ESCUELA POLITÉCNICA NACIONAL
FACULTAD DE INGENIERÍA DE SISTEMAS
INGENIERÍA EN CIENCIAS DE LA COMPUTACIÓN

PERÍODO ACADÉMICO: 2025-A
ASIGNATURA: ICCD412 Métodos Numéricos **GRUPO:** GR2
TIPO DE INSTRUMENTO: Practica 4
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TEMA

Polinomio de Taylor, Lagrange

OBJETIVOS

- Ser capaces de aplicar los métodos de interpolación de Taylor y Lagrange
- Comparar los resultados obtenidos mediante el método con respecto al valor real de la fucnión a interpolar

MARCO TEÓRICO

El teorema de Taylor es de gran valor en el estudio de los métodos numéricos, más específicamente la serie de Taylor proporciona un medio para predecir el valor de una función en un punto en términos del valor de la función y sus derivadas en otro punto. El teorema establece que cualquier función suave puede aproximarse por un polinomio.

Serie de taylor:

$$\sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

Se añade para terminar un término residual para considerar todos los términos desde el $k + 1$ hasta el infinito

Término residual:

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{n+1}$$

Donde el subindice n indica que éste es el residuo de la aproximación de n-ésimo orden y $\xi(x)$ es un valor de x que se encuentra en algún punto entre x_i y x_{i+1}.

Polinomio de Lagrange

El polinomio de interpolacion de Lagrange es simplemente una reformulación del polinomio de Newton que evita el cálculo de las diferencias divididas, y se representa de manera concisa como

$$f_n(x) \sum_{i=0}^{n} L_k(x) f(x_i)$$

donde

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

[1]

DESARROLLO

1. Para las siguientes funciones, usar el polinomio de Taylor:

   a) $e^{\sin x}, \quad x_0 = 1,\ n = 3$

$$P_3(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3$$

Derivadas:

$$
\begin{aligned}
f(x) &= e^{\sin x} \\
f'(x) &= \cos x e^{\sin x} \\
f''(x) &= \cos^2 x e^{\sin x} - \sin x e^{\sin x} \\
f'''(x) &= -\sin(2x)e^{\sin x} + \cos^3 x e^{\sin x} - \frac{\sin(2x)}{2}e^{\sin x} - \cos^2 x e^{\sin x} \\
f^{(4)}(x) &= -\sin(2x)\cos(x)e^{\sin x} + \cos(2x)e^{\sin x} + \cos^4 x e^{\sin x} - 3\cos^2 x \sin x e^{\sin x} \\
&\quad - \frac{\sin(2x)}{2}\cos x e^{\sin x} + \sin(2x)e^{\sin x} - \cos^3 x e^{\sin x}
\end{aligned}
$$

Evuluando para $x_0 = 1$:

$$
\begin{aligned}
f(1) &= e^{\sin(1)} \approx 2{,}31977 \\
f'(1) &= \cos(1)e^{\sin(1)} \approx 1{,}25338 \\
f''(1) &= \cos^2(1)e^{\sin(1)} - \sin(1)e^{\sin(1)} \approx -1{,}27482 \\
f'''(1) &= -\sin(2(1))e^{\sin(1)} + \cos^3(1)e^{\sin(1)} - \frac{\sin(2(1))}{2}e^{\sin(1)} - \cos^2(1)e^{\sin(1)} \approx -3{,}47536
\end{aligned}
$$

Construimos el polinomio de Taylor:

$$P_3(x) = 2{,}31977 + 1{,}25338(x-1) - \frac{1{,}27482}{2!}(x-1)^2 - \frac{3{,}47536}{3!}(x-1)^3$$

$$P_3(x) = 2{,}31977 + 1{,}25338(x-1) - \frac{1{,}27482}{2!}(x^2-2x+1) - \frac{3{,}47536}{3!}(x^3-3x^2+3x-1)$$

$$P_3(x) = 2{,}31977 + 1{,}25338(x-1) - 0{,}63741(x^2-2x+1) - 0{,}57922(x^3-3x^2+3x-1)$$

Construimos el término residual $R_n(x)$:

$$R_3(x) = \frac{f^{(4)}(\xi(x))}{4!}(x-1)^4$$

$$R_3(x) = \frac{f^{(4)}(\xi(x))}{4!}(x^4 - 4x^3 + 6x^2 - 4x + 1)$$

Por problemas con el tamaño de la función solo dejaré expresado $f^{(4)}(\xi(x))$ pero no lo reemplazaré en la formula de $R_3(x)$:

$$
\begin{aligned}
f^{(4)}(\xi(x)) &= -\sin(2\xi(x))\cos(\xi(x))e^{\sin(\xi(x))} + \cos(2\xi(x))e^{\sin(\xi(x))} + \cos^4(\xi(x))e^{\sin(\xi(x))} \\
&\quad - 3\cos^2(\xi(x))\sin(\xi(x))e^{\sin(\xi(x))} - \frac{\sin(2\xi(x))}{2}\cos(\xi(x))e^{\sin(\xi(x))} \\
&\quad + \sin(2\xi(x))e^{\sin(\xi(x))} - \cos^3(\xi(x))e^{\sin(\xi(x))}
\end{aligned}
$$

$$f(x) = P_n(x) + R_n(x)$$

$$f(x) = P_3(x) + R_3(x)$$

$$
\begin{aligned}
f(x) &= 2{,}31977 + 1{,}25338(x-1) - 0{,}63741(x^2-2x+1) - 0{,}57922(x^3-3x^2+3x-1) \\
&\quad + \frac{f^{(4)}(\xi(x))}{24}(x^4-4x^3+6x^2-4x+1)
\end{aligned}
$$

b) $\ln(1+x), \quad x_0 = 2,\ n = 5$

$$
\begin{aligned}
P_5(x) &= f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x-x_0)^4 \\
&\quad + \frac{f^{(5)}(x_0)}{5!}(x-x_0)^5
\end{aligned}
$$

Derivadas:

$$
\begin{aligned}
f(x) &= \ln(1+x) \\
f'(x) &= \frac{1}{(1+x)} \\
f''(x) &= -\frac{1}{(1+x)^2} \\
f'''(x) &= \frac{2}{(1+x)^3} \\
f^{(4)}(x) &= -\frac{6}{(1+x)^4} \\
f^{(5)}(x) &= \frac{24}{(1+x)^5} \\
f^{(6)}(x) &= -\frac{120}{(1+x)^6}
\end{aligned}
$$

Evaluando para $x_0 = 2$

$$
\begin{aligned}
f(2) &= \ln(1+2) \approx 1{,}09861 \\
f'(2) &= \frac{1}{(1+2)} \approx 0{,}33333 \\
f''(2) &= -\frac{1}{(1+2)^2} \approx -0{,}11111 \\
f'''(2) &= \frac{2}{(1+2)^3} \approx 0{,}07407 \\
f^{(4)}(2) &= -\frac{6}{(1+2)^4} \approx -0{,}07407 \\
f^{(5)}(2) &= \frac{24}{(1+2)^5} \approx 0{,}09876
\end{aligned}
$$

Construimos el polinomio de Taylor:

$$P_5(x) = 1{,}09861 + 0{,}33333(x-2) - \frac{0{,}11111}{2!}(x-2)^2 + \frac{0{,}07407}{3!}(x-2)^3$$
$$- \frac{0{,}07407}{4!}(x-2)^4 + \frac{0{,}09876}{5!}(x-2)^5$$

$$P_5(x) = 1{,}09861 + 0{,}33333(x-2) - 0{,}05555(x^2-4x+4) + 0{,}01234(x^3-6x^2+12x-8)$$
$$-0{,}00308(x^4-8x^3+24x^2-32x+16) + 0{,}00082(x^5-5x^4+40x^3-80x^2+80x-32)$$

Construimos el término residual $R_n(x)$:

$$R_5(x) = \frac{f^{(6)}(\xi(x))}{6!}(x-2)^6$$

$$R_5(x) = -\frac{1}{6(1+\xi(x))^6}(x^6-12x^5+60x^3-160x^3+240x^2-192x+64)$$

$$f(x) = P_n(x) + R_n(x)$$
$$f(x) = P_5(x) + R_5(x)$$
$$f(x) = 1{,}09861 + 0{,}33333(x-2) - 0{,}05555(x^2-4x+4) + 0{,}01234(x^3-6x^2+12x-8)$$
$$-0{,}00308(x^4-8x^3+24x^2-32x+16) + 0{,}00082(x^5-5x^4+40x^3-80x^2+80x-32)$$
$$-\frac{1}{6(1+\xi(x))^6}(x^6-12x^5+60x^3-160x^3+240x^2-192x+64)$$

c) $\cos 2x, \quad x_0 = \frac{\pi}{2},\ n = 3$

$$P_3(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

Derivadas:

$$f(x) = \cos(2x)$$
$$f'(x) = -2\,\mathrm{sen}(2x)$$
$$f''(x) = -4\cos(2x)$$
$$f'''(x) = 8\,\mathrm{sen}(2x)$$
$$f^{(4)} = 16\cos(2x)$$

Evaluando para $x_0 = \frac{\pi}{2}$

$$
\begin{aligned}
f\left(\frac{\pi}{2}\right) &= \cos\left(2\cdot\frac{\pi}{2}\right) = \cos(\pi) = -1\\
f'\left(\frac{\pi}{2}\right) &= -2\sin\left(2\cdot\frac{\pi}{2}\right) = -2\sin(\pi) = 0\\
f''\left(\frac{\pi}{2}\right) &= -4\cos\left(2\cdot\frac{\pi}{2}\right) = -4\cos(\pi) = 4\\
f'''\left(\frac{\pi}{2}\right) &= 8\sin\left(2\cdot\frac{\pi}{2}\right) = 8\sin(\pi) = 0
\end{aligned}
$$

Construimos el polinomio de Taylor:

$$
\begin{aligned}
P_3(x) &= -1 + (0)(x-\frac{\pi}{2}) + \frac{4}{2!}(x-\frac{\pi}{2})^2 + \frac{(0)}{3!}(x-\frac{\pi}{2})^3\\
&= -1 + 2(x^2 - \pi x + \frac{\pi^2}{4})\\
&= -1 + 2(x^2 - 3{,}14159x + 2{,}46740)
\end{aligned}
$$

Construimos el término residual $R_n(x)$:

$$R_3(x) = \frac{f^{(4)}(\xi(x))}{4!}(x-\frac{\pi}{2})^4$$

$$R_3(x) = \frac{16\cos(2\xi(x))}{24}(x^4 - 2\pi x^3 + \tfrac{3\pi^2}{2}x^2 - \tfrac{\pi^3}{2}x + \tfrac{\pi^4}{16})$$

$$
\begin{aligned}
f(x) &= P_n(x) + R_n(x)\\
f(x) &= P_3(x) + R_3(x)
\end{aligned}
$$

$$f(x) = -1 + 2(x^2 - 3{,}14159x + 2{,}46740) + \frac{2\cos(2\xi(x))}{3}(x^4 - 2\pi x^3 + \tfrac{3\pi^2}{2}x^2 - \tfrac{\pi^3}{2}x + \tfrac{\pi^4}{16})$$

d) $\sqrt[3]{x}, \quad x_0 = 2,\ n = 4$

$$P_4(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x-x_0)^4$$

Derivadas:

$$
\begin{aligned}
f(x) &= x^{1/3}\\
f'(x) &= \frac{1}{3}x^{-2/3}\\
f''(x) &= -\frac{2}{9}x^{-5/3}\\
f'''(x) &= \frac{10}{27}x^{-8/3}\\
f^{(4)} &= -\frac{80}{81}x^{-11/3}\\
f^{(5)} &= \frac{880}{243}x^{-14/3}
\end{aligned}
$$

Evaluando en $x_0 = 2$:

$$\begin{aligned}
f(2) &= 2^{1/3} \approx 1{,}25992 \\
f'(2) &= \frac{1}{3} \cdot 2^{-2/3} \approx 0{,}20998 \\
f''(2) &= -\frac{2}{9} \cdot 2^{-5/3} \approx -0{,}06999 \\
f'''(2) &= \frac{10}{27} \cdot 2^{-8/3} \approx 0{,}05832 \\
f^{(4)}(2) &= -\frac{80}{81} \cdot 2^{-11/3} \approx -0{,}07777 \\
f^{(5)}(2) &= \frac{880}{243} \cdot 2^{-14/3} \approx 0{,}14258
\end{aligned}$$

Construimos el polinomio de Taylor:

$$P_4(x) = 1{,}25992 + 0{,}20998(x-2) - \frac{0{,}06999}{2!}(x-2)^2 + \frac{0{,}05832}{3!}(x-2)^3 - \frac{0{,}07777}{4!}(x-2)^4$$

$$P_4(x) = 1{,}25992 + 0{,}20998(x-2) - \frac{0{,}06999}{2}(x^2-4x+4) + \frac{0{,}05832}{6}(x^3-6x^2+12x-8) - \frac{0{,}07777}{24}(x^4-8x^3+24x^2-32x+16)$$

$$P_4(x) = 1{,}25992 + 0{,}20998(x-2) - 0{,}03499(x^2-4x+4) + 0{,}00972(x^3-6x^2+12x-8) - 0{,}00324(x^4-8x^3+24x^2-32x+16)$$

Construimos el término residual $R_n(x)$:

$$R_4(x) = \frac{f^{(5)}(\xi(x))}{5!}(x-2)^5$$

$$R_4(x) = \frac{55\xi(x)^{-14/3}}{182}(x^5-10x^4+40x^3-80x^2+80x-32)$$

$$f(x) = P_n(x) + R_n(x)$$

$$f(x) = P_4(x) + R_4(x)$$

$$f(x) = 1{,}25992 + 0{,}20998(x-2) - 0{,}03499(x^2-4x+4) + 0{,}00972(x^3-6x^2+12x-8)$$
$$-0{,}00324(x^4-8x^3+24x^2-32x+16) + \frac{55\xi(x)^{-14/3}}{182}(x^5-10x^4+40x^3-80x^2+80x-32)$$

e) $\cos(\pi x^2), \quad x_0 = 0, \; n = 6$

$$P_6(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x - x_0)^4$$
$$+ \frac{f^{(5)}(x_0)}{5!}(x - x_0)^5 + \frac{f^{(6)}(x_0)}{6!}(x - x_0)^6$$

Derivadas:

$$\begin{aligned}
f(x) &= \cos(\pi x^2) \\
f'(x) &= -2\pi x \sin(\pi x^2) \\
f''(x) &= -2\pi \left[\sin(\pi x^2) + 2\pi x^2 \cos(\pi x^2)\right] \\
f'''(x) &= 8\pi^3 x^3 \sin(\pi x^2) - 12\pi^2 x \cos(\pi x^2) \\
f^{(4)}(x) &= 48\pi^3 x^2 \sin(\pi x^2) + 16\pi^4 x^4 \cos(\pi x^2) - 12\pi^2 \cos(\pi x^2) \\
f^{(5)}(x) &= 120\pi^3 x \sin(\pi x^2) + 96\pi^3 x^3 \cos(\pi x^2) + 32\pi^2 x \cos(\pi x^2) - 32\pi^3 x^3 \sin(\pi x^2) \\
f^{(6)}(x) &= 120\pi^3 \sin(\pi x^2) + 240\pi^4 x^2 \cos(\pi x^2) + 288\pi^3 x^2 \cos(\pi x^2) - 192\pi^3 x^4 \sin(\pi x^2) \\
&\quad + 32\pi^2 \cos(\pi x^2) - 160\pi^3 x^2 \sin(\pi x^2) - 64\pi^4 x^4 \cos(\pi x^2) \\
f^{(7)}(x) &= 240\pi^4 x \cos(\pi x^2) + 480\pi^4 x \cos(\pi x^2) - 480\pi^5 x^3 \operatorname{sen}(\pi x^2) + 560\pi^3 x \cos(\pi x^2) \\
&\quad - 560\pi^4 x^3 \operatorname{sen}(\pi x^2) - 768\pi^3 x^3 \sin(\pi x^2) + 384\pi^4 x^5 \cos(\pi x^2) - 64\pi^3 x \sin(\pi x^2) \\
&\quad - 320\pi^3 x \sin(\pi x^2) - 320\pi^4 x^3 \cos(\pi x^2) - 256\pi^4 x^3 \cos(\pi x^2) + 128\pi^5 x^5 \sin(\pi x^2)
\end{aligned}$$

Evaluando en $x_0 = 0$:

$$\begin{aligned}
f(0) &= \cos(\pi 0^2) = \cos(0) = 1 \\
f'(0) &= -2\pi 0 \sin(\pi 0^2) = -2\pi 0 \sin(0) = 0 \\
f''(0) &= -2\pi \left[\sin(\pi 0^2) + 2\pi 0^2 \cos(\pi 0^2)\right] = -2\pi \left[\sin(0) + 0\right] = 0 \\
f'''(0) &= 8\pi^3 0^3 \sin(\pi 0^2) - 12\pi^2 0 \cos(\pi 0^2) = 0 - 0 = 0 \\
f^{(4)}(0) &= 48\pi^3 0^2 \sin(\pi 0^2) + 16\pi^4 0^4 \cos(\pi 0^2) - 12\pi^2 \cos(\pi 0^2) \\
&= 0 + 0 - 12\pi^2 \cos(0) = -12\pi^2 \approx -118{,}43525 \\
f^{(5)}(0) &= 120\pi^3 0 \sin(\pi 0^2) + 96\pi^3 0^3 \cos(\pi 0^2) + 32\pi^2 0 \cos(\pi 0^2) - 32\pi^3 0^3 \sin(\pi 0^2) = 0 \\
f^{(6)}(0) &= 120\pi^3 \sin(\pi 0^2) + 240\pi^4 0^2 \cos(\pi 0^2) + 288\pi^3 0^2 \cos(\pi 0^2) - 192\pi^3 0^4 \sin(\pi 0^2) \\
&\quad + 32\pi^2 \cos(\pi 0^2) - 160\pi^3 0^2 \sin(\pi 0^2) - 64\pi^4 0^4 \cos(\pi 0^2) \\
&= 0 + 0 + 0 + 0 + 32\pi^2 - 0 - 0 = 32\pi^2 \approx 315{,}82734
\end{aligned}$$

Construimos el polinomio de Taylor:

$$P_6(x) = 1 + 0(x - 0) + \frac{0}{2!}(x - 0)^2 + \frac{0}{3!}(x - 0)^3 - \frac{118{,}43525}{4!}(x - 0)^4$$
$$+ \frac{0}{5!}(x - 0)^5 + \frac{315{,}82734}{6!}(x - 0)^6$$

$$P_6(x) = 1 - 4{,}93480x^4 + 0{,}43864x^6$$

Construimos esl término residual $R_n(x)$ (debido al espacio que ocupa $f^{(7)}(\xi(x))$) dejaré la función evaluada a parte de la formula de $R_6(x)$

$$R_6(x) = \frac{f^{(7)}(\xi(x))}{7!}(x-0)^7$$
$$R_6(x) = \frac{f^{(7)}(\xi(x))}{5040}x^7$$

$$\begin{aligned} f^{(7)}(\xi(x)) = {} & 240\pi^4\xi(x)\cos(\pi\xi(x)^2) + 480\pi^4\xi(x)\cos(\pi\xi(x)^2) - 480\pi^5\xi(x)^3\operatorname{sen}(\pi\xi(x)^2) \\ & + 560\pi^3\xi(x)\cos(\pi\xi(x)^2) - 560\pi^4\xi(x)^3\operatorname{sen}(\pi\xi(x)^2) - 768\pi^3\xi(x)^3\sin(\pi\xi(x)^2) \\ & + 384\pi^4\xi(x)^5\cos(\pi\xi(x)^2) - 64\pi^3\xi(x)\sin(\pi\xi(x)^2) - 320\pi^3\xi(x)\sin(\pi\xi(x)^2) \\ & - 320\pi^4\xi(x)^3\cos(\pi\xi(x)^2) - 256\pi^4\xi(x)^3\cos(\pi\xi(x)^2) + 128\pi^5\xi(x)^5\sin(\pi\xi(x)^2) \end{aligned}$$

$$f(x) = P_n(x) + R_n(x)$$
$$f(x) = P_6(x) + R_6(x)$$
$$f(x) = 1 - 4{,}93480x^4 + 0{,}43864x^6 + \frac{f^{(7)}(\xi(x))}{5040}x^7$$

f) $\dfrac{x}{e^x}, \quad x_0 = 3, \ n = 3$

$$P_3(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

Derivadas:

$$\begin{aligned} f(x) &= e^{-x}\cdot x \\ f'(x) &= e^{-x}(1-x) \\ f''(x) &= -e^{-x}(2-x) \\ f'''(x) &= e^{-x}(3-x) \\ f^{(4)} &= -e^{-x}(4-x) \end{aligned}$$

Evaluando en $x_0 = 3$:

$$\begin{aligned} f(3) &= e^{-3}\cdot 3 \approx 0{,}14936 \\ f'(3) &= e^{-3}(1-3) \approx -0{,}09957 \\ f''(3) &= -e^{-3}(2-3) \approx 0{,}4978 \\ f'''(3) &= e^{-3}(3-3) = 0 \end{aligned}$$

Construimos el polinomio de Taylor:

$$P_3(x) = 0{,}14936 - 0{,}09957(x-3) + \frac{0{,}4978}{2!}(x-3)^2 + \frac{0}{3!}(x-3)^3$$

$$P_3(x) = 0{,}14936 - 0{,}09957(x-3) + 0{,}02489(x^2 - 6x + 9)$$

Construimos el término residual $R_n(x)$

$$R_3(x) = \frac{f^{(4)}(\xi(x))}{4!}(x-3)^4$$
$$R_3(x) = -\frac{e^{-\xi(x)}(4-\xi(x))}{24}(x^4 - 12x^3 + 54x^2 - 108x + 81)$$

$$f(x) = P_n(x) + R_n(x)$$
$$f(x) = P_3(x) + R_3(x)$$
$$f(x) = 0{,}14936 - 0{,}09957(x-3) + 0{,}02489(x^2 - 6x + 9)$$
$$- \frac{e^{-\xi(x)}(4-\xi(x))}{24}(x^4 - 12x^3 + 54x^2 - 108x + 81)$$

2. Determine para el ejercicio 1.a el polinomio de Taylor dado que $x = 1{,}5$ y un error relativo de 0,0001.

   Previamente se ha calculado las primeras cuatro derivadas de la función $f(x) = e^{\sin x}$.

   Derivadas:

   $f^{(5)}(x) = e^{\operatorname{sen}(x)}\left(3\operatorname{sen}^2(x) + \left(1 - 6\cos^2(x)\right)\operatorname{sen}(x) + \cos^4(x) - 4\cos^2(x)\right)$
   $f^{(6)}(x) = e^{\operatorname{sen}(x)}\cos(x)\left(15\operatorname{sen}^2(x) + \left(15 - 10\cos^2(x)\right)\operatorname{sen}(x) + \cos^4(x) - 10\cos^2(x) + 1\right)$
   $f^{(7)}(x) = -e^{\operatorname{sen}(x)}\left(15\operatorname{sen}^3(x) + \left(15 - 45\cos^2(x)\right)\operatorname{sen}^2(x) + \left(15\cos^4(x) - 75\cos^2(x) + 1\right)\operatorname{sen}(x)\right.$
   $\left. - \cos^6(x) + 20\cos^4(x) - 16\cos^2(x)\right)$

   Evaluando en $x = 1{,}5$

$$f(1{,}5) \approx 2{,}71148$$
$$f'(1{,}5) \approx 0{,}19180$$
$$f''(1{,}5) \approx -2{,}69112$$
$$f'''(1{,}5) \approx -0{,}766480$$
$$f^{(4)}(1{,}5) \approx 10{,}12832$$
$$f^{(5)}(1{,}5) \approx 10{,}66303$$
$$f^{(6)}(1{,}5) \approx 5{,}90511$$
$$f^{(7)}(1{,}5) \approx -81{,}7035$$

   Construimos el polinomio de Taylor:

$$
\begin{aligned}
P_7(x) &= f(1{,}5) + f'(1{,}5)(x-1{,}5) + \frac{f''(1{,}5)}{2!}(x-1{,}5)^2 + \frac{f'''(1{,}5)}{3!}(x-1{,}5)^3 \\
&+ \frac{f^{(4)}(1{,}5)}{4!}(x-1{,}5)^4 + \frac{f^{(5)}(1{,}5)}{5!}(x-1{,}5)^5 + \frac{f^{(6)}(1{,}5)}{6!}(x-1{,}5)^6 \\
&+ \frac{f^{(7)}(1{,}5)}{7!}(x-1{,}5)^7
\end{aligned}
$$

$$
\begin{aligned}
P_7(x) &= 2{,}71148 + 0{,}19180(x-1{,}5) - \frac{2{,}69112}{2!}(x-1{,}5)^2 - \frac{0{,}76480}{3!}(x-1{,}5)^3 \\
&+ \frac{10{,}12832}{4!}(x-1{,}5)^4 + \frac{10{,}66303}{5!}(x-1{,}5)^5 + \frac{5{,}90511}{6!}(x-1{,}5)^6 \\
&- \frac{81{,}7035}{7!}(x-1{,}5)^7
\end{aligned}
$$

$$
\begin{aligned}
P_7(x) &= 2{,}71148 + 0{,}19180(x-1{,}5) - 1{,}34556(x-1{,}5)^2 - 0{,}12746(x-1{,}5)^3 \\
&+ 0{,}42201(x-1{,}5)^4 + 0{,}88858(x-1{,}5)^5 + 0{,}00820(x-1{,}5)^6 \\
&- 0{,}01621(x-1{,}5)^7
\end{aligned}
$$

Desarrollando los polinomios y reduciendo términos semejantes:

$$P_7(x) = -0{,}0401\,x^7 + 0{,}31047\,x^6 - 0{,}82277\,x^5 + 0{,}89986\,x^4 - 0{,}85289\,x^3 + 0{,}93485\,x^2 + 0{,}87464\,x + 1{,}015$$

$P_7(x)$ está en función de x y si la evaluamos en 1.5 obtenemos p^*

$$p^* = P_7(1{,}5) \approx 2{,}71148$$

Ya con los datos y siendo $p = e^{\sin x}$ como valor real, sacamos el error relativo

$$Error_{relativo} = \left| \frac{e^{\sin x} - 2{,}71148}{e^{\sin x}} \right| \approx 0{,}00001$$

ya que el error es menor que 0.0001 dejamos de iterar.

3. Dados los siguientes puntos, usar el polinomio de Lagrange:

   a) $(2, 1{,}43); (3{,}20, 2{,}79); (4, 3{,}56)$

$$P(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$L_0 = \prod_{\substack{i=0\\i\neq 0}}^{2} \frac{x-x_i}{x_0-x_i} = \frac{(x-3{,}2)(x-4)}{(2-3{,}2)(2-4)} = \frac{x^2-7{,}2x+12{,}8}{2{,}4}$$

$$L_1 = \prod_{\substack{i=0\\i\neq 1}}^{2} \frac{x-x_i}{x_1-x_i} = \frac{(x-2)(x-4)}{(3{,}2-2)(3{,}2-4)} = -\frac{x^2-6x+8}{0{,}96}$$

$$L_2 = \prod_{\substack{i=0\\i\neq 2}}^{2} \frac{x-x_i}{x_2-x_i} = \frac{(x-2)(x-3{,}2)}{(4-2)(4-3{,}2)} = \frac{x^2-5{,}2x+6{,}4}{1{,}6}$$

Reemplazando en el polinomio:

$$\begin{aligned}
P(x) &= (1{,}43)\frac{x^2-7{,}2x+12{,}8}{2{,}4} - (2{,}79)\frac{x^2-6x+8}{0{,}96} + (3{,}56)\frac{x^2-5{,}2x+6{,}4}{1{,}6} \\
&= 0{,}59583x^2 - 4{,}28997x + 7{,}62662 - 2{,}90625x^2 + 17{,}4375x - 23{,}25 + 2{,}225x^2 \\
&- 11{,}57x + 14{,}24 \\
&= -0{,}08542x^2 + 1{,}57753x - 1{,}38338
\end{aligned}$$

Calculamos $R_n(x)$:

$$\begin{aligned}
R_2(x) &= \frac{f^{(3)}(\xi(x))}{3!}(x-2)(x-3{,}2)(x-4) \\
&= \frac{f^{(3)}(\xi(x))}{6}(x^3-9{,}2x^2+27{,}2x-25{,}6)
\end{aligned}$$

$$f(x) = P_n(x) + R_n(x)$$
$$f(x) = P_2(x) + R_2(x)$$

$$f(x) = -0{,}08542x^2 + 1{,}57753x - 1{,}38338 + \frac{f^{(3)}(\xi(x))}{3!}(x^3-9{,}2x^2+27{,}2x-25{,}6)$$

b) $(1,10);(-4,10);(-7,34)$

$$P(x) = y_0L_0(x) + y_1L_1(x) + y_2L_2(x)$$

$$L_0 = \prod_{\substack{i=0\\i\neq 0}}^{2} \frac{x-x_i}{x_0-x_i} = \frac{(x+4)(x+7)}{(1+4)(1+7)} = \frac{x^2+11x+28}{40}$$

$$L_1 = \prod_{\substack{i=0\\i\neq 1}}^{2} \frac{x-x_i}{x_1-x_i} = \frac{(x-1)(x+7)}{(-4-1)(-4+7)} = -\frac{x^2+6x-7}{15}$$

$$L_2 = \prod_{\substack{i=0\\i\neq 2}}^{2} \frac{x-x_i}{x_2-x_i} = \frac{(x-1)(x+4)}{(-7-1)(-7+4)} = \frac{x^2+3x-4}{24}$$

Reemplazando en el polinomio:

$$\begin{aligned}
P(x) &= 10\frac{x^2+11x+28}{40} - 10\frac{x^2+6x-7}{15} + 34\frac{x^2+3x-4}{24} \\
&= 0{,}25x^2 + 2{,}75x + 7 - 0{,}66666x^2 - 4x + 4{,}66666 + 1{,}41666x^2 + 4{,}25x - 5{,}66664 \\
&= x^2 - 2{,}99998x + 6
\end{aligned}$$

Calculaos $R_n(x)$:

$$\begin{aligned}
R_2(x) &= \frac{f^{(3)}(\xi(x))}{3!}(x-1)(x+4)(x+7) \\
&= \frac{f^{(3)}(\xi(x))}{6}(x^3+10x^2+17x-28)
\end{aligned}$$

$$f(x) = P_n(x) + R_n(x)$$

$$f(x) = P_2(x) + R_2(x)$$

$$f(x) = x^2 - 2{,}99998x + 6 + \frac{f^{(3)}(\xi(x))}{6}(x^3+10x^2+17x-28)$$

c) $(4,808); (0,4); (-6,1438); (-4,160)$

$$P(x) = y_0L_0(x) + y_1L_1(x) + y_2L_2(x) + y_3L_3(x)$$

$$L_0 = \prod_{\substack{i=0\\i\neq 0}}^{3}\frac{x-x_i}{x_0-x_i} = \frac{(x-0)(x+6)(x+4)}{(4-0)(4+6)(4+4)} = \frac{x^3+10x^2+24x}{320}$$

$$L_1 = \prod_{\substack{i=0\\i\neq 1}}^{3}\frac{x-x_i}{x_1-x_i} = \frac{(x-4)(x+6)(x+4)}{(0-4)(0+6)(0+4)} = -\frac{x^3+6x^2-16x-96}{96}$$

$$L_2 = \prod_{\substack{i=0\\i\neq 2}}^{3}\frac{x-x_i}{x_2-x_i} = \frac{(x-4)(x-0)(x+4)}{(-6-4)(-6-0)(-6+4)} = -\frac{x^3-16x}{120}$$

$$L_3 = \prod_{\substack{i=0\\i\neq 3}}^{3}\frac{x-x_i}{x_3-x_i} = \frac{(x-4)(x-0)(x+6)}{(-4-4)(-4-0)(-4-6)} = \frac{x^3+2x^2-24x}{64}$$

Reemplazando en el polinomio:

$$\begin{aligned}
P(x) &= 808\frac{x^3+10x^2+24x}{320} - 4\frac{x^3+6x^2-16x-96}{96} - 1438\frac{x^3-16x}{120} + 160\frac{x^3+2x^2-24x}{64} \\
&= 2{,}525x^3 + 25{,}25x^2 + 60{,}6x - 0{,}04166x^3 - 0{,}25x^2 + 0{,}66666x + 4 - 11{,}98333x^3 \\
&+ 191{,}73333x + 2{,}5x^3 + 5x^2 - 60x \\
&= -6{,}99999x^3 + 30x^2 + 192{,}99999x + 4
\end{aligned}$$

Calculamos $R_n(x)$

$$
\begin{aligned}
R_3(x) &= \frac{f^{(4)(\xi(x))}}{4!}(x-4)(x-0)(x+6)(x+4) \\
&= \frac{f^{(4)(\xi(x))}}{24}(x^4+6x^3-16x^2-96x) \\
f(x) &= P_n(x) + R_n(x) \\
f(x) &= P_3(x) + R_3(x)
\end{aligned}
$$

$$
f(x) = -6{,}99999x^3 + 30x^2 + 192{,}99999x + 4 + \frac{f^{(4)}(\xi(x))}{24}(x^4+6x^3-16x^2-96x)
$$

CONCLUSIONES

- El polinomio de Taylor nos ayuda a aproximar funciones cuando se tenga conocimiento de las derivadas
- El polinomio de Lagrange nos da directamente sin necesidad de derivadas

RECOMENDACIONES

- Se recoienda usar la serie de Taylor siempre que se tenga conocimiento de las derivadas.
- Cuando no es posible conocer la derivada se recoimienda usar Lagrange

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