ESCUELA POLITÉCNICA NACIONAL
FACULTAD DE INGENIERÍA DE SISTEMAS
INGENIERÍA EN CIENCIAS DE LA COMPUTACIÓN

PERÍODO ACADÉMICO: 2025-A
ASIGNATURA: ICCD412 Métodos Numéricos **GRUPO:** GR2
TIPO DE INSTRUMENTO: Practica 4
FECHA DE ENTREGA LÍMITE: 2/06/2025
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TEMA

Splines Cúbicos

OBJETIVOS

- Poder comprender el método de interpolacion mediante splines cúbicos.
- Aproximar funciones mediante el uso de splines cúbicos

DESARROLLO

1. Dado los puntos $x = [-2,\ -1,\ 1,\ 3]$, $y = [3,\ 1,\ 2,\ -1]$

   Splines:

$$[-2,\ -1] \quad [-1,\ 1] \quad [1,\ 3]$$

$$
\begin{aligned}
S_0(x) &= a_0 + b_0(x - x_0) + c_0(x - x_0)^2 + d_0(x - x_0)^3 = y_0 \\
S_1(x) &= a_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3 = y_1 \\
S_2(x) &= a_2 + b_2(x - x_2) + c_2(x - x_2)^2 + d_2(x - x_2)^3 = y_2
\end{aligned}
$$

Coincidencian con los puntos de datos.

- $S_0(x)$

$$
\begin{aligned}
S_0(x_0) &= a_0 + b_0(x_0 - x_0) + c_0(x_0 - x_0)^2 + d_0(x_0 - x_0)^3 = y_0 \\
S_0(x_0) &= a_0 + b_0(0) + c_0(0)^2 + d_0(0)^3 = a_0 = 3 \\
a_0 &= 3 \quad (1) \\
S_0(x_1) &= a_0 + b_0(x_1 - x_0) + c_0(x_1 - x_0)^2 + d_0(x_1 - x_0)^3 = y_1 \\
S_0(x_1) &= 3 + b_0(-1 + 2) + c_0(-1 + 2)^2 + d_0(-1 + 2)^3 = 1 \\
S_0(x_1) &= b_0 + c_0 + d_0 = -2 \\
b_0 + c_0 + d_0 &= -2 \quad (2)
\end{aligned}
$$

- $S_1(x)$

$$
\begin{aligned}
S_1(x_1) &= a_1 + b_1(x_1 - x_1) + c_1(x_1 - x_1)^2 + d_1(x_1 - x_1)^3 = y_1 \\
S_1(x_1) &= a_1 + b_1(0) + c_1(0)^2 + d_1(0)^3 = a_1 = 1 \\
a_1 &= 1 \quad (3) \\
S_1(x_2) &= a_1 + b_1(x_2 - x_1) + c_1(x_2 - x_1)^2 + d_1(x_2 - x_1)^3 = y_2 \\
S_1(x_2) &= 1 + b_1(1 + 1) + c_1(1 + 1)^2 + d_1(1 + 1)^3 = 2 \\
S_1(x_2) &= 2b_1 + 4c_1 + 8d_1 = 1 \\
2b_1 + 4c_1 + 8d_1 &= 1 \quad (4)
\end{aligned}
$$

- $S_2(x)$

$$
\begin{aligned}
S_2(x_2) &= a_2 + b_2(x_2 - x_2) + c_2(x_2 - x_2)^2 + d_2(x_2 - x_2)^3 = y_2 \\
S_2(x_2) &= a_2 + b_2(0) + c_2(0)^2 + d_2(0)^3 = 2 \\
a_2 &= 2 \quad (5) \\
S_2(x_3) &= a_2 + b_2(x_3 - x_2) + c_2(x_3 - x_2)^2 + d_2(x_3 - x_2)^3 = y_3 \\
S_2(x_3) &= 2 + b_2(3 - 1) + c_2(3 - 1)^2 + d_2(3 - 1)^3 = -1 \\
S_2(x_3) &= 2b_2 + 4c_2 + 8d_2 = -3 \quad (6)
\end{aligned}
$$

Continuidad de la primera derivada:

$$S_j'(x) = b_j + 2c_j(x - x_j) + 3d_j(x - x_j)^2$$

- $S_0'(x_1) = S_1'(x_1)$

$$
\begin{aligned}
b_0 + 2c_0(x_1 - x_0) + 3d_0(x_1 - x_0)^2 &= b_1 + 2c_1(x_1 - x_1) + 3d_1(x_1 - x_1)^2 \\
b_0 + 2c_0(-1 + 2) + 3d_0(-1 + 2)^2 &= b_1 + 2c_1(0) + 3d_1(0)^2 \\
b_0 + 2c_0 + 3d_0 &= b_1 \quad (7)
\end{aligned}
$$

- $S_1'(x_2) = S_2'(x_2)$

$$b_1 + 2c_1(x_2 - x_1) + 3d_1(x_2 - x_1)^2 = b_2 + 2c_2(x_2 - x_2) + 3d_2(x_2 - x_2)^2$$
$$b_1 + 2c_1(1 + 1) + 3d_1(1 + 1)^2 = b_2 + 2c_2(0) + 3d_2(0)^2$$
$$b_1 + 4c_1 + 12d_1 = b_2 \quad (8)$$

Continuidad de la segunda derivada:

$$S_j''(x) = 2c_j + 6d_j(x - x_j)$$

- $S_0''(x_1) = S_1''(x_1)$

$$2c_0 + 6d_0(x_1 - x_0) = 2c_1 + 6d_1(x_1 - x_1)$$
$$2c_0 + 6d_0(-1 + 2) = 2c_1 + 6d_1(0)$$
$$c_0 + 3d_0 = c_1 \quad (9)$$

- $S_1''(x_2) = S_2''(x_2)$

$$2c_1 + 6d_1(x_2 - x_1) = 2c_2 + 6d_2(x_2 - x_2)$$
$$2c_1 + 6d_1(1 + 1) = 2c_2 + 6d_2(0)$$
$$c_1 + 6d_1 = c_2 \quad (10)$$

a) Determine el spline cúbico con frontera natural

Frontera Natural

$$S_0''(x_0) = S_{n-1}''(x_n) = 0$$

- $S_0''(x_0) = S_2''(x_3) = 0$

$$2c_0 + 6d_0(x_0 - x_0) = 2c_2 + 6d_2(x_3 - x_2) = 0$$
$$2c_0 + 6d_0(0) = 2c_2 + 6d_2(3 - 1) = 0$$
$$c_0 = 0 \quad (11a) \wedge c_2 + 6d_2 = 0 \quad (12a)$$

Resolviendo el sistema de ecuaciones:

Incognitas:

a_0	3	c_1	1.63636
b_0	-2.54545	d_1	-0.46590
c_0	0	a_2	2
d_0	0.54545	b_2	0.04545
a_1	1	c_2	-1.15909
b_1	-0.90909	d_2	0.19318

$$S_0(x) = 3 - 2{,}54545(x + 2) + 0{,}54545(x + 2)^3$$
$$S_1(x) = 1 - 0{,}90909(x + 1) + 1{,}63636(x + 1)^2 - 0{,}46590(x + 1)^3$$
$$S_2(x) = 2 + 0{,}04545(x - 1) - 1{,}15909(x - 1)^2 + 0{,}19318(x - 1)^3$$

$b_0 = -2 - d_0 \qquad 3d_0 = c_1 \qquad b_1 = b_0 + 3d_0$

$b_1 = (-2 - d_0) + 3d_0$

$b_1 = -2 + 2d_0$

$2b_1 + 4c_1 + 8d_1 = 1$

$2(-2 + 2d_0) + 4(3d_0) + 8d_1 = 1$

$d_1 = \frac{1}{8}(5 - 16d_0)$

$c_2 = c_1 + 6d_1$

$c_2 = 3d_0 + 6\left(\frac{1}{8}(5 - 16d_0)\right)$

$c_2 = -9d_0 + \frac{15}{4}$

$b_2 = b_1 + 4c_1 + 12d_1$

$b_2 = (-2 + 2d_0) + 4(3d_0) + 12\left(\frac{1}{8}(5 - 16d_0)\right)$

$b_2 = -10d_0 + \frac{11}{2}$

$2b_2 + 4c_2 + 8d_2 = -3$

$2\left(-10d_0 + \frac{11}{2}\right) + 4\left(-9d_0 + \frac{15}{4}\right) + 8d_2 = -3$

$d_2 = \frac{1}{8}(-29 + 56d_0)$

$c_2 + 6d_2 = 0$

$\left(-9d_0 + \frac{15}{4}\right) + 6\left(\frac{1}{8}(-29 + 56d_0)\right) = 0$

$33d_0 - 18 = 0 \quad ; \quad d_0 = \frac{6}{11}$

$b_0 = -2 - d_0 = -2 - \frac{6}{11} = -\frac{28}{11}$

$b_0 = -\frac{28}{11}$

$b_1 = -2 + 2d_0 = -2 + 2\left(\frac{6}{11}\right) = -\frac{10}{11}$

$b_1 = -\frac{10}{11}$

$c_1 = 3\left(\frac{6}{11}\right) = \frac{18}{11}$

$c_1 = \frac{18}{11}$

$d_1 = \frac{1}{8}(5 - 16d_0)$

$d_1 = \frac{1}{8}\left(5 - 16\left(\frac{6}{11}\right)\right)$

$d_1 = \frac{5}{8} - \frac{12}{11} = \frac{55 - 96}{88}$

$d_1 = -\frac{41}{88}$

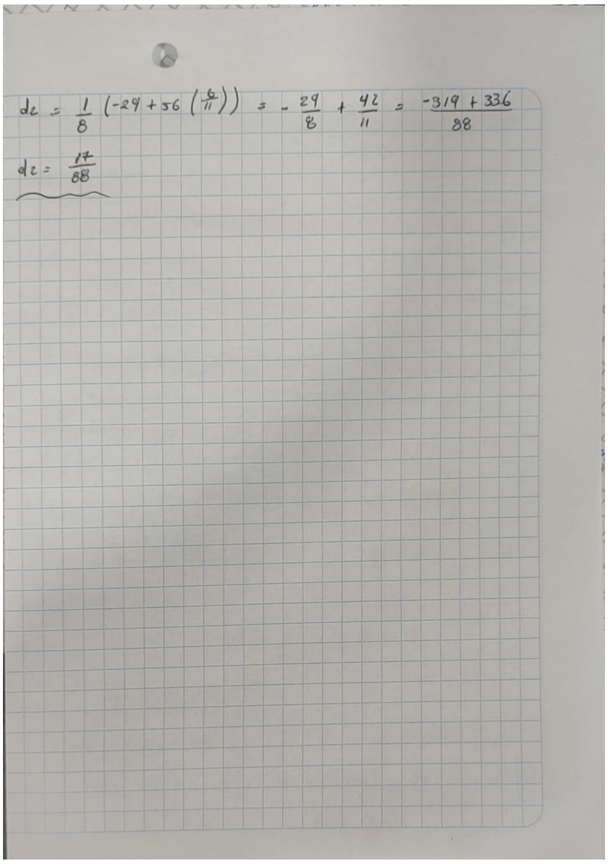
$b_2 = -10d_0 + \frac{11}{2} = -10\left(\frac{6}{11}\right) + \frac{11}{2} = -\frac{60}{11} + \frac{11}{2}$

$b_2 = \frac{-120 + 121}{22} = \frac{1}{22}$

$b_2 = \frac{1}{22}$

$c_2 = -9d_0 + \frac{15}{4} = -9\left(\frac{6}{11}\right) + \frac{15}{4} = \frac{15}{4} - \frac{54}{11} = \frac{165 - 216}{44}$

$c_2 = -\frac{51}{44}$



$$d_2 = \frac{1}{8}\left(-29 + 56\left(\frac{6}{11}\right)\right) = -\frac{29}{8} + \frac{42}{11} = \frac{-319 + 336}{88}$$

$$d_2 = \frac{17}{88}$$

b) Determine el spline cúbico con frontera condicionada

$$B_0 = 1 B_n = -1$$

Frontera Condicionada:

$$S_0'(x_0) = f'(x_0) = B_0$$
$$S_{n-1}'(x_n) = f'(x_n) = B_n$$

- $S_0'(x_0) = f'(x_0) = 1$

$$b_0 + 2c_0(x_0 - x_0) + 3d_0(x_0 - x_0)^2 = 1$$
$$b_0 + 2c_0(0) + 3d_0(0)^2 = 1$$
$$b_0 = 1 \quad (11b)$$

- $S_2'(x_3) = f'(x_3) = -1$

$$b_2 + 2c_2(x_3 - x_2) + 3d_2(x_3 - x_2)^2 = -1$$
$$b_2 + 2c_2(3 - 1) + 3d_2(3 - 1)^2 = -1$$
$$b_2 + 4c_2 + 12d_2 = -1 \quad (12b)$$

Resolviendo el sistema de ecuaciones: Incognitas:

$a_0 = 3$ $\quad$ $b_0 + 2c_0 + 3d_0 = b_1$

$b_0 + c_0 + d_0 = -2$ $\quad$ $b_1 + 4c_1 + 12d_1 = b_2$

$a_1 = 1$ $\quad$ $c_0 + 3d_0 = c_1$

$2b_1 + 4c_1 + 8d_1 = 1$ $\quad$ $c_1 + 6d_1 = c_2$

$a_2 = 2$ $\quad$ $b_0 = 1$

$2b_2 + 4c_2 + 8d_2 = -3$ $\quad$ $b_2 + 4c_2 + 12d_2 = -1$

$c_0 + d_0 = -3$ $\quad$ $b_1 = 1 + 2(-3 - d_0) + 3d_0$

$c_0 = -3 - d_0$ $\quad$ $\cancel{b_1 =}$

$b_1 = 1 - 6 - 2d_0 + 3d_0$

$b_1 = -5 + d_0$

$c_1 = (-3 - d_0) + 3d_0$

$c_1 = 2d_0 - 3$

$2(-5 + d_0) + 4(2d_0 - 3) + 8d_1 = 1$

$8d_1 = 1 - 2(-5 + d_0) - 4(2d_0 - 3)$

$8d_1 = 1 + 10 - 2d_0 - 8d_0 + 12$

$6d_1 = \cancel{8}\ 23 - 10d_0$

$d_1 = \frac{1}{8}(23 - 10d_0) = 2.875 - 1.25d_0.$

$(-5 + d_0) + 4(2d_0 - 3) + 12(2.875 - 1.25d_0) = b_2$

$-5 + d_0 + 8d_0 - 12 + 34.5 - 15d_0 = b_2$

$-6d_0 + 17.5 = b_2$

$(2d_0 - 3) + 6(2.875 - 1.25d_0) = c_2$

$2d_0 - 3 + 17.25 - 7.5d_0 = c_2$

$c_2 = 14.25 - 5.5d_0$

$2(-6d_0 + 17.5) + 4(14.25 - 5.5d_0) + 8d_2 = -3$

$-12d_0 + 35 + 57 - 22d_0 + 8d_2 = -3$

$-34d_0 + 8d_2 = -95$

$8d_2 = 34d_0 - 95$

$d_2 = 4.25d_0 - 11.875$

$-6d_0 + 17.5 + 4(14.25 - 5.5d_0) + 12(4.25d_0 - 11.875) = -1$

$-6d_0 + 17.5 + 57 - 22d_0 + 51d_0 - 142.5 = -1$

$23d_0 = 67$

$d_0 = \frac{67}{23} = 2.913043$

$b_0 = 1$ ~~$c_0 = -3 - d_0 = -3 - 2.91304$~~
~~$c_0 = 0.08696$~~

$c_0 = -3 - d_0 = -3 - 2.91304$

$c_0 = -5.91304$

$b_1 = -5 + d_0 = -5 + 2.91304$

$b_1 = -2.08696$

$c_1 = 2d_0 - 3 = 2(2.91304) - 3$

$c_1 = 2.82608$

$d_1 = 2.875 - 1.25 d_0 = 2.875 - 1.25(2.91304)$

$d_1 = -0.76630$

$b_2 = -6d_0 + 17.5 = -6(2.91304) + 17.5 = 0.02176$

$b_2 = 0.02176$

$c_2 = 14.25 - 5.5 d_0 = 14.25 - 5.5(2.91304)$

$c_2 = -1.77172$

$d_2 = 4.25 d_0 - 11.875$

$d_2 = 0.50542$

a_0	3	c_1	2.82608
b_0	1	d_1	-0.76630
c_0	-5.91304	a_2	2
d_0	2.91304	b_2	0.02173
a_1	1	c_2	-1.77173
b_1	-2.08695	d_2	0.50543

$$
\begin{aligned}
S_0(x) &= 3 + 1(x+2) - 5{,}91304(x+2)^2 + 2{,}91304(x+2)^3 \\
S_1(x) &= 1 - 2{,}08695(x+1) + 2{,}82608(x+1)^2 - 0{,}76630(x+1)^3 \\
S_2(x) &= 2 + 0{,}02173(x-1) - 1{,}77173(x-1)^2 + 0{,}50543(x-1)^3
\end{aligned}
$$

2. Dado los puntos $(0,1); (1,5); (2,3)$, determine el spline cúbico.

$$x = [0,1,2] \quad y = [1,5,3]$$

$$
\begin{aligned}
S_0(x) &= a_0 + b_0(x-x_0) + c_0(x-x_0)^2 + d_0(x-x_0)^3 = y_0 \\
S_1(x) &= a_1 + b_1(x-x_1) + c_1(x-x_1)^2 + d_1(x-x_1)^3 = y_1
\end{aligned}
$$

Coincidencia con los puntos de datos.

- $S_0(x)$

$$
\begin{aligned}
S_0(x_0) &= a_0 + b_0(x_0-x_0) + c_0(x_0-x_0)^2 + d_0(x_0-x_0)^3 = y_0 \\
S_0(x_0) &= a_0 + b_0(0) + c_0(0)^2 + d_0(0)^3 = a_0 = 1 \\
a_0 &= 1 \quad (1) \\
S_0(x_1) &= a_0 + b_0(x_1-x_0) + c_0(x_1-x_0)^2 + d_0(x_1-x_0)^3 = y_1 \\
S_0(x_1) &= 1 + b_0(1-0) + c_0(1-0)^2 + d_0(1-0)^3 = 5 \\
S_0(x_1) &= b_0 + c_0 + d_0 = 4 \\
b_0 + c_0 + d_0 &= 4 \quad (2)
\end{aligned}
$$

- $S_1(x)$

$$
\begin{aligned}
S_1(x_1) &= a_1 + b_1(x_1-x_1) + c_1(x_1-x_1)^2 + d_1(x_1-x_1)^3 = y_1 \\
S_1(x_1) &= a_1 + b_1(0) + c_1(0)^2 + d_1(0)^3 = a_1 = 5 \\
a_1 &= 5 \quad (3) \\
S_1(x_2) &= a_1 + b_1(x_2-x_1) + c_1(x_2-x_1)^2 + d_1(x_2-x_1)^3 = y_2 \\
S_1(x_2) &= 5 + b_1(2-1) + c_1(2-1)^2 + d_1(2-1)^3 = 3 \\
S_1(x_2) &= b_1 + c_1 + d_1 = -2 \\
b_1 + c_1 + d_1 &= -2 \quad (4)
\end{aligned}
$$

Continuidad de la primera derivada:

$$S'_j(x) = b_j + 2c_j(x-x_j) + 3d_j(x-x_j)^2$$

- $S'_0(x_1) = S'_1(x_1)$

$$
\begin{aligned}
b_0 + 2c_0(x_1-x_0) + 3d_0(x_1-x_0)^2 &= b_1 + 2c_1(x_1-x_1) + 3d_1(x_1-x_1)^2 \\
b_0 + 2c_0(1-0) + 3d_0(1-0)^2 &= b_1 + 2c_1(0) + 3d_1(0)^2 \\
b_0 + 2c_0 + 3d_0 &= b_1 \quad (7)
\end{aligned}
$$

Continuidad de la segunda derivada:

$$S''_j(x) = 2c_j + 6d_j(x-x_j)$$

a_0	1
b_0	5.5
c_0	0
d_0	-1.5
a_1	5
b_1	1
c_1	-4.5
d_1	1.5

- $S_0''(x_1) = S_1''(x_1)$

$$2c_0 + 6d_0(x_1 - x_0) = 2c_1 + 6d_1(x_1 - x_1)$$
$$2c_0 + 6d_0(1 - 0) = 2c_1 + 6d_1(0)$$
$$c_0 + 3d_0 = c_1 \quad (9)$$

por Frontera Natural:

$$S_0''(x_0) = S_1''(x_2) = 0$$
$$2c_0 + 6d_0(x_0 - x_0) = 2c_1 + 6d_1(x_2 - x_1) = 0$$
$$2c_0 + 6d_0(0) = 2c_2 + 6d_2(2 - 1) = 0$$
$$2c_0 = 2c_1 + 6d_1(1) = 0$$
$$c_0 = 2c_1 + 6d_1 = 0$$
$$c_0 = 0 \quad (7) \wedge c_1 + 3d_1 = 0 \quad (8)$$

Incognitas:

$$S_0(x) = 1 + 5{,}5\,x - 1{,}5\,x^3$$
$$S_1(x) = 5 + 1(x-1) - 4{,}5(x-1)^2 + 1{,}5(x-1)^3$$

3. Dado los puntos $(-1, 1); (1, 3); (0{,}5, 4{,}8)$, determine el spline cúbico sabiendo que

$$f'(x_0) = 1, \quad f'(x_n) = 2.$$

$$x = [-1, 0{,}5, 1] \quad y = [1, 4{,}8, 3]$$
$$S_0(x) = a_0 + b_0(x - x_0) + c_0(x - x_0)^2 + d_0(x - x_0)^3 = y_0$$
$$S_1(x) = a_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3 = y_1$$

Coincidencia con los puntos de datos.

- $S_0(x)$

$$\begin{aligned}
S_0(x_0) &= a_0 + b_0(x_0 - x_0) + c_0(x_0 - x_0)^2 + d_0(x_0 - x_0)^3 = y_0 \\
S_0(x_0) &= a_0 + b_0(0) + c_0(0)^2 + d_0(0)^3 = a_0 = 1 \\
a_0 &= 1 \quad (1) \\
S_0(x_1) &= a_0 + b_0(x_1 - x_0) + c_0(x_1 - x_0)^2 + d_0(x_1 - x_0)^3 = y_1 \\
S_0(x_1) &= 1 + b_0(0{,}5 + 1) + c_0(0{,}5 + 1)^2 + d_0(0{,}5 + 1)^3 = 4{,}8 \\
S_0(x_1) &= 1{,}5b_0 + 2{,}25c_0 + 3{,}375d_0 = 3{,}8 \\
1{,}5b_0 &+ 2{,}25c_0 + 3{,}375d_0 = 3{,}8 \quad (2)
\end{aligned}$$

- $S_1(x)$

$$\begin{aligned}
S_1(x_1) &= a_1 + b_1(x_1 - x_1) + c_1(x_1 - x_1)^2 + d_1(x_1 - x_1)^3 = y_1 \\
S_1(x_1) &= a_1 + b_1(0) + c_1(0)^2 + d_1(0)^3 = a_1 = 4{,}8 \\
a_1 &= 4{,}8 \quad (3) \\
S_1(x_2) &= a_1 + b_1(x_2 - x_1) + c_1(x_2 - x_1)^2 + d_1(x_2 - x_1)^3 = y_2 \\
S_1(x_2) &= 4{,}8 + b_1(1 - 0{,}5) + c_1(1 - 0{,}5)^2 + d_1(1 - 0{,}5)^3 = 3 \\
S_1(x_2) &= 0{,}5b_1 + 0{,}25c_1 + 0{,}125d_1 = -1{,}8 \\
0{,}5b_1 &+ 0{,}25c_1 + 0{,}125d_1 = -1{,}8 \quad (4)
\end{aligned}$$

Continuidad de la primera derivada:

$$S'_j(x) = b_j + 2c_j(x - x_j) + 3d_j(x - x_j)^2$$

- $S'_0(x_1) = S'_1(x_1)$

$$\begin{aligned}
b_0 + 2c_0(x_1 - x_0) + 3d_0(x_1 - x_0)^2 &= b_1 + 2c_1(x_1 - x_1) + 3d_1(x_1 - x_1)^2 \\
b_0 + 2c_0(0{,}5 + 1) + 3d_0(0{,}5 + 1)^2 &= b_1 + 2c_1(0) + 3d_1(0)^2 \\
b_0 + 3c_0 + 6{,}75d_0 &= b_1 \quad (5)
\end{aligned}$$

Continuidad de la segunda derivada:

$$S''_j(x) = 2c_j + 6d_j(x - x_j)$$

- $S''_0(x_1) = S''_1(x_1)$

$$\begin{aligned}
2c_0 + 6d_0(x_1 - x_0) &= 2c_1 + 6d_1(x_1 - x_1) \\
2c_0 + 6d_0(0{,}5 + 1) &= 2c_1 + 6d_1(0) \\
2c_0 + 9d_0 &= 2c_1 \quad (6)
\end{aligned}$$

por Frontera Condicionada:

- $S'_0(x_0) = f'(x_0) = B_0$

$$\begin{aligned}
&b_0 + 2c_0(x_0 - x_0) + 3d_0(x_0 - x_0)^2 = 1 \\
&b_0 + 2c_0(0) + 3d_0(0)^2 = 1 \\
&b_0 = 1
\end{aligned}$$

a_0	1
b_0	1
c_0	6.38333
d_0	-3.57
a_1	4.8
b_1	-3.975
c_1	-9.7
d_1	20.9

- $S_1'(x_2) = f'(x_2) = 2$

$$\begin{aligned}
&b_2 + 2c_1(x_2 - x_1) + 3d_1(x_2 - x_1)^2 = 2 \\
&b_2 + 2c_1(1 - 0{,}5) + 3d_1(1 - 0{,}5)^2 = 2 \\
&b_2 + c_1 + 0{,}75d_1 = 2 \quad (8)
\end{aligned}$$

Incognitas:

$$\begin{aligned}
S_0(x) &= 1 + 1(x + 1) + 6{,}38333(x + 1)^2 - 3{,}57407(x + 1)^3 \\
S_1(x) &= 4{,}8 - 3{,}975(x - 0{,}5) - 9{,}7(x - 0{,}5)^2 + 20{,}9(x - 0{,}5)^3
\end{aligned}$$