Actividad 8
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

Actividad.

1) Resolver el problema del valor inicial haciendo uso de la transformada de Laplace.

$z'' + 16z = \cos(4t) \quad z(0)=0 \quad z'(0)=1$

$\mathcal{L}\{z''\} + \mathcal{L}16\{z\} = \mathcal{L}\{\cos(4t)\}$

$\mathcal{L}\{z''\} = s^2F(s) - sF(0) - F'(0) = s^2 z(s) - s\cancel{z(0)}^{0} - z'(0)^{1}$

$\mathcal{L}\{z\} = z(s)$

$\mathcal{L}\{\cos(4t)\} = \left(\frac{s}{s^2+16}\right)$

$s^2 z(s) - 0 - 1 + 16 z(s) = \frac{s}{s^2+16}$

$z(s)(s^2+16) = \frac{s}{s^2+16} + 1$

$z(s) = \left(\frac{s+s^2+16}{s^2+16}\right)\left(\frac{1}{s^2+16}\right) = \frac{s+s^2+16}{(s^2+16)^2}$

$z(s) = \mathcal{L}^{-1}\left\{\frac{s+s^2+16}{(s^2+16)^2}\right\}$

$\frac{s^2+s+16}{(s^2+16)^2} = \frac{As+B}{s^2+16} + \frac{Cs+D}{(s^2+16)^2}$

$s^2+s+16 = (As+B)(s^2+16) + Cs + D$

$s^2+s+16 = \underline{As^3} + \underline{16As} + \underline{Bs^2} + \underline{16B} + \underline{Cs} + \underline{D}$

$s^2 = Bs^2 = 1 \rightarrow B = 1$

$0s^3 = As^3 = 0 \rightarrow A = 0$

$s = s(16A + C) \quad C = 1$

$1 = 16A + C \quad D = 0$

$C = 1$

$16 = 16B + D$

$16 = 16 + D$

$z(s) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+16} + \frac{s}{(s^2+16)^2}\right\}$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+16}\right\} = \frac{1}{4}\,\text{sen}(4t)$

$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+16)^2}\right\} = \frac{1}{8}\,t\,\text{sen}(4t)$

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$$y(t) = \frac{1}{4}\operatorname{sen}(4t) + \frac{1}{8}t\operatorname{sen}(4t)$$ #

2) $\mathcal{L}\{t^4 \operatorname{sen}(2t)\}$

$$\mathcal{L}\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$= (-1)^4 \frac{d^4}{ds^4} \mathcal{L}\{\operatorname{sen}(2t)\} = \frac{d^4}{ds^4}\left\{\frac{2}{s^2+4}\right\} = 2(s^2+4)^{-1}$$

$$2\frac{d^4}{ds^4}(s^2+4)^{-1} = \frac{d}{ds}2(s^2+4)^{-1} = 2(-1)(s^2+4)^{-2}(2s)$$

$$-\frac{d}{ds}\frac{4s}{(s^2+4)^2} = \frac{(s^2+4)^{\cancel{2}}(4) - (4s)(2)\cancel{(s^2+4)}(2s)}{(s^2+4)^4}$$

$$= \frac{(s^2+4)(4) - (16s^2)}{(s^2+4)^3} = \frac{4s^2+16-16s^2}{(s^2+4)^3} = \frac{-12s^2+16}{(s^2+4)^3}$$

$$\frac{d}{ds}\frac{12s^2}{(s^2+4)^3} = \frac{(s^2+4)^{\cancel{3}}(24s) - (12s^2)(3)\cancel{(s^2+4)^2}(2s)}{(s^2+4)^6}$$

$$= \frac{(s^2+4)(24s) - 72s^3}{(s^2+4)^4} = \frac{24s^3+96s-72s^3}{(s^2+4)^4}$$

$$= \frac{-48s^3+96s}{(s^2+4)^4}$$

$$\frac{d}{ds}\left(-\frac{48s^3}{(s^2+4)^4} + \frac{96s}{(s^2+4)^4}\right)$$

$$-\frac{d}{ds}\frac{48s^3}{(s^2+4)^4} = \frac{(s^2+4)^{\cancel{4}}(144s^2) - (48s^3)(4)\cancel{(s^2+4)^3}(2s)}{(s^2+4)^8}$$

$$= \frac{(s^2+4)(144s^2) - (48s^3)(8s)}{(s^2+4)^5} = \frac{144s^4 - 384s^4 + 576s^2}{(s^2+4)^5}$$

Actividad 8
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$$\frac{240s^4-576s^2}{(s^2+4)^5} + \frac{d}{ds}\frac{96s}{(s^2+4)^4}$$

$$\frac{d}{ds}\left(\frac{96s}{(s^2+4)^4}\right) = \frac{(s^2+4)^{\cancel{4}}(96)-(96s)(4)\cancel{(s^2+4)^3}(2s)}{(s^2+4)^{\cancel{8}}}$$

$$= \frac{(s^2+4)(96)-(96s)(8s)}{(s^2+4)^5} = 96s^2+384-768s^2 = \frac{-672s^2+384}{(s^2+4)^5}$$

$$\mathcal{L}\{t^4 \operatorname{sen}(2t)\} = \frac{240s^4-576s^2-672s^2+384}{(s^2+4)^5}$$

$$= \frac{240s^4-1248s^2+384}{(s^2+4)^5} +$$

$$\frac{d}{ds}\frac{+16}{(s^2+4)^3} = 16(s^2+4)^{-3} = (16)(+3)(s^2+4)^{-4}(2s) = \frac{+96s}{(s^2+4)^4}$$

$$+\frac{d}{ds}\frac{96s}{(s^2+4)^4} = \frac{(s^2+4)^{\cancel{4}}(96)-(96s)(4)\cancel{(s^2+4)^3}(2s)}{(s^2+4)^{\cancel{8}}}$$

$$= 96s^2+384-768s^2 = \frac{-672s^2+384}{(s^2+4)^5}$$

$$\mathcal{L}\{t^4 \operatorname{sen}(2t)\} = \frac{240s^4-1920s^2+768}{(s^2+4)^5}$$

3) $$\mathcal{L}\left\{\int_0^t \operatorname{sen}(2\tau)\,d\tau\right\} = \frac{1}{s}\mathcal{L}\{F(\tau)\} = \frac{F(s)}{s}$$

$$\frac{1}{s}\left\{\frac{2}{s^2+4}\right\} = \frac{2}{s(s^2+4)}$$

4) $$\mathcal{L}\left\{\int_0^t \cos(3\tau)\,d\tau\right\} = \left(\frac{1}{s}\right)\left\{\frac{s}{s^2+9}\right\} = \frac{s}{s(s^2+9)} = \frac{1}{s^2+9}$$

Actividad 8
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Grupo: 25

5) $\mathcal{L}^{-1}\left\{\frac{1}{s(s-2)^2}\right\} = \mathcal{L}\left\{\frac{F(s)}{s}\right\} = \int_0^t \mathcal{L}^{-1}\{F(s)\}\,\partial\tau$

$$= \int_0^t f(\tau)\,d\tau$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s-2)^2}\right\} = \int_0^t \mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}\partial\tau$$

$$\int_0^t te^{2t}\,\partial\tau = \int_0^t \tau e^{2\tau}\,d\tau =$$

$u = \tau$ $\quad \int dv = \int e^{2\tau}\,d\tau$

$du = d\tau$ $\quad v = \frac{1}{2}e^{2\tau}$

$$\int_0^t \tau e^{2\tau}d\tau = \frac{\tau e^{2\tau}}{2} - \frac{1}{2}\int e^{2\tau}\,d\tau$$

$$\int_0^t \frac{\tau e^{2\tau}}{2} - \frac{1}{4}e^{2\tau} = \frac{te^{2t}}{2} - \frac{e^{2t}}{4} + \frac{1}{4}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s-2)^2}\right\} = \underline{\frac{te^{2t}}{2} - \frac{e^{2t}}{4} + \frac{1}{4}}$$

6) $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2-1)}\right\} = \int_0^t \mathcal{L}^{-1}\left\{\frac{1}{s^2-1}\right\}d\tau$

$$\int_0^t \operatorname{senh}(\tau)\,d\tau = \cosh(\tau)\Big|_0^t = \cosh(t) - \overset{1}{\cosh(0)}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2-1)}\right\} = \underline{\cosh(t) - 1}$$

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