Ejercicio 2

$\mathcal{L} y' = sY(s) - f(0) \qquad \mathcal{L} y'' = s^2 Y(s) - s f(0) - f'(0)$

2a) $y'' - 4y' = 6e^{3t} - 3e^{-t} \qquad y(0) = 1 \qquad y'(0) = -1$

$\mathcal{L} y'' = s^2 Y(s) - s + 1$

$\mathcal{L} -4y' = -4.(sY(s) - 1)$

$\mathcal{L}\, 6e^{3t} = 6\frac{1}{s-3} \qquad \mathcal{L} -3e^{-t} = -3\frac{1}{s+1}$

$(s^2 Y(s) - s + 1) - 4(sY(s) - 1) = \frac{6}{s-3} - \frac{3}{s+1}$

$s^2 Y(s) - s + 1 - 4sY(s) + 4 = \frac{6}{s-3} - \frac{3}{s+1}$

$Y(s)(s^2 - 4s) - s + 5 = \frac{6}{s-3} + \frac{3}{s+1}$

$= \frac{6}{s-3} - \frac{3}{s+1} + s - 5$

$Y(s) = \frac{6}{(s-3)(s^2-4s)} - \frac{3}{(s+1)(s^2-4s)} + \frac{s-5}{(s^2-4s)}$

$= \frac{6(s+1) - 3(s-3) + (s-5)(s-3)(s+1)}{(s-3)(s^2-4s)(s+1)}$

$6(s+1) - 3(s-3) + (s-5)(s-3)(s+1) = \frac{A}{s-3} + \frac{B}{s} + \frac{C}{s-4} + \frac{D}{s+1}$

$6(s+1) - 3(s-3) + (s-5)(s-3)(s+1) = A\, s(s-4)(s+1) +$
$B(s-3)(s-4)(s+1) +$
$C(s-3).s.(s+1) +$
$D(s-3).s.(s-4)$

$s = 3$ $\quad$ $s = 0$ $\quad$ $s = 4$

$24 = A\,{-12}$ $\quad$ $30 = B12$ $\quad$ $22 = C \cdot 20$

$C = \frac{22}{20} = \frac{11}{10}$

$A = -\frac{24}{12} = -2$ $\quad$ $B = \frac{30}{12}$

$\boxed{A = -2}$ $\quad$ $B = \frac{5}{2}$

$s = -1$

$12 = D.(-20)$

$D = \frac{12}{20} = -\frac{3}{5}$

$$\mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1}\frac{-2}{s-3} + \mathcal{L}^{-1}\frac{5}{2}\cdot\frac{1}{s} + \mathcal{L}^{-1}\frac{11}{10}\cdot\frac{1}{s-4} + \mathcal{L}^{-1}\frac{3}{5}\cdot\frac{1}{s+1}$$

$$-2e^{3t} + \frac{5}{2} + \frac{11}{10}e^{4t} - \frac{3}{5}e^{-t}$$

MATLAB

$$\frac{11}{10}e^{4t} - \frac{e^{-t}\cdot\left(e^{4t}\cdot 40 - 50e^{t} + 12\right)}{20}$$

$$\frac{11}{10}e^{4t} - \frac{40e^{3t}}{20} + \frac{50}{20} - \frac{12e^{-t}}{20}$$

$$\frac{11}{10}e^{4t} - 2e^{3t} + \frac{5}{2} - \frac{3}{5}e^{-t}$$

```
>> y=dsolve('D2y-4*Dy=6*exp(3*t)-3*exp(-1*t)','y(0)=1','Dy(0)=-1')

y =

(11*exp(4*t))/10 - (exp(-t)*(40*exp(4*t) - 50*exp(t) + 12))/20

>> simplify(y)

ans =
```

(11*exp(4*t))/10 - 2*exp(3*t) - (3*exp(-t))/5 + 5/2

Ejercicio 4

$\mathcal{L}y' = sY(s) - f(0) \qquad \mathcal{L}y'' = s^2Y(s) - sf(0) - f'(0)$

4) $y'' - 4y' + 4y = t^3e^{2t} \qquad y(0) = 0 \quad y'(0) = 0.$

$\mathcal{L}y'' = s^2Y(s) - s.0 - 0 = s^2Y(s)$

$\mathcal{L}y' = sY(s) - 0 = sY(s)$

$\mathcal{L}t^3e^{2t} = \frac{3!}{(s-2)^4}$

$s^2Y(s) - 4sY(s) + 4Y(s) = \frac{6}{(s-2)^4}$

$Y(s)\cdot(s^2 - 4s + 4) = \frac{6}{(s-2)^4}$

$Y(s) = \frac{6}{(s-2)^4}\cdot\frac{1}{(s-2)^2}$

$\mathcal{L}^{-1}Y(s) = \mathcal{L}\frac{6}{(s-2)^4}\cdot\frac{1}{(s-2)^2}$

$y(s) = t^3e^{2t} * te^{2t}$

$f*g = \int_0^t f(u)\cdot g(t-u)\,du$

$f(t) = t^3e^{2t} \qquad g(t) = te^{2t}$

$\int_0^t u^3e^{2u}\cdot(t-u)e^{2(t-u)}\,du =$

$\int_0^t u^3\cdot \cancel{e^{2u}}\cdot(t-u)\cdot e^{2t}\cdot \cancel{e^{-2u}}\,du =$

$\int_0^t u^3te^{2t} - u^4e^{2t}\,du$

$$\int_0^t u^3 t e^{2t} - \int_0^t u^4 e^{2t} =$$

$$t e^{2t} \int_0^t u^3 - e^{2t} \int_0^t u^4$$

$$t e^{2t} \left. \frac{u^4}{4} \right|_0^t - e^{2t} \left. \frac{u^5}{5} \right|_0^t$$

$$t e^{2t} \cdot \frac{t^4}{4} - e^{2t} \frac{t^5}{5} = \frac{1}{4} t^5 e^{2t} - \frac{1}{5} t^5 e^{2t}$$

$$= \frac{1}{20} t^5 e^{2t}$$

```
y=dsolve('D2y-4*Dy+4*y=t^3*exp(2*t)','y(0)=0','Dy(0)=0')

y =

(t^5*exp(2*t))/20
```

Ejercicio 6

6 $\mathcal{L}y' = sY(s) - f(0)$ $\quad \mathcal{L}y'' = s^2 Y(0) - s f(0) - f'(0)$

$y'' + y = \sqrt{2}\ \text{sen}\sqrt{2}t \quad y(0)=10 \quad y'(0)=0$

$\mathcal{L}y'' = s^2 Y(s) - 10s$

$$\mathcal{L}\sqrt{2}\ \text{sen}\sqrt{2}t = \sqrt{2}\cdot\frac{\sqrt{2}}{s^2+(\sqrt{2})^2} = \frac{2}{s^2+2}$$

$\mathcal{L}(y''+y) = \mathcal{L}\sqrt{2}\ \text{sen}\sqrt{2}t$

$$s^2 Y(s) - 10s + Y(s) = \frac{2}{s^2+2}$$

$$Y(s)(s^2+1) - 10s = \frac{2}{s^2+2}$$

$$Y(s)(s^2+1) = \frac{2}{s^2+2} + 10s.$$

$$Y(s) = \frac{2}{(s^2+2)(s^2+1)} + \frac{10s}{s^2+1}$$

$$\mathcal{L}^{-1}\frac{10s}{s^2+1} = 10\mathcal{L}^{-1}\frac{s}{s^2+1} = 10\cos t.$$

$$\mathcal{L}^{-1}\frac{2}{(s^2+2)(s^2+1)} \Rightarrow \frac{2}{(s^2+2)(s^2+1)} = \frac{As+B}{s^2+2} + \frac{Cs+D}{s^2+1}$$

$$2 = (As+B)(s^2+1) + (Cs+D)(s^2+2)$$

$$2 = As^3 + s^2 B + As + B + Cs^3 + Ds^2 + 2sC + 2D$$

$$2 = s^3(A+C) + s^2(B+D) + s(A+2C) + (B+2D)$$

$$\begin{array}{l} A+C=0 \\ B+D=0 \\ A+2C=0 \\ B+2D=2 \end{array} \qquad \begin{array}{cccc} A & B & C & D \end{array}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

MatLaB.

$A=0 \quad B=-2 \quad C=0 \quad D=2$

$$\mathcal{L}^{-1}\frac{2}{(s^2+2)(s^2+1)} = \mathcal{L}^{-1}\frac{As}{s^2+2} + \mathcal{L}^{-1}\frac{B}{s^2+2} + \mathcal{L}\frac{Cs}{s^2+1} + \mathcal{L}\frac{D}{s^2+1}$$

$$= -2\,\mathcal{L}^{-1}\frac{1}{s^2+2} + 2\,\mathcal{L}^{-1}\frac{1}{s^2+1}$$

$$= \frac{2}{\sqrt{2}}\mathcal{L}\frac{\sqrt{2}}{s^2+2} + 2 \cdot \operatorname{sen} t.$$

$$= \sqrt{2}\operatorname{sen}\sqrt{2}t + 2\operatorname{sen} t$$

$$\mathcal{L}^{-1} y(s) = \frac{2}{(s^2+2)(s^2+1)} - \frac{10s}{s^2+1}$$

$$\boxed{y(s) = \sqrt{2}\operatorname{sen}\sqrt{2}t + 2\operatorname{sen} t + 10\cos t}$$

```
>> coef=[1 0 1 0;0 1 0 1;1 0 2 0;0 1 0 2]

coef =

    1    0    1    0
    0    1    0    1
```

```
     1     0     2     0
     0     1     0     2

>> res=[0;0;0;2]

res =

     0
     0
     0
     2

>> r=inv(coef)*res

r =

     0
    -2
     0
     2
y=dsolve('D2y+ y = sqrt(2)*sin(sqrt(2)*t)','y(0)=10','Dy(0)=0')
 y =
 10*cos(t) - sin(t)*((2^(1/2)*cos(t*(2^(1/2) - 1)))/(2*2^(1/2) - 2) + (2^(1/2)*cos(t*(2^(1/2) + 1)))/(2*2^(1/2) + 2)) + sin(t)*(2^(1/2)/(2*2^(1/2) - 2) + 2^(1/2)/(2*2^(1/2) + 2) + (2^(1/2)*(2^(1/2) - 1))/(2*2^(1/2) - 2) - (2^(1/2)*(2^(1/2) + 1))/(2*2^(1/2) + 2)) - cos(t)*((2^(1/2)*sin(t*(2^(1/2) - 1)))/(2*2^(1/2) - 2) - (2^(1/2)*sin(t*(2^(1/2) + 1)))/(2*2^(1/2) + 2))
 >> simplify(y)
 ans =
 10*cos(t) + 2*sin(t) - 2^(1/2)*sin(2^(1/2)*t)
```