Díaz Hernández Marcos Bryan

1) Sea la superficie $S: x^2+y^2=z+1$

a) Vector normal S en coordenadas cilíndricas

$x^2+y^2=z+1$

$(r\cos\theta)^2+(r\,sen\theta)^2=z+1$

$r^2(1)=z+1$

$r^2=z+1$

$z=r^2-1$

$x=r\cos\theta$
$y=r\,sen\theta$
$z=z$

$\hat{r}=\cos\theta\hat{i}+sen\theta\hat{j}$ $\quad h_r=1$
$\hat{\theta}=-sen\theta\hat{i}+\cos\theta\hat{j}$ $\quad h_\theta=r$
$\hat{z}=\hat{k}$ $\quad h_z=1$

$S: x\hat{i}+y\hat{j}+z\hat{k} \longrightarrow r\cos\theta\hat{i}+r\,sen\theta\hat{j}+z\hat{k}=S$

$S=r(\cos\theta\hat{i}+sen\theta\hat{j})+z\hat{z}$

$S=r(\hat{r})+z\hat{z}$

S depende de: $r,\theta$ $\therefore$ $S=r\hat{r}+(r^2-1)\hat{z}$

$$\frac{\partial s}{\partial r}=\frac{\partial}{\partial r}\left(r\hat{r}+(r^2-1)\hat{z}\right)=\hat{r}\frac{\partial r}{\partial r}+\hat{z}\frac{\partial(r^2-1)}{\partial r}$$

$$=\hat{r}+\hat{z}(2r)$$

$$\frac{\partial s}{\partial\theta}=\frac{\partial}{\partial\theta}\left(r\hat{r}+(r^2-1)\hat{z}\right)=(r)\frac{\partial}{\partial\theta}\left(\cos\theta\hat{i}+sen\theta\hat{j}\right)+\frac{\partial}{\partial\theta}\cancel{(r^2-1)}^{0}\hat{z}$$

$$=r(-sen\theta\hat{i}+\cos\theta\hat{j})+0=r\hat{\theta}$$

$$\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{z}\\ 1 & 0 & 2r\\ 0 & r & 0\end{vmatrix}=-2r^2\hat{r}+r\hat{z}$$

b) Divergencia $\nabla\cdot n$ en cilíndricas: $P=-2r^2,\ Q=0,\ R=r$

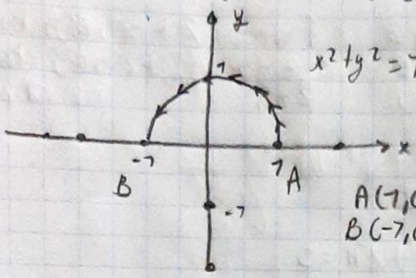
$$\frac{1}{h_rh_\theta h_z}\left(\frac{\partial}{\partial r}h_\theta h_zP+\frac{\partial}{\partial\theta}h_rh_zQ+\frac{\partial}{\partial z}h_rh_\theta R\right)$$

$$=\frac{1}{r}\left(\frac{\partial}{\partial r}(r)(-2r^2)+\frac{\partial}{\partial\theta}\cancel{(1)}^{0}(0)+\frac{\partial}{\partial z}\cancel{(r)}^{0}(r)\right)=\frac{1}{r}(-6r^2)=-6r$$

Díaz Hernández Marcos Bryan

2) Calcula la integral de Línea de campo vectorial

$F(x,y) = (x+2y)\hat{i} + (x-y)\hat{j}$ , a lo largo de la trayectoria:




$x^2+y^2=1$

A(1,0)
B(-1,0)

Parametrizar:

$$x^2+y^2=1$$
$$\cos^2 t + \operatorname{sen}^2 t = 1 \quad \begin{cases} \cos t = x \\ \operatorname{sen} t = y \end{cases}$$

$$r(t) = x\hat{i} + y\hat{j} \rightarrow \cos t\,\hat{i} + \operatorname{sen} t\,\hat{j} = r(t)$$
$$r'(t) = -\operatorname{sen}(t)\hat{i} + \cos(t)\hat{j}$$

$$F(r(t)) = F(\cos t, \operatorname{sen} t) = (\cos t + 2\operatorname{sen} t)\hat{i} + (\cos t - \operatorname{sen} t)\hat{j}$$

valores de t: $\left.\begin{matrix}\cos t = 1 \\ \operatorname{sen} t = 0\end{matrix}\right\} \begin{matrix} t=0 \\ A \end{matrix}$ $\quad \left.\begin{matrix}\cos t = -1 \\ \operatorname{sen} t = 0\end{matrix}\right\} \begin{matrix} t=\pi \\ B \end{matrix}$

$$F(r(t)) \cdot r'(t) = (\cos t + 2\operatorname{sen} t)\hat{i} + (\cos t - \operatorname{sen} t)\hat{j} \cdot (-\operatorname{sen}(t)\hat{i} + \cos t\,\hat{j})$$
$$= -\operatorname{sen} t\cos t - 2\operatorname{sen}^2 t + \cos^2 t - \operatorname{sen} t\cos t$$
$$= -2\operatorname{sen} t\cos t - 2\operatorname{sen}^2 t + \cos^2 t$$

$$I = \int_0^{\pi} -2\operatorname{sen} t\cos t - 2\operatorname{sen}^2 t + \cos^2 t \, dt$$

$$\cos^2 t\Big|_0^{\pi} - 2\int_0^{\pi} \frac{1}{2} - \frac{\cos(2t)}{2}\,dt + \int_0^{\pi} \frac{1}{2} + \frac{\cos(2t)}{2}\,dt$$

$$-2\left(\frac{t}{2} + \frac{\operatorname{sen}(2t)}{(2)(2)}\right)\Big|_0^{\pi} + \left(\frac{t}{2} - \frac{\operatorname{sen}(2t)}{4}\Big|_0^{\pi}\right)$$

$$-(\pi) + (\pi/2)$$
$$= \pi/2 - \pi = -\pi/2$$