

Actividad 5.

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Ecuaciones Diferenciales Grupo: 25

Actividad 5

Sea $\frac{d^2x}{dt^2} + 64x = 16 \cos(8t)$

v.o.: x

v.i.: t

Q(t) = 16 cos(8t)

1) $\frac{d^2x}{dt^2} + 64x = 0 \rightarrow D^2x + 64x = 0 \rightarrow (D^2 + 64)x = 0$

$\lambda^2 + 64 = 0 \quad \lambda = \sqrt{-64} \quad \lambda_{1,2} = \pm 8i$

$x_h = e^0 (C_1 \cos(8t) + C_2 \sin(8t))$

$x_h = C_1 \cos(8t) + C_2 \sin(8t)$

2) $D^2x + 64x = 16 \cos(8t) \quad (D^2 + 64)^2 = 2(0)$
 $\lambda = 0$
 $n = 2$
 $p = 8$

$(D^2 + 64)(D^2x + 64x) = (D^2 + 64)(16 \cos(8t))$
 $= D^2 16 \cos(8t) + 1024 \cos(8t)$
 $= D(16)(-8) \sin(8t) + 1024 \cos(8t)$
 $= (16)(-64) \cos(8t) + 1024 \cos(8t)$
 $= -1024 \cos(8t) + 1024 \cos(8t) = 0$

$(\lambda^2 + 64)(\lambda^2 + 64)x = 0$
 $x_{pnh} \quad x_h$

$\lambda_{3,4} = \pm 8i$

$x_{pnh} = e^0 (C_3 t \cos(8t) + C_4 t \sin(8t))$

$x' = C_3 [(t)(-8 \sin(8t)) + \cos(8t)] + C_4 [t(8) \cos(8t) + \sin(8t)]$
 $= -8C_3 t \sin(8t) + C_3 \cos(8t) + 8tC_4 \cos(8t) + C_4 \sin(8t)$

$x'' = (-8C_3) [(t)(8) \cos(8t) + \sin(8t)] + (38 \sin(8t) + (8t)(8) \sin(8t) + 8C_4 \cos(8t))$
 $(8C_4) [(t)(-8) \sin(8t) + \cos(8t)] + 8C_4 \cos(8t)$

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$$\text{Subt} = -64(c_3 t \cos(8t) - 8(c_3 \sin(8t) - c_3 8 \sin(8t)) - 64(c_4 t \sin(8t) + 8(c_4 \cos(8t) + 8(c_4 \cos(8t)))$$

$$= -64(c_3 t \cos(8t) - 8(c_3 \sin(8t) - c_3 8 \sin(8t)) - 64(c_4 t \sin(8t) + 16(c_4 \cos(8t) + 64(c_3 t \cos(8t) + c_4 t \sin(8t))) = 16 \cos(8t)$$

$$= -64(c_3 t \cos(8t) + 64(c_3 t \cos(8t) + 64 t \sin(8t) - 64(c_4 t \sin(8t))$$

$$= -16(c_3 \sin(8t) + 16(c_4 \cos(8t)) = 16 \cos(8t)$$

$$1) 16(c_4 \cos(8t)) = 16 \cos(8t)$$

$$c_4 = \frac{16 \cos(8t)}{16 \cos(8t)}$$

$$c_4 = 1$$

$$2) -16(c_3 \sin(8t)) = 0$$

$$c_3 = 0$$

$$x_{\text{part}} = 0(t \cos(8t)) + t \sin(8t)$$

$$x_g = c_1 \cos(8t) + c_2 \sin(8t) + t \sin(8t)$$

$$x_g \text{ tal que } x(0) = 0 \Rightarrow x_g(0) = c_1 \cos(0) + c_2 \sin(0) + t \sin(0) = 0$$

$$= c_1 = 0$$

$$x'_g = -c_1 8 \sin(8t) + c_2 8 \cos(8t) + (t)(8) \cos(8t) + \sin(8t)$$

$$x'_g(0) = -8(c_1 \sin(0)) + c_2 8(\cos(0)) + (0)(8) \cos(0) + \sin(0)$$

$$= 8c_2$$

$$= c_2$$

$$c_2 = 0$$

$$x_p = t \sin(8t)$$

Sol. particular