Nonlinearities in Production Network Economies with Distortions

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Alejandro Rojas-Bernal*[1] and Ali Karimirad†[2]

[1]Vancouver School of Economics, The University of British Columbia

[2]Vancouver School of Economics, The University of British Columbia

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Abstract

We offer the first non-linear analysis of how microeconomic disruptions in an inefficient production network economy impact aggregate TFP. Our decompositions, applicable to any general equilibrium economy, provide non-parametric insights. We identify essential general equilibrium metrics for capturing the non-linear repercussions of microeconomic fluctuations through network connections. Our findings encompass firm-level productivity shocks, wasted distortions, and rebated distortions. We reveal that substantial shocks or high production process complementarity/substitution introduce substantial bias in the linear approximation of distortions and productivity shocks found in the literature. Our non-linear second-order effects substantially mitigate this bias.

*Email: alejandrorojasecon@gmail.com

†Email: akarimir@mail.ubc.ca

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1 Introduction

The neoclassic theory classifies economic agents as either producers or consumers. On the production side, firms use factors to produce goods and services. Other firms further use some of this output as intermediate inputs. The production possibility frontier is an analytical device that characterizes the range of attainable consumption bundles influenced by varying allocations of resources and inputs among firms. The limits for this frontier depend on the firms' technologies and productivities, and the abundance of primary factors. On the consumption side, a welfare function aggregates households' preferences on what to consume and how much of the available factors to supply. Aggregate output depends on an equilibrium allocation and a system of prices that solves both firms' and households' problems while adhering to the feasibility constraints. Using the aggregate production function, we can break down the equilibrium output into two main parts: the aggregate factoral component and the aggregate efficiency, often referred to as TFP (Total Factor Productivity). In the absence of distortions, the equilibrium allocation is at a point of the production possibility frontier that generates an efficient level of aggregate output.

For efficient input-output economies featuring a representative household, Hulten (1978) established that the first-order variation of aggregate TFP depends only on a weighted sum of firms' productivity shocks, with weights given by the sales shares or Domar weights. Consequently, up to the first order, the sales distribution is a sufficient statistic, the microeconomic structure of the network is irrelevant, and the reallocation of resources in response to a shock is neutral. Baqaee & Farhi (2019) expand the aggregate TFP decomposition to a second-order. Their findings underscore the importance of the microeconomic intricacies of the network in grasping the nonlinear impact of individual firm productivity on aggregate efficiency. Thus, accounting for nonlinearities, elements like network linkages, micro-level elasticities of substitution, returns to scale, and resource reallocation are pivotal in comprehending the macroeconomic consequences of microeconomic disturbances.

When an economy faces distortions, the equilibrium vector of prices and its corresponding allocation yield a suboptimal level of aggregate output. Furthermore, the nature of distortions matters. Distortions that waste resources or production will bring the equilibrium within the boundaries of the efficient production possibility frontier. Distortions that rebate resources, e.g., profits or taxes, will move the equilibrium along the edges of the efficient production possibility frontier. With distortions, understanding how microeconomic shocks influence TFP becomes a more complex task that requires tracking how the reallocation of resources moves the equilibrium within and along the boundaries of the production possibility frontier. For inefficient input-output economies with a representative household, Liu (2019) and Baqaee & Farhi (2020) characterize the aggregate TFP first-order variation. Now, to understand aggregate effects up to the first order, the network structure becomes relevant, and the reallocation of resources in response to microeconomic fluctuations is no longer neutral.

This paper provides the first nonlinear decomposition for aggregate TFP in a distorted input-output economy with a representative household. These decompositions allow us to identify non-parametric sufficient statistics that account for second-order effects from microeconomic fluctuations through network linkages. Our decompositions are non-parametric and apply to any general equilibrium economy. Additonally, we account for firm level shocks in productivites, wasted distortions, and rebated distortions. Using a simple economy as an example, we show that for large idiosyncratic shocks in productivities and distortions, the linear approximation is biased, and the nonlinear second-order effects we account for significantly reduce this bias.

Related Literature

This paper contributes to the literature on shock propagation in production networks. These models builds on the canonical multisector models from Hulten (1978) and Long & Plosser (1983). These models have been used to study the linear propagation of sectoral productivity shocks (Foerster et al., 2011; Horvath, 1998, 2000; Dupor, 1999; Acemoglu et al., 2012, 2016; Carvalho et al., 2021) and distortions (Basu, 1995; Ciccone, 2002; Yi, 2003; Jones, 2011, 2013; Asker et al., 2014; Baqaee, 2018; Liu, 2019; Baqaee & Farhi, 2020; Bigio & La'O, 2020; Rojas-Bernal, 2023). Baqaee & Farhi (2019) were the first ones to consider to nonlinear effects from microeconomic productivity shocks in efficient input-output economies. To the best of our knowledge, this is the first study that accounts for nonlinearities in general equilibrium input-output environments with distortions. Our results nest all of the previous models and results as specific cases.

Layout

The structure of the paper is as follows. Section 2 introduces the multisector input-output model. Section 3 characterizes the equilibrium and the network centrality measures. Section 4 introduces the first and second order TFP decomposition for efficient input-output economies. Section 5 shows the linear and nonlinear effects of the TFP decomposition for an inefficient input-output economy when distortions are wasted. Section 6 shows for a simple economy that the bias from the linear approximation increases with the magnitude of the shock, and that the nonlinear effects from our decompostion go to great extent in solving this bias. Section 7 presents the linear and nonlinear effects of the TFP decomposition when distortions are rebated. Section 8 concludes.

2 The Environment

In this section, I set up a static nonparametric general equilibrium model with constant-returns-to-scale (CRS) for economies with N sectors and a representative household. Sector $i \in \mathscr{N} = \{1, \cdots, N\}$ consists of two types of firms: (i) a unit mass of monopolistic competitive firms indexed by $z_i \in [0, 1]$ producing differentiated goods, and (ii) a perfectly competitive producer that aggregates the industry's differentiated goods into a uniform sectoral good that can be consumed by households or used by other firms as intermediate inputs. Firms differ along four dimensions; first, monopolistic firms across sectors operate under different technologies; second, monopolistic firms within sectors have heterogeneous input demand; third, sectoral aggregators face different distortions; and (iv) for each sector, a fraction of their output is wasted. The representative household endogenously supply factors $f \in \mathscr{F} = \{1, \ldots, F\}$, receive a fraction ϕ_i of sector i's profits, and use this income to consume sectoral goods.

2.1 Production

Monopolistic firms within sectors produce differentiated goods using the same technology. The production for firm z_i in sector i follows

$$y_{z_i} = \mathscr{A}_i\, Q_i\left(L_{z_i}, X_{z_i}\right), \quad L_{z_i} = A_i^{\ell}\, Q_i^{\ell}\left(\left\{A_{if}^{\ell}\, \ell_{z_i f}\right\}_{f \in \mathscr{F}}\right), \quad X_{z_i} = A_i^{x}\, Q_i^{x}\left(\left\{A_{ij}^{x}\, x_{z_i j}\right\}_{j \in \mathscr{N}}\right), \quad (1)$$

where y_{z_i} stands for output, $\mathscr{A}_i$ is the sector-specific Hicks-neutral productivity term. L_{z_i} is the factoral composite that depends on the productivity A_i^{ℓ}. $\ell_{z_i f}$ is the rented amount of type f factors and is influenced by the productivity A_{if}^{ℓ}. X_{z_i} is the intermediate input composite that depends on the productivity A_i^{x}. $x_{z_i j}$ is the amount of intermediate input goods purchased from sector j and is influenced by the productivity A_{ij}^{x}.

The technologies $Q_i : \mathbb{R}_+^2 \to \mathbb{R}_+$, $Q_i^{\ell} : \mathbb{R}_+^H \to \mathbb{R}_+$, and $Q_i^{x} : \mathbb{R}_+^N \to \mathbb{R}_+$ are neoclassical and satisfy the following regularity conditions: they are positive, finite, and for the set of factors and intermediate inputs for which there is effective demand, they are monotonically increasing, twice continuously differentiable, strictly concave, and the Inada conditions hold.

The profits for firms z_i are given by

$$\pi_{z_i} = \phi_i \left(p_{z_i} y_{z_i} - \underbrace{\sum_{f \in \mathscr{F}} w_f\, \ell_{z_i f}}_{= p_{z_i}^{\ell} L_{z_i}} - \underbrace{\sum_{j \in \mathscr{N}} p_j\, x_{z_i j},}_{= p_{z_i}^{x} X_{z_i}} \right) \quad (2)$$

where p_{z_i} is the price of its output, $p_{z_i}^{\ell}$ is the price for the labor composite, $p_{z_i}^{x}$ is the price for the intermediate input composite, w_h is the wage received by households of type h, p_j is the market price for the good produced by the competitive aggregator in sector j, and $\phi_i \in [0,1]$ stands for the sectoral share of profits rebated back to households.

The competitive firm in sector i guarantees a homogeneous good by aggregating sectoral production using the following CES production function

$$y_i = \left(\int {y_{z_i}}^{\mu_i} \, dz_i \right)^{\frac{1}{\mu_i}}, \tag{3}$$

where $\mu_i \in (0,1]$ stands for the sector-specific markdown, and y_{z_i} represents the demand of goods produced by firm z_i. The aggregator takes prices as given and maximizes profits given by $\bar{\pi}_i = p_i y_i - \int p_{z_i} y_{z_i} \, dz_i$.

2.2 Consumption

The representative household's preferences are captured by the utility function $U\,(Y,\, L)$, where Y stands for real GDP, and L for the aggregate factoral supply. The utility $U_h : \mathbb{R}^2_+ \to \mathbb{R}_+$ satisfies the following regularity conditions: $U_Y > 0$, $U_L < 0$, twice continuously differentiable, strictly concave, and the Inada conditions hold. Real GDP is a composite $Y = Q\left(\{C_h\}_{i \in \mathscr{N}}\right)$ that depends on the final consumption C_i of goods from sector i. Aggregate factoral supply L is a composite $L = F\left(\{L_f\}_{f \in \mathscr{F}}\right)$. The consumption aggregation technology $Q : \mathbb{R}^N_+ \to \mathbb{R}_+$ and factoral aggregation technology $F : \mathbb{R}^F_+ \to \mathbb{R}_+$ are neoclassical: positive, finite, homogeneous of degree one, and for the set of goods for which there is effective final demand, it is monotonically increasing, twice continuously differentiable, strictly concave, and the Inada conditions hold. The representative household aggregates a unit mass continuous of infinitesimal symmetric households that take prices and wages as given.

GDP and GNI given by

$$GDP = p_Y\, Y = \sum_{i \in \mathscr{N}} p_i\, C_i \le GNI = \sum_{f \in \mathscr{F}} J_f + \sum_{i \in \mathscr{N}} \left(\bar{\pi}_i + \int \pi_{z_i} \, dz_i \right). \tag{4}$$

GDP must not be greater than GNI; the latter includes factoral income $J_f = w_f\, L_f$ and dividends.

2.3 Market Clearing

For this economy, the technologies, productivities, and markdowns are primitives. Monopolistic competition and wasted resources are the only sources of distortion. Hence, goods and labor market clearing conditions are given by

$$y_i = C_i + \sum_{j \in \mathscr{N}} x_{ji} + h_i \qquad \forall i \in \mathscr{N},$$

$$L_f = \sum_{i \in \mathscr{N}} \int \ell_{z_i f} \, d\, z_i \qquad \forall f \in \mathscr{F},$$

where $\ell_{if} = \int \ell_{z_i f} \, d\, z_i$ and $x_{ji} \equiv \int x_{z_j i} \, dz_j$ are respectively the total amount of factor f and intermediate inputs j demanded by sector i. h_i stands for the real unit of goods produced by sector i that are destroyed or wasted in the process of production. The nominal value of these wasted resources needs to satisfy

$$\sum_{i \in \mathscr{N}} p_i \, h_i = \sum_{i \in \mathscr{N}} (1 - \phi_i) \, (1 - \mu_i) \, p_i \, y_i.$$

From this condition, for a given vector of markdowns $\mu = [\, \mu_1, \ldots, \mu_N]'$ and rebated profits $\phi = [\, \phi_1, \ldots, \phi_N]'$, sectoral wasted resources are indeterminate. We are going to assume that distortions in one sector generate wasted resources in the same sector, i.e.,

$$h_i = (1 - \phi_i) \, (1 - \mu_i) \, y_i.$$

Following McKenzie (1959), this model also applies to economies with variable (increasing or decreasing) return to scale, which can be handled by appropriately introducing producer-specific fixed entrepreneurial factors in a constant return model.

3 Equilibrium and Centrality Measures

In this section, first, we characterize the equilibrium for this economy. Second, we introduce measures of bilateral centrality across firms and the representative households, and measures of aggregate centrality that portray each firm role in the economy. This section is essential to understand the second-order approximations that make up the main contribution of this paper.

3.1 Equilibrium Characterization

Let $e \equiv (\mathscr{A}, \mu, \phi)$ represent the aggregate state, and $\mathscr{E}$ denote the measurable collection of all possible realizations for this state. The matrix $\mathscr{A} \equiv (\mathcal{A}, A_\ell, A_x, \underline{A}_\ell, \underline{A}_x)$ collects all productivity measures,[1], sectoral markdowns are captured by μ, and rebated profits shares by ϕ. $\mathbb{1}_N$ is an N sized vector of ones.

For this economy, a mapping of the realization of the aggregate state to an allocation $\vartheta = (\vartheta(e))_{e\in\mathscr{E}}$ and the price system $\rho = (\rho(e))_{e\in\mathscr{E}}$ is represented by the set of functions

$$\vartheta(e) \equiv \left\{Y(e), L(e), \left\{\left(y_{z_i}(e), \{\ell_{z_i f}(e)\}_{f\in\mathscr{F}}, \{x_{z_i j}(e)\}_{j\in\mathscr{N}}\right)_{z_i\in[0,1]}, y_i(e), C_i(e), h_i(e)\right\}_{i\in\mathscr{N}}\right\},$$

$$\rho(e) \equiv \left\{\left\{\left(p_{z_i}(e), p^{\ell}_{z_i}(e), p^{x}_{z_i}(e)\right)_{z_i\in[0,1]}, p_i(e)\right\}_{i\in\mathscr{N}}, \{w_f(e)\}_{f\in\mathscr{F}}, p_Y(e)\right\}.$$

To make the notation cleaner, the definitions and implementation of the model that follows are conditional in a specific aggregate state $e \in \mathscr{E}$, e.g., $\mu(e)$ is portrayed by μ.

Definition 1. For any realization of the aggregate state e in the state space $\mathscr{E}$, an equilibrium is the combination of an allocation and a price system (ϑ, ρ) such that:

(i) given rates $\{w_f\}_{f\in\mathscr{F}}$ and prices $\{p_j\}_{j\in\mathscr{N}}$, monopolistically competitive firms' labor $\{\ell_{z_i f}\}_{f\in\mathscr{F}}$ and intermediate input demand $\{x_{z_i j}\}_{j\in\mathscr{N}}$, output y_{z_i}, and price p_{z_i} maximize their profits;

(ii) given prices $[p_{z_i}]_{z_i\in[0,1]}$, aggregator firms' good demand $[y_{z_i}]_{z_i\in[0,1]}$, and output y_i maximize their profits;

(iii) given prices $\{p_i\}_{i\in\mathscr{N}}$ and rates $\{w_f\}_{f\in\mathscr{F}}$, the representative household's consumption $\{C_i\}_{i\in\mathscr{N}}$ and factor supply $\{L_f\}_{f\in\mathscr{F}}$ maximize utility while satisfying their budget constraint;

(iv) goods and factor markets clear.

We will abstract from within sector firm heterogeneity by imposing the assumption of symmetry, i.e., $\ell_{ih} = \ell_{z_i h}$, and $x_{ij} = x_{z_i j}$ $\forall z_i \in [0,1]$, $\forall i, j \in \mathscr{N}$ and $\forall h \in \mathscr{H}$.[2] For this reason, I will refer indistinguishably to firm z_i as firm i.

Proposition 1. The set of functions (ϑ, ρ) are an equilibrium if and only if the following set of conditions are jointly satisfied $\forall e \in \mathscr{E}$

$$\frac{Y_{C_j}}{Y_{C_i}} = \mu_i \frac{\partial\, y_i}{\partial\, x_{ij}} \quad \forall i, j \in \mathscr{N}, \quad \text{such that} \quad C_i > 0,\ C_j > 0,\ \text{and}\ x_{ij} > 0, \tag{5}$$

$$-\frac{w_b}{w_f}\frac{U_L}{U_Y}\frac{L_{L_f}}{Y_{C_i}} = \mu_i \frac{\partial\, y_i}{\partial\, \ell_{ib}} \quad \forall i \in \mathscr{N},\ \forall b, f \in \mathscr{F}, \quad \text{such that} \quad C_i > 0,\ \text{and}\ \ell_{ib} > 0, \tag{6}$$

[1] $\mathcal{A} \equiv (\mathcal{A}_1, \cdots, \mathcal{A}_N)'$, $A_\ell = \left(A^\ell_1, \cdots, A^\ell_N\right)'$, $A_x \equiv (A^x_1, \cdots, A^x_N)'$, $\underline{A}_\ell = \left(\underline{A}^\ell_1, \cdots, \underline{A}^\ell_N\right)'$, $\underline{A}_x = (\underline{A}^x_1, \cdots, \underline{A}^x_N)'$, $\underline{A}^\ell_i = \left(A^\ell_{i1}, \cdots, A^\ell_{iH}\right)'$, and $\underline{A}^x_i = (A^x_{i1}, \cdots, A^x_{iN})'$.

[2] As a consequence $y_i = y_{z_i}$, $p_i = p_{z_i}$, $L_i = L_{z_i}$, and $X_i = X_{z_i}$.

where $Y_{C_i} = \partial Y / \partial C_i$, $L_{L_f} = \partial L / \partial L_f$, and resource constraints

$$\begin{aligned} \chi_i \, y_i &= C_i + \sum_{j \in \mathscr{N}} x_{ji} \qquad \forall i \in \mathscr{N}, \\ \text{and} \qquad L_h &= \sum_{i \in \mathscr{N}} \ell_{ih} \qquad \forall h \in \mathscr{H}, \end{aligned} \tag{7}$$

where $\chi_i = \mu_i + \phi_i \, (1 - \mu_i)$ represents the share of production that is not wasted.

Proposition 1 identifies the set of equilibrium allocations. In equation (5), the aggregate marginal rate of substitution between goods i and j has to equal the firm i's markdown-adjusted marginal productivity from using the good from sector j as an intermediate input. In equation (6), the firm i's markdown-adjusted marginal productivity from using factor b has to equal the aggregate price-adjusted marginal rate of substitution between the consumption of the good from sector i and the supply of factor f.

Notice that in the set of conditions captured by equation (6), the only thing that is necessary for the existence of an equilibrium relationship between the factor demand from firm i and the supply of factor f, is the consumption from the representative household of the good supplied by sector i. Whenever firm i rents factor b, and $b \neq h$, the differential price adjustement w_b / w_h arises in the equilibrium conditions. A higher w_b / w_h is isomoprhic to an increase in the marginal rates of susbtition between consumption C_i and factor supply L_h, and in equilibrium, it requires a higher marginal productivity in firm i of the supply of factor b. This price ration is a point of difference with Bigio & La'O (2020), where they only consider the endogenous supply by the representative household of one factor. Additionally, there is an isomorphism between distortionary markdown increases and positive productivity shocks in equations (5) and (6): both will increase the markdown-adjusted marginal productivities from labor and intermediate inputs.

3.2 Measures of Centrality

For the following measures, downstream or cost centrality refers to the propagation of costs from the supply of factors or intermediate inputs through supply chains, and upstream or revenue centrality refers to the propagation of money flows from the demand for labor and goods through payment chains. Table 1 summarizes the direct centralities and Table 2 the network centralities. The notation in this section will follow Rojas-Bernal (2023).

3.2.1 Direct Centralities

The vectors $\omega_\ell \equiv \left(\omega_1^\ell, \cdots, \omega_N^\ell\right)'$ and $\omega_x \equiv \left(\omega_1^x, \cdots, \omega_N^x\right)'$ portray the direct cost centralities from composites. Its elements $\omega_i^\ell \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_i^\ell} = \frac{p_i^\ell L_i}{c_i(\vartheta,\rho)}$ and $\omega_i^x \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_i^x} = \frac{p_i^x X_i}{c_i(\vartheta,\rho)}$ capture respectively firm i's cost elasticities to p_i^ℓ and p_i^x, and in equilibrium they equal the cost share of the factor and intermediate input composites. For this reason, $\omega_i^\ell + \omega_i^x = 1$.

Table 1: Direct Centralities

Matrix	***Definition***	***In Equilibrium***	***Properties***
ω_ℓ	$\omega_i^\ell \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_i^\ell}$	Cost share of L_i	$\omega_i^\ell + \omega_i^x = 1$
ω_x	$\omega_i^x \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_i^x}$	Cost share of X_i	
$S = p \circ \chi \circ y$	$S_i \equiv p_i\, \chi_i\, y_i$	Sales from sector i	$GDP = \sum_{i\in\mathscr{N}} \left(1 - \omega_i^x \frac{\mu_i}{\chi_i}\right) S_i$
$\widetilde{\Omega}_\ell$	$\widetilde{\Omega}_{if}^\ell \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log w_f}$	Cost share of ℓ_{if}	$\sum\limits_{f\in\mathscr{F}} \widetilde{\Omega}_{if}^\ell + \sum\limits_{j\in\mathscr{N}} \widetilde{\Omega}_{ij}^x = 1$
$\widetilde{\Omega}_x$	$\widetilde{\Omega}_{ij}^x \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_j}$	Cost share of x_{ij}	
$diag\,(\omega_\ell)\,\alpha = \widetilde{\Omega}_\ell$	$\alpha_{if} \equiv \frac{\partial \log p_i^\ell L_i}{\partial \log w_f}$	Cost share of ℓ_{if} in L_i	$\sum\limits_{f\in\mathscr{F}} \alpha_{if} = 1$
$diag\,(\omega_x)\,\mathscr{W} = \widetilde{\Omega}_x$	$\omega_{ij} \equiv \frac{\partial \log p_i^x X_i}{\partial \log p_j}$	Cost share of x_{ij} in X_i	$\sum\limits_{j\in\mathscr{N}} \omega_{ij} = 1$
β	$\beta_i \equiv \frac{\partial \log GDP}{\partial \log p_i}$	Cost share of C_i	$\sum\limits_{i\in\mathscr{N}} \beta_i = 1$
$diag\,(\chi)\,\Omega_\ell \equiv diag\,(\mu)\,\widetilde{\Omega}_\ell$	$\Omega_{if}^\ell \equiv \frac{\partial \log S_i}{\partial \log w_f}$	Share of S_i for ℓ_{if}	$\sum\limits_{f\in\mathscr{F}} \Omega_{if}^\ell + \sum\limits_{j\in\mathscr{N}} \Omega_{ij}^x + \Omega_i^\pi = 1$
$diag\,(\chi)\,\Omega_x \equiv diag\,(\mu)\,\widetilde{\Omega}_x$	$\Omega_{ij}^x \equiv \frac{\partial \log S_i}{\partial \log p_j}$	Share of S_i for x_{ij}	
$\Omega_\pi = diag\,(\mathbb{1}_N - \mu)\,\phi$	$\Omega_i^\pi = \frac{\pi_i}{S_i}$	Share of S_i for π_i	

The matrices $\widetilde{\Omega}_\ell$ and $\widetilde{\Omega}_x$ depict direct labor and intermediate input downstream centralities. Its elements $\widetilde{\Omega}_{if}^\ell \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log w_f} = \frac{w_f\, \ell_{if}}{c_i(\vartheta,\rho)}$ and $\widetilde{\Omega}_{ij}^x \equiv \frac{\partial \log c_i(\vartheta,\rho)}{\partial \log p_j} = \frac{p_j\, x_{ij}}{c_i(\vartheta,\rho)}$ capture respectively firm i's cost elasticities to w_f and p_j, and in equilibriym they equal the cost share of the factor f and the good from sector j. The fact that $\sum_{f\in\mathscr{F}} \widetilde{\Omega}_{if}^\ell + \sum_{j\in\mathscr{N}} \widetilde{\Omega}_{ij}^x = 1$ indicate that all costs come from factors or intermediate inputs.

Using these definitions, we obtain the factor network $\alpha \equiv diag\,(\omega_\ell)^{-1}\, \widetilde{\Omega}_\ell$ and the input-output network $\mathscr{W} \equiv diag\,(\omega_x)^{-1}\, \widetilde{\Omega}_x$, where $diag$ stands for the diagonal operator. Its elements $\alpha_{if} \equiv \frac{\partial \log p_i^\ell L_i}{\partial \log w_f} = \frac{w_f\, \ell_{if}}{p_i^\ell L_i}$ and $\omega_{ij} \equiv \frac{\partial \log p_i^x X_i}{\partial \log p_j} = \frac{p_j\, x_{ij}}{p_i^x X_i}$ capture respectively firm i's composite cost elasticities to w_f and p_j, and in equilibrium they equal the corresponding composites' cost share of the factor f and the good from sector j. Notice that $\sum_{f\in\mathscr{F}} \alpha_{if} = 1$ and $\sum_{j\in\mathscr{N}} \omega_{ij} = 1$.

From here, we can define the revenue-based upstream centrality matrices

$$\Omega_\ell \equiv diag\,(\mu)\, diag\,(\chi)^{-1}\, \widetilde{\Omega}_\ell \qquad \text{and} \qquad \Omega_x \equiv diag\,(\mu)\, diag\,(\chi)^{-1}\, \widetilde{\Omega}_x,$$

with $\chi = [\chi_1, \ldots, \chi_N]'$. Its elements $\Omega^{\ell}_{if} \equiv \frac{\partial \log S_i}{\partial \log w_f} = \frac{w_f\, \ell_{if}}{S_i}$ and $\Omega^{x}_{ij} \equiv \frac{\partial \log S_i}{\partial \log p_j} = \frac{p_j\, x_{ij}}{S_i}$ capture respectively the elasticities of firm i's sales $S_i = p_i\, \chi_i\, y_i$ to w_f and p_j, and in equilibrium they equal the sales share of payments for factor f and goods from firm j. Additionally, $\Omega^{\pi}_{i} = \frac{\pi_i}{S_i}$ portrays the equilibrium sales share of firm i's profits rebated back to households of type h. The fact that $\sum_{f\in\mathscr{F}} \Omega^{\ell}_{if} + \sum_{j\in\mathscr{N}} \Omega^{x}_{ij} + \Omega^{\pi}_{i} = 1$ indicate that all revenue generated by firm i ends as payments for factors, intermediate inputs, or dividends.

Finally, for the representative household, the consumption vector $\beta = (\beta_1, \cdots, \beta_N)'$ contains $\beta_i \equiv \frac{\partial \log GDP}{\partial \log p_i} = \frac{p_i\, C_i}{GDP}$ captures the GDP elasticity to p_i, and in equilibrium they equal the aggregate expenditure share on the good supplied by sector i. For this reason $\sum_{i\in\mathscr{N}} \beta_i = 1$.

3.2.2 Network Adjusted Centralities

The firm-to-firm downstream centrality matrix or cost-based Leontief inverse matrix is given by $\widetilde{\Psi}_x \equiv \left(I - \widetilde{\Omega}_x\right)^{-1} \equiv \sum_{q=0}^{\infty} \widetilde{\Omega}^{q}_{x}$. Its element $\widetilde{\psi}^{x}_{ij}$ captures the centrality of intermediate inputs supplied by firm j on the costs of firm i. Similarly, I define the firm-to-firm upstream centrality matrix or revenue-based Leontief inverse matrix $\Psi_x \equiv (I - \Omega_x)^{-1} \equiv \sum_{q=0}^{\infty} \Omega^{q}_{x}$, where its element ψ^{x}_{ij} represents the revenue share from firm i that through the payment of intermediate input reaches sales of firm j.

The cost-based sales Domar weights are given by $\widetilde{\lambda} = \widetilde{\Psi}'_x\, \beta$. Its element $\widetilde{\lambda}_i = \sum_{j\in\mathscr{N}} \beta_j\, \widetilde{\psi}^{x}_{ji}$ captures all direct and indirect paths through which the costs of firm i can reach the representative household's expenditure. This is equivalent to the share of aggregate value-added that passes through sector i. For this reason, $\omega^{\ell}_{i}\, \widetilde{\lambda}_i$ stands for the aggregate share of value-added that is extracted from factors by firm i, and $\sum_{i\in\mathscr{N}} \omega^{\ell}_{i}\, \widetilde{\lambda}_i = 1$. The summation of the cost-based Domar weights is the aggregate network multiplier $\xi = \sum_{i\in\mathscr{N}} \widetilde{\lambda}_i$. The revenue-based sales Domar weights are given by $\lambda = \Psi'_x\, \beta$. Its element $\lambda_i = \sum_{j\in\mathscr{N}} \beta_j\, \psi^{x}_{ji} = S_i/GDP$ stands for the share of aggregate expenditure that reaches revenue from firm i. These definitions generalize the supplier centrality vector from Baqaee (2018) and Baqaee & Farhi (2020), or the influence vector from Acemoglu et al. (2012), to an environment with wasted and rebated distortions.

The worker-to-firm downstream centrality matrix is given by $\widetilde{\Psi}_{\ell} \equiv \widetilde{\Psi}_x\, \widetilde{\Omega}_{\ell}$. Given that $\sum_{f\in\mathscr{F}} \widetilde{\psi}^{\ell}_{if} = 1$, all costs for a firm can be traced back through the production network to some original factor cost. As a consequence, $\widetilde{\psi}^{\ell}_{if}$ is the value-added share by factor f on the production process of firm i. In the same way, I define the firm-to-worker upstream centrality matrix $\Psi_{\ell} \equiv \Psi_x\, \Omega_{\ell}$, where the element ψ^{ℓ}_{if} represents the revenue share from firm i that reaches compensation for factor f. The payment centrality $\psi^{\ell}_{i} = \sum_{f\in\mathscr{F}} \psi^{\ell}_{if}$ captures the share of revenue from firm i that reaches factoral compensation.

Table 2: **Network Adjusted Centralities**

Matrix	*Definition in Equilibrium*	*Properties*
	Downstream or Cost-Based Centralities	
$\widetilde{\Psi}_x = \left(I - \widetilde{\Omega}_x\right)^{-1}$	$\widetilde{\psi}^x_{ij}$ *firm-to-firm* Centrality of j in the costs of i	
$\widetilde{\Psi}_\ell = \widetilde{\Psi}_x \, \widetilde{\Omega}_\ell$	$\widetilde{\psi}^\ell_{if}$ *factor-to-firm* Value-added share by h in the production of i	$\sum_{f \in \mathscr{F}} \widetilde{\psi}^\ell_{if} = 1$
$\widetilde{\lambda} = \widetilde{\Psi}'_x \, \beta$	$\widetilde{\lambda}_i$ *cost-based Domar weight* Share of aggregate value-added that passes through i	$\sum_{i \in \mathscr{N}} \omega^\ell_i \, \widetilde{\lambda}_i = 1$
$\widetilde{\Lambda} = \widetilde{\Psi}'_\ell \, \beta$	$\widetilde{\Lambda}_f$ *cost-based factor share* Share of aggregate value-added generated by f	$\sum_{f \in \mathscr{F}} \widetilde{\Lambda}_f = 1$
	Upstream or Revenue-Based Centralities	
$\Psi_x = (I - \Omega_x)^{-1}$	ψ^x_{ij} *firm-to-firm* Share of S_i that reaches S_j	
$\Psi_\ell = \Psi_x \, \Omega_\ell$	ψ^ℓ_{if} *firm-to-factor* Share of S_i that reaches J_f	
$\lambda = \Psi'_x \, \beta$	λ_i *revenue-based Domar weight* Aggregate sales share S_i/GDP	
$\Lambda = \Psi'_\ell \, \beta$	Λ_f *revenue-based factor share* Factor income share J_f/GDP	$\Gamma = \sum_{f \in \mathscr{F}} \Lambda_f \leq 1$
	Other Definitions	
$\xi = \mathbb{1}'_N \, \widetilde{\lambda}$	ξ *Network Multiplier*	$\xi = \sum_{i \in \mathscr{N}} \widetilde{\lambda}_i \geq 1$
$\psi_\ell = \Psi_\ell \, \mathbb{1}_F$	ψ^ℓ_i *payment centrality* Share of S_i that reaches Γ	$\psi^\ell_i = \sum_{f \in \mathscr{F}} \psi^\ell_{if}$

The cost-based factor Domar weights are given by $\widetilde{\Lambda} = \widetilde{\Psi}'_\ell \, \beta$. Its element $\widetilde{\Lambda}_f = \sum_{i \in \mathscr{N}} \beta_i \, \widetilde{\psi}^\ell_{if}$ captures all direct and indirect paths through which the cost of factor i can reach the representative household's expenditure. Consequently, $\widetilde{\Lambda}_f$ is the share of aggregate value-added by factor f. All the costs from this economy originate in factor, and for this reason, $\sum_{f \in \mathscr{F}} \widetilde{\Lambda}_f = 1$. The revenue-based factor Domar weights are given by $\Lambda = \Psi'_\ell \, \beta$. Its element $\Lambda_f = \sum_{i \in \mathscr{N}} \beta_i \, \psi^\ell_{if} = J_f/GDP$ stands for the share of aggregate expenditure that reaches compensation for factor f. The summation of the revenue-based Domar weights is the aggregate labor share $\Gamma = \sum_{f \in \mathscr{F}} \Lambda_f$.

3.3 Price Variation

Proposition 2 captures the network-adjusted response of prices to supply shocks. These shocks propagate downstream through the costs of intermediate inputs and final goods, and the cost-based firm-to-firm and firm-to-consumer centrality measures capture their magnitude.

Proposition 2. The change in sector i's prices, household h's price index, and country r's GDP deflator in response to productivity, markdown, and factor cost shocks are, to a first-order,

$$d\log p_i = -\sum_{j\in\mathcal{N}} \widetilde{\psi}^x_{ij}\, d\log A_j\, \mu_j + \sum_{f\in\mathscr{F}} \widetilde{\psi}^\ell_{if}\, d\log w_f,$$
$$d\log p_Y = -\sum_{i\in\mathcal{N}} \widetilde{\lambda}_i\, d\log A_i\, \mu_i + \sum_{f\in\mathscr{F}} \widetilde{\Lambda}_f\, d\log w_f,$$

where $d\log A_i = d\log \mathcal{A}_i + \omega^\ell_i\, d\log A^\ell_i + \omega^x_i\, d\log A^x_i + \sum_{h\in\mathscr{H}} \widetilde{\Omega}^\ell_{ih}\, d\log A^\ell_{ih} + \sum_{j\in\mathcal{N}} \widetilde{\Omega}^x_{ij}\, d\log A^x_{ij}$.

First, firm i's compound measure of productivity $d\log A_i$ incorporates Hicks-neutral, factor-specific, and input-specific augmenting productivity shocks, and its effect on prices across all firms and households is isomorphic to an increase in the markdown for firm i. Second, factor costs have a direct effect on the factor bundle price that propagates through the supply of intermediate inputs to other firms and finally reaches the representative household. Third, the GDP deflator for country depends negatively in productivity and markdown shocks, and positively on wages. The elasticities from these shocks on the GDP deflator are respectively equal to the cost-based sales and factor Domar weights.

4 Nonlinearities in Efficient Production Networks

In this section, we present the second-order for the aggregate efficiency wedge that already exist in the literature. We start with Hulten's (1978) theorem. This result characterizes the first-order variation for aggregate TFP around the efficient equilibrium. Then, we introduce the second-order approximation for TFP from Baqaee & Farhi (2019).

4.1 Hulten's Theorem

Theorem 1 characterizes real GDP in equilibrium and its first-order variation around the efficient equilibrium.

Theorem 1. Hulten's (1978). In equilibrium, real GDP satisfies

$$Y = Q\left(\{C_i\}_{i\in\mathcal{N}}\right) = TFP\; F\left(\{L_f\}_{f\in\mathscr{F}}\right), \tag{8}$$

where TFP captures total factor productivity and F satisfies $d\log F/d\log L_f = \Lambda_f$.

The change in Y and TFP are, to a first-order

$$d\,log\,Y = d\,log\,TFP + \sum_{f\in\mathscr{F}} \Lambda_f \, d\,log\,L_f, \tag{9}$$

$$d\,log\,TFP \approx \overbrace{\sum_{i\in\mathscr{N}} \lambda_i \, d\,log\,A_i}^{d\,log\ Technology}. \tag{10}$$

From equation (8), real GDP in equilibrium has two representations. First, as a CRS function Q that aggregates sectoral consumption. Second, as the product of TFP, and the CRS function F that aggregates factors with elasticities equal to the value-added weights $\widetilde{\Lambda}$. Equation (9) segments the output response into a TFP and a factoral component. Equation (10) is Hulten's theorem, i.e., the first-order variation for TFP corresponds to the Domar weighted variation of productivity shocks. The implication from this result is that the sales distribution is a sufficient statistic for the aggregate efficiency wedge variation. Consequently, the network details are unnecessary to gauge the aggregate effects from microeoconomic shocks.

In its original version, Hulten (1978) assumed an inelastic factor supply. Under this assumption, this theorem is a macroeconomic envelope condition for the production possibility frontier. Theorem 1 shows that once factors are allowed to be elastic, Hulten's result still holds. However, now the theorem characterizes a macroeconomic envelope condition for the aggregate efficiency wedge. As noticed by Bigio & La'O (2020), what is trully surprising about Hulten's theorem is that its local implications hold even for distortionary shocks that drive the economy away from efficiency, i.e., shocks in μ, ϕ, χ.

4.2 Beyond Hulten's Theorem

For an input-output economy without distortions and inelastic factors, Baqaee & Farhi (2019) estimate the second-order approximation for real GDP. In this section, we are going to characterize the second-order approximation for TFP in a production network economy with elastic factors. For these approximation we require general elasticities of substitution.

The Morishima (1967) elasticity of substitution

$$\frac{1}{\rho^M_{ji}} = \frac{\partial\,log\,\left(\frac{\partial GDP/\partial p_i}{\partial GDP/\partial p_j}\right)}{\partial\,log\,(p_j/p_i)} = \frac{\partial\,log\,(C_i/C_j)}{\partial\,log\,(p_j/p_i)} = \frac{\partial\,log\,(C_j/C_i)}{\partial\,log\,p_i}.$$

This elasticity measures the variation in the representative household's demand ratios C_i/C_j with respect to changes for corresponding input price ratios p_j/p_i, holding output constant, allowing only p_i variations, and letting all other quantities to adjust optimally (Blackorby &

Russell, 1989).[3] The assumption of constant output implies that this is a net elasticity. However, when the output is allowed to adjust, then a gross elasticity is the appropriate measure. Net an gross elasticities are equivalent when the aggregator Q is CRS (Karney, 2016). Goods i and j are *Morishima complements* if $\rho^M_{ji} < 0$ and *Morishima substitutes* if $\rho^M_{ji} > 0$. For example, in response to a p_i increase, goods i and j are *Morishima complements* if C_j/C_i falls, and *Morishima substitutes* if C_j/C_i increases.

Baqaee & Farhi (2019) introduce an analogous *pseudo elasticity of substitution*

$$\frac{1}{\rho^{BF}_{ji}} = \frac{\partial \log \left(\frac{\partial Y/\partial A_i}{\partial Y/\partial A_j}\right)}{\partial \log \left(A_j/A_i\right)} = \frac{\partial \log \left(Y_{A_i}/Y_{A_j}\right)}{\partial \log \left(A_j/A_i\right)} = \frac{\partial \log \left(Y_{A_j}/Y_{A_i}\right)}{\partial \log A_i}.$$

This elasticity measures the variation in the marginal productivity ratios Y_{A_i}/Y_{A_j} on aggregate output with respect to changes in corresponding productivity ratios A_j/A_i, allowing only A_i variations, and letting all other quantities adjust optimally.

Here we introduce the *technological pseudo elasticities of substitution*

$$\frac{1}{\rho_{ji}} = \frac{\partial \log \left(\frac{\partial \, Technology/\partial A_i}{\partial \, Technology/\partial A_j}\right)}{\partial \log \left(A_j/A_i\right)} = \frac{\partial \log \left(T_{A_i}/T_{A_j}\right)}{\partial \log \left(A_j/A_i\right)} = \frac{\partial \log \left(T_{A_j}/T_{A_i}\right)}{\partial \log A_i}.$$

This elasticity measures the variation in the marginal productivity ratios on technology T_{A_i}/T_{A_j} with respect to changes in corresponding productivity ratios A_j/A_i, allowing only A_i variations, and letting all other quantities adjust optimally. Under the Baqaee & Farhi (2019) assumptions of no distortions and an inelastic factor suppy $\rho_{ji} = \rho^{BF}_{ji}$.

This elasticity of substitution allows us to characterize changes in the relative shares of value-added that pass through sector j and i in response to productivity shocks A_i

$$\begin{aligned}\frac{\partial \log \left(\widetilde{\lambda}_i/\widetilde{\lambda}_j\right)}{\partial \log A_i} &= \frac{\partial \log \left(\left(A_i T_{A_i}\right)/\left(A_j T_{A_j}\right)\right)}{\partial \log A_i} \\ &= 1 - \frac{\partial \log \left(T_{A_j}/T_{A_i}\right)}{\partial \log A_i} = 1 - \frac{1}{\rho_{ji}}.\end{aligned}$$

In the absence of rebated distortions (i.e., $\phi_i = 0 \ \forall i \in \mathcal{N}$), revenue- and cost-based Domar weights are the same ($\lambda = \widetilde{\lambda}$ and $\Lambda = \widetilde{\Lambda}$), and ρ_{ij} allows us to characterize changes in the relative sales shares of sectors j and i. We will call goods i and j *technological complements* if $\rho_{ji} \in (0, 1)$ and *technological substitutes* if $\rho_{ji} < 0$ or $\rho_{ji} > 1$.

Theorem 2. Baqaee & Farhi's (2019). The second-order impact of idiosyncratic produc-

[3] $\frac{\partial GDP}{\partial p_i} = C_i$ due to Shepard's Lemma.

tivity shocks on TFP for an economy without distortions are given by[4]

$$\frac{d^2 \log TFP}{d \log A_j \, d \log A_i} = \frac{d \lambda_i}{d \log A_j} = \lambda_i \sum_{m \in \mathcal{N}} \lambda_m \frac{d \omega_m^x}{d \log A_j} + \mathbb{1}\{i = j\} \lambda_i \sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \omega_m^\ell \lambda_m \left(1 - \frac{1}{\rho_{mi}}\right)$$
$$+ \mathbb{1}\{i \neq j\} \lambda_i \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \lambda_m \left(\frac{1}{\rho_{ij}} - \frac{1}{\rho_{mj}}\right) + \omega_i^x \lambda_j \left(1 - \frac{1}{\rho_{ij}}\right) \right). \tag{11}$$

Therefore

$$d \log TFP \approx \sum_i \lambda_i \, d \log A_i + \frac{1}{2} \sum_{i \in \mathcal{N}} \lambda_i \left(\sum_{m \in \mathcal{N}} \lambda_m \frac{d \omega_m^x}{d \log A_i} + \sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \omega_m^\ell \lambda_m \left(1 - \frac{1}{\rho_{mi}}\right) \right) d \log A_i^2$$
$$+ \frac{1}{2} \sum_{i \in \mathcal{N}} \lambda_i \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \left(\sum_{m \in \mathcal{N}} \lambda_m \frac{d \omega_m^x}{d \log A_j} + \sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \lambda_m \left(\frac{1}{\rho_{ij}} - \frac{1}{\rho_{mj}}\right) + \omega_i^x \lambda_j \left(1 - \frac{1}{\rho_{ij}}\right) \right) d \log A_i \, d \log A_j.$$

Theorem 2 characterizes the second-order effect on TFP from firm level productivity shocks. The sales shares λ, the technological pseudo elasticities, and the semi elasticities of the intermediate input cost intensities are sufficient statistics for the aggregate efficiency wedge variation to productivity shocks.

For a Cobb-Douglas economy, the *technological pseudo elasticities of substitution* are unitary, the intermediate input cost intentity distribution ω_x is fixed, and the first-order approximation from Theorem 1 is globally accurate.

[4] To be precise, in Baqaee & Farhi (2019)

$$\frac{d^2 \log TFP}{d \log A_j \, d \log A_i} = \frac{d \lambda_i}{d \log A_j} = \mathbb{1}\{i = j\} \frac{\lambda_i}{\xi} \sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \lambda_m \left(1 - \frac{1}{\rho_{mi}}\right) + \lambda_i \frac{d \log \xi}{d \log A_i}$$
$$+ \mathbb{1}\{i \neq j\} \lambda_i \left(\frac{\lambda_i}{\xi} \sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \lambda_m \left(\frac{1}{\rho_{ij}} - \frac{1}{\rho_{mj}}\right) - \frac{\lambda_i \lambda_j}{\xi} \left(1 - \frac{1}{\rho_{ij}}\right) \right).$$

This is because their proof starts from the condition $\xi = \sum_{i \in \mathcal{N}} \lambda_i$, while the proof from Theorem 2 utilizes the $1 = \sum_{i \in \mathcal{N}} \omega_i^\ell \lambda_i$ condition. While in their paper it is necessary to track the input-output multiplier, in equation (11), we track the distribution of intermediate input cost intensities ω_x.

5 Nonlinearities with Distortions

In this section, we present the second-order effects for the aggregate efficiency wedge in an input-output economy with distortions. We start by introducing the first-order approximation for TFP in an environment with wasted and rebated distortions. Then we proceed to show the second order effects with fully wasted distortions and fully rebated distortion. This section constitutes the main contribution from this paper.

5.1 Aggregate Efficiency Wedge with Inelastic Factoral Supply

Theorem 3. Baqaee & Farhi (2020). The change in Y and TFP are, to a first-order

$$d\log Y \approx \overbrace{\sum_{i\in\mathscr{N}} \widetilde{\lambda}_i \, d\log A_i}^{d\log\, Technology} + \overbrace{\sum_{i\in\mathscr{N}} \widetilde{\lambda}_i \, d\log \mu_i}^{d\log\, Competitiveness} - \overbrace{\sum_{f\in\mathscr{F}} \widetilde{\Lambda}_f \, d\log \Lambda_f}^{d\log\, Distribution} . \tag{12}$$

Theorem 3 characterizes the first-order variation for real GDP around a distorted equilibrium when the supply of factors is inelastic. This result comes from accounting identities, and normalization relative to the price of a numeraire is unnecessary. This lack of normalization is a point of difference with the comparable theorem from Baqaee & Farhi (2020), who instead assume a fixed nominal GDP. Using nominal GDP as the numeraire creates uncertainty about the fundamental real unit of account, as real GDP will no longer be neutral to pure nominal variations, e.g., Y has to increase as P_Y falls. Rojas-Bernal (2023) shos that the nominal GDP normalization used by Baqaee & Farhi (2020, 2023) is non-neutral on TFP whenever the substitution and income effects on the labor supply are asymmetric.

Equation (12) divides the first-order variation of TFP into three components. First, *technology* captures the direct effect of changes in productivity under a fixed allocation of resources. Second, *competitiveness* portrays the reallocation effects from distortions assuming that there are no variations in the factoral income distribution. These two components tell us that in the absence of distributional reallocation, the effects on TFP of productivity and markdown changes in sector i are proportional to its cost-based sales Domar weight $\widetilde{\lambda}_i$. Third, *distribution* portrays the aggregate efficiency losses from reallocating inputs in response to variations in the factoral income distribution. The last two components capture the effects on TFP from the reallocation of factos and intermediate inputs across firms arising from exogenous variations in distortions and endogenous changes in the factoral income shares. Baqaee & Farhi (2020) label the last two components as the variation in *allocative efficiency*.

5.2 Aggregate Efficiency Wedge with Elastic Factoral Supply

Theorem 4 characterizes the equilibrium factoral supply. This theorem represents an extension of the labor wedge decompositions from Bigio & La'O (2020) to an environment with multiple factors and a distorted equilibrium. For the supply of factor f, the factor wedge Γ_f gauges how the whole set of economic distortions influences its supply decision.

Theorem 4. Factor Income. In equilibrium, the supply of factor f satisfies

$$\frac{U_L}{U_C} + \Gamma_f \frac{Y}{L_f} = 0 \qquad \text{with} \qquad \Gamma_f = \frac{\Lambda_f}{\widetilde{\Lambda}_f}. \tag{13}$$

The variation of Λ_h in response any shock is given by

$$\begin{aligned} d\,\Lambda_h = & \overbrace{\sum_{i \in \mathcal{N}} \psi^{\ell}_{if}\, d\,\beta_i}^{\substack{\textit{Final Demand} \\ \textit{Recomposition}_f}} + \overbrace{\sum_{i \in \mathcal{N}} \psi^{\ell}_{if} \sum_{j \in \mathcal{N}} \mu_j\, \lambda_j\, d\,\widetilde{\Omega}^{x}_{ji}}^{\substack{\textit{Intermediate Demand} \\ \textit{Recomposition}_f}} + \overbrace{\sum_{i \in \mathcal{N}} \mu_i\, \lambda_i\, d\,\widetilde{\Omega}^{\ell}_{if}}^{\substack{\textit{Factoral Demand} \\ \textit{Recomposition}_f}} \\ & + \underbrace{\sum_{i \in \mathcal{N}} \psi^{\ell}_{if}\, \lambda_i\, \frac{\phi_i}{\chi_i}\, d\log \mu_i}_{\textit{Competitive Income}_f} - \underbrace{\sum_{i \in \mathcal{N}} \psi^{\ell}_{if}\, (1 - \mu_i)\, \lambda_i\, \frac{\phi_i}{\chi_i}\, d\log \phi_i}_{\textit{Rebated Income}_f}. \end{aligned} \tag{14}$$

The factoral wedge Γ_h from equation (13) relates the aggregate marginal rate of substitution between Y and L with the factor's average rate of transformation on real GDP, i.e., Y/L_f. In equilibrium, the factoral wedge equals the share of income to value-added, i.e., $\Gamma_f = \Lambda_f/\widetilde{\Lambda}_f$. In Rojas-Bernal (2023) the inverse of this ratio is called *distortion centrality*. A factor is overvalued when $\Gamma_f > 1$ and undervalued when $\Gamma_f < 1$. For economies without distortions or with fully wasted distortions $\Gamma_f = 1$.

Equation (14) segments the first-order variation of the factoral income shares into five channels. The *final* and *intermediate demand recomposition* characterize the factoral income distribution effects from the reallocation of the household's expenditure on final goods, and firms's expenditure on intermediate inputs. The income share for factor f increases as the household's consumption patterns or the firms' cost structure shifts towards sectors with a high firm-to-factor centrality on L_f. For example, Λ_f rises in response to a cost reallocation from good j to good i, by households or firms, if $\psi^{\ell}_{if} > \psi^{\ell}_{jf}$. The *factoral demand recomposition* portrays the influence on the labor income share from higher factoral demand; the magnitude of this effect is more prominent for big and relatively undistorted sectors. The *competitive income* tells us that lower profit margins in a sector will increase the income share for factor f in a magnitude proportional to the sector's size, its centrality on the income for this factor, and the ratio of rebated to wasted distortions. Finally, *rebated income* captures how the factor income shares fall in response to higher shares of profits rebated back to households. This latter effect is proportional to the sector's size, profit margin, its centrality on the income for the factor, and the ratio of rebated to wasted distortions. Collectivelly, the sales shares λ, the markdowns μ, the rebated shares ϕ, the wasted ratios χ, the firm-to factor centrality matrix Ψ_ℓ, and the changes for the expenditure shares β, $\widetilde{\Omega}_\ell$ and $\widetilde{\Omega}_x$, are sufficient statistics for the factoral income distribution variations.

Theorem 5 describes the equilibrium composition for the aggregate factoral supply. For L, the aggregate factor wedge Γ gauges how the whole set of economic distortions influences its supply decision.

Theorem 5. Aggregate Factor Share. In equilibrium, the aggregate factoral supply satisfies

$$\frac{U_L}{U_C} + \Gamma\,\frac{Y}{L} = 0 \qquad \text{with} \qquad \Gamma = \sum_{f\in\mathscr{F}} \Lambda_f \tag{15}$$

The composition of the aggregate factoral supply needs to satisfy

$$\Gamma = \Gamma_f = \frac{\Lambda_f}{\widetilde{\Lambda}_f} \qquad \forall f \in \mathscr{F}. \tag{16}$$

The variation of the aggregate factor share Γ in response any shock is given by

$$\begin{aligned} d\,\Gamma = {} & \overbrace{\sum_{i\in\mathscr{N}} \psi_i^\ell \, d\,\beta_i}^{\substack{\text{Final Demand}\\ \text{Recomposition}}} + \overbrace{\sum_{i\in\mathscr{N}} \psi_i^\ell \sum_{j\in\mathscr{N}} \mu_j\,\lambda_j\,d\,\widetilde{\Omega}_{ji}^x}^{\substack{\text{Intermediate Demand}\\ \text{Recomposition}}} + \overbrace{\sum_{i\in\mathscr{N}} \mu_i\,\lambda_i\,d\,\omega_i^\ell}^{\substack{\text{Factoral Demand}\\ \text{Recomposition}}} \\ & \underbrace{+ \sum_{i\in\mathscr{N}} \psi_i^\ell\,\lambda_i\,\frac{\phi_i}{\chi_i}\,d\log\mu_i}_{\text{Competitive Income}} \underbrace{- \sum_{i\in\mathscr{N}} \psi_i^\ell\,(1-\mu_i)\,\lambda_i\,\frac{\phi_i}{\chi_i}\,d\log\phi_i}_{\text{Rebated Income}}\,. \end{aligned} \tag{17}$$

Equation (15) characterizes the aggregate factor supply and its wedge. The aggregate factor wedge Γ relates the aggregate marginal rate of substitution with the economy's marginal rate of transformation Y/L. This wedge equals the aggregate factor income share. Equation (16) tells us that in equilibrium, all factors will have a symmetric factoral wedge, i.e., all factors will be equally undervalued or overvalued. This implies that, conditional on the distortions μ, ϕ, and χ, this is the point at which the composition of L is optimal. Consequently, combinations of factors that violate this symmetry in factoral wedges will be inefficient from the representative household's perspective.

Equation (17) segments the variation for the aggregate factor income share into five sources analogous to the channels in Theorem 4. The difference now is that the vector of sectoral payment centralities $\psi_\ell = \left(\psi_1^\ell, \ldots, \psi_N^\ell\right)'$ replaces the matrix of firm-to-factor centralities Ψ_ℓ as a sufficient statistic.

Furthermore, in the absence of distortions (i.e., $\mu = \phi = \chi = \mathbb{1}_N$), the effect from markdowns on the factoral income share is sufficiently captured by the Domar weights, i.e., $\frac{d\,log\,\Gamma}{d\,log\,\mu_i} = \lambda_i$. This local variation is the main result from Bigio & La'O (2020), and Theorem 4 and Theorem 5 capture the extension from their findings to a distorted equilibrium with multiple factors.

Corollary 1. *dlogTFP* with elastic factor supply. With elastic factor supply

$$d\,log\,Distribution = \sum_{f\in\mathscr{F}} \widetilde{\Lambda}_f\, d\,log\,\Lambda_f = \sum_{f\in\mathscr{F}} \frac{\widetilde{\Lambda}_f}{\Lambda_f}\, d\,\Lambda_f = \frac{1}{\Gamma}\sum_{f\in\mathscr{F}} d\,\Lambda_f = d\,log\,\Gamma. \qquad (18)$$

The variation in Y and TFP are, to a first-order

$$d\,log\,Y = d\,log\,TFP + \sum_{f\in\mathscr{F}} \widetilde{\Lambda}_f\, d\,log\,L_f,$$

$$d\,log\,TFP \approx \overbrace{\sum_{i\in\mathscr{N}} \widetilde{\lambda}_i\, d\,log\,A_i}^{d\,log\ Technology} + \overbrace{\sum_{i\in\mathscr{N}} \widetilde{\lambda}_i\, d\,log\,\mu_i}^{d\,log\ Competitiveness} - \ d\,log\,\Gamma.$$

Corollary 1 introduces in Theorem 3 the results from Theorem 5. Equation (18) establishes that under endogenous labor supply, there is a tight connection between the TFP decomposition from Baqaee & Farhi (2020) and the first-order variation of the aggregate factor income share. For a representative household economy with endogenous factor supply, the TFP decomposition is simplified, as the misallocation driven by distributional variations correspond to the aggregate factoral income share growth. It is no longer necessary to trace the dynamics for every component in the distribution $\{\Lambda_f\}_{f\in\mathscr{F}}$, but only for the aggregate factor wedge Γ. Furthermore, this result associates in a single equation the two equilibrium objects that, according to Chari et al. (2007), account for the bulk of business cycle fluctuations.

5.3 Nonlinearities with Wasted Distortions

Fully wasted distortion can be seen in the literature from forgone risk premium paid by firms in Liu to iceberg cost in the trade literature. When distortions are fully wasted (i.e., $\phi_i = 0$), the markdown and the share of non-wasted production are symmetric, i.e., $\mu_i = \chi_i$. Wasted distortions do not dilute revenue as consumption expenditure flows upstream in a production network. These distortions are isomorphic to a productivity shock, e.g., firm i requires χ_i^{-1} times the amount of inputs to produce one unit of y_i relative to the case with no wasted distortions. Consequently, all income is factoral income ($\Gamma = 1$), the value-added by a factor corresponds to its revenue ($\widetilde{\Lambda} = \Lambda$), and the value-added that passes through a sector equals the sector's revenue ($\widetilde{\lambda} = \lambda$). However, the allocation of factors and intermediate inputs is not the same as in the undistorted equilibrium. For this reason, there is a misallocation relative to efficient equilibrium.

Corollary 2. $d\,log\,TFP$ with wasted distortions.

$$d\,log\,TFP \approx \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i\, d\,log\,A_i + \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i\, d\,log\,\mu_i.$$

Corollary 2 represents the first-order variation for TFP in an economy with fully wasted distortions. Productivity and markdown variations have an isomorphic effect on TFP with elasticities equal to the cost-based Domar weights $\widetilde{\lambda}$.

The *markdown pseudo elasticities of substitution* are given by

$$\frac{1}{\vartheta_{ji}} = \frac{\partial\, log\left(\frac{\partial\, Technology/\partial\, A_i}{\partial\, Technology/\partial\, A_j}\right)}{\partial\, log\,\left(\mu_j/\mu_i\right)} = \frac{\partial\, log\,\left(T_{A_i}/T_{A_j}\right)}{\partial\, log\,\left(\mu_j/\mu_i\right)} = \frac{\partial\, log\,\left(T_{A_j}/T_{A_i}\right)}{\partial\, log\,\mu_i}.$$

This elasticity measures the variation in the marginal productivity ratios on technology T_{A_i}/T_{A_j} with respect to changes in corresponding markdown ratios μ_j/μ_i, allowing only μ_i variations, and letting all other quantities adjust optimally.

This elasticity of substitution allows us to characterize changes in the relative shares of value-added that pass through sector j and i in response to markdown shocks μ_i

$$\frac{\partial\, log\,\left(\widetilde{\lambda}_i/\widetilde{\lambda}_j\right)}{\partial\, log\,\mu_i} = \frac{\partial\, log\,\left(\left(A_i\, T_{A_i}\right)/\left(A_j\, T_{A_j}\right)\right)}{\partial\, log\,\mu_i} = \frac{\partial\, log\,\left(T_{A_i}/T_{A_j}\right)}{\partial\, log\,\mu_i} = -\frac{1}{\vartheta_{ji}}.$$

We will call goods i and j *markdown complements* if $\vartheta_{ji} > 0$ and *markdown substitutes* if $\vartheta_{ji} < 0$.

Theorem 6. Second-Order TFP impact of Microeconomic Shocks. The second-order impact of idiosyncratic productivity shocks on TFP for an economy with wasted distortions are given by

$$\begin{aligned}\frac{d^2\, log\,TFP}{d\,log\,A_j\; d\,log\,A_i} = \frac{d^2\, log\,TFP}{d\,log\,A_j\; d\,log\,\mu_i} &= \frac{d\,\widetilde{\lambda}_i}{d\,log\,A_j} \\ &= \widetilde{\lambda}_i \sum_{m \in \mathcal{N}} \widetilde{\lambda}_m \frac{d\,\omega^x_m}{d\,log\,A_j} + \mathbb{1}\left\{i = j\right\} \widetilde{\lambda}_i \sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \omega^{\ell}_m\, \widetilde{\lambda}_m \left(1 - \frac{1}{\rho_{mi}}\right) \\ &+ \mathbb{1}\left\{i \neq j\right\} \widetilde{\lambda}_i \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega^{\ell}_m\, \widetilde{\lambda}_m \left(\frac{1}{\rho_{ij}} - \frac{1}{\rho_{mj}}\right) + \omega^x_i\, \widetilde{\lambda}_j \left(1 - \frac{1}{\rho_{ij}}\right)\right),\end{aligned}$$

$$
\begin{aligned}
\frac{d^2 \log TFP}{d \log \mu_j \, d \log A_i} = \frac{d^2 \log TFP}{d \log \mu_j \, d \log \mu_i} &= \frac{d\,\widetilde{\lambda}_i}{d \log \mu_j} \\
&= \widetilde{\lambda}_i \sum_{m \in \mathcal{N}} \widetilde{\lambda}_m \frac{d\,\omega_m^x}{d \log \mu_j} - \mathbb{1}\{i = j\}\, \widetilde{\lambda}_i \sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \omega_m^\ell \frac{\widetilde{\lambda}_m}{\vartheta_{mi}} \\
&\quad - \mathbb{1}\{i \neq j\}\, \widetilde{\lambda}_i \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \widetilde{\lambda}_m \left(\frac{1}{\vartheta_{ij}} - \frac{1}{\vartheta_{mj}} \right) + \omega_i^x \frac{\widetilde{\lambda}_j}{\vartheta_{ij}} \right),
\end{aligned}
$$

Therefore

$$
\begin{aligned}
d \log TFP &\approx \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \, d \log A_i + \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \, d \log \mu_i \\
&+ \frac{1}{2} \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \omega_m^\ell \widetilde{\lambda}_m \left(1 - \frac{1}{\rho_{mi}} \right) + \sum_{m \in \mathcal{N}} \omega_m^x \widetilde{\lambda}_m \frac{d \log \omega_m^x}{d \log A_i} \right) (d \log A_i)^2 \\
&+ \frac{1}{2} \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \widetilde{\lambda}_m \left(\frac{1}{\rho_{ij}} - \frac{1}{\rho_{mj}} \right) - \omega_j^\ell \widetilde{\lambda}_j \left(1 - \frac{1}{\rho_{ij}} \right) + \sum_{m \in \mathcal{N}} \omega_m^x \widetilde{\lambda}_m \frac{d \log \omega_m^x}{d \log A_j} \right) d \log A_i \, d \log A_j \\
&- \frac{1}{2} \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i}} \frac{\omega_m^\ell \widetilde{\lambda}_m}{\vartheta_{mi}} + \sum_{m \in \mathcal{N}} \omega_m^\ell \widetilde{\lambda}_m \frac{d \log \omega_m^\ell}{d \log \mu_j} \right) (d \log \mu_i)^2 \\
&- \frac{1}{2} \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \widetilde{\lambda}_m \left(\frac{1}{\vartheta_{mj}} - \frac{1}{\vartheta_{ij}} \right) - \frac{\omega_j^\ell \widetilde{\lambda}_j}{\vartheta_{ij}} + \sum_{m \in \mathcal{N}} \omega_m^\ell \widetilde{\lambda}_m \frac{d \log \omega_m^\ell}{d \log \mu_j} \right) d \log \mu_i \, d \log \mu_j \\
&+ \sum_{i \in \mathcal{N}} \widetilde{\lambda}_i \sum_{j \in \mathcal{N}} \left(\sum_{\substack{m \in \mathcal{N} \\ m \neq i,j}} \omega_m^\ell \widetilde{\lambda}_m \left(\frac{1}{\rho_{ij}} - \frac{1}{\rho_{mj}} \right) - \omega_j^\ell \widetilde{\lambda}_j \left(1 - \frac{1}{\rho_{ij}} \right) + \sum_{m \in \mathcal{N}} \omega_m^x \widetilde{\lambda}_m \frac{d \log \omega_m^x}{d \log A_j} \right) d \log \mu_i \, d \log A_j.
\end{aligned}
$$

Theorem 6 characterizes the second-order effect on TFP from firm level productivity and markdown shocks when distortions are fully wasted. The cost-based Domar weights $\widetilde{\lambda}$, the technological and markdown pseudo elasticities, and the semi elasticities of the intermediate input cost intensities are sufficient statistics for the aggregate efficiency wedge variation to productivity and markdown shocks.

6 Illustrative Examples

To illustrate the impact of Theorem 6 on the approximation of TFP responses to sectoral productivity and markdown shocks, we analyze a simple one-factor, two-sector economy with inelastic labor supply. In this model, sector i's production function is defined as:

$$y_i = A_i \left(\omega_\ell \, L_i^{\frac{\gamma-1}{\gamma}} + \omega_x \, x_{ij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \quad \omega_\ell + \omega_x = 1$$

Here, γ represents the elasticity of substitution between labor and intermediate inputs. Firms in each sector purchase all their intermediate inputs from the other sector, leading to the market clearing condition for sectoral good i:

$$y_i = C_i + x_{ji,j\neq i}$$

Aggregate consumption is a CES aggregation of sectoral goods produced for households:

$$C = \left(\omega_{c1} \, C_1^{\frac{\theta-1}{\theta}} + \omega_{c2} \, C_2^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \omega_{c1} + \omega_{c2} = 1$$

where θ denotes the elasticity of substitution between different final goods. Figure 1 is a visual representation of this model. It is important to note that the model exhibits symmetry in markdown, productivity, elasticities, β, ω_c, and ω_l at the steady state. To simulate the effects of a sectoral shock to sector 1, please see Figure 2. This figure illustrates the actual real GDP response, along with the first-order and second-order responses based on Theorem 6, under varying levels of productivity shocks and assuming full labor reallocation.

Figure 1: Two-sector economy with one inelastic factor

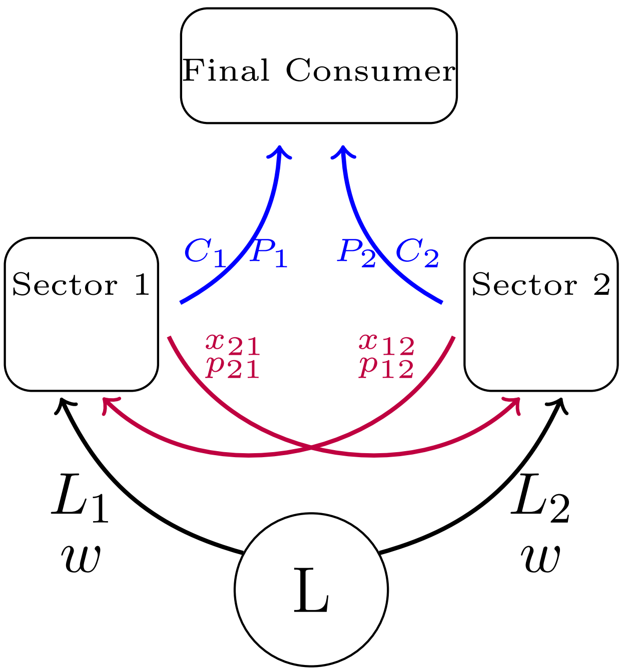


Figure 2A examines a symmetric economy characterized by high substitutability between labor and intermediate inputs, as well as between final goods. A positive sectoral shock to sector 1 leads to labor reallocation to this sector, resulting in a significant increase in the production of final goods. High substitutability implies that the ratios $\frac{p_1}{p_2}$ and $\frac{\widetilde{\lambda}_1}{\widetilde{\lambda}_2}$ decrease substantially, causing a large General Equilibrium (GE) elasticity of ρ_{12}. Consequently, the first-order approximation becomes less accurate, and the second-order term gains prominence.

The high elasticity of substitution allows the economy to respond significantly to positive sectoral shocks. Conversely, when sector 1 faces a negative shock, resources shift towards sector 2, mitigating the impact of this adverse shock. In the presence of severe productivity shocks, the first-order approximation, which matches the economy's response to a Cobb-Douglas economy where $\theta = \gamma = 1$, fails to capture the real GDP response. Instead, the second-order approximation performs much better because $\widetilde{\lambda}_1$ increases significantly, while $\widetilde{\lambda}_2$ decreases considerably, resulting in a substantial GE elasticity of ρ_{12}, thereby making the second-order term crucial.

Moving to Figure 2B, we explore a symmetric economy characterized by high complementarity between labor and intermediate inputs, as well as between final goods. Due to this high complementarity, the economy allocates more resources to the sector with relatively lower productivity when faced with sectoral shocks. In such cases, $\widetilde{\lambda}_i$ and $\widetilde{\lambda}_j$ respond similarly to shocks, resulting in $\frac{\partial \log(\widetilde{\lambda}_i/\widetilde{\lambda}_j)}{\partial \log A_i}$ remaining close to zero, and the second-order effect becomes less significant.

Figure 3 illustrates the economy's response, along with the first-order and second-order responses to sectoral shocks when there is no labor reallocation. In this scenario, firms can only choose their intermediate inputs. With high substitutability between labor and intermediate inputs and between final goods, the economy cannot fully benefit from positive sectoral shocks or mitigate the impact of negative shocks. Consequently, $\widetilde{\lambda}_1$ decreases slightly, while $\widetilde{\lambda}_2$ increases slightly, leading to a small GE elasticity of ρ_{12}, thereby diminishing the significance of the second-order term. Conversely, in the presence of high complementarity, the sectoral responses to a sectoral shock differ. $\widetilde{\lambda}_1$ increases more, while $\widetilde{\lambda}_2$ increases less in the opposite direction of sectoral shocks, resulting in a relatively larger GE elasticity of ρ_{12} compared to full labor allocation in Figure 2, making the second-order effect more critical.

Figure 4 presents the actual real GDP response, along with the first-order and second-order responses, based on Theorem 6, for varying levels of distortion shocks, assuming full labor reallocation. At the steady state, μ_1 and μ_2 equal 0.9. The distortion represents the inefficiency in resource allocation, with $1 - \mu$ of sectoral output being wasted. This can be seen as the forgone risk premium firms pay in Liu (2019) or iceberg trade costs. Sector 1 experiences a distortion shock, causing μ_1 to deviate from its steady-state value.

In Figure 4A, we focus on a symmetric economy characterized by high substitutability between labor and intermediate inputs and between final goods. A high elasticity of substitution enables the economy to benefit significantly from the reduction in sectoral distortion. When sector 1 experiences a reduction in distortion (higher markdown), resources shift towards sector 1, leading to a substantial increase in the production of final goods compared to the steady state. In severe distortion shocks, the first-order approximation matching with the unitary elastic economy ($\theta = \gamma = 1$) response fails to accurately capture the real GDP response. Instead, the second-order approximation performs much better because $\widetilde{\lambda}_1$ decreases significantly, while $\widetilde{\lambda}_2$ increases significantly, resulting in a significant GE elasticity of ν_{12}. Consequently, the second-order effect becomes crucial.

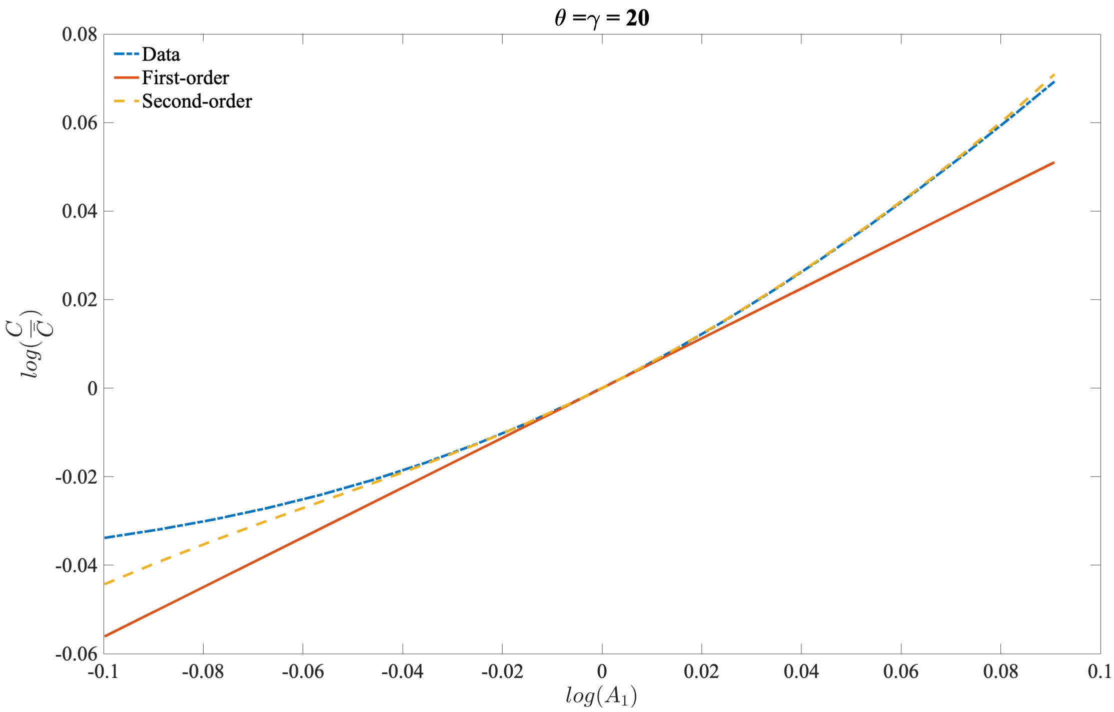
Notably, Baqaee & Farhi (2020) only captures the first-order effect of distortion shocks, making it a weak approximation for the economy's response with full reallocation. When there is no labor reallocation, distortion shocks lead to less movements in $\widetilde{\lambda}_1$ and $\widetilde{\lambda}_2$ compared to the economy with full reallocation. This results in a smaller ν_{12} and a diminished second-order term. Therefore, the first-order approximation becomes excellent, and the second-order contribution is close to zero.

Figure 4B illustrates a symmetric economy characterized by high complementarity between labor and intermediate inputs and between final goods. Due to this high complementarity, the economy allocates more resources to the sector with relatively higher distortion. In such cases, $\widetilde{\lambda}_1$ and $\widetilde{\lambda}_2$ respond similarly to sectoral distortion shocks, resulting in $\frac{\partial \log(\widetilde{\lambda}_1/\widetilde{\lambda}_2)}{\partial \log \mu_1}$ remaining close to zero, and the second-order effect becomes less significant. Therefore, the first-order approximation, similar to Baqaee & Farhi (2020), accurately captures the effect of distortion shocks.

However, in scenarios without labor reallocation, distortion shocks lead to a much larger increase in $\widetilde{\lambda}_1$ compared to $\widetilde{\lambda}_2$, resulting in a larger ν_{12} and a more substantial role for the second-order term in an economy with high complementarity.

Figure 2: Real GDP response to sectoral shocks in a symmetric two-sector economy with inelastic labor with full reallocation/constant return to scale. $\mu = 0.9$

A. Extreme substitution



B. Extreme complementarity

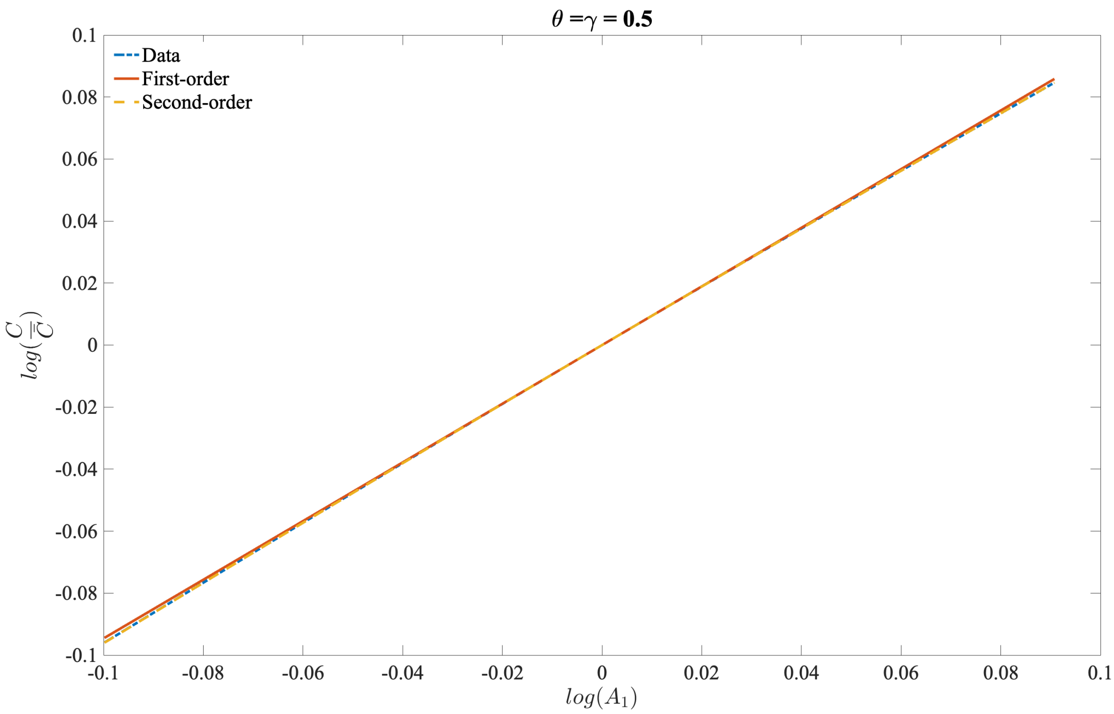
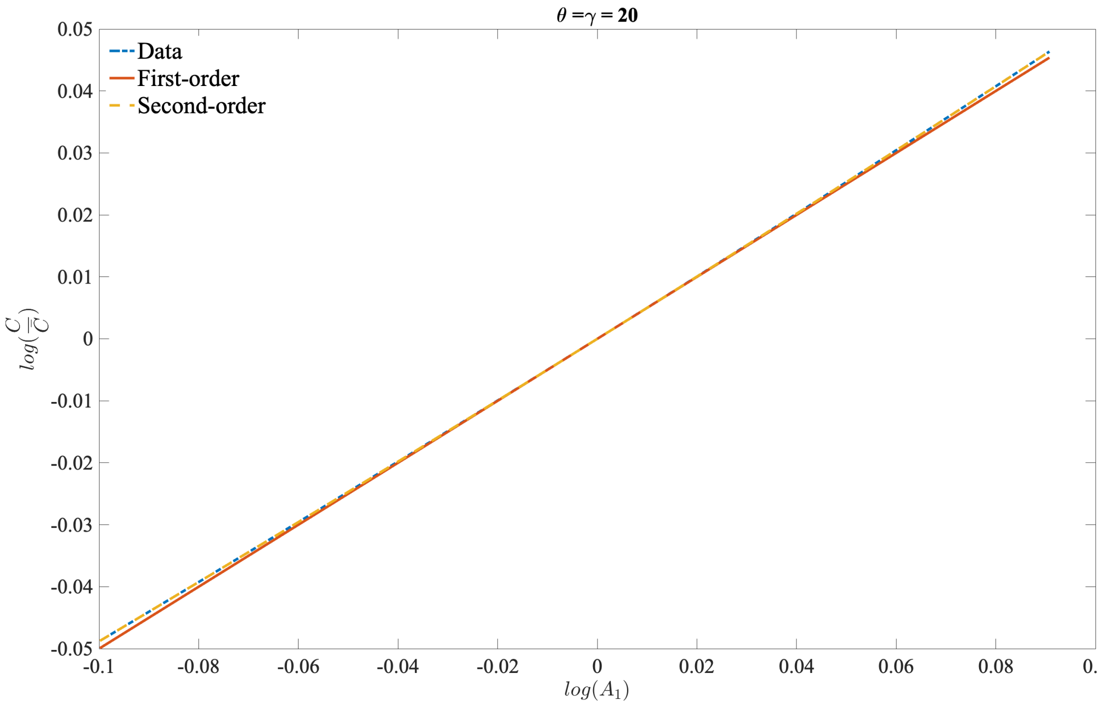


Figure 3: Real GDP response to sectoral shocks in a symmetric two-sector economy with inelastic labor and no reallocation/extreme decreasing return to scale. $\mu = 0.9$

A. Extreme substitution



B. Extreme complementarity

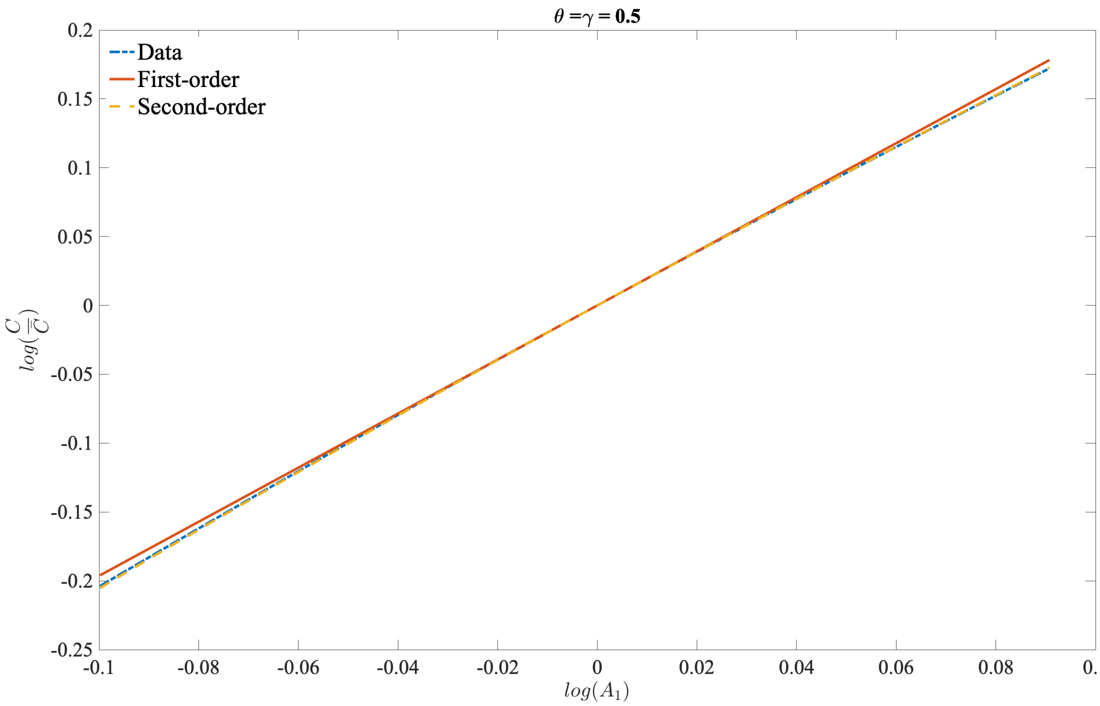
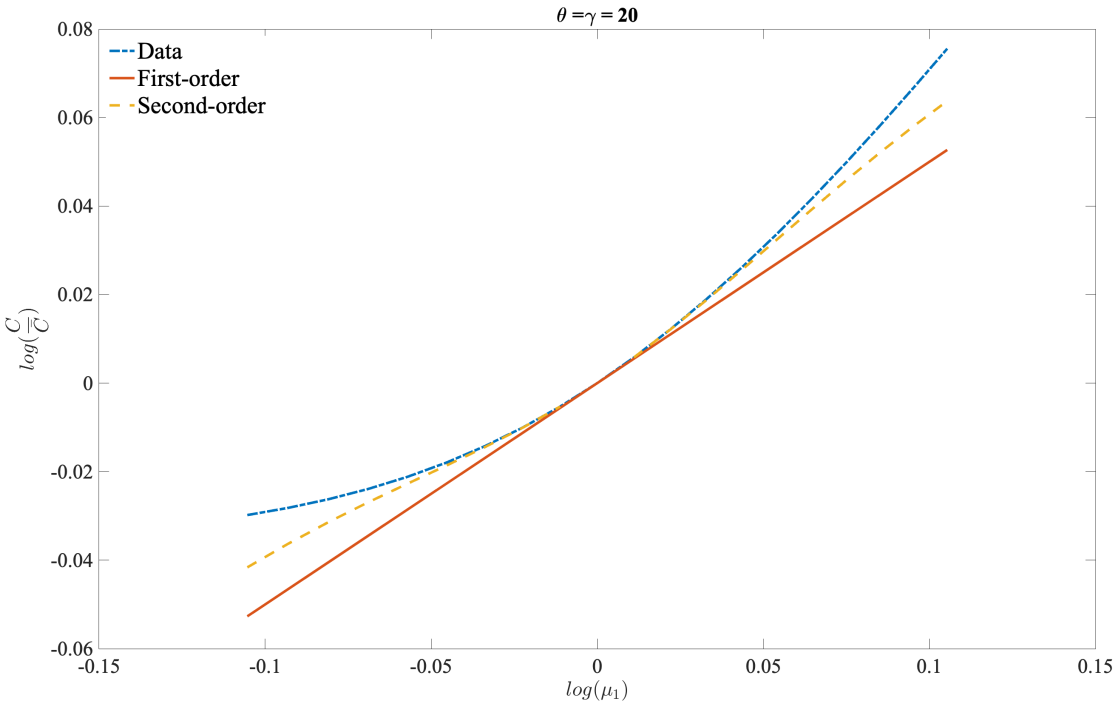


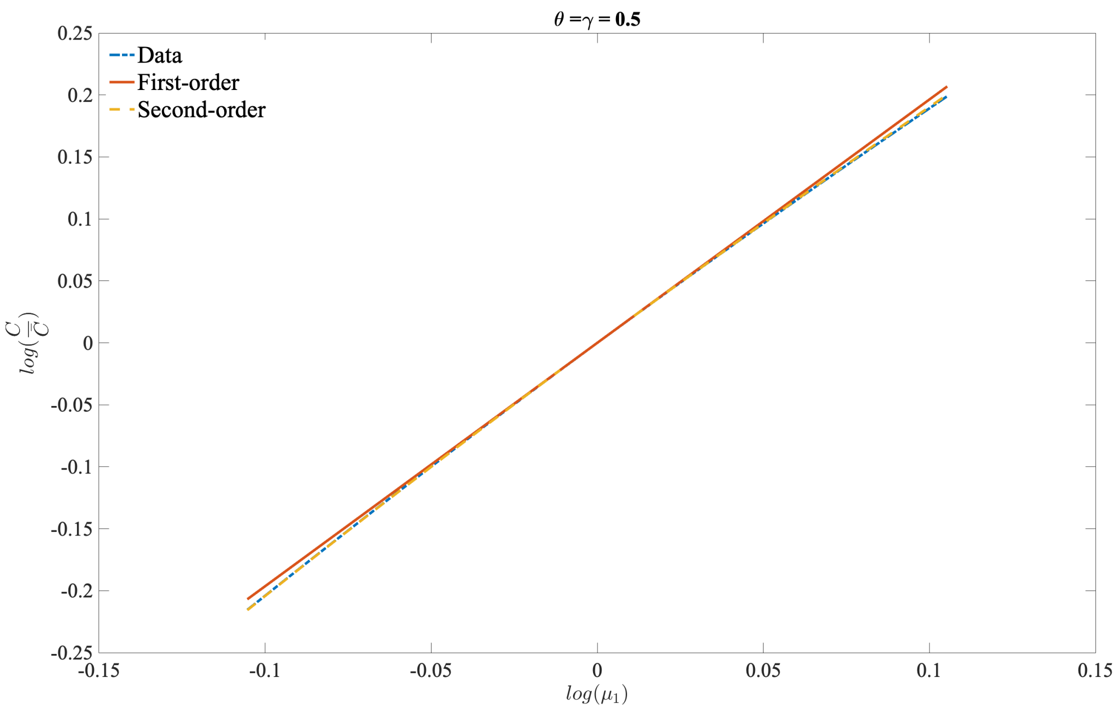
Figure 4: Real GDP response to distortion shocks in a Symmetric two-sector economy with inelastic labor with full reallocation/constant return to scale.

$\mu = 0.9$

A. Extreme substitution



B. Extreme complementarity



7 Rebated Distortions

Work in Progress...

8 Conclusion

This paper offers a novel perspective on the impact of microeconomic disturbances in an inefficient production network economy on aggregate TFP. We present non-linear decompositions designed to apply across various general equilibrium settings, enabling the identification of essential metrics for capturing the non-linear consequences of microeconomic fluctuations. These consequences encompass firm-level productivity shocks, wasted distortions, and rebated distortions.

Our findings underscore that significant shocks or pronounced complementarity/substitution in production processes introduce considerable bias into the linear approximations found in the literature regarding distortions and productivity shocks. Importantly, our non-linear second-order effects act as a potent counterbalance to this bias. The model builds upon established multisector models and advances the understanding of non-linear effects in general equilibrium input-output environments with distortions.

Appendix

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