Primer Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

Díaz Hernandez Marcos Bryan – Ecuaciones Diferenciales.

1)

- $\frac{dy}{dx} = 2e^{x}$ — V.D: y; V.I: x; Tipo: ordinaria — Orden: 1; Grado: 1; Lineal: si
- $\frac{dy}{dt} = 2t - \frac{dy}{ds}$ — V.D: y; V.I: t, s; Tipo: parcial — Orden: 1; Grado: 1; Lineal: si
- $x^2y'' + xy' + y = xy$ — V.D: y; V.I: x; Tipo: ordinaria — Orden: 2; Grado: 1; Lineal: no
- $(y'')^2 - 3yy' + xy = 0$ — V.D: y; V.I: x; Tipo: ordinaria — Orden: 2; Grado: 2; Lineal: No
- $\left(\frac{d^2y}{dx^2}\right)^{3/2} + y = x$ — V.D: y; V.I: x; Tipo: ordinaria — Orden: 2; Grado: 3/2; Lineal: No
- $xy''' + x^2y'' - xy' + \text{sen}(y) = 0$ — V.D: y; V.I: x; Tipo: ordinaria — Orden: 3; Grado: No aplica; Lineal: No aplica

2) Obtener $y(x)$ de la sig. e.d:

$$(xy^2 - x - y^2 + 1)\,dx = (xy + x - y - 1)\,dy$$

$$\frac{dx}{dy} = \frac{(xy + x - y - 1)}{(xy^2 - x - y^2 + 1)} \rightarrow \frac{dx}{dy} = \frac{y(x-1) + 1(x-1)}{y^2(x-1) - 1(x-1)}$$

$$\frac{dx}{dy} = \frac{\cancel{(y+1)}\cancel{(x-1)}}{\cancel{(y+1)}(y-1)\cancel{(x-1)}} \rightarrow \int dx = \int \frac{dy}{y-1} \qquad e^{x+C} = e^{\ln(y-1)}$$

$$y - 1 = e^{x}e^{C} \rightarrow \boxed{y = Ce^{x} + 1} \quad \text{sol. general.}$$

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3) Resolver: $r\cos\theta\, d\theta + (r - \operatorname{sen}\theta)\, dr = 0$

$$\frac{dM}{dr} = \cos\theta \qquad \frac{dN}{d\theta} = -\cos\theta$$

$$\mu(r) = e^{\int \frac{1}{r\cos\theta}(-\cos\theta - \cos\theta)\,dr} \rightarrow e^{\int -\frac{2\cancel{\cos\theta}}{r\cancel{\cos\theta}}\,dr} \rightarrow e^{-2\int \frac{1}{r}\,dr}$$

$$\mu(r) = e^{-2\ln(r)} = \frac{1}{r^2}$$

$$\frac{1}{r^2}\Big(r\cos\theta\, d\theta + (r - \operatorname{sen}\theta)\,dr\Big) = 0 \rightarrow \frac{\cos\theta\, d\theta}{r} + \frac{(r - \operatorname{sen}\theta)}{r^2}\,dr = 0$$

$$\frac{dM}{dr} = \frac{-\cos\theta}{r^2} \qquad \frac{dN}{\partial\theta} = \frac{-\cos\theta}{r^2} \quad \therefore \text{ exacta}$$

$$F(\theta, r) = \int \frac{\cos\theta}{r}\,d\theta + h(r) \rightarrow \frac{\operatorname{sen}\theta}{r} + h(r)$$

$$\frac{dF}{dr} = -\frac{\operatorname{sen}\theta}{r^2} + h'(r) \rightarrow \left(\frac{1}{r} - \cancel{\frac{\operatorname{sen}\theta}{r^2}}\right) = \cancel{-\frac{\operatorname{sen}\theta}{r^2}} + h'(r)$$

$$\frac{dh}{dr} = \frac{1}{r} \rightarrow \int dh = \int \frac{dr}{r} \rightarrow h(r) = \ln(r)$$

$$F(\theta, r) = \frac{\operatorname{sen}\theta}{r} + \ln(r) + C \rightarrow \boxed{C = \frac{\operatorname{sen}\theta}{r} + \ln(r)} \quad \text{sol. general}$$

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4) Resolver: $\left(y^2 - \frac{xy}{1+x} + xy^2\right) + \left(2xy - x + \ln(x+1) + x^2y + \frac{y^3}{y^8-2}\right)y' = 0$

$$\left(y^2 - \frac{xy}{1+x} + xy^2\right)dx + \left(2xy - x + \ln(x+1) + x^2y + \frac{y^3}{y^8-2}\right)dy = 0$$

$$\frac{dM}{dy} = 2y - \frac{x}{x+1} + 2yx \qquad \frac{dN}{dx} = 2y + 2xy - \frac{x}{x+1} \quad \therefore \text{ exacta}$$

$$F(x,y) = \int M\,dx + h(y) \to \int \left(y^2 - \frac{xy}{1+x} + xy^2\right)dx + h(y)$$

$$F(x,y) = y^2x - y\int \frac{x}{1+x}\,dx \to y^2x - y\left((1+x) - (\ln(1+x))\right) + h(y)$$

$$F(x,y) = y^2x - y(1+x) + y(\ln(x+1)) + \frac{x^2y^2}{2} + h(y)$$

$$\frac{dF}{dy} = 2yx - (1+x) + \ln(x+1) + \frac{2x^2y}{2} + h'(y)$$

$$2\cancel{xy} - \cancel{x} + \cancel{\ln}(x+1) + \cancel{x^2}y + \frac{y^3}{y^8-2} = 2\cancel{xy} - 1 - \cancel{x} + \cancel{\ln}(x+1) + \cancel{x^2}y + h'(y)$$

$$h'(y) = \frac{y^3}{y^8-2} + 1 \to \frac{dh}{dy} = \frac{y^3}{y^8-2} + 1 \to h(y) = \int \frac{y^3}{y^8-2} + 1\,dy$$

$$h(y) = \frac{\ln(y^4-\sqrt{2}) - \ln(y^4+\sqrt{2})}{2^{7/2}} + y$$

$$F(x,y) = y^2x - y(1+x) + y(\ln(x+1)) + \frac{x^2y^2}{2} + \frac{\ln(y^4-\sqrt{2}) - \ln(y^4+\sqrt{2})}{2^{7/2}} + y + C$$

$$\boxed{C = y^2x - y(1+x) + y(\ln(x+1)) + \frac{x^2y^2}{2} + \frac{\ln(y^4-\sqrt{2}) - \ln(y^4+\sqrt{2})}{2^{7/2}} + y}$$

$\therefore$ sol general

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5) Resolver el valor inicial: $xy' - y = x^2 \operatorname{sen}(\ln(x))$ $y(1) = 0$

$y' - \frac{y}{x} = x \operatorname{sen}(\ln(x))$ $p(x) = -\frac{1}{x}$ $F(x) = x \operatorname{sen}(\ln(x))$

$y = e^{\int \frac{1}{x} dx} \int e^{-\int \frac{dx}{x}} x \operatorname{sen}(\ln(x)) dx + c e^{\int \frac{1}{x} dx}$

$y(x) = (x)(\int \frac{1}{x}(x \operatorname{sen}(\ln(x))) dx + cx$ $v = \ln(x)$ $dv = \frac{1}{x} dx$ $e^{v} = x$
$dx = x\,dv$

$y(x) = (x) \int \operatorname{sen}(\ln(x)) dx + cx$

$y(x) = (x) \int \operatorname{sen}(v) e^{v} dv + cx \rightarrow \frac{e^{v}[\operatorname{sen}(v) - \cos(v)]}{2}$

$y(x) = x^2 \frac{[\operatorname{sen}(\ln(x)) - \cos(\ln(x))]}{2} + cx$ $y(1) = 0$

$0 = \frac{(1)}{2}(\operatorname{sen}(0) - \cos(0)) + c \rightarrow 0 = -\frac{1}{2} + c$ $c = 1/2$

$$\boxed{y(x) = \frac{x^2 [\operatorname{sen}(\ln(x)) - \cos(\ln(x))]}{2} + \frac{x}{2}}$$ sol. particular.

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6) Resolver el problema de valor inicial:

$$(1+2e^{x/y})dx + 2e^{x/y}(1-x/y)dy = 0$$

$$\frac{dM}{dy} = \frac{-2xe^{x/y}}{y^2} \qquad \frac{dN}{dx} = -\frac{2x}{y^2}e^{x/y} \quad \therefore \text{ exacta.}$$

$$F(x,y) = \int M\,dx + h(y) \to \int(1+2e^{x/y})dx + h(y)$$

$$F(x,y) = x+2\int e^{x/y} \to \int e^{v}dx \to \begin{array}{l} v = x/y \\ dv = \frac{dx}{y} \\ dx = y\,dv \end{array} \quad \int e^{v}y\,dv$$

$$F(x,y) = x+2(e^{v}y) \to x+2y(e^{x/y}) + h(y)$$

$$\frac{dF}{dy} = 2\left[y(e^{x/y})\left(-\frac{x}{y^2}\right) + (e^{x/y})\right] + h'(y)$$

$$\frac{dF}{dy} = -2y\frac{xe^{x/y}}{y^2} + 2e^{x/y} + h'(y) = -\frac{2xe^{x/y}}{y} + 2e^{x/y} + h'(y)$$

$$\cancel{2e^{x/y}} - \cancel{\frac{x2e^{x/y}}{y}} = -\cancel{\frac{2xe^{x/y}}{y}} + \cancel{2e^{x/y}} + h'(y) \qquad h'(y) = 0$$

$$\int dh = \int 0\,dy \to h(y) = C$$

$$F(x,y) = x + 2ye^{x/y} + C \to C = x+2y\,e^{x/y} \qquad y(0) = 1$$

$$C = 0 + 2(1) = 2$$

$$\boxed{2 = x+2y\,e^{x/y}}$$

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7) Resolver: $(3x-6y+4)\frac{dy}{dx}+x-2y+3=0$

$(3(x-2y)+4)\frac{dy}{dx}+(x-2y)+3=0$ $\quad v=x-2y$ $\quad dv=dx-2dy$ $\quad dy=\frac{dv-dx}{-2}$ $\quad dy=\frac{dx-dv}{2}$ $\quad \frac{dy}{dx}=(\frac{1}{2})(1-\frac{dv}{dx})$

$(3(v)+4)(\frac{1}{2})(1-\frac{dv}{dx})+(v)+3=0$

$(\frac{3v+4}{2})(1-\frac{dv}{dx})+v+3 \to \frac{3v+4}{2}-(\frac{3v+4}{2})(\frac{dv}{dx})+v+3=0$

$\frac{3v}{2}+2-(\frac{3v}{2}+2)(\frac{dv}{dx})+v+3=0 \to \frac{3v}{2}+v+5-(\frac{3v}{2}+2)(\frac{dv}{dx})=0$

$\frac{5v}{2}+5-(\frac{3v}{2}+2)(\frac{dv}{dx})=0 \to (\frac{5v}{2}+5)(dx)=(\frac{3v}{2}+2)dv$

$\int dx=\int\frac{(\frac{3}{2}v+2)}{(\frac{5v}{2}+5)}dv \to \int\frac{3v+4}{5v+10}dv=x+C \to \frac{1}{5}\int\frac{3v+4}{v+2}dv=x+C$

$w=v+2$
$dw=dv$
$v=w-2$

$+\frac{1}{5}\int\frac{3v\,dv}{v+2}+\frac{1}{5}\int\frac{4}{v+2}dv \to \frac{1}{5}\int\frac{3(w-2)dw}{w}+\frac{1}{5}\int\frac{4}{w}dw$

$(\frac{3}{5})(\int\frac{w-2}{w}dw)+\frac{4}{5}\int\frac{1}{v}dw \to (\frac{3}{5})(\int dw-2\int\frac{dw}{w})+\frac{4}{5}\int\frac{dw}{w}$

$(\frac{3}{5})(w-2\ln(w))+\frac{4}{5}\ln(w) \to \frac{3}{5}(x-2y+2-2\ln(x-2y+2))+\frac{4}{5}(\ln(x-2y+2))$

$(\frac{3}{5})(x-2y+2-2\ln(x-2y+2))+\frac{4}{5}(\ln(x-2y+2))=x+C$

$\frac{1}{5}(3x-6y-6\ln(x-2y+2)+4\ln(x-2y+2))=x+C$

$\frac{1}{5}(3x-6y-2\ln(x-2y+2))=x+C$

$\boxed{x=\frac{1}{5}(3x-6y-2\ln(x-2y+2))+C}$ Sol. general.

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8) Obtener $M(x,y)$ y resolver: $M(x,y)\,dx + \left(\sec^2 y - \frac{x}{y}\right)dy = 0$

$\frac{dM}{dx} = \frac{dN}{dy} = -\frac{1}{y} \rightarrow \int -\frac{1}{y}\,dy \rightarrow -\ln(y) = M(x,y)$

$\frac{dM}{dy} = -\frac{1}{y} = -\frac{1}{y}$ ∴ exacta.

$F(x,y) = -\int \ln(y)\,dx + h(x) \rightarrow F = -\ln(y)x + h(x)$

$\frac{dF}{dy} = -\frac{x}{y} + h'(y) \rightarrow \sec^2 y \cancel{-\frac{x}{y}} = \cancel{-\frac{x}{y}} + h'(y)$

$dh = \sec^2 y\,dy \rightarrow h = \int \sec^2 y\,dy \rightarrow h = \tan(y)$

$F(x,y) = -\ln(y)x + \tan(y) + C \rightarrow \boxed{C = -\ln(y)x + \tan(y)}$

sol. general.

9) Resolver: $\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = x\,\text{sen}(x)$

$p(x) = -\frac{2}{x}$

$f(x) = x^2\,\text{sen}(x)$

$\frac{dy}{dx} - \frac{\frac{2}{x^2}}{\frac{1}{x}}(y) = \frac{x\,\text{sen}(x)}{\frac{1}{x}} \rightarrow \frac{dy}{dx} - \frac{2}{x}y = x^2\,\text{sen}(x)$

$y(x) = e^{\int \frac{2}{x}dx}\int e^{-\int \frac{2}{x}dx}\, x^2\,\text{sen}(x)\,dx + C e^{\int \frac{2}{x}dx}$

$= (x^2)\int \frac{\cancel{x^2}\,\text{sen}(x)}{\cancel{x^2}}\,dx + Cx^2$

$= (x^2)(-\cos(x)) + Cx^2$

$\boxed{y(x) = -x^2\cos(x) + Cx^2}$ sol. general.

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10) Determinar la solucion general: $sen(\omega)\frac{dy}{d\omega} - 2\cos(\omega)y = -sen(\omega)\cos(\omega)$

$\frac{dy}{d\omega} - 2\cot(\omega)y = -\cos(\omega)$ $\quad P(\omega) = -2\cot(\omega)$

$F(\omega) = -\cos(\omega)$

$$y = e^{2\int \cot(\omega)d\omega}\int e^{-2\int \cot(\omega)d\omega} - \cos(\omega)\,d\omega + ce^{2\int \cot(\omega)d\omega}$$

$$y(\omega) = e^{2\ln(sen(\omega))}\int e^{-2\ln(sen(\omega))} - \cos(\omega)\,d\omega + ce^{2\ln(sen(\omega))}$$

$$y(\omega) = (sen^2(\omega))\int \frac{1}{sen^2(\omega)} - \cos(\omega)\,d\omega + c\,sen^2(\omega)$$

$v = sen(\omega)$

$dv = \cos(\omega)\,d\omega$

$\int \frac{dv}{v^2}$

$$y(\omega) = (sen^2(\omega))(\csc(\omega) + c\,sen^2(\omega)$$

$\boxed{y(\omega) = (sen(\omega)) + c\,sen^2(\omega)}$ sol. general.