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1) Resuelve la integral de superficie de la esfera $x^2+y^2+z^2=9$ entre $z=0$ y $z=\sqrt{5}$

$S=\iint_{\Omega} \left\| \frac{d\bar{r}}{du} \times \frac{d\bar{r}}{dv} \right\| du\,dv$

$\rho^2\cos^2\theta\,sen^2\varphi+\rho^2 sen^2\theta\,sen^2\varphi+\rho^2\cos^2\varphi=9$

$\rho^2 sen^2\varphi(\cos^2\theta+sen^2\theta)+\rho^2\cos^2\varphi=9$

$\rho^2(sen^2\varphi+\cos^2\varphi)=9$

$\rho^2=9$

$\rho=3$

$x=\rho\cos\theta\,sen\varphi$

$y=\rho\,sen\theta\,sen\varphi$

$z=\rho\cos\varphi$

$\bar{r}(\theta,\varphi)=x\hat{i}+y\hat{j}+z\hat{k}=3\cos\theta\,sen\varphi\,\hat{i}+3\,sen\theta\,sen\varphi\,\hat{j}+3\cos\varphi\,\hat{k}$

$\frac{d\bar{r}}{d\theta}=-3\,sen\theta\,sen\varphi\,\hat{i}+3\cos\theta\,sen\varphi\,\hat{j}+0\hat{k}$

$\frac{d\bar{r}}{d\varphi}=3\cos\theta\cos\varphi\,\hat{i}+3\,sen\theta\cos\varphi\,\hat{j}-3\,sen\varphi\,\hat{k}$

$\begin{vmatrix} -3\,sen\theta\,sen\varphi & 3\cos\theta\,sen\varphi & 0 \\ 3\cos\theta\cos\varphi & 3\,sen\theta\cos\varphi & -3\,sen\varphi \end{vmatrix}=(-9\cos\theta\,sen^2\varphi)\hat{i}-(9\,sen\theta\,sen^2\varphi)\hat{j}+(-9\,sen^2\theta\,sen\varphi\cos\varphi-9\cos^2\theta\cos\varphi\,sen\varphi)$

$\bar{n}=-9\cos\theta\,sen^2\varphi\,\hat{i}-9\,sen\theta\,sen^2\varphi\,\hat{j}-9\,sen\varphi\cos\varphi\,\hat{k}$

$\|\bar{n}\|=\sqrt{81\cos^2\theta\,\underline{sen^4\varphi}+81\,sen^2\theta\,\underline{sen^4\varphi}+81\,sen^2\varphi\cos^2\varphi}$

$\|\bar{n}\|=9\sqrt{sen^4\varphi+sen^2\varphi\cos^2\varphi}$

$\|\bar{n}\|=9\sqrt{sen^2\varphi(sen^2\varphi+\cos^2\varphi)}=9\sqrt{sen^2\varphi}$

$\|\bar{n}\|=9\,sen\varphi$

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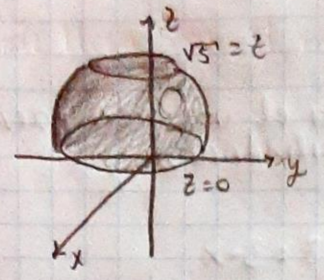

$z = 3\cos\varphi$

1) $z=0$ $0 = 3\cos\varphi$
$\arg\cos(0) = \varphi$
$\varphi = \pi/2$

2) $z=\sqrt{5}$ $\sqrt{5} = 3\cos\varphi$
$\arg\cos\left(\frac{\sqrt{5}}{3}\right) = \varphi$
$\varphi = 41.81$

$$\lim \begin{cases} 0 \le \rho \le 3 \\ 41.81 \le \varphi \le \pi/2 \\ 0 \le \theta \le 2\pi \end{cases}$$

$$\int_0^{2\pi}\int_{\arg\cos\left(\frac{\sqrt{5}}{3}\right)}^{\pi/2} 9\,\mathrm{sen}\,\varphi \; d\varphi\, d\theta =$$

$$\int_0^{2\pi} -9\cos\varphi \Big|_{\arg\cos\left(\frac{\sqrt{5}}{3}\right)}^{\pi/2} d\theta = \int_0^{2\pi} -9\left[\cos(\pi/2) - \cos\left(\arg\cos\left(\frac{\sqrt{5}}{3}\right)\right)\right] d\theta$$

$$-9\int_0^{2\pi}\left[0 - \frac{\sqrt{5}}{3}\right] d\theta = \frac{9\sqrt{5}}{3}\int_0^{2\pi} 1\, d\theta = 3\sqrt{5}\,[2\pi - 0] = \underline{6\sqrt{5}\,\pi \; u^2}$$

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2) Mediante la integral triple, calcula el volumen encerrado en la porción de cilindro $\rho = \cos\theta$ (cilíndricas) acotado por $z=0$ y $x+z=3$

$r = \cos\theta$

$x = r\cos\theta$
$y = r\,\text{sen}\,\theta$
$z = z$

$h_r = 1$
$h_\theta = r$
$h_z = 1$

$$\int_{c_1}^{c_2}\int_{g_2}^{g_1}\int_{r_1}^{r_2} h_r h_\theta h_z \, dr\, d\theta\, dz$$

$0 \le z \le 3 - r\cos\theta$
$0 \le \theta \le 2\pi$
$0 \le r \le \cos\theta$

$$\int_0^{2\pi}\int_0^{3-r\cos\theta}\int_0^{\cos\theta} r\, dr\, dz\, d\theta$$

∴ la "r" de dz no se elimina después

$$\int_0^{2\pi}\int_0^{\cos\theta}\int_0^{3-r\cos\theta} r\, dz\, dr\, d\theta = \int_0^{2\pi}\int_0^{\cos\theta} r\left(z\Big|_0^{3-r\cos\theta}\right) dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{\cos\theta} r(3 - r\cos\theta)\, dr\, d\theta = \int_0^{2\pi}\int_0^{\cos\theta} 3r - r^2\cos\theta \, dr\, d\theta$$

$$\int_0^{2\pi}\left[\frac{3r^2}{2} - \frac{r^3\cos\theta}{3}\right]\Big|_0^{\cos\theta} d\theta = \int_0^{2\pi}\left[\frac{3}{2}\cos^2\theta - \frac{\cos^4\theta}{3}\right] - [0-0]\, d\theta$$

$$\int_0^{2\pi} \frac{3}{2}\cos^2\theta - \frac{\cos^4\theta}{3}\, d\theta \qquad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

$$\underbrace{\int_0^{2\pi} \frac{3}{2}\left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right) d\theta}_{[\text{uno}]} - \underbrace{\int_0^{2\pi} \frac{\cos^4\theta}{3}\, d\theta}_{[\text{dos}]}$$

$$\frac{3}{2}\int_0^{2\pi} \frac{1}{2}\theta + \frac{1}{4}\operatorname{sen}(2\theta) = \frac{3}{2}\left(\frac{1}{2}\theta + \frac{1}{4}\operatorname{sen}(2\theta)\right)\Big|_0^{2\pi} \quad [\text{Uno}]$$

$$\frac{3}{2}\left(\pi + \frac{\operatorname{sen}(4\pi)}{2}\right) - (0+0) = \frac{3\pi}{2}$$

$$-\frac{1}{3}\int_0^{2\pi} \cos^4\theta \, d\theta = -\frac{1}{3}\int_0^{2\pi} (\cos^2\theta)^2 \, d\theta \quad [\text{Dos}]$$

$$(\cos^2\theta)^2 = \frac{1}{2} + \frac{1}{2}\cos(2\theta) = \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right)^2$$

$$\frac{1}{4} + \frac{1}{2}\cos(2\theta) + \frac{1}{4}\cos^2(2\theta)$$

$$-\frac{1}{3}\int_0^{2\pi} \frac{1}{4} + \frac{1}{2}\cos(2\theta) + \frac{1}{4}\cos^2(2\theta)\, d\theta$$

$$-\frac{1}{3}\left[\frac{1}{4}\theta + \frac{1}{4}\operatorname{sen}(2\theta)\right]\Big|_0^{2\pi} = -\frac{1}{3}\left[\frac{\pi}{2}\right]$$

$$-\frac{1}{3}\int_0^{2\pi} \frac{1}{4}\cos^2(2\theta)\, d\theta = -\frac{1}{12}\pi$$

$$-\frac{1}{3}\int_0^{2\pi} \cos^4(\theta)\, d\theta = -\frac{1}{3}\frac{\pi}{2} - \frac{1}{12}\pi = \pi\left(-\frac{1}{6} - \frac{1}{12}\right) = \pi\left(-\frac{3}{12}\right)$$

$$V = \frac{3\pi}{2} - \frac{\pi 3}{12} = \frac{18\pi - 3\pi}{12} = \underline{\frac{15\pi}{12}[u^3]}$$