

Resuelve la ecuación: $(1 + \sin(y))dx = [2y \cos(y) - x(\sec(y) + \tan(y))]dy$

$$(1 + \sin(y))dx + [2y \cos(y) + x(\sec(y) + \tan(y))]dy = 0 \quad y(0) = 1$$

$$\frac{\partial M}{\partial y} = \cos(y) \quad \frac{\partial N}{\partial x} = \sec(y) + \tan(y) \quad : \text{no exacta}$$

$$\alpha(y) = \int \frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dy$$

$$= \int \frac{1}{1 + \sin(y)} \left(\sec(y) + \tan(y) - \cos(y) \right) dy$$

$$= \int \frac{(\sec(y) + \tan(y) - \cos(y))}{1 + \sin(y)} dy$$

$$= \int \frac{\left(\frac{1}{\cos(y)} + \frac{\sin(y)}{\cos(y)} - \cos(y) \right)}{1 + \sin(y)} dy$$

$$\text{Integral} \int \frac{1 + \sin(y)}{\cos(y)} dy - \int \frac{\cos(y)}{1 + \sin(y)} dy$$

$$= \int \frac{1}{\cos(y)} dy - \int \frac{\cos(y)}{1 + \sin(y)} dy \quad \begin{matrix} v = 1 + \sin(y) \\ dv = \cos(y) dy \end{matrix}$$

$$= \int \sec(y) dy - \int \frac{dv}{v}$$

$$= \ln(\sec(y) + \tan(y)) - \ln(1 + \sin(y))$$

$$= \ln \left(\frac{\sec(y) + \tan(y)}{1 + \sin(y)} \right)$$

$$= e^{\ln \left(\frac{\sec(y) + \tan(y)}{1 + \sin(y)} \right)}$$

$$\alpha(y) = \frac{\sec(y) + \tan(y)}{1 + \sin(y)} = \alpha(y) = \frac{1 + \sin(y)}{\cos(y)} = \alpha(y) = \left(\frac{1}{\cos(y)} \right)$$

$$\frac{d(y)}{d(y)} [M(x,y) dx + N(x,y) dy] = 0$$

$$\frac{1}{\cos(y)} \left((1 + \sin(y)) dx - [2y \cos(y) - x(\sec y + \tan y)] dy \right) = 0$$

$$\frac{1 + \sin(y)}{\cos(y)} dx - \left[\frac{2y \cos(y)}{\cos(y)} - \frac{x(\sec y + \tan y)}{\cos(y)} \right] dy = 0$$

$$\begin{aligned} \frac{\partial M}{\partial y} &= \frac{-\cos(y)(\cos(y)) + (1 + \sin(y))(\sin(y))}{\cos^2(y)} = \frac{\cos^2(y) + \sin^2(y) + \sin(y)}{\cos^2(y)} \\ &= \frac{1 + \sin(y)}{\cos^2(y)} \end{aligned}$$

$$\frac{\partial N}{\partial x} = \frac{\sec y + \tan y}{\cos(y)} = \frac{1 + \sin(y)}{\cos(y)} = \frac{1 + \sin(y)}{\cos^2(y)} \quad ; \text{ son exactas}$$

$$F(x,y) = C \Rightarrow \frac{dF}{dt} = 0 \Rightarrow \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy = 0 \quad \begin{cases} \frac{\partial F}{\partial x} = M \\ \frac{\partial F}{\partial y} = N \end{cases}$$

$$\frac{dF}{dx} = M \Rightarrow \int dF = \int M dx$$

$$F = \int \left(\frac{1 + \sin(y)}{\cos(y)} \right) dx + h(y)$$

$$F = x \left(\frac{1 + \sin(y)}{\cos(y)} \right) + h(y) \dots A$$

$$\frac{dF}{dy} = x \left(\frac{\cos(y)(\cos(y)) + (1 + \sin(y))(\sin(y))}{\cos^2(y)} \right)$$

$$\frac{dF}{dy} = x \left(\frac{\cos^2(y) + \sin^2(y) + \sin(y)}{\cos^2(y)} \right) = x \left(\frac{1 + \sin(y)}{\cos^2(y)} \right) + h'(y)$$

$$-2y + x \left(\frac{\sec y + \tan y}{\cos(y)} \right) = x \left(\frac{1 + \sin(y)}{\cos^2(y)} \right) + h'(y)$$

$$-2y + x \left(\frac{1 + \sin(y)}{\cos^2(y)} \right) = x \left(\frac{1 + \sin(y)}{\cos^2(y)} \right) + h'(y)$$

$$-2y = h'(y)$$

$$\frac{dh}{dy} = -2y$$

$$\int dh = \int -2y \, dy$$

$$h = -y^2$$

en A) ...

$$F(x,y) = x \left(\frac{1 + \sin(y)}{\cos(y)} \right) - y^2 + C$$

$$C = x \left(\frac{1 + \sin(y)}{\cos(y)} \right) - y^2 \quad \text{condición } y(0) = 1 \quad \begin{cases} x=0 \\ y=1 \end{cases}$$

$$C = 0 - 1$$

$$C = -1$$

$$-1 = x \left(\frac{1 + \sin(y)}{\cos(y)} \right) - y^2$$
