Tercer Examen Parcial
Díaz Hernández Marcos Bryan
Ecuaciones Diferenciales
Grupo: 25

1) Haciendo uso de la definición obtener la transformada de Laplace de $f(t)$

$$F(t) = \begin{cases} 0 \ ; & t<5 \\ t^2 + e^t \ ; & t \geq 5 \end{cases} \qquad \mathcal{L}\{F(t)\} = \lim_{b\to\infty} \int_0^b e^{-st} F(t)\,dt$$

$$\mathcal{L}\{F(t)\} = \int_0^5 e^{-st}\,0\,dt + \int_5^\infty e^{-st}(t^2+e^t)\,dt$$

$$= \int_0^5 0\,dt + \lim_{b\to\infty} \int_5^b e^{-st}(t^2+e^t)\,dt$$

$$= 0 + \lim_{b\to\infty} \int_5^b e^{-st}t^2 + e^{-st+t}\,dt$$

$$\lim_{b\to\infty} \int_5^b \overset{1)}{e^{-st}t^2\,dt} + \lim_{b\to\infty} \int_5^b \overset{2)}{e^{-st+t}}\,dt$$

$$\lim_{b\to\infty} \int_5^b e^{-st}t^2\,dt = \quad \begin{matrix} u = t^2 & \int v' = \int e^{-st}dt \\ du = 2t\,dt & v = -\frac{1}{s}e^{-st} \end{matrix}$$

$$(t^2)\left(-\frac{1}{s}e^{-st}\right) + \frac{2}{s}\int te^{-st}dt \qquad \begin{matrix} u = t & \int v' = \int e^{-st}dt \\ du = dt & v = -\frac{1}{s}e^{-st} \end{matrix}$$

$$\int te^{-st}dt = (t)\left(-\frac{1}{s}e^{-st}\right) - \int -\frac{1}{s}e^{-st}dt$$

$$\int -\frac{e^{-st}}{s}dt \quad \begin{matrix} v = -st \\ dv = -s\,dt \\ dt = \frac{dv}{-s} \end{matrix} = \int \frac{1}{s^2}e^{v}dv = \frac{1}{s^2}e^{v} = \frac{1}{s^2}e^{-st}$$

$$\lim_{b\to\infty} \left((t^2)\left(-\frac{1}{s}e^{-st}\right)\right) + \frac{2}{s}\left(t\left(-\frac{1}{s}e^{-st}\right) - \frac{1}{s^2}e^{-st}\right) = \lim_{b\to\infty}\int_5^b e^{-st}t^2\,dt$$

$$\lim_{b\to\infty}\int_5^b e^{-st}t^2\,dt = \lim_{b\to\infty} -\frac{t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\Big|_5^b$$

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$$\lim_{b\to\infty} -\frac{t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2}{s^3}e^{-st}\Big|_5^b$$

$$\lim_{b\to\infty}\left(-\frac{b^2e^{-sb}}{s} - \frac{2be^{-sb}}{s^2} - \frac{2e^{-sb}}{s^3}\right) - \left(-\frac{25e^{-5s}}{s} - \frac{10e^{-5s}}{s^2} - \frac{2e^{-5s}}{s^3}\right)$$

$$\lim_{b\to\infty} -\frac{b^2}{se^{sb}} - \frac{2b}{s^2e^{sb}} - \frac{2}{s^3e^{sb}} = -\frac{\infty^2}{se^{\infty}} - \frac{\infty}{s^2e^{\infty}} - \cancel{\frac{2}{s^3e^{\infty}}}\to 0$$

$\lim_{b\to\infty} \frac{-b^2}{se^{sb}}$. Aplica L'Hopital $\lim_{b\to\infty}\frac{-2b}{s^2e^{sb}} = \lim_{b\to\infty}\frac{-2}{s^3e^{sb}} = \cancel{\frac{-2}{s^3e^{\infty}}}\to 0$

$\lim_{b\to\infty} \frac{-2b}{s^2e^{sb}}$ Aplica L'Hopital $\lim_{b\to\infty}\frac{-2}{s^3e^{sb}} = \lim_{b\to\infty}\frac{-2}{s^3e^{sb}} = \cancel{\frac{-2}{s^3e^{\infty}}}\to 0$

$$\lim_{b\to\infty}\int_5^\infty e^{-st}t^2\,dt = \frac{25e^{-5s}}{s} + \frac{10e^{-5s}}{s^2} + \frac{2e^{-5s}}{s^3}$$

$$\lim_{b\to\infty}\int_5^b e^{-st}e^t\,dt = \lim_{b\to\infty}\int_5^b e^{-ts+t}\,dt = \lim_{b\to\infty}\int_5^b e^{t(-s+1)}\,dt$$

$\int_5^b e^{t(-s+1)}\,dt$ $\quad v = t(-s+1)$, $dv = (-s+1)dt$ $\quad \int_5^b \frac{e^v\,dv}{(-s+1)} = \frac{e^v}{-s+1} = \frac{e^{t(-s+1)}}{-s+1}$

$$\lim_{b\to\infty}\frac{1}{(-s+1)}e^{t(-s+1)}\Big|_5^b = \lim_{b\to\infty}\frac{1}{(-s+1)}\left(e^{b(-s+1)} - e^{5(-s+1)}\right)$$

$$\frac{1}{(-s+1)}\lim_{b\to\infty} e^{-b(+s-1)} - e^{5(-s+1)} = \lim_{b\to\infty}\frac{1}{e^{b(s-1)}} = \cancel{\frac{1}{e^{\infty}}}\to 0$$

$$\lim_{b\to\infty}\int_5^b e^{-st}e^t\,dt = \frac{1}{(-s+1)}\left(-e^{5(-s+1)}\right) = \frac{e^{-5s}e^5}{(s-1)}$$

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$$\mathcal{L}\{f(t)\} = \frac{25e^{-5s}}{s} + \frac{10e^{-5s}}{s^2} + \frac{2e^{-5s}}{s^3} + \frac{e^{-5s}e^{5}}{(s-7)}$$

$$= e^{-5s}\left(\frac{25}{s} + \frac{10}{s^2} + \frac{2}{s^3} + \frac{e^5}{s-7}\right)$$

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2) Haciendo uso de laplace resolver

$$y''' + 6y'' + 12y' + 8y = 0 \qquad y(0)=4 \quad y'(0)=-12 \quad y''(0)=34$$

$$\mathcal{L}\{y''' + 6y'' + 12y' + 8y = 0\}$$

$$\mathcal{L}\{y'''\} = s^3 y(s) - s^2(\overset{4}{\cancel{y(0)}}) - s\,\overset{-12}{\cancel{y'(0)}} - \overset{34}{\cancel{y''(0)}}$$

$$= s^3 y(s) - 4s^2 + 12s - 34$$

$$\mathcal{L}\{6y''\} = 6(s^2 y(s) - s\,\overset{4}{\cancel{y(0)}} - \overset{-12}{\cancel{y'(0)}})$$

$$= 6(s^2 y(s) - 4s + 12) = 6s^2 y(s) - 24s + 72$$

$$\mathcal{L}\{12y'\} = 12(s y(s) - \overset{4}{\cancel{y(0)}}) = 12(sy(s) - 4) = 12sy(s) - 48$$

$$\mathcal{L}\{8y\} = 8y(s)$$

$$\mathcal{L}\{y''' + 6y'' + 12y' + 8y = 0\} = s^3 y(s) - 4s^2 + 12s - 34 + 6s^2 y(s) - 24s + 72 + 12sy(s) - 48 + 8y(s)$$

$$s^3 y(s) - 4s^2 + 12s - 34 + 6s^2 y(s) - 24s + 72 + 12sy(s) - 48 + 8y(s)$$

$$s^3 y(s) + 6s^2 y(s) + 12sy(s) + 8y(s) - 4s^2 - 12s - 10 = 0$$

$$y(s)(s^3 + 6s^2 + 12s + 8) = \frac{4s^2 + 12s + 10}{1}$$

$$y(s) = \frac{4s^2 + 12s + 10}{s^3 + 6s^2 + 12s + 8} \qquad \pm 1 \pm 2 \pm 4 \pm 8$$

$$\begin{array}{rrrr|r} 1 & 6 & 12 & 8 & -2 \\ & -2 & -8 & -8 & \\ \hline 1 & 4 & 4 & 0 & \end{array}$$

$$s^2 + 4s + 4 = (s+2)(s+2)$$

$$s_{1,2,3} = -2$$

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$$y(s) = \frac{4s^2+12s+10}{(s+2)^3} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)^3}$$

$$4s^2+12s+10 = A(s+2)^2 + B(s+2) + C$$

$$4s^2+12s+10 = A(s^2+4s+4) + B(s+2) + C$$

$$= \underline{As^2} + \underline{A4s} + 4A + \underline{Bs} + 2B + C$$

$$s^2(A) = 4s^2 \longrightarrow A = 4$$

$$s(4A+B) = 12s$$

$$4A+2B+C = 10$$

$$4A+B=12$$
$$4(4)+B=12$$
$$B=12-16$$
$$B=-4$$

$$4A+2B+C=10$$
$$4(4)+2(-4)+C=10$$
$$16-8+C=10$$
$$8+C=10$$
$$C=2$$

$$y(s) = \frac{4}{(s+2)} - \frac{4}{(s+2)^2} + \frac{2}{(s+2)^3}$$

$$\mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{s+2}\right\} - \mathcal{L}^{-1}\left\{\frac{4}{(s+2)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^3}\right\}$$

$$y(t) = (4)(e^{-2t}) - (4)(t\,e^{-2t}) + \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^3}\right\}$$

$$y(t) = 4e^{-2t} - 4te^{-2t} + (1)\,\mathcal{L}^{-1}\left\{\frac{2}{(s+2)^3}\right\}$$

$$\underline{y(t) = 4e^{-2t} - 4te^{-2t} + (t^2 e^{-2t})}$$

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4) 1) $x' + 2y' = e^t$ $\qquad x(0) = 1$

2) $2x' + y' = sen(t)$ $\qquad y(0) = -1$

$\mathcal{L}\{x' + 2y' = e^t\} = sx(s) - x(0) + 2(sy(s) - y(0)) = \frac{1}{s-1}$

$\mathcal{L}\{x'\} = sx(s) - \cancel{x(0)}^{1} = sx(s) - 1$

$\mathcal{L}\{2y'\} = 2(sy(s) - \cancel{y(0)}^{-1}) = 2sy(s) + 2$

$\mathcal{L}\{e^t\} = \frac{1}{s-1}$

$\mathcal{L}\{x' + 2y' = e^t\} = sx(s) + 2sy(s) + 1 = \frac{1}{s-1}$

$\mathcal{L}\{2x' + y' = sen(t)\}$

$\mathcal{L}\{2x'\} = 2(sx(s) - \cancel{x(0)}^{1}) = 2sx(s) - 2$

$\mathcal{L}\{y'\} = sy(s) - \cancel{y(0)}^{-1} = sy(s) + 1$

$\mathcal{L}\{sen(t)\} = \frac{1}{s^2+1}$

$\mathcal{L}\{2x' + y' = sen(t)\} = 2sx(s) + sy(s) - 1 = \frac{1}{s^2+1}$

3) $sx(s) + 2sy(s) + 1 = \frac{1}{s-1} \quad (-2)$

4) $2sx(s) + sy(s) - 1 = \frac{1}{s^2+1}$

$$\left.\begin{aligned} -2sx(s) - 4sy(s) - 2 &= -\frac{2}{s-1} \\ 2sx(s) + sy(s) - 1 &= \frac{1}{s^2+1} \end{aligned}\right\} = 0x(s) - 3sy(s) - 3 = \frac{-2}{s-1} + \frac{1}{s^2+1}$$

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$$-3sy(s) - 3 = \left(-\frac{2}{s-1}\right) + \left(\frac{1}{s^2+1}\right)$$

$$y(s) = \frac{\frac{1}{s^2+1}}{-3s} - \frac{\frac{2}{s-1}}{-3s} + \frac{3}{-3s}$$

$$y(s) = \frac{2}{(s-1)(3s)} - \frac{1}{(s^2+1)(3s)} - \frac{1}{s}$$

$$\frac{2}{(s-1)(3s)} = \frac{A}{(s-1)} + \frac{B}{3s}$$

$$= A(3s) + B(s-1)$$

$$= 3As + Bs - B$$

$s(3A+B) = 0s$ $\quad B = -2 \quad 3A + B = 0$

$-B = 2 \quad\quad 3A - 2 = 0$

$A = 2/3$

$$\frac{+1}{(s^2+1)(3s)} = \frac{As+B}{s^2+1} + \frac{C}{3s}$$

$$+1 = (As+B)(3s) + C(s^2+1)$$

$$= 3As^2 + 3Bs + Cs^2 + C$$

$(3A+C)s^2 = 0s^2 \quad B = 0 \quad 3A + C = 0$

$3Bs = 0s \quad C = +1 \quad 3A + 1 = 0$

$C = +1 \quad\quad A = -1/3$

$$y(s) = \frac{2}{3(s-1)} - \frac{2}{3s} + \frac{1}{3}\frac{s}{(s^2+1)} - \frac{1}{3s} - \frac{1}{s}$$

$$\mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{3}\frac{1}{(s-1)}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{3}\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{3}\frac{s}{s^2+1}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$y(t) = \frac{2}{3}e^t - \frac{2}{3} + \frac{1}{3}(\cos t) - \frac{1}{3} - 1$$

$$\underline{y(t) = 2/_3 e^t + 1/_3 \cos(t) - 2}$$

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$$y(t) = 2/3\,e^{t} + \tfrac{1}{3}\cos(t) - 2$$

3) $x' + 2y' = e^{t} \qquad y' = 2/3\,e^{t} - \tfrac{1}{3}\operatorname{sen}(t)$

$$x' + 2\left(\tfrac{2}{3}e^{t} - \tfrac{1}{3}\operatorname{sen}(t)\right) = e^{t}$$

$$x' + \tfrac{4}{3}e^{t} - \tfrac{2}{3}\operatorname{sen}(t) = e^{t}$$

$$x' = e^{t} - \tfrac{4}{3}e^{t} + \tfrac{2}{3}\operatorname{sen}(t)$$

$$\mathcal{L}\{x'\} = \mathcal{L}\{-\tfrac{1}{3}e^{t} + \tfrac{2}{3}\operatorname{sen}(t)\}$$

$$\mathcal{L}\{x'\} = sX(s) - \cancel{x(0)}^{\,1} = sX(s) - 1$$

$$sX(s) - 1 = -\tfrac{1}{3}\left(\frac{1}{s-1}\right) + \tfrac{2}{3}\left(\frac{1}{s^2+1}\right)$$

$$X(s) = -\tfrac{1}{3}\left(\frac{\frac{1}{s-1}}{s}\right) + \tfrac{2}{3}\left(\frac{\frac{1}{s^2+1}}{s}\right) + 1$$

$$X(s) = -\tfrac{1}{3}\left(\frac{1}{s(s-1)}\right) + \tfrac{2}{3}\left(\frac{1}{(s^2+1)(s)}\right) + \frac{1}{s}$$

$$\frac{1}{s(s-1)} = \frac{A}{(s)} + \frac{B}{(s-1)}$$

$1 = A(s-1) + B(s)$
$1 = As - A + Bs$
$(A+B)(s) = 0s \qquad A+B=0$
$-A = 1 \qquad -1+B=0$
$A = -1 \qquad B = 1$

$$\frac{1}{s(s^2+1)} = \frac{As+B}{(s^2+1)} + \frac{C}{s}$$

$1 = (As+B)(s) + C(s^2+1)$
$1 = As^2 + Bs + Cs^2 + C$
$s^2(A+C) = 0s^2 \qquad B=0 \qquad A+C=0$
$Bs = 0s \qquad C=1 \qquad A+1=0$
$C = 1 \qquad A=-1$

$$X(s) = -\tfrac{1}{3}\left(-\frac{1}{s} + \frac{1}{s-1}\right) + \tfrac{2}{3}\left(-\frac{1s}{s^2+1} + \frac{1}{s}\right) + \frac{1}{s}$$

$$\mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\{\tfrac{1}{3}\tfrac{1}{s}\} - \mathcal{L}^{-1}\{\tfrac{1}{3}\tfrac{1}{s-1}\} - \mathcal{L}^{-1}\{\tfrac{2}{3}\tfrac{s}{s^2+1}\} + \mathcal{L}^{-1}\{\tfrac{2}{3}\tfrac{1}{s}\} + \mathcal{L}^{-1}\{\tfrac{1}{s}\}$$

$$x(t) = \tfrac{1}{3} - \tfrac{1}{3}e^{t} - \tfrac{2}{3}\cos t + \tfrac{2}{3} + 1 = \underline{-\tfrac{1}{3}e^{t} - \tfrac{2}{3}\cos t + 2}$$ //