Díaz Hernández Marcos Bryan – N° de Lista: 12

Díaz Hernández Marcos Bryan Ma-Ju N.L: 12

Tarea: 17

-Ejercicio 46, página 238, Godínez

Determina los valores y vectores característicos de la transformación lineal $T: M \to M$, donde

$M = \left\{ \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ y T esta definida por

$$T\left(\begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \right) = \begin{bmatrix} x+2y & y+2z \\ 0 & 2z \end{bmatrix} \qquad F\left(\begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \right) = (x, y, z)$$

$$T((x,y,z)) = (x+2y, y+2z, 2z)$$

$B = \{ (1,0,0), (0,1,0) (0,0,1) \}$

$T(1,0,0) = (1,0,0)$

$T(0,1,0) = (2,1,0)$

$T(0,0,1) = (0,2,2)$

$$M(T) = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

$(A - \lambda I)v =$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (1-\lambda)^2(2-\lambda)$$

$\underline{\lambda_1 = 2 \quad \lambda_{2,3} = 1}$

• $\lambda_1 = 2$

$$\left(\begin{array}{ccc|c} -1 & 2 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{cases} -a + 2b = 0 \\ -b + 2c = 0 \end{cases} \quad \begin{cases} a = 2b \\ b \in \mathbb{R} \\ c = b/2 \end{cases}$$

$V.C(2) = \{ (2b, b, b/2) \mid b \in \mathbb{R} \} - \{ \bar{0} \}$

$\underline{V.C(2) = \left\{ \begin{bmatrix} 2b & b \\ 0 & b/2 \end{bmatrix} \mid b \in \mathbb{R} \right\} - \{ \bar{0} \}}$

• $\lambda_{2,3} = 1$

$$\left[\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \underset{(2)}{\sim} \left[\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{cases} 2b = 0 \\ c = 0 \\ a \in \mathbb{R} \end{cases}$$

$V.C(1) = \{ (a, 0, 0) \mid a \in \mathbb{R} \} - \{ \bar{0} \}$

$\underline{V.C(1) = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \mid a \in \mathbb{R} \right\} - \{ \bar{0} \}}$

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– Ejercicio 2, 2012-1, 1° Final, Tipo A

Sean el espacio vectorial real $P=\{a+bx+cx^2 \mid a,b,c\in\mathbb{R}\}$ y el operador lineal $T: P\to P$ tal que $T(a+bx+cx^2) = b + cx + (2a-5b+4c)x^2$

a) Una matriz asociada

b) Los espacios característicos

c) Si T es diagonalizable

$F(a+bx+cx^2) = (a,b,c)$

$T(a+bx+cx^2) = (b,\ c,\ 2a-5b+4c)$

$B=\{(1,0,0)(0,1,0)(0,0,1)\}$ $\quad M(T)=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{bmatrix}$

$T(1,0,0)=(0,0,2)$

$T(0,1,0)=(1,0,-5)$

$T(0,0,1)=(0,1,4)$ $\quad \lambda=1$

$-\lambda^3+4\lambda^2-5\lambda+2=0$

$\pm 2 = \pm 1 \ \pm 2$

$$\begin{array}{r|rrrr} 2 & -1 & 4 & -5 & 2 \\ & & -2 & 4 & -2 \\ \hline 1 & -1 & 2 & -1 & \boxed{0} \\ & & -1 & 1 & \\ \hline & -1 & 1 & \boxed{0} & \end{array}$$

$\lambda_1=2$

$\lambda_{2,3}=1$

$-x+1=0$

$x=1$

b) $\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 2 & -5 & 4-\lambda \end{vmatrix} = (\lambda^2)(4-\lambda)+2-(5\lambda)$

$4\lambda^2-\lambda^3-5\lambda+2=0$

• $\lambda=2$

$$\left[\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 2 & 0 \end{array}\right]_{(1)} \left[\begin{array}{ccc|c} 0 & -4 & 2 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 2 & 0 \end{array}\right](-2) \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 2 & 0 \end{array}\right](-2) \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & -1 & 0 & 0 \end{array}\right]$$

$\begin{cases} -2b+c=0 \\ 2a-b=0 \end{cases}$ $\quad c=2b$, $\quad a=b/2$, $\quad b\in\mathbb{R}$

$E(2)=\{(b/2,\ b,\ 2b) \mid b\in\mathbb{R}\}$ $\quad E(2)=\{b/2+bx+2bx^2\}$

• $\lambda=1$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & -5 & 3 & 0 \end{array}\right] (2)\ (-3) \left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\begin{cases} -a+b=0 \\ -b+c=0 \\ b\in\mathbb{R} \end{cases}$

$E(1)=\{(b,b,b) \mid b\in\mathbb{R}\}$

$E(1)=\{b+bx+bx^2 \mid b\in\mathbb{R}\}$

c) 1) $\lambda_1\neq\lambda_2=\lambda_3$ ∴ No cumple

2) $\text{Dim } P=3$ $\quad \text{Dim } E(2)+\text{Dim } E(1)=2$ ∴ $3\neq 2$

3) $B=\{(x^2+x+1),\ (4x^2+2x+1),\ (2x^2+2x+2)\}$ ∴ Son L.D

No es diagonalizable

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- Ejercicio 7, 2004-2, 2° Final, Tipo A

Sea $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ la matriz asociada al operador lineal $T: \mathbb{R}^3 \to \mathbb{R}^3$ respecto a la base canónica de $\mathbb{R}^3$.

a) Los valores característicos

b) Espacios característicos

c) Matriz P tal que $P^{-1}AP$ es una matriz diagonal

$$\begin{vmatrix} 3-\lambda & 1 & -1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)^2 \Rightarrow \lambda_1 = 3$$

a) $\underline{\lambda_{2,3} = 2}$

- $\lambda = 3$

$$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \begin{array}{c} \\ (1) \\ (-1) \end{array} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$$\begin{cases} -b = 0 \\ -c = 0 \\ a \in \mathbb{R} \end{cases} \qquad \underline{E(3) = \{(a,0,0) \mid a \in \mathbb{R}\}}$$

- $\lambda = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{cases} a+b-c = 0 \\ a = -b+c \end{cases} \quad \begin{array}{l} b \in \mathbb{R} \\ c \in \mathbb{R} \end{array}$$

$$\underline{E(2) = \{(-b+c, b, c) \mid b, c \in \mathbb{R}\}}$$

c)

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \underline{P = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \qquad B = \{(1,0,0), (-1,1,0), (1,0,1)\}$$

$$\alpha(1,0,0) + \beta(-1,1,0) + \gamma(1,0,1) = (0,0,0)$$

- $\lambda_1 = 3$ con $a = 1$
- $\lambda_2 = 2$ con $b = 1$ $c = 0$
- $\lambda_3 = 2$ con $b = 0$ $c = 1$

$\alpha - \beta + \gamma = 0$ $\quad \alpha = 0$

$\beta = 0$ $\quad \beta = 0$ $\quad \therefore$ son L.I

$\gamma = 0$ $\quad \gamma = 0$