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Tarea: 27

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- Ejercicio 6, 2014-2, 2° Final, Tipo A

Sean el espacio vectorial $\mathbb{R}^2$ con producto interno usual y el operador lineal $S: \mathbb{R}^2 \to \mathbb{R}^2$ cuya regla de correspondencia es: $S(x,y) = (4x+y, x+4y)$ obtener la descomposición espectral.

$$M(S) = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \quad (4-\lambda)(4-\lambda)-1$$

$$16 - 8\lambda + \lambda^2 - 1$$

$$(\lambda-3)(\lambda-5)$$

$B = \{(1,0), (0,1)\}$

$T(1,0) = (4,1)$

$T(0,1) = (1,4)$

- $\lambda = 3$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{matrix} x+y=0 \\ x+y=0 \end{matrix} \quad x=-y \quad E(3) = \{(-y,y) \mid y \in \mathbb{R}\}$$

- $\lambda = 5$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{matrix} -x+y=0 \\ x-y=0 \end{matrix} \quad x=y \quad E(5) = \{(x,x) \mid x \in \mathbb{R}\}$$

$P_1(a,b) = (-a,a)$ $\quad (x,y) = (-a+b, a+b)$ $\quad x = -a+b$ $\quad b = x+a$ $\to a = \frac{y-x}{2}$

$P_2(a,b) = (b,b)$ $\quad y = a+b$ $\quad y = x+a+a$ $\quad y = x+2a$

$P_1(a,b) = \left(\frac{x-y}{2}, \frac{y-x}{2}\right)$ $\quad a = \frac{y-x}{2}$ $\quad b = x + \frac{y-x}{2} = \frac{2x+y-x}{2} = \frac{x+y}{2}$

$P_2(a,b) = \left(\frac{x+y}{2}, \frac{x+y}{2}\right)$

$$T(x,y) = \lambda P_1 + \lambda P_2 = 3\left(\frac{x-y}{2}, \frac{y-x}{2}\right) + 5\left(\frac{x+y}{2}, \frac{x+y}{2}\right)$$

$$= \left(\frac{3x-3y+5x+5y}{2}, \frac{3y-3x+5x+5y}{2}\right)$$

$$= \left(\frac{8x+2y}{2}, \frac{8y+2x}{2}\right) = (4x+y, 4y+x) \text{ Se cumple}$$

- Ejercicio 6, 2016-2, 2° Final, Tipo C.
  $\quad \to 1°$

Sea el espacio vectorial $\mathbb{C}^2$ con producto interno definido: $(\bar{z} \mid \bar{w}) = z_1\bar{w}_1 + z_2\bar{w}_2$

- Determinar la descomposición espectral del operador lineal $S: \mathbb{C}^2 \to \mathbb{C}^2$ cuya regla de correspondencia es $S(z_1, z_2) = (-iz_2, iz_1)$.

$B = \{(1,0), (0,1)\}$

$T(1,0) = (0, i)$ $\quad M(S) = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\quad (-\lambda)^2 - 1 = 0$

$T(0,1) = (-i, 0)$ $\quad (\lambda+1)(\lambda-1) = 0$

∘ $\lambda = 1$

$\begin{bmatrix} -1 & -i \\ i & -1 \end{bmatrix} =$ $\begin{matrix} -x - iy = 0 \\ ix - y = 0 \end{matrix}$ $\quad y = ix$ $\quad E(1) = \{(x, ix) \mid x \in \mathbb{C}\}$ $\quad B = \{(1, i)/\sqrt{2}\}$ $\quad \|(1,i)\| = 1+1$

∘ $\lambda = -1$

$\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$ $\begin{matrix} x - iy = 0 \\ ix + y = 0 \end{matrix}$ $\quad \begin{matrix} x = iy \\ -y + y = 0 \end{matrix}$ $\quad E(-1) = \{(iy, y) \mid y \in \mathbb{C}\}$ $\quad B = \{(i, 1)/\sqrt{2}$

$P_2((x,y)) \| \frac{1}{\sqrt{2}}(i,1) | \frac{1}{\sqrt{2}}(1,1) = \frac{1}{2}(-ix+y)(i,1)$

$= \dfrac{(+x+yi, -ix+y)}{2}$

$P_1(x,y) \mid \frac{1}{\sqrt{2}}(1,i) \frac{1}{\sqrt{2}}(1,i) = \frac{1}{2}(x-yi)(1,i)$

$= \left(\dfrac{x-yi, xi+y}{2}\right)$

$S(x,y) = 1\left(\dfrac{x-yi}{2}, \dfrac{xi+y}{2}\right) - 1\left(\dfrac{x+yi}{2}, \dfrac{-ix+y}{2}\right)$

$= \left(\dfrac{\cancel{x-x} - yi - yi}{2}, \dfrac{xi + \cancel{y} + ix - \cancel{y}}{2}\right)$

$= \underline{(-iy, xi)}$ ∴ cumple