

# Inequality and Misallocation under Production Networks

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# Total Factor Productivity & Aggregation

**1.** In standard models TFP is given

- Solow & Ramsey growth model
  - RBC & New Keynesian models

## Total Factor Productivity & Aggregation

1. In standard models TFP is given
    - Solow & Ramsey growth model
    - RBC & New Keynesian models
  2. Through Aggregation
    - Multiple Firms  $\Rightarrow$  Allocation
    - Production Networks  $\Rightarrow$  Amplification

# Aggregate TFP is endogenous

## Research Question & Motivation

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## **Higher Expenditure Share in Education, Entertainment, Pensions**

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### Data from Consumer Expenditure Survey

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## Data from Consumer Expenditure Survey

Income share for the top has increased

## In this Presentation

In economies with distortions, variations in distributions (factoral income & expenditure) can influence misallocation

Novel TFP decomposition that measures aggregate misallocations effects

Implementation of the model with US data

# Approach

# Contribution

$\Delta$  Consumption &  $\Delta$  Demand  
 $\Delta$  Distribution  $\Delta$  Structure



$\Delta$  Income Distribution

## Approach

## Contribution

## $\Delta$ Consumption Distribution & $\Delta$ Demand Structure

↓

## △ Labor Income Shares

## △ Income Distribution

## Approach

$\Delta$  Consumption Distribution &  $\Delta$  Demand Structure



## △ Income Distribution

## Contribution

Bigio & La'O (2020)

- (i) Rep Household
  - (ii) Efficient Equilibrium

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## My Model

- (i) Het Households
  - (ii) Any Equilibrium

## Approach

## △ Consumption Distribution & △ Demand Structure



## △ Income Distribution



## △ Misallocation



# Δ TFP

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$\Delta$  Income Distribution



$\Delta$  Misallocation



$\Delta$  TFP

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**Baqae & Farhi (2020)**

- (i) Rep Household
- (ii) Exogenous  $L$



## My Model

- (i) Het Households
- (ii) Endogenous  $L$

## Static General Equilibrium Model with...

**Two Firms** :  $\begin{cases} \text{More Competitive: } H \\ \text{Less Competitive: } L \end{cases}$  + **Two Workers** :  $\begin{cases} \text{High-Skill: } h \\ \text{Low-Skill: } l \end{cases}$

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**1.** Good markets face exogenous distortions

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**2.** Labor markets are competitive

**3.** Labor supply is endogenous

**4. Correlations:**

- $H$  has high  $\mu$
- $H$  requires more  $h$
- $h$  have a higher expenditure in  $H$

## Mechanism's Intuition

1.  $\mu$  heterogeneity —→ allocates more workers to  $H$ 
  - $H$  operates with low marginal productivity
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2. **Skill-bias heterogeneity** → asymmetries in the income exposure in response to local perturbations
3. **Preference heterogeneity** expenditure flows
  - As  $h$  income increase, expenditure in  $H$  rises
  - Workers relocate from  $L$  to  $H$
  - Misallocation is accentuated

# Firm Heterogeneity $i \in \{H, L\}$

{
a. Skill Bias
b. Distortions

$$\underset{y_i, \ell_{ih}, \ell_{il}}{\text{Max}} \quad \pi_i = p_i y_i - w_h \ell_{ih} - w_l \ell_{il}$$

$$y_i = A_i \ell_{ih}^{\alpha_i} \ell_{il}^{1-\alpha_i}$$

**Skill Bias**  
 $\alpha_L \leq \alpha_H$

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## Markdown

$$0 < \mu_L \leq \mu_H \leq 1$$

$$\text{Cost}_i = \mu_i \times \text{Revenue}_i$$

Alternative Narrative: Sectoral Dixit-Stiglitz Aggregation

## Household Heterogeneity

$$r \in \{h, l\}$$



Graphic Argument



$$\underset{C_r, L_r}{\text{Max}} U_r(C_r, L_r) \quad \text{s.t.} \quad \frac{C_r}{\bar{C}_r} = \left( \beta_r \left( \frac{C_r H}{\bar{C}_r H} \right)^{\frac{\rho-1}{\rho}} + (1 - \beta_r) \left( \frac{C_r L}{\bar{C}_r L} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

$$E_r = p_r^c C_r = p_H C_{rH} + p_L C_{rL} \leq \mathbf{w}_r \mathbf{L}_r + 0.5 \text{ profits}$$

## Consumption Bias

$$\beta_l \leq \beta_h$$

## Equilibrium Definition

For  $(A, \mu, \beta, \alpha)$ , prices and allocations:

- (i) **Firms'** labor demand and output decisions maximize profits;
- (ii) **Households'** consumption and labor supply maximize utility satisfying budget constraints;
- (iii) Goods and labor markets clear.

## Solve for Equilibrium Distributions

From **FOC** of households and firms

$$p_H C_{rH} = \beta_r p_r^c C_r$$

$$w_h \ell_{ih} = \alpha_i \mu_i p_i y_i$$

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$$y_i = C_{hi} + C_{li} \quad L_r = \ell_{Hr} + \ell_{Lr}$$

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**Equilibrium** in terms of

$$\lambda_i = \frac{p_i y_i}{GDP}$$

Sales (Domar weights)

$$\Lambda_h = \frac{w_h L_h}{GDP} = \sum \alpha_i \mu_i \lambda_i$$

Labor income

$$\chi_r = \frac{p_r^c C_r}{GDP}$$

Expenditure

$$\tilde{\Lambda}_h = \sum \alpha_i \lambda_i$$

Value added

# Source of Misallocation

## Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

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$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

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### Additional Restriction

$$\mu_H + \mu_L = 1$$

[Solution](#)

[Allocative ≠](#)

## What I Don't Do

- Misallocation literature **distorted vs. efficient** equilibrium
- Parametric assumptions (usually CD) → analytic TFP
- Evaluate how getting rid of distortions has an effect on TFP

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## What I Do

- Local TFP  $\Delta$  around distorted equilibrium to any perturbation
- Distributional  $\Delta \rightarrow$  Misallocation  $\Delta \rightarrow \Delta$  TFP
- To illustrate:  $d \log A_L = 1\%$

## Local Variation to $d \log A_L = 1\%$

$$\frac{d \chi_h}{d \log A_L} = \frac{(\alpha_H - \alpha_L)}{2} \frac{d \lambda_H}{d \log A_L}$$

Expenditure elasticity requires  $\alpha_H \neq \alpha_L$

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**Expenditure elasticity requires  $\alpha_H \neq \alpha_L$**

$$\frac{d \lambda_H}{d \log A_L} = - \frac{2(\rho - 1)\beta_h\beta_l}{2 - (\alpha_H - \alpha_L)(\beta_h - \beta_l) + 2(\rho - 1) \frac{\beta_h\beta_l}{1 + \zeta^w} \left( \frac{\alpha_H - \mu_L}{\Lambda_h} + \frac{\alpha_H - \mu_H}{\Lambda_l} + \frac{\zeta^e}{2} \frac{\alpha_H - \alpha_L}{\chi_h \chi_l} \right)}$$

**Sales elasticity requires  $\rho \neq 1$**

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**Sales elasticity requires  $\rho \neq 1$**

$\begin{cases} \text{Under } \rho > 1: \text{ consumers increase expenditure on } L \text{ & } \lambda_H \downarrow \\ \text{Under } \rho < 1: \underbrace{\text{consumers increase expenditure on } H \text{ & } \lambda_H \uparrow}_{\text{Baumol's Cost Disease}} \end{cases}$

## In This Section

- First-order local  $\Delta$  **Income Distribution**

$$d\Lambda_h, \quad d\Lambda_I$$

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- First-order local  $\Delta$  **Income Distribution**

$$d\Lambda_h, \quad d\Lambda_I$$

- Decomposition of  $d\Lambda$  in

$\Delta$  **Consumption  
Distribution**

$\Delta$  **Demand  
Structure**

&

Keep  $\beta$  fix  
& change  $\chi$

Keep  $\chi$  fix  
& change  $\beta$

# Income Distribution & Bilateral Centralities

$$\Lambda_h = \overbrace{\left( \underbrace{\alpha_H \mu_H}_{f_{H \rightarrow h}} \beta_h + \underbrace{\alpha_L \mu_L}_{f_{L \rightarrow h}} (1 - \beta_h) \right)}^{m_{h \rightarrow h}} \chi_h \\ + \overbrace{\left( \underbrace{\alpha_H \mu_H}_{f_{H \rightarrow h}} \beta_I + \underbrace{\alpha_L \mu_L}_{f_{L \rightarrow h}} (1 - \beta_I) \right)}^{m_{I \rightarrow h}} \chi_I$$

$m_{I \rightarrow h}$     % of expenditure from I reaching  $\Lambda_h$   
 $f_{L \rightarrow h}$     revenue from L

## Comparative Statics

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{I \rightarrow h} \chi_I$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h} \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

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Take total derivative

$$d \Lambda_h = \underbrace{m_{h \rightarrow h} d \chi_h + m_{I \rightarrow h} d \chi_I}_{\text{Distributive Income}_h} + \underbrace{\chi_h d m_{h \rightarrow h} + \chi_I d m_{I \rightarrow h}}_{\text{Income Centrality}_h}$$

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**We know:**  $d \chi_h + d \chi_I = 0$  &  $d m_{r \rightarrow h} = (f_{H \rightarrow h} - f_{L \rightarrow h}) d \beta_r$

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We know:  $d \chi_h + d \chi_I = 0$  &  $d m_{r \rightarrow h} = (f_{H \rightarrow h} - f_{L \rightarrow h}) d \beta_r$

$$\overbrace{(\beta_h - \beta_I)}^{\geq 0} \overbrace{(\alpha_H - \mu_L)}^{\geq 0}$$

$$\overbrace{(\alpha_H - \mu_L)}^{\geq 0}$$

$$d \Lambda_h = \underbrace{(m_{h \rightarrow h} - m_{I \rightarrow h}) d \chi_h}_{\text{Distributive Income}_h} + \underbrace{(f_{H \rightarrow h} - f_{L \rightarrow h}) \sum \chi_r d \beta_r}_{\text{Income Centrality}_h}$$

## Baqae & Fahri (2020)

A.

$$\begin{aligned} d \log Y &= d \log GDP - d \log P_Y \\ &= \mathbf{d \log TFP} + \sum_{r \in \{h,l\}} \tilde{\Lambda}_r d \log L_r \end{aligned}$$

Intermediate Steps

## Baqae & Fahri (2020)

A.

$$\begin{aligned} d \log Y &= d \log GDP - d \log P_Y \\ &= \textcolor{brown}{d \log TFP} + \sum_{r \in \{h,l\}} \tilde{\Lambda}_r d \log L_r \end{aligned}$$

B.

Intermediate Steps

$$\begin{aligned} \textcolor{brown}{d \log TFP} &= \underbrace{\lambda_H d \log A_H + \lambda_L d \log A_L}_{\text{Technology}} \\ &+ \underbrace{\lambda_H d \log \mu_H + \lambda_L d \log \mu_L}_{\text{Competitiveness}} \\ &- \underbrace{(\tilde{\Lambda}_h d \log \Lambda_h + \tilde{\Lambda}_l d \log \Lambda_l)}_{\text{Misallocation}} \end{aligned}$$

Without distortions → Hulten (1978)

## Distortion Centralities $\delta$

$$\begin{aligned} \textbf{\textit{Misallocation}} &= \tilde{\Lambda}_h d \log \Lambda_h + \tilde{\Lambda}_I d \log \Lambda_I \\ &= \frac{\tilde{\Lambda}_h}{\Lambda_h} d \Lambda_h + \frac{\tilde{\Lambda}_I}{\Lambda_I} d \Lambda_I \\ &= \delta_h d \Lambda_h + \delta_I d \Lambda_I \end{aligned}$$

## Distortion Centralities $\delta$

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 \textbf{\textit{Misallocation}} &= \tilde{\Lambda}_h d \log \Lambda_h + \tilde{\Lambda}_I d \log \Lambda_I \\
 &= \frac{\tilde{\Lambda}_h}{\Lambda_h} d \Lambda_h + \frac{\tilde{\Lambda}_I}{\Lambda_I} d \Lambda_I \\
 &= \delta_h d \Lambda_h + \delta_I d \Lambda_I
 \end{aligned}$$

$\delta$  measures how **undervalue** a worker is

$$\delta_I - \delta_h > 0$$

## Introduce $d\Lambda_h$ and $d\Lambda_I$

$$\text{Misallocation} = \underbrace{(\mathcal{M}_h - \mathcal{M}_I) \mathbf{d} \boldsymbol{\chi}_h}_{1. \text{ Distributive}} + \underbrace{(\mathcal{F}_H - \mathcal{F}_L) \sum \boldsymbol{\chi}_r \mathbf{d} \boldsymbol{\beta}_r}_{2. \text{ Final Demand}}$$

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## Sufficient Statistics

## Expenditure Centrality $M$

$$M_r = m_{r \rightarrow h} \delta_h + m_{r \rightarrow I} \delta_I$$

## Introduce $d\Lambda_h$ and $d\Lambda_I$

$$\text{Misallocation} = \underbrace{(M_h - M_I) d\chi_h}_{1. \text{ Distributive}} + \underbrace{(F_H - F_L) \sum \chi_r d\beta_r}_{2. \text{ Final Demand}}$$

## Sufficient Statistics

**Expenditure Centrality  $M$**     **Revenue Centrality  $F$**

$$M_r = m_{r \rightarrow h} \delta_h + m_{r \rightarrow I} \delta_I \quad F_i = f_{i \rightarrow h} \delta_h + f_{i \rightarrow I} \delta_I$$

## Introduce $d\Lambda_h$ and $d\Lambda_I$

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## Sufficient Statistics

**Expenditure Centrality  $M$**     **Revenue Centrality  $F$**

$$M_r = m_{r \rightarrow h} \delta_h + m_{r \rightarrow I} \delta_I \quad F_i = f_{i \rightarrow h} \delta_h + f_{i \rightarrow I} \delta_I$$

1.  $M_r$  is high for households that consume from relatively competitive supply chains that demand workers with high  $\delta$
2.  $F_i$  is high for firms that operate in relatively competitive supply chains and directly or indirectly demand high  $\delta$  workers

**Distributive**  $\uparrow \rightarrow$  Misallocation  $\uparrow \rightarrow$  TFP  $\downarrow$

$$\underbrace{(M_H - M_L) d \chi_h}_{\text{Distributive}} \quad d \chi_h = \frac{(\alpha_H - \alpha_L)}{2} d \lambda_H$$

**Distributive**  $\uparrow \rightarrow$  Misallocation  $\uparrow \rightarrow$  TFP  $\downarrow$

$$\underbrace{(M_H - M_L)}_{\text{Distributive}} d \chi_h = \frac{(\alpha_H - \alpha_L)}{2} d \lambda_H$$

$$\overbrace{M_H - M_L}^{\geq 0} = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \overbrace{(\beta_h - \beta_l)}^{\geq 0}$$

$$\times \underbrace{[\delta_l + (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) a]}_{> 0}$$

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	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Expenditure Redistribution	$\times$	$\times$	$\times$

***Final Demand*  $\uparrow \rightarrow$  Misallocation  $\uparrow \rightarrow$  TFP  $\downarrow$**

$$\underbrace{(F_H - F_L) \sum \chi_r d \beta_r}_{\text{Final Demand}}$$

**Final Demand**  $\uparrow \rightarrow$  Misallocation  $\uparrow \rightarrow$  TFP  $\downarrow$

$$\underbrace{(\mathcal{F}_H - \mathcal{F}_L) \sum \chi_r \mathbf{d} \beta_r}_{\text{Final Demand}}$$

$$\overbrace{\mathcal{F}_H - \mathcal{F}_L}^{\geq 0} = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \times \underbrace{[\delta_I + (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) \mathbf{a}]}_{> 0}$$

**Final Demand**  $\uparrow \rightarrow$  Misallocation  $\uparrow \rightarrow$  TFP  $\downarrow$

$$\underbrace{(\mathcal{F}_H - \mathcal{F}_L) \sum \chi_r \mathbf{d} \beta_r}_{\text{Final Demand}}$$

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	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Final Demand Recomposition			

## Requirements in Heterogeneity

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
1. Expenditure Redistribution	✗	✗	✗
2. Final Demand Recomposition	✗	✓	✓

## Representative Household

Assume instead a representative household

$$\underset{Y, L, C_H, C_L, L_h, L_I}{\text{Max}} \quad U(Y, L) \quad s.t. \quad Y = Q(C_H, C_L),$$

$$p_Y Y = p_H C_H + p_L C_L$$

$$\leq w_h L_h + w_l L_l + (1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L$$

## Representative Household

Assume instead a representative household

$$\underset{Y, L, C_H, C_L, L_h, L_I}{\text{Max}} \quad U(Y, L) \quad \text{s.t.} \quad Y = Q(C_H, C_L),$$

$$\begin{aligned} p_Y Y &= p_H C_H + p_L C_L \\ &\leq w_h L_h + w_I L_I + (1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L \end{aligned}$$

The **first-order conditions** imply that

$$\delta_h = \delta_I = \Lambda^{-1} \quad \Lambda = \Lambda_h + \Lambda_I$$

The effects from one additional percentage point of labor income share on TFP are equalized

## Misallocation under a Representative Household

From  $\delta_h = \delta_I = \Lambda^{-1}$

$$\delta_h d \Lambda_h + \delta_I d \Lambda_I = \frac{d \Lambda_h + d \Lambda_I}{\Lambda} = d \log \Lambda$$

Track only one element of the distribution!

## Misallocation under a Representative Household

From  $\delta_h = \delta_I = \Lambda^{-1}$

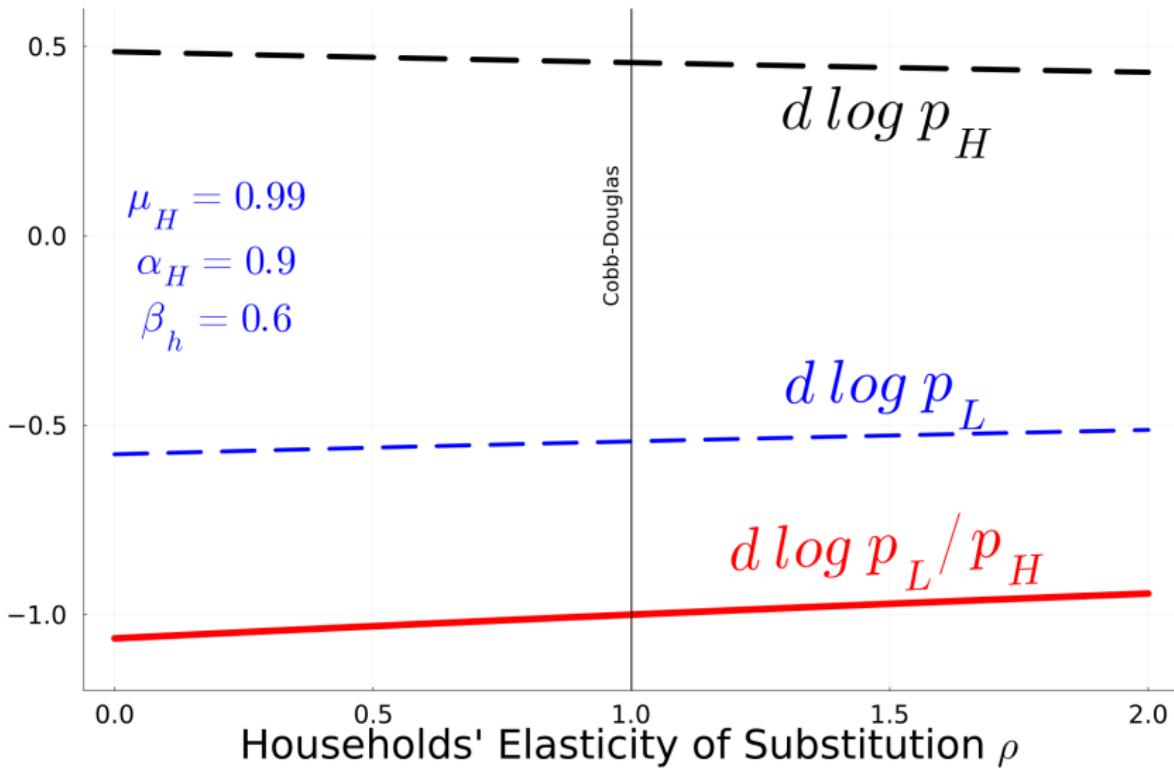
$$\delta_h d \Lambda_h + \delta_I d \Lambda_I = \frac{d \Lambda_h + d \Lambda_I}{\Lambda} = d \log \Lambda$$

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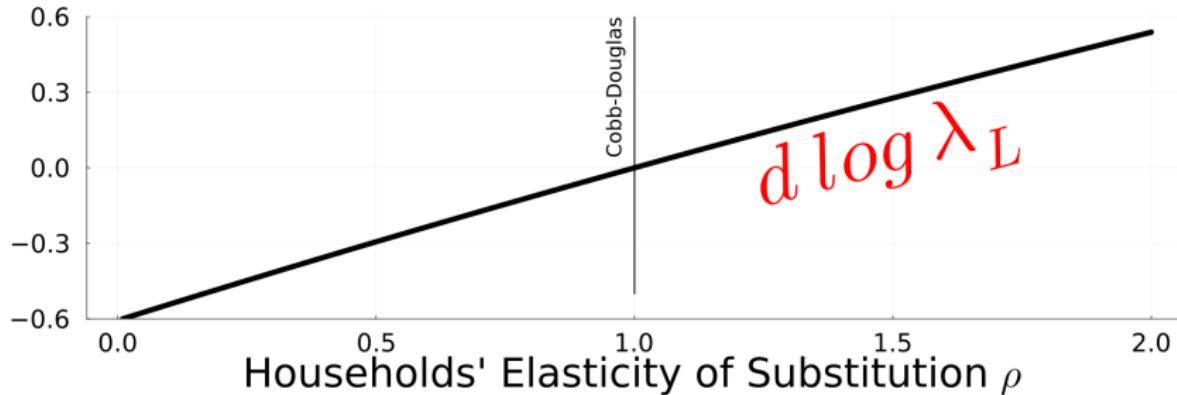
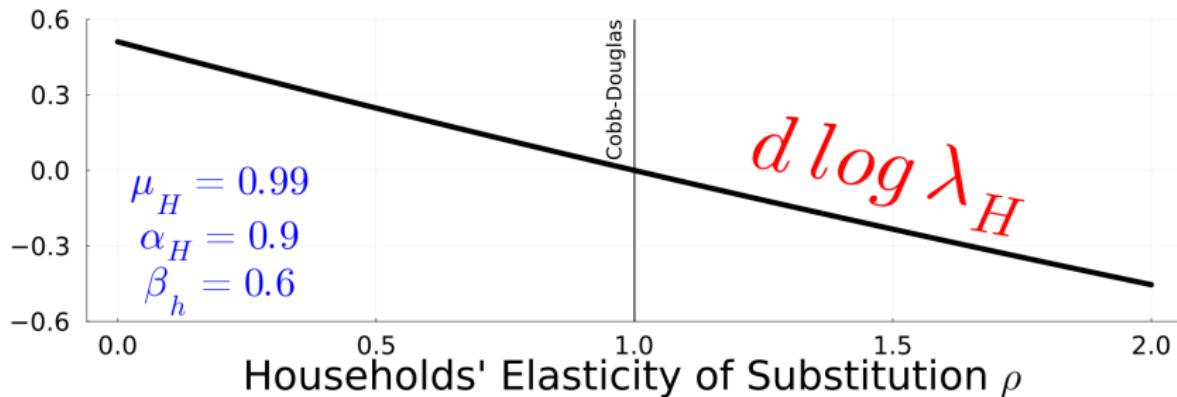
In decomposition -  $d \chi = 0$

$$M \underbrace{\mathbf{d} \chi}_{=0} + (F_H - F_L) \mathbf{d} \beta = \frac{\mu_H - \mu_L}{\Lambda} \mathbf{d} \beta$$

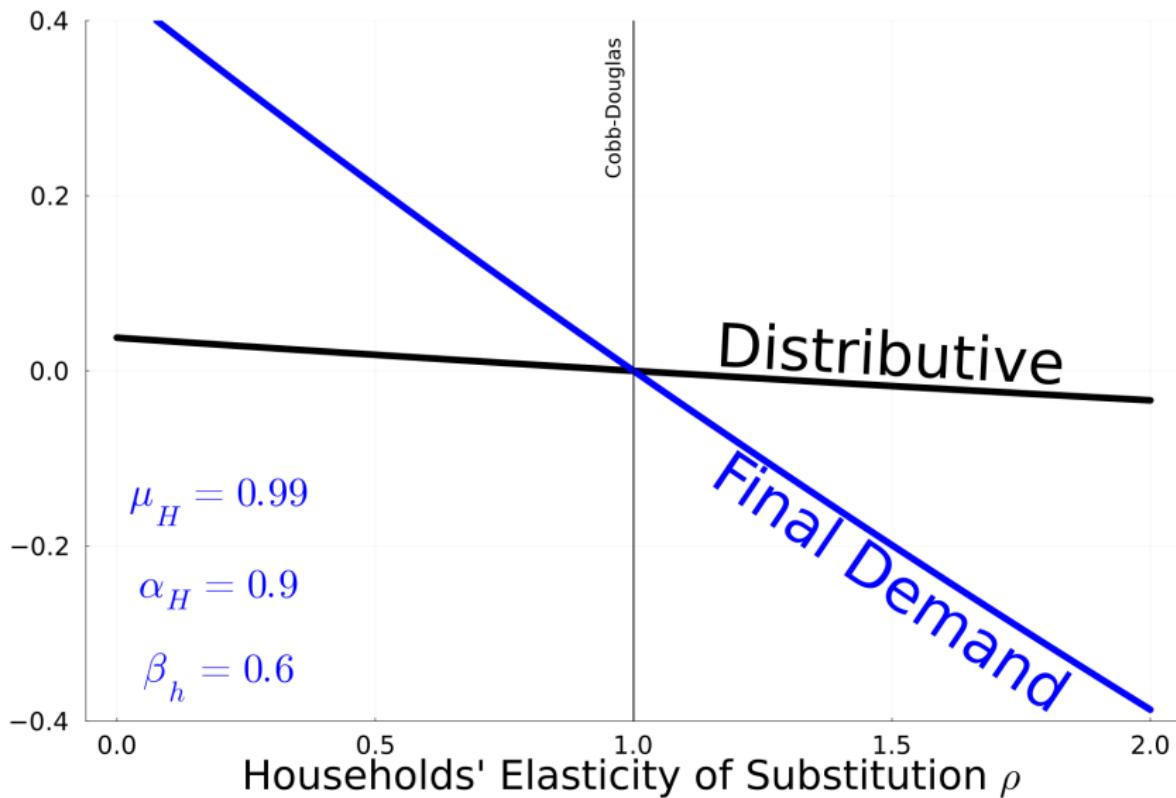
$p_L \downarrow$  &  $p_H \uparrow$



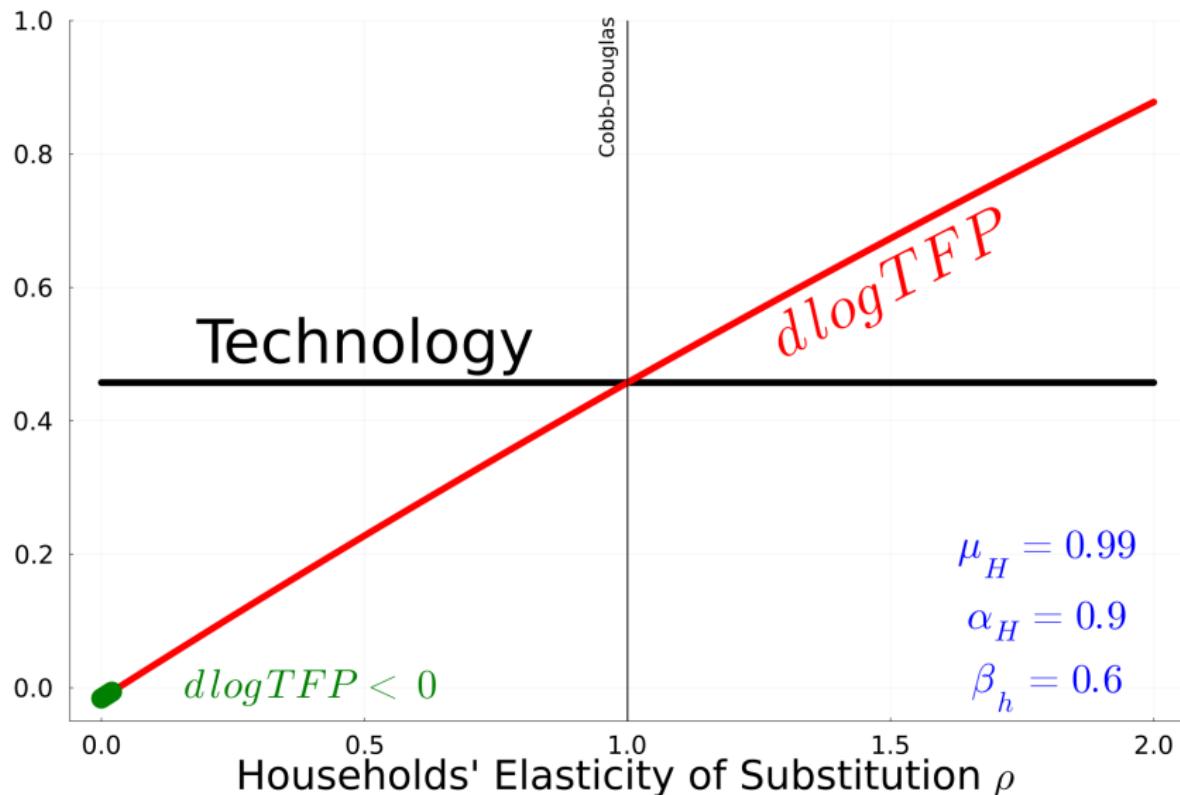
**Under  $\rho > 1$ :**  $\lambda_L \uparrow$  &  $\lambda_H \downarrow$



## Under $\rho > 1$ : Distributive $\downarrow$ & Final Demand $\downarrow$



## Under $\rho > 1$ : Misallocation $\downarrow$ & $d \log TFP > \lambda_L$



## In the Paper...

- General Non-Parametric CRS model for production & consumption
- General Input-Output Networks
- General Equity Distribution

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## Additional Channels

1.  $\mu$   $\uparrow$  & stronger for sectors with high  $\lambda_i F_i$
2.  $\alpha$   $\uparrow$  for high  $\delta$  workers & stronger if  $\mu_i \lambda_i$  high
3. **Intermediate demand**  $\uparrow$  on sectors with high  $F_i$

## Data Requirements

### 3 Types of Money Flows...

1. Household-to-Firm: Final consumption
2. Firm-to-Firm: Intermediate inputs
3. Firm-to-workers: Labor market

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### Measures of Shocks...

- A. Productivity shocks
- B. Markdown shocks
- C. Distributional variations

## Data for money flows from 1997 to 2021 Household to Firm

### 1. State level Personal Consumption Expenditure (BEA)

$\beta_{state, industry}$  : { PCE provides expenditure on types of goods  
IO Make matrix: type of good → industry

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### 1. State level Personal Consumption Expenditure (BEA)

$\beta_{state, industry}$  : { PCE provides expenditure on types of goods  
IO Make matrix: type of good → industry

### Firm to Firm

### 2. Input-Output tables (BEA) for 66 NAICS industries

$$\mu_i = \frac{\text{Total Cost}_i}{\text{Sales}_i}$$

$$\text{Intermediate Intensity } ij = \frac{p_j x_{ij}}{\text{Total Cost}_i}$$

$$\text{Total Cost}_i = \text{Labor Costs}_i + \text{Intermediate Cost}_i$$

## Firm-to-household

### 3. County Business Patterns (Census)

Industry specific geographic (state) bias in labor

Antisupression Algorithm

Missing Private Employment

## Firm-to-household

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### 4. Occupational Employment & Wage Statistics

- Industry specific **occupational** demand bias
- State specific **occupational** supply bias

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From 3 & 4 → industry specific heterogeneity by worker type. Worker type comes from State & Occupational interactions  $H = 38,189$  ( $\approx 1.5$  bill m's)

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e.g. Finance's labor demand intensity for economists in Maine

$$\underbrace{\alpha_{ir} \propto \text{Finance's share of labor expenditure in Maine}}_{\text{Spatial Demand (CBP)}} \times \underbrace{\text{Finance's share of labor expenditure in economists}}_{\text{Occupational Demand (OEWS)}} \times \underbrace{\text{Maine's share of labor income from economists}}_{\text{Occupational Supply (OEWS)}} .$$

Motivation  
oooo

Model  
oooooo

Solution  
ooooo

Income  
ooo

TFP  
oooooo

Novelty  
oo

Example  
oooooo

Data  
ooo●

Empirics  
ooooo

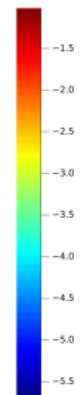
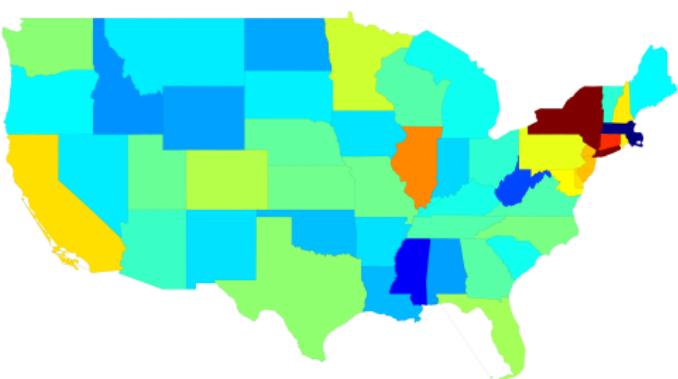
Policy  
ooo

Distributive  
ooo

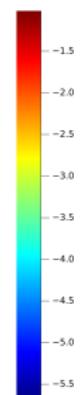
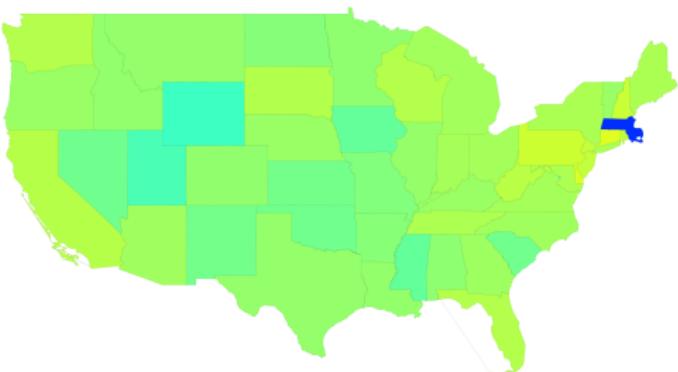
Conclusion  
oo

Appendix

## Financial Sector in Economists



## Ambulatory Health in Dentists



## Data for Shocks

### A. Industry Level Production Accounts (BEA)

$$d \log A_i$$

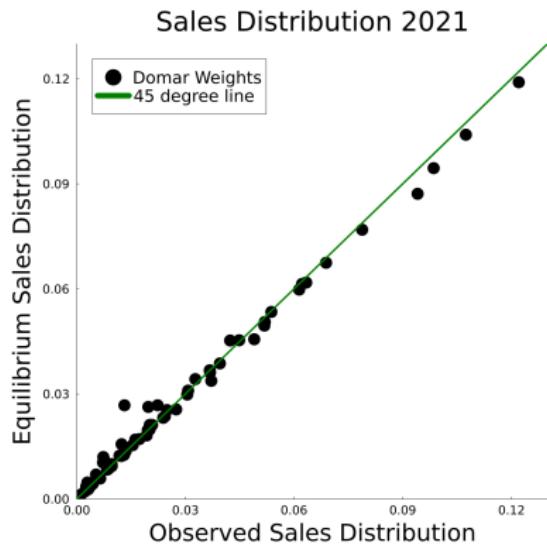
### B. Input Output Tables

$$\mu_i \longrightarrow d \log \mu_i$$

### C. CBP + OEWS

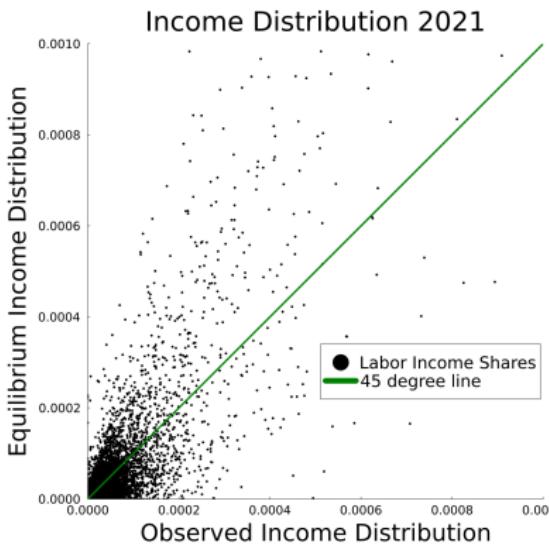
$$\Lambda_r \longrightarrow d \log \Lambda_r$$

# Moments with Heterogeneous Households



<i>Observed <math>\lambda</math> on</i>	
Model $\lambda$	1.021*** (0.003)

$R^2$  0.981  
 $N$  1,650



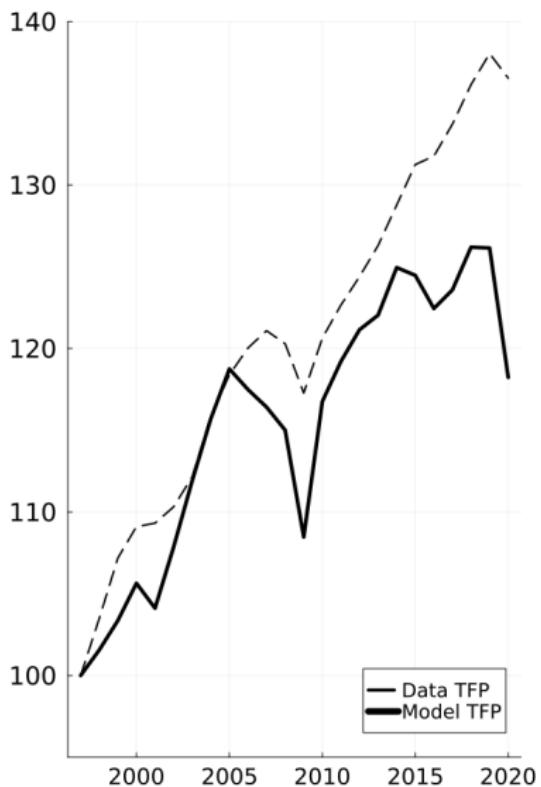
<i>Observed <math>\Lambda</math> on</i>	
Model $\Lambda$	0.435*** (0.0015)

$R^2$  0.682  
 $N$  38,189

## Implementation

$$d \log TFP_t = \underbrace{\sum_i \tilde{\lambda}_{i,t-1} d \log A_{i,t}}_{\text{Technology}_t} + \underbrace{\sum_i \tilde{\lambda}_{i,t-1} d \log \mu_{i,t}}_{\text{Competitiveness}_t} - \underbrace{\sum_r \tilde{\Lambda}_{r,t-1} d \log \Lambda_{r,t}}_{\text{Misallocation}_t}$$

# $R^2$ rises from 5% to 50% with IO Networks

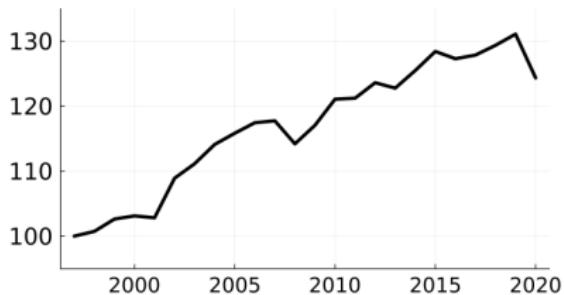


	<i>Observed d log TFP on</i>	
	Without Networks	With Networks
$d \log \widehat{\text{TFP}}$	(1) -0.265 (0.264)	(2) 0.311*** (0.069)
$Adj R^2$	0.048	0.499
Technology	(3) 0.847*** (0.289)	(4) 0.413*** (0.082)
Competitive	0.986 (0.695)	0.342*** (0.054)
Misallocation	-0.105 (0.360)	0.0168 (0.125)
$Adj R^2$	0.562	0.732

***d log TFP = Technology + Competitiveness – Misallocation***

**Technology ↑**

$$\sum_{i \in \mathcal{N}} \tilde{\lambda}_i d \log A_i$$



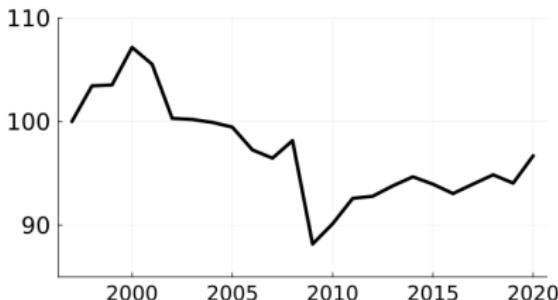
**Between 1997 and 2020**

---

Oil & gas extraction	<b>-11.1%</b>
Computer & electronic	<b>-6.6%</b>

**Competitiveness ↓**

$$\sum_{i \in \mathcal{N}} \tilde{\lambda}_i d \log \mu_i$$



**Between 1997 and 2020**

---

Credit intermediation	<b>4.1%</b>
-----------------------	-------------

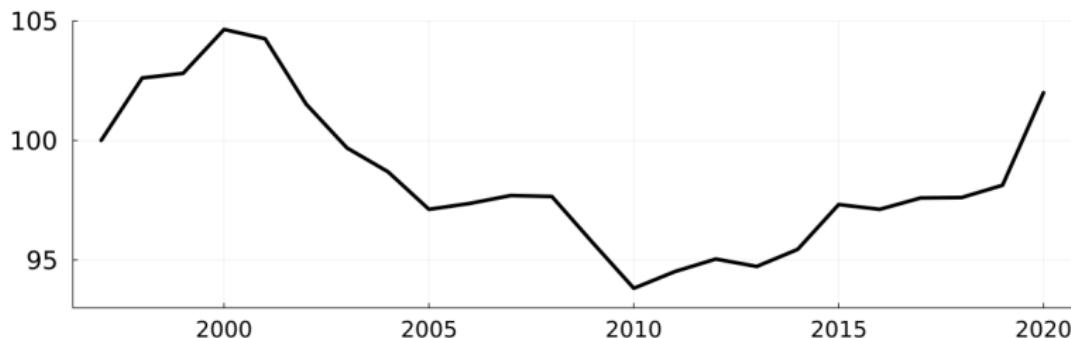
**Between 2002 and 2009**

---

Oil & gas extraction	<b>6.6%</b>
----------------------	-------------

# Without Misallocation↑ after 2009, TFP↑ 7.5%

- Misallocation↓ between 2001 and 2010 by **-8.2%**
- Misallocation↑ between 2010 and 2020 by **7.5%**



## Increasing profit margins

- Oil & gas extraction: **-1.5%**
- Computer & electronics: **-1.1%**

## Increasing labor demand

- Credit intermediation: **2.4%**

## Final and Intermediate Demand

- Wholesale Trade: **2.2%**

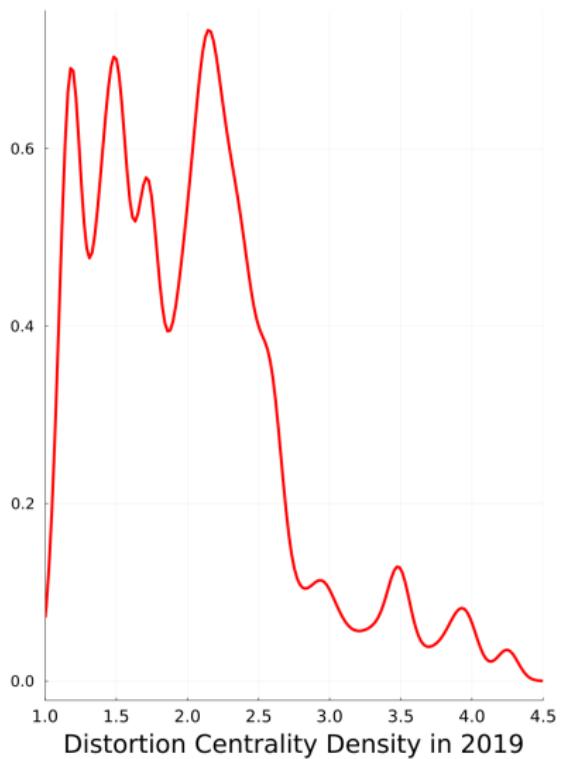
Sources of Misallocation (Graph)

Sources of Misallocation (Counterfactual)

Industry variation from  $\mu$  and Labor Demand

Industry variation from Final and Intermediate Demand

## Distortion Centralities $\delta$



<b>Lowest <math>\delta</math></b>	
Nursing Assistant	1.05 - 1.08
Residential Advisor	1.06 - 1.22
Rehabilitation Counselor	1.07 - 1.08
Recreational Therapist	1.07 - 1.09
Food Server	1.07 - 1.45

<b>Highest <math>\delta</math></b>	
Teller	4.27 - 4.28
New Accounts Clerk	4.24 - 4.27
Loan Interviewer	4.21 - 4.26
Loan Officer	4.23 - 4.26
Credit Analyst	3.89 - 4.22

## Normalized Nested CES

Introduced by **de La Grandville (1989)** and **Klump & de La Grandville (1989)** and as in **Baqae & Farhi (2019,a,b, 2020, 2022)**

Normalized Nested CES

Parameters - **Atalay (2017), Boehm et al. (2014)**

1. Elasticity of substitution **between worker types**: 1.0
2. Elasticity of substitution **between sectoral intermediate inputs**: 0.2
3. Elasticity of substitution **between labor and intermediate inputs**: 0.5
4. Elasticity of substitution in **final consumption**: 0.9
5. **Substitution effect** in labor supply  $\zeta_h^w = 2$
6. **Income effect** in labor supply  $\zeta_h^e = 2$

$$d \log TFP = \underbrace{\textcolor{red}{Technology}}_{=1\%} - \textcolor{black}{Misallocation}$$

<b>Best Sectors</b>	<b>d log TFP</b>	<b>d log TFP on</b>				
1. Nursing & Residential Care	1.041%	$\mu_i$	0.359*** (0.09)			0.207 (0.13)
2. Social Assistance	1.039%	$\lambda_i$		0.170 (0.56)		0.854* (0.50)
3. General Merchandise Store	1.029%	$F_i$			0.212*** (0.05)	0.148** (0.07)
4. Ambulatory health care	1.027%					
5. Hospitals	1.026%					
<b>Worst Sectors</b>	<b>d log TFP</b>	$R^2$	0.20	$1e^{-3}$	0.21	0.27
1. Oil & Gas extraction	0.587%	$N$			66	
2. Primary Metals	0.610%					
3. Chemical Products	0.618%					
4. Mining, except Oil & Gas	0.630%					
5. Utilities	0.647%					

We want productivity shocks in  
sectors with high  $F_i$ !

$$d \log TFP = \underbrace{\text{Competitiveness} - \text{Misallocation}}_{= 1\%}$$

<b>Best Sectors</b>		<b>d log TFP</b>	<b>d log TFP on</b>			
1.	Housing	0.766%	$\mu_i$	-0.974*** (0.12)		-0.919*** (0.18)
2.	Credit Intermediation	0.414%	$\lambda_i$	1.351 (0.95)		-0.132 (0.73)
3.	Oil & Gas extraction	0.384%	$F_i$		-0.427*** (0.08)	-0.046 (0.11)
4.	Furniture	0.370%				
5.	Mining, except Oil & Gas	0.364%				
<b>Worst Sectors</b>		<b>d log TFP</b>	$R^2$	0.48	0.03	0.29
1.	Nursing & Residential Care	-0.329%	$N$		66	0.48
2.	Social Assistance	-0.303%				
3.	General Merchandise Store	-0.274%				
4.	Hospitals	-0.219%				
5.	Ambulatory health care	-0.201%				

We want competition shocks in sectors with low  $\mu_i$ !

## Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_I)$$

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$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_I)$$

$$d \log TFP = \sum \chi_r d \log \mathbf{PTT}_r$$

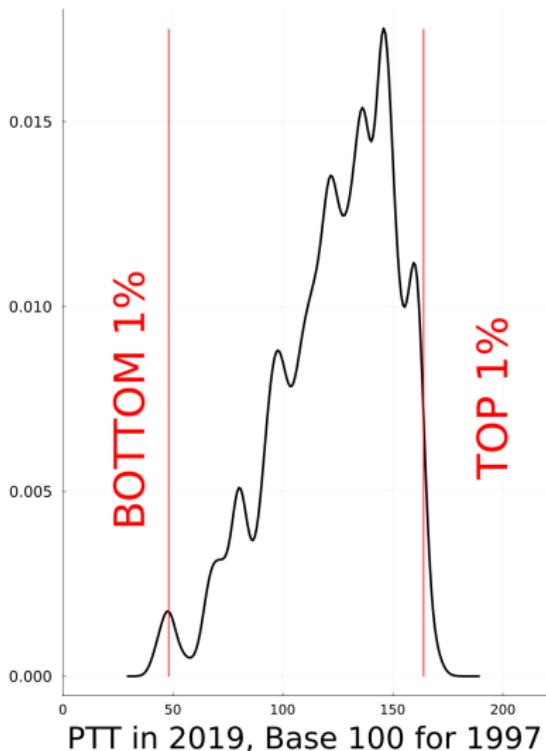
## Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_I)$$

$$d \log TFP = \sum \chi_r d \log \mathbf{PTT}_r$$

$$\begin{aligned} d \log \mathbf{PTT}_h &= \underbrace{\beta_h d \log A_H + (1 - \beta_h) d \log A_L}_{\text{Technology}_h} \\ &\quad + \underbrace{\beta_h d \log \mu_H + (1 - \beta_h) d \log \mu_L}_{\text{Competitiveness}_h} \\ &\quad - \underbrace{\left( \frac{\tilde{m}_{h \leftarrow h}}{\Lambda_h} d \Lambda_h + \frac{\tilde{m}_{h \leftarrow I}}{\Lambda_I} d \Lambda_I - d \log \chi_h \right)}_{\text{Misallocation}_h} \end{aligned}$$

$$C_r = \text{Positional Terms of Trade}_r \times f_r(L_h, L_l)$$

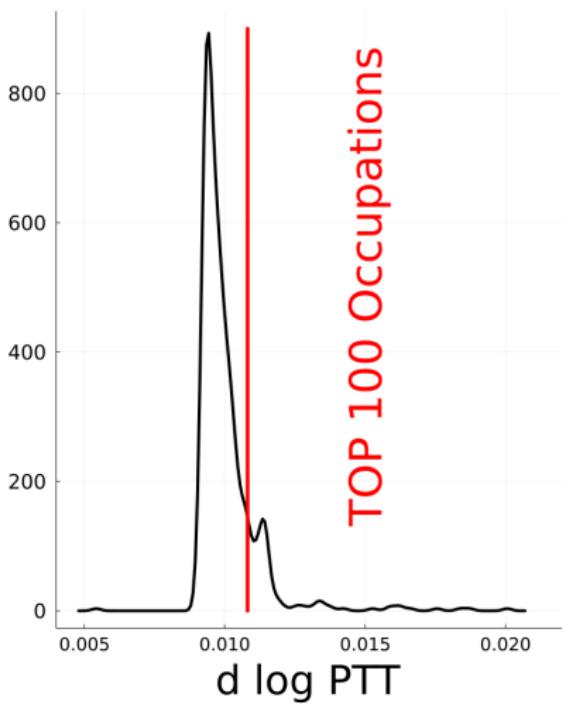


<b><i>Top 1% Occupation</i></b>	
Logging Workers	37%
Computer Occupations	13%
Mathematical Sciences Occupations	10%
Compensation Managers	7%

<b><i>Bottom 1% Occupation</i></b>	
Printing Workers	40%
Shoe & Leather Operator	26%
Textile Machine Operator	15%
Miscellaneous Textile	12%

# Effects from more competition in Housing



<i>Top 100 occupations</i>	
<b>Construction workers</b>	48
Painters, Carpet Installer, Tile Setter, Stonemason, Plasterer, Drywall Installer, Septic Servicer, Construction Supervisor	
<b>Financial specialist</b>	7
Property appraiser, Loan Officer Credit Analyst, Financial Examiner	
<b>Extraction Workers</b>	7
Rock Splitter, Roof Bolter	
<b>Woodworkers</b>	6
Cabinetmaker, Furnite Finisher	
<b>Installation &amp; Maintenance</b>	5
Heating & AC, Mobile Home Installer	

## Conclusion

- First comprehensive study for joint heterogeneity in multisector economies with distortions and input-output networks
- **Theoretical Contribution** in production network + distortions + heterogeneous households:
  - Variation of the income distribution
  - Variations for TFP
- **Empirical Contribution:** First implementation of a production network model with household heterogeneity for the US
  - In the absence of distributional sources of misallocation, TFP would have grown 7.5% more after Great Recession

# Pipeline

## Working Papers

1. In **International Misallocation and Comovement under Production Networks**, I obtain the first decomposition for a distorted open economy production network when there is cross-country factor allocation and ownership of firms

## Pipeline

### Working Papers

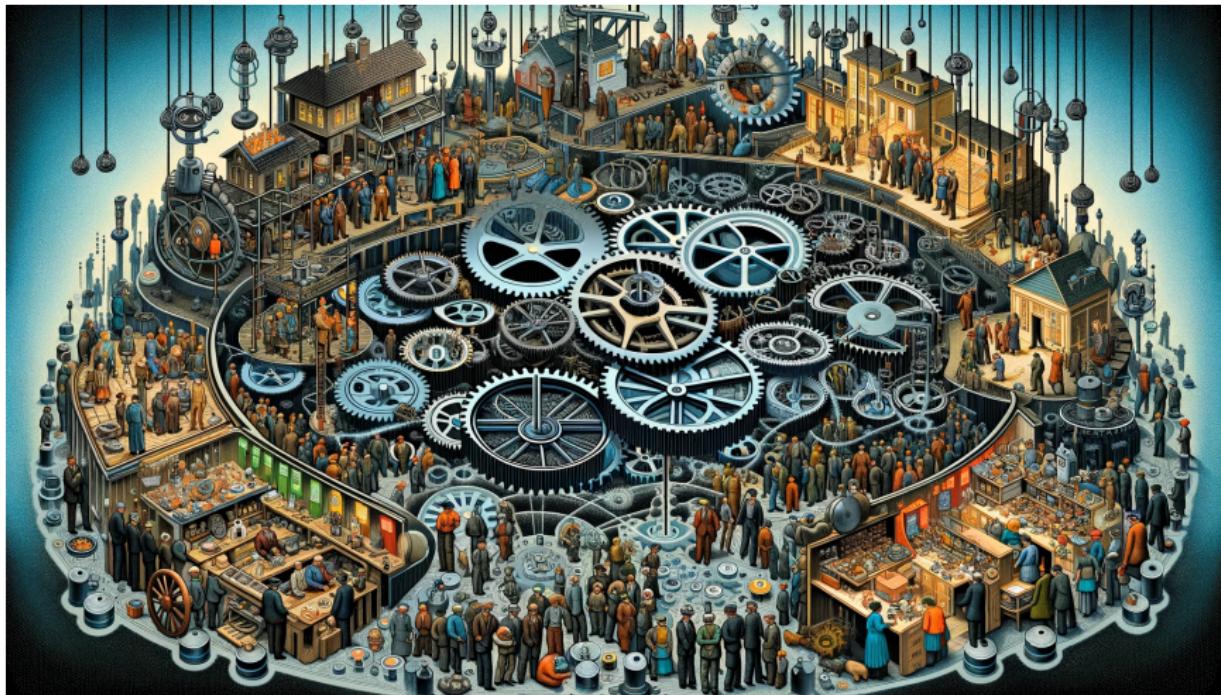
1. In **International Misallocation and Comovement under Production Networks**, I obtain the first decomposition for a distorted open economy production network when there is cross-country factor allocation and ownership of firms
2. In **Growth Through Industrial Linkages**, we evaluate how variations in global production networks have lifted up the growth for emerging economies

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2. In **Growth Through Industrial Linkages**, we evaluate how variations in global production networks have lifted up the growth for emerging economies
3. In **Nonlinearities in Production Networks with Distortions**, we obtain the first second-order approximation for a aggregate TFP in a production network economy with distortions

# Thank you!



Output from DALL-E after introducing title and abstract

# Upper Decile vs The Rest

## (Consumer Expenditure Survey 2021)

Higher Expenditure Share in

- Education: 3.4% vs 1.3%
- Entertainment: 6.5% vs 4.9%
- Pensions: 17.4% vs 9.1%
- Lodging: 2.6% vs 1.1%

Lower Expenditure Share in

- Shelter: 17.6% vs 20.5%
- Home Food: 5.9% vs 8.5%
- Utilities: 4.1% vs 7.0%
- Healthcare: 6.2% vs 8.3%

From 2004 to 2019

Income share for top quintile ↑ from 48% to 53%

# Literature Review

- **Disaggregated National Accounts**

Cantillon (1756), Quesnay (1758), Leontief (1928), Meade & Stone (1941), Kuznetz (1946), Stone (1961), Andersen et al. (2022)

- **Production Networks**

Hulten (1978), Long & Plosset (1983), Gabaix (2011), Jones (2011, 2013), Acemoglu et al. (2012), Baqaee (2018), Baqaee & Farhi (2019, 2020, 2023), Bigio & La'O (2020)

- **Growth Accounting**

Solow (1957), Domar (1961), Jorgenson et al. (1987), Basu & Fernanld (2022), Petrin & Levinsohn (2012), Baqaee & Farhi (2020)

## Dixit-Stiglitz Aggregation

- Sector  $i$  has a sectoral aggregator for  $z_i \in [0, 1]$

$$y_i = \left( \int y_{z_i}^{\mu_i} d z_i \right)^{\frac{1}{\mu_i}}$$

- Demand for varieties

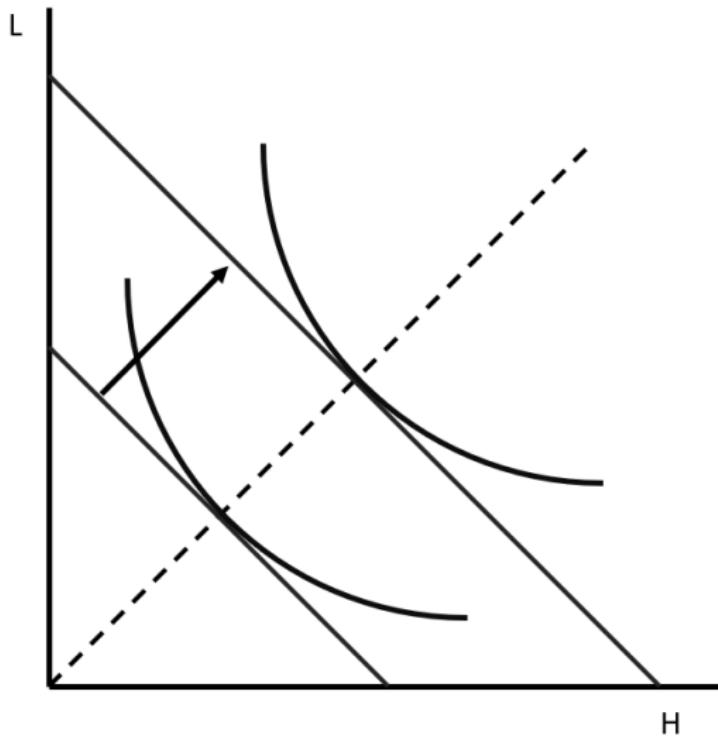
$$y_{z_i} = \left( \frac{p_i}{p_{z_i}} \right)^{\frac{1}{1-\mu_i}} y_i$$

- Intermediate's problem

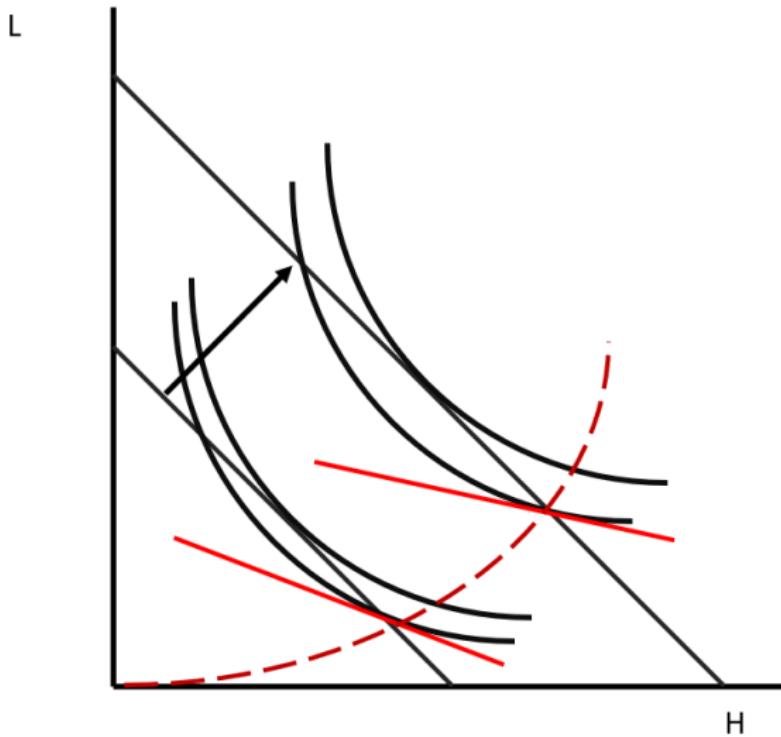
$$\underset{p_{z_i}, y_{z_i}, \ell_{z_i h}, \ell_{z_i I}}{\text{Max}} \quad \pi_{z_i} = p_{z_i} y_{z_i} - w_h \ell_{z_i h} - w_I \ell_{z_i I}$$

$$y_{z_i} = A_i \ell_{z_i h}^{\alpha_i} \ell_{z_i I}^{1-\alpha_i}$$

## Aggregate Non-Homotheticity



## Aggregate Non-Homotheticity



## Equilibrium Definition

$e = (A, \mu, \beta, \alpha) \in \mathcal{E}$  into

$$\vartheta \equiv \left\{ \left\{ y_i, \{\ell_{ir}, C_{ri}\}_{r \in \{h, l\}} \right\}_{i \in \{H, L\}}, \{C_r, L_r\}_{r \in \{h, l\}} \right\}$$

$$\rho \equiv \{p_H, p_L, w_h, w_l, p_h^c, p_l^c\}$$

## Necessary & sufficient equilibrium conditions

$(\vartheta, \rho)$  are an equilibrium iff

$$-\frac{w_b}{w_r} \frac{U_{Lr}}{U_{Cr_i}} = \mu_i \frac{\partial y_i}{\partial \ell_{ib}} \quad i \in \{H, L\}, r, b \in \{h, l\},$$

such that  $C_{ri} > 0$ , and  $\ell_{ib} > 0$ ,

and resource constraints

$$y_i(e) = C_{hi}(e) + C_{li}(e) \quad i \in \{H, L\}$$

$$L_r(e) = \ell_{Hr}(e) + \ell_{Lr}(e) \quad r \in \{h, l\}.$$

## Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

## Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

## Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

## Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

## Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

## Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2} \left( (1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L \right)$$

## Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

## Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

## Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2} \left( (1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L \right)$$

## Value Added Share

$$\tilde{\Lambda}_h = \alpha_H \lambda_H + \alpha_L \lambda_L \quad \tilde{\Lambda}_h + \tilde{\Lambda}_I = 1$$

## Sales Distribution

$$\lambda_H = \frac{\theta}{\left( \frac{1}{2 - (\underbrace{\beta_h - \beta_l}_{\geq 0})(\underbrace{\alpha_H - \alpha_L}_{\geq 0})} \right)}$$

**Amplification Effect**

**Contractionary Effect**

$\theta \geq 1/2$

## Consumption Expenditure Distribution

$$\chi_h = \theta \left( 1 - \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(\beta_h - \mu_H)}_{?} \right)$$

## Labor Income Distribution

$$\Lambda_h = \theta \left[ \underbrace{\alpha_L + \mu_H (\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(1 - \mu_L (\beta_h - \beta_I))}_{\geq 0} \right]$$

$$\Lambda_I = \theta \left[ \underbrace{\alpha_H - \mu_H (\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(1 + \mu_L (\beta_h - \beta_I))}_{\geq 0} \right]$$

## Value-Added Distribution [Back](#)

$$\begin{aligned}\tilde{\Lambda}_h &= \alpha_H \lambda_H + \alpha_L \lambda_L \\ &= \theta \left( 1 - \underbrace{(\beta_h - \beta_I)}_{\geq 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{\left( \alpha_H - \mu_H (\alpha_H - \alpha_L) \right)}_{\geq 0} \right)\end{aligned}$$

$$\tilde{\Lambda}_I = \theta \left( 1 - \underbrace{(\beta_h - \beta_I)}_{\geq 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{\left( \alpha_L + \mu_H (\alpha_H - \alpha_L) \right)}_{\geq 0} \right)$$

## 3 Effects from Distortions on Labor

### 1. Misallocation comes from *MRS* wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

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**2. Allocative differences  $\neq$  Misallocation**

$$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$$

**Intuition**  
 For the undistorted case  
 $\mu_H = \mu_L = 1/2$   
 there is a continuum  
 of property rights on firms

Cases

Back

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 there is a continuum  
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Cases

**3. Distorted Labor Supply  $\Gamma_r$**

$$-\frac{U_{Lr}}{U_{Cr}} = \frac{\Lambda_r}{\chi_r} \frac{C_r}{L_r}$$

$$= \Gamma_r$$

Back

$\frac{\ell_{Hh}}{L_H} \neq \alpha_H$  not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

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$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

$\mu_H = \mu_L$   
Symmetric  $\pi$

Case 2

All  $\pi$  for h

Case 3

Case 4

$\frac{\ell_{Hh}}{L_H} \neq \alpha_H$  not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

Case 2

Case 3

Case 4

$$\mu_H = \mu_L$$

Symmetric  $\pi$

All  $\pi$  for h

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_L)$$

$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$  not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

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$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{L_I}}{U_{C_I}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

$$\frac{\ell_{Hh}}{L_h} \neq \alpha_H \quad \text{not the same}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

**Case 1**

$$\mu_H = \mu_L$$

Symmetric  $\pi$

**Case 2**

All  $\pi$  for h

**Case 3**

$$\alpha_H = \alpha_L \quad \beta_h = \beta_I$$

Symmetric  $\pi$

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_I)$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$-\frac{U_{Lh}}{U_{Ch}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{LI}}{U_{CI}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$  not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

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Case 3

$$\alpha_H = \alpha_L \quad \beta_h = \beta_I$$

Symmetric  $\pi$

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_I)$$

$$\frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

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$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

Case 2

Case 3

Case 4

$$\mu_H = \mu_L$$

$$\alpha_H = \alpha_L \quad \beta_h = \beta_I$$

Symmetric  $\pi$

All  $\pi$  for h

Symmetric  $\pi$

$$\frac{\ell_{Hh}}{L_h} = \alpha_H \quad \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_I)$$

$$\frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

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$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{L_I}}{U_{C_I}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

# Linear Approximation in response to $d \log A_L$

■  $\lambda_H d \log S_H = \beta_h \chi_h d \log E_h + \beta_I \chi_I d \log E_I$

$$- (\rho - 1) \beta_h \beta_I \left( d \log A_L + (\alpha_H - \alpha_L) d \log \frac{w_h}{w_I} \right)$$

■  $\lambda_L d \log S_L = (1 - \beta_h) \chi_h d \log E_h + (1 - \beta_I) \chi_I d \log E_I$

$$+ (\rho - 1) \beta_h \beta_I \left( d \log A_L + (\alpha_H - \alpha_L) d \log \frac{w_h}{w_I} \right)$$

■  $d \log E_r = \frac{(1 + \zeta^w) \Gamma_r}{1 + \zeta^e \Gamma_r} d \log w_r + \frac{1}{2} \frac{\sum \lambda_i (1 - \mu_i) d \log S_i}{(1 + \zeta^e \Gamma_r) \chi_r}$

■  $d \log w_r = \frac{\zeta^e}{1 + \zeta^w} d \log E_r + \frac{\sum f_{ir} \lambda_i d \log S_i}{(1 + \zeta^w) \Lambda_r}$

## Bilateral Centralities

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{l \rightarrow h} \chi_l$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h}, \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

3 definitions for  $m_{r \rightarrow h}$

1. **Partial equilibrium effect** on  $h$ 's labor income from one additional **expenditure** unit from  $r$

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1. **Partial equilibrium effect** on  $h$ 's labor income from one additional **expenditure** unit from  $r$
2. **Share** of **expenditure** from  $r$  that reaches  $\Lambda_h$

## Bilateral Centralities

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3.  $\{m_{h \rightarrow h}, m_{l \rightarrow h}\}$  is a **ranking** for **expenditure** relevance on  $\Lambda_h$

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3 definitions for  $m_{r \rightarrow h}$

1. **Partial equilibrium effect** on  $h$ 's labor income from one additional **expenditure** unit from  $r$
2. **Share** of **expenditure** from  $r$  that reaches  $\Lambda_h$
3.  $\{m_{h \rightarrow h}, m_{l \rightarrow h}\}$  is a **ranking** for **expenditure** relevance on  $\Lambda_h$ 
  - Similar 3 definitions for  $f_{i \rightarrow h}$  but for **revenue** of  $i$

## Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\text{Shephard's Lemma}} = \frac{p_H C_{rH}}{p_r^c C_r} = \beta_r \text{ In equilibrium as parameter}$$

$$\frac{d p_r^c C_r}{d p_H} = C_{rH}$$

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1. New equilibrium with local approximations keep  $\alpha$  and  $\beta$  fixed

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**Exact delta hat - Dekle, Eaton & Kortum (2008)**

$$\frac{p_H C_{rH}}{E_r} = \beta_r^\rho \left( \frac{p_r^c \bar{C}_r}{p_H \bar{C}_{rH}} \right)^{\rho-1} \rightarrow d \beta_r = (\rho - 1) \beta_r (1 - \beta_r) \underbrace{d \log \frac{p_L}{p_H}}_{\text{Increases under substitutability when } p_L/p_H \uparrow}$$

## Theorem 1: labor income share variation

$$d \Lambda_I = \underbrace{\left( m_{h \rightarrow I} - m_{I \rightarrow h} \right) d \chi_h}_{\text{Distributive Income}_I} + \underbrace{\left( f_{H \rightarrow L} - f_{L \rightarrow H} \right) \sum \chi_r d \beta_r}_{\text{Income Centrality}_I}$$

$\overbrace{(\beta_h - \beta_I)}^{\geq 0} \overbrace{(\mu_H - \alpha_H)}^{?}$ 
 $\overbrace{(\mu_H - \alpha_H)}^{?}$   
 $\overbrace{(f_{H \rightarrow L} - f_{L \rightarrow H})}^{?}$

## Labor Wedge

For factors with endogenous supply...

$$-\frac{U_{L_h}}{U_{C_h}} = \Gamma_h \frac{C_h}{L_h}$$

with

$$\Gamma_h = \frac{\Lambda_h}{\chi_h}$$

Proof

*d log  $\Gamma_h$*  - Extension of Bigio & La'O (2020)

- |                                   |   |                             |
|-----------------------------------|---|-----------------------------|
| (i) Representative Household      | → | (i) Heterogenous Households |
| (ii) Around Efficient Equilibrium |   | (ii) Any Equilibrium        |

$$d \log \Gamma_h = d \log \Lambda_h - d \log \chi_h$$

## Proof of Theorem 1 for $d \log \Gamma_h$

**From goods market clearing**

$$\begin{pmatrix} y_H \\ y_L \end{pmatrix} = \begin{pmatrix} C_{hH} + C_{IH} \\ C_{hL} + C_{IL} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hL}} y_L \end{pmatrix} = \begin{pmatrix} \frac{\beta_h}{C_{hH}} (C_{hH} + C_{IH}) \\ \frac{(1-\beta_h)}{C_{hL}} (C_{hL} + C_{IL}) \end{pmatrix}$$

From FOC and equilibrium  $\beta_h \frac{\chi_h}{C_{hH}} = p_H = \beta_I \frac{\chi_I}{C_{IH}}$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hH}} y_L \end{pmatrix} = \begin{pmatrix} \beta_h \frac{\chi_h}{\chi_h} + \beta_I \frac{\chi_I}{\chi_h} \\ (1 - \beta_h) \frac{\chi_h}{\chi_h} + (1 - \beta_I) \frac{\chi_I}{\chi_h} \end{pmatrix}$$

Back

## Proof of Theorem 1 for $d \log \Gamma_h$

From FOC and equilibrium  $-\frac{1}{\beta_h} \frac{U_{L_h}}{U_{C_h}} \frac{C_{hH}}{C_h} = \frac{w_h}{p_H} = \mu_H \alpha_H \frac{y_H}{\ell_{Hh}}$

$$\begin{pmatrix} \ell_{Hh} \\ \ell_{Lh} \end{pmatrix} = \begin{pmatrix} -\frac{U_{C_h}}{U_{L_h}} \alpha_H \mu_H y_H \beta_h \frac{C_h}{C_{hH}} \\ -\frac{U_{C_h}}{U_{L_h}} \alpha_L \mu_L y_L (1 - \beta_h) \frac{C_h}{C_{hL}} \end{pmatrix}$$

**From labor market clearing condition**

$$L_h = \ell_{Hh} + \ell_{Lh} = -\frac{U_{C_h}}{U_{L_h}} C_h (\alpha_H \mu_H - \alpha_L \mu_L) \begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hH}} y_L \end{pmatrix}$$

$$= -\frac{U_{C_h}}{U_{L_h}} C_h \underbrace{\left( \alpha_H \mu_H \sum_{r \in \{h,l\}} \beta_r \frac{\chi_r}{\chi_h} + \alpha_L \mu_L \sum_{r \in \{h,l\}} (1 - \beta_r) \frac{\chi_r}{\chi_h} \right)}_{=\Gamma_h} \quad \text{Back}$$

## Δ TFP

A.

$$GDP = P_Y Y = p_h^c C_h + p_l^c C_l$$

$$d \log GDP = \chi_h d \log p_h^c C_h + \chi_l d \log p_l^c C_l$$

## Δ TFP

A.

$$GDP = P_Y Y = p_h^c C_h + p_l^c C_l$$

$$d \log GDP = \chi_h d \log p_h^c C_h + \chi_l d \log p_l^c C_l$$

B. Divisia Index GDP deflator

$$\begin{aligned} d \log P_Y &\equiv \chi_h d \log p_h^c + \chi_l d \log p_l^c \\ &= \tilde{\Lambda}_h d \log w_h + \tilde{\Lambda}_l d \log w_l \\ &\quad - \lambda_H d \log (A_H \times \mu_H) - \lambda_L d \log (A_L \times \mu_L) \end{aligned}$$

## Additional Steps for $d \log P_Y$

Start from

$$p_H = \frac{w_h \ell_{Hh} + w_l \ell_{Hl}}{\mu_H A_H \ell_{Hh}^{\alpha_H} \ell_{Hl}^{1-\alpha_H}}$$

Take first-order approximation

$$\hat{p}_H = -\hat{A}_H - \hat{\mu}_H + \alpha_H \hat{\alpha}_{Hh} + (1 - \alpha_H) \hat{\alpha}_{Hl}$$

Do the same for bundle prices

$$\hat{p}_h^c = -\beta_h (\hat{A}_H + \hat{\mu}_H) - (1 - \beta_h) (\hat{A}_L + \hat{\mu}_L) + \tilde{\mathcal{C}}_{hh} \hat{w}_h + \tilde{\mathcal{C}}_{hl} \hat{w}_l$$

# Distortion Centrality Heterogeneity

$$d \Lambda = d \Lambda_h + d \Lambda_I$$

$$\text{Misallocation} = \overbrace{(\delta_I - \delta_h)}^{\geq 0} d \Lambda_I + \delta_h d \Lambda$$

$$\delta_I - \delta_h = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \overbrace{(\alpha_H - \alpha_L)}^{\geq 0} \overbrace{a}^{> 0}$$

$$a = \frac{1 + (\beta_h - \beta_I)(\alpha_H - \alpha_L) \left( 1 + \overbrace{\mu_H \mu_L (\beta_h - \beta_I)(\alpha_H - \alpha_L)}^b \right)}{(\alpha_H \mu_H + \alpha_L \mu_L - b)(\alpha_H \mu_L + \alpha_L \mu_H - b)}$$

## Constant *a*

$$a = \frac{1 + (\beta_h - \beta_l)(\alpha_H - \alpha_L) \left( 1 + \overbrace{\mu_H \mu_L (\beta_h - \beta_l)(\alpha_H - \alpha_L)}^b \right)}{(\alpha_H \mu_H + \alpha_L \mu_L - b)(\alpha_H \mu_L + \alpha_L \mu_H - b)}$$

## Alternatives: Income distribution → Output

In Auclert & Rognlie (2020)

- Negative Correlation between income and MPC
- + Wage rigidities
- Aggregate Demand  $\downarrow$  & Keynesian unemployment

In my model

- Static model, MPC equals 1
- No nominal rigidities
- Supply effect due to Misallocation

# Income Centrality

$$\begin{aligned}
 \text{Income Centrality}_h &= \overbrace{\sum_{i \in \mathcal{N}} \mu_i \lambda_i d \tilde{\Omega}_{ih}^{\ell}}^{\text{Labor Demand Recomposition}_h} + \overbrace{\sum_{i \in \mathcal{N}} f_{ih}^{\ell} \lambda_i d \log \mu_i}^{\text{Competitive Income}_h} \\
 &\quad + \overbrace{\sum_{i \in \mathcal{N}} f_{ih}^{\ell} \sum_{b \in \mathcal{H}} \chi_b d \beta_{bi}}^{\text{Final Demand Recomposition}_h} + \overbrace{\sum_{i \in \mathcal{N}} f_{ih}^{\ell} \sum_{j \in \mathcal{N}} \mu_j \lambda_j d \tilde{\Omega}_{ji}^x}^{\text{Intermediate Demand Recomposition}_h}
 \end{aligned}$$

Share of sector  
 i's revenue reaching  
 worker h's income

Sales Share  $\lambda_j = \frac{\text{Sales}_j}{GDP}$

## Misallocation Decomposition

1. Misallocation  $\uparrow$  as expenditure rises for households with high  $M_h$
2. Misallocation  $\uparrow$  as labor demand for workers with high  $\delta$  rises
3. Misallocation  $\uparrow$  as profit margins fall in sector with high  $F_i$

$$\begin{aligned}
 & \underbrace{\sum_{h \in \mathcal{H}} M_h d \chi_h}_{\text{Distributive}} + \underbrace{\sum_{i \in \mathcal{N}} \mu_i \lambda_i \sum_{h \in \mathcal{H}} \delta_h d \tilde{\Omega}_{ih}^\ell}_{\text{Labor Demand}} + \underbrace{\sum_{i \in \mathcal{N}} \lambda_i F_i d \log \mu_i}_{\text{Competitive}} \\
 & + \underbrace{\sum_{h \in \mathcal{H}} \chi_h \sum_{i \in \mathcal{N}} F_i d \beta_{hi}}_{\text{Final Demand}} + \underbrace{\sum_{i \in \mathcal{N}} \mu_i \lambda_i \sum_{j \in \mathcal{N}} F_j d \tilde{\Omega}_{ij}^x}_{\text{Intermediate Demand}}
 \end{aligned}$$

4. Misallocation  $\uparrow$  as demand of goods  $\uparrow$  from sectors with high  $F_i$

## Antisupression Algorithm

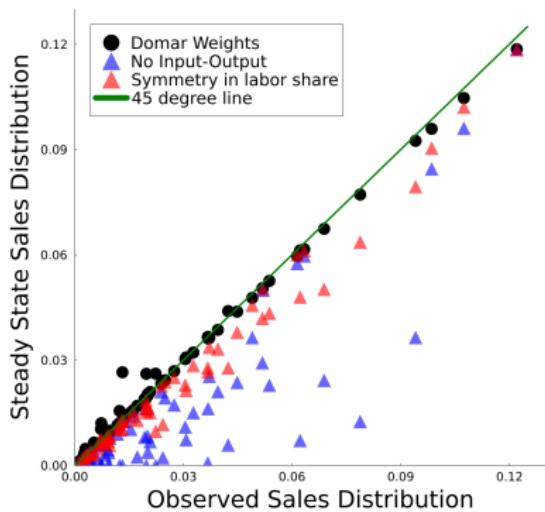
1. Significant portion of data suppressed to protect confidentiality
2. Since 2007 non-suppressed observations have a random noise infusion multiplier
3. Use information available due to the industrial and geographical hierarchical nature → manifold of bound and aggregation constraints across hierarchies
4. Two gold standards:
  - i. Two-staged algorithm from Isserman & Westervelt (2006)
  - ii. Linear programming solution from Eckert et al. (2020)
5. These two methods estimate the number of workers, not their compensation. I develop a three-staged algorithm that starting from the guess Eckert et al. (2020) extends Isserman & Westervelt (2006) to the estimation of labor compensation

## Missing Private Employment

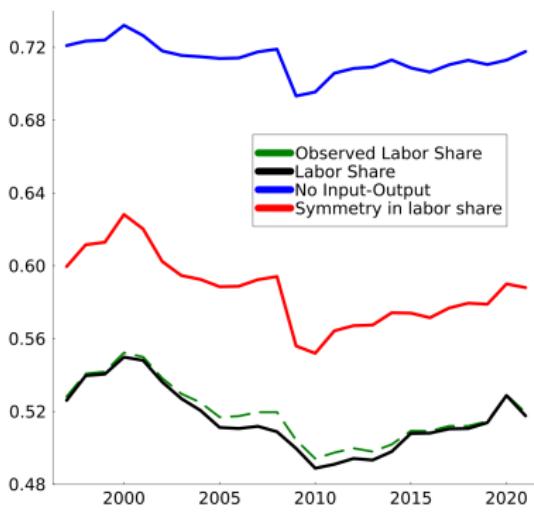
1. The CBP only covers some forms of private employment
2. It does not include workers in
  - Agriculture production
  - Railroads
  - Government
  - Private household
3. To fill this gap, I use the BEA's Regional Economic Information System to obtain state-level employment and income measures for agricultural and production workers
4. Data sources for REIS are the Quarterly Census of Employment and Statistics from the BLS
5. Main limitation from REIS is that it is only provided at the 2-digit NAICS level

# Moments under Representative Household

Sales Distribution 2021



Labor Share



$R^2$  on sales distribution

$R^2$  on labor cost share

	$R^2$ on sales distribution	$R^2$ on labor cost share
<b>Base Model</b>	0.994	0.981
<b>No Input-Output</b>	0.730	0.733
<b>Symmetry in Labor</b>	0.978	0.933

## Contribution from each component

Table: Counterfactual TFP Growth Differential in the Absence of Components

### A. Between 1997 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>Misallocation</i>
-23.4%	2.5%	2.8%

### B. Between 2002 and 2009

<i>Technology</i>	<i>Competitiveness</i>	<i>Misallocation</i>
-13.0%	19.3%	-8.2%

### C. Between 2010 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>Misallocation</i>
-6.3%	-9.8%	7.6%

# Covariance Decomposition

Table: Covariance Decomposition

## A. Between 1997 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>-Misallocation</i>
44.4%	34.6%	21.0%

## B. Between 2002 and 2009

<i>Technology</i>	<i>Competitiveness</i>	<i>-Misallocation</i>
28.3%	61.2%	10.5%

## C. Between 2010 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>-Misallocation</i>
58.1%	4.9%	37.0%

# Model without Intermediate Inputs

	<i>Rep.</i>	<i>Household</i>	<i>Occupation</i>	<i>County</i>	<i>State &amp; Occupation</i>			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>dlog TFP</i>	0.523 (0.366)		0.503 (0.350)		0.388 (0.316)		-0.265 (0.264)	
Technology		1.341*** (0.308)		0.789*** (0.267)		0.796*** (0.266)		0.847*** (0.289)
Competitiveness		0.212 (0.423)		0.320 (0.489)		0.454 (0.373)		0.986 (0.695)
Misallocation		0.573* (0.329)		0.450 (0.437)		0.335 (0.315)		-0.105 (0.360)
Intercept	0.012*** (3.2e-3)	0.011*** (2.0e-3)	0.012*** (3.2e-3)	0.012*** (2.2e-3)	0.013*** (3.2e-3)	0.012*** (2.1e-3)	0.015*** (3.0e-3)	0.012*** (2.2e-3)
Observations					22			
N					66			
H		1		750		3,136		38,190
<i>R</i> <sup>2</sup>	9.2%	71.4%	9.35%	62.4%	7.00%	62.5%	4.8%	60.4%
<i>Adj. R</i> <sup>2</sup>	9.2%	68.4%	9.35%	58.4%	7.00%	58.6%	4.8%	56.2%

# Model with Intermediate Inputs

	<i>Rep.</i>	<i>Household</i>	<i>Occupation</i>	<i>County</i>	<i>State &amp; Occupation</i>			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>dlog TFP</i>	0.370*** (0.072)		0.311*** (0.069)		0.316*** (0.065)		0.311*** (0.069)	
Technology		0.478*** (0.097)		0.414*** (0.081)		0.416*** (0.083)		0.413*** (0.082)
Competitiveness		0.398*** (0.062)		0.341*** (0.054)		0.350*** (0.053)		0.342*** (0.054)
Misallocation		0.074 (0.138)		0.172 (0.125)		0.164 (0.135)		0.168 (0.125)
Intercept	0.010*** (2.1e-3)	0.009 (2.0e-3)	0.011*** (2.2e-3)	0.010*** (1.8e-3)	0.011*** (2.1e-3)	0.010*** (1.9e-3)	0.011*** (2.3e-3)	0.010*** (1.9e-3)
Observations					22			
N					66			
H		1		750		3,136		38,190
<i>R</i> <sup>2</sup>	56.9%	75.2%	49.9%	75.8%	54.0%	75.4%	49.9%	75.5%
<i>Adj. R</i> <sup>2</sup>	56.9%	72.6%	49.9%	73.3%	54.0%	72.8%	49.9%	73.2%

# Technological Sources

## A. Between 1998 and 2020

1	Oil & gas extraction	-11.11%
2	Computer & electronics	-6.64%
3	Telecommunications	-2.85%
4	Computer systems design	-2.30%
5	Administrative services	-1.74%
6	Insurance carriers	-1.45%
7	Farms	-1.34%
8	Primary metals	-1.28%
	:	
63	Rental & leasing	1.41%
64	Credit intermediation	1.77%
65	Chemical Products	2.84%
66	Construction	2.87%

## C. Between 2010 and 2020

1	Oil & gas extraction	-5.41%
2	Computer systems design	-1.29%
3	Management of companies	-1.26%
4	Housing	-1.14%
5	Other real estate	-1.01%
	:	
64	Air transportation	1.03%
65	Chemical products	1.90%
66	Credit intermediation	2.73%

## B. Between 2002 and 2009

1	Oil & gas extraction	-5.35%
2	Computer & electronics	-2.84%
3	Telecommunications	-2.27%
4	Utilities	-1.92%
5	Administrative services	-1.06%
	:	
66	Construction	1.76%

# Competitiveness Sources

## A. Between 1998 and 2020

1	Housing	-1.65%
2	Insurance carriers	-1.53%
3	Misc. professional services	-1.10%
4	Other services	-0.89%
	:	
63	Publishing industries	0.80%
64	Computer and electronics	1.34%
65	Chemical products	2.57%
66	Credit intermediation	4.10%

## C. Between 2010 and 2020

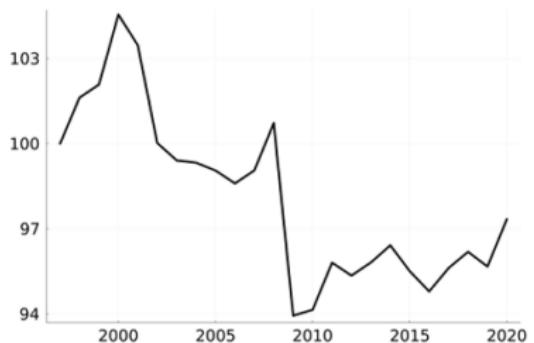
1	Oil & gas extraction	-6.34%
2	Housing	-3.09%
3	Insurance carriers	-0.98%
4	Misc. professional services	-0.87%
5	Administrative services	-0.82%
	:	
64	Primary metals	0.80%
65	Chemical products	0.84%
66	Credit intermediation	3.86%

## B. Between 2002 and 2009

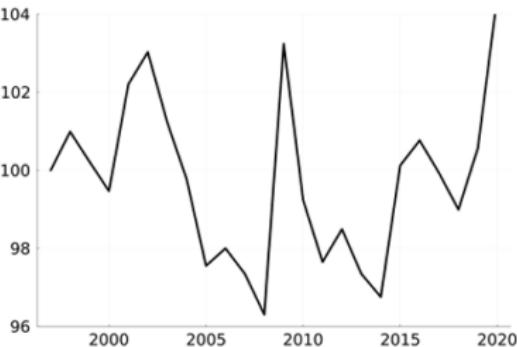
1	Securities & investment	-0.86%
	:	
58	Wholesale trade	0.92%
59	Publishing industries	0.93%
60	Internet, & inf. services	0.99%
61	Chemical products	1.35%
62	Telecommunications	1.43%
63	Computer and electronics	1.48%
64	Housing	1.57%
65	Utilities	1.87%
66	Oil & gas extraction	6.59%

# Sources of Misallocation

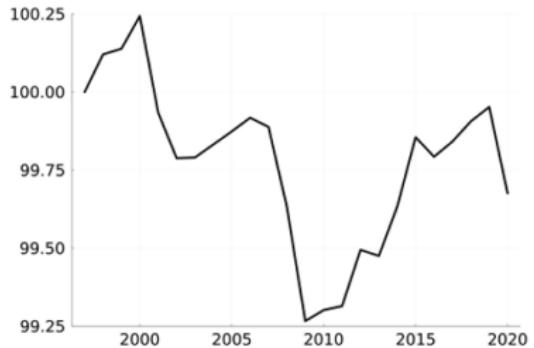
Competitive Terms of Trade



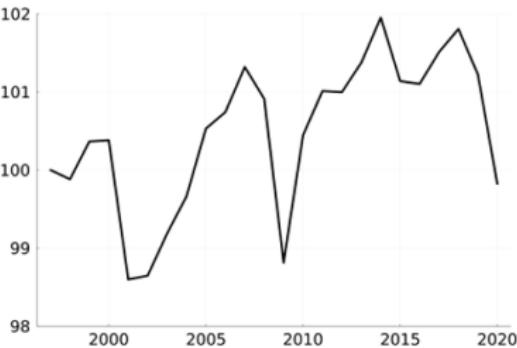
Labor Demand Terms of Trade



Final Demand Terms of Trade



Intermediate Demand Terms of Trade



**Table 11: Counterfactual TFP Growth Differential in the Absence of Misallocation Components**

**A. Between 1998 and 2020**

<i>Heterogeneity</i>	<i>Distributive</i>	<i>Competitive</i>	<i>Labor</i>	<i>Final</i>	<i>Intermediate</i>
	<i>TT</i>	<i>TT</i>	<i>DTT</i>	<i>DTT</i>	<i>DTT</i>
Rep. Household	0%	-3.4%	6.3%	0.4%	-1.3%
Occupation	0%	-5.9%	15.1%	-2.0%	-4.2%
County	0.1%	-5.2%	14.2%	-0.9%	-4.4%
State & Occupation	0.1%	-5.9%	15.6%	-2.6%	-4.5%

**B. Between 2002 and 2009**

<i>Heterogeneity</i>	<i>Distributive</i>	<i>Competitive</i>	<i>Labor</i>	<i>Final</i>	<i>Intermediate</i>
	<i>TT</i>	<i>TT</i>	<i>DTT</i>	<i>DTT</i>	<i>DTT</i>
Rep. Household	0%	-9.3%	1.1%	-0.9%	-0.2%
Occupation	0%	-11.0%	3.4%	-1.9%	-0.8%
County	0.1%	-10.4%	3.4%	-0.7%	-1.0%
State & Occupation	0.1%	-11.1%	3.4%	-2.0%	-0.9%

**C. Between 2010 and 2020**

<i>Heterogeneity</i>	<i>Distributive</i>	<i>Competitive</i>	<i>Labor</i>	<i>Final</i>	<i>Intermediate</i>
	<i>TT</i>	<i>TT</i>	<i>DTT</i>	<i>DTT</i>	<i>DTT</i>
Rep. Household	0%	3.9%	1.2%	1.7%	0.9%
Occupation	0%	2.9%	7.2%	0.2%	-1.8%
County	0.1%	3.0%	3.5%	2.1%	-1.5%
State & Occupation	0.1%	2.8%	7.4%	-0.1%	-1.7%

**Table 12: Counterfactual TFP Growth Without Sectoral Competitive TT**

<b>A. Between 1998 and 2020</b>		
1	Credit intermediation	-2.16%
2	Chemical products	-1.06%
3	Computer & electronics	-0.98%
4	Publishing industries	-0.80%
5	Internet & inf. services	-0.69%
	:	
64	Insurance carriers	0.77%
65	Other services	0.81%
66	Misc. professional services	0.87%

**B. Between 2002 and 2009**

<b>B. Between 2002 and 2009</b>		
1	Oil & gas extraction	-1.46%
2	Computer & electronics	-1.11%
3	Internet & inf. services	-1.01%
4	Wholesale trade	-0.92%
5	Telecommunications	-0.86%
6	Utilities	-0.84%
7	Publishing industries	-0.82%

**C. Between 2010 and 2020**

<b>C. Between 2010 and 2020</b>		
1	Credit intermediation	-2.0%
2	Securities & investment	-0.52%
	:	
64	Administrative services	0.62%
65	Misc. professional services	0.70%
66	Oil & gas extraction	1.91%

**Table 13: Counterfactual TFP Growth Without Sectoral Labor Demand TT**

<b>A. Between 1998 and 2020</b>		
1	Wholesale trade	-1.62%
2	Insurance carriers	-1.61%
3	Other retail	-1.07%
	:	
61	Utilities	0.69%
62	Computer systems design	0.82%
63	Publishing industries	1.34%
64	Oil & gas extraction	1.79%
65	Computer & electronics	2.28%
66	Credit intermediation	2.40%

**B. Between 2002 and 2009**

<b>B. Between 2002 and 2009</b>		
1	Securities & investment	-0.96%
	:	
64	Computer & electronicss	0.85%
65	Utilities	1.02%
66	Oil & gas extraction	2.20%

**C. Between 2010 and 2020**

<b>C. Between 2010 and 2020</b>		
1	Wholesale trade	-1.70%
2	Insurance carriers	-1.03%
3	Administrative services	-0.93%
4	Other retail	-0.83%
	:	
64	Publishing industries	0.89%
65	Computer & electronics	0.98%
66	Credit intermediation	2.44%

**Table 14: Counterfactual TFP Growth Without Sectoral Final Demand TT**

<b>A. Between 1998 and 2020</b>		
1	Computer & electronics	-1.50%
2	Motor vehicles	-0.91%
3	Machinery	-0.88%
4	Apparel & leather	-0.51%
	:	
62	Securities & investment	0.87%
63	Misc. professional services	0.94%
64	Hospitals	0.95%
65	Internet & inf. services	1.01%
66	Wholesale trade	1.18%

<b>B. Between 2002 and 2009</b>		
1	Construction	-1.22%
2	Motor vehicles	-0.82%
	:	
66	Hospitals	0.58%

<b>C. Between 2010 and 2020</b>		
1	Computer & electronics	-0.52%
	:	
63	Other retail	0.59%
64	Internet & inf. services	0.60%
65	Construction	0.89%
66	Wholesale trade	1.08%

**Table 15: Counterfactual TFP Growth Without Sectoral Intermediate Demand TT**

<b>A. Between 1998 and 2020</b>		
1	Computer & electronics	-1.24%
2	Credit intermediation	-0.90%
3	Publishing industries	-0.76%
4	Computer systems design	-0.45%
5	Ambulatory health	-0.42%
	:	
61	Telecommunications	0.52%
62	Administrative services	0.54%
63	Hospitals	0.56%
64	Insurance carriers	0.74%
65	Other retail	0.90%
66	Wholesale trade	1.21%

<b>B. Between 2002 and 2009</b>		
1	Computer & electronics	-0.48%
	:	
66	Securities & investment	0.49%

<b>C. Between 2010 and 2020</b>		
1	Credit intermediation	-0.97%
2	Publishing industries	-0.51%
3	Computer & electronics	-0.49%
	:	
63	Insurance carriers	0.52%
64	Administrative services	0.63%
65	Other retail	0.66%
66	Wholesale trade	1.12%

## Normalized nested CES environment - Firms

### Firms

$$\frac{y_i}{\bar{y}_i} = A_i \left( \sum_{h \in \mathcal{H}} \tilde{\Omega}_{ih}^{\ell} \left( \frac{\ell_{ih}}{\bar{\ell}_{ih}} \right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{j \in \mathcal{N}} \tilde{\Omega}_{ij}^x \left( \frac{x_{ij}}{\bar{X}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}$$

Back

# Normalized nested CES environment - Households

## Households

$$U_h(c_h, \tilde{L}_h) = \frac{\left(c_h \left(1 - E_h^{-\gamma_h} \tilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1 - \sigma} \quad s.t. \quad \frac{C_h}{\bar{C}_h} = \left(\sum_{i \in \mathcal{N}} \beta_{hi} \left(\frac{C_{hi}}{\bar{C}_{hi}}\right)^{\frac{\rho_h-1}{\rho_h}}\right)^{\frac{\rho_h}{\rho_h-1}}$$

with  $C_h = n_h c_h$  and  $L_h = n_h \tilde{L}_h$

## Normalized nested CES environment - Households

### Households

$$U_h \left( c_h, \tilde{L}_h \right) = \frac{\left( c_h \left( 1 - E_h^{-\gamma_h} \tilde{L}_h \right)^{\varphi_h} \right)^{1-\sigma} - 1}{1 - \sigma} \quad s.t. \quad \frac{C_h}{\bar{C}_h} = \left( \sum_{i \in \mathcal{N}} \beta_{hi} \left( \frac{C_{hi}}{\bar{C}_{hi}} \right)^{\frac{\rho_h - 1}{\rho_h}} \right)^{\frac{\rho_h}{\rho_h - 1}}$$

with  $C_h = n_h c_h$  and  $L_h = n_h \tilde{L}_h$

The change in labor supply from type  $h$  workers is, to a first-order

$$d \log L_h = \zeta_h^n d \log n_h + \zeta_h^w d \log w_h - \zeta_h^e d \log E_h$$

Where the corresponding elasticities are given by

$$\zeta_h^n = \frac{E_h^{\gamma_h}}{1 - \varphi_h \gamma_h} \frac{n_h}{L_h}, \quad \zeta_h^w = \frac{1}{1 - \varphi_h \gamma_h} \frac{\varphi_h}{\Gamma_h}, \quad \zeta_h^e = \zeta_h^w - \gamma_h \zeta_h^n.$$

# Solution - Expenditure & Wages

$$\begin{aligned}
 d \log E_h &= \underbrace{\frac{\zeta_h^n \Gamma_h}{1 + \zeta_h^e \Gamma_h} d \log n_h}_{\text{Demographic Effect on Expenditure (PE)}} + \underbrace{\frac{(1 + \zeta_h^w) \Gamma_h}{1 + \zeta_h^e \Gamma_h} d \log w_h}_{\text{Wage Effect on Expenditure (GE)}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{\kappa_{ih} \lambda_i}{(1 + \zeta_h^e \Gamma_h) \chi_h} ((1 - \mu_i) d \log S_i - \mu_i d \log \mu_i)}_{\text{Corporate Income Effect on Expenditure (PE + GE)}} \\
 d \log w_h &= \underbrace{\frac{\zeta_h^e}{1 + \zeta_h^w} d \log E_h}_{\text{Expenditure Effect on Wages (GE)}} - \underbrace{\frac{\zeta_h^n}{1 + \zeta_h^w} d \log n_h}_{\text{Demographic Effect on Wages (PE)}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} ((\theta_i - 1) d \log A_i + \theta_i d \log \mu_i)}_{\text{Direct Effect on Wages (PE)}} \\
 &\quad - \underbrace{\sum_{j \in \mathcal{N}} \left( \sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} (\theta_i - 1) \tilde{\psi}_{ij}^x \right) (d \log A_j + d \log \mu_j)}_{\text{Supplier Effect on Wages (PE)}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} d \log S_i}_{\text{Sales Effect on Wages (GE)}} \\
 &\quad - \underbrace{\left( \sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} (\theta_i - 1) \right) d \log w_h}_{\text{Direct Substitution Effect on Wages (GE)}} + \underbrace{\sum_{b \in \mathcal{B}} \left( \sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} (\theta_i - 1) \tilde{\psi}_{ib}^\ell \right) d \log w_b}_{\text{Supplier Substitution Effect on Wages (GE)}}
 \end{aligned}$$

## Solution - Sales

$$\begin{aligned}
 d \log S_i = & \underbrace{\sum_{h \in \mathcal{H}} \frac{\beta_{hi} \chi_h}{\lambda_i} d \log E_h}_{\text{Expenditure Effect on Sales (GE)}} + \underbrace{\sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} d \log S_j}_{\text{Sales Effect on Sales (GE)}} + \underbrace{\sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} \left( (\theta_j - 1) d \log A_j + \theta_j d \log \mu_j \right)}_{\text{Direct Effect on Sales (PE)}} \\
 & + \underbrace{\sum_{j \in \mathcal{N}} \left( \sum_{h \in \mathcal{H}} \frac{\beta_{hi} \chi_h}{\lambda_i} (\rho_h - 1) (\tilde{\psi}_{ij}^x - \tilde{\mathcal{B}}_{hj}) + \sum_{q \in \mathcal{N}} \frac{\Omega_{qi}^x \lambda_q}{\lambda_i} (\theta_q - 1) (\tilde{\psi}_{ij}^x - \tilde{\psi}_{qj}^x) \right) (d \log A_j + d \log \mu_j)}_{\text{Supplier Effect on Sales (PE)}} \\
 & + \underbrace{\sum_{h \in \mathcal{H}} \left( \sum_{b \in \mathcal{H}} \frac{\beta_{bi} \chi_b}{\lambda_i} (\rho_b - 1) (\tilde{\mathcal{C}}_{bh} - \tilde{\psi}_{ih}^\ell) + \sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} (\theta_j - 1) (\tilde{\psi}_{jh}^\ell - \tilde{\psi}_{ih}^\ell) \right) d \log w_h}_{\text{Supplier Substitution Effect on Sales (GE)}}
 \end{aligned}$$