1. Calcule los diferentes tipos de errores en las aproximaciones de p por p^* tome en cuenta 8 cifras significativas, 8 cifras por redondeo y 8 cifras por truncamiento

$$p = \phi = \frac{1+\sqrt{5}}{2} \quad , \quad p^* = \frac{13}{8}$$

Cambio a 2 cifras

$p = 1.61803398875$ $\qquad p^* = 1.625 \approx 1.63$

• Redondeo

$\text{Error real} = 1.61803398875 - 1.63 \approx -0.012$

$\text{Error absoluto} = |\text{Error real}| \approx 0.012$

$$\text{Error relativo} = \frac{\text{Error absoluto}}{|p|} \approx 0.0074$$

$\text{Error porcentual} = \text{Error relativo} \cdot 100\% \approx 0.74\%$

• Truncamiento

$p = 1.61803398875$ $\qquad p^* = 1.625 \approx 1.62$

$\text{Error real} = 1.61803398875 - 1.62 \approx -0.0019$

$\text{Error absoluto} = |\text{Error real}| \approx 0.0019.$

$$\text{Error relativo} = \frac{\text{Error absoluto}}{|p|} \approx 0.0012$$

$\text{Error porcentual} = \text{Error relativo} \cdot 100 \approx 0.12\%$

• Significativos

$p = 1.61803398875$ $\qquad p^* = 1.625 \approx 1.6$

$\text{Error Real} = 1.61803398875 - 1.6 \approx 0.018$

$\text{Error absoluto} = |\text{Error real}| \approx 0.018$

$$\text{Error relativo} = \frac{\text{Error absoluto}}{|p|} \approx 0.011 \qquad \text{Error porcentual} \approx 1.1\%$$

2. Pasar 76.14810 al formato IEEE 754 de 32 bits

76	0
38	0
19	1
9	1
4	0
2	0
1	1
0	0

$76_{10} = 1001100_2$

$14810_{10} = 00100101111010011$

$0.1481 \cdot 2 = 0$
$0.2962 \cdot 2 = 0$
$0.5924 \cdot 2 = 1$
$0.1848 \cdot 2 = 0$
$0.3696 \cdot 2 = 0$
$0.7392 \cdot 2 = 1$
$0.4784 \cdot 2 = 0$
$0.9568 \cdot 2 = 1$
$0.9136 \cdot 2 = 1$
$0.8272 \cdot 2 = 1$
$0.6544 \cdot 2 = 1$
$0.3088 \cdot 2 = 0$
$0.6176 \cdot 2 = 1$
$0.2352 \cdot 2 = 0$
$0.4704 \cdot 2 = 0$
$0.9408 \cdot 2 = 1$
$0.8816 \cdot 2 = 1$
$0.7632 \cdot 2 = 1$

$76.14810_{10} = 1001100.00100101111010011$

$76.14810_{10} = 1.\underbrace{00110000100101111010011}_{\text{Mantisa}} \cdot 2^6$

Exponente = 127 + 6 = 133

$133_{10} = 10000101_2$

133	1
66	0
33	1
16	0
8	0
4	0
2	0
1	1
0	0

Exponente = 100 00 101

IEE 754 : + 76.14810

S	Exponente	Mantisa
0	10000101	00110000100101111010011

3. Pasar de formato IEEE 754 11000001100010010011001100110011 a decimal

1 | 10000011 | 00010010011001100110011

$10000011_2 = 131$; Exponente: $127 + x$

$131 = 127 + x$

$x = 131 - 127 = 4$

$2^{131-127}$

Mantisa:

$10001_2 = 17_{10}$

1.00010010011001100110011

10001.0010011001100110011 $0.0010011001100110011_2 = 0.$

$\to 0.1499996185\,3$

$-17.1499996185\,3$

4. Suponga que $a = \frac{4}{9}$, $b = \frac{2}{5}$, $c = 0.81234$, $d = 45932.7$

$e = 0.22222 \cdot 10^{-3}$, resuelva haciendo uso de 5 cifras por redondeo:

$a = \frac{4}{9} \approx 0.44444$; $b = \frac{2}{5} = 0.4$; $c = 0.81234$; $d = 45932.7 \approx 0.45933 \cdot 10^{5}$

a) $a \oplus c = 0.44444 + 0.81234 = 1.25678 \approx 0.12568 \cdot 10^{1}$

b) $(a \ominus c) \otimes e = (0.44444 - 0.81234) \cdot 0.2222 \cdot 10^{-3} \approx -0.817547 \cdot 10^{-4}$

c) $d \oslash b = \dfrac{0.45933 \cdot 10^{5}}{0.4} \approx 0.11483 \cdot 10^{6}$

d) $(b \otimes e) \oplus [(a \oslash d) \ominus b] = (0.4 \cdot 0.22222 \cdot 10^{-3}) + [(0.44444 / 0.45933 \cdot 10^{5}) - 0.4]$

≈ -0.3999

5. Dada la función $f(x) = x^4 - x - 1$, use método de la bisección para los intervalos $[-1,0]$ y $[1,2]$, obtener soluciones precisas dentro de 10^{-4} como tolerancia, trabajo con 8 cifras decimales por redondeo, muestre tabla de valores

* $[-1;0]$

a	b	p	f(a)	f(b)	f(p)	Tol
-1	0	-0.5	1	-1	-0.4375	0.5
-1	-0.5	-0.75	1	-0.4375	0.06640625	0.25
-0.75	-0.5	-0.625	0.06640625	-0.4375	-0.22241211	0.125
-0.75	-0.625	-0.6875	0.06640625	-0.22241211	-0.08909607	0.0625
-0.75	-0.6875	-0.71875	0.06640625	-0.08909607	-0.0143728	0.03125
-0.75	-0.71875	-0.734375	0.06640625	-0.0143728	0.02522468	0.015625
-0.734375	-0.71875	-0.7265625	0.02522468	-0.01437282	0.00523359	$7.8125 \cdot 10^{-3}$
-0.7265625	-0.71875	-0.72265625	0.00523359	-0.01437282	-0.00461743	$3.90625 \cdot 10^{-3}$
-0.7265625	-0.72265625	-0.72460938	0.00523359	-0.00461743	0.00029606	$1.95313 \cdot 10^{-3}$
-0.72460938	-0.72265625	-0.72363281	0.00029606	-0.00461743	-0.00216369	$9.7656 \cdot 10^{-4}$
-0.72460938	-0.72363281	-0.72412109	0.00029606	-0.00216369	-0.00093457	$4.8828 \cdot 10^{-4}$
-0.72460938	-0.72412109	-0.72436524	0.00029606	-0.00093457	-0.00031942	$2.4414 \cdot 10^{-4}$
-0.72460938	-0.72436524	-0.72448731	0.00029606	-0.00031942	-0.00001172	$1.2207 \cdot 10^{-4}$
-0.72460938	-0.72448731	-0.72454834	0.00029606	-0.00001172	0.00014215	$6.103 \cdot 10^{-5}$
-0.72454834	-0.72448731	-0.72451782	0.00014215	-0.00001172	0.0000652	$3.052 \cdot 10^{-5}$
-0.72451782	-0.72448731	-0.72450256	0.0000652	-0.00001172	0.00002673	$1.525 \cdot 10^{-5}$
-0.72450256	-0.72448731	-0.72449494	0.00002673	-0.00001172	0.00000752	$7.63 \cdot 10^{-6}$
-0.72449494	-0.72448731	-0.72449113	0.00000752	-0.00001172	-0.00000209	$3.82 \cdot 10^{-6}$
-0.72449494	-0.72449113	-0.72449304	0.00000752	-0.00000209	0.00000273	$1.91 \cdot 10^{-6}$
-0.72449304	-0.72449113	-0.72449209	0.00000273	-0.00000209	0.00000053	$9.6 \cdot 10^{-7}$

[1 ; 2]

a	b	c	f(a)	f(b)	f(c)	TOL
1	2	1.5	-1	13	2.5625	0.5
1	1.5	1.25	-1	2.5625	0.19140625	0.25
1	1.25	1.125	-1	0.19140625	-0.33319336	0.125
1.125	1.25	1.1875	-0.33319336	0.19140625	[illegible]	0.0625
1.1875	1.25	1.21875	[illegible]	0.19140625	-0.0114807[illegible]	0.03125
1.21875	1.25	1.234375	-0.01148074	0.19140625	0.08[illegible]	0.015625
1.21875	1.234375	1.2265625	-0.01148074	0.08[illegible]	0.0358[illegible]	0.0078125
1.21875	1.2265625	1.22265625	-0.01148074	0.0358[illegible]	0.0110[illegible]	0.00390625
1.21875	1.22265625	1.22070312	-0.01148074	0.0110[illegible]	-0.00025711	0.00195312
1.22070312	1.22265625	1.22167969	-0.00025711	0.0110348	0.00538024	0.00097656
1.22070312	1.22167969	1.2211914	-0.00025711	0.00538024	0.00280945	0.00048828
1.22070312	1.2211914	1.22094726	-0.00025711	0.00280495	0.00127054	0.00024414
1.22070312	1.22094726	1.22082519	-0.00025711	0.00127564	0.00050913	0.00012207
1.22070312	1.22082519	1.22076415	-0.00025711	0.00050913	0.00011095	$6.103 \cdot 10^{-5}$
1.22070312	1.22076415	1.22073363	-0.00025711	0.00011545	-0.00006962	$3.051 \cdot 10^{-5}$
1.22073363	1.22076415	1.22074889	-0.00006962	0.00011545	0.00003016	$1.526 \cdot 10^{-5}$
1.22073363	1.22074889	1.22074126	-0.00006962	0.00003016	-0.00001773	$7.63 \cdot 10^{-6}$
1.22074126	1.22074889	1.22074507	-0.00001773	0.00003016	0.00000619	$3.81 \cdot 10^{-6}$
1.22074126	1.22074507	1.22074317	-0.00001773	0.00000619	-0.00000057	$1.9 \cdot 10^{-6}$
1.22074317	1.22074507	1.22074412	-0.00000057	0.00000619	$2.2 \cdot 10^{-7}$	$9.5 \cdot 10^{-7}$

6. Dada la función $f(x) = x^4 + 4x^3 - 10$, determine el número de iteraciones necesarias con presición 10^{-5}, $a=1$ y $b=2$, trabaje con 8 cifras significativas.

error abs $< 10^{-5}$ $\qquad |p_n - p| \leq \frac{b-a}{2^n}$

$$\frac{b-a}{2^n} < 10^{-5} \;;\; \frac{1}{2^n} < 10^{-5} \;;\; 2^{-n} < 10^{-5}$$

$$\log_2(2^{-n}) < \log_2(10^{-5}) \;;\; -n \cancel{\log_2(2)} < -5 \log_2(10)$$

$$-n < -5\log_2(10) \;;\; n > 5\log_2(10)$$

$$n > 16.6096 \;;\; n \approx 17$$

7. Dada la función $f(x) = x^3+x-1$, use el método del punto fijo donde p se encuentra en $[0;1]$, y obtenga soluciones precisas dentro de 10^{-3}, trabaje con 8 cifras decimales por truncamiento, muestre tabla de valores

1) $x = g(x)$

$$f(x) = \frac{x^3+x-1}{3} = 0$$

$$x^3+x-1 = 0 \;;\; x(x^2+1) = 1 \;;\; x = \frac{1}{x^2+1}$$

2) $x_{n+1} = g(x_n)$

$$x_{n+1} = \frac{1}{x_n^2+1}$$

3) $$\text{error} = \left| \frac{x_{n+1} - x_n}{x_{n+1}} \right|$$

x_n	x_{n+1}	error
1	0.5	0.5
0.5	0.8	0.375
0.8	0.6097561	0.31199999
0.6097561	0.72896791	0.11921181
0.72896791	0.65249972	0.11633724
0.65249972	0.70106138	0.06855557
0.70106138	0.67047179	0.04562398

x_n	x_{n+1}	Error
0.67047179	0.68487764	0.02817794
0.68487764	0.67753838	0.0182119
0.67753838	0.68537359	0.0114 2242
0.68537359	0.68031366	$7.3192 \cdot 10^{-3}$
0.68039366	0.68355712	$4.6279 \cdot 10^{-3}$
0.68355712	0.68154696	$2.7499 \cdot 10^{-3}$
0.68154696	0.68282398	$1.8703 \cdot 10^{-3}$
0.68282398	0.68201259	$1.1897 \cdot 10^{-3}$
0.68201259	0.68232808	$7.553 \cdot 10^{-4}$ ← Condición de parada

8) Dada la función $f(x) = x^4 - x - 1$, $p_0 = 1$. Use el método de Newton para obtener soluciones precisas con tolerancia 10^{-7}, trabaje con 8 cifras decimales por truncamiento. Muestre tabla de valores.

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$$f(x) = x^4 - x - 1$$

$$f'(x) = 4x^3 - 1$$

x_{n-1}	x_n	$f(x_{n-1})$	$f'(x_{n-1})$	ToL	$\lvert p_n - p_{n-1} \rvert$
1	1.33333333	-1	3	0.33333	
1.33333333	1.23580135	0.82716047	8.48148141	0.09752547	
1.23580786	1.22105849	0.04659633	6.5490719	0.01474897	
1.22105849	1.22074423	0.00197745	6.28232783	0.00031476	
1.22074423	1.22074408	0.00000091	6.27669265	$1.5 \cdot 10^{-7}$	← Condición de parada

9. Dada la función $f(x) = x^4 - x - 1$, $p_0 = 1$ y $p_1 = 1.4$, use el método de la secante obtener soluciones precisas con tolerancia 10^{-6} trabajo con 8 cifras decimales por truncamiento. Muestre tabla de valores.

$$x_n = x_{n-1} - f(x_{n-1}) \cdot \frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})}$$

x_{n-2}	x_{n-1}	x_n	$f(x_{n-2})$	$f(x_{n-1})$	$f(x_n)$	TOL
1	1.4	1.163827	-1	1.4416	-0.32917494	0.236173
1.4	1.163827	1.20772494	1.4416	-0.32917494	-0.08018212	0.04390294
1.163827	1.20772494	1.22186782	-0.32917494	-0.08018212	0.00706464	0.01413788
1.20772494	1.22186781	1.22072303	-0.08018212	0.00706464	-0.00013215	0.00114479
1.22186781	1.22072303	1.22074405	0.00706464	-0.00013215	$-2.2 \cdot 10^{-7}$	$2.102 \cdot 10^{-5}$
1.22072303	1.22074405	1.22074479	-0.00013215	$-2.2 \cdot 10^{-7}$	$3 \cdot 10^{-8}$	$4 \cdot 10^{-8}$

Condición de parada

10. Dada la función $f(x) = x^4 - x - 1$, $p_0 = 1$ y $p_1 = 1.4$, Use el método de la posición falsa, obtener soluciones precisas con tolerancia 10^{-6} trabaje con 8 cifras decimales por redondeo. Muestre tabla de valores.

$$x_n = x_{n-1} - f(x_{n-1}) \cdot \frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})} \quad ; \operatorname{sgn} f(p_2) \cdot \operatorname{sgn} f(p_1) < 0$$

x_{n-2}	x_{n-1}	x_n	$f(x_{n-2})$	$f(x_{n-1})$	$f(x_n)$	TOL
1	1.4	1.163827	-1	1.4416	-0.32291494	0.236173
1.4	1.163827	1.20772494	1.4416	-0.32291494	-0.0818212	0.04390244
1.4	1.20772494	1.21786058	1.4416	-0.0818212	-0.01802464	0.01013064
1.4	1.21786058	1.22010979	1.4416	-0.01802464	-0.00397767	0.00224921
1.4	1.22010979	1.22060478	1.4416	-0.00397767	-0.0008742	0.00049499

x_{n-2}	x_{n-1}	x_n	$f(x_{n-2})$	$f(x_{n-1})$	$f(x_n)$	TOL
1.4	1.22060478	1.22047135	1.4416	-0.00008742	-0.00049146	0.00010872
1.4	1.22047135	1.22073737	1.4416	-0.000[illegible]	$-4.78 \cdot 10^{-5}$	$2.387 \cdot 10^{-5}$
1.4	1.22073737	1.22074261	1.4416	$-4.31 \cdot 10^{-5}$	$-9.5 \cdot 10^{-6}$	$5.14 \cdot 10^{-6}$
1.4	1.22074261	1.22074376	1.4416	$-4.2 \cdot 10^{-6}$	$-2.4 \cdot 10^{-8}$	$1.15 \cdot 10^{-6}$
1.4	1.22074376	1.22074401	1.4416	$-2.4 \cdot 10^{-6}$	$-4.7 \cdot 10^{-9}$	$9.5 \cdot 10^{-9}$

condición de parada

11. Dada la función $f(x) = x^4 - x - 1$ justifique cuál método es mejor y porqué

El método de Newton-Raphson es el mejor para la función $x^4 - x - 1$ porque sacar su función derivada no es complicado y converge más rápido que los demás métodos.