Díaz Hernández Marcos Bryan | Ma – Ju | Número de lista: 12

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Díaz Hernández Marcos Bryan  Ma - Ju  N.L:12

Tarea: 28

- Ejercicio 6, 2014-1, 2° Final, Tipo A

6. Sea la curva $C$: $5x^2+4xy+5y^2-21=0$

a) Matriz simétrica asociada a la forma cuadrática

b) Unas ecuaciones de $C$ que no contengan término cruzado

c) Trazar la curva $C$.

$\bar{x}^T A \bar{x} + K\bar{x} + f = 0$

$A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$ $\bar{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ $K = [0\ 0]$

$(\lambda-5)^2-4 = \lambda^2-10\lambda+25-4$

$\lambda^2-10\lambda+21 = (\lambda-3)(\lambda-7)$

$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{2}+\frac{1}{2}=1$

- $\lambda = 3$ $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $2x+2y=0$, $2x+2y=0$, $x=-y$ $V(3)=\{(-y,y)\,|\,y\in\mathbb{R}\}-\{\bar{0}\}$

- $\lambda = 7$ $\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$ $-2x+2y=0$, $2x-2y=0$, $y=x$ $V(7)=\{(x,x)\,|\,x\in\mathbb{R}\}-\{\bar{0}\}$

$B = \left\{ \frac{(-1,1)}{\sqrt{2}}, \frac{(1,1)}{\sqrt{2}} \right\}$

a) $A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$

$\bar{x} = Px'$

$(Px')^T A (Px') + 0(Px') = 21$

$(x')^T D (x') + 0 = 21$

b) $[x'\ y'] \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = 21$

$1\times2 \quad 2\times2$

$[3x'\ 7y'] \begin{bmatrix} x' \\ y' \end{bmatrix} = 21$

$\frac{3(x')^2}{21} + \frac{7(y')^2}{21} = \frac{21}{21}$

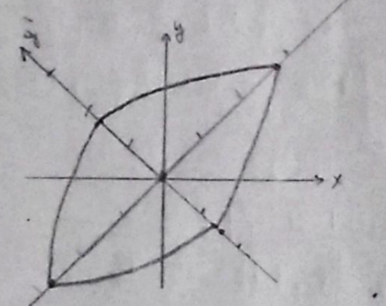
1) $\frac{(x')^2}{7} + \frac{(y')^2}{3} = 1$

$\cos\theta = \frac{(1,1)\cdot(1,0)}{\|(1,1)\|\,\|(1,0)\|} = \frac{1}{\sqrt{2}} = 45°$

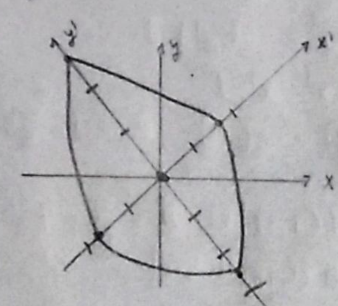
2) $\frac{(x')^2}{3} + \frac{(y')^2}{7} = 1$

c)

1)




2)



Díaz Hernández Marcos Bryan

-Ejercicio 39, Página 458, Barrera.

Para la cónica: $x^2+2xy+y^2-\frac{10}{\sqrt{2}}x+\frac{2}{\sqrt{2}}y+14=0$

a) Determine la matriz P que corresponde al giro que hace paralelos.

b) Ecuación de la cónica, en $(x'', y'')$ que no tenga término mixto ni lineales.

c) Ángulo

d) Dibuje la cónica, así como los sistemas de referencia

$\bar{x}^T A\bar{x} + h\bar{x} + f = 0$

$A=\begin{bmatrix}1&1\\1&1\end{bmatrix}$ $h=\begin{bmatrix}-\frac{10}{\sqrt{2}} & \frac{2}{\sqrt{2}}\end{bmatrix}$ $\bar{x}=\begin{bmatrix}x\\y\end{bmatrix}$

$(1-\lambda)^2-1=\lambda^2-2\lambda+1-1=\lambda(\lambda-2)=0$ $\quad \lambda=0$, $\lambda=2$

$\lambda=0$

$\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{matrix}x+y=0\\x+y=0\end{matrix}$ $\quad x=-y \quad V(0)=\{(-y,y)\mid y\in\mathbb{R}\}-\{\bar{0}\}$

$\lambda=2$

$\begin{bmatrix}-1&1\\1&-1\end{bmatrix}=\begin{matrix}-x+y=0\\x+(-y)=0\end{matrix}$ $\quad x=y \quad V(2)=\{(x,x)\mid x\in\mathbb{R}\}-\{\bar{0}\}$

$B=\left\{\frac{(-1,1)}{\sqrt{2}}, \frac{(1,1)}{\sqrt{2}}\right\}$

a) $\underline{P=\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}}\end{bmatrix}}_{//}=\frac{1}{2}+\frac{1}{2}=1$

$\bar{x}=Px'$

$(Px')^T A(Px')+h(Px')+f=0$

$(x')^T D x' + hPx' + f = 0$

$\begin{bmatrix}x' & y'\end{bmatrix}\begin{bmatrix}2&0\\0&0\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix}+\begin{bmatrix}-\frac{10}{\sqrt{2}} & \frac{2}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix}+14=0$

$\begin{bmatrix}2x'\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix}+\begin{bmatrix}(-5+1) & (5+1)\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix}+14=0$

$2(x')^2 + -4(x') + 6(y') + 14 = 0$

$(x')^2-2(x')+3(y')+7=0$

$((x')^2-2(x')+1)=-3y'-6$

$(x'-1)^2=-3(y'+2)$

$y''=y'+2 \quad x''=x'-1$

b) $\underline{(x'')^2=-3(y'')}_{//}$ $\quad$ Parábola $y=-\frac{x^2}{3}$

c) $\cos\theta=\frac{|(1,1)\cdot(1,0)|}{\|(1,1)\|\,\|(1,0)\|}=\frac{1}{\sqrt{2}}=45°$

d) Gráfico: