

$$5) \quad y'' - 3y' = 8e^{3x} + 4\cos(x) \quad Z(n) = (D-3)(D^2+1)$$

$$\begin{array}{l} n=1 \quad n=1 \\ \alpha=3 \quad \alpha=0 \\ \quad \quad \beta=1 \end{array}$$

$$(D-3)(D^2+1)(D^2-3D)y = (D-3)(D^2+1)^0 (8e^{3x} + 4\cos(x))$$

$$(\lambda-3)(\lambda^2+1)(\lambda^2-3\lambda) = 0$$

$$y_h = \lambda^2 - 3\lambda = 0 \quad \lambda(\lambda-3) = 0 \quad \begin{array}{l} \lambda_1 = 0 \\ \lambda_2 = 3 \end{array}$$

$$y_h = c_1 e^{0x} + c_2 e^{3x}$$

$$y_{pnb} = (\lambda-3)(\lambda^2+1) = 0 \quad \lambda_3 = 3$$

$$\left. \begin{array}{l} \lambda_4 = -i \\ \lambda_5 = +i \end{array} \right\} \begin{array}{l} \alpha=0 \\ \beta=1 \end{array}$$

$$y_{pnb} = c_3 x e^{3x} + c_4 \cos(x) + c_5 \sin(x)$$

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$$y_{ph} = c_3 x e^{3x} + c_4 \cos(x) + c_5 \sin(x)$$

$$y'_{ph} = c_3 [(x)(3e^{3x}) + (e^{3x})] - c_4 \sin(x) + c_5 \cos(x)$$

$$y''_{ph} = 3c_3 [(x)(3e^{3x}) + (e^{3x})] + 3c_3 e^{3x} - c_4 \cos(x) - c_5 \sin(x)$$

$$9c_3 x e^{3x} + 3c_3 e^{3x} + 3c_3 e^{3x} - c_4 \cos(x) - c_5 \sin(x)$$

$$- 3c_3 [3x e^{3x} + e^{3x} - c_4 \sin(x) + c_5 \cos(x)]$$

$$3c_3 e^{3x} - c_4 \cos(x) - c_5 \sin(x) + 3c_4 \sin(x) + 3c_5 \cos(x) = 8e^{3x} + 4\sin(x)$$

$$3c_3 e^{3x} = 8e^{3x} \quad c_3 = 8/3$$

$$-c_4 \cos(x) - 3c_5 \cos(x) - c_5 \sin(x) + 3c_4 \sin(x) = 4\sin(x) + 0\cos(x)$$

$$\cos(x)(-c_4 - 3c_5) = 0 \cos(x) \quad -c_4 - 3c_5 = 0$$

$$\sin(x)(-c_5 + 3c_4) = 4\sin(x) \quad -c_5 + 3c_4 = 4$$

$$c_4 = -3c_5 \rightarrow -c_5 + 3(-3c_5) = 4 \rightarrow -c_5 - 9c_5 = 4 \quad c_5 = -4/10$$

$$-c_5 = 2/5 \quad c_4 = -3(-2/5) \quad c_4 = +6/5$$

$$c_3 = 8/3 \quad c_4 = 6/5 \quad c_5 = -2/5$$

$$y_p = \frac{8}{3} x e^{3x} + \frac{6}{5} \cos(x) + \frac{-2}{5} \sin(x)$$

$$y_g = c_1 + c_2 e^{3x} + \frac{8}{3} x e^{3x} + \frac{6}{5} \cos(x) - \frac{2}{5} \sin(x)$$

$$6) y'' + y = x \cos x - \cos x \quad \mathcal{L}(0) = (D^2 + 1)^2$$

$$\left. \begin{array}{l} n=2 \\ \alpha=0 \\ \beta=1 \end{array} \right\} \begin{array}{l} n=1 \\ \alpha=0 \\ \beta=1 \end{array} \left. \begin{array}{l} \text{iguales} \\ \text{cos}(x) (x-1) \end{array} \right\}$$

$$(D^2 + 1)^2 (D^2 + 1) y = (D^2 + 1)^2 \overset{\rightarrow 0}{x \cos x - \cos x}$$

$$(\lambda^2 + 1)^2 (\lambda^2 + 1) = 0$$

$$y_h = \left. \begin{array}{l} \lambda_1 = +i \\ \lambda_2 = -i \end{array} \right\} \begin{array}{l} \alpha=0 \\ \beta=1 \end{array} \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} y_h = e^{0x} (c_1 \cos(x) + c_2 \sin(x))$$

$$y_p = (\lambda^2 + 1)^2 = 0 \quad (\lambda^2 + 1)(\lambda^4 + 1) = 0 \quad \left. \begin{array}{l} \lambda_3 = +i \\ \lambda_4 = -i \\ \lambda_5 = +i \\ \lambda_6 = -i \end{array} \right\} \begin{array}{l} \alpha=0 \\ \beta=1 \end{array}$$

$$y_{ph} = c_3 x \cos(x) + c_4 x \sin(x) + c_5 x^2 \cos(x) + c_6 x^2 \sin(x)$$

$$y'_{ph} = c_3 [-\sin(x)(x) + \cos(x)] + c_4 [(x) \cos(x) + \sin(x)]$$

$$+ c_5 [(x^2)(-\sin(x) + \cos(x)(2x))] + c_6 [(x^2)(\cos(x) + \sin(x)(2x))]$$

$$y''_{ph} = -c_3 [(x) \cos(x) + \sin(x)] - c_3 \sin(x) + c_4 [(x)(-\sin(x) + \cos(x))] + c_4 \cos(x) +$$

$$+ c_5 [(x^2)(\cos(x) + \sin(x)(2x))] +$$

$$+ 2c_5 [(x)(-\sin(x) + \cos(x))] + c_6 [(x^2)(-\sin(x) + \cos(x)(2x))] +$$

$$+ 2c_6 [(x)(\cos(x) + \sin(x))]$$

$$= -c_3 x \cos(x) - c_3 \sin(x) - c_3 \sin(x) - c_4 x \sin(x) + c_4 \cos(x) + c_4 \cos(x) +$$

$$- c_5 x^2 \cos(x) - c_5 2x \sin(x) - 2c_5 x \sin(x) + 2c_5 \cos(x) +$$

$$- c_6 x^2 \sin(x) + 2c_6 x \cos(x) + 2c_6 x \cos(x) + 2c_6 \sin(x) +$$

$$c_3 x \cos(x) + c_4 x \sin(x) + c_5 x^2 \cos(x) + c_6 x^2 \sin(x)$$

$$- c_3 x \cos(x) - 2c_3 \sin(x) - c_4 x \sin(x) + 2c_4 \cos(x) - c_5 x^2 \cos(x) +$$

$$- 4c_5 x \sin(x) + 2c_5 \cos(x) - c_6 x^2 \sin(x) + 4c_6 x \cos(x) + 2c_6 \sin(x) +$$

$$- 2c_3 \sin(x) - 4c_5 x \sin(x) + 2c_6 \sin(x) = 0 \sin(x)$$

$$2c_4 \cos(x) + 2c_5 \cos(x) + 4c_6 x \cos(x) = \cos(x)(x-1)$$

$$7) \quad 2C_4 + 2C_5 + 4C_6x = x - 7 \quad 2) \quad -2C_3 - 4C_5x + 2C_6 = 0$$

$$4C_6x = x$$

$$C_6 = 1/4$$

$$-2C_3 - 4C_5x = -1/2$$

$$2C_4 + 2C_5 = -7$$

$$2C_4 = -1$$

$$C_4 = -1/2$$

$$-4C_5x = 0x$$

$$C_5 = 0$$

$$-2C_3 = -1/2$$

$$C_3 = 1/4$$

$$C_3 = 1/4 \quad C_4 = -1/2 \quad C_5 = 0 \quad C_6 = 1/4$$

$$y_{ph} = \frac{1}{4}x \cos(x) - \frac{1}{2}x \sin(x) + 0x^2 \cos(x) + \frac{1}{4}x^2 \sin(x)$$

$$y_g = \frac{1}{4} \cos(x) + \frac{1}{2} \sin(x) + \frac{1}{4}x \cos(x) - \frac{1}{2}x \sin(x) + \frac{1}{4}x^2 \sin(x)$$

$$7) \quad y'' - 2y' + y = 10e^{-2x} \cos(x) \quad \mathcal{L}(D) = (D^2 + 4D + 4 + 1)$$

$$n=1 \quad \mathcal{L}(D) = (D^2 + 4D + 5)$$

$$\alpha = -2$$

$$\beta = 1$$

$$(D^2 + 4D + 5)(D^2 - 2D + 1)y = (D^2 + 4D + 5)10e^{-2x} \cos(x)$$

$$(\lambda^2 + 4\lambda + 5)(\lambda^2 - 2\lambda + 1) = 0$$

$$y_h: \lambda^2 - 2\lambda + 1 = (\lambda - 1)(\lambda - 1) = \lambda_1 = 1$$

$$y_h = C_1 e^x + C_2 x e^x$$

$$y_{ph} = \lambda^2 + 4\lambda + 5 = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

$$\lambda_3 = -2 + i \quad \alpha = -2$$

$$\lambda_4 = -2 - i \quad \beta = 1$$

$$y_{ph} = e^{-2x}(C_3 \cos(x) + C_4 \sin(x))$$

$$y'_{ph} = (e^{-2x})(-C_3 \sin(x) + C_4 \cos(x)) + (C_3 \cos(x) + C_4 \sin(x))(-2e^{-2x})$$

$$y''_{ph} = (e^{-2x})(-C_3 \cos(x) - C_4 \sin(x)) + (-C_3 \sin(x) + C_4 \cos(x))(-2e^{-2x})$$

$$+ (C_3 \cos(x) + C_4 \sin(x))(-4e^{-2x}) + (-2e^{-2x})(-C_3 \sin(x) + C_4 \cos(x))$$

$$\begin{aligned}
& - (e^{-2x})(3 \cos(x)) - (e^{-2x})(4 \sin(x)) + 2e^{-2x}(3 \sin(x)) - 2e^{-2x}(4 \cos(x)) \\
& + 4e^{-2x}(3 \cos(x)) + 4e^{-2x}(4 \sin(x)) + 2e^{-2x}(3 \sin(x)) - 2e^{-2x}(4 \cos(x)) \\
& = 3e^{-2x}(3 \cos(x)) + 4e^{-2x}(3 \sin(x)) + 3e^{-2x}(4 \sin(x)) - 4e^{-2x}(4 \cos(x)) \\
& \quad - 2(-e^{-2x}(3 \sin(x)) + e^{-2x}(4 \cos(x)) - 2e^{-2x}(3 \cos(x)) - 2e^{-2x}(4 \sin(x))) \\
& = 3e^{-2x}(3 \cos(x)) + 4e^{-2x}(3 \sin(x)) + 3e^{-2x}(4 \sin(x)) - 4e^{-2x}(4 \cos(x)) \\
& \quad + 2e^{-2x}(3 \sin(x)) - 2e^{-2x}(4 \cos(x)) + 4e^{-2x}(3 \cos(x)) + 4e^{-2x}(4 \sin(x)) \\
& = 7e^{-2x}(3 \cos(x)) + 7e^{-2x}(4 \sin(x)) - 6e^{-2x}(4 \cos(x)) + 6e^{-2x}(3 \sin(x)) \\
& \quad + e^{-2x}(3 \cos(x)) + e^{-2x}(4 \sin(x)) \\
& 8e^{-2x}(3 \cos(x)) + 8e^{-2x}(4 \sin(x)) - 6e^{-2x}(4 \cos(x)) + 6e^{-2x}(3 \sin(x)) \\
& 8e^{-2x}(3 \cos(x)) - 6e^{-2x}(4 \cos(x)) = 10e^{-2x} \cos(x) \\
& 8(3) - 6(4) = 10 \\
& 8e^{-2x}(4 \sin(x)) + 6e^{-2x}(3 \sin(x)) = 0e^{0x} \sin(x) \\
& 8(4) + 6(3) = 0 \quad 8(3) = 10 + 6(4) \quad C_3 = \frac{5 + 3(4)}{4} \\
& 8(3) - 6(4) = 10 \quad 8(4) + 6\left(\frac{5 + 3(4)}{4}\right) = 0 \\
& 6C_3 = -8(4) \quad 8(4) + \frac{30 + 18(4)}{4} \rightarrow 8(4) + \frac{15}{2} + \frac{9(4)}{2} = 0 \\
& 6C_3 = -8\left(-\frac{3}{8}\right) \quad \frac{25(4)}{2} = -\frac{15}{2} \\
& C_3 = \frac{24}{(6)(3)} = \frac{4}{5} \quad C_4 = -\frac{15}{25} = -\frac{3}{5} \\
& y_{p1} = e^{-2x} \left(\frac{4}{5} \cos(x) - \frac{3}{5} \sin(x) \right) \\
& \underline{y_g = (c_1 e^x + c_2 x e^x + e^{-2x} \left(\frac{4}{5} \cos(x) - \frac{3}{5} \sin(x) \right))}
\end{aligned}$$