Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

Díaz Hernández Marcos Bryan - Ecuaciones Diferenciales

1) Resolver: $y'' + y = \sec(\theta)\tan(\theta)$ $\quad (1/\cos)\left(\frac{sen}{\cos}\right) = \frac{sen}{\cos^2}$

$y'' + y = \frac{sen}{\cos^2}$ $\quad$ VD = y $\quad$ VI = θ

$-Q(\theta) = 0 \quad D^2y + y = 0 \rightarrow (D^2+1)y = 0 \quad \lambda^2 + 1 = 0 \quad \lambda^2 = -1$

$\left.\begin{array}{l} r_1 = +i \\ r_2 = -i \end{array}\right\} \alpha = 0$

$\left.\begin{array}{l} yh = C_1\cos(\theta) + C_2 sen(\theta) \\ yp = v(\theta)\cos(\theta) + v(\theta)sen(\theta) \end{array}\right\}$ $\quad \begin{array}{l} v'y_1 + v'y_2 = 0 \\ v'y'_1 + v'y_2 = Q(x) \end{array}$

$$\begin{bmatrix} \cos(\theta) & sen(\theta) \\ -sen(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} v' \\ v' \end{bmatrix} = \begin{bmatrix} 0 \\ Q(x) \end{bmatrix} \quad v' = \frac{w_1}{T_1} \quad v' = \frac{w_2}{J_2}$$

$$w_T = \begin{bmatrix} \cos(\theta) & sen(\theta) \\ -sen(\theta) & \cos(\theta) \end{bmatrix} = \cos^2\theta + sen^2\theta = 1$$

$$w_1 = \begin{bmatrix} 0 & sen \\ \frac{sen}{\cos^2} & \cos \end{bmatrix} = \frac{-sen^2}{\cos^2} \quad v' = \frac{-sen^2}{\cos^2} \quad v' = \frac{sen}{\cos}$$

$$w_2 = \begin{bmatrix} \cos & 0 \\ -sen & \frac{sen}{\cos^2} \end{bmatrix} = \frac{sen}{\cos}$$

$$-\int \frac{sen^2\theta}{\cos^2\theta}\,d\theta = -\int \tan^2\theta\,d\theta = -\int \sec^2\theta - 1\,d\theta = -\tan\theta + \theta$$

$$\int \tan\,d\theta = -\int \frac{sen}{\cos}\,d\theta \quad \begin{array}{l} v = \cos \\ dv = -sen\,d\theta \end{array} = -\int \frac{1}{v}\,dv = -\ln(\cos\theta)$$

Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

$$y_p = (-\tan\theta + \theta)(\cos\theta) + \ln(\cos\theta)(\operatorname{sen}(\theta))$$

$$y_g = y_h + y_p = C_1\cos\theta + C_2\operatorname{sen}\theta - \tan\theta\cos\theta - \theta\cos\theta - \ln\cos\theta\,\text{se}$$

$$y_g = C_1\cos\theta + C_2\operatorname{sen}\theta - \cancel{\operatorname{sen}}\theta + \theta\cos\theta - \ln((\cos\theta))\operatorname{sen}(\theta)$$

$$y_g = C_1\cos\theta + C_2\operatorname{sen}\theta + \theta\cos\theta - \ln(\cos\theta)\operatorname{sen}(\theta)$$

Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

2) $x^2D^2y + xD(xy) - xy = x^2(e^{-x} + \cos(2x))$

$-xy + \frac{x^2D^2y}{x^2} + \frac{x(y + xy')}{x^2} = \frac{x^2(e^{-x} + \cos(2x))}{x^2}$

$D^2y + Dy = e^{-x} + \cos(2x) \rightarrow r^2 + r = 0 \quad r(r+1) = 0$

$r_1 = 0 \quad y_h = c_1 + c_2 e^{-x}$
$r_2 = -1 \quad y_p = v(x) + u(x) e^{-x}$

$v'y_1 + u'y_2 = 0$
$v'y_1' + u'y_2' = Q(x)$

$$\begin{bmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{bmatrix} \begin{bmatrix} v'(x) \\ u'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ e^{-x} + \cos(2x) \end{bmatrix}$$

$W_T = -e^{-x}$ $\quad v' = \frac{+e^{-2x} + e^{-x}\cos(2x)}{+e^{-x}} = e^{-x} + \cos(2x)$

$W_1 = -e^{-x}(e^{-x} + \cos(2x))$

$W_2 = e^{-x} + \cos(2x)$ $\quad u' = \frac{e^{-x} + \cos(2x)}{-e^{-x}} = -1 + \cos(2x)e^{x}$

$\int e^{-x} + \cos(2x)\,dx = -e^{-x} + \frac{\operatorname{sen}(2x)}{2}$

$\int -1 - \cos(2x)e^{x}\,dx = -x - \int \cos(2x)e^{x}\,dx$ $\quad u = \cos(2x) \quad du = -2\operatorname{sen}(2x) \quad \int dv = \int e^{x}dx \quad v = e^{x}$

$\int \cos(2x)e^{x} = \cos(2x)(e^{x}) + \int 2\operatorname{sen}(2x)(e^{x})\,dx$ $\quad u = \operatorname{sen}(2x) \quad du = 2\cos(2x) \quad \int dv = \int e^{x}dx \quad v = e^{x}$

$\cos(2x)(e^{x}) + 2(\operatorname{sen}(2x))(e^{x}) - 4\int \cos(2x)e^{x}\,dx$

$-\left(\frac{2\operatorname{sen}(2x)e^{x} + \cos(2x)e^{x}}{5}\right)$

$v = -e^{-x} + \frac{\operatorname{sen}(2x)}{2} \qquad u = -x - \frac{e^{x}(2\operatorname{sen}(2x) + \cos(2x))}{5}$

Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

$$yp = -e^{-x} + \frac{\operatorname{sen}(2x)}{2} + -xe^{-x} - \frac{2\operatorname{sen}(2x)}{5} - \frac{\cos(2x)}{5}$$

$$yp = -e^{-x} + e^{-x}x + \frac{\operatorname{sen}(2x)}{10} - \frac{\cos(2x)}{5}$$

$$yg = C_1 + C_2 e^{-x} - xe^{-x} + \frac{\operatorname{sen}(2x)}{10} - \frac{\cos(2x)}{5}$$

Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

3) $y = x\cos(2x) + 4\cos^2(x) + 4$ es una solucion particular de una ecuacion lineal homogenea y coeficientes constantes

$ay'' + by' + c = 0 \quad Q(x) = 0$

a) Obtener la ecuacion diferencial correspondiente de menor orden y al sustituir en $ay'' + by' + c = 0$ (como forma de la ecuación) debe ser igual a 0.

Para $y = 0 \quad x\cos(2x) + 4\cos^2(x) + 4 = 0 \qquad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

$n = 2$
$\alpha = 0$
$\beta = 2$

$x\cos(2x) + 4\left(\frac{1}{2}\right)(1 + \cos(2x)) + 4 = x\cos(2x) + 2 + 2\cos(2x) + 4$

$y = x\cos(2x) + 2\cos(2x) + 6 \qquad (D^2+4)^2 D \cdot \div 2(D)$

$\left.\begin{matrix} n=2 \\ \alpha=0 \\ \beta=2 \end{matrix}\right\} \quad \begin{matrix} n=1 \\ \alpha=0 \\ \beta=2 \end{matrix} \qquad n=1$

$(D^2+4)^2 D y = (D^2+4)^2 D \left(x\cos(2x) + 6 + 2\cos(2x)\right) \to 0$

$(D^4 + 8D^2 + 16) D y = 0 \to (D^5 + 8D^3 + 16D) y = 0$

a) $y^5 + 8y''' + 16y' = 0$

b) Solucion general: $(D^2+4)^2 D y = 0 \quad (\lambda^2+4)(\lambda^2+4)(\lambda) = 0$

$\lambda_{1,2} = \pm 2i \qquad y_g = C_1 e^{0x} + e^{0x}\left(C_2\cos(2x) + C_3\,\mathrm{sen}(2x)\right)$

$\lambda_{3,4} = \pm 2i \qquad + x e^{0x}\left(C_4\cos(2x) + C_5\,\mathrm{sen}(2x)\right)$

$\lambda = 0 \qquad y_g = C_1 + C_2\cos(2x) + C_3\,\mathrm{sen}(2x) + x C_4\cos(2x)$
$+ x C_5\,\mathrm{sen}(2x)$

Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

4) resolver: $y'' - 4y' + 8y = 4e^{2x}\sec(2x) - 4e^{2x}\csc(2x)$

1) $Q(x)=0 \qquad D^2y - 4Dy + 8y = 0 \qquad (D^2 - 4D + 8)y = 0$

$$r^2 - 4r + 8 = 0 \Big\} \; \frac{4 \pm \sqrt{16-(4)(8)}}{2} = \frac{4 \pm \sqrt{16-32}}{2}$$

$$\frac{4 \pm \sqrt{-16}}{2} = \frac{(2)(2) \pm 4i}{2} = 2 \pm 2i \qquad r_1 = 2+2i, \quad r_2 = 2-2i$$

$$y_h = e^{2x}\left(C_1\cos(2x) + C_2\,\mathrm{sen}(2x)\right) \Big\} \; v'y_1 + v'y_2 = 0$$

$$y_p = e^{2x}v(x)\cos(2x) + e^{2x}v(x)\,\mathrm{sen}(2x) \Big\} \; v'y'_1 + v'y'_2 = 0$$

$$\begin{bmatrix} e^{2x}\cos(2x) & e^{2x}\,\mathrm{sen}(2x) \\ -2e^{2x}\,\mathrm{sen}(2x) + \cos(2x)(2e^{2x}) & 2e^{2x}\cos(2x) + \mathrm{sen}(2x)(2e^{2x}) \end{bmatrix}$$

$$W = 2\cos^2(2x)e^{4x} + \cancel{2e^{4x}\cos(2x)\,\mathrm{sen}(2x)} + 2e^{4x}\,\mathrm{sen}^2(2x)$$
$$- \cancel{\cos(2x)\,\mathrm{sen}(2x)(2e^{4x})} = 2\cos^2(2x)e^{4x} + 2e^{4x}\,\mathrm{sen}^2(2x)$$
$$= 2e^{4x}\left(\cos^2(2x) + \mathrm{sen}^2(2x)\right) = 2e^{4x}$$

$$W_1 = -\left(e^{2x}\,\mathrm{sen}(2x)\right)\left(4e^{2x}\sec(2x) - 4e^{2x}\csc(2x)\right)$$
$$= -4e^{4x}\tan(2x) + 4e^{4x}$$

Segundo Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

$$w_2 = (e^{2x}\cos(2x))(4e^{2x}\sec(2x) - 4e^{2x}\csc(2x))$$

$$w_2 = 4e^{4x} - 4e^{4x}\cot(2x)$$

$$u' = \frac{-4\cancel{e^{4x}}\tan(2x) + 4\cancel{e^{4x}}}{2\cancel{e^{4x}}} = -2\tan(2x) + 2$$

$$v = \frac{4\cancel{e^{4x}} - 4\cancel{e^{4x}}\cot(2x)}{2\cancel{e^{4x}}} = 2 - 2\cot(2x)$$

$$u = \int -2\tan(2x) + 2\,dx = \frac{-2}{2}\int \tan(v)\,dv \quad \begin{matrix} v = 2x \\ dv = 2dx \end{matrix} \quad -\int \frac{sen(v)}{\cos(v)}dv$$

$$+\int \frac{sen(v)}{v}dv \quad \begin{matrix} v = \cos(v) \\ dv = -sen(v)dv \end{matrix} \quad \int \frac{1}{v}dv = \ln(\cos(v))$$

$$u = \ln(\cos(2x)) + 2x$$

$$v = \int -2\cot(2x) + 2\,dx = -2\int \frac{\cos(2x)}{sen(2x)}dx = -\ln(sen(2x)) + 2x$$

$$y_p = e^{2x}\cos(2x)(\ln(\cos(2x)) + 2x) + e^{2x}sen(2x)(-\ln(sen(2x)) + 2x)$$

$$y_g = e^{2x}(C_1\cos(2x) + C_2 sen(2x)) + e^{2x}\cos(2x)\ln(\cos(2x)) +$$
$$2xe^{2x}\cos(2x) - e^{2x}sen(2x)\ln(sen(2x)) + 2xe^{2x}sen(2x)$$