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Tarea: 22

- Ejercicio 7, 2007-2, 2° Final, Tipo A

Sea el espacio vectorial $\mathbb{R}^3$ con el producto interno definido por: $(\bar{x}|\bar{y}) = x_1y_1 + 2x_2y_2 + x_3y_3 \ \forall\, \bar{x},\bar{y} \in \mathbb{R}^3$ y sea $W=\{(x,y,z) \mid x+2y-z=0, x,y,z\in\mathbb{R}\}$ un subespacio de $\mathbb{R}^3$. Determinar el complemento ortogonal de W.

$W=\{(x,y,x+2y) \mid x,y\in\mathbb{R}\}$ $\quad ((a,b,c)\mid(1,0,1)) = a+c=0$ $\quad c=-a$

$B_W=\{(1,0,1),(0,1,2)\}$ $\quad ((a,b,c)\mid(0,1,2)) = 2b+2c=0$ $\quad 2b-2a=0$, $2b=+2a$, $b=+a$

$\operatorname{Dim} V = \operatorname{Dim} W + \operatorname{Dim} W^{\perp}$ $\quad \underline{W^{\perp}=\{(a,a,-a) \mid a\in\mathbb{R}\}}$

$3=2+1$

$3=3$ $\quad \bar{v}=(1,1,3)$ $\quad (1,1,3)\mid 1,1,-1 = 1+2-3=0$

$\bar{v}=(1,1,-1)$ Se cumple

- Ejercicio 4, 2003-1, 1° Final, Tipo A.

Sea $W=\{(x,y,z) \mid x+y+z=0, x,y,z\in\mathbb{R}\}$ un subespacio de $\mathbb{R}^3$. En W se considera el producto interno usual en $\mathbb{R}^3$, y una base de W es $\{(1,-1,0),(0,1,-1)\}$. Obtener la proyección ortogonal del vector $\bar{v}=\{(5,2,3)\}$ sobre W y calcular la distancia mínima entre $\bar{v}$ y W.

$W=\{(1,-1,0),(0,1,-1)\}$ $\quad W_0=\{(1,-1,0),(1/2,1/2,-1)\}$ $\quad W_n=\left\{\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\right), \left(\frac{\sqrt{2}}{2\sqrt{3}},\frac{\sqrt{2}}{2\sqrt{3}},-\frac{\sqrt{2}}{\sqrt{3}}\right)\right\}$

$\bar{w}_1=(1,-1,0)$

$\bar{w}_2=(0,1,-1)-\dfrac{((0,1,-1)\mid(1,-1,0))}{(1,-1,0)\mid(1,-1,0)}(1,-1,0) = (0,1,-1)+\dfrac{1}{2}(1,-1,0)$

$= (1/2, 1/2, -1)$

$\sqrt{(1/2,1/2,-1)\mid(1/2,1/2,-1)} = \sqrt{1/4+1/4+1} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}}$

$\bar{w}=\left((5,2,3)\mid \frac{1}{\sqrt{2}}(1,-1,0)\right)\left(\frac{1}{\sqrt{2}}(1,-1,0)\right)+\left((5,2,3)\mid \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{1}{2},\frac{1}{2},-1\right)\right)\left(\frac{\sqrt{2}}{\sqrt{3}}\left(\frac{1}{2},\frac{1}{2},-1\right)\right)$

$\bar{w}=\frac{1}{2}(5-2)(1,-1,0)+\frac{2}{3}\left(\frac{5}{2}+1-3\right)(1/2,1/2,-1) = \left(\frac{3}{2},-\frac{3}{2},0\right)+\left(\frac{2}{12},\frac{2}{12},-\frac{2}{6}\right)$

$\underline{\bar{w}=\left(\frac{5}{3},-\frac{4}{3},-\frac{1}{3}\right)}$ $\rightsquigarrow$ proyección ortogonal

$d(\bar{v}-\bar{w}) = \|\bar{v}-\bar{w}\|$

$= \|(5,2,3)-(\frac{5}{3},-\frac{9}{3},-\frac{1}{3})\| = \|(\frac{10}{3},\frac{10}{3},\frac{10}{3})\| = \sqrt{\frac{100}{9}+\frac{100}{9}+\frac{100}{9}} = \frac{10\sqrt{3}}{3}\,u$