

Inequality and Misallocation under Production Networks

Alejandro Rojas-Bernal

The University of British Columbia

January 2024

Total Factor Productivity & Aggregation

1. In standard models TFP is given

- Solow & Ramsey growth model
 - RBC & New Keynesian models

Total Factor Productivity & Aggregation

1. In standard models TFP is given
 - Solow & Ramsey growth model
 - RBC & New Keynesian models
 2. Through Aggregation
 - Multiple Firms \Rightarrow Allocation
 - Production Networks \Rightarrow Amplification

Aggregate TFP is endogenous

Research Question & Motivation

Research Question

What is the effect of variations in the distributions of labor income and consumption expenditure on TFP?

Research Question & Motivation

Research Question

What is the effect of variations in the distributions of labor income and consumption expenditure on TFP?

Upper Decile vs The Rest

Higher Expenditure Share in Education, Entertainment, Pensions

Lower Expenditure Share in Shelter, Utilities, Healthcare

Data from Consumer Expenditure Survey

Research Question & Motivation

Research Question

What is the effect of variations in the distributions of labor income and consumption expenditure on TFP?

Upper Decile vs The Rest

Higher Expenditure Share in
Education, Entertainment, Pensions

Lower Expenditure Share in
Shelter, Utilities, Healthcare

Data from Consumer Expenditure Survey

Income share for the top has increased

In this Presentation

In economies with distortions, variations in distributions (labor income & expenditure) can influence misallocation

Novel TFP decomposition that measures aggregate misallocations effects

Implementation of the model with US data

Approach

Contribution

Δ Consumption Distribution & Δ Demand Structure



△ Income Distribution

Approach

Δ Consumption Distribution & Δ Demand Structure



Contribution

△ Labor Income Shares

△ Income Distribution

Approach

Δ Consumption Distribution & Δ Demand Structure



△ Income Distribution

Contribution

Bigio & La'O (2020)

- (i) Rep Household
 - (ii) Efficient Equilibrium

↓ **Δ Labor Income Shares**

Approach

Δ Consumption Distribution & Δ Demand Structure



△ Income Distribution

Contribution

Bigio & La’O (2020)

- (i) Rep Household
 - (ii) Efficient Equilibrium

↓ **Labor Income Shares**

My Model

- (i) Het Households
 - (ii) Any Equilibrium

Approach

Δ Consumption Distribution & Δ Demand Structure



△ Income Distribution



△ Misallocation



Δ TFP

Contribution

Bigio & La’O (2020)

- (i) Rep Household
 - ii) Efficient Equilibrium

↓ **Δ Labor Income Shares**

My Model

- (i) Het Households
 - (ii) Any Equilibrium

Approach

Δ Consumption Distribution & Δ Demand Structure



△ Income Distribution



△ Misallocation



△ TFP

Contribution

Bigio & La'O (2020)

- (i) Rep Household
 - ii) Efficient Equilibrium



My Model

- (i) Het Households
 - (ii) Any Equilibrium

Baqae & Farhi (2020)

- (i) Rep Household
 - (ii) Exogenous L



My Model

- (i) Het Households
 - (ii) Endogenous L

Static General Equilibrium Model with...

Two Firms : $\begin{cases} \text{More Competitive: } H \\ \text{Less Competitive: } L \end{cases}$ + **Two Workers** : $\begin{cases} \text{High-Skill: } h \\ \text{Low-Skill: } l \end{cases}$

Caveat: Paper is more general than this case

Static General Equilibrium Model with...

Two Firms : $\begin{cases} \text{More Competitive: } H \\ \text{Less Competitive: } L \end{cases}$ + **Two Workers** : $\begin{cases} \text{High-Skill: } h \\ \text{Low-Skill: } l \end{cases}$

Caveat: Paper is more general than this case

1. Good markets face exogenous distortions

$$Cost = \mu \times Revenue$$

Static General Equilibrium Model with...

Two Firms : $\begin{cases} \text{More Competitive: } H \\ \text{Less Competitive: } L \end{cases}$ + **Two Workers** : $\begin{cases} \text{High-Skill: } h \\ \text{Low-Skill: } l \end{cases}$

Caveat: Paper is more general than this case

1. Good markets face exogenous distortions

$$\text{Cost} = \mu \times \text{Revenue}$$

2. Labor markets are competitive

Static General Equilibrium Model with...

Two Firms : $\begin{cases} \text{More Competitive: } H \\ \text{Less Competitive: } L \end{cases}$ + Two Workers : $\begin{cases} \text{High-Skill: } h \\ \text{Low-Skill: } l \end{cases}$

Caveat: Paper is more general than this case

1. Good markets face exogenous distortions

$$Cost = \mu \times Revenue$$

2. Labor markets are competitive
3. Labor supply is endogenous

Static General Equilibrium Model with...

Two Firms : $\begin{cases} \text{More Competitive: } H \\ \text{Less Competitive: } L \end{cases}$ + **Two Workers** : $\begin{cases} \text{High-Skill: } h \\ \text{Low-Skill: } l \end{cases}$

Caveat: Paper is more general than this case

1. Good markets face exogenous distortions

$$\text{Cost} = \mu \times \text{Revenue}$$

2. Labor markets are competitive

3. Labor supply is endogenous

4. Correlations:

- H has high μ
- H requires more h
- h have a higher expenditure in H

Mechanism's Intuition

1. μ heterogeneity —→ allocates more workers to H
 - H operates with low marginal productivity
 - L operates with high marginal productivity

Mechanism's Intuition

1. **μ heterogeneity** → allocates more workers to H
 - H operates with low marginal productivity
 - L operates with high marginal productivity
2. **Skill-bias heterogeneity** → asymmetries in the income exposure in response to local perturbations

Mechanism's Intuition

1. **μ heterogeneity** → allocates more workers to H
 - H operates with low marginal productivity
 - L operates with high marginal productivity
2. **Skill-bias heterogeneity** → asymmetries in the income exposure in response to local perturbations
3. **Preference heterogeneity** expenditure flows
 - As h income increase, expenditure in H rises
 - Workers relocate from L to H
 - Misallocation is accentuated

Firm Heterogeneity $i \in \{H, L\}$

{
a. Skill Bias
b. Distortions

$$\underset{y_i, \ell_{ih}, \ell_{il}}{\text{Max}} \quad \pi_i = p_i y_i - w_h \ell_{ih} - w_l \ell_{il}$$

$$y_i = A_i \ell_{ih}^{\alpha_i} \ell_{il}^{1-\alpha_i}$$

Skill Bias
 $\alpha_L \leq \alpha_H$

Firm Heterogeneity $i \in \{H, L\}$

a. Skill Bias
 b. Distortions

$$\underset{y_i, \ell_{ih}, \ell_{il}}{\text{Max}} \quad \pi_i = p_i y_i - w_h \ell_{ih} - w_l \ell_{il}$$

$$y_i = A_i \ell_{ih}^{\alpha_i} \ell_{il}^{1-\alpha_i}$$

Skill Bias
 $\alpha_L \leq \alpha_H$

Markdown

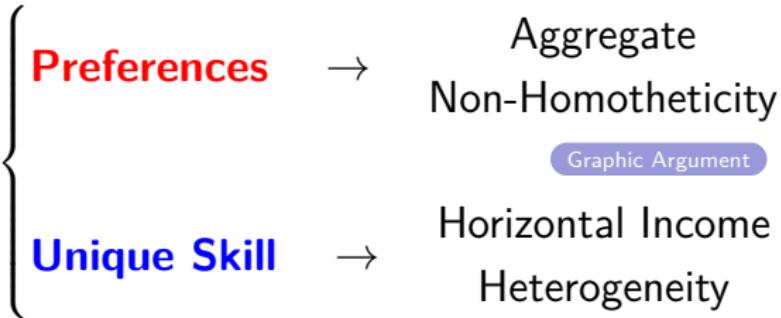
$$0 < \mu_L \leq \mu_H \leq 1$$

$$\text{Cost}_i = \mu_i \times \text{Revenue}_i$$

Alternative Narrative: Sectoral Dixit-Stiglitz Aggregation

Household Heterogeneity

$$r \in \{h, l\}$$



Graphic Argument



$$\underset{C_r, L_r}{\text{Max}} U_r(C_r, L_r) \quad \text{s.t.} \quad \frac{C_r}{\bar{C}_r} = \left(\beta_r \left(\frac{C_r H}{\bar{C}_r H} \right)^{\frac{\rho-1}{\rho}} + (1 - \beta_r) \left(\frac{C_r L}{\bar{C}_r L} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

$$E_r = p_r^c C_r = p_H C_{rH} + p_L C_{rL} \leq \mathbf{w}_r \mathbf{L}_r + 0.5 \text{ profits}$$

Consumption Bias

$$\beta_l \leq \beta_h$$

Equilibrium Definition

For (A, μ, β, α) , prices and allocations:

- (i) **Firms'** labor demand and output decisions maximize profits;
- (ii) **Households'** consumption and labor supply maximize utility satisfying budget constraints;
- (iii) Goods and labor markets clear.

Solve for Equilibrium Distributions

From **FOC** of households and firms

$$p_H C_{rH} = \beta_r p_r^c C_r$$

$$w_h \ell_{ih} = \alpha_i \mu_i p_i y_i$$

Solve for Equilibrium Distributions

From **FOC** of households and firms

$$p_H C_{rH} = \beta_r p_r^c C_r \quad w_h \ell_{ih} = \alpha_i \mu_i p_i y_i$$

In **market clearing conditions**

$$y_i = C_{hi} + C_{li} \quad L_r = \ell_{Hr} + \ell_{Lr}$$

Solve for Equilibrium Distributions

From **FOC** of households and firms

$$p_H C_{rH} = \beta_r \ p_r^c \ C_r \qquad \qquad w_h \ell_{ih} = \alpha_i \ \mu_i \ p_i \ y_i$$

In market clearing conditions

$$y_j = C_{hj} + C_{lj} \quad L_r = \ell_{Hr} + \ell_{Lr}$$

Equilibrium in terms of

$$\lambda_i = \frac{p_i y_i}{GDP}$$

Sales (Domar weights)

$$\Lambda_h = \frac{w_h L_h}{GDP} = \sum \alpha_i \mu_i \lambda_i$$

Labor income

$$\chi_r = \frac{p_r^c C_r}{GDP}$$

Expenditure

$$\tilde{\Lambda}_h = \sum \alpha_i \lambda_i$$

Value added

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

Undistorted Benchmark: $\mu_H = \mu_L = 1$

$$\lambda_H = \Lambda_r = \chi_r = \frac{1}{2}$$

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

Undistorted Benchmark: $\mu_H = \mu_L = 1$

$$\lambda_H = \Lambda_r = \chi_r = \frac{1}{2}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

Undistorted Benchmark: $\mu_H = \mu_L = 1$

$$\lambda_H = \Lambda_r = \chi_r = \frac{1}{2}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

Source of Misallocation

Parameter Space Restrictions

$$\alpha_H + \alpha_L = \beta_h + \beta_l = 1$$

Undistorted Benchmark: $\mu_H = \mu_L = 1$

$$\lambda_H = \Lambda_r = \chi_r = \frac{1}{2}$$

$$\frac{U_{C_{rH}}}{U_{C_{rl}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

Additional Restriction

$$\mu_H + \mu_L = 1$$

What I Don't Do

- Misallocation literature **distorted vs. efficient** equilibrium
- Parametric assumptions (usually CD) → analytic TFP
- Evaluate how getting rid of distortions has an effect on TFP

What I Don't Do

- Misallocation literature **distorted vs. efficient** equilibrium
- Parametric assumptions (usually CD) → analytic TFP
- Evaluate how getting rid of distortions has an effect on TFP

What I Do

- Local TFP Δ around distorted equilibrium to any perturbation
- Distributional $\Delta \rightarrow$ Misallocation $\Delta \rightarrow \Delta$ TFP
- To illustrate: $d \log A_L = 1\%$

Local Variation to $d \log A_L = 1\%$

$$\frac{d \chi_h}{d \log A_L} = \frac{(\alpha_H - \alpha_L)}{2} \frac{d \lambda_H}{d \log A_L}$$

Expenditure elasticity requires $\alpha_H \neq \alpha_L$

Local Variation to $d \log A_L = 1\%$

$$\frac{d \chi_h}{d \log A_I} = \frac{(\alpha_H - \alpha_L)}{2} \frac{d \lambda_H}{d \log A_I}$$

Expenditure elasticity requires $\alpha_H \neq \alpha_L$

$$\frac{d \lambda_H}{d \log A_L} = -\frac{2(\rho-1)\beta_h\beta_l}{2 - (\alpha_H - \alpha_L)(\beta_h - \beta_l) + 2(\rho-1)\frac{\beta_h\beta_l}{1+\zeta^w}\left(\frac{\alpha_H - \mu_L}{\Lambda_h} + \frac{\alpha_H - \mu_H}{\Lambda_l} + \frac{\zeta^e}{2}\frac{\alpha_H - \alpha_L}{\chi_h\chi_l}\right)}$$

Sales elasticity requires $\rho \neq 1$

Local Variation to $d \log A_L = 1\%$

$$\frac{d \chi_h}{d \log A_L} = \frac{(\alpha_H - \alpha_L)}{2} \frac{d \lambda_H}{d \log A_L}$$

Expenditure elasticity requires $\alpha_H \neq \alpha_L$

$$\frac{d \lambda_H}{d \log A_L} = - \frac{2(\rho - 1)\beta_h\beta_l}{2 - (\alpha_H - \alpha_L)(\beta_h - \beta_l) + 2(\rho - 1) \frac{\beta_h\beta_l}{1 + \zeta^w} \left(\frac{\alpha_H - \mu_L}{\Lambda_h} + \frac{\alpha_H - \mu_H}{\Lambda_l} + \frac{\zeta^e}{2} \frac{\alpha_H - \alpha_L}{\chi_h\chi_l} \right)}$$

Sales elasticity requires $\rho \neq 1$

$\begin{cases} \text{Under } \rho > 1: \text{ consumers increase expenditure on } L \text{ & } \lambda_H \downarrow \\ \text{Under } \rho < 1: \underbrace{\text{consumers increase expenditure on } H \text{ & } \lambda_H \uparrow}_{\text{Baumol's Cost Disease}} \end{cases}$

In This Section

- First-order local Δ **Income Distribution**

$$d\Lambda_h, \quad d\Lambda_I$$

In This Section

- First-order local Δ **Income Distribution**

$$d\Lambda_h, \quad d\Lambda_I$$

- Decomposition of $d\Lambda$ in

Δ **Consumption
Distribution**

Δ **Demand
Structure**

&

Keep β fix
& change χ

Keep χ fix
& change β

Income Distribution & Bilateral Centralities

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{I \rightarrow h} \chi_I$$

$m_{I \rightarrow h}$ % of expenditure from / reaching Λ_h

Alternative Definitions for m 's

Bilateral Centralities

$$\Lambda_h = \overbrace{\left(\underbrace{\alpha_H \mu_H}_{f_{H \rightarrow h}} \beta_h + \underbrace{\alpha_L \mu_L}_{f_{L \rightarrow h}} (1 - \beta_h) \right)}^{m_{h \rightarrow h}} \chi_h$$

$$+ \overbrace{\left(\underbrace{\alpha_H \mu_H}_{f_{H \rightarrow I}} \beta_I + \underbrace{\alpha_L \mu_L}_{f_{L \rightarrow I}} (1 - \beta_I) \right)}^{m_{I \rightarrow h}} \chi_I$$

$m_{I \rightarrow h}$ % of expenditure from /
 $f_{L \rightarrow h}$ revenue from L reaching Λ_h

Comparative Statics

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{I \rightarrow h} \chi_I$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h} \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

Comparative Statics

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{I \rightarrow h} \chi_I$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h} \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

Take total derivative

$$d \Lambda_h = \underbrace{m_{h \rightarrow h} d \chi_h + m_{I \rightarrow h} d \chi_I}_{\text{Distributive Income}_h} + \underbrace{\chi_h d m_{h \rightarrow h} + \chi_I d m_{I \rightarrow h}}_{\text{Income Centrality}_h}$$

Comparative Statics

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{I \rightarrow h} \chi_I$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h} \quad \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

Take total derivative

$$\mathbf{d} \Lambda_h = \underbrace{m_{h \rightarrow h} \mathbf{d} \chi_h + m_{I \rightarrow h} \mathbf{d} \chi_I}_{\text{Distributive Income}_h} + \underbrace{\chi_h \mathbf{d} m_{h \rightarrow h} + \chi_I \mathbf{d} m_{I \rightarrow h}}_{\text{Income Centrality}_h}$$

We know: $d\chi_h + d\chi_I = 0$ & $d m_{r \rightarrow h} = (f_{H \rightarrow h} - f_{L \rightarrow h}) d\beta_r$

Comparative Statics

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{I \rightarrow h} \chi_I$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h} \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

Take total derivative

$$d \Lambda_h = \underbrace{m_{h \rightarrow h} d \chi_h + m_{I \rightarrow h} d \chi_I}_{\text{Distributive Income}_h} + \underbrace{\chi_h d m_{h \rightarrow h} + \chi_I d m_{I \rightarrow h}}_{\text{Income Centrality}_h}$$

We know: $d \chi_h + d \chi_I = 0$ & $d m_{r \rightarrow h} = (f_{H \rightarrow h} - f_{L \rightarrow h}) d \beta_r$

$$\overbrace{(\beta_h - \beta_I)}^{\geq 0} \overbrace{(\alpha_H - \mu_L)}^{\geq 0}$$

$$d \Lambda_h = \underbrace{(m_{h \rightarrow h} - m_{I \rightarrow h}) d \chi_h}_{\text{Distributive Income}_h} + \underbrace{(f_{H \rightarrow h} - f_{L \rightarrow h}) \sum \chi_r d \beta_r}_{\text{Income Centrality}_h}$$

Baqae & Fahri (2020)

A.

$$\begin{aligned} d \log Y &= d \log GDP - d \log P_Y \\ &= \mathbf{d \log TFP} + \sum_{r \in \{h,l\}} \tilde{\Lambda}_r d \log L_r \end{aligned}$$

Intermediate Steps

Baqaee & Fahri (2020)

A.

$$d \log Y = d \log GDP - d \log P_Y$$

$$= d \log TFP + \sum_{r \in \{h, l\}} \tilde{\Lambda}_r d \log L_r$$

B.

Intermediate Steps

$$d \log TFP = \underbrace{\lambda_H d \log A_H + \lambda_L d \log A_L}_{\text{Technology}} + \underbrace{\lambda_H d \log \mu_H + \lambda_L d \log \mu_L}_{\text{Competitiveness}} - \underbrace{(\tilde{\Lambda}_H d \log \Lambda_H + \tilde{\Lambda}_L d \log \Lambda_L)}_{\text{Misallocation}}$$

Without distortions → Hulten (1978)

Distortion Centralities δ

$$\begin{aligned} \textbf{\textit{Misallocation}} &= \tilde{\Lambda}_h d \log \Lambda_h + \tilde{\Lambda}_I d \log \Lambda_I \\ &= \frac{\tilde{\Lambda}_h}{\Lambda_h} d \Lambda_h + \frac{\tilde{\Lambda}_I}{\Lambda_I} d \Lambda_I \\ &= \delta_h d \Lambda_h + \delta_I d \Lambda_I \end{aligned}$$

Distortion Centralities δ

$$\begin{aligned} \textbf{\textit{Misallocation}} &= \tilde{\Lambda}_h d \log \Lambda_h + \tilde{\Lambda}_l d \log \Lambda_l \\ &= \frac{\tilde{\Lambda}_h}{\Lambda_h} d \Lambda_h + \frac{\tilde{\Lambda}_l}{\Lambda_l} d \Lambda_l \\ &= \delta_h d \Lambda_h + \delta_l d \Lambda_l \end{aligned}$$

δ measures how **undervalue** a worker is

$$\delta_l - \delta_h > 0$$

Introduce $d\Lambda_h$ and $d\Lambda_I$

$$\text{Misallocation} = \underbrace{(\mathcal{M}_h - \mathcal{M}_I) \mathbf{d} \chi_h}_{1. \text{ Distributive}} + \underbrace{(\mathcal{F}_H - \mathcal{F}_L) \sum \chi_r \mathbf{d} \beta_r}_{2. \text{ Final Demand}}$$

Introduce $d\Lambda_h$ and $d\Lambda_I$

$$\text{Misallocation} = \underbrace{(\mathcal{M}_h - \mathcal{M}_I) \mathbf{d} \chi_h}_{1. \text{ Distributive}} + \underbrace{(\mathcal{F}_H - \mathcal{F}_L) \sum \chi_r \mathbf{d} \beta_r}_{2. \text{ Final Demand}}$$

Sufficient Statistics

Expenditure Centrality M

$$M_r = m_{r \rightarrow h} \delta_h + m_{r \rightarrow I} \delta_I$$

Introduce $d\Lambda_h$ and $d\Lambda_l$

$$\text{Misallocation} = \underbrace{(\textcolor{red}{M_h} - \textcolor{blue}{M_l}) \, d \, \chi_h}_{\text{1. Distributive}} + \underbrace{(\textcolor{brown}{F_H} - \textcolor{teal}{F_L}) \sum \chi_r \, d \, \beta_r}_{\text{2. Final Demand}}$$

Sufficient Statistics

Expenditure Centrality M **Revenue Centrality F**

$$M_r = m_{r \rightarrow h} \delta_h + m_{r \rightarrow l} \delta_l \quad F_i = f_{i \rightarrow h} \delta_h + f_{i \rightarrow l} \delta_l$$

Introduce $d\Lambda_h$ and $d\Lambda_l$

$$\text{Misallocation} = \underbrace{(M_h - M_l) d \chi_h}_{\text{1. Distributive}} + \underbrace{(F_H - F_L) \sum \chi_r d \beta_r}_{\text{2. Final Demand}}$$

Sufficient Statistics

Expenditure Centrality M **Revenue Centrality F**

$$M_r = m_{r \rightarrow h} \delta_h + m_{r \rightarrow l} \delta_l \quad F_i = f_{i \rightarrow h} \delta_h + f_{i \rightarrow l} \delta_l$$

- M_r is high for households that consume from relatively competitive supply chains that demand workers with high δ
 - F_i is high for firms that operate in relatively competitive supply chains and directly or indirectly demand high δ workers

Distributive $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP \downarrow

$$\underbrace{(M_h - M_l) d \chi_h}_{\text{Distributive}} \quad d \chi_h = \frac{(\alpha_H - \alpha_L)}{2} d \lambda_H$$

Distributive $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP \downarrow

$$\underbrace{(\mathcal{M}_h - \mathcal{M}_l) d \chi_h}_{\text{Distributive}} \quad d \chi_h = \frac{(\alpha_H - \alpha_L)}{2} d \lambda_H$$

$$\overbrace{M_h - M_l}^{\geq 0} = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \overbrace{(\beta_h - \beta_l)}^{\geq 0}$$

$$\times \underbrace{[\delta_l + (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) \mathbf{a}]}_{\geq 0}$$

Distributive ↑ → Misallocation ↑ → TFP ↓

$$\underbrace{(\mathcal{M}_H - \mathcal{M}_L) d\chi_h}_{\text{Distributive}} \quad d\chi_h = \frac{(\alpha_H - \alpha_L)}{2} d\lambda_H$$

$$\overbrace{M_h - M_l}^{\geq 0} = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \overbrace{(\beta_h - \beta_l)}^{\geq 0}$$

$$\times \underbrace{[\delta_I + (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) \mathbf{a}]}_{\geq 0}$$

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Expenditure Redistribution	X	X	X

***Final Demand* $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP \downarrow**

$$\underbrace{(F_H - F_L) \sum \chi_r d \beta_r}_{\text{Final Demand}}$$

Final Demand $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP \downarrow

$$\underbrace{(\mathcal{F}_H - \mathcal{F}_L)}_{\textcolor{red}{Final Demand}} \sum \chi_r \, \mathbf{d} \, \boldsymbol{\beta}_r$$

$$\overbrace{F_H - F_L}^{\geq 0} = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \times \underbrace{[\delta_I + (\alpha_H - \alpha_L)(\alpha_H \mu_H - \alpha_L \mu_L) \mathbf{a}]}_{\geq 0}$$

Final Demand $\uparrow \rightarrow$ Misallocation $\uparrow \rightarrow$ TFP \downarrow

$$\underbrace{(\mathcal{F}_H - \mathcal{F}_L)}_{\textcolor{red}{Final Demand}} \sum \chi_r \, \mathbf{d} \, \boldsymbol{\beta}_r$$

$$\overbrace{F_H - F_L}^{\geq 0} = \overbrace{(\mu_H - \mu_L)}^{\geq 0}$$

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
Final Demand Recomposition	X	✓	✓

Requirements in Heterogeneity

	$\mu_H = \mu_L$	$\alpha_H = \alpha_L$	$\beta_h = \beta_l$
1. Expenditure Redistribution	✗	✗	✗
2. Final Demand Recomposition	✗	✓	✓

Representative Household

Assume instead a representative household

$$\underset{Y, L, C_H, C_L, L_h, L_I}{\text{Max}} \quad U(Y, L) \quad s.t. \quad Y = Q(C_H, C_L),$$

$$p_Y Y = p_H C_H + p_L C_L$$

$$\leq w_h L_h + w_l L_l + (1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L$$

Representative Household

Assume instead a representative household

$$\underset{Y, L, C_H, C_L, L_h, L_I}{\text{Max}} \quad U(Y, L) \quad \text{s.t.} \quad Y = Q(C_H, C_L),$$

$$\begin{aligned} p_Y Y &= p_H C_H + p_L C_L \\ &\leq w_h L_h + w_I L_I + (1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L \end{aligned}$$

The **first-order conditions** imply that

$$\delta_h = \delta_I = \Lambda^{-1} \quad \Lambda = \Lambda_h + \Lambda_I$$

The effects from one additional percentage point of labor income share on TFP are equalized

Misallocation under a Representative Household

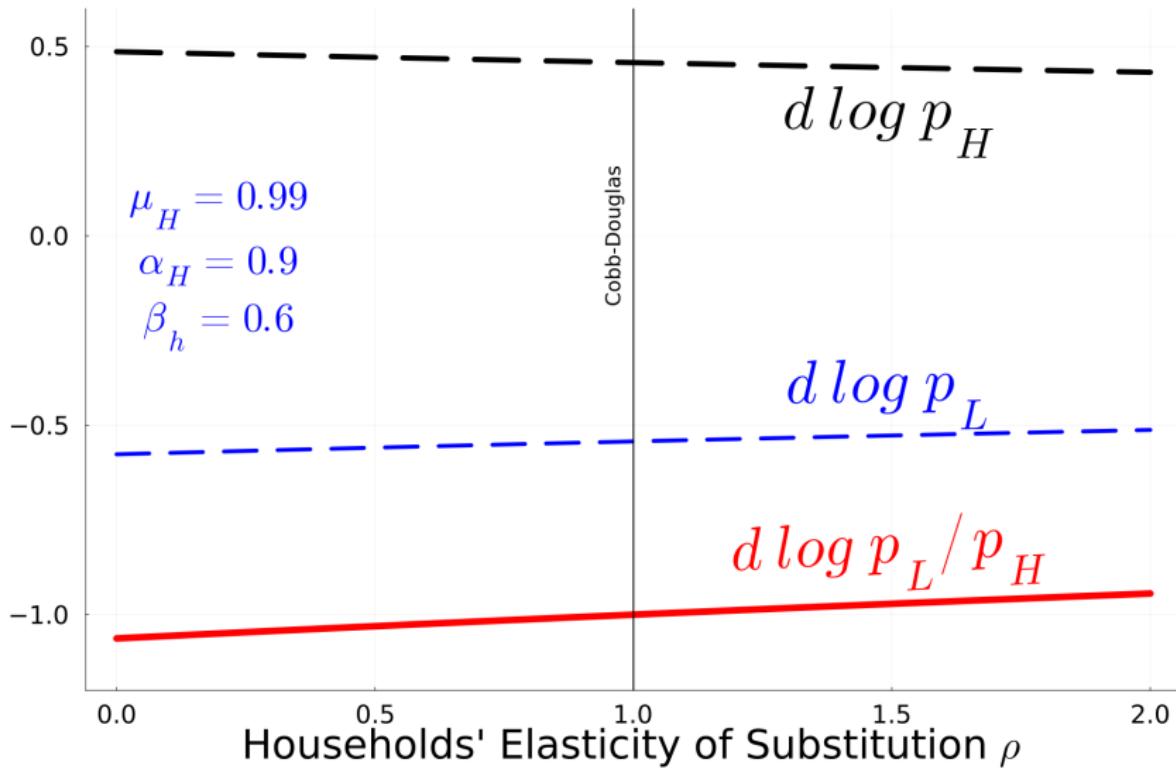
From $\delta_h = \delta_I = \Lambda^{-1}$

$$\delta_h d \Lambda_h + \delta_I d \Lambda_I = \frac{d \Lambda_h + d \Lambda_I}{\Lambda} = d \log \Lambda$$

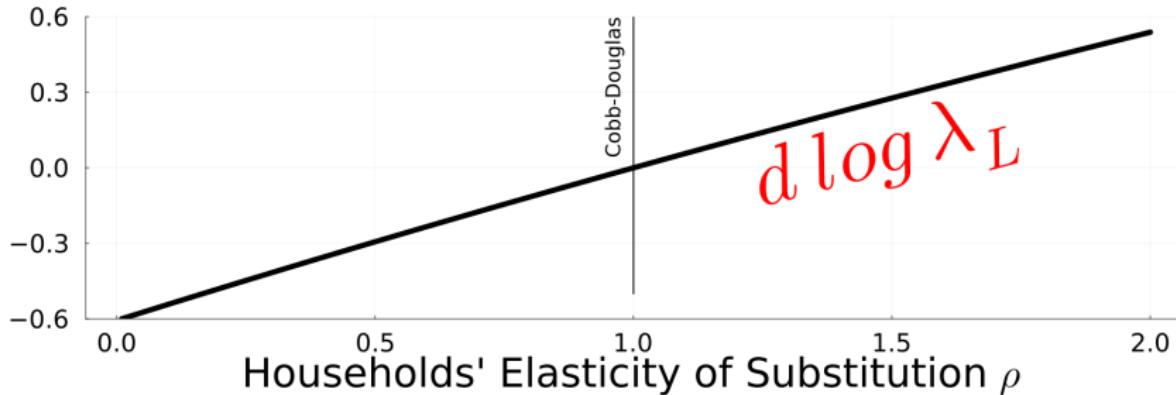
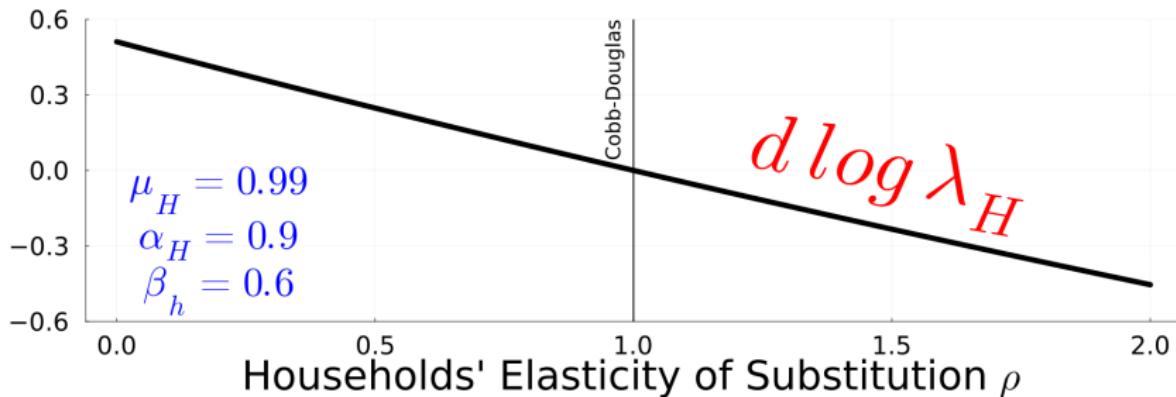
Track only one element of the distribution!

Additional Contribution Relative to HANK models

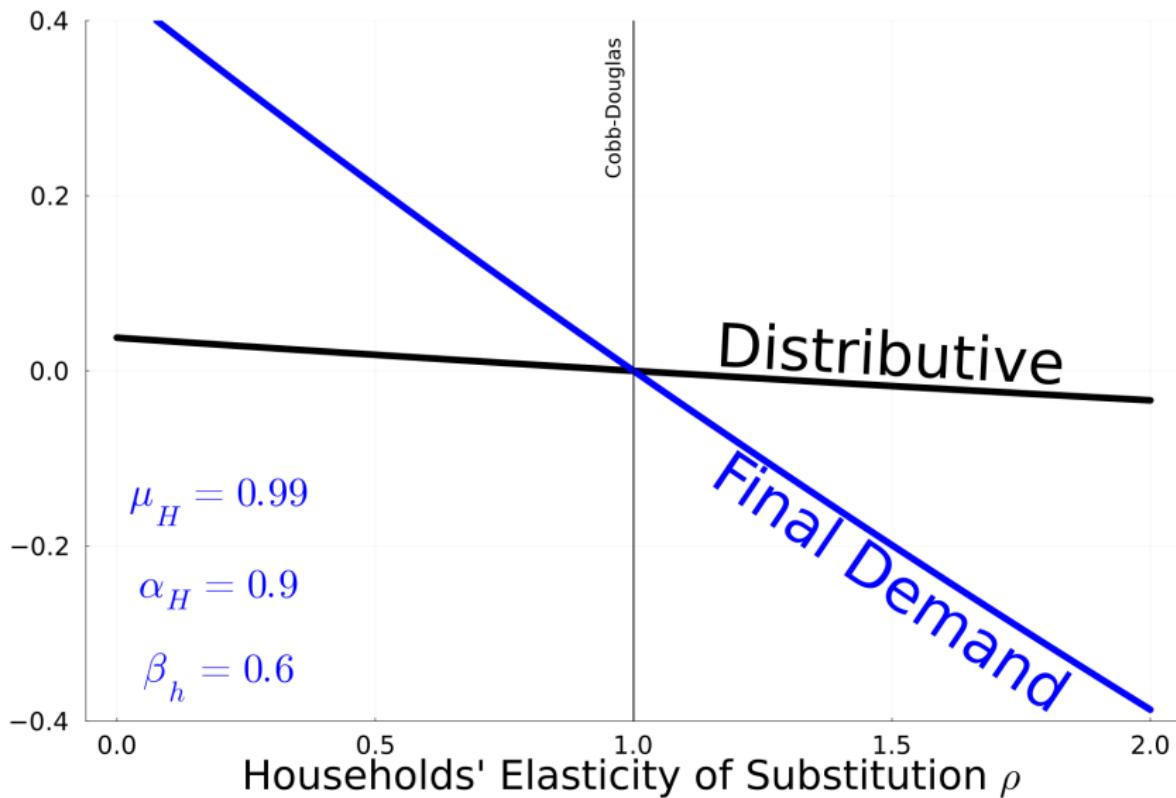
$p_L \downarrow$ & $p_H \uparrow$



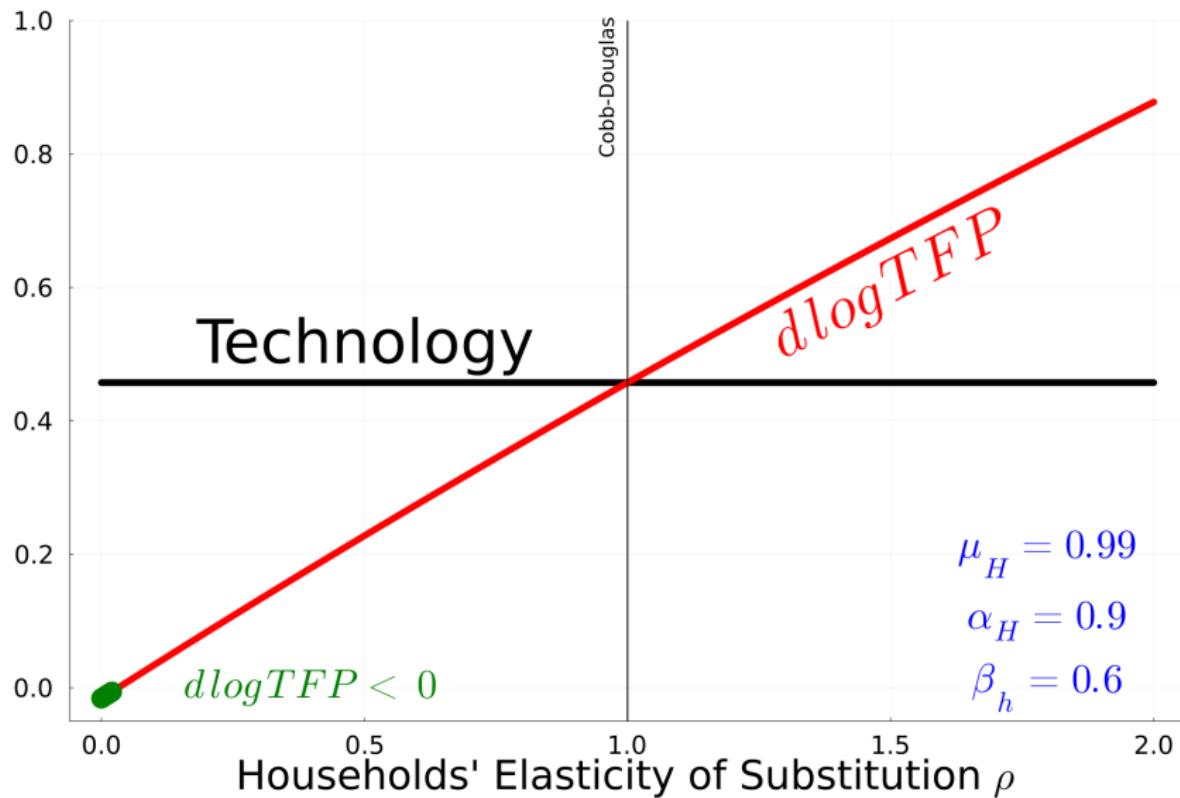
Under $\rho > 1$: $\lambda_L \uparrow$ & $\lambda_H \downarrow$



Under $\rho > 1$: Distributive \downarrow & Final Demand \downarrow



Under $\rho > 1$: Misallocation \downarrow & $d \log TFP > \lambda_L$



In the Paper...

- General Non-Parametric CRS model for production & consumption
- General Input-Output Networks
- General Equity Distribution

In the Paper...

- General Non-Parametric CRS model for production & consumption
- General Input-Output Networks
- General Equity Distribution

Additional Channels

1. μ \uparrow & stronger for sectors with high $\lambda_i F_i$
2. α \uparrow for high δ workers & stronger if $\mu_i \lambda_i$ high
3. **Intermediate demand** \uparrow on sectors with high F_i

Data Requirements

3 Types of Money Flows...

1. Household-to-Firm: Final consumption
2. Firm-to-Firm: Intermediate inputs
3. Firm-to-workers: Labor market

Data Requirements

3 Types of Money Flows...

1. Household-to-Firm: Final consumption
2. Firm-to-Firm: Intermediate inputs
3. Firm-to-workers: Labor market

Measures of Shocks...

- A. Productivity shocks
- B. Markdown shocks
- C. Distributional variations

Data for money flows from 1997 to 2021 Household to Firm

1. State level Personal Consumption Expenditure (BEA)

$\beta_{state, industry}$: { PCE provides expenditure on types of goods
IO Make matrix: type of good → industry

Data for money flows from 1997 to 2021 Household to Firm

1. State level Personal Consumption Expenditure (BEA)

$\beta_{state, industry}$: { PCE provides expenditure on types of goods
 IO Make matrix: type of good → industry

Firm to Firm

2. Input-Output tables (BEA) for 66 NAICS industries

$$\mu_i = \frac{\text{Total Cost}_i}{\text{Sales}_i}$$

$$\text{Intermediate Intensity } ij = \frac{p_j x_{ij}}{\text{Total Cost}_i}$$

$$\text{Total Cost}_i = \text{Labor Costs}_i + \text{Intermediate Cost}_i$$

Firm-to-household

3. County Business Patterns (Census)

Industry specific geographic (state) bias in labor

Antisupression Algorithm

Missing Private Employment

Firm-to-household

3. County Business Patterns (Census)

Industry specific **geographic (state)** bias in labor

Antisupression Algorithm

Missing Private Employment

4. Occupational Employment & Wage Statistics

- Industry specific **occupational** demand bias
- State specific **occupational** supply bias

Firm-to-household

3. County Business Patterns (Census)

Industry specific geographic (state) bias in labor

Antisupression Algorithm

Missing Private Employment

4. Occupational Employment & Wage Statistics

- Industry specific **occupational** demand bias
- State specific **occupational** supply bias

From 3 & 4 → industry specific heterogeneity by worker type. Worker type comes from State & Occupational interactions $H = 38,189$ (≈ 1.5 bill m's)

Firm-to-household

3. County Business Patterns (Census)

Industry specific geographic (state) bias in labor

Antisupression Algorithm

Missing Private Employment

4. Occupational Employment & Wage Statistics

- Industry specific **occupational** demand bias
- State specific **occupational** supply bias

From 3 & 4 → industry specific heterogeneity by worker type. Worker type comes from State & Occupational interactions $H = 38,189$ (≈ 1.5 bill m's)

e.g. Finance's labor demand intensity for economists in Maine

$$\alpha_{ir} \propto \underbrace{\text{Finance's share of labor expenditure in Maine}}_{\text{Spatial Demand (CBP)}} \times \underbrace{\text{Finance's share of labor expenditure in economists}}_{\text{Occupational Demand (OEWS)}} \times \underbrace{\text{Maine's share of labor income from economists}}_{\text{Occupational Supply (OEWS)}} .$$

Motivation
oooo

Model
oooooo

Solution
oooo

Income
oooo

TFP
oooooo

Novelty
oo

Example
oooooo

Data
ooo●

Empirics
ooooo

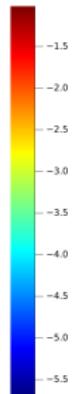
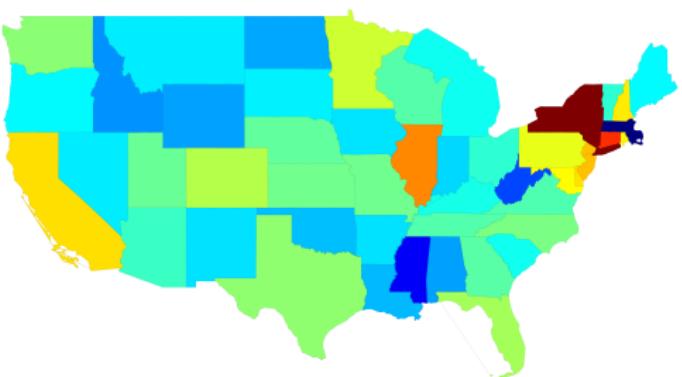
Policy
ooo

Distributive
ooo

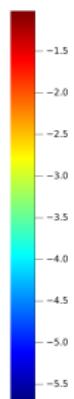
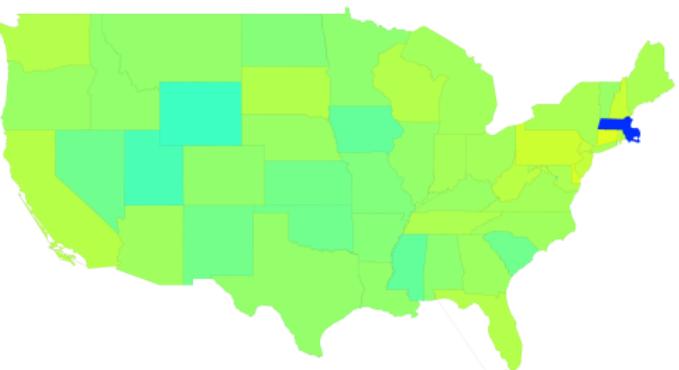
Conclusion
oo

Appendix

Financial Sector in Economists



Ambulatory Health in Dentists



Data for Shocks

A. Industry Level Production Accounts (BEA)

$$d \log A_i$$

B. Input Output Tables

$$\mu_i \longrightarrow d \log \mu_i$$

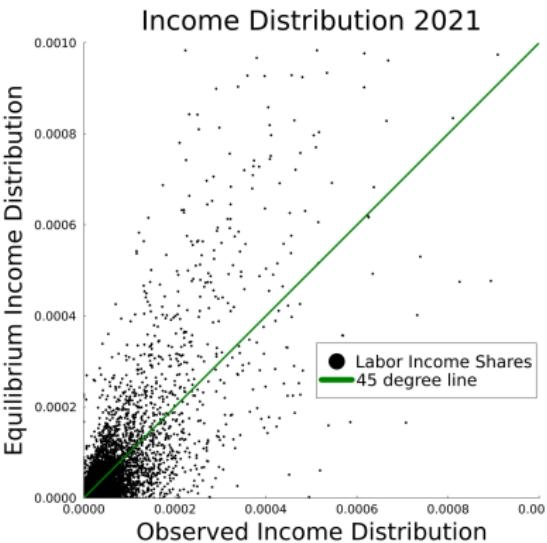
C. CBP + OEWS

$$\Lambda_r \longrightarrow d \log \Lambda_r$$

Moments with Heterogeneous Households



<i>Observed</i> λ <i>on</i>	
Model λ	1.021*** (0.003)
R^2	0.981
N	1,650

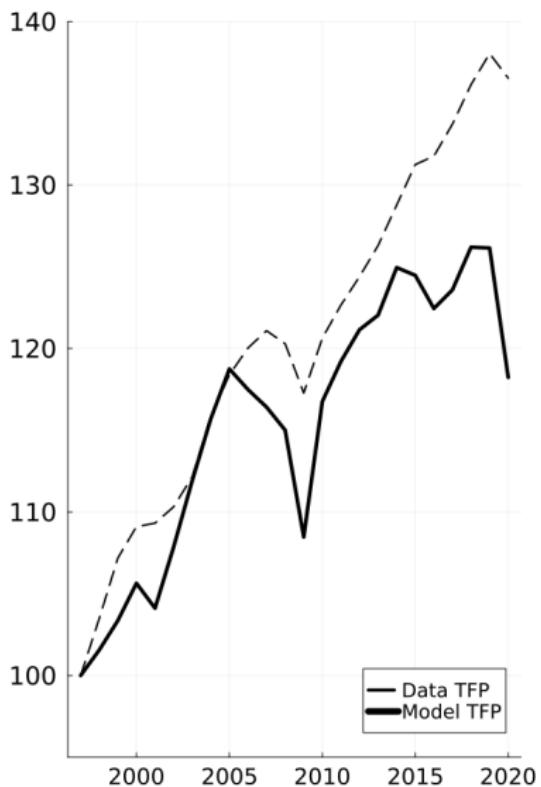


<i>Observed</i> \wedge <i>on</i>	
Model \wedge	0.435*** (0.0015)
R^2	0.682
N	38,189

Implementation

$$d \log TFP_t = \underbrace{\sum_i \tilde{\lambda}_{i,t-1} d \log A_{i,t}}_{\text{Technology}_t} + \underbrace{\sum_i \tilde{\lambda}_{i,t-1} d \log \mu_{i,t}}_{\text{Competitiveness}_t} - \underbrace{\sum_r \tilde{\Lambda}_{r,t-1} d \log \Lambda_{r,t}}_{\text{Misallocation}_t}$$

R^2 rises from 5% to 50% with IO Networks

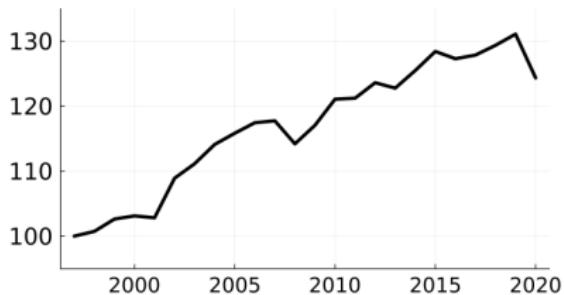


<i>Observed d log TFP on</i>		
	Without Networks	With Networks
<i>d log \widehat{TFP}</i>	(1) -0.265 (0.264)	(2) 0.311*** (0.069)
<i>Adj R</i> ²	0.048	0.499
Technology	(3) 0.847*** (0.289)	(4) 0.413*** (0.082)
Competitive	0.986 (0.695)	0.342*** (0.054)
Misallocation	-0.105 (0.360)	0.0168 (0.125)
<i>Adj R</i> ²	0.562	0.732

d log TFP = Technology + Competitiveness – Misallocation

Technology ↑

$$\sum_{i \in \mathcal{N}} \tilde{\lambda}_i d \log A_i$$

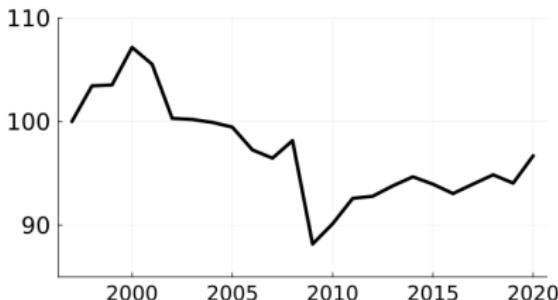


Between 1997 and 2020

Oil & gas extraction	-11.1%
Computer & electronic	-6.6%

Competitiveness ↓

$$\sum_{i \in \mathcal{N}} \tilde{\lambda}_i d \log \mu_i$$



Between 1997 and 2020

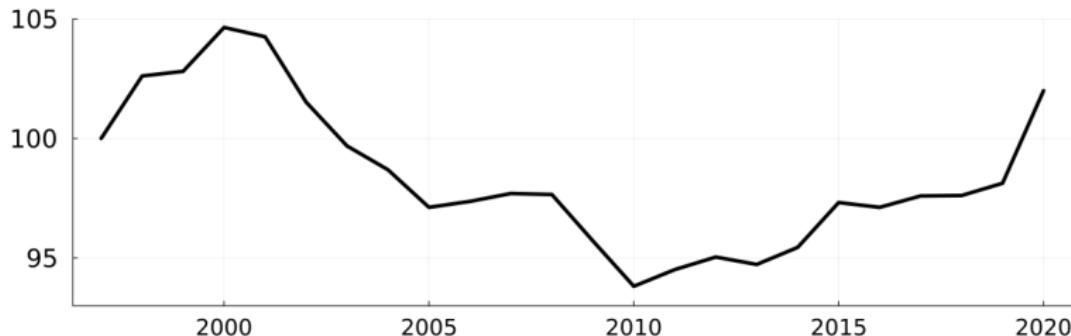
Credit intermediation	4.1%
-----------------------	-------------

Between 2002 and 2009

Oil & gas extraction	6.6%
----------------------	-------------

Without Misallocation↑ after 2009, TFP↑ 7.5%

- Misallocation↓ between 2001 and 2010 by **-8.2%**
- Misallocation↑ between 2010 and 2020 by **7.5%**



Increasing profit margins

- Oil & gas extraction: **-1.5%**
- Computer & electronics: **-1.1%**

Increasing labor demand

- Credit intermediation: **2.4%**

Final and Intermediate Demand

- Wholesale Trade: **2.2%**

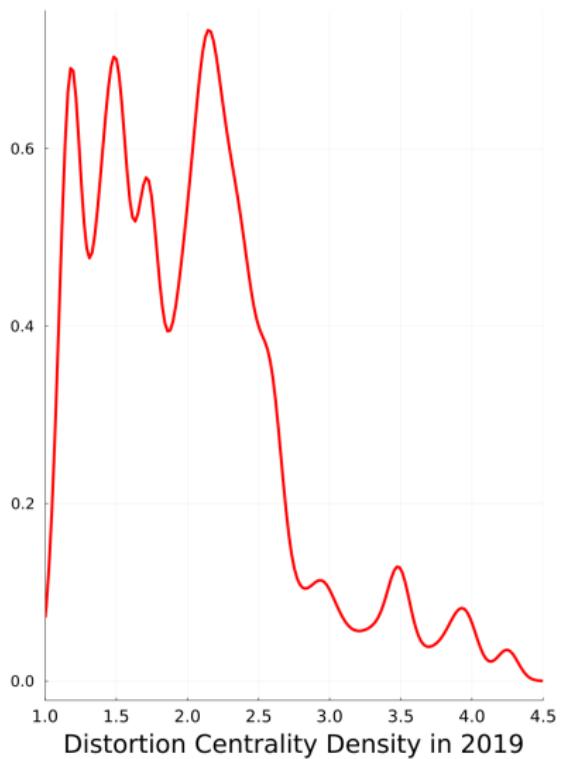
Sources of Misallocation (Graph)

Sources of Misallocation (Counterfactual)

Industry variation from μ and Labor Demand

Industry variation from Final and Intermediate Demand

Distortion Centralities δ



<i>Lowest δ</i>	
Nursing Assistant	1.05 - 1.08
Residential Advisor	1.06 - 1.22
Rehabilitation Counselor	1.07 - 1.08
Recreational Therapist	1.07 - 1.09
Food Server	1.07 - 1.45

<i>Highest δ</i>	
Teller	4.27 - 4.28
New Accounts Clerk	4.24 - 4.27
Loan Interviewer	4.21 - 4.26
Loan Officer	4.23 - 4.26
Credit Analyst	3.89 - 4.22

Normalized Nested CES

Introduced by **de La Grandville (1989)** and **Klump & de La Grandville (1989)** and as in **Baqae & Farhi (2019,a,b, 2020, 2022)**

Normalized Nested CES

Parameters - **Atalay (2017), Boehm et al. (2014)**

1. Elasticity of substitution **between worker types**: 1.0
2. Elasticity of substitution **between sectoral intermediate inputs**: 0.2
3. Elasticity of substitution **between labor and intermediate inputs**: 0.5
4. Elasticity of substitution in **final consumption**: 0.9
5. **Substitution effect** in labor supply $\zeta_h^w = 2$
6. **Income effect** in labor supply $\zeta_h^e = 2$

$$d \log TFP = \underbrace{\textcolor{red}{Technology}}_{=1\%} - \textcolor{black}{Misallocation}$$

Best Sectors	d log TFP	d log TFP on				
1. Nursing & Residential Care	1.041%	μ_i	0.359*** (0.09)			0.207 (0.13)
2. Social Assistance	1.039%	λ_i		0.170 (0.56)		0.854* (0.50)
3. General Merchandise Store	1.029%	F_i			0.212*** (0.05)	0.148** (0.07)
4. Ambulatory health care	1.027%					
5. Hospitals	1.026%					
Worst Sectors	d log TFP	R^2	0.20	$1e^{-3}$	0.21	0.27
1. Oil & Gas extraction	0.587%	N			66	
2. Primary Metals	0.610%					
3. Chemical Products	0.618%					
4. Mining, except Oil & Gas	0.630%					
5. Utilities	0.647%					

We want productivity shocks in
sectors with high F_i !

$$d \log TFP = \underbrace{\text{Competitiveness} - \text{Misallocation}}_{= 1\%}$$

Best Sectors		d log TFP	d log TFP on			
1.	Housing	0.766%	μ_i	-0.974*** (0.12)		-0.919*** (0.18)
2.	Credit Intermediation	0.414%	λ_i	1.351 (0.95)		-0.132 (0.73)
3.	Oil & Gas extraction	0.384%	F_i		-0.427*** (0.08)	-0.046 (0.11)
4.	Furniture	0.370%				
5.	Mining, except Oil & Gas	0.364%				
Worst Sectors		d log TFP	R^2	0.48	0.03	0.29
1.	Nursing & Residential Care	-0.329%	N		66	0.48
2.	Social Assistance	-0.303%				
3.	General Merchandise Store	-0.274%				
4.	Hospitals	-0.219%				
5.	Ambulatory health care	-0.201%				

We want competition shocks in sectors with low μ_i !

Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_I)$$

Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_I)$$

$$d \log TFP = \sum \chi_r d \log \mathbf{PTT}_r$$

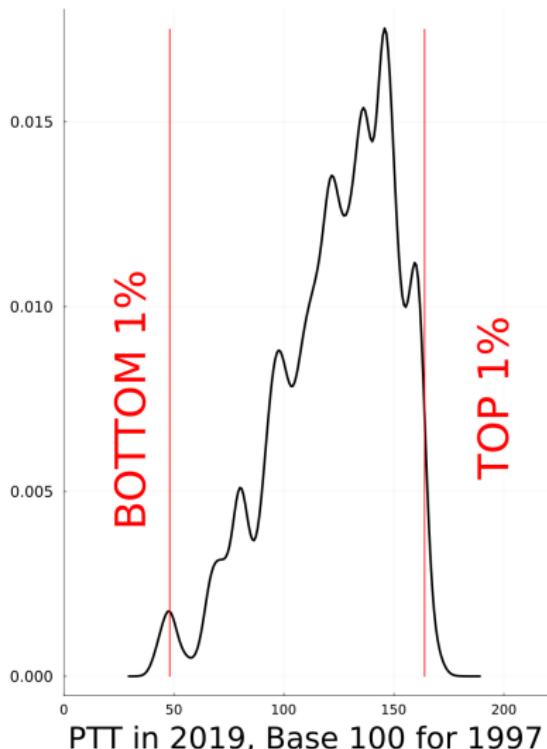
Positional Terms of Trade

$$C_r = \mathbf{PTT}_r \times f_r(L_h, L_I)$$

$$d \log TFP = \sum \chi_r d \log \mathbf{PTT}_r$$

$$\begin{aligned} d \log \mathbf{PTT}_h &= \underbrace{\beta_h d \log A_H + (1 - \beta_h) d \log A_L}_{\text{Technology}_h} \\ &\quad + \underbrace{\beta_h d \log \mu_H + (1 - \beta_h) d \log \mu_L}_{\text{Competitiveness}_h} \\ &\quad - \underbrace{\left(\frac{\tilde{m}_{h \leftarrow h}}{\Lambda_h} d \Lambda_h + \frac{\tilde{m}_{h \leftarrow I}}{\Lambda_I} d \Lambda_I - d \log \chi_h \right)}_{\text{Misallocation}_h} \end{aligned}$$

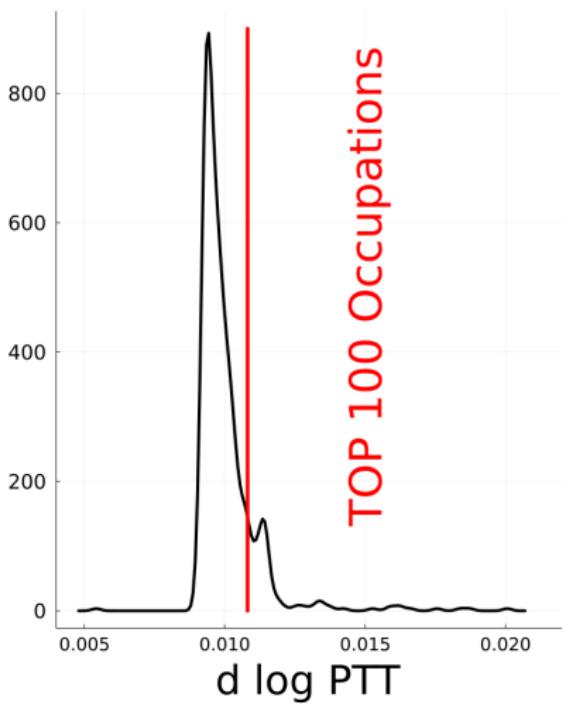
$$C_r = \text{Positional Terms of Trade}_r \times f_r(L_h, L_l)$$



<i>Top 1% Occupation</i>	
Logging Workers	37%
Computer Occupations	13%
Mathematical Sciences Occupations	10%
Compensation Managers	7%

<i>Bottom 1% Occupation</i>	
Printing Workers	40%
Shoe & Leather Operator	26%
Textile Machine Operator	15%
Miscellaneous Textile	12%

Effects from more competition in Housing



<i>Top 100 occupations</i>	
Construction workers	48
Painters, Carpet Installer, Tile Setter, Stonemason, Plasterer, Drywall Installer, Septic Servicer, Construction Supervisor	
Financial specialist	7
Property appraiser, Loan Officer Credit Analyst, Financial Examiner	
Extraction Workers	7
Rock Splitter, Roof Bolter	
Woodworkers	6
Cabinetmaker, Furnite Finisher	
Installation & Maintenance	5
Heating & AC, Mobile Home Installer	

Conclusion

- First comprehensive study for joint heterogeneity in multisector economies with distortions and input-output networks
- **Theoretical Contribution** in production network + distortions + heterogeneous households:
 - Variation of the income distribution
 - Variations for TFP
 - Variations for PTT
- **Empirical Contribution:** First implementation of a production network model with household heterogeneity for the US
 - In the absence of distributional sources of misallocation, TFP would have grown 7.5% more after Great Recession

Pipeline

Working Papers

1. In **International Misallocation and Comovement under Production Networks**, I obtain the first decomposition for a distorted open economy production network when there is cross-country factor allocation and ownership of firms

Pipeline

Working Papers

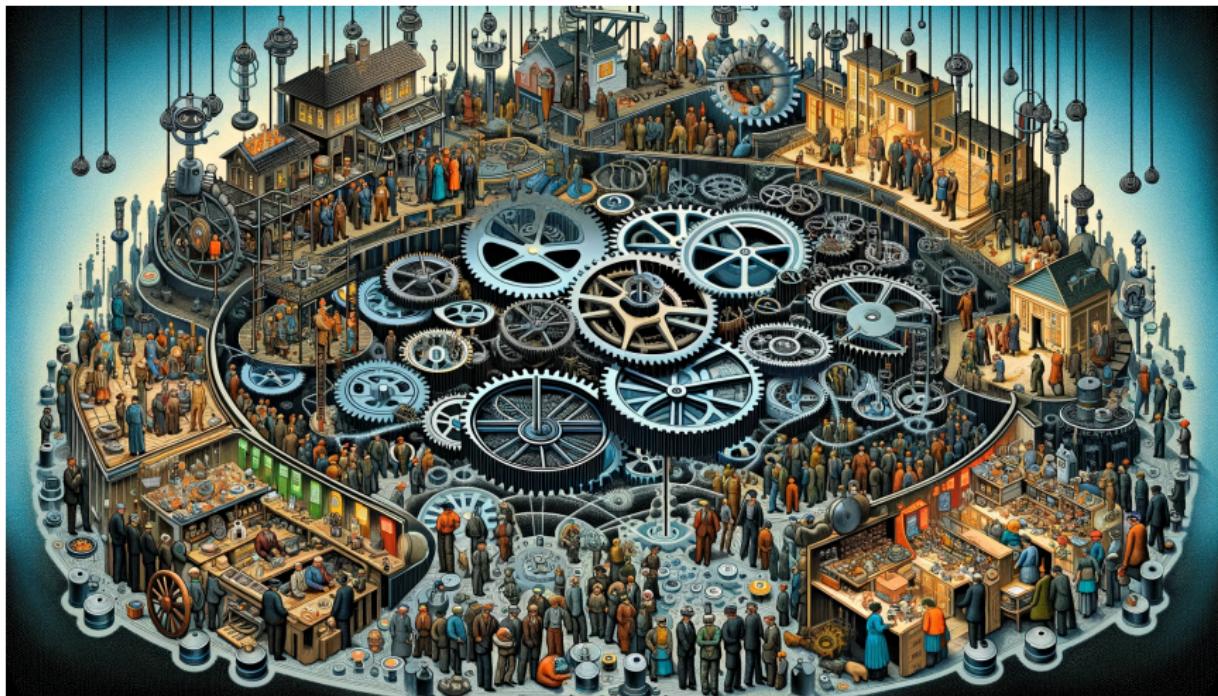
1. In **International Misallocation and Comovement under Production Networks**, I obtain the first decomposition for a distorted open economy production network when there is cross-country factor allocation and ownership of firms
2. In **Growth Through Industrial Linkages**, we evaluate how variations in global production networks have lifted up the growth for emerging economies

Pipeline

Working Papers

1. In **International Misallocation and Comovement under Production Networks**, I obtain the first decomposition for a distorted open economy production network when there is cross-country factor allocation and ownership of firms
2. In **Growth Through Industrial Linkages**, we evaluate how variations in global production networks have lifted up the growth for emerging economies
3. In **Nonlinearities in Production Networks with Distortions**, we obtain a novel second-order approximation for a aggregate TFP in a production network economy with distortions

Thank you!



Output from DALL-E after introducing title and abstract

Upper Decile vs The Rest

(Consumer Expenditure Survey 2021)

Higher Expenditure Share in

- Education: 3.4% vs 1.3%
- Entertainment: 6.5% vs 4.9%
- Pensions: 17.4% vs 9.1%
- Lodging: 2.6% vs 1.1%

Lower Expenditure Share in

- Shelter: 17.6% vs 20.5%
- Home Food: 5.9% vs 8.5%
- Utilities: 4.1% vs 7.0%
- Healthcare: 6.2% vs 8.3%

From 2004 to 2019

Income share for top quintile ↑ from 48% to 53%

Literature Review

- **Disaggregated National Accounts**

Cantillon (1756), Quesnay (1758), Leontief (1928), Meade & Stone (1941), Kuznetz (1946), Stone (1961), Andersen et al. (2022)

- **Production Networks**

Hulten (1978), Long & Plosser (1983), Gabaix (2011), Jones (2011, 2013), Acemoglu et al. (2012), Baqaee (2018), Baqaee & Farhi (2019, 2020, 2023), Bigio & La'O (2020)

- **Growth Accounting**

Solow (1957), Domar (1961), Jorgenson et al. (1987), Basu & Fernanld (2022), Petrin & Levinsohn (2012), Baqaee & Farhi (2020)

Dixit-Stiglitz Aggregation

- Sector i has a sectoral aggregator for $z_i \in [0, 1]$

$$y_i = \left(\int y_{z_i}^{\mu_i} d z_i \right)^{\frac{1}{\mu_i}}$$

- Demand for varieties

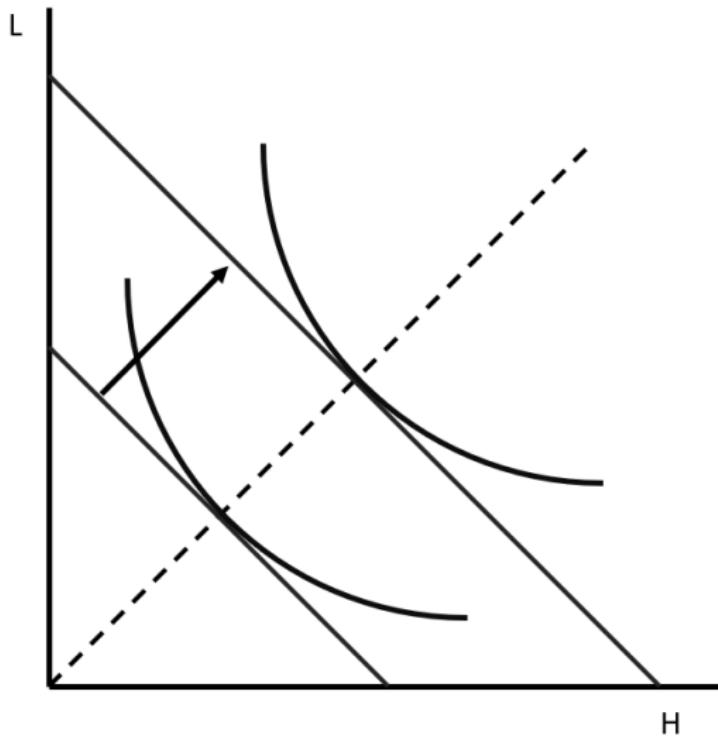
$$y_{z_i} = \left(\frac{p_i}{p_{z_i}} \right)^{\frac{1}{1-\mu_i}} y_i$$

- Intermediate's problem

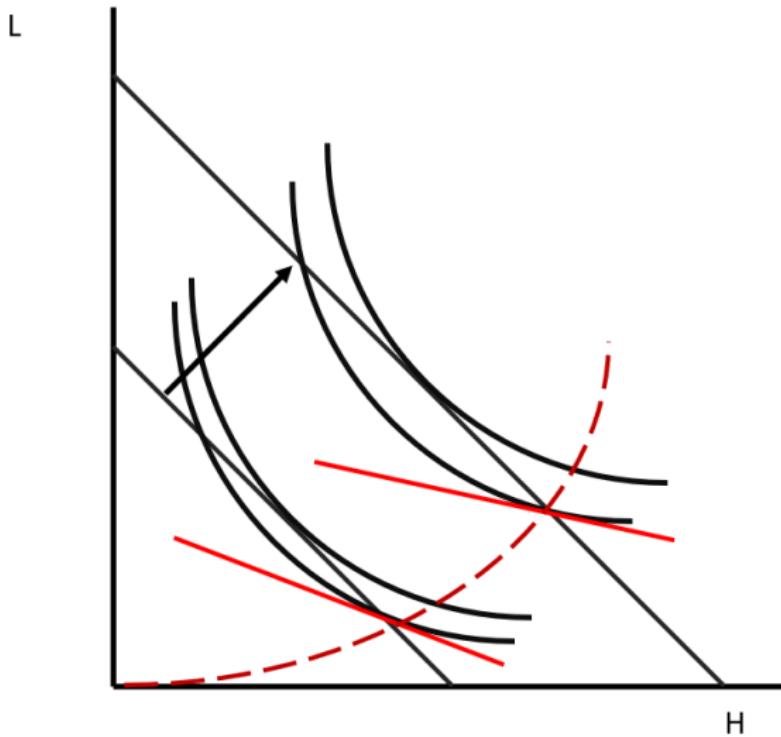
$$\underset{p_{z_i}, y_{z_i}, \ell_{z_i h}, \ell_{z_i I}}{\text{Max}} \quad \pi_{z_i} = p_{z_i} y_{z_i} - w_h \ell_{z_i h} - w_I \ell_{z_i I}$$

$$y_{z_i} = A_i \ell_{z_i h}^{\alpha_i} \ell_{z_i I}^{1-\alpha_i}$$

Aggregate Non-Homotheticity



Aggregate Non-Homotheticity



Equilibrium Definition

$e = (A, \mu, \beta, \alpha) \in \mathcal{E}$ into

$$\vartheta \equiv \left\{ \left\{ y_i, \{\ell_{ir}, C_{ri}\}_{r \in \{h, l\}} \right\}_{i \in \{H, L\}}, \{C_r, L_r\}_{r \in \{h, l\}} \right\}$$

$$\rho \equiv \{p_H, p_L, w_h, w_l, p_h^c, p_l^c\}$$

Necessary & sufficient equilibrium conditions

(ϑ, ρ) are an equilibrium iff

$$-\frac{w_b}{w_r} \frac{U_{Lr}}{U_{Cr_i}} = \mu_i \frac{\partial y_i}{\partial \ell_{ib}} \quad i \in \{H, L\}, r, b \in \{h, l\},$$

such that $C_{ri} > 0$, and $\ell_{ib} > 0$,

and resource constraints

$$y_i(e) = C_{hi}(e) + C_{li}(e) \quad i \in \{H, L\}$$

$$L_r(e) = \ell_{Hr}(e) + \ell_{Lr}(e) \quad r \in \{h, l\}.$$

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2} \left((1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L \right)$$

Sales Share

$$\lambda_H = \beta_h \chi_h + \beta_I \chi_I \quad \lambda_L = 1 - \lambda_H$$

Labor Income Share

$$\Lambda_h = \alpha_H \mu_H \lambda_H + \alpha_L \mu_L \lambda_L$$

Expenditure Share

$$\chi_h = \Lambda_h + \frac{1}{2} \left((1 - \mu_H) \lambda_H + (1 - \mu_L) \lambda_L \right)$$

Value Added Share

$$\tilde{\Lambda}_h = \alpha_H \lambda_H + \alpha_L \lambda_L \quad \tilde{\Lambda}_h + \tilde{\Lambda}_I = 1$$

Sales Distribution

$$\lambda_H = \frac{\theta}{\left(\frac{1}{2 - (\underbrace{\beta_h - \beta_l}_{\geq 0})(\underbrace{\alpha_H - \alpha_L}_{\geq 0})} \right)}$$

Amplification Effect

Contractionary Effect

$\theta \geq 1/2$

Consumption Expenditure Distribution

$$\chi_h = \theta \left(1 - \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(\beta_h - \mu_H)}_{?} \right)$$

Labor Income Distribution

$$\Lambda_h = \theta \left[\underbrace{\alpha_L + \mu_H (\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(1 - \mu_L (\beta_h - \beta_I))}_{\geq 0} \right]$$

$$\Lambda_I = \theta \left[\underbrace{\alpha_H - \mu_H (\alpha_H - \alpha_L)}_{\geq 0} \underbrace{(1 + \mu_L (\beta_h - \beta_I))}_{\geq 0} \right]$$

Value-Added Distribution [Back](#)

$$\begin{aligned}\tilde{\Lambda}_h &= \alpha_H \lambda_H + \alpha_L \lambda_L \\ &= \theta \left(1 - \underbrace{(\beta_h - \beta_I)}_{\geq 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{\left(\alpha_H - \mu_H (\alpha_H - \alpha_L) \right)}_{\geq 0} \right)\end{aligned}$$

$$\tilde{\Lambda}_I = \theta \left(1 - \underbrace{(\beta_h - \beta_I)}_{\geq 0} \underbrace{(\alpha_H - \alpha_L)}_{\geq 0} \underbrace{\left(\alpha_L + \mu_H (\alpha_H - \alpha_L) \right)}_{\geq 0} \right)$$

3 Effects from Distortions on Labor

1. Misallocation comes from *MRS* wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

3 Effects from Distortions on Labor

1. Misallocation comes from *MRS* wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

2. Allocative differences \neq Misallocation

$$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$$

Intuition
 For the undistorted case
 $\mu_H = \mu_L = 1/2$
 there is a continuum
 of property rights on firms

Cases

Back

3 Effects from Distortions on Labor

1. Misallocation comes from *MRS* wedges

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

2. Allocative differences \neq Misallocation

$$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$$

Intuition
 For the undistorted case
 $\mu_H = \mu_L = 1/2$
 there is a continuum
 of property rights on firms

Cases

3. Distorted Labor Supply Γ_r

$$-\frac{U_{Lr}}{U_{Cr}} = \frac{\Lambda_r}{\chi_r} \frac{C_r}{L_r}$$

$$= \Gamma_r$$

Back

$\frac{\ell_{Hh}}{L_H} \neq \alpha_H$ not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

$\frac{\ell_{Hh}}{L_H} \neq \alpha_H$ not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

$\mu_H = \mu_L$
Symmetric π

Case 2

All π for h

Case 3

Case 4

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

$$\mu_H = \mu_L$$

Symmetric π

Case 2

$$\mu_H = \mu_L$$

All π for h

Case 3

Case 4

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$ not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

Case 2

Case 3

Case 4

$$\mu_H = \mu_L$$

Symmetric π

All π for h

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$\frac{\ell_{Hh}}{L_h} \neq \alpha_H$ not the same

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

Case 2

Case 3

Case 4

$$\mu_H = \mu_L$$

Symmetric π

All π for h

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{L_I}}{U_{C_I}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Misallocation

Case 1

$$\mu_H = \mu_L$$

Symmetric π

Case 2

All π for h

Case 3

$$\alpha_H = \alpha_L \quad \beta_h = \beta_L$$

Symmetric π

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{L_I}}{U_{C_I}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

$$\mu_H = \mu_L$$

Symmetric π

Case 2

All π for h

Case 3

$$\alpha_H = \alpha_L \quad \beta_h = \beta_L$$

Symmetric π

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

$$\frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{L_I}}{U_{C_I}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

$$\frac{\ell_{Hh}}{L_H} \neq \alpha_H \quad \text{not the same}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \underbrace{\frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}}_{\text{Misallocation}}$$

Case 1

$$\mu_H = \mu_L$$

Symmetric π

Case 2

All π for h

Case 3

$$\alpha_H = \alpha_L \quad \beta_h = \beta_l$$

Symmetric π

$$\frac{\ell_{Hh}}{L_h} = \alpha_H - \alpha_H + \alpha_H \alpha_L (\beta_h - \beta_l)$$

$$\frac{\ell_{Hh}}{L_h} > \alpha_H$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$\frac{U_{C_{rH}}}{U_{C_{rL}}} = \frac{\mu_L}{\mu_H} \frac{dy_L/d\ell_{Lr}}{dy_H/d\ell_{Hr}}$$

$$-\frac{U_{L_h}}{U_{C_h}} = \underbrace{(\Lambda_h/\chi_h)}_{\Gamma_h} \frac{C_h}{L_h}$$

$$-\frac{U_{L_I}}{U_{C_I}} = \underbrace{(\Lambda_I/\chi_I)}_{\Gamma_I} \frac{C_I}{L_I}$$

Linear Approximation in response to $d \log A_L$

■ $\lambda_H d \log S_H = \beta_h \chi_h d \log E_h + \beta_I \chi_I d \log E_I$

$$- (\rho - 1) \beta_h \beta_I \left(d \log A_L + (\alpha_H - \alpha_L) d \log \frac{w_h}{w_I} \right)$$

■ $\lambda_L d \log S_L = (1 - \beta_h) \chi_h d \log E_h + (1 - \beta_I) \chi_I d \log E_I$

$$+ (\rho - 1) \beta_h \beta_I \left(d \log A_L + (\alpha_H - \alpha_L) d \log \frac{w_h}{w_I} \right)$$

■ $d \log E_r = \frac{(1 + \zeta^w) \Gamma_r}{1 + \zeta^e \Gamma_r} d \log w_r + \frac{1}{2} \frac{\sum \lambda_i (1 - \mu_i) d \log S_i}{(1 + \zeta^e \Gamma_r) \chi_r}$

■ $d \log w_r = \frac{\zeta^e}{1 + \zeta^w} d \log E_r + \frac{\sum f_{ir} \lambda_i d \log S_i}{(1 + \zeta^w) \Lambda_r}$

Bilateral Centralities

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{l \rightarrow h} \chi_l$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h}, \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

3 definitions for $m_{r \rightarrow h}$

1. **Partial equilibrium effect** on h 's labor income from one additional **expenditure** unit from r

Bilateral Centralities

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{l \rightarrow h} \chi_l$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h}, \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

3 definitions for $m_{r \rightarrow h}$

1. **Partial equilibrium effect** on h 's labor income from one additional **expenditure** unit from r
2. **Share** of **expenditure** from r that reaches Λ_h

Bilateral Centralities

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{l \rightarrow h} \chi_l$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h}, \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

3 definitions for $m_{r \rightarrow h}$

1. **Partial equilibrium effect** on h 's labor income from one additional **expenditure** unit from r
2. **Share** of **expenditure** from r that reaches Λ_h
3. $\{m_{h \rightarrow h}, m_{l \rightarrow h}\}$ is a **ranking** for **expenditure** relevance on Λ_h

Bilateral Centralities

$$\Lambda_h = m_{h \rightarrow h} \chi_h + m_{l \rightarrow h} \chi_l$$

$$m_{r \rightarrow h} = \beta_r f_{H \rightarrow h} + (1 - \beta_r) f_{L \rightarrow h}, \quad f_{i \rightarrow h} = \alpha_i \mu_i$$

3 definitions for $m_{r \rightarrow h}$

1. **Partial equilibrium effect** on h 's labor income from one additional **expenditure** unit from r
2. **Share** of **expenditure** from r that reaches Λ_h
3. $\{m_{h \rightarrow h}, m_{l \rightarrow h}\}$ is a **ranking** for **expenditure** relevance on Λ_h
 - Similar 3 definitions for $f_{i \rightarrow h}$ but for **revenue** of i

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\text{Shephard's Lemma}} = \frac{p_H C_{rH}}{p_r^c C_r} = \beta_r \text{ In equilibrium as parameter}$$

$$\frac{d p_r^c C_r}{d p_H} = C_{rH}$$

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\text{Shephard's Lemma}} = \frac{p_H C_{rH}}{p_r^c C_r} = \beta_r \text{ In equilibrium as parameter}$$

$$\frac{d p_r^c C_r}{d p_H} = C_{rH}$$

1. New equilibrium with local approximations keep α and β fixed

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\text{Shephard's Lemma}} = \frac{p_H C_{rH}}{p_r^c C_r} = \beta_r \text{ In equilibrium as parameter}$$

$$\frac{d p_r^c C_r}{d p_H} = C_{rH}$$

1. New equilibrium with local approximations keep α and β fixed
2. Estimate β 's consistent with the new equilibrium

Substitution Effects

$$\beta_r \equiv \underbrace{\frac{d \log p_r^c C_r}{d \log p_H}}_{\text{Shephard's Lemma}} = \underbrace{\frac{p_H C_{rH}}{p_r^c C_r}}_{\frac{d p_r^c C_r}{d p_H} = C_{rH}} = \beta_r \text{ In equilibrium as parameter}$$

1. New equilibrium with local approximations keep α and β fixed
2. Estimate β 's consistent with the new equilibrium

Exact delta hat - Dekle, Eaton & Kortum (2008)

$$\frac{p_H C_{rH}}{E_r} = \beta_r^\rho \left(\frac{p_r^c \bar{C}_r}{p_H \bar{C}_{rH}} \right)^{\rho-1} \rightarrow d \beta_r = \underbrace{(\rho - 1) \beta_r (1 - \beta_r)}_{\text{Increases under substitutability when } p_L/p_H \uparrow} \frac{d \log \frac{p_L}{p_H}}{d \log \frac{p_L}{p_H}}$$

Theorem 1: labor income share variation

$$d \Lambda_I = \underbrace{\left(m_{h \rightarrow I} - m_{I \rightarrow h} \right) d \chi_h}_{\text{Distributive Income}_I} + \underbrace{\left(f_{H \rightarrow L} - f_{L \rightarrow H} \right) \sum \chi_r d \beta_r}_{\text{Income Centrality}_I}$$

$\overbrace{(\beta_h - \beta_l)}^{\geq 0} \overbrace{(\mu_H - \alpha_H)}^{?}$
 $\overbrace{(\mu_H - \alpha_H)}^{?}$
 $\overbrace{(f_{H \rightarrow L} - f_{L \rightarrow H})}^{?}$

Labor Wedge

For factors with endogenous supply...

$$-\frac{U_{L_h}}{U_{C_h}} = \Gamma_h \frac{C_h}{L_h}$$

with

$$\Gamma_h = \frac{\Lambda_h}{\chi_h}$$

Proof

d log Γ_h - Extension of Bigio & La'O (2020)

- | | | |
|-----------------------------------|---|-----------------------------|
| (i) Representative Household | → | (i) Heterogenous Households |
| (ii) Around Efficient Equilibrium | | (ii) Any Equilibrium |

$$d \log \Gamma_h = d \log \Lambda_h - d \log \chi_h$$

Proof of Theorem 1 for $d \log \Gamma_h$

From goods market clearing

$$\begin{pmatrix} y_H \\ y_L \end{pmatrix} = \begin{pmatrix} C_{hH} + C_{IH} \\ C_{hL} + C_{IL} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hL}} y_L \end{pmatrix} = \begin{pmatrix} \frac{\beta_h}{C_{hH}} (C_{hH} + C_{IH}) \\ \frac{(1-\beta_h)}{C_{hL}} (C_{hL} + C_{IL}) \end{pmatrix}$$

From FOC and equilibrium $\beta_h \frac{\chi_h}{C_{hH}} = p_H = \beta_I \frac{\chi_I}{C_{IH}}$

$$\begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hH}} y_L \end{pmatrix} = \begin{pmatrix} \beta_h \frac{\chi_h}{\chi_h} + \beta_I \frac{\chi_I}{\chi_h} \\ (1 - \beta_h) \frac{\chi_h}{\chi_h} + (1 - \beta_I) \frac{\chi_I}{\chi_h} \end{pmatrix}$$

Back

Proof of Theorem 1 for $d \log \Gamma_h$

From FOC and equilibrium $-\frac{1}{\beta_h} \frac{U_{L_h}}{U_{C_h}} \frac{C_{hH}}{C_h} = \frac{w_h}{p_H} = \mu_H \alpha_H \frac{y_H}{\ell_{Hh}}$

$$\begin{pmatrix} \ell_{Hh} \\ \ell_{Lh} \end{pmatrix} = \begin{pmatrix} -\frac{U_{C_h}}{U_{L_h}} \alpha_H \mu_H y_H \beta_h \frac{C_h}{C_{hH}} \\ -\frac{U_{C_h}}{U_{L_h}} \alpha_L \mu_L y_L (1 - \beta_h) \frac{C_h}{C_{hL}} \end{pmatrix}$$

From labor market clearing condition

$$L_h = \ell_{Hh} + \ell_{Lh} = -\frac{U_{C_h}}{U_{L_h}} C_h (\alpha_H \mu_H - \alpha_L \mu_L) \begin{pmatrix} \frac{\beta_h}{C_{hH}} y_H \\ \frac{(1-\beta_h)}{C_{hH}} y_L \end{pmatrix}$$

$$= -\frac{U_{C_h}}{U_{L_h}} C_h \underbrace{\left(\alpha_H \mu_H \sum_{r \in \{h, l\}} \beta_r \frac{\chi_r}{\chi_h} + \alpha_L \mu_L \sum_{r \in \{h, l\}} (1 - \beta_r) \frac{\chi_r}{\chi_h} \right)}_{=\Gamma_h} \quad \text{Back}$$

Δ TFP

A.

$$GDP = P_Y Y = p_h^c C_h + p_l^c C_l$$

$$d \log GDP = \chi_h d \log p_h^c C_h + \chi_l d \log p_l^c C_l$$

Δ TFP

A.

$$GDP = P_Y Y = p_h^c C_h + p_l^c C_l$$

$$d \log GDP = \chi_h d \log p_h^c C_h + \chi_l d \log p_l^c C_l$$

B. Divisia Index GDP deflator

$$d \log P_Y \equiv \chi_h d \log p_h^c + \chi_l d \log p_l^c$$

$$= \tilde{\Lambda}_h d \log w_h + \tilde{\Lambda}_l d \log w_l$$

$$- \lambda_H d \log (A_H \times \mu_H) - \lambda_L d \log (A_L \times \mu_L)$$

Additional Steps for $d \log P_Y$

Start from

$$p_H = \frac{w_h \ell_{Hh} + w_l \ell_{Hl}}{\mu_H A_H \ell_{Hh}^{\alpha_H} \ell_{Hl}^{1-\alpha_H}}$$

Take first-order approximation

$$\hat{p}_H = -\hat{A}_H - \hat{\mu}_H + \alpha_H \hat{\alpha}_{Hh} + (1 - \alpha_H) \hat{\alpha}_{Hl}$$

Do the same for bundle prices

$$\hat{p}_h^c = -\beta_h (\hat{A}_H + \hat{\mu}_H) - (1 - \beta_h) (\hat{A}_L + \hat{\mu}_L) + \tilde{\mathcal{C}}_{hh} \hat{w}_h + \tilde{\mathcal{C}}_{hl} \hat{w}_l$$

Distortion Centrality Heterogeneity

$$d \Lambda = d \Lambda_h + d \Lambda_l$$

$$\text{Misallocation} = \overbrace{(\delta_l - \delta_h)}^{\geq 0} d \Lambda_l + \delta_h d \Lambda$$

$$\delta_l - \delta_h = \overbrace{(\mu_H - \mu_L)}^{\geq 0} \overbrace{(\alpha_H - \alpha_L)}^{\geq 0} \overbrace{a}^{> 0}$$

$$a = \frac{1 + (\beta_h - \beta_l)(\alpha_H - \alpha_L) \left(1 + \overbrace{\mu_H \mu_L (\beta_h - \beta_l)(\alpha_H - \alpha_L)}^b \right)}{(\alpha_H \mu_H + \alpha_L \mu_L - b)(\alpha_H \mu_L + \alpha_L \mu_H - b)}$$

Constant *a*

$$a = \frac{1 + (\beta_h - \beta_l)(\alpha_H - \alpha_L) \left(1 + \overbrace{\mu_H \mu_L (\beta_h - \beta_l)(\alpha_H - \alpha_L)}^b \right)}{(\alpha_H \mu_H + \alpha_L \mu_L - b)(\alpha_H \mu_L + \alpha_L \mu_H - b)}$$

Alternatives: Income distribution → Output

In Auclert & Rognlie (2020)

- Negative Correlation between income and MPC
- + Wage rigidities
- Aggregate Demand \downarrow & Keynesian unemployment

In my model

- Static model, MPC equals 1
- No nominal rigidities
- Supply effect due to Misallocation

Income Centrality

$$\begin{aligned}
 \text{Income Centrality}_h &= \overbrace{\sum_{i \in \mathcal{N}} \mu_i \lambda_i d \tilde{\Omega}_{ih}^\ell}^{\text{Labor Demand Recomposition}_h} + \overbrace{\sum_{i \in \mathcal{N}} f_{ih} \lambda_i d \log \mu_i}^{\text{Competitive Income}_h} \\
 &\quad + \overbrace{\sum_{i \in \mathcal{N}} f_{ih} \sum_{b \in \mathcal{H}} \chi_b d \beta_{bi}}^{\text{Final Demand Recomposition}_h} + \overbrace{\sum_{i \in \mathcal{N}} f_{ih} \sum_{j \in \mathcal{N}} \mu_j \lambda_j d \tilde{\Omega}_{ji}^x}^{\text{Intermediate Demand Recomposition}_h}
 \end{aligned}$$

Share of sector
 i's revenue reaching
 worker h's income

Sales Share $\lambda_j = \frac{\text{Sales}_j}{\text{GDP}}$

Misallocation Decomposition

1. Misallocation \uparrow as expenditure rises for households with high M_h
2. Misallocation \uparrow as labor demand for workers with high δ rises
3. Misallocation \uparrow as profit margins fall in sector with high F_i

$$\begin{aligned}
 & \underbrace{\sum_{h \in \mathcal{H}} M_h d \chi_h}_{\text{Distributive}} + \underbrace{\sum_{i \in \mathcal{N}} \mu_i \lambda_i \sum_{h \in \mathcal{H}} \delta_h d \tilde{\Omega}_{ih}^\ell}_{\text{Labor Demand}} + \underbrace{\sum_{i \in \mathcal{N}} \lambda_i F_i d \log \mu_i}_{\text{Competitive}} \\
 & + \underbrace{\sum_{h \in \mathcal{H}} \chi_h \sum_{i \in \mathcal{N}} F_i d \beta_{hi}}_{\text{Final Demand}} + \underbrace{\sum_{i \in \mathcal{N}} \mu_i \lambda_i \sum_{j \in \mathcal{N}} F_j d \tilde{\Omega}_{ij}^x}_{\text{Intermediate Demand}}
 \end{aligned}$$

4. Misallocation \uparrow as demand of goods \uparrow from sectors with high F_i

Antisupression Algorithm

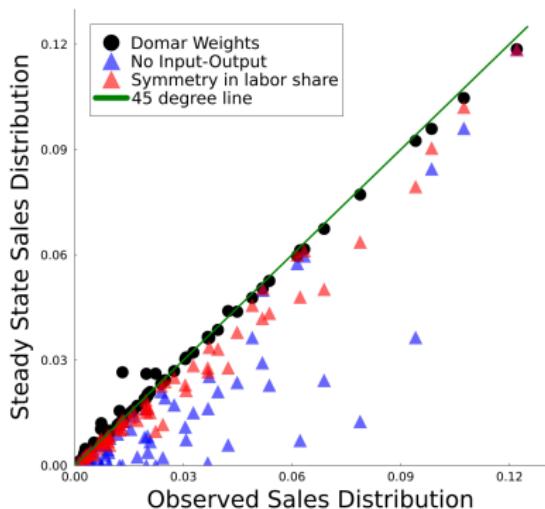
1. Significant portion of data suppressed to protect confidentiality
2. Since 2007 non-suppressed observations have a random noise infusion multiplier
3. Use information available due to the industrial and geographical hierarchical nature → manifold of bound and aggregation constraints across hierarchies
4. Two gold standards:
 - i. Two-staged algorithm from Isserman & Westervelt (2006)
 - ii. Linear programming solution from Eckert et al. (2020)
5. These two methods estimate the number of workers, not their compensation. I develop a three-staged algorithm that starting from the guess Eckert et al. (2020) extends Isserman & Westervelt (2006) to the estimation of labor compensation

Missing Private Employment

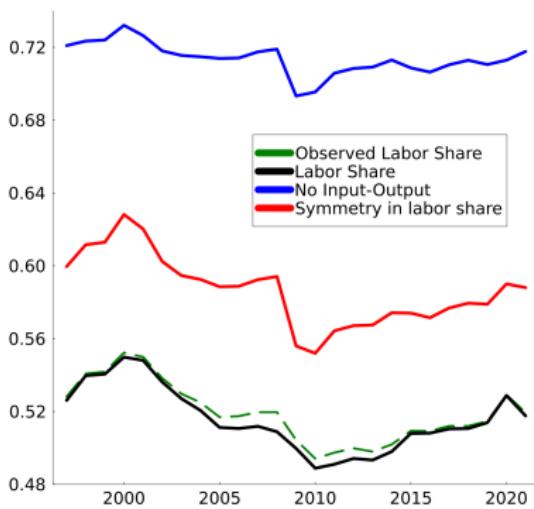
1. The CBP only covers some forms of private employment
2. It does not include workers in
 - Agriculture production
 - Railroads
 - Government
 - Private household
3. To fill this gap, I use the BEA's Regional Economic Information System to obtain state-level employment and income measures for agricultural and production workers
4. Data sources for REIS are the Quarterly Census of Employment and Statistics from the BLS
5. Main limitation from REIS is that it is only provided at the 2-digit NAICS level

Moments under Representative Household

Sales Distribution 2021



Labor Share



R^2 on sales distribution

R^2 on labor cost share

	R^2 on sales distribution	R^2 on labor cost share
Base Model	0.994	0.981
No Input-Output	0.730	0.733
Symmetry in Labor	0.978	0.933

Contribution from each component

Table: Counterfactual TFP Growth Differential in the Absence of Components

A. Between 1997 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>Misallocation</i>
-23.4%	2.5%	2.8%

B. Between 2002 and 2009

<i>Technology</i>	<i>Competitiveness</i>	<i>Misallocation</i>
-13.0%	19.3%	-8.2%

C. Between 2010 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>Misallocation</i>
-6.3%	-9.8%	7.6%

Covariance Decomposition

Table: Covariance Decomposition

A. Between 1997 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>-Misallocation</i>
44.4%	34.6%	21.0%

B. Between 2002 and 2009

<i>Technology</i>	<i>Competitiveness</i>	<i>-Misallocation</i>
28.3%	61.2%	10.5%

C. Between 2010 and 2020

<i>Technology</i>	<i>Competitiveness</i>	<i>-Misallocation</i>
58.1%	4.9%	37.0%

Model without Intermediate Inputs

	<i>Rep.</i>	<i>Household</i>	<i>Occupation</i>	<i>County</i>	<i>State & Occupation</i>			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>dlog TFP</i>	0.523 (0.366)		0.503 (0.350)		0.388 (0.316)		-0.265 (0.264)	
Technology		1.341*** (0.308)		0.789*** (0.267)		0.796*** (0.266)		0.847*** (0.289)
Competitiveness		0.212 (0.423)		0.320 (0.489)		0.454 (0.373)		0.986 (0.695)
Misallocation		0.573* (0.329)		0.450 (0.437)		0.335 (0.315)		-0.105 (0.360)
Intercept	0.012*** (3.2e-3)	0.011*** (2.0e-3)	0.012*** (3.2e-3)	0.012*** (2.2e-3)	0.013*** (3.2e-3)	0.012*** (2.1e-3)	0.015*** (3.0e-3)	0.012*** (2.2e-3)
Observations					22			
N					66			
H		1		750		3,136		38,190
<i>R</i> ²	9.2%	71.4%	9.35%	62.4%	7.00%	62.5%	4.8%	60.4%
<i>Adj. R</i> ²	9.2%	68.4%	9.35%	58.4%	7.00%	58.6%	4.8%	56.2%

Model with Intermediate Inputs

	<i>Rep.</i>	<i>Household</i>	<i>Occupation</i>	<i>County</i>	<i>State & Occupation</i>			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>dlog TFP</i>	0.370*** (0.072)		0.311*** (0.069)		0.316*** (0.065)		0.311*** (0.069)	
Technology		0.478*** (0.097)		0.414*** (0.081)		0.416*** (0.083)		0.413*** (0.082)
Competitiveness		0.398*** (0.062)		0.341*** (0.054)		0.350*** (0.053)		0.342*** (0.054)
Misallocation		0.074 (0.138)		0.172 (0.125)		0.164 (0.135)		0.168 (0.125)
Intercept	0.010*** (2.1e-3)	0.009 (2.0e-3)	0.011*** (2.2e-3)	0.010*** (1.8e-3)	0.011*** (2.1e-3)	0.010*** (1.9e-3)	0.011*** (2.3e-3)	0.010*** (1.9e-3)
Observations					22			
N					66			
H		1		750		3,136		38,190
<i>R</i> ²	56.9%	75.2%	49.9%	75.8%	54.0%	75.4%	49.9%	75.5%
<i>Adj. R</i> ²	56.9%	72.6%	49.9%	73.3%	54.0%	72.8%	49.9%	73.2%

Technological Sources

A. Between 1998 and 2020

1	Oil & gas extraction	-11.11%
2	Computer & electronics	-6.64%
3	Telecommunications	-2.85%
4	Computer systems design	-2.30%
5	Administrative services	-1.74%
6	Insurance carriers	-1.45%
7	Farms	-1.34%
8	Primary metals	-1.28%
	:	
63	Rental & leasing	1.41%
64	Credit intermediation	1.77%
65	Chemical Products	2.84%
66	Construction	2.87%

C. Between 2010 and 2020

1	Oil & gas extraction	-5.41%
2	Computer systems design	-1.29%
3	Management of companies	-1.26%
4	Housing	-1.14%
5	Other real estate	-1.01%
	:	
64	Air transportation	1.03%
65	Chemical products	1.90%
66	Credit intermediation	2.73%

B. Between 2002 and 2009

1	Oil & gas extraction	-5.35%
2	Computer & electronics	-2.84%
3	Telecommunications	-2.27%
4	Utilities	-1.92%
5	Administrative services	-1.06%
	:	
66	Construction	1.76%

Competitiveness Sources

A. Between 1998 and 2020

1	Housing	-1.65%
2	Insurance carriers	-1.53%
3	Misc. professional services	-1.10%
4	Other services	-0.89%
	:	
63	Publishing industries	0.80%
64	Computer and electronics	1.34%
65	Chemical products	2.57%
66	Credit intermediation	4.10%

C. Between 2010 and 2020

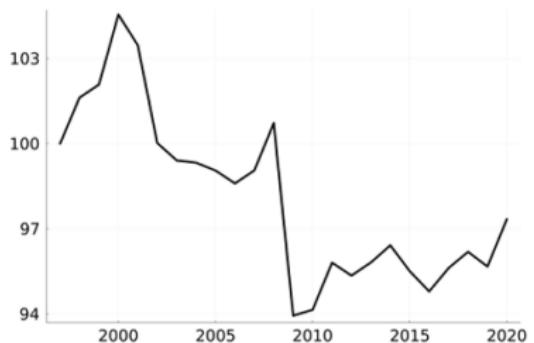
1	Oil & gas extraction	-6.34%
2	Housing	-3.09%
3	Insurance carriers	-0.98%
4	Misc. professional services	-0.87%
5	Administrative services	-0.82%
	:	
64	Primary metals	0.80%
65	Chemical products	0.84%
66	Credit intermediation	3.86%

B. Between 2002 and 2009

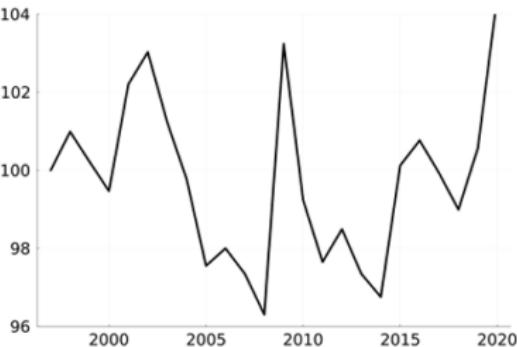
1	Securities & investment	-0.86%
	:	
58	Wholesale trade	0.92%
59	Publishing industries	0.93%
60	Internet, & inf. services	0.99%
61	Chemical products	1.35%
62	Telecommunications	1.43%
63	Computer and electronics	1.48%
64	Housing	1.57%
65	Utilities	1.87%
66	Oil & gas extraction	6.59%

Sources of Misallocation

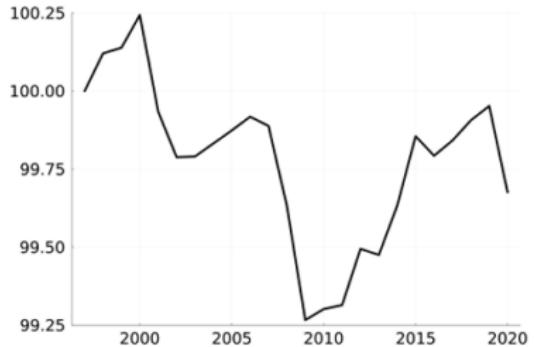
Competitive Terms of Trade



Labor Demand Terms of Trade



Final Demand Terms of Trade



Intermediate Demand Terms of Trade

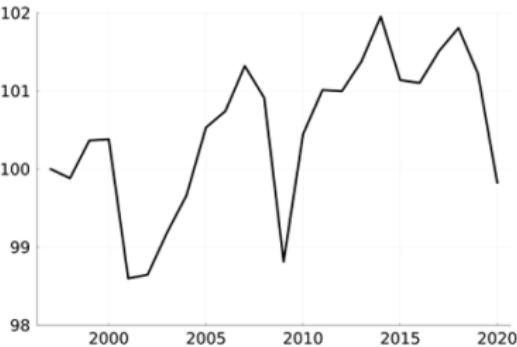


Table 11: Counterfactual TFP Growth Differential in the Absence of Misallocation Components

A. Between 1998 and 2020

<i>Heterogeneity</i>	<i>Distributive</i>	<i>Competitive</i>	<i>Labor</i>	<i>Final</i>	<i>Intermediate</i>
	<i>TT</i>	<i>TT</i>	<i>DTT</i>	<i>DTT</i>	<i>DTT</i>
Rep. Household	0%	-3.4%	6.3%	0.4%	-1.3%
Occupation	0%	-5.9%	15.1%	-2.0%	-4.2%
County	0.1%	-5.2%	14.2%	-0.9%	-4.4%
State & Occupation	0.1%	-5.9%	15.6%	-2.6%	-4.5%

B. Between 2002 and 2009

<i>Heterogeneity</i>	<i>Distributive</i>	<i>Competitive</i>	<i>Labor</i>	<i>Final</i>	<i>Intermediate</i>
	<i>TT</i>	<i>TT</i>	<i>DTT</i>	<i>DTT</i>	<i>DTT</i>
Rep. Household	0%	-9.3%	1.1%	-0.9%	-0.2%
Occupation	0%	-11.0%	3.4%	-1.9%	-0.8%
County	0.1%	-10.4%	3.4%	-0.7%	-1.0%
State & Occupation	0.1%	-11.1%	3.4%	-2.0%	-0.9%

C. Between 2010 and 2020

<i>Heterogeneity</i>	<i>Distributive</i>	<i>Competitive</i>	<i>Labor</i>	<i>Final</i>	<i>Intermediate</i>
	<i>TT</i>	<i>TT</i>	<i>DTT</i>	<i>DTT</i>	<i>DTT</i>
Rep. Household	0%	3.9%	1.2%	1.7%	0.9%
Occupation	0%	2.9%	7.2%	0.2%	-1.8%
County	0.1%	3.0%	3.5%	2.1%	-1.5%
State & Occupation	0.1%	2.8%	7.4%	-0.1%	-1.7%

Table 12: Counterfactual TFP Growth Without Sectoral Competitive TT

A. Between 1998 and 2020		
1	Credit intermediation	-2.16%
2	Chemical products	-1.06%
3	Computer & electronics	-0.98%
4	Publishing industries	-0.80%
5	Internet & inf. services	-0.69%
	:	
64	Insurance carriers	0.77%
65	Other services	0.81%
66	Misc. professional services	0.87%

B. Between 2002 and 2009

B. Between 2002 and 2009		
1	Oil & gas extraction	-1.46%
2	Computer & electronics	-1.11%
3	Internet & inf. services	-1.01%
4	Wholesale trade	-0.92%
5	Telecommunications	-0.86%
6	Utilities	-0.84%
7	Publishing industries	-0.82%

C. Between 2010 and 2020

C. Between 2010 and 2020		
1	Credit intermediation	-2.0%
2	Securities & investment	-0.52%
	:	
64	Administrative services	0.62%
65	Misc. professional services	0.70%
66	Oil & gas extraction	1.91%

Table 13: Counterfactual TFP Growth Without Sectoral Labor Demand TT

A. Between 1998 and 2020		
1	Wholesale trade	-1.62%
2	Insurance carriers	-1.61%
3	Other retail	-1.07%
	:	
61	Utilities	0.69%
62	Computer systems design	0.82%
63	Publishing industries	1.34%
64	Oil & gas extraction	1.79%
65	Computer & electronics	2.28%
66	Credit intermediation	2.40%

B. Between 2002 and 2009

B. Between 2002 and 2009		
1	Securities & investment	-0.96%
	:	
64	Computer & electronicss	0.85%
65	Utilities	1.02%
66	Oil & gas extraction	2.20%

C. Between 2010 and 2020

C. Between 2010 and 2020		
1	Wholesale trade	-1.70%
2	Insurance carriers	-1.03%
3	Administrative services	-0.93%
4	Other retail	-0.83%
	:	
64	Publishing industries	0.89%
65	Computer & electronics	0.98%
66	Credit intermediation	2.44%

Table 14: Counterfactual TFP Growth Without Sectoral Final Demand TT

A. Between 1998 and 2020		
1	Computer & electronics	-1.50%
2	Motor vehicles	-0.91%
3	Machinery	-0.88%
4	Apparel & leather	-0.51%
	:	
62	Securities & investment	0.87%
63	Misc. professional services	0.94%
64	Hospitals	0.95%
65	Internet & inf. services	1.01%
66	Wholesale trade	1.18%
B. Between 2002 and 2009		
1	Construction	-1.22%
2	Motor vehicles	-0.82%
	:	
66	Hospitals	0.58%

C. Between 2010 and 2020		
1	Computer & electronics	-0.52%
	:	
63	Other retail	0.59%
64	Internet & inf. services	0.60%
65	Construction	0.89%
66	Wholesale trade	1.08%

Table 15: Counterfactual TFP Growth Without Sectoral Intermediate Demand TT

A. Between 1998 and 2020		
1	Computer & electronics	-1.24%
2	Credit intermediation	-0.90%
3	Publishing industries	-0.76%
4	Computer systems design	-0.45%
5	Ambulatory health	-0.42%
	:	
61	Telecommunications	0.52%
62	Administrative services	0.54%
63	Hospitals	0.56%
64	Insurance carriers	0.74%
65	Other retail	0.90%
66	Wholesale trade	1.21%
B. Between 2002 and 2009		
1	Computer & electronics	-0.48%
	:	
66	Securities & investment	0.49%

C. Between 2010 and 2020		
1	Credit intermediation	-0.97%
2	Publishing industries	-0.51%
3	Computer & electronics	-0.49%
	:	
63	Insurance carriers	0.52%
64	Administrative services	0.63%
65	Other retail	0.66%
66	Wholesale trade	1.12%

Normalized nested CES environment - Firms

Firms

$$\frac{y_i}{\bar{y}_i} = A_i \left(\sum_{h \in \mathcal{H}} \tilde{\Omega}_{ih}^{\ell} \left(\frac{\ell_{ih}}{\bar{\ell}_{ih}} \right)^{\frac{\theta_i-1}{\theta_i}} + \sum_{j \in \mathcal{N}} \tilde{\Omega}_{ij}^x \left(\frac{x_{ij}}{\bar{X}_{ij}} \right)^{\frac{\theta_i-1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i-1}}$$

Back

Normalized nested CES environment - Households

Households

$$U_h(c_h, \tilde{L}_h) = \frac{\left(c_h \left(1 - E_h^{-\gamma_h} \tilde{L}_h\right)^{\varphi_h}\right)^{1-\sigma} - 1}{1 - \sigma} \quad s.t. \quad \frac{C_h}{\bar{C}_h} = \left(\sum_{i \in \mathcal{N}} \beta_{hi} \left(\frac{C_{hi}}{\bar{C}_{hi}}\right)^{\frac{\rho_h-1}{\rho_h}}\right)^{\frac{\rho_h}{\rho_h-1}}$$

with $C_h = n_h c_h$ and $L_h = n_h \tilde{L}_h$

Normalized nested CES environment - Households

Households

$$U_h \left(c_h, \tilde{L}_h \right) = \frac{\left(c_h \left(1 - E_h^{-\gamma_h} \tilde{L}_h \right)^{\varphi_h} \right)^{1-\sigma} - 1}{1 - \sigma} \quad s.t. \quad \frac{C_h}{\bar{C}_h} = \left(\sum_{i \in \mathcal{N}} \beta_{hi} \left(\frac{C_{hi}}{\bar{C}_{hi}} \right)^{\frac{\rho_h - 1}{\rho_h}} \right)^{\frac{\rho_h}{\rho_h - 1}}$$

with $C_h = n_h c_h$ and $L_h = n_h \tilde{L}_h$

The change in labor supply from type h workers is, to a first-order

$$d \log L_h = \zeta_h^n d \log n_h + \zeta_h^w d \log w_h - \zeta_h^e d \log E_h$$

Where the corresponding elasticities are given by

$$\zeta_h^n = \frac{E_h^{\gamma_h}}{1 - \varphi_h \gamma_h} \frac{n_h}{L_h}, \quad \zeta_h^w = \frac{1}{1 - \varphi_h \gamma_h} \frac{\varphi_h}{\Gamma_h}, \quad \zeta_h^e = \zeta_h^w - \gamma_h \zeta_h^n.$$

Solution - Expenditure & Wages

$$d \log E_h = \underbrace{\frac{\zeta_h^n \Gamma_h}{1 + \zeta_h^e \Gamma_h} d \log n_h}_{\text{Demographic Effect on Expenditure (PE)}} + \underbrace{\frac{(1 + \zeta_h^w) \Gamma_h}{1 + \zeta_h^e \Gamma_h} d \log w_h}_{\text{Wage Effect on Expenditure (GE)}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{\kappa_{ih} \lambda_i}{(1 + \zeta_h^e \Gamma_h) \chi_h} ((1 - \mu_i) d \log S_i - \mu_i d \log \mu_i)}_{\text{Corporate Income Effect on Expenditure (PE + GE)}}$$

$$d \log w_h = \underbrace{\frac{\zeta_h^e}{1 + \zeta_h^w} d \log E_h}_{\text{Expenditure Effect on Wages (GE)}} - \underbrace{\frac{\zeta_h^n}{1 + \zeta_h^w} d \log n_h}_{\text{Demographic Effect on Wages (PE)}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} ((\theta_i - 1) d \log A_i + \theta_i d \log \mu_i)}_{\text{Direct Effect on Wages (PE)}}$$

$$- \sum_{j \in \mathcal{N}} \left(\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} (\theta_i - 1) \tilde{\psi}_{ij}^x \right) (d \log A_j + d \log \mu_j) + \underbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} d \log S_i}_{\text{Supplier Effect on Wages (PE)}} + \underbrace{\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} d \log S_i}_{\text{Sales Effect on Wages (GE)}}$$

$$- \underbrace{\left(\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} (\theta_i - 1) \right) d \log w_h}_{\text{Direct Substitution Effect on Wages (GE)}} + \underbrace{\sum_{b \in \mathcal{B}} \left(\sum_{i \in \mathcal{N}} \frac{\Omega_{ih}^\ell \lambda_i}{(1 + \zeta_h^w) \Lambda_h} (\theta_i - 1) \tilde{\psi}_{ib}^\ell \right) d \log w_b}_{\text{Supplier Substitution Effect on Wages (GE)}}$$

Solution - Sales

$$\begin{aligned}
 d \log S_i = & \underbrace{\sum_{h \in \mathcal{H}} \frac{\beta_{hi} \chi_h}{\lambda_i} d \log E_h}_{\text{Expenditure Effect on Sales (GE)}} + \underbrace{\sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} d \log S_j}_{\text{Sales Effect on Sales (GE)}} + \underbrace{\sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} \left((\theta_j - 1) d \log A_j + \theta_j d \log \mu_j \right)}_{\text{Direct Effect on Sales (PE)}} \\
 & + \underbrace{\sum_{j \in \mathcal{N}} \left(\sum_{h \in \mathcal{H}} \frac{\beta_{hi} \chi_h}{\lambda_i} (\rho_h - 1) (\tilde{\psi}_{ij}^x - \tilde{\mathcal{B}}_{hj}) + \sum_{q \in \mathcal{N}} \frac{\Omega_{qi}^x \lambda_q}{\lambda_i} (\theta_q - 1) (\tilde{\psi}_{ij}^x - \tilde{\psi}_{qj}^x) \right) (d \log A_j + d \log \mu_j)}_{\text{Supplier Effect on Sales (PE)}} \\
 & + \underbrace{\sum_{h \in \mathcal{H}} \left(\sum_{b \in \mathcal{H}} \frac{\beta_{bi} \chi_b}{\lambda_i} (\rho_b - 1) (\tilde{\mathcal{C}}_{bh} - \tilde{\psi}_{ih}^\ell) + \sum_{j \in \mathcal{N}} \frac{\Omega_{ji}^x \lambda_j}{\lambda_i} (\theta_j - 1) (\tilde{\psi}_{jh}^\ell - \tilde{\psi}_{ih}^\ell) \right) d \log w_h}_{\text{Supplier Substitution Effect on Sales (GE)}}
 \end{aligned}$$