Díaz Hernández Marcos Bryan Ma-Ju N.L: 12
Tarea: 23

- Ejercicio 6, 2001-2, 2° final, Tipo B

Sea $W = \{(a,b,c,d) \mid c=-d;\ a,b,d \in \mathbb{R}\}$ un subespacio de $\mathbb{R}^4$. Para el producto interno usual de $\mathbb{R}^4$, expresar $\bar{v} = (\sqrt{3}, -\frac{4}{5}, 7, 0)$ como la suma de un vector $\bar{a}$ de W y otro $\bar{b}$ del complemento ortogonal de W.

$\bar{v} = (\sqrt{3}, -\frac{4}{5}, 7, 0)$ $\quad w = \{(a,b,-d,d) \mid a,b,d \in \mathbb{R}\}$ $\quad B_G = \{(1,0,0,0),(0,1,0,0),(0,0,-7,7)\}$

$\bar{v} = \bar{v}_1 + \bar{v}_2$

$(1,0,0,0) \mid (0,1,0,0) = 0$

$(1,0,0,0) \mid 0,0,-7,7) = 0$

$(0,1,0,0) \mid (0,0,-7,-7) = 0$

$\|(1,0,0,0)\| = \sqrt{(1,0,0,0) \mid (1,0,0,0)} = 7$ $\quad \|(0,0,-7,7)\| = \sqrt{1+7} = \sqrt{2}$

$B_n = \{(1,0,0,0); (0,1,0,0), \frac{1}{\sqrt{2}}(0,0,-7,7)\}$

$\bar{v}_1 = ((\sqrt{3}, -4/5, 7, 0) \mid (1,0,0,0))(1,0,0,0) + ((\sqrt{3}, -4/5, 7, 0) \mid (0,1,0,0))(0,7,0,0)$
$+ ((\sqrt{3}, -4/5, 7, 0) \mid \frac{1}{\sqrt{2}}(0,0,-7,7))(0,0,-7,7)\frac{1}{\sqrt{2}}$
$= (\sqrt{3},0,0,0) + (0,-4/5,0,0) + \frac{1}{2}(0,0,7,-7) = (\sqrt{3}, -4/5, 1/2, -1/2)$

$(\sqrt{3}, -4/5, 7, 0) = (\sqrt{3}, -4/5, 1/2, -1/2) + \bar{v}_2$
$\bar{v}_2 = (\sqrt{3}-\sqrt{3}, -4/5+4/5, 7-1/2, +1/2)$

$\bar{v}_2 = (0,0,+1/2,1/2)$

$(a,b,c,d) \mid (1,0,0,0) = a = 0$
$(a,b,c,d) \mid (0,1,0,0) = b = 0$
$(a,b,c,d) \mid (0,0,-7,7) = -c+d = 0$
$c = d \in \mathbb{R}$

$\bar{v} = \bar{v}_1 + \bar{v}_2$
$\bar{v} = (\sqrt{3}, -4/5, 1/2, -1/2) + (0,0,1/2,1/2)$

$(\sqrt{3}, -4/5, 7, 0) = (\sqrt{3}, -4/5, 7, 0)$ //

$w^{\perp} = \{(0,0,c,c) \mid c \in \mathbb{R}\}$
$\bar{v}_2 = (0,0,1/2,1/2) \in w^{\perp}$

- Ejercicio 16, 2008-1, 1°Parcial, Tipo 1

Sean P_2 el espacio de los polinomios de grado menor o igual a dos con coeficientes reales, $W = \{ax^2 \mid a \in \mathbb{R}\}$ un subespacio de P_2 y el producto interno en P_2 definido por:

$$(p|q) = \sum_{i=-1}^{1} p(i)q(i) \quad \forall p, q \in P_2 \qquad 2x^2$$

El polinomio $h(x) \in W$ más próximo a $m(x) = x^2+1$ es.

$B = \{x^2\} \quad \|x^2\| = \sqrt{(x^2)|(x^2)} = \sqrt{1+1} = \sqrt{2}$

$B_n = \{\frac{1}{\sqrt{2}}x^2\} \quad \bar{w} = ((x^2+1)|\frac{1}{\sqrt{2}}x^2)(\frac{1}{\sqrt{2}}x^2) \quad \bar{w} = \frac{1}{2}(2+2)(x^2 = \frac{4x^2}{2} \quad \underline{\bar{w} = 2x^2}$

- Ejercicio 53, Página 370, Barrera.

Obtenga, empleando el método de mínimos cuadrados, una solución aproximada al sistema de ecuaciones.

$$\begin{cases} -a+b+c=0 \\ a-b+c=0 \\ b+c=-1 \\ a+c=1 \end{cases} \quad \underbrace{\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\bar{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}}_{\bar{y}} \qquad A\bar{x} = \bar{y}, \quad A^TA\bar{x} = A^T\bar{y}$$

$$A^T = \underbrace{\begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{3\times 4} \underbrace{\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{4\times 3} = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \begin{aligned} 3a-2b+c &= 1 \\ -2a+3b+c &= -1 \\ a+b+4c &= 0 \end{aligned} \qquad \left|\begin{matrix} 3 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 4 \end{matrix}\right|\left.\begin{matrix} 1 \\ -1 \\ 0 \end{matrix}\right| (2)(-3)$$

$$\left|\begin{matrix} 0 & -5 & -11 \\ 0 & 5 & 9 \\ 1 & 1 & 4 \end{matrix}\right|\left.\begin{matrix} 1 \\ -1 \\ 0 \end{matrix}\right| (1) \quad \left|\begin{matrix} 0 & 0 & -2 \\ 0 & 5 & 9 \\ 1 & 1 & 4 \end{matrix}\right|\left.\begin{matrix} 0 \\ -1 \\ 0 \end{matrix}\right| (-1/2)(-9)(-4) \quad \left|\begin{matrix} 0 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 1 & 0 \end{matrix}\right|\left.\begin{matrix} 0 \\ -1 \\ 0 \end{matrix}\right| (1/5)(-1) \quad \left|\begin{matrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{matrix}\right|\left.\begin{matrix} 0 \\ -1/5 \\ 1/5 \end{matrix}\right|$$

$c = 0$

$b = -1/5$

$\underline{a = 1/5}$