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Ecuaciones Diferenciales
Participación Extra de la Transformada de Laplace

$$\mathcal{L}\{\cos(wt)\} = \frac{s}{s^2+w^2}$$

$$\mathcal{L}\{\cos(wt)\} = \lim_{b\to\infty}\int_0^b e^{-st}\cos(wt)\,dt \to \int_0^b e^{-st}\cos(wt)\,dt$$

Integral: $\int_0^b e^{-st}\cos(wt)\,dt$ $\quad u = e^{-st}$ $\quad dv = \cos(wt)\,dt$

$du = -se^{-st}\,dt$ $\quad dv = \frac{\cos(a)}{w}\,da$

$$\int e^{-st}\cos(wt) = (e^{-st})\left(\frac{\operatorname{sen}(wt)}{w}\right) + \int s\frac{e^{-st}\operatorname{sen}(wt)}{w}\,dt \quad v = \frac{\operatorname{sen}(wt)}{w}$$

$$\int e^{-st}\cos(wt) = \frac{e^{-st}\operatorname{sen}(wt)}{w} + \frac{s}{w}\int e^{-st}\operatorname{sen}(wt)\,dt \quad u = e^{-st}$$

$du = -se^{-st}\,dt$

$$\frac{s}{w}\left(-\frac{e^{-st}\cos(wt)}{w} - \int s\frac{e^{-st}\cos(wt)}{w}\,dt\right)$$

$dv = \operatorname{sen}(wt)\,dt$

$v = -\frac{\cos(wt)}{w}$

$$\int e^{-st}\cos(wt) = \frac{e^{-st}\operatorname{sen}(wt)}{w} - \frac{s}{w^2}\left(e^{-st}\cos(wt)\right) - \frac{s^2}{w^2}\int e^{-st}\cos(wt)\,dt$$

$$\left(\int e^{-st}\cos(wt)\,dt\right)\left(1+\frac{s^2}{w^2}\right) = \frac{e^{-st}\operatorname{sen}(wt)}{w} - \frac{s}{w^2}\left(e^{-st}\cos(wt)\right)$$

$$\lim_{b\to\infty} \frac{e^{-st}\operatorname{sen}(wt)}{w} - \frac{s}{w^2}\left(e^{-st}\cos(wt)\right)\left(\frac{w^2+s^2}{w^2}\right)\Big|_0^b$$

$$\left[\frac{w^2}{w^2+s^2}\right]\lim_{b\to\infty} \frac{e^{-sb}\overset{\to 0}{\cancel{\operatorname{sen}(wb)}}}{w} - \frac{s}{w^2}\left(e^{-sb}\overset{\to 0}{\cancel{\cos(wb)}}\right) + \left(\frac{s}{w^2}\right)$$

$$\left(\frac{w^2}{w^2+s^2}\right)\lim_{b\to\infty}\left(\frac{s}{w^2}\right) = \left(\frac{\cancel{w^2}}{w^2+s^2}\right)\left(\frac{s}{\cancel{w^2}}\right) = \left(\frac{s}{w^2+s^2}\right) \quad s>0$$

$$\mathcal{L}\{\cos(wt)\} = \frac{s}{w^2+s^2} \quad s>0$$