ESCUELA POLITÉCNICA NACIONAL
FACULTAD DE INGENIERÍA DE SISTEMAS
INGENIERÍA EN CIENCIAS DE LA COMPUTACIÓN

PERÍODO ACADÉMICO: 2025-A
ASIGNATURA: ICCD412 Métodos Numéricos **GRUPO:** GR2
TIPO DE INSTRUMENTO: Tarea3
FECHA DE ENTREGA LÍMITE: 04/05/2025
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TEMA

Aritmética de dígitos finitos

OBJETIVOS

- Comprender la importancia de la representacion finita de números en los errores por redondeo y truncamiento.
- Aplicar operaciones aritméticas básicas utilizando dígitos finitos para conocer el su porcentaje de error.

DESARROLLO

Operaciones Aritméticas de dígitos finitos

$$x \oplus y = fl(fl(x) + fl(y)),$$

$$x \ominus y = fl(fl(x) - fl(y)),$$
$$x \otimes y = fl(fl(x) \cdot fl(y)),$$
$$x \oslash y = fl\left(\frac{fl(x)}{fl(y)}\right).$$

Suponga que $x = 5/7$, $y = 1/3$, $u = 0{,}714251$, $v = 98765{,}9$, y $w = 0{,}111111 \cdot 10^{-4}$, resuelva haciendo uso de corte de 5 cifras significativas:

$x = 0,7142857142857143$, $y = 0{,}333\ldots$

1. $x \otimes u$

$$fl(x) = 0{,}71428 \cdot 10^0 \quad y \quad fl(u) = 0{,}71425 \cdot 10^0$$
$$\begin{aligned} x \otimes u &= fl(fl(x) \cdot fl(u)) \\ &= fl(0{,}71428 \cdot 10^0 \cdot 0{,}71425 \cdot 10^0) \\ &= fl(0{,}51017449 \cdot 10^0) \\ &= 0{,}51017 \cdot 10^0 \end{aligned}$$

2. $(x \oslash u) \oplus w$

$$\begin{aligned} (x \oslash u) &= fl\left(\frac{fl(x)}{fl(u)}\right) \\ &= fl\left(\frac{0{,}71428 \cdot 10^0}{0{,}71425 \cdot 10^0}\right) \\ &= fl(1{,}000042002100105 \cdot 10^0) \\ &= 0{,}1 \cdot 10^1 \end{aligned}$$
$$\begin{aligned} (x \oslash u) \oplus w &= fl(fl(x \oslash u) + fl(w)) \\ &= fl(fl(0{,}1 \cdot 10^1) + fl(0{,}111111 \cdot 10^{-4})) \\ &= fl(0{,}1 \cdot 10^1 + 0{,}11111 \cdot 10^{-4}) \\ &= fl(1{,}000011111) \\ &= 0{,}1 \cdot 10^1 \end{aligned}$$

3. $v \oslash u$

$$\begin{aligned} (v \oslash u) &= fl\left(\frac{fl(v)}{fl(u)}\right) \\ &= fl\left(\frac{fl(98765{,}9)}{fl(0{,}714251)}\right) \\ &= fl\left(\frac{0{,}98765 \cdot 10^5}{0{,}71425 \cdot 10^0}\right) \\ &= fl(138277{,}91389569477) \\ &= 0{,}13827 \cdot 10^6 \end{aligned}$$

4. $(y \otimes w) \oplus [(x \oslash v) \oslash y]$

$$\begin{aligned}(x \oslash v) &= fl\left(\frac{fl(x)}{fl(v)}\right)\\ &= fl\left(\frac{0{,}71428 \cdot 10^{0}}{0{,}98765 \cdot 10^{5}}\right)\\ &= fl(0{,}000072321166405103 03)\\ &= 0{,}72321 \cdot 10^{-4}\end{aligned}$$

$$\begin{aligned}(x \oslash v) \oslash y &= fl\left(\frac{fl(x \oslash v)}{fl(y)}\right)\\ &= fl\left(\frac{fl(0{,}72321 \cdot 10^{-4})}{fl(0{,}333\ldots)}\right)\\ &= fl\left(\frac{0{,}72321 \cdot 10^{-4}}{0{,}33333 \cdot 10^{0}}\right)\\ &= fl(0{,}0002169651696516965)\\ &= 0{,}21696 \cdot 10^{-3}\end{aligned}$$

$$\begin{aligned}y \otimes w &= fl(fl(y) \cdot fl(w))\\ &= fl(0{,}33333 \cdot 10^{0} \cdot 0{,}11111 \cdot 10^{-4})\\ &= fl(0{,}00000370362963000000 05)\\ &= 0{,}37036 \cdot 10^{-5}\end{aligned}$$

$$\begin{aligned}(y \otimes w) \oplus [(x \oslash v) \oslash y] &= fl(fl(y \otimes w) + fl((x \oslash v) \oslash y))\\ &= fl(fl(0{,}37036 \cdot 10^{-5}) + fl(0{,}21696 \cdot 10^{-3}))\\ &= fl(0{,}0002206635999999 9997)\\ &= 0{,}22066 \cdot 10^{-3}\end{aligned}$$

Cálculo de errores

Dados los ejercicios anteriores calcule los 4 tipos de errores. $valorAproximado = r^*$ y $valorReal = r$

1. $x \otimes u$

$$\begin{aligned}r = x \cdot u &= \frac{5}{7} \cdot 0{,}714251\\ &= 0{,}5101792857142857\end{aligned}$$

$$r^* = x \otimes u = 0{,}51017 \cdot 10^{0}$$

$$\begin{aligned}Error_{Real} &= r - r^*\\ &= 0{,}5101792857142857 - 0{,}51017\\ &= 0{,}0000092857142857\end{aligned}$$

$$
\begin{aligned}
Error_{Absoluto} &= |r - r^*| \\
&= |0{,}5101792857142857 - 0{,}51017| \\
&= |0{,}0000092857142857| \\
&= 0{,}0000092857142857
\end{aligned}
$$

$$
\begin{aligned}
Error_{Relativo} &= \left| \frac{r - r^*}{r} \right| \\
&= \left| \frac{0{,}5101792857142857 - 0{,}51017}{0{,}5101792857142857} \right| \\
&= |0{,}00001820088456296176| \\
&= 0{,}00001820088456296176
\end{aligned}
$$

$$
\begin{aligned}
Error_{Porcentual} &= \left| \frac{r - r^*}{r} \right| \cdot 100\,\% \\
&= 0{,}00001820088456296176 \cdot 100\,\% \\
&= 0{,}001820088456296176\,\% \\
&\approx 0{,}002\,\%
\end{aligned}
$$

2. $(x \oslash u) \oplus w$

$$
\begin{aligned}
r = \frac{x}{u} + w &= \frac{\frac{5}{7}}{0{,}714251} + 0{,}111111 \cdot 10^{-4} \\
&= 1{,}000059713462075 \\
r^* = (x \oslash u) \oplus w &= 0{,}1 \cdot 10^1
\end{aligned}
$$

$$
\begin{aligned}
Error_{Real} &= r - r^* \\
&= 1{,}000059713462075 - 0{,}1 \cdot 10^1 \\
&= 0{,}000059713462075
\end{aligned}
$$

$$
\begin{aligned}
Error_{Absoluto} &= |r - r^*| \\
&= |1{,}000059713462075 - 0{,}1 \cdot 10^1| \\
&= |0{,}000059713462075| \\
&= 0{,}000059713462075
\end{aligned}
$$

$$
\begin{aligned}
Error_{Relativo} &= \left| \frac{r - r^*}{r} \right| \\
&= \left| \frac{0{,}000059713462075}{1{,}000059713462075} \right| \\
&= |0{,}00005970989659035445| \\
&= 0{,}00005970989659035445
\end{aligned}
$$

$$
\begin{aligned}
Error_{Porcentual} &= \left| \frac{r - r^*}{r} \right| \cdot 100\,\% \\
&= 0{,}00005970989659035445 \cdot 100\,\% \\
&= 0{,}005970989659035445\,\% \\
&\approx 0{,}006\,\%
\end{aligned}
$$

3. $v \oslash u$

$$\begin{aligned} r = \frac{v}{u} &= \frac{98765{,}9}{0{,}714251} \\ &= 138278{,}9803584454 \end{aligned}$$

$$r^* = v \oslash u = 0{,}13827 \cdot 10^6$$

$$\begin{aligned} Error_{Real} &= r - r^* \\ &= 138278{,}9803584454 - 0{,}13827 \cdot 10^6 \\ &= 8{,}980358445405727 \end{aligned}$$

$$\begin{aligned} Error_{Absoluto} &= |r - r^*| \\ &= |138278{,}9803584454 - 0{,}13827 \cdot 10^6| \\ &= |8{,}980358445405727| \\ &= 8{,}980358445405727 \end{aligned}$$

$$\begin{aligned} Error_{Relativo} &= \left|\frac{r - r^*}{r}\right| \\ &= \left|\frac{8{,}980358445405727}{138278{,}9803584454}\right| \\ &= |0{,}00006494377107877806| \\ &= 0{,}00006494377107877806 \end{aligned}$$

$$\begin{aligned} Error_{Porcentual} &= \left|\frac{r - r^*}{r}\right| \cdot 100\,\% \\ &= 0{,}00006494377107877806 \cdot 100\,\% \\ &= 0{,}006494377107877806\,\% \\ &\approx 0{,}006\,\% \end{aligned}$$

4. $(y \otimes w) \oplus [(x \oslash v) \oslash y]$

$$r = (y \cdot w) + \left[\frac{(\frac{x}{v})}{y}\right] = (\frac{1}{3} \cdot 0{,}111111 \cdot 10^{-4}) + \left[\frac{(\frac{\frac{5}{7}}{8765{,}9})}{\frac{1}{3}}\right]$$

$$= 0{,}000248157451794698$$

$$r^* = (y \otimes w) \oplus [(x \oslash v) \oslash y] = 0{,}22066 \cdot 10^{-3}$$

$$\begin{aligned} Error_{Real} &= r - r^* \\ &= 0{,}000248157451794698 - 0{,}22066 \cdot 10^{-3} \\ &= 0{,}000027497451794698005 \end{aligned}$$

$$\begin{aligned} Error_{Absoluto} &= |r - r^*| \\ &= |0{,}000248157451794698 - 0{,}22066 \cdot 10^{-3}| \\ &= |0{,}000027497451794698005| \\ &= 0{,}000027497451794698005 \end{aligned}$$

$$
\begin{aligned}
Error_{Relativo} &= \left| \frac{r - r^*}{r} \right| \\
&= \left| \frac{0{,}000027497451794698005}{0{,}000248157451794698} \right| \\
&= |0{,}11080647224507606| \\
&= 0{,}11080647224507606
\end{aligned}
$$

$$
\begin{aligned}
Error_{Porcentual} &= \left| \frac{r - r^*}{r} \right| \cdot 100\,\% \\
&= 0{,}11080647224507606 \cdot 100\,\% \\
&= 11{,}080647224507606\,\% \\
&\approx 11{,}1\,\%
\end{aligned}
$$