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7) Mediante Multiplicadores de Lagrange obten el punto mas cercano al origen

Res: $x + 2y + 3z = 6$

Fo: $d(x,y,z) = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$

1) $\frac{\partial f}{\partial x} = 2x = \alpha$

2) $\frac{df}{dy} = 2y = 2\alpha$

3) $\frac{df}{dy} = 2z = 3\alpha$

4) $x + 2y + 3z - 6 = 0$

$\left.\begin{array}{l} 2x = \alpha \\ 2y = 2\alpha \\ 2z = 3\alpha \end{array}\right\}$ $y = \alpha$ $\begin{array}{l} 2x = y \\ 2z = 3y \\ 2z = 6x \\ z = 3x \end{array}$ $\begin{array}{l} y = 2x \\ z = 3x \end{array}$

en 4) $x + 4x + 9x - 6 = 0$

$14x = 6$

$x = 6/14$

$x = 3/7$

$\left.\begin{array}{l} x = 3/7 \\ y = 6/7 \\ z = 9/7 \end{array}\right\}$ Solución

$3/7 + 12/7 + 27/7 - 6 = 0$

$\frac{15+27}{7} = \frac{42}{7} = 6$

$P_0 (3/7, 6/7, 9/7)$ } solución

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2) $r(t) = e^t\hat{i} + te^t\hat{j} + e^t\hat{k}$ [m] $T(s)$ $P(1,0,1)$

$r'(t) = e^t\hat{i} + [(te^t) + e^t]\hat{j} + e^t\hat{k}$ $\qquad t = e^t = 1 \quad \ln(e^t) = \ln(1)$

$te^t = 0 \qquad t = 0$

$r''(t) = e^t\hat{i} + [te^t + e^t + e^t]\hat{j} + e^t\hat{k}$ $\qquad e^t = 1$

$r'''(t) = e^t\hat{i} + [te^t + e^t + e^t + e^t]\hat{j} + e^t\hat{k}$

$\bar{r}'(0) = \hat{i} + \hat{j} + \hat{k}$

$\bar{r}''(0) = \hat{i} + 2\hat{j} + \hat{k}$ $\qquad \hat{T} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ a)

$\bar{r}'''(0) = \hat{i} + 3\hat{j} + \hat{k}$

$$\hat{B} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \frac{-\hat{i} + 0\hat{j} + \hat{k}}{\sqrt{2}} = \frac{-\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$$ a)

$$\hat{N} = \begin{vmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}} = \frac{-\hat{i}}{\sqrt{6}} + \frac{2\hat{j}}{\sqrt{6}} - \frac{\hat{k}}{\sqrt{6}}$$ a)

$\bar{c} = \bar{p} + \rho\hat{N}$ $\quad \rho = 1/k$

$k = \frac{\sqrt{2}}{3\sqrt{3}}$ $\quad \rho = \frac{3\sqrt{3}}{\sqrt{2}}$

$$c = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \frac{3\sqrt{3}}{\sqrt{2}}\begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{matrix} \frac{-3\sqrt{3}}{2\sqrt{3}} \\ \frac{6\sqrt{3}}{2\sqrt{3}} \\ \frac{-3\sqrt{3}}{2\sqrt{3}} \end{matrix}$$

$$C = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{bmatrix} -3/2 \\ 3 \\ -3/2 \end{bmatrix}$$

$$C = \begin{bmatrix} -1/2 \\ 3 \\ -1/2 \end{bmatrix}$$ c)

$r'' = \bar{a}_T + \bar{a}_N$

$$\bar{a}_T = \text{Proy}_{\bar{r}'}\bar{r}'' = \frac{\bar{r}'' \cdot \bar{r}'}{\bar{r}' \cdot \bar{r}'}\bar{r}'$$

$$= \frac{(1,1,1) \cdot (1,2,1)}{(1,1,1) \cdot (1,1,1)}(1,1,1)$$

$\bar{a}_T = \frac{4}{3}(1,1,1) = (4/3, 4/3, 4/3)$: componente tangencial a)

$\bar{a}_N = \bar{r}'' - a_T = (1,2,1) - (4/3, 4/3, 4/3) = (-1/3, 2/3, -1/3)$: comp. normal