hulten (1978); Jorgenson et al. (1987); Hall & Diamond (1990); Basu & Fernald (2002); Petrin & Levinsohn (2012); Osotimehin (2019); Baqaee & Farhi (2020), I develop a segmentation of the allocative component from the aggregate TFP that depends on the variations in distortions, the demand structure, and the consumption distribution. The aggregate and distributional decomposition of the effects from the reallocation of resources relates my model with the misallocation literature (Restuccia & Rogerson, 2008; Hsieh & Klenow, 2009).

Layout

The structure of the paper is as follows. Section 2 introduces the multisector input-output model with heterogeneous households and distortions. Section 3 characterizes the equilibrium and the centrality measures. Section 4 presents sufficient statistics for aggregate TFP and household-level PTTs under a nonparametric environment. Section 5 characterizes how the equilibrium and the sufficient statistics would change if households' decisions were centralized in a constrained social planner. Section 6.1 describes the data and the quantitative implementation for aggregate TFP and distributional PTTs. Section 4 introduces a parametric setting that disciplines endogenous variations. Section 8 presents the most simple economy for which the distributional effects on TFP will show up. Section 9 evaluates the aggregate and distributional effects from a manifold of sectoral shocks in productivities and markdowns. Finally, Section 10 identifies four general classes of economies for which there are zero first-order distributional reallocation gains on TFP, which allows me to understand the economic structure and primitives necessary for variations in the income and consumption distributions that allow for non-technological growth.

2 General Framework

In this section, I set up a static nonparametric general equilibrium model with constant-returns-to-scale (CRS) for economies with N sectors and H types of households. Sector $i \in \mathscr{N} = \{1, \cdots, N\}$ consists of two types of firms: (i) a unit mass of monopolistic competitive firms indexed by $z_i \in [0, 1]$ producing differentiated goods, and (ii) a perfectly competitive producer that aggregates the industry's differentiated goods into a uniform sectoral good that can be consumed by households or used by other firms as intermediate inputs. Firms differ along three dimensions; first, monopolistic firms across sectors operate under different technologies; second, monopolistic firms within sectors have heterogeneous input demand; and third, sectoral aggregators face different distortions. Households of type $h \in \mathscr{H} = \{1, \cdots, H\}$ consume sectoral goods using the income received from their endogenous labor supply and rebated profits. Households differ along three dimensions; first, their preferences; second, a type-specific horizontally differentiated labor supply; and third, the composition of their equity portfolio. Financial markets are incomplete, and households cannot cross-insure their idiosyncratic income shocks.

2.1 Production

Monopolistic firms within sectors produce differentiated goods using the same technology. The production for firm z_i in sector i follows

$$y_{z_i} = A_i\, Q_i\left(L_{z_i}, X_{z_i}\right), \quad L_{z_i} = A_i^\ell\, Q_i^\ell\left(\left\{A_{ih}^\ell\, \ell_{z_i h}\right\}_{h\in\mathscr{H}}\right), \quad X_{z_i} = A_i^x\, Q_i^x\left(\left\{A_{ij}^x\, x_{z_i j}\right\}_{j\in\mathscr{N}}\right), \tag{1}$$

where y_{z_i} stands for output, A_i is the sector-specific Hicks-neutral productivity term. L_{z_i} is the labor composite that depends on the productivity A_i^ℓ. $\ell_{z_i h}$ is the amount of labor hired from household h and is influenced by the productivity A_{ih}^ℓ. X_{z_i} is the intermediate input composite that depends on the productivity A_i^x. $x_{z_i j}$ is the amount of intermediate input goods purchased from sector j and is influenced by the productivity A_{ij}^x.

The technologies $Q_i : \mathbb{R}_+^2 \to \mathbb{R}_+$, $Q_i^\ell : \mathbb{R}_+^H \to \mathbb{R}_+$, and $Q_i^x : \mathbb{R}_+^N \to \mathbb{R}_+$ are neoclassical and satisfy the usual regularity conditions, that is, they are positive, finite, and for the set of labor types and intermediate inputs for which there is effective demand, they are monotonically increasing, twice continuously differentiable, strictly concave, and the Inada conditions hold.

The profits for firms z_i are given by

$$\pi_{z_i} = p_{z_i} y_{z_i} - \underbrace{\sum_{h\in\mathscr{H}} w_h\, \ell_{z_i h}}_{=p_{z_i}^\ell L_{z_i}} - \underbrace{\sum_{j\in\mathscr{N}} p_j\, x_{z_i j}}_{=p_{z_i}^x X_{z_i}}, \tag{2}$$

where p_{z_i} is the price of its output, $p_{z_i}^\ell$ is the price for the labor composite, $p_{z_i}^x$ is the price for the intermediate input composite, w_h is the wage received by households of type h, and p_j is the market price for the good produced by the competitive aggregator in sector j.

The competitive firm in sector i guarantees a homogeneous good by aggregating sectoral production using the following CES production function

$$y_i = \left(\int {y_{z_i}}^{\mu_i}\, dz_i\right)^{\frac{1}{\mu_i}}, \tag{3}$$

where $\mu_i \leq 1$ stands for the sector-specific markdown, and y_{z_i} represents the demand of goods produced by firm z_i. The aggregator takes prices as given and maximizes profits given by $\bar{\pi}_i = p_i y_i - \int p_{z_i} y_{z_i}\, dz_i$.

2.2 Households

Households of type h share the preference utility function $U_h(C_h, L_h)$, where C_h stands for real consumption, and L_h for the labor supply. The utility $U_h : \mathbb{R}^2_+ \to \mathbb{R}_+$ satisfies the usual regularity conditions: $U_{C_h} > 0$, $U_{L_h} < 0$, twice continuously differentiable, strictly concave, and the Inada conditions hold. The composite real consumption $C_h = Q^c_h\left(\{C_{hi}\}_{i\in\mathscr{N}}\right)$ depends on the final consumption C_{hi} of goods from sector i. The consumption aggregation technology $Q^c_h : \mathbb{R}^N_+ \to \mathbb{R}_+$ is neoclassical: positive, finite, homogeneous of degree one, and for the set of goods for which there is effective final demand, it is monotonically increasing, twice continuously differentiable, strictly concave, and the Inada conditions hold.

Each household is infinitesimal, and for this reason, they take prices and wages as given. Consequently, for any two households with type h, their choices are equivalent, and the notation of the model becomes simpler by assuming a type-specific representative household with a budget constraint given by

$$E_h = p^c_h C_h = \sum_{i\in\mathscr{N}} p_i C_{hi} \leq J_h + \Pi_h, \qquad \text{and} \qquad \Pi_h = \sum_{i\in\mathscr{N}} \kappa_{ih}\left(\bar{\pi}_i + \int \pi_{z_i}\, dz_i\right). \tag{4}$$

Expenditure E_h must not be greater than income; the latter includes labor income $J_h = w_h L_h$, and dividend income Π_h. Households of type h own a fraction κ_{ih} of the firms in sector i.

2.3 Market Clearing

For this economy, the technologies, productivities, markdowns, and ownership distributions are primitives. Monopolistic competition is the only source of market imperfections. These distortions reallocate resources across firms and imply no wasted resources. Hence, the goods market clearing is given by

$$y_i = \sum_{h\in\mathscr{H}} C_{hi} + \sum_{j\in\mathscr{N}} x_{ji} \quad \forall i \in \mathscr{N}, \tag{5}$$

where $x_{ji} \equiv \int x_{z_j i}\, dz_j$ is the total amount of intermediate inputs from sector i bought by all monopolistic firms in sector j. Labor market clearing requires $L_h = \ell_h \ \forall h \in \mathscr{H}$, with $\ell_h = \sum_{i\in\mathscr{N}} \int \ell_{z_i h}\, dz_i$.

2.4 Remarks

This environment also applies to the following three generalizations. First, following McKenzie (1959), economies with variable (increasing or decreasing) return to scale can be handled by appropriately introducing producer-specific fixed entrepreneurial factors in a constant return model. Second, without loss of generality, the model and the following results apply to any production factor, not only labor. Finally, the effect of markdowns in the results from my model is isomorphic to other distortions that deviate the system of prices from its first-best solution, such as taxes and financial constraints.

A potential limitation of my model is that I assume segmentation of the labor supply across types of households. The parsimony from this premise allows me to bypass three problems. First, I do not need to consider an ownership matrix that specifies the factor share supplied by each household type. Second, I do not need to consider the cross-elasticities in preferences that arise from the supply of multiple factors by the same household. Third, I can abstract from strategic complementarities between multiple types of households in the supply of the same factor.

3 Equilibrium, Centralities, and Information Theory

In this section, first, I characterize the equilibrium for this economy. Second, I introduce measures of bilateral centrality across firms and households, and measures of aggregate centrality that portray each firm or household's role in the economy. Third, I explain how the concept of relative entropy borrowed from information theory serves as a measure of statistical distance between distributions. This section is essential to understand the first-order approximations that make up the main contribution of this paper.

3.1 Equilibrium Characterization

Let $e \equiv (\mathscr{A}, \mu, \kappa)$ represent the aggregate state, and $\mathscr{E}$ denote the measurable collection of all possible realizations for this state. The matrix $\mathscr{A} \equiv (A, A_\ell, A_x, \underline{A}_\ell, \underline{A}_x)$ collects all productivity measures,[1] and sectoral markdowns are captured by $\mu \equiv (\mu_1, \cdots, \mu_N)'$. The equity matrix $\kappa \equiv (\kappa_1, \cdots, \kappa_N)'$ of size $N \times H$ contains the ownership distribution of firms in sector i represented by the vector $\kappa_i \equiv (\kappa_{i1}, \cdots, \kappa_{iH})'$, with $\kappa_i' \mathbb{1}_H = 1$, and where $\mathbb{1}_H$ is an H sized vector of ones.

For this economy, a mapping of the realization of the aggregate state to an allocation $\vartheta = (\vartheta(e))_{e\in\mathscr{E}}$ and the price system $\rho = (\rho(e))_{e\in\mathscr{E}}$ is represented by the set of functions

$$\vartheta(e) \equiv \left\{ \left\{ \left(y_{z_i}(e), \{\ell_{z_i h}(e)\}_{h\in\mathscr{H}}, \{x_{z_i j}(e)\}_{j\in\mathscr{N}} \right)_{z_i\in[0,1]}, y_i(e), \{C_{hi}(e)\}_{h\in\mathscr{H}} \right\}_{i\in\mathscr{N}}, \{C_h(e), L_h(e)\}_{h\in\mathscr{H}} \right\},$$

$$\rho(e) \equiv \left\{ \left\{ \left(p_{z_i}(e), p^{\ell}_{z_i}(e), p^{x}_{z_i}(e) \right)_{z_i\in[0,1]}, p_i(e) \right\}_{i\in\mathscr{N}}, \{w_h(e), p^{c}_h(e)\}_{h\in\mathscr{H}} \right\}.$$

Definition 1. For any realization of the aggregate state e in the state space $\mathscr{E}$, an equilibrium is the combination of an allocation and a price system (ϑ, ρ) such that: (i) monopolistically competitive firms' labor and intermediate input demand, output, and price decisions maximize their profits; (ii) given prices, aggregator firms' good demand, and output decisions maximize their profits; (iii) given prices, households' consumption bundles and labor supply decisions maximize utility while satisfying their budget constraints; (iv) goods and labor markets clear.

Proposition 1. The set of functions (ϑ, ρ) are an equilibrium if and only if the following set of

[1] $A \equiv (A_1, \cdots, A_N)'$, $A_\ell = \left(A^\ell_1, \cdots, A^\ell_N\right)'$, $A_x \equiv (A^x_1, \cdots, A^x_N)'$, $\underline{A}_\ell = \left(\underline{A}^\ell_1, \cdots, \underline{A}^\ell_N\right)'$, $\underline{A}_x = (\underline{A}^x_1, \cdots, \underline{A}^x_N)'$, $\underline{A}^\ell_i = \left(A^\ell_{i1}, \cdots, A^\ell_{iH}\right)'$, and $\underline{A}^x_i = (A^x_{i1}, \cdots, A^x_{iN})'$.

conditions are jointly satisfied

$$\frac{\partial C_h(e)/\partial C_{hj}(e)}{\partial C_h(e)/\partial C_{hi}(e)} = \mu_i(e)\left(\frac{y_i(e)}{y_{z_i}(e)}\right)^{1-\mu_i(e)} \frac{\partial y_{z_i}(e)}{\partial x_{z_i j}(e)} \quad \forall i,j \in \mathscr{N},\ \forall z_i \in [0,1], \tag{6}$$
$$\forall h \in \mathscr{H},\ \text{and}\ \forall e \in \mathscr{E}\ \text{such that}\ C_{hi}(e) > 0,\ C_{hj}(e) > 0,\ \text{and}\ x_{z_i j}(e) > 0,$$

$$-\frac{w_b(e)}{w_h(e)}\frac{U_{L_h}}{U_{C_{hi}}} = \mu_i(e)\left(\frac{y_i(e)}{y_{z_i}(e)}\right)^{1-\mu_i(e)} \frac{\partial y_i(e)}{\partial \ell_{ib}(e)} \quad \forall i \in \mathscr{N},\ \forall z_i \in [0,1], \tag{7}$$
$$\forall h,b \in \mathscr{H},\ \text{and}\ \forall e \in \mathscr{E}\ \text{such that}\ C_{hi}(e) > 0,\ \text{and}\ \ell_{ib}(e) > 0,$$

and resource constraints

$$\begin{aligned} y_i(e) &= \sum_{h\in\mathscr{H}} C_{hi}(e) + \sum_{j\in\mathscr{N}} \int x_{z_j i}(e)\, dz_j \quad \forall i \in \mathscr{N}, \\ \text{and} \qquad L_h(e) &= \sum_{i\in\mathscr{N}} \int \ell_{z_i h}(e)\, dz_i \quad \forall h \in \mathscr{H}. \end{aligned} \tag{8}$$

Proposition 1 identifies the set of equilibrium allocations. In equation (6), for a firm z_i, the markdown-adjusted marginal productivity from using the good from sector j as an intermediate input has to equate for every household the marginal rate of substitution between goods i and j.[2] In equation (7), for a firm z_i, the markdown-adjusted marginal productivity from using the labor supplied by households of type b, has the equate for every household a wage-adjusted marginal rate of substitution between the consumption of the good from sector i and their labor supply.

Notice that in the set of conditions captured by equation (7), the only thing that is necessary for the existence of an equilibrium relationship between the labor demand from firm z_i and the labor supply from households of type h, is the consumption from the latter of the goods supplied by sector i. Whenever firm z_i hires households of type b, and $b \neq h$, the differential wage adjustment w_b/w_h arises in these equilibrium conditions. This wage ratio is a point of difference with Bigio & La'O's (2020) representative-household economy, where they only consider the endogenous supply of one factor. For households of type h, a higher w_b/w_h is isomorphic to an increase in the marginal rate of substitution between consumption and labor supply, and in equilibrium, it requires a higher marginal productivity in firm z_i of the labor supplied by households of type b. Additionally, there is an isomorphism between distortionary markdown increases and positive productivity shocks in equations (6) and (7): both will increase the markdown-adjusted marginal productivities from labor and intermediate goods.

Furthermore, a relevant technicality is that Proposition 1 does not require final consumption in each sector. The usual assumption for this type of proof in the production network literature is that $\forall i \in \mathscr{N}$, the representative household's consumption technology satisfies $\partial C/\partial C_i > 0$ (see Bigio & La'O (2020) and La'O & Tahbaz-Salehi (2022)). The equivalent assumption under heterogeneous

[2] In the right-hand side of equation (6), notice that for fixed marginal rates of substitution, and under no variation in the relative production of firms within a sector (i.e., $y_i(e)/y_{z_i}(e)$ fixed $\forall z_i \in [0,1]$), an increase in $\mu_i(e)$ has a heterogeneous effect across firms in sector i. On the one hand, for firms with relatively low levels of production (more precisely $1 < \mu_i \, log\,(y_i/y_{z_i})$) the markdown increase forces a reduction in the demand for intermediate inputs. On the other hand, it increases the demand for intermediate inputs for the rest of the firms. Furthermore, notice that for fixed marginal rates of substitution and markdown $\mu_i(e)$, an increase in $y_i(e)/y_{z_i}(e)$ requires a reduction in the demand for intermediate inputs. The same analysis holds for equation (7).

households is that $\forall i \in \mathscr{N}$, there $\exists h \in \mathscr{H}$ such that $\partial C_h/\partial C_{hi} > 0$, but this assumption does not match the empirical input-output tables, where it is not uncommon to find sectors for which there is no direct registered final consumption, e.g., oil and gas extraction. The less stringent assumption that I make instead is that $\forall h \in \mathscr{H}$, there $\exists i \in \mathscr{N}$ such that for all the firms in this sector, it is possible to establish a direct or indirect demand of labor supplied by workers of type h.

To make the notation cleaner, the definitions and implementation of the model in the following sections are conditional in a specific aggregate state $e \in \mathscr{E}$, e.g., $\mu(e)$ is portrayed by μ. Finally, I will abstract from within sector firm heterogeneity by imposing the assumption of symmetry, i.e., $\ell_{ih} = \ell_{z_i h}$, and $x_{ij} = x_{z_i j}$ $\forall z_i \in [0,1]$, $\forall i, j \in \mathscr{N}$ and $\forall h \in \mathscr{H}$.[3] For this reason, I will refer indistinguishably to firm z_i as firm i.

3.2 Measures of Centrality

For the following measures, downstream or cost centrality refers to the propagation of costs from the supply of labor or intermediate inputs through supply chains, and upstream or revenue centrality refers to the propagation of money flows from the demand for labor and goods through payment chains.

3.2.1 Direct Centralities

The vectors $\omega_\ell \equiv \left(\omega_1^\ell, \cdots, \omega_N^\ell\right)'$ and $\omega_x \equiv \left(\omega_1^x, \cdots, \omega_N^x\right)'$ portray the direct cost centralities from composites. Its elements $\omega_i^\ell \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_i^\ell} = \frac{p_i^\ell L_i}{c_i(\vartheta,\rho)}$ and $\omega_i^x \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_i^x} = \frac{p_i^x X_i}{c_i(\vartheta,\rho)}$ capture respectively firm i's cost elasticities to p_i^ℓ and p_i^x, and in equilibrium they equal the cost share of the labor and intermediate input composites. For this reason, $\omega_i^\ell + \omega_i^x = 1$.

The matrices $\widetilde{\Omega}_\ell$ and $\widetilde{\Omega}_x$ depict direct labor and intermediate input downstream centralities. Its elements $\widetilde{\Omega}_{ih}^\ell \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log w_h} = \frac{w_h \ell_{ih}}{c_i(\vartheta,\rho)}$ and $\widetilde{\Omega}_{ij}^x \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_j} = \frac{p_j x_{ij}}{c_i(\vartheta,\rho)}$ capture respectively firm i's cost elasticities to w_h and p_j, and in equilibriym they equal the cost share of the labor supplied by households of type h and the good from firm j. The fact that $\sum_{h\in\mathscr{H}} \widetilde{\Omega}_{ih}^\ell + \sum_{j\in\mathscr{N}} \widetilde{\Omega}_{ij}^x = 1$ indicate that all costs come from labor or intermediate inputs.

Using these definitions, I obtain the labor network $\alpha \equiv diag(\omega_\ell)^{-1}\widetilde{\Omega}_\ell$ and the input-output network $\mathscr{W} \equiv diag(\omega_x)^{-1}\widetilde{\Omega}_x$, where $diag$ stands for the diagonal operator. Its elements $\alpha_{ih} \equiv \frac{\partial \log p_i^\ell L_i}{\partial \log w_h} = \frac{w_h \ell_{ih}}{p_i^\ell L_i}$ and $\omega_{ij} \equiv \frac{\partial \log p_i^x X_i}{\partial \log p_j} = \frac{p_j x_{ij}}{p_i^x X_i}$ capture respectively firm i's composite cost elasticities to w_h and p_j, and in equilibrium they equal the corresponding composites' cost share of the labor supplied by households of type h and the good from firm j. Notice that $\sum_{h\in\mathscr{H}} \alpha_{ih} = 1$ and $\sum_{j\in\mathscr{N}} \omega_{ij} = 1$.

From here, I can define the revenue-based upstream centrality matrices $\Omega_\ell \equiv diag(\mu)\widetilde{\Omega}_\ell$ and $\Omega_x \equiv diag(\mu)\widetilde{\Omega}_x$. Since $\mu_i \in (0,1]$ $\forall i \in \mathscr{N}$, $\widetilde{\Omega}_\ell \succcurlyeq \Omega_\ell$ and $\widetilde{\Omega}_x \succcurlyeq \Omega_x$, where $\succcurlyeq$ stands for elementwise greater than or equal to. Its elements $\Omega_{ih}^\ell \equiv \frac{\partial \log S_i}{\partial \log w_h} = \frac{w_h \ell_{ih}}{S_i}$ and $\Omega_{ij}^x \equiv \frac{\partial \log S_i}{\partial \log p_j} = \frac{p_j x_{ij}}{S_i}$ capture respectively the elasticities of firm i's sales to w_h and p_j, and in equilibrium they equal the sales share

[3]As a consequence $y_i = y_{z_i}$, $p_i = p_{z_i}$, $L_i = L_{z_i}$, and $X_i = X_{z_i}$.

of payments for labor supplied by workers of type h and goods from firm j. Additionally, $\Omega^{\pi}_{ih} = \frac{\kappa_{ih}\pi_i}{S_i}$ portrays the equilibrium sales share of firm i's profits rebated back to households of type h. The fact that $\sum_{h\in\mathscr{H}} \Omega^{\ell}_{ih} + \sum_{j\in\mathscr{N}} \Omega^{x}_{ij} + \sum_{b\in\mathscr{H}} \Omega^{\pi}_{ib} = 1$ indicate that all revenue generated by firm i ends as payments for labor, intermediate inputs, or dividends.

Finally, for households, the consumption network $\beta = (\beta_1, \cdots, \beta_H)'$ contains the vectors $\beta_h \equiv (\beta_{h1}, \cdots, \beta_{hN})'$. Its element $\beta_{hi} \equiv \frac{\partial \log E_h}{\partial \log p_i} = \frac{p_i C_{hi}}{E_h}$ captures the expenditure elasticity for households of type h to p_i, and in equilibrium they equal the expenditure share on the good supplied by firm i. For this reason $\sum_{i\in\mathscr{N}} \beta_{hi} = 1$.

3.2.2 Network Adjusted Centralities

The firm-to-firm downstream centrality matrix or cost-based Leontief inverse matrix is given by $\widetilde{\Psi}_x \equiv \left(I - \widetilde{\Omega}_x\right)^{-1} \equiv \sum_{q=0}^{\infty} \widetilde{\Omega}^{q}_x$. Its element $\widetilde{\psi}^{x}_{ij}$ captures the centrality of intermediate inputs supplied by firm j on the costs of firm i. Similarly, I define the firm-to-firm upstream centrality matrix or revenue-based Leontief inverse matrix $\Psi_x \equiv (I - \Omega_x)^{-1} \equiv \sum_{q=0}^{\infty} \Omega^{q}_x$, where its element ψ^{x}_{ij} represents the revenue share from firm i that through the payment of intermediate input reaches sales of firm j.

The firm-to-consumer downstream centrality matrix is given by $\widetilde{\mathscr{B}} \equiv \beta\, \widetilde{\Psi}_x$. Its element $\widetilde{\mathscr{B}}_{hi} = \sum_{j\in\mathscr{N}} \beta_j\, \widetilde{\psi}^{x}_{ji}$ captures all direct or indirect paths through which the costs of firm i can reach the expenditure for households of type h. The cost-based sales Domar weight $\widetilde{\lambda}_i = \sum_{h\in\mathscr{H}} \chi_h\, \widetilde{\mathscr{B}}_{hi}$ stands for the average firm-to-consumer centrality from sector i, where $\chi_h = E_h/GDP$ represents the absorption share for households of type h. Likewise, I define the consumer-to-firm upstream centrality matrix $\mathscr{B} \equiv \beta\, \Psi_x$, where its element $\mathscr{B}_{hi} = \sum_{j\in\mathscr{N}} \beta_j\, \psi^{x}_{ji}$ represents the share of expenditure from households of type h that through the payment chain reaches the revenue of firm i. The revenue-based sales Domar weight $\lambda_i = \sum_{h\in\mathscr{H}} \chi_h\, \mathscr{B}_{hi} = S_i/GDP$ stands for the average consumer-to-firm centrality towards sector i, and in equilibrium it coincides with the ratio of sales to GDP. These definitions generalize the supplier centrality vector from Baqaee (2018), or the influence vector from Acemoglu et al. (2012), to an environment with heterogeneous households and distortions.

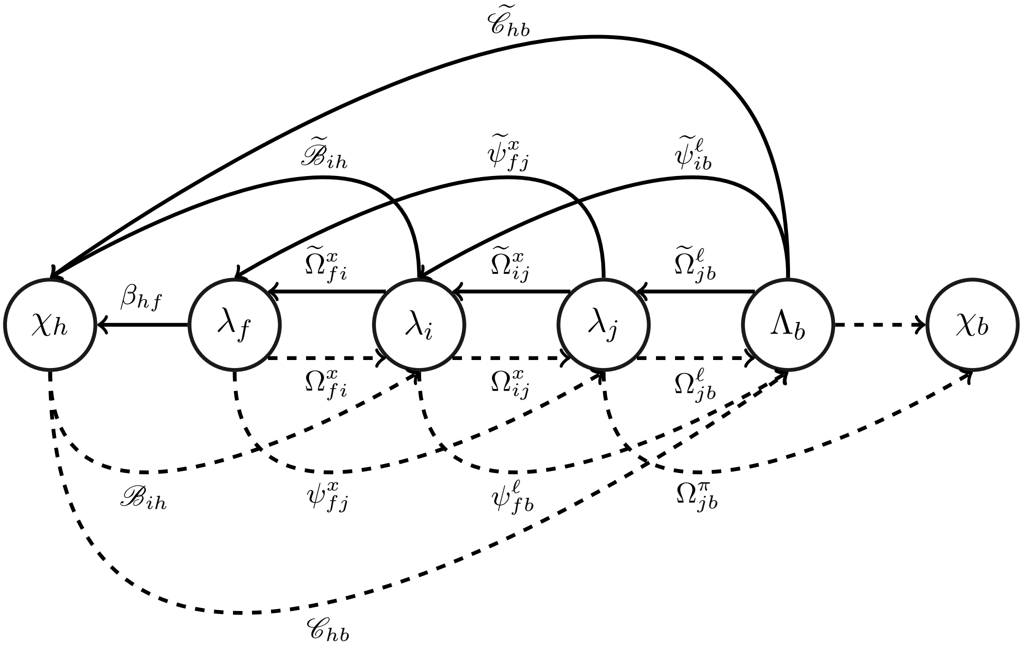
The worker-to-firm downstream centrality matrix is given by $\widetilde{\Psi}_\ell \equiv \widetilde{\Psi}_x\, \widetilde{\Omega}_\ell$. Given that $\sum_{h\in\mathscr{H}} \widetilde{\psi}^{\ell}_{ih} = 1$, all costs for a firm can be traced back through the production network to some original labor cost. As a consequence, $\widetilde{\psi}^{\ell}_{ih}$ is the value-added share by workers of type h on the production process of firm i. In the same way, I define the firm-to-worker upstream centrality matrix $\Psi_\ell \equiv \Psi_x\, \Omega_\ell$, where the element ψ^{ℓ}_{ih} represents the revenue share from firm i that reaches labor income for workers of type h.

The worker-to-consumer downstream centrality matrix is given by $\widetilde{\mathscr{C}} \equiv \beta\, \widetilde{\Psi}_\ell$. Given that $\sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb} = 1$, its element $\widetilde{\mathscr{C}}_{hb}$ represents the value-added share for households of type h attributed to workers of type b. The cost-based factor Domar weight $\widetilde{\Lambda}_h = \sum_{b\in\mathscr{H}} \chi_b \widetilde{\mathscr{C}}_{bh}$ stands for the average worker-to-consumer centrality from workers of type h. Consequently, $\widetilde{\Lambda}_h$ is the share of aggregate value-added by their labor. All the costs from this economy originate in labor costs, and for this reason, $\sum_{h\in\mathscr{H}} \widetilde{\Lambda}_h = 1$. Similarly, the consumer-to-worker upstream centrality matrix is given by $\mathscr{C} \equiv \beta\, \Psi_\ell$, where its element $\mathscr{C}_{hb}$ portrays the share of consumption expenditure from households of type h that reaches labor income for workers of type b. The revenue-based factor Domar weight $\Lambda_h = \sum_{b \in \mathscr{H}} \chi_b \mathscr{C}_{bh} = J_h/GDP$ stands for the average consumer-to-worker centrality towards workers of type h. In equilibrium Λ_h coincides with the ratio of labor income to GDP.

Cost-based centralities are greater than or equal to revenue-based centralities, i.e., $\widetilde{\Psi}_x \succcurlyeq \Psi_x$, $\widetilde{\mathscr{B}} \succcurlyeq \mathscr{B}$, $\widetilde{\Psi}_\ell \succcurlyeq \Psi_\ell$, $\widetilde{\mathscr{C}} \succcurlyeq \mathscr{C}$, $\widetilde{\lambda} \succcurlyeq \lambda$, and $\widetilde{\Lambda} \succcurlyeq \Lambda$. For this reason, for workers of type h, $\delta_h = \widetilde{\Lambda}_h/\Lambda_h \geq 1$ is a measure of distortion centrality that captures how undervalued a worker is in the market. When workers supply their labor to sectors that operate in heavily distorted supply chains, their distortion centrality will be high, and a higher share of their value-added will reach households' income via rebated distortions. Additionally, for households of type h and firm i, I will respectively use $\mathscr{C}_h = \sum_{b \in \mathscr{H}} \mathscr{C}_{hb}$ and $\psi_i^\ell = \sum_{h \in \mathscr{H}} \psi_{ih}^\ell$ to capture their payment centrality, i.e., the share of their expenditure that reaches households' income via labor income. Notice that the cost-based equivalent for $\mathscr{C}_h$ and ψ_i^ℓ are equal to one, which implies that these measures will shrink as the influence from distortions rises.

Finally, in equilibrium, the absorption shares are connected to the revenue-based Domar weights via the following relationship $\chi_h = \Lambda_h + \sum_{i \in \mathscr{N}} \Omega_{ih}^\pi \lambda_i$, and by definition $\sum_{h \in \mathscr{H}} \chi_h = 1$.

Figure 1: Measures of Centrality



Notes: Continuous and dashed arrows represent the cost-based and revenue-based centrality measures, respectively.

3.2.3 A Diagrammatic Recap

Figure 1 illustrates the centrality measures for firms $f, i, j \in \mathscr{N}$ and households $h, b \in \mathscr{H}$. Household b supplies labor to firm j, firm j supplies intermediate inputs to firm i, firm i supplies intermediate inputs to firm f, and firm f supplies goods to household h. Notice that this is not an exhaustive list of all the cost-based and revenue-based measures of centrality for this economy; for example, it ignores $\widetilde{\mathscr{C}}_{bh}$ and $\mathscr{C}_{bh}$, among others.

3.2.4 Networks as a Markov Chain

Alternatively, the transition matrix that represents the downstream probabilities of cost propagation between agents (firms and households) in a Markov chain, and its corresponding generalized downstream Leontief inverse matrix that gathers the effects from all of the previous cost-based centrality matrices are given by

$$\widetilde{\Omega} = \begin{pmatrix} \widetilde{\Omega}_x & \widetilde{\Omega}_\ell \\ \beta & 0 \end{pmatrix}, \qquad \left(I - \widetilde{\Omega}\right)^{-1} = \begin{pmatrix} \widetilde{\Psi}_x + \widetilde{\Psi}_\ell \left(I - \widetilde{\mathscr{C}}\right)^{-1} \widetilde{\mathscr{B}} & \widetilde{\Psi}_\ell \left(I - \widetilde{\mathscr{C}}\right)^{-1} \\ \left(I - \widetilde{\mathscr{C}}\right)^{-1} \widetilde{\mathscr{B}} & \left(I - \widetilde{\mathscr{C}}\right)^{-1} \end{pmatrix}.$$

Instead of using the absorbing Markov chain, I work independently with the substochastic matrices.[4] In this sense, my notation is closer to the models from Hulten (1978), Long & Plosser (1983), Acemoglu et al. (2012, 2016), and Bigio & La'O (2020), with the added complexity of accounting for consumption and income heterogeneity at the household level. My decision to operate with substochastic matrics differs from Baqaee & Farhi (2019a,b, 2020, 2022), where the Markov transition matrix is the production network, and its Leontief inverse lumps together all of the measures of centrality previously introduced. The segmentation of the production network in its different components allows me to analytically separate the different channels for the propagation of shocks through the economic network and introduce bilateral measures for each firm or household's centrality on every other firm or household across the economy.

3.3 Information Theory

A discrete random variable $\mathcal{Q}$ with F mutually exclusive events is distributed according to the probability vector $q = [q_1, \cdots, q_F]'$. The natural units of information carried by an event f are given by $I\left(f|\mathcal{Q}\right) = -\,log\, q_f$.[5] Shannon's (1948) entropy captures the average amount of information conveyed by a random draw, or similarly the expected surprise from observing an event,[6] and is given by $H\left(q\right) = \sum_{f=1}^{F} q_f\, I\left(f|\mathcal{Q}\right) = -\sum_{f=1}^{F} q_f\, log\, q_f$. The excess surprise from using the distribution $\widetilde{q}$ instead of the true distribution q is given by the Kullback-Leibler (KL) divergence or relative entropy $\mathscr{K}\left(q|\widetilde{q}\right) = -\sum_{f=1}^{F} q_f\, log\, \left(\widetilde{q}_f/q_f\right)$. From Gibbs's inequality $\mathscr{K}\left(q|\widetilde{q}\right) \geq 0$, which captures the idea that using an incorrect probability model $\widetilde{q}$ will introduce a positive bias in the measure of average expected information conveyed by a random draw. This excess surprise measures the statistical distance between the two distributions q and $\widetilde{q}$. However, the KL divergence is not a metric, as it does not satisfy the properties of symmetry and triangle inequality.

[4] The upstream probabilities of money flow between agents are portrayed by the Markov chain $\Omega = \left(\begin{smallmatrix} \Omega_x & \Omega_\ell + \Omega_\pi \\ \beta & 0 \end{smallmatrix}\right)$.

[5] This function satisfied the two properties. First, it is decreasing, i.e., $q_a < q_b$ implies that $I\left(a|\mathcal{Q}\right) > I\left(b|\mathcal{Q}\right)$. Second, it is additive, i.e., $I\left(a\, b|\mathcal{Q}\right) = I\left(a|\mathcal{Q}\right) + I\left(b|\mathcal{Q}\right)$. Monotonicity captures the idea that less probable events convey more information, and additivity means that combined information is the sum of separate information.

[6] In information theory, maximum entropy is equivalent to maximal surprise under current knowledge. For the case of distributions with no prior information, the uniform distribution maximizes the entropy.

The first-order variation on the relative entropy when the distribution $\widetilde{q}$ changes are given by

$$d\,\mathscr{K}\left(q|\widetilde{q}\right) = -\sum_{f=1}^{F} q_f \, d\log \widetilde{q}_f.$$

When $q = \widetilde{q}$ this implies that $d\,\mathscr{K}\left(q|\widetilde{q}\right) = 0$, which reflects that the average expected excess information from changing the model distribution $\widetilde{q}$ around the true distribution does not add any excess surprise up to a first-order. In other words, the information conveyed by $\widetilde{q}$ satisfies an envelope condition around q.[7]

4 Aggregate and Distributional Accounting

In this section, I derive the nonparametric ex-post sufficient statistics necessary to characterize the first-order variations in prices, labor wedges, labor income shares, aggregate TFP, and household-level terms of trade. I call these measures ex-post because they assume that the necessary variations are observable and do not depend on underlying model primitives. First, I present the price variation in response to exogenous shocks and show that these effects propagate downstream through the cost of intermediate and final goods. Second, I characterize the first-order variation for the decentralized labor wedges and the labor income distribution. Third, I decompose the first-order variation for aggregate TFP and the household-level positional terms of trade (PTT) and establish a connection with the decentralized labor wedges that allow me to decompose the aggregate and distributional effects from the endogenous reallocation of labor across firms into variations of (i) exogenous distortions, (ii) endogenous variations in the consumption distribution, and (iii) endogenous demand recomposition from firms and households on goods and labor.

4.1 Price Variation

Proposition 2 captures the network-adjusted response of prices to supply shocks. These shocks propagate downstream through the costs of intermediate inputs and final goods, and the cost-based firm-to-firm and firm-to-consumer centrality measures capture their magnitude.

Proposition 2. The change in sector i's prices and household h's price index in response to productivity, markdown, and factor cost shocks are, to a first-order,

$$\begin{aligned}
d\log p_i^{\ell} &= -d\log A_i^{\ell} - \sum_{h\in\mathscr{H}} \alpha_{ih}\left(d\log A_{ih}^{\ell} - d\log w_h\right),\\
d\log p_i^{x} &= -d\log A_i^{x} - \sum_{j\in\mathscr{N}} \omega_{ij}\ \left(d\log A_{ij}^{x} - d\log p_j\right),\\
d\log p_i &= -\sum_{j\in\mathscr{N}} \widetilde{\psi}_{ij}^{x}\left(d\log \mathscr{A}_j + d\log \mu_j\right) + \sum_{h\in\mathscr{H}} \widetilde{\psi}_{ih}^{\ell}\, d\log w_h,\\
d\log p_h^{c} &= -\sum_{i\in\mathscr{N}} \widetilde{\mathscr{B}}_{hi}\left(d\log \mathscr{A}_i + d\log \mu_i\right) + \sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb}\, d\log w_b,
\end{aligned}$$

[7] For this envelope condition it is required that when $\widetilde{q}$ changes to $\widetilde{q}^{*}$, the new distribution satisfies $\mathbb{1}_F'\,\widetilde{q}^{*} = 1$.

where $d\log \mathscr{A}_i = d\log A_i + \omega_i^\ell \, d\log A_i^\ell + \omega_i^x \, d\log A_i^x + \sum_{h\in\mathscr{H}} \widetilde{\Omega}_{ih}^\ell \, d\log A_{ih}^\ell + \sum_{j\in\mathscr{N}} \widetilde{\Omega}_{ij}^x \, d\log A_{ij}^x$.

First, notice that non-Hicks neutral productivity shocks directly influence firms' composite bundle prices. Second, firm i's compound measure of productivity $d\log \mathscr{A}_i$ incorporates Hicks-neutral, labor-specific, and input-specific augmenting productivity shocks, and its effect on prices across all firms and households is isomorphic to an increase in the markdown for firm i. Third, labor cost shocks directly affect the primary factor bundle price that propagates through the supply of intermediate inputs to other firms and finally reaches consumption bundle prices.

4.2 Labor Wedges and the Income Distribution

Theorem 1 portrays the equilibrium characterization of the households' labor supply and the endogenous variation of the labor income distribution. This theorem represents an extension of the labor wedge decompositions from Bigio & La'O (2020) to an environment with heterogeneous households and a distorted equilibrium.

Theorem 1. In equilibrium, the labor supply from households of type h satisfies

$$\frac{U_{L_h}}{U_{C_h}} + \Gamma_h \frac{C_h}{L_h} = 0 \qquad \text{with} \qquad \Gamma_h = \frac{\Lambda_h}{\chi_h}. \tag{9}$$

The change of Λ_h in response to variations in the consumption distribution and expenditure-to-labor centralities is, to a first-order,

$$d\Lambda_h = \overbrace{\sum_{b\in\mathscr{H}} \mathscr{C}_{bh} \, d\chi_b}^{\substack{\textit{Distributive} \\ \textit{Income}_h}} + \overbrace{\sum_{b\in\mathscr{H}} \chi_b \, d\mathscr{C}_{bh}}^{\substack{\textit{Income} \\ \textit{Centrality}_h}}, \tag{10}$$

$$\substack{\textit{Income} \\ \textit{Centrality}_h} = \overbrace{\sum_{i\in\mathscr{N}} \psi_{ih}^\ell \, \lambda_i \, d\log \mu_i}^{\substack{\textit{Competitive} \\ \textit{Income}_h}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i \, \lambda_i \, d\widetilde{\Omega}_{ih}^\ell}^{\substack{\textit{Labor Demand} \\ \textit{Recomposition}_h}} + \overbrace{\sum_{i\in\mathscr{N}} \psi_{ih}^\ell \sum_{b\in\mathscr{H}} \chi_b \, d\beta_{bi}}^{\substack{\textit{Final Demand} \\ \textit{Recomposition}_h}} + \overbrace{\sum_{i\in\mathscr{N}} \psi_{ih}^\ell \sum_{j\in\mathscr{N}} \mu_j \, \lambda_j \, d\widetilde{\Omega}_{ji}^x}^{\substack{\textit{Intermediate Demand} \\ \textit{Recomposition}_h}}. \tag{11}$$

The decentralized labor wedge Γ_h from equation (9) relates the marginal rate of substitution between consumption and the labor supply with the household's average labor rate of transformation on consumption C_h/L_h. In equilibrium, the decentralized labor wedge equals the share of labor income to consumption expenditure, i.e., J_h/E_h. This wedge is decentralized because each household independently chooses it, and it differs from the centralized labor wedge that in Section 5 is chosen by the constrained social planer. These wedges measure how the whole set of distortions alters labor supply decisions. Consequently, the labor income and consumption distributions will differ from those in an efficient economy.

Equation (10) divides the first-order variation of the labor income share into changes in the consumption distribution and changes in the consumer-to-worker centralities. First, *distributive income* captures how the revenue share for workers of type h increases as the absorption share grows for households whose expenditure has a relatively high upstream centrality on their labor income. For example, Λ_h will increase in response to an endogenous redistribution of expenditure from type q to type b households if $\mathscr{C}_{bh} > \mathscr{C}_{qh}$. Second, *income centrality* portrays how the revenue share for workers of type h increases as the consumer-to-worker centralities on their labor income rise.[8]

The income centrality variation collects four different effects. First, *competitive income* tells us that lower profit margins in a sector will increase the labor income share for workers of type h in a magnitude proportional to the sector's size and the sector's centrality on the labor income of these workers. Second, *labor demand recomposition* portrays the influence on the labor income share from higher labor demand; the magnitude of this effect is more prominent for big and relatively undistorted sectors. Finally, *final* and *intermediate demand recomposition* characterize the income distribution effects from households' and firms' expenditure reallocation, respectively. These two channels convey that the labor revenue share for workers of type h will increase as the household's consumption patterns or the firms' cost structure shift towards sectors with a high firm-to-worker centrality on their labor income. For example, Λ_h rises in response to a cost reallocation from sector j to sector i, by any firm or household, if $\psi^{\ell}_{ih} > \psi^{\ell}_{jh}$.

4.3 Aggregate Accounting

Theorem 2 characterizes aggregate real output Y in equilibrium and its first-order variation around the equilibrium.

Theorem 2. In equilibrium, real GDP satisfies

$$Y = Q_Y\left(\{C_h\}_{h\in\mathscr{H}}\right) = TFP\ F\left(\{L_h\}_{h\in\mathscr{H}}\right), \tag{12}$$

where TFP captures total factor productivity, and Q_Y and F satisfy $d\log Q_Y/d\log C_h = \chi_h$ and $d\log F/d\log L_h = \widetilde{\Lambda}_h$.

The change in Y and TFP are, to a first-order

$$d\log Y = d\log TFP + \sum_{h\in\mathscr{H}} \widetilde{\Lambda}_h\, d\log L_h, \tag{13}$$

$$d\log TFP = \mathit{Technology} + \mathit{Competitiveness} - \mathit{Misallocation}, \tag{14}$$

where

$$\mathit{Technology} = \sum_{i\in\mathscr{N}} \widetilde{\lambda}_i\ d\log \mathscr{A}_i, \qquad \mathit{Competitiveness} = \sum_{i\in\mathscr{N}} \widetilde{\lambda}_i\ d\log \mu_i,$$

[8] The following variations in relative entropies capture these two insights: $\Lambda_h\, d\mathscr{K}\left(\Lambda_h^{-1}\mathscr{C}'\chi|\chi\right) + \mathit{Distributive\ Income}_h = 0$ and $\Lambda_h\, d\mathscr{K}\left(\Lambda_h^{-1}\mathscr{C}'\chi|\mathscr{C}_{\uparrow h}\right) + \mathit{Income\ Centrality}_h = 0$ with $\mathscr{C}_{\uparrow h} = (\mathscr{C}_{1h}, \cdots, \mathscr{C}_{Hh})'$. This underscores how Λ_h increases as the statistical distances of $\Lambda_h^{-1}\mathscr{C}'\chi$ relative to the distributions χ and $\mathscr{C}_{\uparrow h}$ fall.

and *Misallocation* has the following four equivalent definitions

$$1. \quad \overbrace{\sum_{h\in\mathscr{H}} \delta_h \, d\,\Lambda_h}^{\textit{Entropic TT}}, \quad 2. \quad \overbrace{\sum_{h\in\mathscr{H}} (\delta_h - 1)\,\Lambda_h \, d\,log\,J_h}^{\textit{Labor TT}} - \overbrace{\sum_{i\in\mathscr{N}} \lambda_i \left((1-\mu_i)\,d\,log\,S_i - d\,\mu_i\right)}^{\textit{Corporate Income}},$$

$$3. \quad \overbrace{\sum_{h\in\mathscr{H}} \mathcal{Z}^c_h \, d\,\chi_h}^{\textit{Distributive TT}} + \overbrace{\sum_{h\in\mathscr{H}} \chi_h \sum_{b\in\mathscr{H}} \delta_b \, d\,\mathscr{C}_{hb}}^{\textit{Centrality TT}},$$

$$4. \quad \sum_{h\in\mathscr{H}} \mathcal{Z}^c_h \, d\,\chi_h + \overbrace{\sum_{i\in\mathscr{N}} \lambda_i \, \mathcal{Z}^\ell_i \, d\,log\,\mu_i}^{\textit{Competitive TT}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\,\lambda_i \sum_{h\in\mathscr{H}} \delta_h \, d\,\widetilde{\Omega}^\ell_{ih}}^{\textit{Labor Demand TT}}$$
$$+ \overbrace{\sum_{h\in\mathscr{H}} \chi_h \sum_{i\in\mathscr{N}} \mathcal{Z}^\ell_i \, d\,\beta_{hi}}^{\textit{Final Demand TT}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\,\lambda_i \sum_{j\in\mathscr{N}} \mathcal{Z}^\ell_j \, d\,\widetilde{\Omega}^x_{ij}}^{\textit{Intermediate Demand TT}},$$

with $\mathcal{Z}^c_h = \sum_{b\in\mathscr{H}} \mathscr{C}_{hb}\,\delta_b$ and $\mathcal{Z}^\ell_i = \sum_{h\in\mathscr{H}} \psi^\ell_{ih}\,\delta_h$.

From equation (12), real GDP in equilibrium has two representations. First, as a CRS function Q_Y that aggregates consumption across households, with elasticities equal to the absorption shares χ. Second, as the product of TFP, and a CRS function F that aggregates the labor supplies with elasticities equal to the cost-based factor Domar weights $\widetilde{\Lambda}$.

Equation (13) segments the output response into a TFP and a factoral component. Equation (14) divides the first-order variation of TFP into three components. First, *technology* captures the direct effect of changes in productivity under a fixed allocation of resources. Second, *competitiveness* portrays the reallocation effects from distortions in the absence of income distribution variations. These two components tell us that in the absence of distributional reallocation, the effects on TFP of productivity and markdown changes in sector i are proportional to its cost-based sales Domar weight $\widetilde{\lambda}_i$. Third, *misallocation* portrays the aggregate efficiency losses from reallocating labor across firms in response to changes in the income distribution. The last two components capture the effects on TFP from the reallocation of labor across firms arising from exogenous variations in distortions and endogenous changes in the factoral revenue shares. For this reason, Baqaee & Farhi (2020) label them as the variation in *allocative efficiency*. Here, I refrain from using the *efficiency* tag as with endogenous labor, an increase in real GDP is not necessarily welfare-improving. Finally, Hulten's (1978) theorem holds in the absence of distortions (i.e., $d\,log\,TFP = \lambda'\,d\,log\,\mathcal{A}$), which implies that variations in the labor income distribution and distortions around the first-best equilibrium generate reallocation of resources that are allocative-neutral on output (i.e., *competitiveness* = *misallocation*).

Theorem 2 also contains four equivalent definitions for the *misallocation* component, and each one gives us a different intuition about the effects on TFP from changes in the income distribution. All four definitions capture the idea that aggregate labor misallocation rises as workers move away from firms that operate in heavily distorted supply chains. Profits dilute revenue as consumption expenditure flows through the payment chains in a production network. For this reason, upstream firms receive less revenue and demand less labor than in an equivalent economy without distortions.

In the first definition, the *entropic terms of trade* capture a reduction in the statistical distance between the value-added and the labor income distributions as measured by $-d\mathscr{K}(\widetilde{\Lambda}|\Lambda)$. Labor misallocation improves as labor income endogenously shifts from high to low distortion centrality workers, or what is equivalent when the distance between the value-added and the labor income distribution rises. This result coincides with the main theorem from Baqaee & Farhi (2020).

The second definition segments *misallocation* into changes in the *labor terms of trade* and *corporate income*, which are in terms of nominal variations. The *labor terms of trade* tell us that under fixed dividends, misallocation worsens when labor income increases, more so for highly undervalued workers. The magnitude of the aggregate labor misallocation arising in response to increases in the labor income for workers of type h is proportional to their revenue and their distortions centrality. *Corporate income* captures the impact of additional dividends arising from higher sales or distortions. Counterintuitively, an increase in aggregate corporate income reduces labor misallocation due to the role of firms' distortions in the model. As dividends increase due to higher sales or lower markdowns, the paths through which consumption expenditure reaches income are less distorted, as money flows are no longer subject to input payment distortions. Hence, profits are the source of distortion in the equilibrium of this economy, but as profits increase, money flows are extracted "earlier" from the distorted payment chains, and aggregate labor misallocation falls. This second definition shows that any measure of labor misallocation that depends exclusively on nominal variation must consider the changes in workers' and shareholders' income.

Additionally, the second definition clarifies an improvement relative to Baqaee & Farhi (2020, 2022), as in my derivation, any normalization is unnecessary. A normalization only becomes necessary to discipline the variations of endogenous variables under a specific parametric structure. More precisely, using the nominal GDP as the numeraire not only creates uncertainty about the real fundamental unit of account but also implies that the first and second definitions of *misallocation* are equivalent and, as a consequence, that an independent nominal measure of aggregate labor misallocation is unnecessary. Furthermore, as I will show in Section 7, with an endogenous labor supply, the nominal GDP normalization used by Baqaee & Farhi (2020, 2022) is non-neutral for growth whenever the substitution and income effects on the labor supply are asymmetric.

The last two definitions require the labor revenue elasticities from Theorem 1. The third definition splits the *misallocation* effect into variations in the consumption distribution and consumer-to-worker centralities. First, the *distributive terms of trade* imply that labor misallocation worsens as income shifts towards households with a consumption bundle that depends on relatively undistorted supply chains, and the firms in this supply chain compete for workers with high distortion centralities. For household h, $\mathcal{Z}_h^c$ is their expenditure's average weighted distortion centrality with weights given by the vector of consumer-to-worker centralities $\mathscr{C}_{\uparrow h} = (\mathscr{C}_{1h}, \cdots, \mathscr{C}_{Hh})'$. For example, TFP will improve in response to an endogenous redistribution of expenditure from type h to type b households if $\mathcal{Z}_h^c > \mathcal{Z}_b^c$. Second, the *centrality terms of trade* indicate that labor misallocation worsens as the consumer-to-worker centralities from a household increase, and the magnitude of this effect is more prominent when it takes place on workers with high distortion centralities.

Corollary 1. Distributive Neutrality. Endogenous changes in the distribution of consumption expenditure are neutral on TFP if and only if the expenditure average distortion centrality is symmetric across all households, i.e., $\mathcal{Z}_h^c = \mathcal{Z}^c \; \forall h \in \mathscr{H}$. This condition nests the following economic structures: (i) undistorted economy, i.e., $\mu_i = 1 \; \forall i \in \mathscr{N}$; (ii) symmetric consumption bundles, i.e., $\beta_{hf} = \beta_f$ $\forall h \in \mathscr{H}$; (iii) no intermediate inputs and symmetric distortions across firms, i.e., $\omega_i^x = 0$ and $\mu_i = \mu$ $\forall i \in \mathscr{N}$; and (iv) no intermediate inputs and sectoral specific labor supply, i.e., $\omega_i^\ell = \alpha_{ii} = 1 \; \forall i \in \mathscr{N}$.

Corollary 1 establishes the condition under which an aggregate production function can disregard changes in the consumption distribution without introducing first-order biases on TFP. The symmetry in $\mathcal{Z}_h^c$'s across households nests a manifold of common economic structures, including environments where the first welfare theorem holds, economies where all households share the same consumption bundle, and models without intermediate inputs in which either there is a common distortion or labor supply is sector specific. These cases prove that consumption bundle heterogeneity and, as a consequence, aggregate non-homotheticity are necessary but not sufficient for the variations in the consumption expenditure distribution to influence TFP.

The last definition separates the *centrality terms of trade* into four different effects that capture endogenous demand recomposition. First, the *competitive terms of trade* represents how labor misallocation rises as distortions fall. The magnitude of this effect is more prominent for large sectors that operate in a relatively undistorted supply chain, and the firms in this supply chain compete for high distortion centrality workers. For firm i, $\mathcal{Z}_i^\ell$ is their revenue's average weighted distortion centrality. Second, the *labor demand terms of trade* portray how labor misallocation increases as the demand for workers with high distortion centrality rises, mainly by large and relatively undistorted sectors. Finally, the *final demand* and *intermediate demand terms of trade* represent how labor misallocation worsens with an increase in the demand for final or intermediate goods produced by firms whose revenue faces a high average distortion centrality. Corollary 2 establishes the conditions under which changes in aggregate demand, demand from a household, and demand from a firm are neutral on TFP.

Corollary 2. Demand Neutrality.

1. Changes in the demand structure are neutral on TFP around the first-best equilibrium.
2. Changes in the final demand from household h are neutral on TFP if and only if all of the firms from which it demands final goods share the same average weighted distortion centrality, i.e., $\mathcal{Z}_i^\ell = \mathcal{Z}^\ell \; \forall i \in \mathscr{N} : \beta_{hi} > 0$.
3. Changes in the demand from firm i are neutral on TFP if: (a) the firm demands no intermediate inputs and all of its workers have a symmetric distortion centrality, i.e., $\delta_h = \delta \; \forall h \in \mathscr{H} : \widetilde{\Omega}_{ih}^\ell > 0$; (b) the firm demands no labor and all of its intermediate input suppliers share the same average weighted distortion centrality, i.e., $\mathcal{Z}_j^\ell = \mathcal{Z}^\ell \; \forall j \in \mathscr{N} : \widetilde{\Omega}_{ij}^x > 0$; or (c) the distortion centrality from all of its workers and the average weighted distortion centrality from all its suppliers are symmetric, i.e., $\delta_h = \mathcal{Z}_j^\ell \; \forall h \in \mathscr{H} : \widetilde{\Omega}_{ih}^\ell > 0$ and $\forall j \in \mathscr{N} : \widetilde{\Omega}_{ij}^x > 0$.

Corollary 3 segments the *labor terms of trade* into three effects: (i) real income, (ii) consumer price index (CPI), and (iii) real exchange rate (RER). First, the *real labor income effect* captures the aggregate net exposure to real labor income variations. As in its nominal counterpart, the change in household h's real income is proportional to their distortion centrality and labor income share. Consequently, variations on the real labor income for households that face no distortions in their revenue are neutral on TFP. Second, the *CPI effect* shows how misallocation rises more in response to increases in the bundle price for households whose expenditure is heavily dependent on corporate income than for households whose expenditure depends mainly on labor income. Finally, the *RER effect* illustrates that for households of type h, a depreciation in their average bilateral real exchange rate ε_h increases misallocation in a magnitude proportional to their absorption share.[9]

Corollary 3. The variation in the labor terms of trade is given by

$$Labor\ TT = \overbrace{\sum_{h\in\mathscr{H}} (\delta_h - 1)\,\Lambda_h\, d\log\mathcal{J}_h}^{\textit{Real Income effect}} + \overbrace{\sum_{h\in\mathscr{H}} (1-\Gamma_h)\,\chi_h\, d\log p^c_h}^{\textit{CPI effect}} + \overbrace{\sum_{h\in\mathscr{H}} \chi_h\, d\log\varepsilon_h}^{\textit{RER effect}},$$

where $\mathcal{J}_h = J_h/p^c_h$ stands for real income and $d\log\varepsilon_h = \sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb}\, d\log\varepsilon_{hb}$ is the average change in the bilateral real exchange rate for households of type h, with $\varepsilon_{hb} = p^c_b/p^c_h$.

4.4 Distributional Accounting

Theorem 3 characterizes household-level real consumption in equilibrium and its first-order variation. For this, I need to introduce the *positional terms of trade* (PTT) as an equilibrium object that captures the efficiency of the labor supply from all workers on the idiosyncratic real consumption bundle for a specific type of household. I use the term *positional* because it depends on the households' location in the production network, and it serves an analogous function to the TFP from Theorem 2, but for non-producing entities that only consume. $d\log TFP = \sum_{h\in\mathscr{H}} \chi_h\, d\log PTT_h$ captures the relationship between TFP and PTTs, which shows that TFP growth is the aggregation of idiosyncratic efficiency growth.

Theorem 3. In equilibrium, real consumption for households of type h satisfies

$$C_h = Q^c_h\left(\{C_{hi}\}_{i\in\mathscr{N}}\right) = PTT_h\ f_h\left(\{L_b\}_{b\in\mathscr{H}}\right), \tag{15}$$

where PTT_h captures the positional terms of trade, and f_h satisfies $d\log f_h/d\log L_b = \widetilde{\mathscr{C}}_{hb}$.

The change in C_h and PTT_h are, to a first-order

$$d\log C_h = d\log PTT_h + \sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb}\, d\log L_b, \tag{16}$$

$$d\log PTT_h = Technology_h + Competitiveness_h - Misallocation_h, \tag{17}$$

[9] An increase in ε_{hb} captures a depreciation of the bundle of type h relative to type b households.

where

$$\textit{Technology}_h = \sum_{i\in\mathcal{N}} \widetilde{\mathscr{B}}_{hi}\, d\log\mathcal{A}_i, \quad \textit{Competitiveness}_h = \sum_{i\in\mathcal{N}} \widetilde{\mathscr{B}}_{hi}\, d\log\mu_i,$$

and $\textit{Misallocation}_h$ has the following four equivalent definitions

$$1.\ \overbrace{\sum_{b\in\mathscr{H}} \delta_{b|h}\, d\,\Lambda_b}^{\textit{Entropic } TT_h} - d\log\chi_h, \quad 2.\ \overbrace{\sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb}\, d\log J_b - \Gamma_h\, d\log J_h}^{\textit{Labor } TT_h} - \overbrace{\sum_{i\in\mathcal{N}} \kappa_{ih}\frac{\lambda_i}{\chi_h}\left((1-\mu_i)\, d\log S_i - d\,\mu_i\right)}^{\textit{Corporate Income}_h},$$

$$3.\ \overbrace{\sum_{b\in\mathscr{H}} \mathcal{Z}^c_{b|h}\, d\,\chi_b}^{\textit{Distributive } TT_h} + \overbrace{\sum_{b\in\mathscr{H}} \chi_b \sum_{q\in\mathscr{H}} \delta_{q|h}\, d\log\mathscr{C}_{bq}}^{\textit{Centrality } TT_h} - d\log\chi_h,$$

$$4.\ \sum_{b\in\mathscr{H}} \mathcal{Z}^c_{b|h}\, d\,\chi_b + \overbrace{\sum_{i\in\mathcal{N}} \lambda_i\, \mathcal{Z}^\ell_{i|h}\, d\log\mu_i}^{\textit{Competitive } TT_h} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i\,\lambda_i \sum_{b\in\mathscr{H}} \delta_{b|h}\, d\,\widetilde{\Omega}^\ell_{ib}}^{\textit{Labor Demand } TT_h}$$
$$+ \overbrace{\sum_{b\in\mathscr{H}} \chi_b \sum_{i\in\mathcal{N}} \mathcal{Z}^\ell_{i|h}\, d\,\beta_{bi}}^{\textit{Final Demand } TT_h} + \overbrace{\sum_{i\in\mathcal{N}} \mu_i\,\lambda_i \sum_{j\in\mathcal{N}} \mathcal{Z}^\ell_{j|h}\, d\,\widetilde{\Omega}^x_{ij}}^{\textit{Intermediate Demand } TT_h} - d\log\chi_h,$$

with $\delta_{b|h} = \widetilde{\mathscr{C}}_{hb}/\Lambda_b$, $\mathcal{Z}^c_{b|h} = \sum_{q\in\mathscr{H}} \mathscr{C}_{bq}\,\delta_{q|h}$, and $\mathcal{Z}^\ell_{i|h} = \sum_{q\in\mathscr{H}} \psi^\ell_{iq}\,\delta_{q|h}$.

In equation (15), real consumption for households of type h has two representations. First, as a CRS function Q^c_h that aggregates final goods. Second, as the product of PTT_h, and a CRS function f_h that aggregates the labor supplies with elasticities equal to the labor-to-expenditure downstream centralities that reach household h's expenditure, i.e., $\widetilde{\mathscr{C}}_{\downarrow h} = \left(\widetilde{\mathscr{C}}_{h1}, \cdots, \widetilde{\mathscr{C}}_{hH}\right)'$. Equation (16) segments the household-level real consumption response into PTT and a factoral component. Equation (17) divides the first-order variation of the PTT for households of type h into three components. Just as in its aggregate counterpart from Theorem 2, for households of type h, *technology*$_h$ captures the direct effect of changes in productivity under a fixed allocation of resources, and *competitiveness*$_h$ portrays the direct effect from distortions. These two components tell us that in the absence of redistribution, the effects on PTT_h of productivity and markdown changes in sector i are proportional to the firm-to-consumer cost-based centrality $\widetilde{\mathscr{B}}_{hi}$. Third, *misallocation*$_h$ displays the endogenous distributional losses in response to changes in the income distribution. The relationship between aggregate *misallocation* and idiosyncratic *misallocation* is represented by $\textit{misallocation} = \sum_{h\in\mathscr{H}} \chi_h\, \textit{misallocation}_h$. From here, we can see that the allocative-neutrality from Hulten's (1978) theorem implies that idiosyncratic *misallocation* effects from productivity shocks are zero-sum around the first-best equilibrium.

Theorem 3 also contains four equivalent definitions for the idiosyncratic *misallocation* component. All four definitions capture the idea that *misallocation* is favorable for households when their income share rises, or when labor is reallocated in a more favorable manner for their consumption bundle. Equation (18) represents the total income share variation for households of type h, which captures that the absorption share increases as labor or corporate income rise.

$$d\,\chi_h = \Lambda_h\, d\,log\,\Lambda_h + \sum_{i\in\mathscr{N}} \kappa_{ih}\,\lambda_i\left((1-\mu_i)\left(d\,log\,\kappa_{ih} + d\,log\,\lambda_i\right) - \mu_i\, d\,log\,\mu_i\right). \tag{18}$$

First, the *entropic terms of trade* capture a reduction in the statistical distance between the idiosyncratic value-added and the labor income distributions as measured by $-d\,\mathscr{K}\big(\widetilde{\mathscr{C}}_{\downarrow h}|\Lambda\big)$. For households of type h, variations in the income distribution are less favorable when labor income shares increase for workers that, from h's perspective, are relatively undervalued. $\delta_{b|h}$ represents the distortion centrality for workers of type b conditional on the value-added distribution from households of type h. From the perspective of h, a worker is overvalued when $0 \leq \delta_{b|h} < 1$. For this reason, when the labor income endogenously shifts from type b to type q workers, the new allocation of labor makes households of type h worse off as long as $\delta_{b|h} < \delta_{q|h}$. The second definition portrays that endogenous distributional variations are favorable for a household when their labor or corporate income increase or the indirect labor costs from their consumption bundle fall.

The third definition segments the *entropic terms of trade* into changes in the consumption distribution and consumption-to-worker centralities. For households of type h, the *distributive terms of trade* imply that distributional reallocation of labor is favorable as income shifts towards households with an expenditure that, on average, reaches workers that, from h's perspective, are overvalued. For household b, $\mathscr{Z}^c_{b|h}$ is their expenditure's average weighted distortion centrality from the perspective of household h. Notice that the difference between $\mathscr{Z}^c_b$ and $\mathscr{Z}^c_{b|h}$ is that the former employs the aggregate value-added distribution $\widetilde{\Lambda}$, while the latter utilizes household h's value added distribution $\widetilde{\mathscr{C}}_{\downarrow h}$. For households of type h, the *centrality terms of trade* indicate that labor misallocation improves as the consumer-to-worker centralities from any other household fall on workers that produce some value-added in the consumption bundle of h.

The last definition segments the idiosyncratic *centrality terms of trade* into four different effects that capture endogenous demand recomposition. First, for households of type h, the *competitive terms of trade* illustrate that reallocation is favorable as profit margins rise in sectors with revenue that, on average, reaches workers that, from h's perspective, are overvalued. For sector i, $\mathscr{Z}^\ell_{i|h}$ is their revenue's average weighted distortion centrality from the perspective of household h. A sector with a high $\mathscr{Z}^\ell_{i|h}$ operates in a relatively undistorted supply chain, and the firms in this supply chains compete for workers that, from the perspective of h, are highly undervalued. As monopolistic power rises in this sector, their labor demand falls, and workers with high distortion centralities reallocate toward firms that operate in supply chains that are more distorted and essential for the consumption bundle of h. Second, the *labor demand terms of trade* portray how reallocation worsens for households of type h as labor demand rises for workers that, from h's perspective, have a high distortion centrality. Finally, the *final* and *intermediate demand terms of trade* represent how reallocation worsens for households of type h as the demand rises for goods produced by firms that, from h's perspective, face a high average weighted distortion centrality.

Corollary 4. The variation in the labor terms of trade for households of type h is given by

$$Labor\ TT_h = \overbrace{\sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb}\, d\log \mathcal{J}_b - \Gamma_h\, d\log \mathcal{J}_h}^{Real\ Income\ Effect_h} + \overbrace{(1-\Gamma_h)\, d\log p_h^c}^{CPI\ effect_h} + \overbrace{\sum_{b\in\mathscr{H}} \widetilde{\mathscr{C}}_{hb}\, d\log \varepsilon_{hb}}^{RER\ effect_h}.$$

Corollary 4 is the distributional equivalent of Corollary 3, and segments the idiosyncratic *labor terms of trade* for households of type h into three effects: (i) real income, (ii) CPI, and (iii) RER. First, as in its nominal counterpart, the change in household h's real income is proportional to the labor wedge Γ_h, and the variation in the real income for workers of type b is proportional to their value-added contribution $\widetilde{\mathscr{C}}_{hb}$. Second, the *CPI effect* shows how idiosyncratic reallocation worsens as the price of the consumption bundle rises, and the magnitude from this effect is proportional to the household h's share of corporate income. Finally, the *RER effect* illustrates that for households of type h, a depreciation of their consumption bundle relative to households of type b has an effect on the *labor terms of trade* that is proportional to $\widetilde{\mathscr{C}}_{hb}$.

5 Constrained Social Planner Economy

Assume the existence of an aggregate welfare function $W(Y,L)$ where Y and $L = F\left(\{L_h\}_{h\in\mathscr{H}}\right)$ are the same functions as in equation (12). A constrained social planner maximizes $W(Y,L)$ by choosing $Y, L, \left\{C_h, L_h, \{C_{hi}\}_{i\in\mathscr{N}}\right\}_{h\in\mathscr{H}}$, subject to

$$p_Y Y = \sum_{h\in\mathscr{H}} p_h^c C_h = \sum_{h\in\mathscr{H}}\sum_{i\in\mathscr{N}} p_i C_{hi} \leq wL + \Pi = \sum_{h\in\mathscr{H}} (w_h L_h + \Pi_h),$$

taking prices, wages, and profits as given. The constraints from this social planner come from his incapacity to develop policies that tackle distortions directly, e.g., flexible Pigouvian taxes that subsidize heavily distorted firms by taxing relatively undistorted sectors. In this sense, the constrained social planner is closer to a representative household that maximizes welfare by choosing its consumption bundle and labor supply. The difference between a representative household and a constrained social planner is that the latter has to decide how to distribute goods across households.

Theorem 4. In equilibrium, the aggregate output and labor supply satisfies

$$\frac{W_L}{W_Y} + \Gamma\frac{Y}{L} = 0 \qquad \text{with} \qquad \Gamma = \sum_{h\in\mathscr{H}} \Lambda_h = \frac{\Lambda_b}{\tilde{\Lambda}_b} \quad \forall b \in \mathscr{H}, \tag{19}$$

and the change in the *misallocation* component of TFP and Γ are, to a first order

$$Misallocation = d\log\Gamma, \tag{20}$$

$$d\Gamma = \overbrace{\sum_{h\in\mathscr{H}} \mathscr{C}_h\, d\chi_h}^{Distributive\ TT} + \overbrace{\sum_{h\in\mathscr{H}} \chi_h\, \mathscr{C}_h}^{Centrality\ TT}, \tag{21}$$

$$
Centrality\ TT = \overbrace{\sum_{i\in\mathscr{N}} \psi_i^\ell\, \lambda_i\, d\log\mu_i}^{Competitive\ TT} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\, \lambda_i\, d\,\omega_i^\ell}^{Labor\ TT} + \overbrace{\sum_{h\in\mathscr{H}} \chi_h \sum_{i\in\mathscr{N}} \psi_i^\ell\, d\,\beta_{hi}}^{Final\ Demand\ TT} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\, \lambda_i \sum_{j\in\mathscr{N}} \psi_j^\ell\, d\,\widetilde{\Omega}_{ij}^x}^{Intermediate\ Demand\ TT}. \quad (22)
$$

Theorem 4 is the analogous of Theorem 2 for the constrained social planner problem. Equation (19) characterizes the equilibrium output, aggregate labor supply, and labor income distribution for the social planner's solution. The centralized labor wedge Γ relates the marginal rate of substitution between output and the aggregate labor supply with the economy's marginal rate of transformation Y/L that, in equilibrium, equals TFP. In equilibrium, the centralized labor wedge equals the share of labor income to GDP (i.e., $\sum_{h\in\mathscr{H}}\Lambda_h$), and also the inverse of the distortion centralities for all workers. The fact that the social planner will choose idiosyncratic levels of labor supply with a symmetric distortion centrality for all households, i.e., $\Gamma = \delta_h^{-1}\ \forall h \in \mathscr{H}$, indicates that the decentralized solution will be inefficient as long as this form of symmetry is not satisfied. In other words, the decentralized economy withstands externalities on the labor supply as an additional source of distortion. Consequently, the decentralized distributions of labor revenue and consumption expenditure are suboptimal from the social planner's perspective whenever the symmetry in distortion centralities is not satisfied.

Equation (20) shows that for the social planner, there is a monotonic relationship between aggregate labor *misallocation* and the centralized labor wedge. This outcome entails the counterintuitive implication that the allocation of workers improves as the share of corporate income rises. Just as in the *corporate income* component from Theorem 2, as dividends increase, the paths through which aggregate consumption expenditure reaches aggregate income are less distorted, as money flows are no longer subject to input payment distortions. An increase in the share of profits represents an "earlier" extraction of money flows from the distorted payment chains, and as a consequence, endogenous aggregate labor misallocation falls.

Additionally, this result shows that under endogenous labor supply, there is a tight connection between the TFP decomposition from Baqaee & Farhi (2020) and the first-order variation of the labor wedge from Bigio & La'O (2020). Furthermore, it associates in a single equation the two equilibrium objects that, according to Chari et al. (2007), account for the bulk of business cycle fluctuations. In other words, under distortions and endogenous labor supply, if the social planner wanted to increase TFP, they could symmetrically amplify the distortion centralities for all workers and reduce the decentralized labor wedge. This increase in the corporate income share represents a linear drift towards a second-best equilibrium.

This result does not imply that total labor misallocation falls as the profit share rises. *Misallocation* both in Theorems 2 and 4 captures only the effect from endogenous changes in the labor income distribution. The definition of total aggregate labor misallocation is *competitiveness − misallocation*. As a consequence, antitrust policies that aim to reduce firms' profit margins and the aggregate profit share can be successful in increasing TFP, as long as *competitiveness > misallocation*.

Equation (21) describes two mechanisms through which the social planner can increase the aggregate labor wedge. First, the *distributive terms of trade* imply that the aggregate labor income share rises as income shifts towards households whose expenditure has a high payment centrality as measured by $\mathscr{C}_h$. Second, the *centrality terms of trade* show that the aggregate labor income share increases as the payment centrality rises for any household. The magnitude of this effect is proportional to the households' absorption share. Equation (22) further divides the latter effect into four sources of endogenous demand recomposition. First, the *competitive terms of trade* capture the increase in labor share from lower profit margins. The magnitude of this effect is proportional to the Domar weight of the sector and its payment centrality ψ_i^ℓ. Second, the *labor terms of trade* portrays how the labor share increases with labor demand, mainly from large sectors with high payment centralities. Finally, the *final* and *intermediate demand terms of trade* represent how the labor share increases with the demand for goods from sectors with high payment centralities.

Furthermore, in the absence of distortions, the effect from markdowns on the centralized labor wedge is sufficiently captured by the Domar weights, i.e., $\frac{d\log\Gamma}{d\log\mu_i} = \lambda_i$. This sufficient statistic is the main result from Bigio & La'O (2020), and Theorem 4 captures the extension from their findings to any inefficient equilibrium in which a constrained social planner makes the distributional decisions on behalf of heterogenous households.

6 Growth Accounting for the United States in the XXI Century

6.1 Data

This section describes the five data sources I use to identify disaggregated money flows and sectoral shocks for the United States. My model builds upon four types of money flows: (1) firm-to-firm in the supply of intermediate inputs, (2) firm-to-workers in the supply of labor, (3) consumer-to-firm in the supply of final goods, and (4) firm-to-households in the distribution of dividends. I explain the details about the construction of this data match in Section 1 of the Online Appendix.

The first source is the input-output (IO) tables constructed by the Bureau of Economic Analysis (BEA) from 1997 to 2021. These tables measure the intermediate input transactions, labor costs, and final expenditure for 71 North American Industry Classification System (NAICS) 3-digit level industries. As usual, I exclude industries corresponding to federal, state, and local governments, resulting in a matched data set of 66 industries. The IO tables are not readily available, as the BEA provides only IO use and make tables. The use tables depict industrial consumption across multiple categories of goods and services, and the make tables characterize industrial production of multiple categories of goods and services. The interaction between the use and make tables produces the IO network. The BEA has IO use and make tables that go back to 1946, but only after 1997 did these tables start to identify the sectoral labor costs as an independent component of value-added, which is essential for my identification of sectoral distortions. I use this tables to calibrate $\forall i \in \mathscr{N}$

$$\omega_i^\ell = \frac{\text{Labor Cost}_i}{\text{Total Cost}_i}, \qquad \mu_i = \frac{\text{Total Cost}_i}{\text{Sales}_i}, \qquad \omega_{ij} = \frac{\text{Sales from } j \text{ to } i}{\text{Intermediate Cost}_i},$$

$$\text{Total Cost}_i = \text{Labor Cost}_i + \text{Intermediate Cost}_i, \qquad \text{Value Added}_i = \text{Labor Cost}_i + \text{Rents}_i,$$

$$\text{Sales}_i = \text{Value Added}_i + \text{Intermediate Cost}_i.$$

The second source is the 1997 to 2021 county business patterns (CBP) from the Census Bureau. The CBP is an annual series that, for each industry, provides economic data at the county, metropolitan statistical area, state, and national levels. For each subnational level, the CBP includes the number of workers and their income in each NAICS industry up to the 6-digit level. The employment statistics count full- and part-time workers with an active payroll in the pay period that includes March 12 and their average annual income. The CBP draws its information from administrative records of the Internal Revenue Agency, the Bureau of Labor Statistics (BLS), and the Social Security Administration, which gives it a higher degree of trustworthiness than voluntary census responses. There are two widely known issues with the CBP.

The first issue is that the census suppresses a significant proportion of the data to protect individual employers' confidentiality.[10] To make matters worse, since 2007, the non-suppressed observations have included a random noise infusion multiplier that further complicates its implementation. A whole research agenda on antisuppression algorithms tries to fill the gaps in the CBP. The data mining techniques developed by this literature utilize the additional information available due to the industrial and geographical hierarchical nature, which justifies a manifold of bounds and aggregation constraints across hierarchies. Two current gold standards solve this problem: first, the two-staged algorithm from Isserman & Westervelt (2006), and second, the linear programming solution from Eckert et al. (2020).[11] For my calibration, the problem with both identification methods is their emphasis on the number of workers rather than their compensation. For this reason, I develop a three-staged estimation for the average annual payroll. The first and second stages consist of the Isserman & Westervelt (2006) algorithm for the number of workers with an initial guess given by the Eckert et al. (2020) solution. The third stage utilizes the two-staged employment estimates and analogous hierarchical bounds and aggregation constraints for income.

The second issue is that the CBP only covers some forms of private employment. The CBP does not include workers in agriculture production, railroads, government, and private households. I use the BEA's Regional Economic Information System (REIS) to fill this gap and obtain state-level employment and income measures for agricultural production and railroad workers. The REIS uses the Quarterly Census of Employment and Statistics from the Bureau of Labor Statistics (BLS). Its main limitation relative to the CBP is that it only provides 2-digit NAICS statistics.

The third source is the BLS Occupational Employment and Wage Statistics (OEWS). This dataset contains employment and wage estimates for approximately 830 occupations under the Standard Occupational Classification System (SOC). These estimations are available at the level of the country, industries, states, and metropolitan and nonmetropolitan areas. The BLS also suppresses data on the OEWS to protect the confidentiality of employers and workers, although this problem is less pervasive

[10] For example, Isserman & Westervelt (2006) document that for 2002, the suppression rate was two-thirds - almost 1.5 million out of the 2.2 million records.

[11] An alternative and more straightforward solution is the employment estimate using midpoints of establishment size groups as in Clapp et al. (1992), Glaeser et al. (1992), Porter (2003). The issue is that this method does not use all the hierarchical information embedded within the CBP.

than in the CBP. For this reason, I implement a two-stage antisuppression algorithm that depends on hierarchical aggregation constraints, and it is analogous to the one implemented for the CBP.

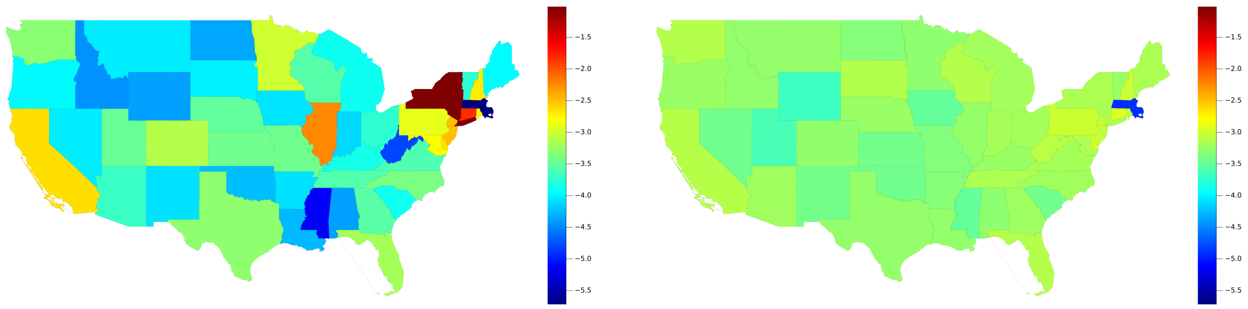
I use the CBP and OEWS to obtain labor network estimates at (a) geographic, (b) occupation, and (c) geographic and occupation levels. For example, to capture, for hospitals (i), the labor share of dentists in Maine (h), I define

$$\alpha_{ih} \propto \overbrace{\begin{array}{c}\text{Hospital's share of}\\ \text{labor expenditure}\\ \text{in Maine}\end{array}}^{\text{Spatial Demand (CBP)}} \times \overbrace{\begin{array}{c}\text{Hospital's share of}\\ \text{labor expenditure}\\ \text{in dentists}\end{array}}^{\text{Occupational Demand (OEWS)}} \times \overbrace{\begin{array}{c}\text{Maine's share of}\\ \text{labor expenditure}\\ \text{in dentists}\end{array}}^{\text{Occupational Supply (OEWS)}}. \tag{23}$$

Each of these three factors portrays a different feature of the labor market. First, *spatial demand* captures sectoral heterogeneity in labor demand at the subnational level. Without this factor, sectors with the same occupational demand would have symmetric labor bundles. Second, *occupational demand* represents sectoral heterogeneity in labor demand across occupations. Without this factor, sectors with the same spatial demand would have symmetric labor bundles. Third, the *occupational supply* illustrates geographic heterogeneity in the availability of occupations. Without this factor, sectors would have the same occupational demand across states. The *demand* and *supply* labels are misnomers, as these three components are all equilibrium objects. However, these labels illustrate how the first two factors capture heterogeneity from the firms, while the last factor represents differences in the availability of occupations across space. For the geographic specifications, I use only the *spatial demand* factor, while for the occupational specifications, I employ the *occupational demand* factor exclusively.

Figure 2: Sectoral Labor Intensity

A. Financial Sector in Economists **B. Ambulatory Health in Dentists**



Note: For comparability, the measures are weighted by the population from each state.

Figure 2 illustrates the implementation of Equation (23) with heatmaps for the estimation of the cost intensity from the financial sector in economists, and from the ambulatory health industry in dentists. On the one hand, there is geographical concentration in the intensity of the financial sector in hiring economists in New York, Connecticut, Illinois, New Jersey, and California. On the other hand, there is no geographic concentration in the intensity of the ambulatory health industry in hiring dentists.

The fourth source is the BEA state-level personal consumption expenditure by product type (PCE). The PCE classifies consumption expenditure into 113 types of products. Of these, only 71 categories are non-redundant or refer to new goods. Using the IO make matrix from the categories left, I build a product-to-sector crosswalk that specifies the state-level final consumption share for each of the 66 sectors in the IO tables. From here, households within the same state will share the same consumption bundle.

The fifth and final data source is the sectoral TFP measure from the BEA's Integrated Industry-Level Production Account (KLEMS). Following La'O & Tahbaz-Salehi (2022), I will use the variations in sectoral TFP as an exogenous measure of productivity variation. Specifically, in my model, sectoral TFP variations differ from sectoral productivity shocks. Still, I equate these two notions in the exogenous variations, not only because it is the standard in the literature but also because the alternative requires having measures of sectoral prices that allow me to directly estimate the sectoral Solow residuals, which I expect could only improve the model's fit with the aggregate data. In this sense, my decision to measure exogenous productivity shocks from sectoral KLEMS's TFP variations imposes the most stringent benchmark for testing the model's implications.

To capture the variations between periods t and $t+1$, I estimate the equilibrium in period t, and introduce the variations captured by the data between period t and $t+1$. For example, the technological component of TFP between period t and $t+1$ is given by

$$Technology_{t+1} = \sum_{i \in \mathcal{N}} \widetilde{\lambda}_{i,t} \, d\, log\, \mathcal{A}_{i,t+1}.$$

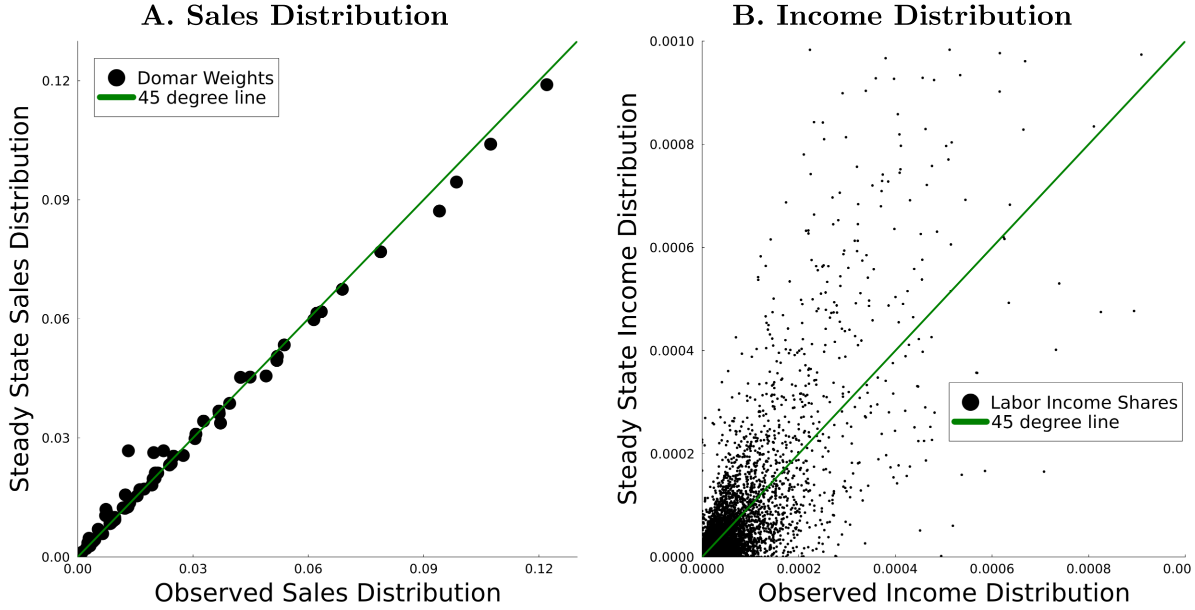
6.2 Quantitative Implementation

6.2.1 Aggregate Accounting

Figure 3 shows two scatterplots that compare the observed sales and income distributions in 2021 with their equivalent equilibrium distributions. The match is almost perfect for the sales distribution, and the R^2 of 0.982 for the OLS regression of the observed λ on its equilibrium equivalent confirms this. The imprecision of the model estimation for the income distribution comes from the uncertainty about the expenditure from each sector on each type of worker. For example, the CBP provides information about the compensation from the financial sector to workers in Illinois, and the OEWS captures the nationwide compensation from the financial sector to economists. However, there is no accessible data that provides compensation from the financial sector to economists in Illinois. For this reason, Equation (23) provides the proxy required for the model implementation. Nevertheless, despite this uncertainty, the R^2 for the OLS regression of the observed Λ on its equilibrium equivalent for 2021 is 0.682, and the t-value for its slope coefficient is 286.

Tables 1 and 2 report the results from two types of OLS regressions for observed TFP growth, first

Figure 3: Sales and Income Distribution



	Observed λ ***on***
Equilibrium λ	1.022***
	(3.4e-3)
Intercept	-5.1e-4***
	(1.2e-4)
R^2	0.982
Observations	1,650

	Observed Λ ***on***
Equilibrium Λ	0.438***
	(1.5e-3)
Intercept	7.6e-6***
	(1.6e-7)
R^2	0.682
Observations	38,189

Notes:

Table 1: Explanatory Power of the Model Without IO Networks

	Rep. Household		***Occupation***		***County***		***State & Ocupation***	
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
$d\log TFP$	0.523		0.503		0.388		-0.265	
	(0.366)		(0.350)		(0.316)		(0.264)	
Technology		1.341***		0.789***		0.796***		0.847***
		(0.308)		(0.267)		(0.266)		(0.289)
Competitiveness		0.212		0.320		0.454		0.986
		(0.423)		(0.489)		(0.373)		(0.695)
Misallocation		0.573*		0.450		0.335		-0.105
		(0.329)		(0.437)		(0.315)		(0.360)
Intercept	0.012***	0.011***	0.012***	0.012***	0.013***	0.012***	0.015***	0.012***
	(3.2e-3)	(2.0e-3)	(3.2e-3)	(2.2e-3)	(3.2e-3)	(2.1e-3)	(3.0e-3)	(2.2e-3)
Observations					22			
N					66			
H	1		750		3,136		38,190	
R^2	9.2%	71.4%	9.35%	62.4%	7.00%	62.5%	4.8%	60.4%
Adj. R^2	9.2%	68.4%	9.35%	58.4%	7.00%	58.6%	4.8%	56.2%

Notes:

on the model prediction of TFP growth, and second on the three components from the decomposition in equation (14). Table 1 uses the regressors from an estimation without input-output networks, while Table 2 allows for intermediate input markets. Each table contains four estimations with different as-

Table 2: **Explanatory Power of the Model With IO Networks**

	Rep. Household		*Occupation*		*County*		*State & Ocupation*	
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
$d\,log\,TFP$	0.370***		0.311***		0.316***		0.311***	
	(0.072)		(0.069)		(0.065)		(0.069)	
Technology		0.478***		0.414***		0.416***		0.413***
		(0.097)		(0.081)		(0.083)		(0.082)
Competitiveness		0.398***		0.341***		0.350***		0.342***
		(0.062)		(0.054)		(0.053)		(0.054)
Misallocation		0.074		0.172		0.164		0.168
		(0.138)		(0.125)		(0.135)		(0.125)
Intercept	0.010***	0.009	0.011***	0.010***	0.011***	0.010***	0.011***	0.010***
	(2.1e-3)	(2.0e-3)	(2.2e-3)	(1.8e-3)	(2.1e-3)	(1.9e-3)	(2.3e-3)	(1.9e-3)
Observations	22							
N	66							
H	1		750		3,136		38,190	
R^2	56.9%	75.2%	49.9%	75.8%	54.0%	75.4%	49.9%	75.5%
Adj. R^2	56.9%	72.6%	49.9%	73.3%	54.0%	72.8%	49.9%	73.2%

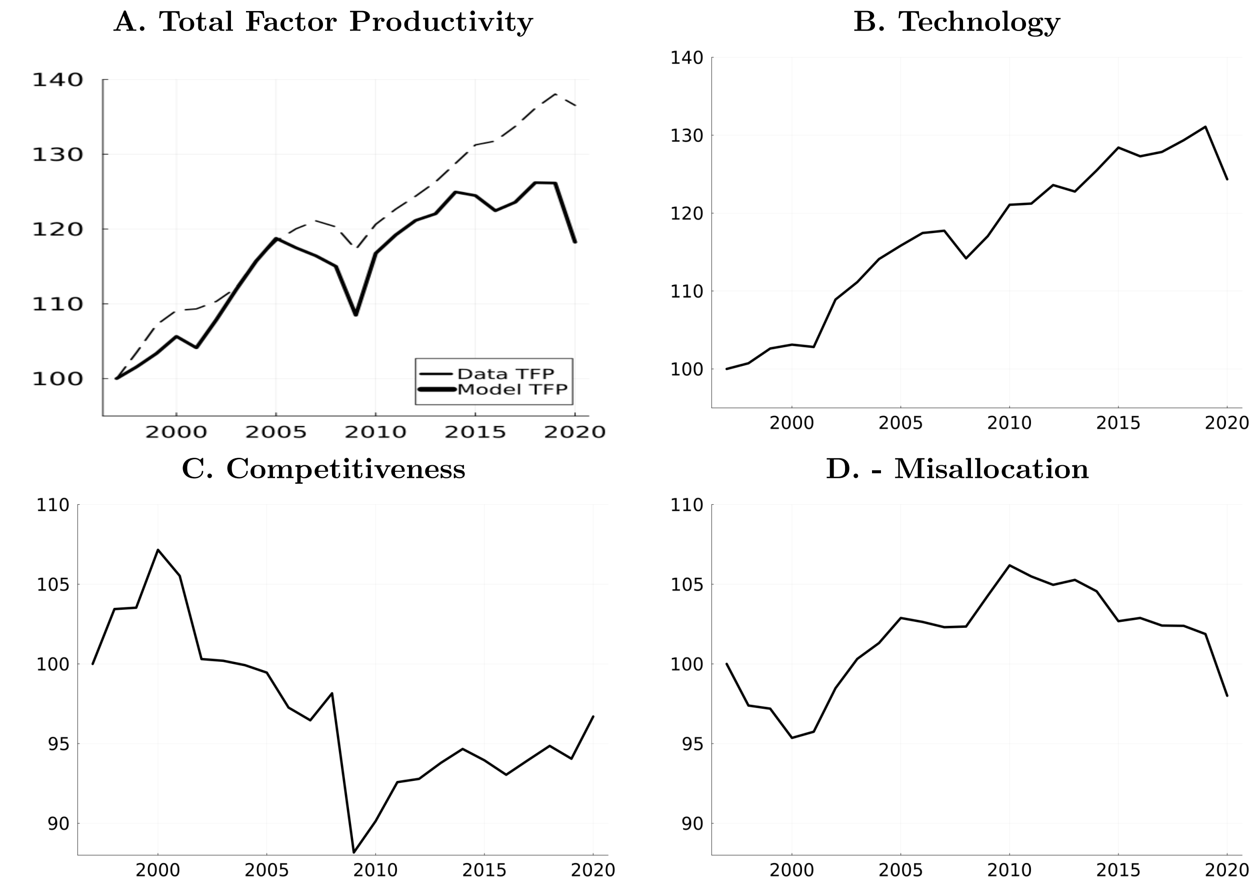
Notes:

sumptions about the number of households: (i) representative household ($H = 1$), (ii) heterogeneity by occupation ($H = 750$), (iii) geographical heterogeneity by county ($H = 3,136$), and (iv) heterogeneity by the interaction of states and occupations ($H = 38,190$). The latter is my preferred specification because it simultaneously accounts for skill and geographic heterogeneity. Under this specification, accounting for intermediate input linkages boosts the R^2 of the model prediction for TFP growth from 5% to 50%. Additionally, in the regression of observed TFP growth on the three components from equation (14) with intermediate input markets, R^2 increases to 75%, and the *technology* and *competitiveness* components are significant at the 1% level.

Figure 4 shows the dynamics for TFP, technology, competitiveness, and misallocation normalizing their initial 1997 level at 100. Table 3 captures the counterfactual growth on TFP relative to the model prediction leaving aside the technology, competitiveness, or misallocation channels. Table 4 and Table 5 portray the counterfactual growth on TFP relative to the model prediction leaving aside the technology or competitiveness effect from a specific industry. Table 6 depicts the variance decomposition for TFP growth, Table 7 the variance decomposition for technology across industries, and Table 8 for competitiveness.

The model prediction for TFP follows observed TFP in terms of levels and variations until 2014 (Figure 4A). After 2014, the model predicts no growth in TFP and a strong reduction in response to the 2020 COVID shock. The static and closed-economy nature of the model is, in my opinion, the reason why the model fails to capture TFP variations after 2014. From 2014 to 2021, the net international investment position as a percentage of GDP almost doubled from -40% to -77.8%. It is reasonable to expect that this increase in external liabilities could be behind an intertemporal demand-driven growth in TFP that this model completely misses. This model emphasizes capturing multiple sources of supply-driven growth that depend on the heterogeneity of firms and households. For this reason, in the absence of measurement errors, this result brings some evidence about the lack

Figure 4: TFP Decomposition



Notes:

of supply-driven sources of growth for TFP after 2014.[12]

From 1998 to 2020, the growth of TFP was mainly attributable to technological shocks, while competitiveness and misallocation had a negative secondary role (Table 3A and Figure 4B). On the one hand, without productivity shocks, TFP in 2020 would have been 23.4% lower. On the other hand, leaving aside the effects of competitiveness or misallocation, TFP would have grown 2.5% and 2.8% more. The productivity shocks in the oil and gas extraction, computer and electronics, telecommunications, and computer system design industries were the main drivers of technologically driven growth. Without them, TFP would have been respectively 11.1%, 6.6%, 2.8%, and 2.3% lower. The productivity shocks in the construction, chemical products, and credit intermediation industries stood in the way of growth. Without them, TFP would have been respectively, 2.9%, 2.8%, and 1.8% higher (Table 4A). Despite the secondary role of aggregate competitiveness, the higher profit margins of the credit intermediation, chemical products, and computer and electronics sectors hindered TFP growth, while

[12] Two potential sources of measurement error are of my concern. First, observed growth in TFP is the difference between growth in real GDP and the labor force. However, from equation (13), the variations in the labor force participation from heterogeneous workers are not symmetric and depend on their aggregate value-added contribution given by the distribution $\widetilde{\Lambda}$. Second, the nature of productivity growth might have changed after 2014 in a way not captured by the BEA's sectoral KLEMS Solow's residual estimation.

Table 3: Counterfactual TFP Growth Differential

A. Between 1998 and 2020

Heterogeneity	*Model*	*Technology*	*Competitiveness*	*Misallocation*
Rep. Household	20.3%	-25.0%	2.3%	2.6%
Occupation	19.0%	-26.8%	4.7%	3.3%
County	18.3%	-26.7%	4.9%	4.0%
State & Occupation	18.2%	-23.4%	2.5%	2.8%

B. Between 2002 and 2009

Heterogeneity	*Model*	*Technology*	*Competitiveness*	*Misallocation*
Rep. Household	5.2%	-13.9%	19.7%	-8.8%
Occupation	6.5%	-17.4%	23.3%	-9.7%
County	5.0%	-17.1%	23.1%	-8.1%
State & Occupation	4.2%	-13.0%	19.3%	-8.2%

C. Between 2010 and 2020

Heterogeneity	*Model*	*Technology*	*Competitiveness*	*Misallocation*
Rep. Household	9.0%	-6.2%	-10.2%	8.0%
Occupation	8.3%	-5.9%	-10.5%	8.8%
County	8.7%	-5.9%	-10.4%	8.3%
State & Occupation	9.0%	-6.3%	-9.8%	7.5%

Notes:

the lower profit margins from the housing and insurance sectors boosted TFP. Without them, TFP would have been respectively, 4.1%, 2.6%, and 1.3% higher, and 1.6% and 1.5% lower (Table 5A).

Furthermore, during the same period, 55.6% of the volatility was attributable to the reallocation of resources. Out of this, 34.6% was due to variations in aggregate competitiveness and 21% due to changes in the income distribution. Productivity shocks explained the remaining 44.4% of the volatility (Table 6A). Productivity shocks in the oil and gas extraction, insurance, air transportation, utilities, and financial sector were the main sources of technological-driven volatility (Table 7A). Variations in the profit margins for the oil and gas extraction, financial, utilities, and chemical product sectors were the main drivers of competitive-driven volatility (Table 8A).

The secondary role of competitiveness and misallocation from 1997 to 2020 reflects a structural change of direction during the Great Recession (GR). Competitiveness and misallocation fell from 2002 to 2009 and increased from 2010 to 2020 (Figures 4C and 4D). The self-compensating nature of competitiveness and misallocation is not surprising, as increases in profit margins are correlated negatively with competitiveness and positively with misallocation.

For the cycle before the Great Recession (2001 to 2009), on the one hand, growth in TFP was driven by technology and reductions in misallocation. Without the growth in productivity or reductions in misallocation, TFP would have been 13% and 8.2% lower, respectively. On the other hand, the reductions in aggregate competitiveness hindered growth, and in their absence, TFP would have been 19.3% higher. The productivity shocks in the oil and gas extraction, computer and electronics, and telecommunication sector were the main drivers of technologically driven growth. Without them, TFP would have been respectively 5.35%, 2.84%, and 2.27% lower (Table 4B). The higher profit margins in oil and gas extraction stood in the way of growth, and in their absence, TFP would have been 6.59% higher (Table 5B).

Table 4: Counterfactual TFP Growth Without Sectoral Technology

A. Between 1998 and 2020

1	Oil & gas extraction	-11.11%
2	Computer & electronics	-6.64%
3	Telecommunications	-2.85%
4	Computer systems design	-2.30%
5	Administrative services	-1.74%
6	Insurance carriers	-1.45%
7	Farms	-1.34%
8	Primary metals	-1.28%
	⋮	
63	Rental & leasing	1.41%
64	Credit intermediation	1.77%
65	Chemical Products	2.84%
66	Construction	2.87%

B. Between 2002 and 2009

1	Oil & gas extraction	-5.35%
2	Computer & electronics	-2.84%
3	Telecommunications	-2.27%
4	Utilities	-1.92%
5	Administrative services	-1.06%
	⋮	
66	Construction	1.76%

C. Between 2010 and 2020

1	Oil & gas extraction	-5.41%
2	Computer systems design	-1.29%
3	Management of companies	-1.26%
4	Housing	-1.14%
5	Other real estate	-1.01%
	⋮	
64	Air transportation	1.03%
65	Chemical products	1.90%
66	Credit intermediation	2.73%

Notes: Observations larger than 1.2% in absolute value for table A and 1% for tables B and C are included.

Table 5: Counterfactual TFP Growth Without Sectoral Competitiveness

A. Between 1998 and 2020

1	Housing	-1.65%
2	Insurance carriers	-1.53%
3	Misc. professional services	-1.10%
4	Other services	-0.89%
	⋮	
63	Publishing industries	0.80%
64	Computer and electronics	1.34%
65	Chemical products	2.57%
66	Credit intermediation	4.10%

B. Between 2002 and 2009

1	Securities & investment	-0.86%
	⋮	
58	Wholesale trade	0.92%
59	Publishing industries	0.93%
60	Internet, & inf. services	0.99%
61	Chemical products	1.35%
62	Telecommunications	1.43%
63	Computer and electronics	1.48%
64	Housing	1.57%
65	Utilities	1.87%
66	Oil & gas extraction	6.59%

C. Between 2010 and 2020

1	Oil & gas extraction	-6.34%
2	Housing	-3.09%
3	Insurance carriers	-0.98%
4	Misc. professional services	-0.87%
5	Administrative services	-0.82%
	⋮	
64	Primary metals	0.80%
65	Chemical products	0.84%
66	Credit intermediation	3.86%

Notes: Observations larger than 0.8% in absolute value are included.

For the cycle after the GR (2010 to 2020), on the one hand, growth in TFP was driven by increases in technology and competitiveness. Without the growth in productivity or competitiveness, TFP would have been 6.3% and 9.8% lower, respectively. On the other hand, the increases in misallocation hindered growth, and in their absence, TFP would have been 7.5% higher. The main growth drivers were the productivity shocks and the reduction in the profit margins from the oil and gas extraction. Without them, TFP would have been respectively 5.41% and 6.34% lower. Additionally, the reductions in the profit margins for the housing sector enabled growth, and in their absence, TFP would have been 3.09% lower. Furthermore, reductions in productivity and higher profits margins for the credit intermediation and the chemical products industries hindered growth (Table 4C) and (Table 5C).

Table 6: TFP Covariance Decomposition

A. Between 1998 and 2020

Heterogeneity	*Technology*	*Competitiveness*	*-Misallocation*
Rep. Household	41.3%	39.3%	19.4%
Occupation	40.1%	41.5%	18.4%
County	37.2%	46.8%	16.1%
State & Occupation	44.4%	34.6%	21.0%

B. Between 2002 and 2009

Heterogeneity	*Technology*	*Competitiveness*	*-Misallocation*
Rep. Household	21.3%	68.6%	10.1%
Occupation	12.7%	85.2%	2.1%
County	10.6%	85.0%	4.4%
State & Occupation	28.3%	61.2%	10.5%

C. Between 2010 and 2020

Heterogeneity	*Technology*	*Competitiveness*	*-Misallocation*
Rep. Household	56.7%	8.2%	35.1%
Occupation	55.3%	13.1%	31.6%
County	55.4%	15.6%	29.0%
State & Occupation	58.1%	4.9%	37.0%

Notes:

Most of the TFP volatility after the GR was attributable to technology and misallocation (Table 6C). Productivity shocks in the air transportation and insurance sectors were the primary technological sources of volatility (Tables 7C).

Figure 5 shows the dynamics for misallocation and its components normalizing their initial 1997 level at 100. Table 3 captures the counterfactual growth on TFP relative to the model prediction leaving aside each one of the components from misallocation. Tables 10-13 portray the counterfactual growth on TFP relative to the model prediction leaving aside the competitive, labor demand, final demand, and intermediate demand terms of trade from a specific industry. Table 14 depicts the variance decomposition for variations in misallocation and Tables 15-18 the variance decomposition for each one of the misallocation components across industries.

From 1998 to 2020, misallocation barely increased, and without its effect on growth, TFP would have grown 2.8% more (Table 3A). However, this apparent lack of variation was due to a worsening in the labor demand terms of trade, partially compensated by the improvement in the competitive, final, and intermediate demand terms of trade. Without the increase in the labor demand terms of trade, TFP would have grown 15.6%, and in the absence of the reduction in the competitive, intermediate, and final demand terms of trade, TFP would have been 5.9%, 4.5%, and 2.6% higher (Table 9A). The worsening in the labor demand terms of trade has its main culprits in the higher labor demand from the credit intermediation, computer and electronics, oil and gas extraction, and publishing sectors. Without them, TFP would have been 2.40%, 2.25%, 1.79%, and 1.34% higher, respectively. Labor demand by the wholesale trade and insurance sectors acted as a buffer, and in their absence, TFP would have been 1.62% and 1.61% lower, respectively. The higher profit margins for the credit intermediation and the chemical products sectors explain the improvement in the competitive

Table 7: Technology Covariance Decomposition by Industry

A. Between 1998 and 2020

1	Oil & gas extraction	12.09%
2	Insurance carriers	9.39%
3	Air transportation	9.32%
4	Utilities	8.83%
5	Securities & investment	5.90%
6	Chemical products	4.84%
7	Motor vehicles	4.33%

B. Between 2002 and 2009

1	Oil & gas extraction	20.71%
2	Securities & investment	20.12%
3	Utilities	17.40%
4	Chemical products	7.64%
5	Insurance carriers	7.31%
6	Motor vehicles	5.18%
7	Internet & inf. services	5.08%
8	Credit intermediation	4.78%
	⋮	
66	Petroleum & coal	-8.64%

C. Between 2010 and 2020

1	Air transportation	14.42%
2	Insurance carriers	12.46%
3	Arts, sports & museums	6.55%
4	Management of companies	5.61%
5	Oil & gas extraction	4.15%
6	Housing	4.63 %
7	Motor vehicles	4.62%
8	Petroleum & coal	4.61%
9	Other real estate	4.60%
10	Food Services	4.26%
	⋮	
66	Farms	-4.58%

Notes: Only sectors with more than 4% in absolute value are included.

Table 8: Competitiveness Covariance Decomposition by Industry

A. Between 1998 and 2020

1	Oil & gas extraction	21.16%
2	Securities & investment	12.00%
3	Utilities	11.03%
4	Chemical products	10.82%
5	Rental & leasing	4.39%

B. Between 2002 and 2009

1	Securities & investment	18.24%
2	Chemical products	14.79%
3	Utilities	14.16%
4	Oil & gas extraction	12.79%
5	Insurance carriers	8.36%
6	Credit intermediation	6.52%
5	Rental & leasing	4.19%
	⋮	
64	Legal services	-4.90%
65	Telecommunications	-5.05%
66	Housing	-9.14%

C. Between 2010 and 2020

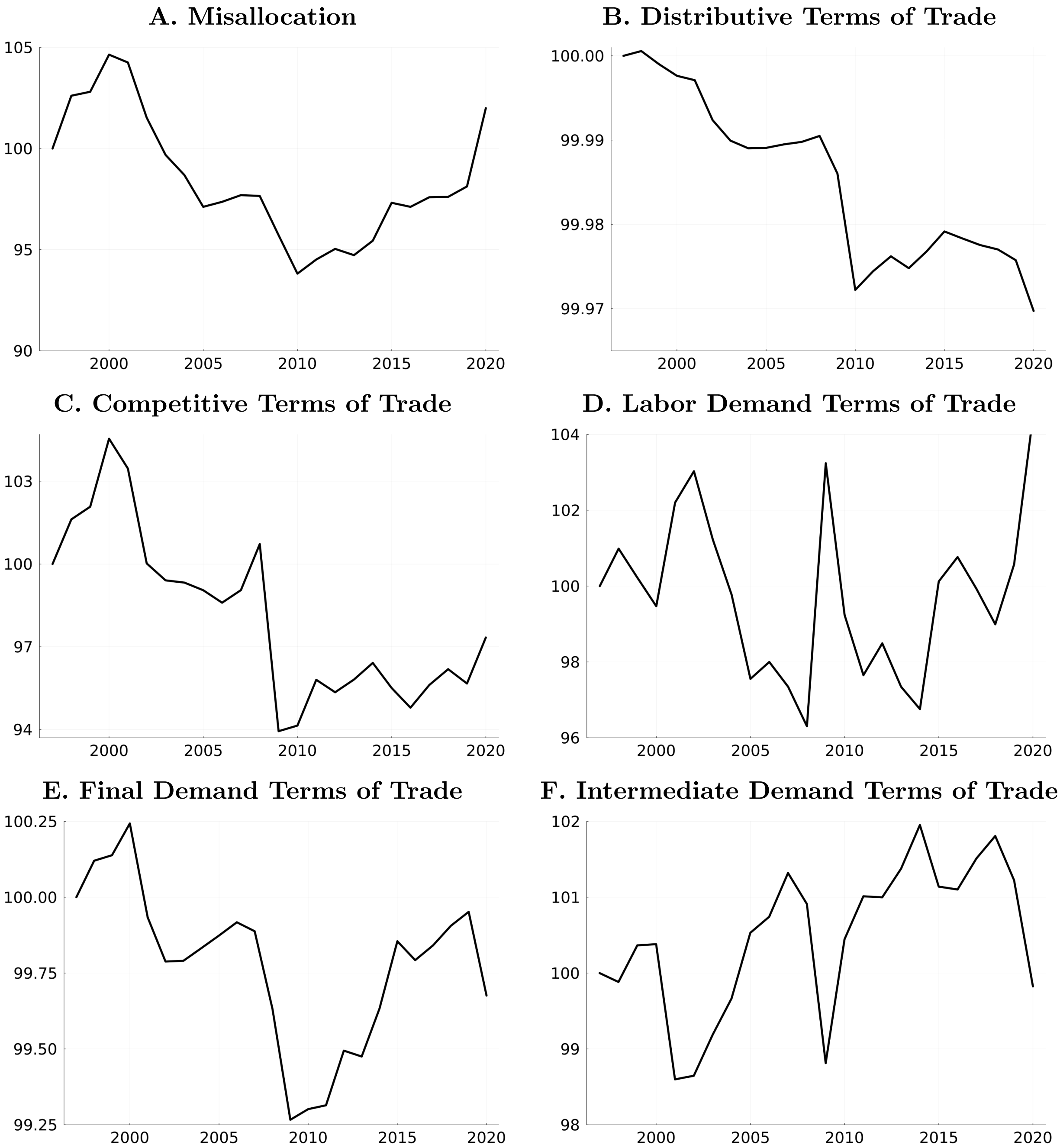
1	Oil & gas extraction	28.82%
2	Chemical products	16.08%
3	Securities & investment	14.94%
4	Rental & leasing	12.91%
5	Insurance carriers	12.57%
6	Telecommunications	9.59%
7	Air transportation	9.04%
8	Food, beverages & tobacco	6.07%
9	Wholesale trade	4.99%
10	Petroleum & coal	4.20%
	⋮	
64	Farms	-9.15%
65	Credit intermediation	-12.62%
66	Other real estate	-20.84%

Notes: Only sectors with more than 4% in absolute value are included.

terms of trade. Without them, TFP would have been 2.16% and 1.06% lower. The shift of final and intermediate demand toward computers and electronics fostered the improvement in the final and intermediate demand terms of trade. In their absence, TFP would have been 1.50% and 1.24% lower, respectively. The shift of final and intermediate demand toward the wholesale trade sector worsened the final and intermediate demand terms of trade. In their absence, TFP would have been 1.18% and 1.21% higher, respectively (Tables 10A-13A). During this period, the primary sources of variation for misallocation were the profit margins from the financial, chemical products, and utilities sectors, and the labor demand from oil and gas extraction (Tables 14A-18A).

Before the GR misallocation improved, and without its effect on growth, TFP would have grown 8.2% less (Table 3B). The improvement in the competitive terms of trade explained the reduction in misallocation. Without it, TFP would have grown 11.1% less (Table 9B). The improvement in the competitive terms of trade mainly originates in the higher profit margins from the oil and gas extraction, computer and electronic, and internet and information services sectors. Withouth them, TFP would have been 1.46%, 1.11%, and 1.01% lower (Table 10B). The main sources of volatility in misallocation were the profit margins for the financial, chemical product, and utility sectors (Tables 14B-15B).

Figure 5: Misallocation Decomposition



Notes:

Table 9: **Counterfactual TFP Growth Differential in the Absence of Misallocation Components**

A. Between 1998 and 2020

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	-3.4%	6.3%	0.4%	-1.3%
Occupation	0%	-5.9%	15.1%	-2.0%	-4.2%
County	0.1%	-5.2%	14.2%	-0.9%	-4.4%
State & Occupation	0.1%	-5.9%	15.6%	-2.6%	-4.5%

B. Between 2002 and 2009

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	-9.3%	1.1%	-0.9%	-0.2%
Occupation	0%	-11.0%	3.4%	-1.9%	-0.8%
County	0.1%	-10.4%	3.4%	-0.7%	-1.0%
State & Occupation	0.1%	-11.1%	3.4%	-2.0%	-0.9%

C. Between 2010 and 2020

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	3.9%	1.2%	1.7%	0.9%
Occupation	0%	2.9%	7.2%	0.2%	-1.8%
County	0.1%	3.0%	3.5%	2.1%	-1.5%
State & Occupation	0.1%	2.8%	7.4%	-0.1%	-1.7%

Notes:

Table 10: Counterfactual TFP Growth Without Sectoral Competitive TT

A. Between 1998 and 2020

1	Credit intermediation	-2.16%
2	Chemical products	-1.06%
3	Computer & electronics	-0.98%
4	Publishing industries	-0.80%
5	Internet & inf. services	-0.69%
	⋮	
64	Insurance carriers	0.77%
65	Other services	0.81%
66	Misc. professional services	0.87%

B. Between 2002 and 2009

1	Oil & gas extraction	-1.46%
2	Computer & electronics	-1.11%
3	Internet & inf. services	-1.01%
4	Wholesale trade	-0.92%
5	Telecommunications	-0.86%
6	Utilities	-0.84%
7	Publishing industries	-0.82%

C. Between 2010 and 2020

1	Credit intermediation	-2.0%
2	Securities & investment	-0.52%
	⋮	
64	Administrative services	0.62%
65	Misc. professional services	0.70%
66	Oil & gas extraction	1.91%

Notes: Only sectors with more than 0.6% in absolute value are included.

Table 11: Counterfactual TFP Growth Without Sectoral Labor Demand TT

A. Between 1998 and 2020

1	Wholesale trade	-1.62%
2	Insurance carriers	-1.61%
3	Other retail	-1.07%
	⋮	
61	Utilities	0.69%
62	Computer systems design	0.82%
63	Publishing industries	1.34%
64	Oil & gas extraction	1.79%
65	Computer & electronics	2.28%
66	Credit intermediation	2.40%

B. Between 2002 and 2009

1	Securities & investment	-0.96%
	⋮	
64	Computer & electronicss	0.85%
65	Utilities	1.02%
66	Oil & gas extraction	2.20%

C. Between 2010 and 2020

1	Wholesale trade	-1.70%
2	Insurance carriers	-1.03%
3	Administrative services	-0.93%
4	Other retail	-0.83%
	⋮	
64	Publishing industries	0.89%
65	Computer & electronics	0.98%
66	Credit intermediation	2.44%

Notes: Only sectors with more than 0.6% in absolute value are included.

Table 12: Counterfactual TFP Growth Without Sectoral Final Demand TT

A. Between 1998 and 2020

1	Computer & electronics	-1.50%
2	Motor vehicles	-0.91%
3	Machinery	-0.88%
4	Apparel & leather	-0.51%
	⋮	
62	Securities & investment	0.87%
63	Misc. professional services	0.94%
64	Hospitals	0.95%
65	Internet & inf. services	1.01%
66	Wholesale trade	1.18%

B. Between 2002 and 2009

1	Construction	-1.22%
2	Motor vehicles	-0.82%
	⋮	
66	Hospitals	0.58%

C. Between 2010 and 2020

1	Computer & electronis	-0.52%
	⋮	
63	Other retail	0.59%
64	Internet & inf. services	0.60%
65	Construction	0.89%
66	Wholesale trade	1.08%

Notes: Only sectors with more than 0.5% in absolute value are included.

Table 13: Counterfactual TFP Growth Without Sectoral Intermediate Demand TT

A. Between 1998 and 2020

1	Computer & electronics	-1.24%
2	Credit intermediation	-0.90%
3	Publishing industries	-0.76%
4	Computer systems design	-0.45%
5	Ambulatory health	-0.42%
	⋮	
61	Telecommunications	0.52%
62	Administrative services	0.54%
63	Hospitals	0.56%
64	Insurance carriers	0.74%
65	Other retail	0.90%
66	Wholesale trade	1.21%

B. Between 2002 and 2009

1	Computer & electronics	-0.48%
	⋮	
66	Securities & investment	0.49%

C. Between 2010 and 2020

1	Credit intermediation	-0.97%
2	Publishing industries	-0.51%
3	Computer & electronics	-0.49%
	⋮	
63	Insurance carriers	0.52%
64	Administrative services	0.63%
65	Other retail	0.66%
66	Wholesale trade	1.12%

Notes: Only sectors with more than 4% in absolute value are included.

Table 14: Misallocation Covariance Decomposition

A. Between 1998 and 2020

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	61.9%	48.9%	3.5%	-14.3%
Occupation	0%	73.1%	37.0%	0.9%	-11.0%
County	0.1%	68.4%	49.6%	-4.8%	-13.3%
State & Occupation	0.1%	73.6%	38.6%	2.3%	-14.6%

B. Between 2002 and 2009

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	136.8%	-30.9%	-7.9%	2.0%
Occupation	0%	234.1%	-160.7%	-2.7%	29.3%
County	0.1%	197.1%	-95.2%	-15.1%	13.1%
State & Occupation	0.1%	155.9%	-78.4%	3.5%	18.9%

C. Between 2010 and 2020

Heterogeneity	***Distributive TT***	***Competitive TT***	***Labor DTT***	***Final DTT***	***Intermediate DTT***
Rep. Household	0%	18.0%	125.8%	-3.2%	-40.6%
Occupation	0%	24.0%	129.5%	-12.8%	-40.7%
County	0.1%	18.9%	140.5%	-17.0%	-42.5%
State & Occupation	0.1%	19.2%	131.4%	-3.1%	-47.6%

Notes:

After the GR misallocation worsened, and without its effect on growth, TFP would have grown 7.5% more (Table 3C). The increase in labor demand and competitive terms of trade explained the rise in misallocation. Without them, TFP would have grown 7.4% and 2.8% more, respectively (Table 9C). The worsening in the labor demand terms of trade has its main culprits in the higher labor demand from the credit intermediation sector and the increase for competitive terms of trade in the lower profit margins for the oil and gas extraction industries. Without them, TFP would have been 2.44% and 1.91% higher, respectively (Tables 10C-11C). The main sources of volatility were the labor demand from the oil and gas extraction and the chemical product sectors (Tables 14C-16B).

The distributive terms of trade had a minuscule role in the misallocation variation and the volatility (Tables 9 and 14). My explanation is the low heterogeneity at the state level in consumption bundles and, consequently, in the average expenditures' average distortion centralities $\mathcal{Z}_h^c$.

Table 15: Competitive TT Covariance Decomposition by Industry

A. Between 1998 and 2020

1	Securities & investment	22.92%
2	Chemical products	12.37%
3	Utilities	10.31%
4	Food, beverages & tobacco	6.87%
5	Oil & gas extraction	6.09%
6	Insurance carriers	5.77%
7	Computer & electronics	4.56%
8	Misc. manufacturing	4.08%

B. Between 2002 and 2009

1	Securities & investment	27.94%
2	Chemical products	14.51%
3	Utilities	11.17%
4	Insurance carriers	7.81%
5	Food, beverages & tobacco	7.62%
6	Misc manufacturing	4.00%

C. Between 2010 and 2020

1	Securities & investment	19.28%
2	Insurance carriers	11.17%
3	Air transportation	9.71%
4	Chemical products	9.57%
5	Food, beverages & tobacco	7.71%
6	Apparel & leather	7.32%
7	Oil & gas	5.07%
8	Misc. manufacturing	4.30%
	⋮	
64	Other real estate	-6.05%
65	Credit intermediation	-6.95%
66	Farms	-8.40%

Notes: Only sectors with more than 4% in absolute value are included.

Table 16: Labor Demand TT Covariance Decomposition by Industry

A. Between 1998 and 2020

1	Oil & gas extraction	18.49%
2	Chemical products	12.03%
3	Utilities	10.18%
4	Securities & investment	6.49%
5	Insurance carriers	4.77%
6	Petroleum & coal	4.16%
7	Computer & electronics	4.11%

B. Between 2002 and 2009

1	Oil & gas extraction	19.61%
2	Utilities	14.01%
3	Chemical products	12.48%
4	Insurance carriers	7.56%
5	Computer & electronicSs	6.21%
6	Securities & investment	5.67%
7	Food, beverages & tobacco	4.03%

C. Between 2010 and 2020

1	Oil & gas extraction	17.45%
2	Chemical production	12.09%
3	Utilities	6.61%
4	Petroleum & coal	4.52%
5	Food, beverages & tobacco	4.30%
6	Primary metals	4.09%

Notes: Only sectors with more than 4% in absolute value are included.

Table 17: Final Demand TT Covariance Decomposition by Industry

A. Between 1998 and 2020

1	Accommodation	31.06%
2	Apparel & leathers	29.34%
3	Arts, sports & museums	25.19%
4	Food services	23.58%
5	Air transportation	21.21%
	⋮	
66	Food, beverages & tobacco	-36.91%

B. Between 2002 and 2009

1	Motor vehicles	51.73%
2	Furniture	39.99%
3	Computer & electronics	25.94%
	⋮	
66	Food, beverages & tobacco	-16.00%

C. Between 2010 and 2020

1	Accommodation	54.88%
2	Arts, sports & museums	50.58%
3	Food services	46.49%
4	Apparel & leather	39.87%
5	Air transportation	33.81%
6	Hospitals	25.26%
7	Misc. professional services	21.43%
8	Recreational & gambling	19.01%
9	Ambulatory health care	18.04%
10	Petroleum & coal	16.49%
	⋮	
64	Misc. manufacturing	-26.47%
65	Computer & electronics	-31.90%
66	Food, beverages & tobacco	-59.61%

Notes: Only sectors with more than 16% in absolute value are included.

Table 18: Intermediate Demand TT Covariance Decomposition by Industry

A. Between 1998 and 2020

1	Securities & investment	12.24%
2	Chemical products	11.36%
3	Oil & gas extraction	9.91%
4	Hospitals	6.31%
5	Apparel & leather	5.61%
6	Utilities	5.25%
7	Misc. manufacturing	4.10%

B. Between 2002 and 2009

1	Chemical products	14.36%
2	Computer & electronics	14.09%
3	Oil & gas extraction	9.13%
4	Apparel & leather	8.96%
5	Movies & music	4.49%
6	Management of companies	4.41%
7	Misc. manufacturing	4.34%

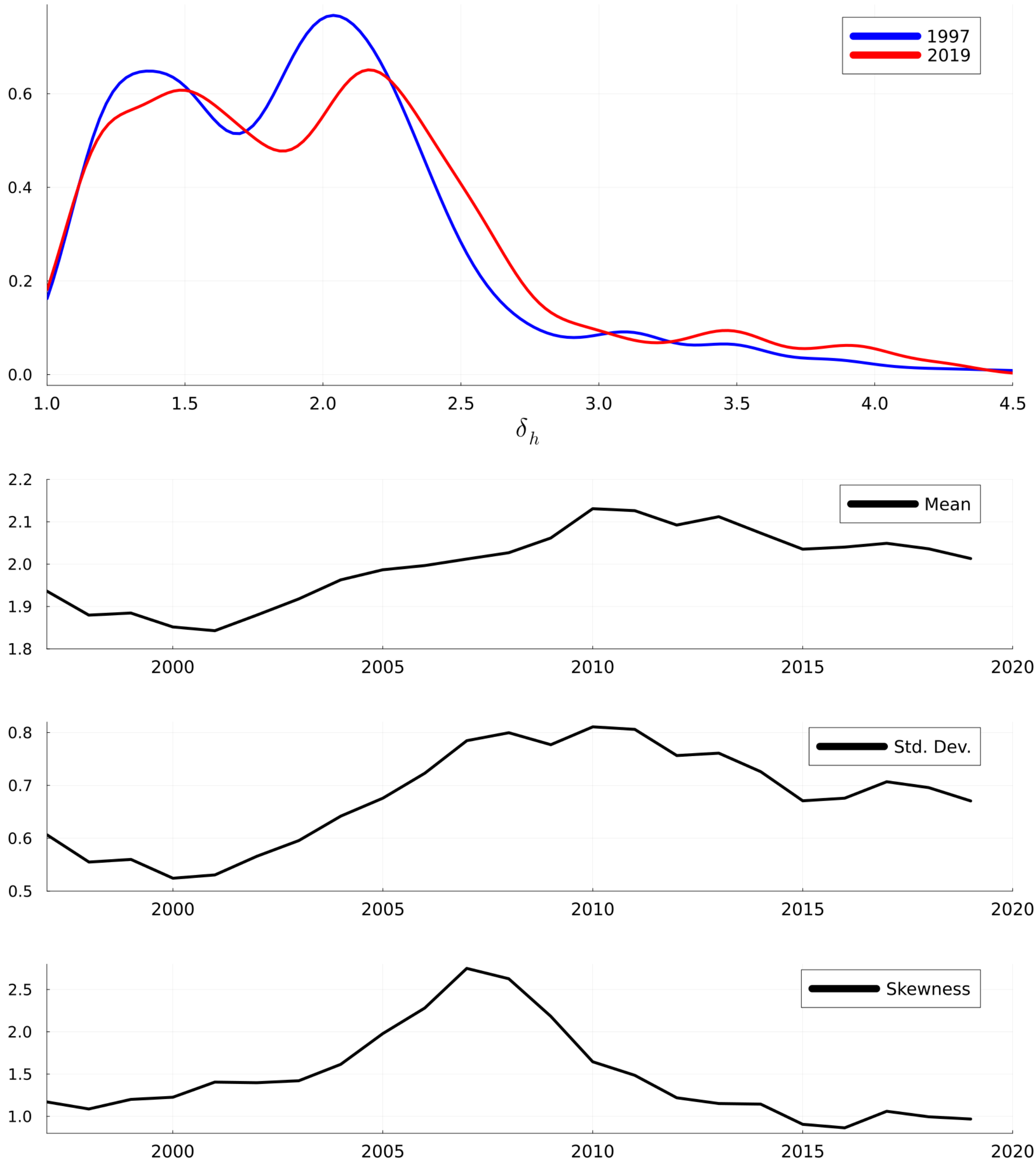
C. Between 2010 and 2020

1	Chemical products	13.71%
2	Oil & gas extraction	11.97%
3	Hospitals	9.52%
4	Accommodation	7.16%
5	Credit intermediation	7.13%
6	Air transportation	5.74%
7	Utilities	5.55%
8	Primary metals	4.50%
9	Ambulatory health care	4.36%
	⋮	
66	Food services	-6.00%

Notes: Only sectors with more than 4% in absolute value are included.

6.2.2 Distributional Accounting

Figure 6: Distortion Centrality Density and Moments

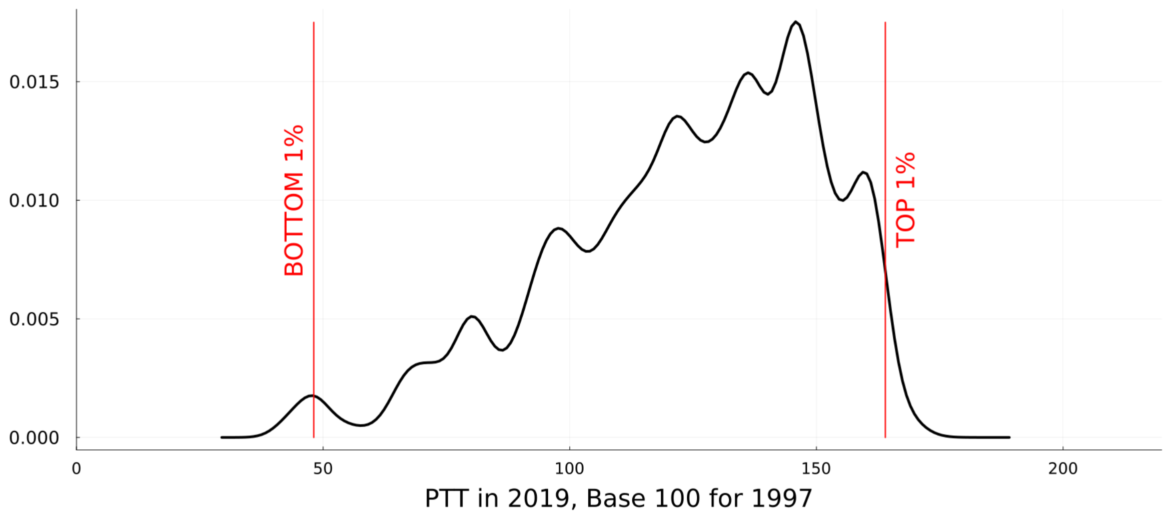


Notes:

Figure 6 portrays the density of the distortion centralities for 1997 and 2019, and the variations across time of its first three moments. These distributions had an average distortion centrality of 2, and their skewness was positive. Before the GR, the three moments increased, and after GR, there was a partial reversal in mean and variance, while the skewness had a full reversal to its original level from its 2007 peak.

Figure 7 portrays the density of PTTs for the state and occupation interaction under the assumption that the 1997 level of PTTs is 100 for all workers. On average, there has been growth in PTTs, and the density has a negative skewness, which captures a heavy left tail of workers for which the shocks from the last two decades have not been favorable. The tails from this distribution tell us that the last two decades of shocks, on the one hand, have favored logging workers, workers with mathematical and computational occupations, and compensation managers, and on the other hand, the same set of shocks have been unfavorable for industrial workers with occupation exposed to the printing, shoe and leather, and textile industries.

Figure 7: Positional Terms of Trade in 2019



Top 1%	
Occupation	
Logging Workers	37%
Computer Occ.	13%
Mathematical Sciences Occ.	10%
Compensation Managers	7%

Bottom 1%	
Occupation	
Printing Workers	40%
Shoe & Leather Operator	26%
Textile Machine Operator	15%
Miscellaneous Textile	12%

Notes:

7 Parametric Accounting

In this section, I derive the parametric ex-ante statistics necessary to characterize the first-order variations derived in Sections 4 and 5. These ex-ante measures depend on the model primitives. In this parametric environment, I identify a linear system of equations that solves the endogenous first-order variations in wages, household expenditure, and sales. This section finishes with a discussion about how the numeraire selection is non-neutral when the labor supply substitution and income effects are asymmetric.

7.1 Normalized CES Environment

Following Baqaee & Farhi (2019a,b, 2020, 2022), I extend the normalized CES function introduced by de La Grandville (1989) and Klump & de La Grandville (2000) to an economy with intermediate goods. The overlined variables correspond to the steady-state values. Firm z_i in sector i uses the normalized CES composite

$$\frac{y_{z_i}}{\overline{y}_{z_i}} = A_i \left(\omega_i^{\ell} \sum_{h \in \mathscr{H}} \alpha_{ih} \left(\frac{\ell_{z_i h}}{\overline{\ell}_{z_i h}} \right)^{\frac{\theta_i - 1}{\theta_i}} + \omega_i^{x} \sum_{j \in \mathscr{N}} \omega_{ij} \left(\frac{x_{z_i j}}{\overline{x}_{z_i j}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}.$$

In this production function, productivity shocks are Hicks-neutral normalized to 1 in equilibrium, and θ_i stands for the elasticity of substitution. Similarly, the consumption aggregator for the representative household of type h is given by

$$\frac{C_h}{\overline{C}_h} = \left(\sum_{i \in \mathscr{N}} \beta_{hi} \left(\frac{C_{hi}}{\overline{C}_{hi}} \right)^{\frac{\varrho_h - 1}{\varrho_h}} \right)^{\frac{\varrho_h}{\varrho_h - 1}},$$

where ϱ_h stands for the elasticity of substitution. The benefit from the normalized CES is that the parameters ω_i^{ℓ}, α_{ih}, ω_i^{x}, ω_{ij}, and β_{hi} have the same interpretation as in Section 3, and do not depend on deep parameters such as the elasticities of substitution (Klump et al., 2012).

Household h, which has an initial population size of n_h, operates under the following utility function

$$U_h\left(c_h, \widetilde{L}_h\right) = \frac{\left(c_h \left(1 - E_h^{-\gamma_h} \widetilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1 - \sigma},$$

with $C_h = n_h c_h$, $L_h = n_h \widetilde{L}_h$, and $\varphi_h > 0$. c_h and $\widetilde{L}_h$ represent the normalized real consumption and labor supply, which makes preferences independent from the population size. This utility function allows for greater flexibility in parametrizing the income and substitution effects on the labor supply.

Proposition 3. The change in labor supply from type h workers in response to demographic, wage, and income shocks is, to a first-order,

$$d\,log\,L_h = \zeta_h^n\, d\,log\,n_h + \zeta_h^w\, d\,log\,w_h - \zeta_h^e\, d\,log\,E_h.$$

Where the corresponding elasticities are given by

$$\zeta_h^n = \frac{E_h^{\gamma_h}}{1 - \varphi_h \gamma_h} \frac{n_h}{L_h}, \qquad \zeta_h^w = \frac{1}{1 - \varphi_h \gamma_h} \frac{\varphi_h}{\Gamma_h}, \qquad \zeta_h^e = \zeta_h^w - \gamma_h \zeta_h^n.$$

Proposition 3 characterizes the endogenous first-order variation of the labor supply in terms of elasticities for the: (1) demographic effect ζ_h^n; (2) substitution effect ζ_h^w; and (3) income effect ζ_h^e. These elasticities depend on equilibrium values and the deep preference parameters γ_h and φ_h.

This utility function nests the following preferences. First, by assuming $\gamma_h = 0$, I obtain King, Plosser, & Rebelo's (1988) preferences with symmetric substitution and income effects, more precisely, $\zeta_h^n = n_h / L_h$ and $\zeta_h^w = \zeta_h^e = \varphi_h / \Gamma_h$. Second, by using the preference parameters that solve $\gamma_h = \frac{1}{2\varphi_h}\left(1 + \Gamma_h^{-1/2}\sqrt{\Gamma_h - 2\varphi_h^2}\right)$, $\varphi_h = \frac{1}{\gamma_h}\left(1 - E_h^{\gamma_h} \frac{n_h}{L_h}\right)$and $\zeta_h^n = 1$, I obtain Greenwood, Hercowitz, & Huffman's (1988) preferences with no income effect, i.e., $\zeta_h^e = 0$. Finally, in its most general form, this utility is inspired by Jaimovich & Rebelo's (2009), and for this reason, it allows for asymmetric income and substitution effects. However, relative to the latter utility preferences, this specification allows for a direct effect from consumption expenditure in labor supply disutility through the parameter γ_h. The disutility effects from increasing the labor supply become weaker as this parameter increases, and as a consequence, there are stronger demographic and substitution effects.

7.2 Sufficient Endogenous Statistics

Theorem 5 characterizes a $2H+N$ linear system of equations that solves for the endogenous first-order variation of consumption expenditure, wages, and sales. These equations capture partial (PE) and general (GE) equilibrium effects.

Theorem 5. In a CES economy, the variation in consumption expenditure, wages, and sales, in response to productivity, distortion, and demographic shocks are, to a first-order,

$$d\log E_h = \overbrace{\frac{\zeta_h^n \Gamma_h}{1+\zeta_h^e \Gamma_h}\, d\log n_h}^{\substack{\textit{Demographic Effect} \\ \textit{on Expenditure (PE)}}} + \overbrace{\frac{(1+\zeta_h^w)\,\Gamma_h}{1+\zeta_h^e \Gamma_h} d\log w_h}^{\substack{\textit{Wage Effect} \\ \textit{on Expenditure (GE)}}} + \overbrace{\sum_{i\in\mathcal{N}} \frac{\kappa_{ih}\lambda_i}{(1+\zeta_h^e\Gamma_h)\,\chi_h}\left((1-\mu_i)\, d\log S_i - \mu_i\, d\log \mu_i\right)}^{\substack{\textit{Corporate Income Effect} \\ \textit{on Expenditure (PE + GE)}}};$$

$$\begin{aligned}
d\log w_h = {}& \overbrace{\frac{\zeta_h^e}{1+\zeta_h^w}\, d\log E_h}^{\substack{\textit{Expenditure Effect} \\ \textit{on Wages (GE)}}} - \overbrace{\frac{\zeta_h^n}{1+\zeta_h^w} d\log n_h}^{\substack{\textit{Demographic Effect} \\ \textit{on Wages (PE)}}} + \overbrace{\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{(1+\zeta_h^w)\,\Lambda_h}\left((\theta_i-1)\, d\log A_i + \theta_i\, d\log \mu_i\right)}^{\substack{\textit{Direct Effect} \\ \textit{on Wages (PE)}}} \\
& \overbrace{-\sum_{j\in\mathcal{N}} \left(\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{(1+\zeta_h^w)\,\Lambda_h}(\theta_i-1)\,\widetilde{\psi}_{ij}^{x}\right)(d\log A_j + d\log \mu_j)}^{\substack{\textit{Supplier Effect} \\ \textit{on Wages (PE)}}} + \overbrace{\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{(1+\zeta_h^w)\,\Lambda_h}\, d\log S_i}^{\substack{\textit{Sales Effect} \\ \textit{on Wages (GE)}}} \\
& \overbrace{-\left(\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{(1+\zeta_h^w)\,\Lambda_h}(\theta_i-1)\right) d\log w_h}^{\substack{\textit{Direct Substitution Effect} \\ \textit{on Wages (GE)}}} + \overbrace{\sum_{b\in\mathcal{H}} \left(\sum_{i\in\mathcal{N}} \frac{\Omega_{ih}^{\ell}\lambda_i}{(1+\zeta_h^w)\,\Lambda_h}(\theta_i-1)\,\widetilde{\psi}_{ib}^{\ell}\right) d\log w_b}^{\substack{\textit{Supplier Substitution Effect} \\ \textit{on Wages (GE)}}};
\end{aligned}$$

$$\begin{aligned}
d\log S_i = {}& \overbrace{\sum_{h\in\mathcal{H}} \frac{\beta_{hi}\chi_h}{\lambda_i}\, d\log E_h}^{\substack{\textit{Expenditure Effect} \\ \textit{on Sales (GE)}}} + \overbrace{\sum_{j\in\mathcal{N}} \frac{\Omega_{ji}^{x}\lambda_j}{\lambda_i}\, d\log S_j}^{\substack{\textit{Sales Effect} \\ \textit{on Sales (GE)}}} + \overbrace{\sum_{j\in\mathcal{N}} \frac{\Omega_{ji}^{x}\lambda_j}{\lambda_i}\left((\theta_j-1)\, d\log A_j + \theta_j\, d\log \mu_j\right)}^{\substack{\textit{Direct Effect} \\ \textit{on Sales (PE)}}} \\
& \overbrace{+\sum_{j\in\mathcal{N}} \left(\sum_{h\in\mathcal{H}} \frac{\beta_{hi}\chi_h}{\lambda_i}(\rho_h-1)\left(\widetilde{\psi}_{ij}^{x} - \widetilde{\mathscr{B}}_{hj}\right) + \sum_{q\in\mathcal{N}} \frac{\Omega_{qi}^{x}\lambda_q}{\lambda_i}(\theta_q-1)\left(\widetilde{\psi}_{ij}^{x} - \widetilde{\psi}_{qj}^{x}\right)\right)(d\log A_j + d\log \mu_j)}^{\substack{\textit{Supplier Effect} \\ \textit{on Sales (PE)}}} \\
& \overbrace{+\sum_{h\in\mathcal{H}} \left(\sum_{b\in\mathcal{H}} \frac{\beta_{bi}\chi_b}{\lambda_i}(\varrho_b-1)\left(\mathscr{C}_{bh} - \widetilde{\psi}_{ih}^{\ell}\right) + \sum_{j\in\mathcal{N}} \frac{\Omega_{ji}^{x}\lambda_j}{\lambda_i}(\theta_j-1)\left(\widetilde{\psi}_{jh}^{\ell} - \widetilde{\psi}_{ih}^{\ell}\right)\right) d\log w_h}^{\substack{\textit{Supplier Substitution Effect} \\ \textit{on Sales (GE)}}}.
\end{aligned}$$

For households of type h, the first-order variation for their consumption expenditure depends on three channels. First, for the *demographic effect*, in response to an increase in their labor force, the factoral supply will rise by ζ_h^n, and its effect on consumption expenditure is proportional to the labor income share Γ_h. Second, for the *wage effect*, a wage increase directly impacts income. However, it additionally triggers a substitution effect on the labor supply captured by ζ_h^w. The magnitude of this substitution effect on consumption expenditure is proportional to the labor income share Γ_h. Finally, for the *corporate income effect*, dividends from sector i depend both on sales and their markdowns: (i) an increase in sales augments dividend income by the rent extraction share $1-\mu_i$, and (ii) an increase in markdowns reduces profits by the cost share μ_i. These two paths for dividend income variation are proportional to the equity participation share κ_{ih} and the sales-to-expenditure ratio λ_i/χ_h. These three channels increase consumption expenditure and trigger an income effect that reduces the labor supply attenuating their magnitudes by $1+\zeta_h^e\Gamma_h$.

For workers of type h, the first-order variation for their wages depends on seven channels. These channels trigger a substitution effect that increases the labor supply and attenuates their influence on wages by $1+\zeta_h^w$. Additionally, the effect on w_h from the channels that depict variations in sector i's labor demand are proportional to the direct revenue-based centrality Ω_{ih}^ℓ and the sales to labor income ratio λ_i/Λ_h. First, for the *expenditure effect*, in response to an increase in their total income, their labor supply falls by ζ_h^e, and wages rise. Second, for the *demographic effect*, in response to an increase in their labor force, their labor supply rises by ζ_h^n, and wages fall. Third, the *direct effect* captures the increase in labor demand for these workers from the firms that receive either productivity or markdown shocks. Firm i increases their demand for workers of type h in response to a positive productivity shock as long as there is substitutability in their production (i.e., $\theta_i > 1$) and in response to lower distortions as long as the production function is not Leontief (i.e., $\theta_i > 0$). Fourth, the *supplier effect* portrays the variations in firms' labor demand in response to productivity and markdown shocks to its intermediate input suppliers. Firm i decreases its demand for workers of type h, as long as there is substitutability in their production, in response to positive productivity shocks and markdown reductions to its direct or indirect intermediate supplier j. The magnitude of this effect is proportional to the cost-based firm-to-firm centrality $\widetilde{\psi}_{ij}^x$. Fifth, the *sales effect* characterizes how sales increases expand labor demand. Sixth, the *direct substitution effect* portrays the variation in firms' labor demand for workers of type h in response to variations in w_h. Firm i increases their demand for workers of type h when w_h falls and there is substitutability in production. Finally, the *supplier substitution effect* captures the variations in firms' labor demand for workers of type h in response to wage changes for all other workers. Firm i increases their demand for workers of type h when the wage from workers of type b rises and there is substitutability in production. The magnitude of this effect is proportional to the cost-based worker-to-firm centrality $\widetilde{\psi}_{ib}^\ell$.

For firms in sector i, the first-order variation for their sales depends on five channels. The channels that represent variation in the demand of final goods by households of type h are proportional to their consumption share β_{hi} and the Domar weight ratio χ_h/λ_i, and those that illustrate changes in the demand for intermediate goods by firms in sector j are proportional to the direct revenue exposure Ω_{ji}^x and the Domar weight ratio λ_j/λ_i. First, the *expenditure effect* captures how higher household expenditure increases demand for final goods. Second, the *sales effect* portrays how higher firms' sales increase demand for intermediate goods. Third, the *direct effect* characterizes the increase in intermediate input demand from firms that receive either productivity or markdown shocks. Firm j increases their demand for good i in response to positive productivity shocks as long as there is substitutability and in response to lower distortions as long as the production function is not Leontief. Fourth, the *supplier effect* characterizes the variations in households' and firms' demand for goods in response to productivity and markdown shocks to its direct or indirect suppliers. Under substitutability, household h and firm q increase their demand for good i in response to increases in productivity or markdowns to its direct or indirect supplier j if their cost-based centrality to firm j is smaller than the one that firms in sector i have. In other words, when firm j reduces its price, the demand by households of type h and firms from sector q for the good i rises if their cost-based exposure to the shock is weaker than the one from firms in sector i, i.e., $\widetilde{\psi}^{x}_{ij} > \widetilde{\mathscr{B}}_{hj}$ and $\widetilde{\psi}^{x}_{ij} > \widetilde{\psi}^{x}_{qj}$. Finally, the *supplier substitution effect* portrays the increase in households' and firms' demand for goods in response to wage variations. Household b and firm j increase their demand for good i in response to the increase in prices from higher wages for workers of type h if there is substitutability and their cost-based centralities to firm j are larger than the one that firms in sector i have, i.e., $\widetilde{\mathscr{C}}_{bh} > \widetilde{\psi}^{\ell}_{ih}$ and $\widetilde{\psi}^{\ell}_{jh} > \widetilde{\psi}^{\ell}_{ih}$.

The solution in Theorem 5 represents an alternative to Baqaee & Farhi's (2022) results for the following five reasons: (1) it does not require the production network covariance operator introduced by Baqaee & Farhi (2019a); (2) it utilizes the measures of centrality from the substochastic Markov chain; (3) it captures the influence of the labor supply demographic, substitution, and income elasticities; (4) it decomposes the channels from productivity, markdown, and wage variations in direct effects, and effects through intermediate input suppliers; and (5) the variations are expressed in nominal terms and not in Domar weights because using the nominal GDP as the numeraire is not required.

7.3 Numeraire Non-Neutrality

From Walras' Law, to solve the model, take $2H + N - 1$ of the equations in Theorem 5, and normalize the variation in this system by using Y as the numeraire, which implies that there are no variations in the global GDP deflator, i.e., $d\log p_Y = 0$. This follows Hulten (1978), Baqaee & Farhi (2019a), and Bigio & La'O (2020) who also use Y as the real unit of account. Now, as mentioned in Section 4, this is not the only normalization that the literature has used, as Baqaee & Farhi (2020, 2022) use nominal GDP as the numeraire.

Proposition 4 portrays the differences in GDP growth and household-level real consumption between using Y or nominal GDP as the numeraire.

Proposition 4. The differences between normalizing with $d\log p_Y = 0$ and $d\log GDP = 0$ are, to a first-order:

- $$\frac{d\log Y | d\log GDP = 0}{d\log Y | d\log p_Y = 0} = \sum_{h\in\mathscr{H}} \widetilde{\Lambda}_h \frac{1+\zeta^e_h}{1+\zeta^w_h};$$

- $(d\log C_h | d\log p_Y = 0) - (d\log C_h | d\log GDP = 0)$
$$= \frac{\sum_{q\in\mathscr{H}} \widetilde{\mathscr{C}}_{hq} \frac{\zeta^w_q - \zeta^e_q}{1+\zeta^w_q}}{\sum_{q\in\mathscr{H}} \widetilde{\Lambda}_q \frac{1+\zeta^e_q}{1+\zeta^w_q}} \left(\sum_{i\in\mathscr{N}} \widetilde{\lambda}_i \left(d\log \mathscr{A}_i + d\log \mu_i\right) + \sum_{b\in\mathscr{H}} \widetilde{\Lambda}_b \frac{\zeta^n_b \, d\log n_b - \zeta^e_b \, d\log \chi_b - d\log \Lambda_b}{1+\zeta^w_b} \right).$$

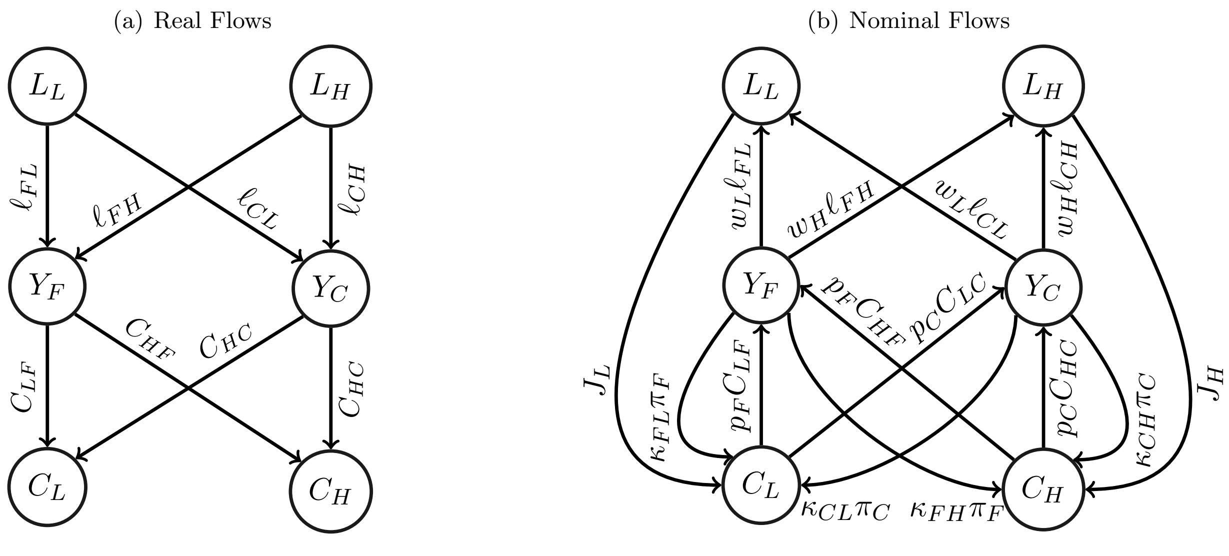
Proposition 4 characterizes the biases in growth between these two numeraire assumptions. The biases exist if there is an endogenous factor supply with asymmetric substitution and income effects. At the aggregate level, the bias is multiplicative, while at the household level, it is additive. The biases from assuming nominal GDP as the unit of account are positive if the income effect strictly dominates the substitution effect, i.e., $\zeta_h^e > \zeta_h^w \ \forall h \in \mathscr{H}$. One advantage of Theorems 1, 2, 3, and 4 over the comparable results in Baqaee & Farhi (2020, 2022), is that my derivations for these sufficient ex-post statistics do not require a normalization assumption. For this reason, they are independent of these biases.

8 Simple Horizontal Economy

In order to simplify the understanding of the main mechanisms behind the effects of the income and consumption expenditure distributions on aggregate TFP, I will use the following horizontal economy with two types of workers and two sectors. This example distills the model from this paper to the most basic structure for which there are still distributional effects on TFP.

Workers can be high- or low-skill, respectively identified with H and L, and firms can operate in the food service or healthcare sectors, i.e., $\mathscr{H} = \{H, L\}$ and $\mathscr{N} = \{F, C\}$. The production from firms within a sector follows a Cobb-Douglas production function $y_{z_i} = A_i \, \ell_{z_i H}^{\alpha_i} \, \ell_{z_i L}^{1-\alpha_i}$, where α_i is the sectoral expenditure share in high-skill workers. The consumption aggregator for each household follows a normalized CES where β_h captures in equilibrium the expenditure intensity in the food industry and ϱ the households' elasticity of substitution. Figure 8 represents the real and nominal flows for this economy.

Figure 8: Horizontal Economy



Note: Low-skill (L), high-skill (H), food industry (F), and healthcare (C).

Let me assume that:

1. Restaurants require more low-skill for the production of food services, while hospitals require more high-skill, i.e., $\alpha_C > \alpha_F$.
2. Low-skill households have a higher share of consumption expenditure in healthcare, while high-skill households have a higher expenditure share in restaurants, i.e., $\beta_H > \beta_L$.
3. Healthcare faces stronger distortions, i.e., $\mu_F > \mu_C$.
4. High- and low-skill households have the same equity, i.e., $\kappa_{F\,L} = \kappa_{C\,L} = 50\%$.

The equilibrium is the solution to a system of six equations on six unknowns that solve for each sector's Domar weights, labor income shares for each worker, and the absorption share for each household. This system is given by

$$\lambda_F = \beta_H\, \chi_H + \beta_L\, \chi_L, \qquad \lambda_C = (1-\beta_H)\, \chi_H + (1-\beta_L)\, \chi_L,$$

$$\Lambda_H = \mu_F\, \alpha_F\, \lambda_F + \mu_C\, \alpha_C\, \lambda_C, \qquad \Lambda_L = \mu_F\, \left(1-\alpha_F\right)\, \lambda_F + \mu_C\, \left(1-\alpha_C\right)\, \lambda_C,$$

$$\chi_h = \Lambda_h + \frac{1}{2} \sum_{i \in \mathscr{N}} (1-\mu_i)\, \lambda_i \quad \text{for} \quad h \in \mathscr{H}.$$

From Walras' law replace one of the last two equations with the accounting identify $\chi_H + \chi_L = 1$.

For this economy, bilateral centralities have an analytical representation given by

$$\mathscr{C}_{hb} = \beta_h\, \psi^{\ell}_{Fb} + (1-\beta_h)\, \psi^{\ell}_{Cb}, \quad \psi^{\ell}_{iH} = \mu_i\, \alpha_i, \quad \psi^{\ell}_{iL} = \mu_i\, (1-\alpha_i) \quad \forall h, b \in \mathscr{H} \quad \text{and} \quad \forall i \in \mathscr{N}.$$

To simplify the exposition, the only exogenous source of variation will be a productivity shock for the healthcare sector such that $d\,log\, A_C = 1\%$. The benchmark scenario will capture markdown, consumption bundle, and skill bias heterogeneity, and its parameters are $\alpha_C = \beta_H = \mu_F = 80\%$, $\alpha_F = \beta_L = \mu_C = 20\%$, $\zeta^w = 2$, and $\zeta^e = 0$. To simplify the exposition, the only exogenous source of variation will be a productivity shock for the healthcare sector such that $d\,log\, A_C = 1\%$. For this economy, the equilibrium is captured by

$$\lambda_F = 45.4\%, \qquad \lambda_C = 54.6\%, \qquad \Lambda_H = 16\%, \qquad \Lambda_L = 31.2\%, \qquad \chi_H = 42.4\%, \qquad \chi_L = 57.6\%,$$

$$\widetilde{\Lambda}_H = 52.7\%, \qquad \widetilde{\Lambda}_L = 47.3\%, \qquad \delta_H = 3.29, \qquad \delta_L = 1.51,$$

$$\mathscr{C}_{HL} = 52\%, \quad \mathscr{C}_{HH} = \mathscr{C}_{LL} = \mathscr{C}_{LH} = 16\%, \quad \psi_{FL} = 64\%, \quad \psi_{FH} = \psi_{CH} = 16\%, \quad \psi_{CL} = 4\%,$$

$$\mathscr{Z}_H = 1.31, \qquad \mathscr{Z}_L = 0.77, \qquad \mathscr{Z}_F = 1.49, \qquad \mathscr{Z}_C = 0.58.$$

The expenditure is biased toward the relatively inefficient healthcare sector. Labor income and consumption expenditure are higher for the low-skill even though the high-skill have a higher contribution in terms of value-added. The high-skill supply most of their labor to the heavily distorted healthcare sector, so their distortion centrality is high. Consequently, the average distortion centralities faced by the expenditure from the high-skill and the revenue from the food industry are higher.

From Theorem 1, the elasticities from the labor income shock in response to healthcare are given by

$$d\,\Lambda_H = \underbrace{\overbrace{(\mathscr{C}_{HH} - \mathscr{C}_{LH})}^{0}\, d\,\chi_H}_{\text{Distributive Income}_H} + \underbrace{\overbrace{(\psi_{FH} - \psi_{CH})}^{0}\,(\chi_H\, d\,\beta_H + \chi_L\, d\,\beta_L)}_{\text{Income Centrality}_H} = 0,$$

$$d\,\Lambda_L = \underbrace{\overbrace{(\mathscr{C}_{HL} - \mathscr{C}_{LL})}^{0.36}\, d\,\chi_H}_{\text{Distributive Income}_L} + \underbrace{\overbrace{(\psi_{FL} - \psi_{CL})}^{0.6}\,(\chi_H\, d\,\beta_H + \chi_L\, d\,\beta_L)}_{\text{Income Centrality}_L}.$$

On the one hand, the labor income share for the high-skill is inelastic. On the other hand, the elasticity from the labor income share for the low-skill increases with the consumption expenditure from the high-skill (*distributive income*) and households' expenditure share in the food industry (*income centrality*). The endogenous adjustment in the expenditure share in the food industry for each household depends on $d\,\beta_h = (\varrho - 1)\,\beta_h\,(1 - \beta_h)\,d\,log\,\frac{p_C}{p_F}\ \forall h \in \mathscr{H}$. According to this equation, under substitutability, households increase the intensity of their expenditure on the food industry as healthcare becomes relatively more expensive than restaurants. Furthermore, if the elasticities of substitution for the firms between types of workers were not unitary, income centrality would also depend on the sectoral variations for the cost shares across types of workers.

From Theorem 2, TFP growth is given by

$$d\,log\,TFP = \lambda_C - \underbrace{\delta_L\, d\,\Lambda_L}_{\text{Misallocation}} = \lambda_C - \underbrace{\overbrace{(\mathscr{Z}_H - \mathscr{Z}_L)}^{0.54}\, d\,\chi_H}_{\text{Distributive TT}} - \underbrace{\overbrace{(\mathscr{Z}_F - \mathscr{Z}_C)}^{0.91}\,(\chi_H\, d\,\beta_H + \chi_L\, d\,\beta_L)}_{\text{Final Demand TT}}.$$

Aggregate misallocation increases as the aggregate labor share rises, which happens when the absorption share for the high-skill (*distributive terms of trade*) or the households' expenditure share in the food industry increases (*final demand terms of trade*).

From Theorem 3, the difference in growth between idiosyncratic PTTs is given by

$$d\,log\,\frac{PTT_L}{PTT_H} = \underbrace{\beta_H - \beta_L}_{\Delta\,\beta} + \underbrace{d\,log\,\chi_L - d\,log\,\chi_H}_{\Delta\,\chi}$$
$$+ \underbrace{\overbrace{(\mathscr{C}_{HL} - \mathscr{C}_{LL})\,(\delta_{L|H} - \delta_{L|L})}^{>0}\, d\,\chi_H}_{\Delta\text{ Distributive TT}} + \underbrace{\overbrace{(\psi_{FL} - \psi_{CL})\,(\delta_{L|H} - \delta_{L|L})}^{>0}\,(\chi_H\, d\,\beta_H + \chi_L\, d\,\beta_L)}_{\Delta\text{ Final Demand TT}},$$

with $\delta_{L|h}\,\Lambda_L = \beta_h\,(1 - \alpha_F) + (1 - \beta_h)\,(1 - \alpha_C)$. Because $\alpha_C > \alpha_F$ and $\beta_H > \beta_L$, it is the case that $\delta_{L|H} > \delta_{L|H}$. The low-skill face a more favorable PTT in response to the productivity shock in healthcare as (i) their intensity in the expenditure in the healthcare sector relative to the high-skill rises, (ii) their income share increases, (iii) the income share for the high-skill rises, or (iv) the expenditure from households shifts towards the food industry.

From Theorem 5, the variations in sectors' sales, households' expenditure, and workers' wages are given by

$$\lambda_F \, d\log S_F = -(\varrho - 1)\,\aleph\,(d\log A_C - (\alpha_C - \alpha_F)\, d\log\,(w_H/w_L)) + \sum_{h\in\mathscr{H}} \beta_h\,\chi_h\, d\log E_h,$$

$$\lambda_C \, d\log S_C = (\varrho - 1)\,\aleph\,(d\log A_C - (\alpha_C - \alpha_F)\, d\log\,(w_H/w_L)) + \sum_{h\in\mathscr{H}} (1-\beta_h)\,\chi_h\, d\log E_h,$$

$$d\log E_h = \frac{(1+\zeta^w)\,\Gamma_h}{1+\zeta^e\Gamma_h} d\log w_h + \frac{1}{2}\frac{\sum_{i\in\mathscr{N}}\lambda_i\,(1-\mu_i)\,d\log S_i}{(1+\zeta^e\Gamma_h)\,\chi_h},$$

$$d\log w_h = \frac{\zeta^e}{1+\zeta^w} d\log E_h + \frac{\sum_{i\in\mathscr{N}}\psi_{ih}\,\lambda_i\,d\log S_i}{(1+\zeta^w)\,\Lambda_h},$$

for $h \in \mathscr{H}$ and where $\aleph = \beta_H\,(1-\beta_H)\,\chi_H + \beta_L\,(1-\beta_L)\,\chi_L$. Using real GDP as the numeraire normalizes variations relative to the GDP deflator, which allows me to replace one of the last equations with $\lambda_C = \sum_{h\in\mathscr{H}} \widetilde{\Lambda}_h \; d\log w_h$. Under substitutability, there is a positive partial equilibrium effect on the sales from healthcare and a negative partial equilibrium effect on the sales from restaurants. When high-skill workers become relatively more expensive, there is a negative general equilibrium on the sales from healthcare and a positive general equilibrium effect on the sales from restaurants.

I will compare this benchmark scenario with the following three alternative specifications for which I shut down one at the time of the types of heterogeneity. Figure 9 portrays for the benchmark and alternative specifications, under different values for ϱ, the elasticities in response to the productivity shock in healthcare for the labor income shares, absorption shares, Domar weights, the distributive terms of trade, the final demand terms of trade, TFP, and the difference in PTTs.

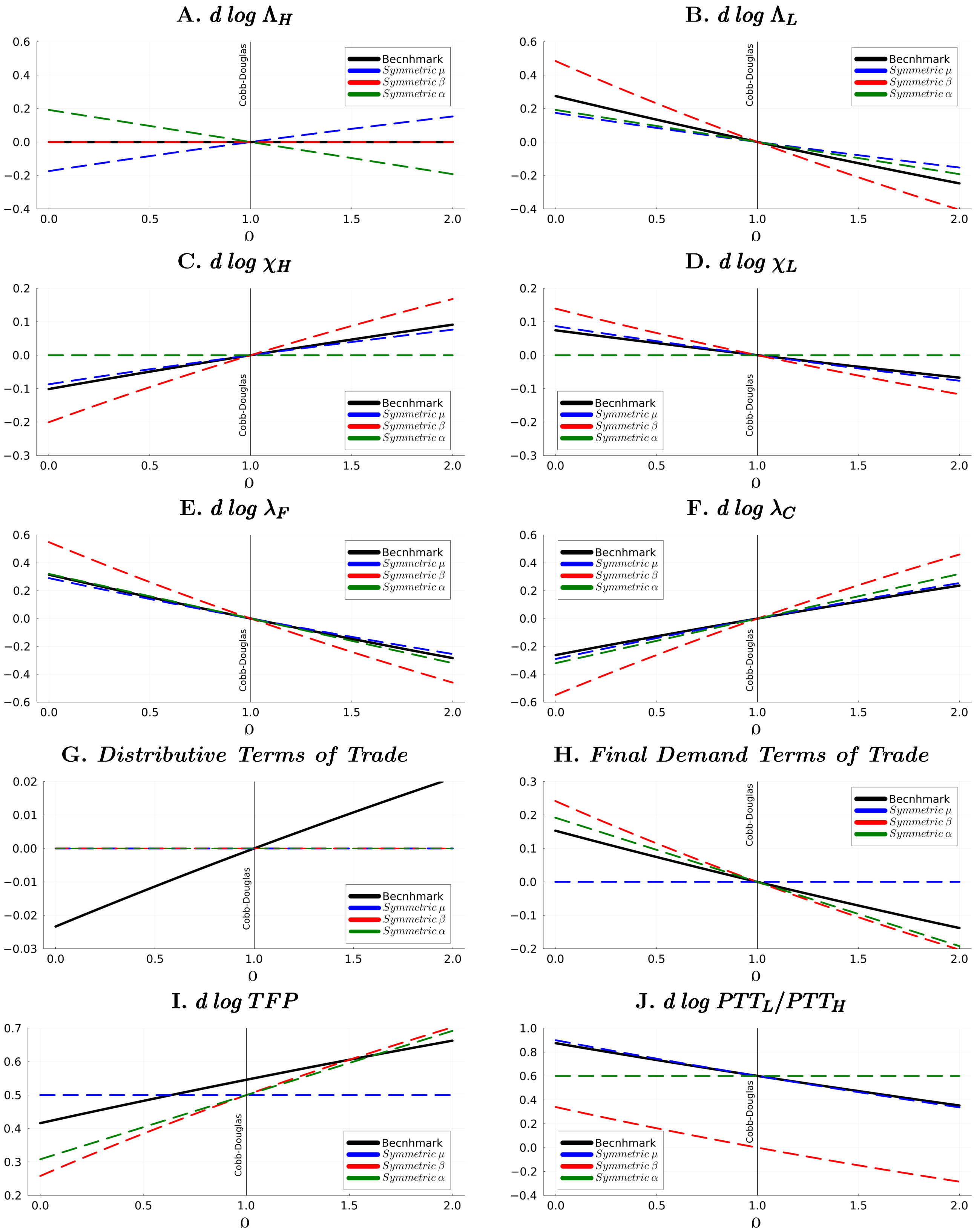
For the benchmark specification, under substitutability, consumers can shift towards the now more abundant healthcare sector. This effect increases the healthcare sector's Domar weight and the absorption share for the high-skill. Consequently, the food industry and the absorption share from the low-skill become smaller. The labor income share for the low-skill falls, while the labor income share for the high-skill is inelastic. The higher absorption share from the high-skill increases misallocation through higher distributive terms of trade. However, this effect is more than compensated by the negative final demand terms of trade from the substitution of expenditure towards the healthcare sector. For this reason, misallocation falls, and the TFP elasticity is higher than the healthcare sector's equilibrium Domar weight. The low-skill households face a more favorable increase in their PTT due to the higher exposure from their consumption bundle to the healthcare sector. However, the increase in the expenditure on healthcare partially reduces the advantage.

Case a: No heterogeneity in markdows, i.e., $\mu_F = \mu_C = 0.5$. For this economy, the equilibrium is given by

$$\lambda_F^a = \lambda_C^a = 50\%, \qquad \Lambda_H^a = \Lambda_L^a = 25\%, \qquad \chi_H^a = \chi_L^a = 50\%,$$

$$\widetilde{\Lambda}_H^a = \widetilde{\Lambda}_L^a = 50\%, \qquad \delta_H^a = \delta_L^a = 2,$$

Figure 9: Simple Horizontal Economy



Notes:

$$\mathscr{C}^a_{HL} = \mathscr{C}^a_{LH} = 34\%, \qquad \mathscr{C}^a_{HH} = \mathscr{C}^a_{LL} = 16\%, \qquad \psi^a_{FH} = \psi^a_{CL} = 10\%, \qquad \psi^a_{FL} = \psi^a_{CH} = 40\%,$$

$$\mathscr{Z}^a_H = \mathscr{Z}^a_L = \mathscr{Z}^a_F = \mathscr{Z}^a_C = 1.$$

Then, from Theorems 1, 2, and 3

$$d\,\Lambda_H + d\,\Lambda_L = 0, \qquad d\,log\,TFP = \lambda_C^a, \qquad d\,log\,\frac{PTT_L}{PTT_H} = \beta_H - \beta_L + d\,log\,\chi_L - d\,log\,\chi_H.$$

Under symmetric markdowns, the household consumption demand is the same as in the first-best equilibrium. Hence, there is no labor misallocation, distortion centralities are symmetric across workers, the aggregate labor income share is inelastic, and the TFP elasticity depends only on the technology effect portrayed by the Domar weight. The favorability of the PTT for the low-skill increases with their absorption share.

Case b: No heterogeneity in consumption bundles, i.e., $\beta_H = \beta_L = 0.5$. For this economy, the equilibrium is given by

$$\lambda_F^b = \lambda_F^b = 50\%, \qquad \Lambda_H^b = 16\%, \qquad \Lambda_L^b = 34\%, \qquad \chi_H^b = 41\%, \qquad \chi_L^b = 59\%,$$

$$\widetilde{\Lambda}_H^b = \widetilde{\Lambda}_L^b = 50\%, \qquad \delta_H^b = 3.12, \qquad \delta_L^b = 1.47,$$

$$\mathscr{C}_{HL}^b = \mathscr{C}_{LL}^b = 34\%, \quad \mathscr{C}_{HH}^b = \mathscr{C}_{LH}^b = 16\%, \quad \psi_{FL}^b = 64\%, \quad \psi_{FH}^b = \psi_{CH}^b = 16\%, \quad \psi_{CL}^b = 4\%,$$

$$\mathscr{Z}_H^b = \mathscr{Z}_L^b = 1, \qquad \mathscr{Z}_F^b = 1.44, \qquad \mathscr{Z}_C^b = 0.55.$$

Then, from Theorems 1, 2, and 3

$$d\,\Lambda_H = 0, \qquad d\,\Lambda_L = \overbrace{\left(\psi_{FL}^b - \psi_{CL}^b\right)}^{0.6}\left(\chi_H\,d\,\beta_H + \chi_L\,d\,\beta_L\right),$$

$$d\,log\,TFP = \lambda_C^c - \underbrace{\overbrace{\left(\mathscr{Z}_F^b - \mathscr{Z}_C^b\right)}^{0.89}\left(\chi_H\,d\,\beta_H + \chi_L\,d\,\beta_L\right)}_{\text{Final Demand TT}},$$

$$d\,log\,\frac{PTT_L}{PTT_H} = d\,log\,\chi_L - d\,log\,\chi_H.$$

Under symmetric consumption bundles, the distributive income effect on the low-skill income share is mute as the bilateral centrality from the expenditure of high- and low-skill households on the low-skill labor income is symmetric. The average expenditure centrality that the expenditure from both households encounters is symmetric; consequently, the distributive terms of trade do not affect TFP. Misallocation increases through the final demand terms of trade as consumption expenditure shifts towards the healthcare sector, which happens when there is complementarity in preferences. The low-skill households no longer face a more favorable PTT than the high-skill, and under substitutability, they are now worse off than the high-skill due to their lower absorption share.

Case c: No skill-bias heterogeneity, i.e., $\alpha_F = \alpha_C = 0.5$. For this economy, the equilibrium is the same as in alternative a. However, the bilateral cetralities now are different

$$\mathscr{C}^c_{HH} = \mathscr{C}^c_{HL} = 34\%, \qquad \mathscr{C}^c_{LH} = \mathscr{C}^c_{LL} = 16\%, \qquad \psi^c_{FH} = \psi^c_{FL} = 40\%, \qquad \psi^c_{CH} = \psi^c_{CL} = 10\%,$$

$$\mathcal{Z}^c_H = 1.36, \qquad \mathcal{Z}^c_L = 0.64, \qquad \mathcal{Z}^c_F = 1.6, \qquad \mathcal{Z}^c_C = 0.4.$$

Then, from Theorems 1, 2, and 3

$$d\,\Lambda_H = d\,\Lambda_L, \qquad d\log TFP = \lambda^c_C - \underbrace{\overbrace{\left(\mathcal{Z}^c_F - \mathcal{Z}^c_C\right)}^{1.2} \left(\chi_H\, d\,\beta_H + \chi_L\, d\,\beta_L\right)}_{\text{Final Demand TT}}, \qquad d\log \frac{PTT_L}{PTT_H} = \beta_H - \beta_L.$$

Under symmetric skill bias, the elasticity in the labor income for high- and low-skill workers is symmetric. Consequently, the absorption shares are inelastic, and the distributive terms of trade are mute. Misallocation increases through the final demand terms of trade as consumption expenditure shifts towards the healthcare sector. The low-skill households face a more favorable increase in their PTT only because of the higher exposure from their consumption bundle to the healthcare sector.

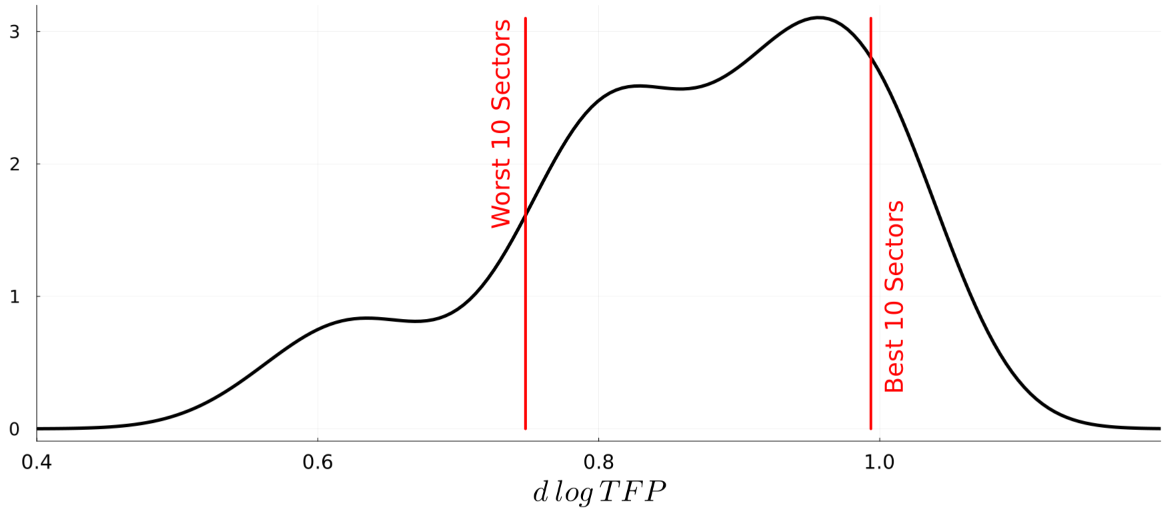
9 Counterfactual Industrial Policy

Section 3 in the Online Appendix provides a nested CES extension to the model from Section 7. This section parameterizes such a model using the following elasticities of substitution, which are consistent with the values estimated and used throught the the inpput-output literature (Boehm et al., 2014; Atalay, 2017; Baqaee, 2018; Baqaee & Farhi, 2020). I assume for all sectors an unitary elasticity of substitution between types of labor, an elasticity of substitution of 0.2 between intermediate inputs, an elasticity of substitution of 0.5 between the labor and intermediate input aggregates, and an elasticity of substitution of 0.9 for the consumption aggregators.

This parametric setting allows me to discipline the endogenous variations in the model and estimate the aggregate and distributional effects from a manifold of sectoral shocks. Here, I evaluate the effects of two shocks: a sectoral productivity shock such that aggregate technology equals 1% and a sectoral increase in markdowns such that aggregate competitiveness equals 1%. The 1% assumption allows me to make their effect comparable, as the differences in TFP will depend exclusively on the asymmetric response from misallocation to the endogenous variations in the income distribution.

Figure 10 displays the density for TFP elasticities in response to sectoral productivity shocks of a magnitude such that aggregate technology equals 1%. If the costs from stimulating a productivity shock of this magnitude were symmetric across sectors, on the one hand, the best technological shocks would be to the healthcare, social assistance, retail, computer design, and education industries. On the other hand, the worst technological shocks would be to extractive, chemical, utilities, farms, real estate, and paper industries. In particular, almost 45% of the initial technological stimulus from a productivity shock to the oil and gas extraction industry is lost due to higher labor misallocation.

Figure 10: $dlogTFP$ density in response to sectoral productivity shock



	Best 10 Sectors	
1	Nursing & residential care	1.035%
2	Social assistance	1.033%
3	Merchandise stores	1.027%
4	Hospital	1.022%
5	Ambulatory health care	1.021%
6	Computer systems design	1.013%
7	Apparel & leather	1.008%
8	Food & beverage stores	1.005%
9	Educational services	0.998%
10	Other retail	0.993%

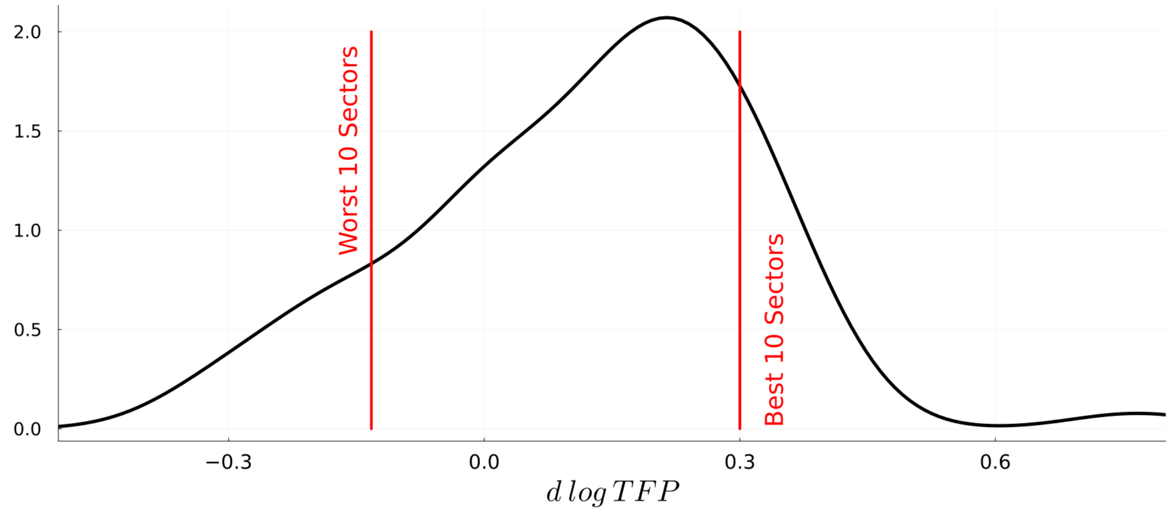
	Worst 10 Sectors	
1	Oil & gas extraction	0.558%
2	Primary metals	0.595%
3	Chemical products	0.601%
4	Mining, except oil & gas	0.616%
5	Utilities	0.628%
6	Petroleum & coal	0.639%
7	Farms	0.658%
8	Rental & leasing	0.680%
9	Other real estate	0.715%
10	Paper products	0.747%

Notes:

Figure 11 displays the density for TFP elasticities in response to sectoral markdown shocks of a magnitude such that aggregate competitiveness equals 1%. If the costs from stimulating competition by this magnitude were symmetric across sectors, on the one hand, the best antitrust interventions would be in the housing, extractive, chemical, and rental industries. On the other hand, it would be an awful idea to push for antitrust interventions in healthcare, social assistance, retail, computer design, recreation, and education sectors. For the last set of industries, the corresponding increase in misallocation more than washes off the gains in aggregate competitiveness.

Table 19 report the results from OLS regressions for the TFP elasticities estimated in Figures 10 and 11 on sectoral Domar weights, markdowns, and the average distortion centrality that the revenue from a sector faces. Not surprisingly, the 1% normalization makes the Domar weights insignificant. Markdowns and the average distortion centrality of the revenue from a sector have a positive correlation with the TFP response to productivity shocks and a negative correlation with the TFP response to higher competition. However, in the multivariate regressions, the average distortion centrality of the revenue from a sector captures the positive correlation with the effect of productivity shocks, and the markdown captures the negative correlation with the TFP response to higher competition. The latter results show that industrial policies that incentivize productivity should aim for high $\mathcal{Z}_i^\ell$ sectors. In contrast, antitrust policies that increase competition should target low μ_i industries.

Figure 11: $d\log TFP$ density in response to sectoral markdow shock



	Best 10 Sectors	
1	Housing	0.766%
2	Credit intermediation	0.409%
3	Furniture	0.376%
4	Pipeline transportation	0.360%
5	Oil & gas extraction	0.355%
6	Mining, except oil & gas	0.349%
7	Primary metals	0.342%
8	Petroleum & coal	0.328%
9	Chemical products	0.316%
10	Rental & leasing	0.300%

	Worst 10 Sectors	
1	Nursing & residential care	-0.329%
2	Social assistance	-0.303%
3	Merchandise stores	-0.274%
4	Hospital	-0.219%
5	Ambulatory health care	-0.201%
6	Educational services	-0.191%
7	Apparel & leather	-0.163%
8	Computer systems design	-0.154%
9	Recreational & gambling	-0.135%
10	Food & beverage stores	-0.132%

Notes:

Table 19: Aggregate TFP on sectoral characteristics

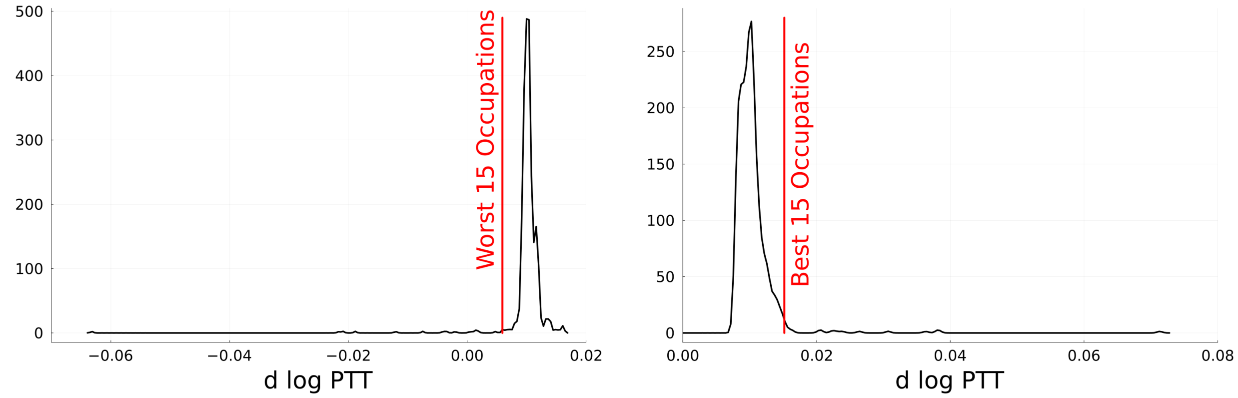
	$\frac{d\log TFP}{d\log A_i}$ *from Figure 10*				$\frac{d\log TFP}{d\log \mu_i}$ *from Figure 11*			
	(1)	**(2)**	**(3)**	**(4)**	**(1)**	**(2)**	**(3)**	**(4)**
λ_i	0.149			0.905*	1.331			-0.084
	(0.591)			(0.523)	(0.941)			(0.736)
μ_i		0.391***		0.198		-0.943***		-0.925***
		(0.094)		(0.134)		(0.125)		(0.189)
Z_i^ℓ			0.241***	0.181**			-0.401***	-0.016
			(0.052)	(0.076)			(0.083)	(0.107)
Intercept	0.860***	0.562***	0.599***	0.486***	0.084**	0.848***	0.563***	0.855***
	(0.022)	(0.074)	(0.059)	(0.079)	(0.035)	(0.098)	(0.094)	(0.111)
N	66				66			
R^2	0.001	0.212	0.245	0.301	0.030	0.471	0.266	0.471
Adj. R^2	0.001	0.245	0.217	0.279	0.030	0.471	0.266	0.454

Notes:

Figure 12 displays the density for the PTT elasticities in response to the productivity and markdown shocks for the oil and gas industry estimated in Figures 10 and 11. In response to the productivity shock, the distribution of PTTs has a negative skewness and a positive skewness in response to the competition shock. The long tails from these distributions capture the effect on fifteen occupations heavily exposed to the oil and gas extraction sector. The complementarity in households' preferences (i.e., $\varrho < 1$) explains the difference in terms of skewness. On the one hand, the productivity shock introduces a supply shock that increases the quantity of goods. However, preference complementarity forces households to substitute their expenditure towards relatively inefficient sectors. Consequently, the final demand for oil and gas extraction falls, and correspondingly, the labor income share for occupations with heavy exposure to this sector shrink. On the other hand, an increase in competition introduces a labor demand shock that raises the labor income share for occupations with heavy exposure to the oil and gas extraction industry. Notice that the occupations that face the worst PTT elasticities in response to the productivity shock are almost the same occupations that face the best PTT elasticities in response to the increase in competition. The bilateral centrality from the revenue of the oil and gas extraction sector on the labor income from these occupations is high.

Figure 12: Density of *d log PTT* to Oil & Gas Extraction Shocks

A. Productivity Shock | **B. Markdown Shock**



Worst 15 Occupations	
1. Wellhead Pumpers	-6.32%
2. Service Unit Operators, Oil & Gas	
3. Petroleum Engineers	-2.15%
4. Rotary Drill Operators, Oil & Gas	-2.08%
5. Roustabouts, Oil & Gas	-1.88%
6. Geoscientists	-1.20%
7. Hydrologic Technicians	-0.74%
8. Geological Technicians	-0.40%
9. Mining & Geological Engineers	-0.35%
10. Gas Compressor & Pumping Station Operators	-0.20%
11. Extraction Workers, All Other	0.04%
12. Rentier	0.14%
13. Gas Plant Operators	0.19%
14. Petroleum Pump System Operators Refinery Operators, and Gaugers	0.47%
15. Pump Operators, Except Wellhead Pumpers	0.59%

Best 15 Occupations	
1. Wellhead Pumpers	7.13%
2. Service Unit Operators, Oil & Gas	3.82%
3. Petroleum Engineers	3.81%
4. Rotary Drill Operators, Oil & Gas	3.63%
5. Roustabouts, Oil & Gas	3.07%
6. Geoscientists	2.64%
7. Hydrologic Technicians	2.38%
8. Geological Technicians	2.28%
9. Mining & Geological Engineers	2.22%
10. Extraction Workers, All Others	2.06%
11. Petroleum Pump System Operators Refinery Operators, and Gaugers	2.04%
12. Gas Plant Operators	1.63%
13. Gas Compressor & Pumping Station Operators	1.60%
14. Pump Operators, Except Wellhead Pumpers	1.55%
15. Pourers and Casters, Metal	1.51%

Notes:

10 Allocative Neutrality

Theorem 6 identifies four general classes of economies for which there are zero first-order aggregate gains from the reallocation of resources. These cases allow me to characterize the primitives necessary to obtain changes in the income and consumption distributions that allow for non-technological growth. By allocative neutrality I mean that $\mathit{Competitiveness} = \mathit{Misallocation}$, and consequently $d\log TFP = \mathit{Technology}$.

Theorem 6. For the following economies and shocks, allocative neutrality is satisfied:

1. For a Cobb-Douglas economy ($\theta_i = \varrho_h = 1\ \forall i \in \mathscr{N}$ and $\forall h \in \mathscr{H}$) in response to productivity or demographic shocks.

2. For a Leontief economy ($\theta_i = \varrho_h = 0\ \forall i \in \mathscr{N}$ and $\forall h \in \mathscr{H}$) with inelastic labor supply in response to a markdown shock if: (i) payment centrality is symmetric across households, i.e., $\mathscr{C}_h = \tau \in (0, 1]\ \forall h \in \mathscr{H}$; or (ii) $\mathscr{C}$ is nonsingular and $\mathscr{B}\,\Omega_\pi$ has its eigenvalues within the unit circle.

3. For a horizontal economy with symmetric distortions ($\mu_i = \mu\ \forall i \in \mathscr{N}$) in response to productivity, markdown, and demographic shocks.

4. In a vertical economy in response to productivity, markdown, and demographic shocks.

1. Cobb-Douglas Neutrality

For the class of Cobb-Douglas economies, there is no first-order variation in aggregate misallocation in response to exogenous supply shocks. Technological shocks that change the productivity from a sector or demographic shocks are allocative-neutral on aggregate TFP because the sales, labor income, and absorption shares are inelastic. Consequently, there is also distributional allocative-neutrality and $d\log PTT_h = \mathit{Technology}_h\ \forall h \in \mathscr{H}$. This result extends the Cobb-Douglas neutrality benchmark from Baqaee & Farhi (2020) to an environment with heterogeneous households and endogenous labor supply.

2. Leontief Neutrality

For the class of Leontief economies with inelastic labor supplies, shocks in markdowns are allocative neutral if one of the two conditions introduced by Theorem 6 are satisfied. However, before discussing the merits and implications of these conditions, let me build up some base intuition. Baqaee & Farhi (2020) establish that markdown shocks are allocative neutral for a representative household Leontief economy with inelastic labor. The reason is that regardless of prices, the household will consume fixed ratios of goods, and the firms will demand fixed ratios of labor and intermediate inputs. As a result, variations in distortions influence prices but not the demand for final goods or intermediate inputs. Consequently, the allocation of workers across firms does not change in response to markdown variations.

Extending this result to an environment with heterogeneous households is more complex. The reason is that any endogenous shift in consumption expenditure between households with heterogenous consumption bundles will modify aggregate final demand and the allocation of workers across firms. However, we know that in response to the markdown shocks, up to a first-order, the endogenous reallocation of real consumption for this class of economies will satisfy

$$0 = \sum_{h \in \mathscr{C}} \chi_h \, \mathscr{C}_h \, d\,log\, C_h. \tag{24}$$

Hence, from Theorem 2, allocative neutrality will be satisfied if the payment centrality from households is symmetric and real GDP will be inelastic, i.e., $d\,log\,Y = \sum_{h \in \mathscr{C}} \chi_h \, d\,log\,C_h = 0$. Notice that payment centralities are symmetry when consumption bundles are homogenous.

Now, the problem is that symmetry between households in their payment centralities is an extremely restrictive condition. In general, Equation (24) implies that

$$d\,log\,Y = \sum_{h \in \mathscr{C}} \chi_h \, (1 - \mathscr{C}_h) \, d\,log\,C_h.$$

This last equation tells us that for a Leontief economy with inelastic labor that faces shocks in markdowns, there is space for GDP growth through the reallocation of workers, when real consumption is endogenously shifted towards households with relatively high consumption expenditure and small payment centralities. The households with small payment centralities are the ones who have a higher share of their expediture reaching income through corporate profits. Consequently, increasing consumption expenditure to these households has the largest negative effect on the aggregate labor wedge.

Furthermore, aggregate distributive neutrality for this class of models is also guaranteed whenever the consumer-to-worker upstream centrality matrix $\mathscr{C}$ is nonsingular and $\mathscr{B}\,\Omega_\pi$ has its eigenvalues within the unit circle. Ω_π is a $N \times H$ matrix, where its ih element is given by $(1 - \mu_i)\,\kappa_{ih}$, i.e., the share of revenue from sector i that reaches income for households of type h through corporate profits. First, the invertibility of $\mathscr{C}$ portrays that there needs to be a sufficiently high level of heterogeneity in consumption bundles. Second, having all of the eigenvalues from $\mathscr{B}\,\Omega_\pi$ within the unit circle implies that its determinant is less than one, and consequently $\sum_{q=0}^{\infty} (\mathscr{B}\,\Omega_\pi)^q = (I - \mathscr{B}\,\Omega_\pi)^{-1}$. Notice that the hb element from $\mathscr{B}\,\Omega_\pi$ is given by $\sum_{i \in \mathscr{N}} \mathscr{B}_{hi}\,(1 - \mu_i)\,\kappa_{ib}$ and captures the share of expenditure from households of type h that reaches the income from households of type b through corporate profits. In other words, the second condition imposes an equilibrium convergence criteria according to which there can be no explosive paths from consumption expenditure to dividend income. Here the Gershgorin circle theorem is useful, as it tells us that all eigenvalues for a matrix are within the unit circle if the off-diagonal element summation for each of its rows is less than one (Gershgorin, 1931). For row h, the off-diagonal elements add up to $\sum_{i \in \mathscr{N}} \mathscr{B}_{hi}\,(1 - \mu_i)\,(1 - \kappa_{ih})$. For an economy without intermediate inputs, this condition holds as $\sum_{i \in \mathscr{N}} \mathscr{B}_{hi} \leq 1$. However, the proof escapes me for an economy with intermediate inputs, mainly because consumer-to-firm upstream centralities can be larger than one.

3. Horizontal Economy

For the class of general horizontal economies with N sectors and H households, allocative neutrality in response to productivity, markdown, and demographic shocks is satisfied if distortions are symmetric across sectors. The reason is that under symmetric markdowns, the final demand from households across sectors is the same as in the first-best equilibrium; distortions cancel out in the households' marginal rates of substitution. For this reason, the allocation of workers across firms is already efficient, and up to a first-order, the endogenous reallocation from workers in response to any of these three types of shocks is neutral on TFP.

Bigio & La'O (2020) prove that for a horizontal representative household economy with symmetric distortions, one type of labor, and endogenous labor supply, shocks in sectoral distortions are neutral on TFP. Theorem 6 extend this result to productivity and demographic shocks and a heterogenous household economy with multiple types of labor.

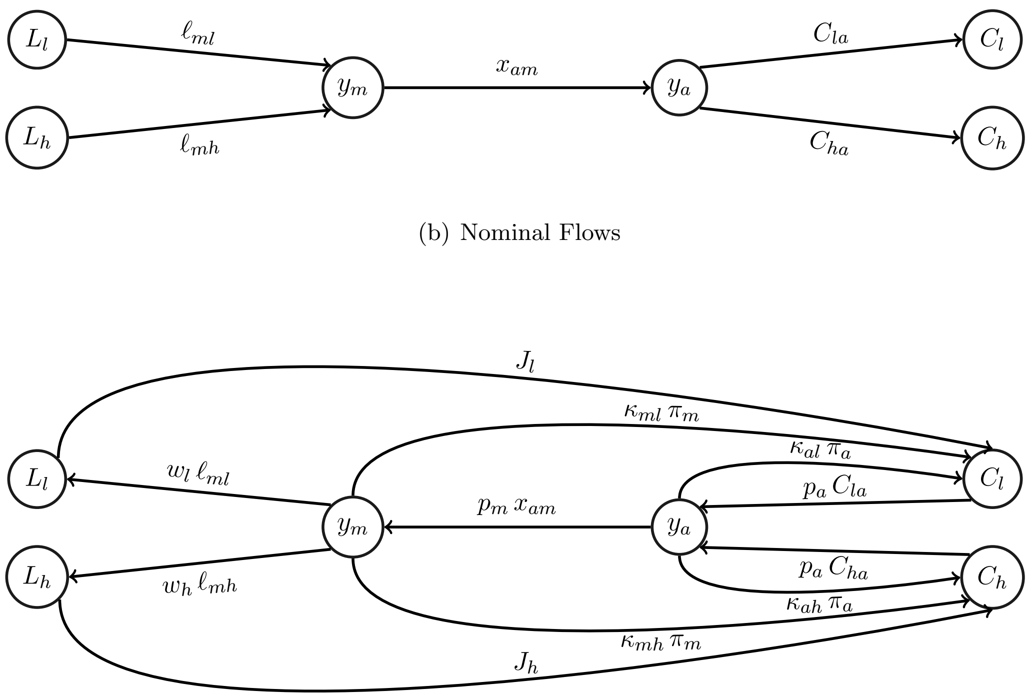
To understand how the presence of intermediate inputs would alter this result, let me get back to the simple horizontal economy introduced in Section 8, with the additional assumption that the healthcare sector demands meals from restaurants. Under symmetric distortions, household final demand is the same as in the first-best equilibrium. However, the distortions alter the healthcare sector's marginal rates of substitution between labor demand and intermediate inputs, and the allocation of workers no longer necessarily coincides with the undistorted economy.

4. Vertical Economy

Figure 13: Vertical Economy

(a) Real Flows

(b) Nominal Flows



Note: Low-skill (l), high-skill (h), manufacturing (m), and agriculture (a).

For the class of vertical economies, productivity, markdown, and demographic shocks are allocative neutral. Notice that this result is independent of the markdowns that sectors face. Figure 13 represents a vertical economy with two firms and two households. The manufacturing firm demands labor from high- and low-skill workers and supplies intermediate inputs to the agricultural firm. Households only consume agricultural goods.

Bigio & La'O (2020) prove that for a vertical representative household economy with one type of endogenous labor, shocks in sectoral distortions are neutral on TFP. Theorem 6 extend this allocative neutrality result to productivity and demographic shocks and to a heterogenous household economy with multiple types of labor. The reason is that in a vertical economy, workers are hired only by the most upstream firm, they have nowhere else to go, and their allocation coincides with the first-best equilibrium.

11 Conclusion

In this paper, I build an aggregation theory for a general production network economy with heterogeneous households and endogenous labor supply. I provide nonparametric characterizations of the local effects that endogenous variations in the income distribution, the consumption expenditure distribution, and the demand structure from firms and households have on aggregate TFP and the households' positional terms of trade. These results show that the channels via which expenditure enters and flows through the economy matter as they influence the allocation of workers across firms. Furthermore, under distortions, the decentralized decision from households about the level of their labor supply introduces externalities on aggregate welfare. A constrained social planner that centralizes household decisions could solve these externalities by making all workers symmetrically undervalued.

The first empirical implementation of a production network environment with heterogenous households for the United States allows me to quantitatively implement the sufficient statistics that decompose the growth of TFP. Not surprisingly, the aggregate increase of TFP during the first two decades of the XXI century has been technologically driven. However, the distributional effects on TFP have been relevant during specific business cycles. Distributionally driven TFP fostered growth and increased TFP by 8.2% before the Great Recession, while it hindered growth and reduced TFP by 7.5% after the Great Recession. The latter result serves as evidence in favor of a distributional explanation behind the lackluster growth that the US economy experienced over the last decade.

Appendix

Table 20: Sectoral Ranking in $Z_i^\ell = \sum_{h \in \mathscr{H}} \psi_{ih}^\ell \, \delta_h$ - Part 1

Sector	***1997***		***2007***		***2019***	
Housing	1	0.14	1	1.16	1	0.16
Rental, leasing, and lessors of intangibles	2	0.66	5	0.77	2	0.66
Pipeline transportation	12	1.08	25	1.16	3	0.73
Petroleum and coal products	4	0.75	2	0.51	4	0.77
Farms	3	0.71	4	0.73	5	0.78
Chemical products	8	0.96	7	0.86	6	0.81
Oil and gas extraction	6	0.81	3	0.56	7	0.83
Utilities	5	0.76	6	0.79	8	0.84
Broadcasting ans telecommunications	9	0.98	9	0.93	9	0.86
Primary metals	13	1.09	8	0.91	10	0.89
Other real estate	7	0.90	13	0.98	11	0.94
Food, beverages, and tobacco products	10	0.98	12	0.95	12	0.97
Data, internet, and information services	59	1.45	24	1.16	13	1.01
Federal reserve banks, and credit intermediation	27	1.20	40	1.27	14	1.07
Air transportation	20	1.15	15	1.07	15	1.08
Mining, except oil and gas	41	1.30	14	1.00	16	1.09
Arts, sports, museums, and related activities	29	1.23	29	1.21	17	1.11
Motion pictures and sound recording	18	1.12	11	0.94	18	1.12
Motor vehicles, bodies and trailers, and parts	21	1.15	23	1.15	19	1.12
Paper products	17	1.12	17	1.12	20	1.13
Ambulatory health care services	15	1.12	22	1.14	21	1.14
Plastics and rubber products	23	1.16	20	1.13	22	1.14
Hospitals	16	1.12	19	1.13	23	1.15
Nonmetallic mineral products	28	1.21	35	1.25	24	1.16
Funds, trusts, and other financial vehicles	34	1.24	38	1.26	25	1.16
Water transportation	11	1.01	10	0.94	26	1.17
Food services and drinking places	24	1.16	28	1.20	27	1.19
Insurance carriers and related activities	30	1.23	21	1.14	28	1.19
Transit and ground passenger transportation	22	1.15	16	1.12	29	1.20
Other retail	46	1.32	41	1.28	30	1.21
Electrical equipment, appliances and components	26	1.18	26	1.18	31	1.22
Rail transportation	58	1.45	45	1.29	32	1.23
Accommodation	35	1.24	31	1.23	33	1.23

Notes:

Table 21: Sectoral Ranking in $Z_i^\ell = \sum_{h \in \mathscr{H}} \psi_{ih}^\ell \, \delta_h$ - Part 2

Sector	***1997***		***2007***		***2019***	
Nursing and residential care facilities	32	1.24	34	1.24	34	1.23
Other transportation equipment	55	1.44	39	1.27	35	1.24
Wholesale trade	47	1.32	47	1.31	36	1.24
Computer and electronic products	36	1.26	42	1.28	37	1.25
Social assistance	25	1.18	27	1.19	38	1.26
Miscellaneous manufacturing	31	1.23	33	1.24	39	1.26
Machinery	45	1.32	36	1.26	40	1.26
Fabricated metal products	37	1.25	32	1.24	41	1.27
Textile mills and textile product mills	33	1.24	30	1.21	42	1.27
Amusements, gambling, and recreational	19	1.13	43	1.28	43	1.29
Printing and related activities	53	1.41	46	1.31	44	1.30
Food and beverage stores	40	1.29	37	1.26	45	1.30
Publishing industries, except internet	38	1.26	18	1.12	46	1.33
Furniture and related products	43	1.31	48	1.35	47	1.35
Truck transportation	49	1.35	44	1.29	48	1.36
Wood products	39	1.29	54	1.43	49	1.36
Waste management and remediation services	42	1.30	51	1.37	50	1.38
Educational services	52	1.38	52	1.40	51	1.41
Motor vehicles and part dealers	50	1.36	56	1.45	52	1.42
Securities, commodity contracts, and investment	61	1.53	64	1.69	53	1.45
General merchandise store	57	1.45	59	1.48	54	1.46
Other services, except government	44	1.32	55	1.44	55	1.47
Miscellaneous professional and technical services	51	1.38	57	1.45	56	1.48
Other transportation and support activities	54	1.44	49	1.36	57	1.49
Warehousing and storage	64	1.61	53	1.42	58	1.50
Administrative and support activities	60	1.50	60	1.50	59	1.50
Legal services	62	1.57	62	1.58	60	1.52
Support activities for mining	63	1.57	58	1.47	61	1.53
Apparel and leather, and allied products	49	1.34	61	1.55	62	1.55
Construction	56	1.45	63	1.61	63	1.59
Forestry, fishing, and related activities	14	1.11	50	1.36	64	1.64
Management of companies and enterprises	65	1.67	65	1.69	65	1.72
Computer systems design and related services	66	1.73	66	1.75	66	1.89

Notes:

Table 22: Sectoral Ranking in $\psi_i^\ell = \sum_{h\in\mathscr{H}} \psi_{ih}^\ell$ - Part 1

Sector	***1997***		***2007***		***2019***	
Housing	1	0.06	1	0.6	1	0.06
Rental, leasing, and lessors of intangibles	2	0.29	6	0.33	2	0.28
Pipeline transportation	12	0.46	15	0.43	3	0.30
Petroleum and coal products	3	0.31	2	0.17	4	0.31
Oil and gas extraction	6	0.32	3	0.18	5	0.33
Farms	4	0.32	4	0.31	6	0.33
Utilities	5	0.32	5	0.31	7	0.34
Chemical products	9	0.43	8	0.39	8	0.35
Broadcasting ans telecommunications	8	0.42	10	0.40	9	0.36
Primary metals	15	0.49	9	0.39	10	0.38
Other real estate	7	0.39	12	0.41	11	0.39
Mining, except oil and gas	22	0.53	7	0.36	12	0.42
Data, internet, and information services	51	0.63	22	0.50	13	0.42
Federal reserve banks, and credit intermediation	14	0.49	29	0.53	14	0.42
Food, beverages, and tobacco products	11	0.45	14	0.42	15	0.43
Air transportation	17	0.51	16	0.46	16	0.48
Nonmetallic mineral products	23	0.53	25	0.51	17	0.48
Funds, trusts, and other financial vehicles	21	0.53	36	0.55	18	0.49
Motion pictures and sound recording	16	0.50	13	0.41	19	0.49
Paper products	18	0.52	19	0.49	20	0.49
Arts, sports, museums, and related activities	28	0.55	30	0.54	21	0.49
Rail transportation	47	0.62	24	0.51	22	0.49
Plastics and rubber products	24	0.54	21	0.50	23	0.50
Motor vehicles, bodies and trailers, and parts	30	0.56	28	0.53	24	0.50
Water transportation	10	0.44	11	0.40	25	0.51
Insurance carriers and related activities	25	0.54	20	0.49	26	0.51
Computer and electronic products	29	0.55	39	0.56	27	0.52
Other transportation equipment	55	0.65	35	0.55	28	0.52
Legal services	33	0.57	34	0.55	29	0.53
Electrical equipment, appliances and components	26	0.55	27	0.53	30	0.53
Transit and ground passenger transportation	19	0.52	23	0.51	31	0.53
Publishing industries, except internet	27	0.55	17	0.48	32	0.54
Wholesale trade	42	0.60	44	0.58	33	0.54

Notes:

Table 23: Sectoral Ranking in $\psi_i^\ell = \sum_{h\in\mathscr{H}} \psi_{ih}^\ell$ - Part 2

Sector	*1997*		*2007*		*2019*	
Machinery	45	0.61	38	0.55	34	0.55
Fabricated metal products	37	0.58	31	0.54	35	0.55
Truck transportation	34	0.57	26	0.51	36	0.55
Miscellaneous manufacturing	32	0.57	37	0.55	37	0.56
Accommodation	35	0.58	42	0.57	38	0.56
Textile mills and textile product mills	36	0.58	32	0.54	39	0.56
Waste management and remediation services	31	0.56	41	0.56	40	0.57
Other retail	50	0.63	48	0.61	41	0.57
Printing and related activities	56	0.65	45	0.59	42	0.57
Construction	40	0.60	40	0.56	43	0.58
Wood products	39	0.59	50	0.62	44	0.59
Support activities for mining	44	0.61	18	0.49	45	0.59
Amusements, gambling, and recreational	20	0.52	43	0.58	46	0.59
Furniture and related products	46	0.62	47	0.60	47	0.60
Securities, commodity contracts, and investment	53	0.65	65	0.75	48	0.61
Miscellaneous professional and technical services	41	0.60	49	0.62	49	0.63
Food services and drinking places	43	0.60	52	0.63	50	0.63
Other transportation and support activities	52	0.65	46	0.59	51	0.64
Food and beverage stores	54	0.65	51	0.63	52	0.65
Other services, except government	38	0.59	53	0.64	53	0.65
Motor vehicles and part dealers	48	0.62	56	0.64	54	0.65
Forestry, fishing, and related activities	13	0.46	33	0.54	55	0.66
Administrative and support activities	59	0.68	55	0.67	56	0.67
Warehousing and storage	66	0.76	54	0.64	57	0.67
Educational services	57	0.67	57	0.68	58	0.69
Ambulatory health care services	58	0.68	58	0.69	59	0.69
Hospitals	62	0.73	61	0.71	60	0.69
Apparel and leather, and allied products	49	0.62	60	0.71	61	0.70
General merchandise store	61	0.71	63	0.73	62	0.72
Management of companies and enterprises	63	0.73	62	0.73	63	0.73
Nursing and residential care facilities	65	0.76	66	0.75	64	0.75
Social assistance	60	0.70	59	0.69	65	0.75
Computer systems design and related services	64	0.74	74	0.75	66	0.77

Notes:

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