Actividad para punto extra.
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Ecuaciones Diferenciales Grupo: 25

Resolver los siguientes ecuaciones lineales diferenciales.

1) $(100+2t)y' + y = 7(100+2t)$

$(100+2t)\frac{dy}{dt} + y = 7(100+2t)$

VD: y $y(t)=$
VI: t

$p(x) = \frac{1}{100+2t}$

$f(x) = 7$

$$\frac{dy}{dt} + \frac{1}{100+2t}y = 7$$

$v = 100+2t$
$dv = 2dt$

$$y(t) = e^{-\frac{1}{2}\int \frac{1}{100+2t}dt} \int e^{\int \frac{1}{100+2t}dt} 7\,dt + Ce^{-\int \frac{1}{100+2t}dt}$$

$$y(t) = \left(e^{-\frac{1}{2}\ln(100+2t)}\right)\left(7\int e^{+\frac{1}{2}\ln(100+2t)}\right)dt + Ce^{-\frac{1}{2}\ln(100+2t)}$$

$$y(t) = \left(\frac{7}{e^{\frac{1}{2}\ln(100+2t)}}\right)\left(\int e^{\ln(100+2t)^{1/2}}\right)dt + \frac{C}{e^{\frac{1}{2}\ln(100+2t)}}$$

$$y(t) = \left(\frac{7}{(100+2t)^{1/2}}\right)\left(\frac{1}{2}\int (100+2t)^{1/2}dt\right)dt + \frac{C}{(100+2t)^{1/2}}$$

$$y(t) = \left(\frac{7}{(100+2t)^{1/2}}\right)\left(\frac{(100+2t)^{3/2}}{3}\right)$$

$$y(t) = \frac{7(100+2t)}{3} + \frac{C}{(100+2t)^{1/2}}$$

2) $x^2y' + 3xy = \frac{\operatorname{sen}x}{x}$

VD: y $y(x)$:
DI $= x$

$$\frac{dy}{dx} + \frac{3}{x}y = \frac{\operatorname{sen}x}{x^3}$$

$$y(x) = \left(e^{-\int \frac{3}{x}dx}\right)\left(\int e^{\int \frac{3}{x}dx} \frac{\operatorname{sen}x}{x^3}dx\right) + Ce^{-\int \frac{3}{x}dx}$$

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$$y(x) = \left(e^{-3\ln(x)}\right)\left(\int e^{3\ln(x)} \frac{\operatorname{sen}(x)}{x^3}\,dx\right) + ce^{-3\ln(x)}$$

$$= \left(e^{\ln(x^{-3})}\right)\left(\int e^{\ln(x^3)} \frac{\operatorname{sen}(x)}{x^3}\,dx\right) + ce^{\ln(x^{-3})}$$

$$= \left(\frac{1}{x^3}\right)\left(\int \cancel{x^3} \frac{\operatorname{sen}(x)}{\cancel{x^3}}\,dx\right) + \frac{c}{x^3}$$

$$= \left(\frac{1}{x^3}\right)\left(\int \operatorname{sen}(x)\,dx\right) + \frac{c}{x^3}$$

$$= \left(\frac{1}{x^3}\right) - \cos(x) + \frac{c}{x^3}$$

$$y(x) = -\frac{\cos(x)}{x^3} + \frac{c}{x^3}$$

3) $\cos(x)\,y' + \operatorname{sen}(x)\,y = x\operatorname{sen}(2x)\cos(x)$ $\left\{\begin{array}{l} \text{VD}: y \\ \text{VI}: x \end{array}\right.$ $y(x)$

$$\frac{dy}{dx} + \tan(x)\,y = x\operatorname{sen}(2x)$$

$$y(x) = \left(e^{+\int \tan(x)dx}\right)\left(\int e^{\int \tan(x)dx}\, x\operatorname{sen}(2x)\,dx\right) + ce^{-\int \tan(x)dx}$$

$\tan = \frac{\operatorname{sen}}{\cos}(x)$ $\quad v = \cos(x)$
$\quad du = -\operatorname{sen}(x)dx$

$$y(x) = \left(e^{\ln(\cos(x))}\right)\left(\int e^{-\ln(\cos(x))}\, x\operatorname{sen}(2x)\,dx\right) + c\,e^{\ln\cos(x)}$$

$$= (\cos(x))\left(\int \frac{1}{\cos(x)}\, x\operatorname{sen}(2x)\,dx\right) + c\cos(x)$$

$$= (\cos(x))\left(+\int \frac{2x\operatorname{sen}(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}\,dx\right) = (\cos(x))\left(+\int 2x\operatorname{sen}(x)\,dx\right)$$

Integral $= -x\cos(x) + \int \cos(x)\,dx = -x\cos(x) + \operatorname{sen}(x)$

$x = v \quad dv = \operatorname{sen}(x)dx$
$dx = du \quad v = -\cos(x)$

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$$y(x) = (\cos(x))(+2)\left[-x\cos(x) + \operatorname{sen}(x)\right] + c\cos(x)$$

$$y(x) = \underline{-2x\cos^2(x) + 2\cos(x)\operatorname{sen}(x) + c\cos(x)}_{/\!/}$$

4) $x' + 2y\,x = y$ $\left\{ \begin{array}{l} VD: x \quad x(y) \\ VI = y \end{array} \right.$

$$\frac{dx}{dy} + 2y\,x = y$$

$$x(y) = \left(e^{-\int 2y\,dy}\right)\left(\int e^{\int 2y\,dy}\, y\,dy\right) + c\,e^{-\int 2y\,dy}$$

$$= \left(e^{-y^2}\right)\left(\int e^{y^2}\, y\,dy\right) + c\,e^{-y^2} \qquad \begin{array}{l} v = y^2 \\ dv = 2y\,dy \end{array}$$

$$= \left(\frac{1}{e^{y^2}}\right)\left(\frac{1}{2}\left(e^{v}\right)\right) + \frac{c}{e^{y^2}}$$

$$x = \underline{\frac{1}{2} + \frac{c}{e^{y^2}}}_{/\!/}$$

5) $\dfrac{dy}{dx} = \dfrac{1}{e^y - x}$ $\left\{ \begin{array}{l} VD = x \quad x(y) \\ VI = y \end{array} \right.$

$$(e^y - x)\frac{dy}{dx} = 1 \Rightarrow (e^y - x)\,dy = dx \Rightarrow e^y - x = \frac{dx}{dy} \Rightarrow \frac{dx}{dy} + x = e^y$$

$$x(y) = \left(e^{-\int 1\,dy}\right)\left(\int e^{\int 1\,dy}\, e^y\,dy\right) + c\,e^{-\int 1\,dy}$$

$$= \left(\frac{1}{e^y}\right)\left(\int e^y e^y\,dy\right) + \frac{c}{e^y} \qquad \begin{array}{l} v = 2y \\ dv = 2\,dy \end{array}$$

$$= \left(\frac{1}{e^y}\right)\left(\frac{1}{2}\right)\left(e^{2y}\right) + \frac{c}{e^y}$$

$$= \underline{\frac{e^y}{2} + \frac{c}{e^y}}_{/\!/}$$

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6) $y' - 2xy = x^3 e^{-x^2}$ con la condicion $y(0) = 1$ $\left\{ \begin{array}{l} VI = x \\ VD = y \end{array} \right.$ $y(x)$

$$y(x) = \left(e^{+\int +2x\,dx}\right)\left(\int e^{\int -2x\,dx} x^3 e^{-x^2} dx\right) + ce^{+\int +2x\,dx}$$

$$y(x) = \left(e^{x^2}\right)\left(\int \frac{1}{e^{x^2}} \frac{x^3}{e^{x^2}} dx\right) + ce^{x^2}$$ $v = 2x^2$, $dv = 4x\,dx$, $\sqrt{v/2} = x$

$$y(x) = \left(e^{x^2}\right)\left(\int \frac{x^3}{e^{2x^2}} dx\right) + ce^{x^2}$$

$$\text{integral} = \frac{1}{4}\int \left(\frac{v}{2}\right)\left(\frac{dv}{e^v}\right) = \frac{1}{8}\int \frac{v\,dv}{e^v} = \frac{1}{8}\int v e^{-v} dv$$ $u = v$, $du = dv$; $\int dv = \int 1/e^v\, dv$, $v = -e^{-v}$

$$\frac{1}{8}\left[(v)(-e^{-v}) - \int -e^{-v} dv\right]$$

$$\frac{1}{8}\left[\frac{-v}{e^v} - e^{-v}\right] \Rightarrow y(x) = \left(e^{x^2}\right)\left(\frac{1}{8}\right)\left(\frac{-2x^2}{e^{2x^2}} - \frac{1}{e^{2x^2}}\right) + ce^{x^2}$$

$$y(x) = -\frac{x^2}{4e^{x^2}} - \frac{1}{8e^{x^2}} + ce^{x^2}$$ si $y(0) = 1$

$1 = 0 - \frac{1}{8} + c$

$1 + 1/8 = c \quad c = 9/8$

$$y(x) = \frac{-x^2}{4e^{x^2}} - \frac{1}{8e^{x^2}} + \frac{9}{8}e^{x^2}$$