

Actividad para punto extra en tercer parcial.

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Ecuaciones Diferenciales

Grupo: 25

### Actividad

- 1) Resolver el sig. sistema de ecuaciones diferenciales haciendo uso de la transformada de Laplace.

$$\frac{d^2 x}{dt^2} + 3 \frac{dy}{dt} + 3y = 0 \quad \text{sujeto a} \quad \begin{matrix} x(0) = 0 & x(t) \\ x'(0) = 2 & y(t) \\ y(0) = 0 \end{matrix}$$

$$\frac{d^2 x}{dt^2} + 3y = t e^{-t}$$

$$\left. \begin{matrix} \frac{d^2 x}{dt^2} + 3 \frac{dy}{dt} + 3y = 0 \\ -\frac{d^2 x}{dt^2} - 3y = -t e^{-t} \end{matrix} \right\} \quad 3 \frac{dy}{dt} = -t e^{-t}$$

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} = \mathcal{L}\left\{-\frac{t}{3} e^{-t}\right\}$$

$$sY(s) - y(0) = -\frac{1}{3} \mathcal{L}\left\{\frac{t}{(s+1)^2}\right\}$$

$$Y(s) = -\frac{1}{3} \frac{1}{s(s+1)^2}$$

$$\begin{aligned} s^2(A+B) &= 0 & A &= 1 \\ s(2A+B+C) &= 0 & B &= -1 \\ A &= 1 & C &= -1 \end{aligned}$$

$$Y(s) = -\frac{1}{3} \left( \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} \right)$$

$$\mathcal{L}^{-1}\{Y(s)\} = -\frac{1}{3} \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{1}{3} \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{3} \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$y(t) = -\frac{1}{3} (1 - e^{-t} - t e^{-t})$$



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substituir en:  $\frac{d^2 x}{dt^2} + 3y = t e^t$

$$\frac{d^2 x}{dt^2} + 3\left(-\frac{1}{3}(1 - e^t - t e^t)\right) = t e^t$$

$$\frac{d^2 x}{dt^2} - 1 + e^t + t e^t = t e^t$$

$$2 \left\{ \frac{d^2 x}{dt^2} = -e^t + 1 \right\}$$

$$s^2 X(s) - s x(0) - \dot{x}(0) = -\frac{1}{s+1} + \frac{1}{s}$$

$$s^2 X(s) = -\frac{1}{(s+1)} + \frac{1}{s} + 2 = \frac{s - (s+1)}{s(s+1)} + 2 = \frac{s-s-1+2s(s+1)}{s(s+1)}$$

$$s^2 X(s) = \frac{2s^2 + 2s + 1}{s(s+1)} \quad F(s) = \frac{2s^2 + 2s + 1}{s^3(s+1)}$$

$$\frac{2s^2 + 2s + 1}{s^3(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1}$$

$$\begin{aligned} 2s^2 + 2s + 1 &= A(s^2)(s+1) + B s(s+1) + C(s+1) + D(s^3) \\ &= A(s^3 + s^2) + B(s^2 + s) + C(s+1) + D s^3 \\ &= A s^3 + A s^2 + B s^2 + B s + C s + C + D s^3 \end{aligned}$$

$$\begin{aligned} s^3(A+D) &= 0 & C &= 1 \quad \left\{ x(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} - \frac{1}{s+1} \right\} \\ s^2(A+B) &= 2 & B &= 1 \\ s(B+C) &= 2 & A &= 1 \\ C &= 1 & D &= -1 \end{aligned}$$

$$x(t) = 1 + t + \frac{1}{2}t^2 - e^t$$

$$y(t) = -\frac{1}{3}(1 - e^t - t e^t)$$

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2) Determinar la solución del sig sistema de ecuaciones diferenciales haciendo uso de la transformada de Laplace sujeto a  $x(0) = y(0) = -1$  obtener  $x(t)$

$$\begin{array}{ll} x' + y' = 1 & 1) \\ x' + 6y - x = 0 & 2) \end{array}$$

$$\mathcal{L}\{x' + y' = 1\} = sX(s) - \overset{-1}{x(0)} + sY(s) - \overset{-1}{y(0)} = 1/s$$

$$sX(s) + sY(s) + 2 = 1/s \quad 3)$$

$$\mathcal{L}\{x' + 6y - x = 0\} = sX(s) - \overset{-1}{x(0)} + 6Y(s) - X(s) = 0$$

$$X(s)(s-1) + 6Y(s) = -1 \quad 4)$$

$$\text{busco } x(t) \rightarrow x(s)$$

$$\text{en 4) } y(s) = \frac{-1 - X(s)(s-1)}{6}$$

$$\text{en 3) } sX(s) + s\left(-\frac{1}{6} - \frac{X(s)(s-1)}{6}\right) = \frac{1}{s} - 2$$

$$sX(s) - \frac{s}{6} - \frac{X(s)(s^2-s)}{6} = \frac{1}{s} - 2$$

$$X(s)\left(s - \frac{(s^2-s)}{6}\right) = \frac{1}{s} - 2 + \frac{s}{6}$$

$$X(s)\left(-\frac{s^2}{6} + \frac{7s}{6}\right) = \frac{1-2s}{s} + \frac{s}{6} = \frac{6-12s+s^2}{6s} = \frac{s^2-12s+6}{6s}$$

$$X(s)\left(\frac{1}{6}\right)(s(7-s)) = \frac{s^2-12s+6}{6s}$$

$$X(s) = \left(\frac{s^2-12s+6}{6s}\right)\left(\frac{6}{s(7-s)}\right)$$

$$X(s) = \frac{s^2-12s+6}{s^2(7-s)}$$



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$$\frac{s^2 - 12s + 6}{s^2(7-s)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{7-s}$$

$$\begin{aligned} s^2 - 12s + 6 &= A(s)(7-s) + B(7-s) + C s^2 \\ &= A(7s - s^2) + 7B - sB + C s^2 \\ &= -A s^2 + A 7s + 7B - sB + C s^2 \end{aligned}$$

$$\begin{aligned} s^2(-A) &= 1 \\ s(7A - B) &= -12 \\ 7B &= 6 \\ B &= 6/7 \end{aligned}$$

$$\begin{aligned} 7A - 6/7 &= -12 \\ 7A &= -12 + 6/7 \\ &= \frac{-84 + 6}{7} \\ A &= -\frac{78}{7} \\ A &= -\frac{78}{49} \end{aligned}$$

$$C + \frac{7B}{49} = 1$$

$$C = 1 - \frac{7B}{49}$$

$$C = \frac{49 - 7B}{49} = \frac{-29}{49}$$

$$\mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\left(-\frac{78}{49}\right)\left(\frac{1}{s}\right) + \left(\frac{6}{7}\right)\left(\frac{1}{s^2}\right) - \left(\frac{29}{49}\right)\left(\frac{1}{7-s}\right)\right\}$$

$$x(t) = \frac{-78}{49} + \frac{6t}{7} + \left(\frac{29}{49}\right)e^{7t}$$

3) Determinar la solución del siguiente sistema de ecuaciones diferenciales haciendo uso de la transformada de Laplace sujeto a  $y(0) = y'(0)$  y  $z(0) = 7$

1)  $y'' + z + y = 0$

2)  $z' + y' = 0$

$$\mathcal{L}\{y'' + z + y = 0\} = s^2 y(s) - s y(0) - y'(0) + z(s) + y(s) = 0$$

$$\mathcal{L}\{z' + y' = 0\} = s z(s) - z(0) + s y(s) - y(0) = s z(s) + s y(s) = 1 \quad 4)$$

3)  $(y(s)(s^2 + 1) + z(s) = 0)(-s) \quad -y(s)(s^3 + s) - s z(s) = 0$

$$(y(s)(s) + s z(s) = 1) \quad y(s)(s) + s z(s) = 1$$

$$-y(s)s^3 = 1$$



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$$-y(s)s^3 = 7$$

$$y(s) = -7/s^3$$

$$\mathcal{L}^{-1}(y(s)) = \mathcal{L}^{-1}\{-7/s^3\}$$

$$y(t) = \frac{-1}{2}t^2 \rightarrow \text{sustituir en 1)}$$

$$z'' + z + y = 0 \Rightarrow y'(t) = -t$$

$$y''(t) = -1$$

$$-1 + z + \frac{-1}{2}t^2 \rightarrow z = \frac{1}{2}t^2 + 1$$

$$z(t) = \frac{1}{2}t^2 + 1$$

$$y(t) = -\frac{1}{2}t^2$$

4) Mediante la transformada de Laplace, obtener la solución del sig. sistema de ecuaciones diferenciales.

$$x' - x + 2y = 0$$

$$3x + y' = 0$$

$$\text{con las condiciones iniciales } x(0) = 0$$

$$y(0) = 1$$

$$3) \mathcal{L}\{x' - x + 2y = 0\}$$

$$= sX(s) - x(0) - X(s) + 2Y(s) = 0$$

$$x(s)(s-1) + 2y(s) = 0$$

$$4) \mathcal{L}\{3x + y' = 0\} = 3x(s) + sY(s) - y(0) = 3x(s) + sY(s) = 1$$

$$sX(s) - x(s) + 2y(s) = 0$$

$$3x(s) + sY(s) = 1$$

$$Y(s) = \frac{1-3x(s)}{s}$$

$$sX(s) - x(s) + \frac{2-6x(s)}{s} = 0$$

$$X(s) = \frac{-2}{s}$$

$$x(s)(s-1) + \frac{2}{s} - \frac{6x(s)}{s} = 0$$

$$\frac{s^2-6-s}{s}$$

$$x(s)(s-1-\frac{6}{s}) + \frac{2}{s} = 0$$

$$X(s) = \frac{-2}{s^2-s-6}$$

$$x(s)(\frac{s^2-6-s}{s}) = -2/s$$

$$x(s) = (-2) \left( \frac{1}{(s-3)(s+2)} \right)$$



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$$x(s) = \frac{-2}{(s-3)(s+2)}$$

$$\begin{aligned}\frac{1}{(s-3)(s+2)} &= \frac{A}{(s-3)} + \frac{B}{(s+2)} \\ &= \frac{A(s+2) + B(s-3)}{(s-3)(s+2)} \\ 1 &= \frac{As + 2A + Bs - 3B}{(s-3)(s+2)}\end{aligned}$$

$$\begin{aligned}5(A+B) &= 0 & A+B &= 0 & A &= -B & A &= 1/5 \\ 2A-3B &= 1 & -5B &= 1 & B &= -1/5\end{aligned}$$

$$x(s) = -2 \left( \frac{1}{5(s-3)} - \frac{1}{5(s+2)} \right)$$

$$\mathcal{L}^{-1}\{x(s)\} = -2 \mathcal{L}^{-1}\left\{ \left( \frac{1}{5} \right) \left( \frac{1}{s-3} \right) - \left( \frac{1}{5} \right) \left( \frac{1}{s+2} \right) \right\}$$

$$x(t) = \frac{-2}{5} e^{3t} + \frac{2}{5} e^{-2t}$$

$$2y(t) = x - x'$$

$$y(t) = \frac{-2}{5} e^{3t} + \frac{2}{5} e^{-2t} + \frac{6}{5} e^{3t} + \frac{4}{5} e^{-2t}$$

$$y(t) = \frac{-1}{5} e^{3t} + \frac{1}{5} e^{-2t} + \frac{3}{5} e^{3t} + \frac{2}{5} e^{-2t}$$

$$y(t) = \frac{3}{5} e^{-2t} + \frac{2}{5} e^{3t}$$