Análisis Numérico

Nombre: Arcos Hernández Raúl
Gómez Luna Alejandro
Numero de lista: 3, 11
Grupo: 11

Fecha: 22/05/2020

---

1. Resuelva la ecuación diferencial

$$y' = \frac{1}{4}(5+x)y$$

Con condición inicial $y(0)=2$, en el intervalo $0 \le x \le 0.8$, para un incremento constante $h=0.2$, por medio de los siguientes métodos.

a) Euler
b) Euler-Gauss
c) Taylor
d) Runge-Kutta de segundo y cuarto orden

Resolución

a) Euler

La solución real es la siguiente:

$$y(x) = 2\,e^{1/8\,x\,(x+10)}$$

Resolución

$$\mathrm{f(x,y)} = \frac{1}{4}(5+\mathrm{x})\mathrm{y}$$

Recordando:

$$\mathrm{y_{i+1}} = \mathrm{y_i} + \mathrm{hf(x_i, y_i)}$$

Para i = 0

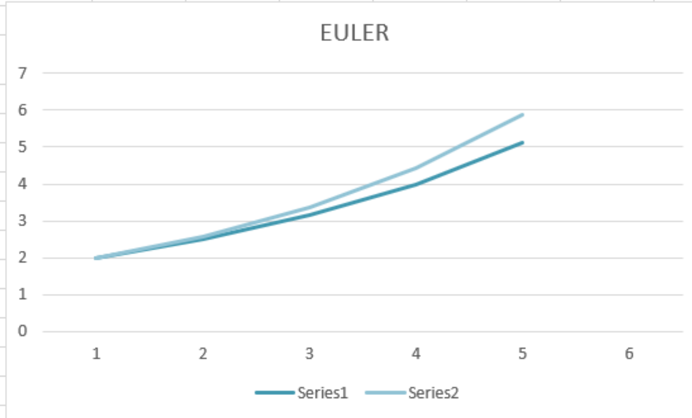
$$\mathrm{y_1} = \mathrm{y_0} + \mathrm{hf(x_0, y_0)} = \mathrm{y_0} + \mathrm{h}\left[\frac{1}{4}(5+\mathrm{x_0})\mathrm{y_0}\right]$$

Sustituyendo valores y condiciones iniciales:

$$\mathrm{y_1} = 2 + (0.2)\mathrm{f}(0,2) = 2 + (0.2)\left[\frac{1}{4}(5+0)2\right] = 2.5$$

Cuadro resumen

	h		
	0.2	$y_0 + h\left[\frac{1}{4}(5+x_0)y_0\right]$	$y(x) = 2\,e^{1/8\,x\,(x+10)}$
i	x	y_aprox	y_real
0	0	2	2
1	0.2	2.5	2.580923242
2	0.4	3.15	3.364055299
3	0.6	4.0005	4.428881994
4	0.8	5.12064	5.889359102



b) Euler-Gauss

Resolución

$$y_{i+1,p} = y(x_i) + hf(x_i, y_i)$$

$$y(x_{i+1}, c) = y(x_i) + \frac{h}{2}\left[f(x_i, y_i) + f(x_{i+1}, y_{i+1,p})\right]$$

$$y' = \frac{1}{4}(5+x)y$$

$$f(x, y) = \frac{1}{4}(5+x)y$$

Para $i = 0$

$$y_{1,p} = y(x_0) + hf(x_0, y_0)$$

$$y_{1,p} = y(x_o) + h\left(\frac{1}{4}(5+x_0)y_0\right) = 2.5$$

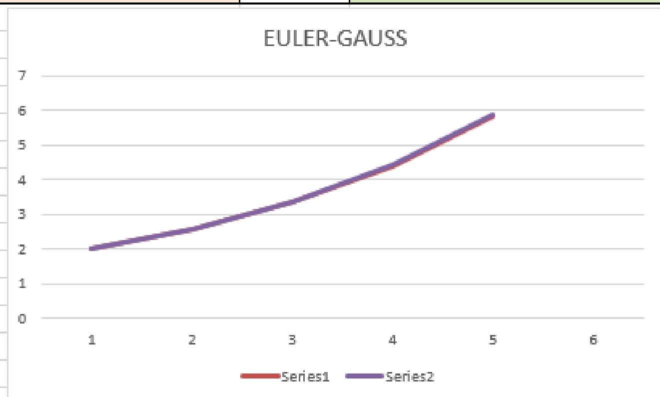
$$y(x_1, c) = y(x_0) + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_{1,p})\right]$$

$$y(x_1, c) = y(x_0) + \frac{h}{2}\left[\left(\frac{1}{4}(5+x_0)y_0\right) + \left(\frac{1}{4}(5+x_1)y_{1,p}\right)\right]$$

$$y(x_1, c) = y(x_0) + \frac{h}{2}\left[\left(\frac{1}{4}(5+0)2\right) + \left(\frac{1}{4}(5+0.2)2.5\right)\right] = 2.575$$

Cuadro resumen

$f(x,y)=\frac{1}{4}(5+x)y$			$y_{1,p}=y(x_0)+hf(x_0,y_0)$		$y(x_1,c)=y(x_0)+\frac{h}{2}[f(x_0,y_0)+f(x_1,y_{1,p})]$	$y(x)=2\,e^{1/8\,x\,(x+10)}$
i	x	f(x0,y0)	Yp	f(xi+1,yi+1,p)	Y	yreal
0	0				2	2
1	0.2	2.5	2.5	3.25	2.575	2.580923242
2	0.4	3.3475	3.2445	4.380075	3.3477575	3.364055299
3	0.6	4.51947263	4.251652025	5.95231284	4.394936046	4.428881994
4	0.8	6.15291046	5.625518139	8.1570013	5.825927223	5.889359102



c) Taylor

Resolución

Proponiendo un polinomio de 3er grado:

$$y(x)=y(x_0)+y'(x_0)(x-x_0)+\frac{y''(x_0)(x-x_0)^2}{2!}+\frac{y'''(x_0)(x-x_0)^3}{3!}$$

Obteniendo derivadas

$$y'=\frac{1}{4}(5+x)y=\frac{5y}{4}+\frac{xy}{4}\quad ;\quad y'=\frac{1}{4}(5+0)*2=2.5$$

$$y''=\frac{5}{4}y'+\frac{y}{4}+\frac{xy'}{4}\quad ;\quad y''=\frac{5}{4}(2.5)+\frac{2}{4}+\frac{0*2.5}{4}=3.625$$

$$y'''=\frac{5}{4}y''+\frac{y'}{2}+\frac{xy''}{4}\quad ;\quad y'''=\frac{5}{4}(3.625)+\frac{2.5}{2}+0=5.78125$$

Sustituyendo datos obtenidos

$$y(x)=y(0)+y'(0)(x-0)+\frac{y''(0)(x-0)^2}{2!}+\frac{y'''(0)(x-0)^3}{3!}$$
$$=2+2.5x+\frac{3.625}{2}x^2+\frac{5.78125x^3}{6}$$

Cuadro resumen

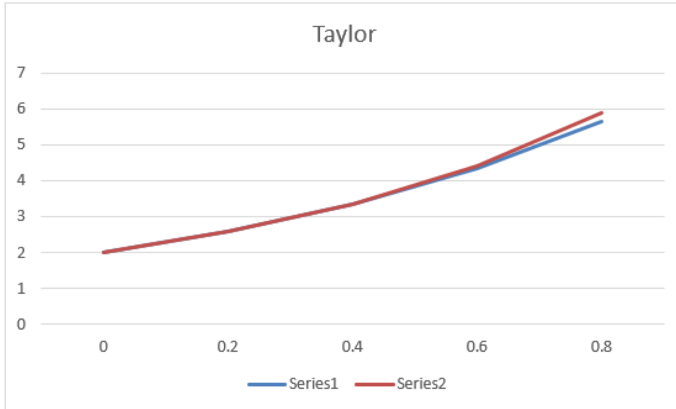
Intervalo

a = 0

b = 0.8

h = 0.2

i	X	$2+2.5x+\frac{3.625}{2}x^2+\frac{5.78125x^3}{6}$	$y(x)=2e^{1/8\,x\,(x+10)}$
i	X	Yt	Yreal
0	0	2	2
1	0.2	2.580208333	2.580923242
2	0.4	3.351666667	3.364055299
3	0.6	4.360625	4.428881994
4	0.8	5.653333333	5.889359102



d) Rungen Kutta

2do orden

Recordando

$$
\begin{aligned}
&y_{i+1} = y_i + \frac{h}{2}(k_1 + k_2)\\
&k_1 = f(x_i, y_i)\\
&k_2 = f(x_i + h, y_i + hk_1)\\
&para \quad k = 1,2,3,\cdots
\end{aligned}
$$

Resolución

$$\mathrm{f(x,y) = y' = \frac{1}{4}(5+x)y}$$

Para i=0 tenemos

$$K_1 = f(x_0, y_0) = f(0,2) = \frac{1}{4}(5+0)2 = 2.5$$

$$K_2 = f(x_o + h, y_0 + hK_1) = f(0+0.2, 2+0.2*2.5) = \frac{1}{4}(5+0.2)\big(2+(0.2)(2.5)\big)$$
$$= 3.25$$

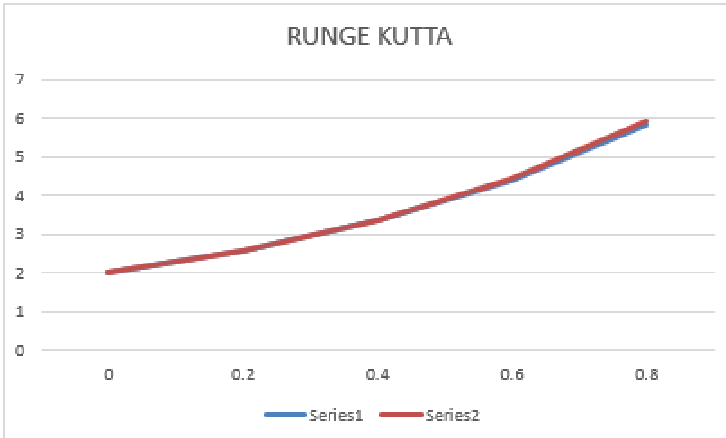
$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2) = 2 + \frac{0.2}{2}(2.5 + 3.25) = 2.575$$

Cuadro resumen

Componentes para K2

i	x	K1=f(xi,yi)	Xi+h	Yi+hk1	K2=F(xi+h,yi+hk1)	Y=yi+(h/2)(k1+k2)	$y(x) = 2e^{1/8\,x(x+10)}$	Error
0	0					2	2	
1	0.2	2.5	0.2	2.5	3.25	2.575	2.580923242	0.005923242
2	0.4	3.3475	0.4	3.2445	4.380075	3.3477575	3.364055299	0.016297799
3	0.6	4.51947263	0.6	4.25165203	5.952312835	4.394936046	4.428881994	0.033945948
4	0.8	6.15291046	0.8	5.62551814	8.157001301	5.825927223	5.889359102	0.06343188

h = 0.2
Intervalo
a= 0
b= 0.8



4to orden

$$k_1 = f(x_0, y_0) = f(0,2) = \frac{1}{4}(5+0)2 = 2.5$$

$$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_1\right) = f\left(0 + \frac{0.2}{2}, 2 + \frac{0.2}{2}(2.5)\right)$$
$$= \frac{1}{4}\left(5 + \left(\frac{0.2}{2}\right)\right)\left(2 + \frac{0.2}{2} * 2.5\right) = 2.8687$$

$$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_2\right) = f\left(0 + \frac{0.2}{2}, 2 + \frac{0.2}{2}(2.8687)\right)$$
$$= \frac{1}{4}\left(5 + \left(\frac{0.2}{2}\right)\right)\left(2 + \frac{0.2}{2}(2.8687)\right) = 2.9157$$

$$k_4 = f(x_0 + h, y_0 + hk_3) = f\big(0 + 0.2, 2 + (0.2)(2.9157)\big)$$
$$= \frac{1}{4}(5 + 0.2)\big(\,2 + (0.2)(2.9157)\big) = 3.358$$

$$y_1 = y_0 + \left(\left(\frac{h}{6}\right) * (k_1 + 2k_2 + 2k_3 + k_4)\right) = 2.5808$$

Cuadro resumen

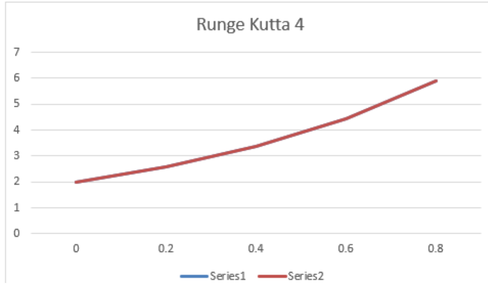
				K2			K3			K4				
i	x	$k_1 = f(x_i, y_i)$	xi+(h/2)	yi+(hk1/2)	$k_2 = f(x_i+\frac{h}{2}, y_i+\frac{h}{2}k_1)$	xi+(h/2)	yi+(hk2/2)	$k_3 = f(x_i+\frac{h}{2}, y_i+\frac{h}{2}k_2)$	xi+h	yi+hk3	$k_4 = f(x_i+h, y_i+hk_3)$	$y_{i+1} = y_i+\frac{h}{6}(k_1+2k_2+2k_3+k_4)$	$y(x) = 2\,e^{1/8\,x\,(x+10)}$	Error
0	0											2	2	
1	0.2	2.5	0.1	2.25	2.86875	0.1	2.286875	2.91576563	0.2	2.58315313	3.35809906	2.58090434	2.580923242	1.8898E-05
2	0.4	3.35517565	0.3	2.91642191	3.86425903	0.3	2.96733025	3.93171258	0.4	3.36724686	4.54578326	3.36400108	3.364055299	5.42184E-05
3	0.6	4.54140146	0.5	3.81814123	5.24994419	0.5	3.8889955	5.34736881	0.6	4.43347484	6.20686478	4.42876416	4.428881994	0.000117838
4	0.8	6.20026982	0.7	5.04879114	7.19452737	0.7	5.14821689	7.33620907	0.8	5.89600597	8.54920866	5.8891292	5.889359102	0.000229901

Intervalo

a= 0

b= 0.8

h= 0.2



2. Dada la ecuación diferencial $3y' - 4xy + e^x = 0$, con condición inicial $y(0) = 0.1$. Obtenga su solución en el intervalo $0 \le x \le 0.5$, con un incremento constante en la variable independiente $h = 0.05$, utilice los método de:

a) Serie de Taylor, considerando un polinomio de tercer grado.
b) Euler Mejorado
c) Runge Kutta de 2° orden

Resolución

Despejando la ecuación diferencial

$$y' = \frac{4xy}{3} - \frac{e^x}{3}$$

a) Taylor

Resolución

Proponiendo un polinomio de 3er grado:

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)(x - x_0)^2}{2!} + \frac{y'''(x_0)(x - x_0)^3}{3!}$$

Obteniendo derivadas

$$y' = \frac{4xy}{3} - \frac{e^x}{3} \quad ; \quad y' = \frac{4(0)(0.1)}{3} - \frac{e^0}{3} = -\frac{1}{3}$$

$$y'' = \frac{4y}{3} + \frac{4xy'}{3} - \frac{e^x}{3} \quad ; \quad y'' = \frac{4(0.1)}{3} + \frac{4(0)\left(\frac{1}{3}\right)}{3} - \frac{e^0}{3} = -\frac{1}{5}$$

$$y''' = \frac{8y'}{3} + \frac{4xy''}{3} - \frac{e^x}{3} \; ; \; y''' = \frac{8\left(-\frac{1}{3}\right)}{3} + \frac{4(0)(-0.2)}{3} - \frac{e^0}{3} = \frac{-11}{9}$$

Sustituyendo datos:

$$y(x) = 0.1 + \left(\frac{1}{3}\right)x + \frac{\left(-\frac{1}{5}\right)x^2}{2} + \frac{\left(\frac{-11}{9}\right)x^3}{6}$$

Cuadro resumen

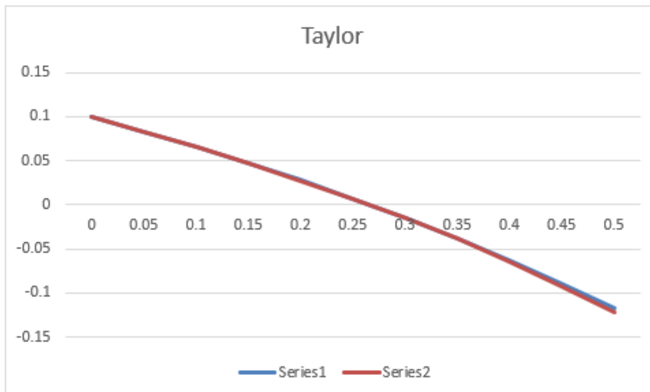
Intervalo

a = 0

b = 0.5

h = 0.05

i	X	Yt	$y(x) = e^{(2x^2)/3}$ Yreal
0	0	0.1	0.100000342
1	0.05	0.08305787	0.083057894
2	0.1	0.065462963	0.065457989
3	0.15	0.0470625	0.047034441
4	0.2	0.027703704	0.027609353
5	0.25	0.007233796	0.006990517
6	0.3	-0.0145	-0.015031466
7	0.35	-0.037650463	-0.038686399
8	0.4	-0.06237037	-0.064228103
9	0.45	-0.0888125	-0.091938521
10	0.5	-0.11712963	-0.122132402



b) Euler Mejorado

Resolución

$$\mathrm{y}_{\mathrm{i+1,p}} = \mathrm{y}(\mathrm{x_i}) + \mathrm{hf}(\mathrm{x_i}, \mathrm{y_i})$$

$$\mathrm{y}(\mathrm{x_{i+1}}, \mathrm{c}) = \mathrm{y}(\mathrm{x_i}) + \frac{\mathrm{h}}{2}\left[\mathrm{f}(\mathrm{x_i}, \mathrm{y_i}) + \mathrm{f}(\mathrm{x_{i+1}}, \mathrm{y_{i+1,p}})\right]$$

$$\mathrm{f}(\mathrm{x}, \mathrm{y}) = y' = \frac{4xy}{3} - \frac{e^x}{3}$$

Para i = 0

$$\mathrm{y}_{1,\mathrm{p}} = \mathrm{y}(\mathrm{x_0}) + \mathrm{hf}(\mathrm{x_0}, \mathrm{y_0})$$

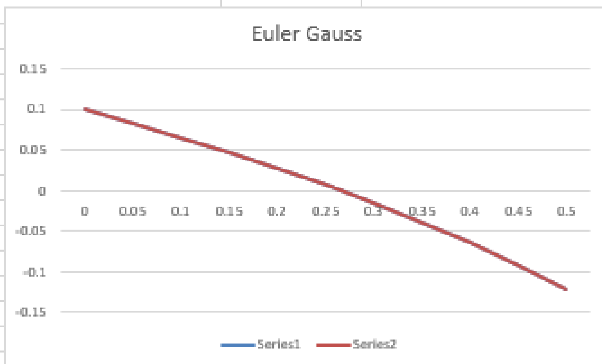
$$\mathrm{y}_{1,\mathrm{p}} = \mathrm{y}(\mathrm{x_o}) + \mathrm{h}\left(\frac{4x_0 y(x_0)}{3} - \frac{e^{x_0}}{3}\right) = \frac{1}{12}$$

$$\mathrm{y}(\mathrm{x_1}, \mathrm{c}) = \mathrm{y}(\mathrm{x_0}) + \frac{\mathrm{h}}{2}\left[\mathrm{f}(\mathrm{x_0}, \mathrm{y_0}) + \mathrm{f}\left(\mathrm{x_1}, \mathrm{y_{1,p}}\right)\right]$$

$$= \mathrm{y}(\mathrm{x_0}) + \frac{\mathrm{h}}{2}\left[\left(\frac{4x_0 y(x_0)}{3} - \frac{e^{x_0}}{3}\right) + \left(\frac{4x_1 y_{1,p}}{3} - \frac{e^{x_1}}{3}\right)\right] = 0.08304496$$

Cuadro resumen

		$\frac{4xy}{3}-\frac{e^x}{3}$	$y_{1,p} = y(x_0) + hf(x_0, y_0)$		$y(x_1, c) = y(x_0) + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_{1,p})\right]$	$y(x) = e^{2x^2/3}\left(-0.526417\,\mathrm{erf}\left(\frac{4x-3}{2\sqrt{6}}\right)-0.222969\right)$
i	x	f(x0,y0)	Yp	f(xi+1,yi+1,p)	Y	yreal
0	0				0.1	0.100000342
1	0.1	-0.333333333	0.083333333	-0.344868143	0.083044963	0.083057894
2	0.1	-0.344887368	0.065800595	-0.359616893	0.065432357	0.065457989
3	0.2	-0.359665992	0.047449057	-0.37778827	0.046996	0.047034441
4	0.2	-0.377878881	0.028102056	-0.399640371	0.027558019	0.027609353
5	0.3	-0.399785448	0.007568746	-0.425485557	0.006926244	0.006990517
6	0.3	-0.425699724	-0.014358743	-0.455696433	-0.01510866	-0.015031466
7	0.4	-0.4559964	-0.03790848	-0.49071314	-0.038776399	-0.038686399
8	0.4	-0.491118169	-0.063332307	-0.53105213	-0.064330656	-0.064228103
9	0.5	-0.531584583	-0.090909885	-0.57731666	-0.092053187	-0.091938521
10	0.5	-0.578002641	-0.120953319	-0.630209303	-0.122258486	-0.122132402



c) Runge Kutta

Resolución

$$f(x, y) = y' = \frac{4xy}{3} - \frac{e^x}{3}$$

Para i=0 tenemos

$$K_1 = f(x_0, y_0) = f(0{,}0.1) = \frac{4(0)(0.1)}{3} - \frac{e^0}{3} = -\frac{1}{3}$$

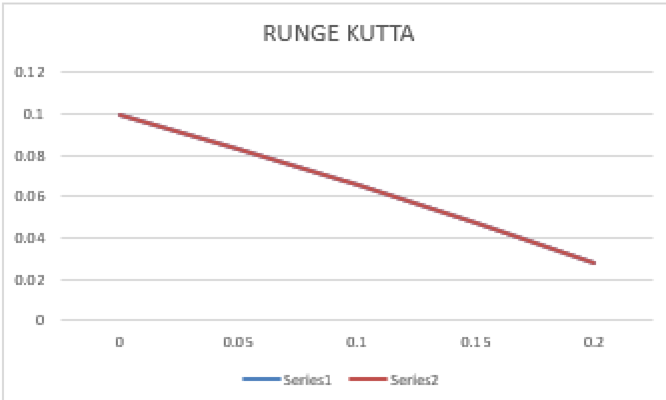
$$K_2 = f(x_o + h, y_0 + hK_1) = f\left(0 + 0.05, 0.1 - \frac{0.05}{3}\right)$$

$$= \frac{4(0 + 0.05)\left(0.1 - \frac{0.05}{3}\right)}{3} - \frac{e^{(0+0.05)}}{3} = -0.3448$$

$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2) = 0.1 + \frac{0.05}{2}\left(-\frac{1}{3} - 0.3448\right) = 0.0830$$

Cuadro resumen

			Componentes para K2					
i	x	K1=f(xi,yi)	Xi+h	Yi+hk1	K2=F(xi+h,yi+hk1)	Y=yi+(h/2)(k1+k2)	[illegible]	Error
0	0					0.1	0.100000342	
1	0.1	-0.33333	0.05	0.083333	-0.344868143	0.083044963	0.083057894	1.293E-05
2	0.1	-0.34489	0.1	0.065801	-0.359616893	0.065432357	0.065457989	2.563E-05
3	0.2	-0.35967	0.15	0.047449	-0.37778827	0.046996	0.047034441	3.844E-05
4	0.2	-0.37788	0.2	0.028102	-0.399640371	0.027558019	0.027609353	5.133E-05
5	0.3	-0.39979	0.25	0.007569	-0.425485557	0.006926244	0.006990517	6.427E-05
6	0.3	-0.4257	0.3	-0.01436	-0.455696433	-0.01510866	-0.015031466	7.719E-05
7	0.4	-0.456	0.35	-0.03791	-0.49071314	-0.038776399	-0.038686399	9E-05
8	0.4	-0.49112	0.4	-0.06333	-0.53105213	-0.064330656	-0.064228103	0.0001026
9	0.5	-0.53158	0.45	-0.09091	-0.57731666	-0.092053187	-0.091938521	0.0001147
10	0.5	-0.578	0.5	-0.12095	-0.630209303	-0.122258486	-0.122132402	0.0001261



3. El sistema de ecuaciones diferenciales

$$T_1' = \frac{5}{6}(T_2 - T_1)$$

$$T_2' = \frac{3}{4}(T_1 - 2T_2 + 293)$$

Representa el modelo matemático para obtener la temperatura T_1 y T_2 en un condensador de superficie. Considere que inicialmente T_1 es de 800 K y T_2 es de 300 K. Obtenga los valores de las temperaturas T_1 y T_2 durante el primer minuto de funcionamiento del equipo. Utilice el método de Euler.

Resolución

Se considerará un aumento h = 1 [s]

$$T_1' = f(T_1, T_2) = \frac{5}{6}(T_2 - T_1)$$

$$T_2' = f(T_1, T_2) = \frac{3}{4}(T_1 - 2T_2 + 293)$$

Recordando

$$y_{i+1} = y_i + hf(x_i, y_i)$$

Para i = 0

$$T_{1_1} = T_{1_0} + hf(T_{1_0}, T_{2_0}) = 800 + (1)\left(\frac{5}{6}(T_{2_0} - T_{1_0})\right) = 800 + \frac{5}{6}(300 - 800) = \frac{1150}{3}$$

$$T_{2_1} = T_{2_0} + hf(T_{1_0}, T_{2_0}) = 300 + (1)\left(\frac{3}{4}(T_{1_0} - 2T_{2_0} + 293)\right)$$
$$= 300 + \frac{3}{4}(800 - (2 * 300) + 293) = \frac{2679}{4}$$

Cuadro resumen

	h		
	1	$T_{1_1} = T_{1_0} + hf(T_{1_0}, T_{2_0})$	$T_{2_1} = T_{2_0} + hf(T_{1_0}, T_{2_0})$
i	x	T_1	T_2
0	0	800	300
1	1	383.3333333	669.75
2	2	622.0138889	172.375
3	3	247.3148148	600.0729167
4	4	541.2798997	105.1996528
5	5	177.8796939	573.1100984
6	6	507.2383643	66.60472126
7	7	140.0436618	566.8764126
8	8	495.7376208	41.34454002
9	9	117.0767202	570.8809456
10	10	495.246908	22.11706732
56	56	885.4182264	-553.8991846
57	57	-314.0129494	1160.763262
58	58	914.9672268	-596.1413431
59	59	-344.2899148	1204.046092
60	60	945.9900906	-640.4904819

Conclusión

La temperatura $T_1 = 945.99\ [K]$ y $T_2 = -640.49\ [K]$

5. Encontrar la solución del sistema de ecuaciones lineales siguiente:

$$y' = sen(2t) - ty \qquad z' = cos(2t) - tz$$

Sujeto a $y(0) = -0.5$, $z(0) = 0.5$

En el intervalo $0 \le t \le 1$

considerando h=0.1, utilizando los siguientes métodos:

d) Método de la serie de Taylor, considerando un polinomio de tercer grado.
e) Euler
f) Método de Runge-Kutta de segundo orden.

Desarrollo

d) Para poder resolver el ejercicio tenemos:

$$y(t) = y(t_0) + y'(t_0)(t - t_0) + \frac{y''(t_0)(t-t_0)^2}{2!} + \frac{y'''(t_0)(t-t_0)^3}{3!} + \cdots$$

$$z(t) = z(t_0) + z'(t_0)(t - t_0) + \frac{z''(t_0)(t-t_0)^2}{2!} + \frac{z'''(t_0)(t-t_0)^3}{3!} + \cdots$$

$$y' = sen(2t) - ty = sen(0) - (0)(-0.5) = 0$$

$$z' = cos(2t) - tz = \cos(0) - (0))0.5) = 1$$

Derivando y evaluando

$$y'' = 2cos(2t) - y - ty' = 2\cos(0) + 0.5 - (0)(0) = 2.5$$

$$z'' = -2sen(2t) - z - tz' = -2sen(0) - 0.5 - (0)(1) = -0.5$$

Derivando y evaluando nuevamente

$$y''' = -4sen(2t) - y' - y' - ty'' = -4sen(0) - 2(0) - (0)(2.5) = 0$$

$$z''' = -4cos(2t) - z' - z' - tz'' = -4cos(0) - 2(1) - (0)(-0.5) = -6$$

Derivando y evaluando en y para obtener un polinomio de tercer grado.

$$y^{iv} = -8cos(2t) - 2y'' - y'' - ty''' = -8cos(0) - 3(2.5) - (0)(0) = -15.5$$

$$y^{v} = 16sen(2t) - 3y''' - y''' - ty^{iv} = 16sen(0) - 4(0) - (0)(-15.5) = 0$$

$$y^{vi} = 32cos(2t) - 4y^{iv} - y^{iv} - ty^{v} = 32cos(0) - 5(-15.5) - (0)(0) = 109.5$$

Sustituyendo

$$y(t) = -0.5 + \frac{5}{4}t^2 - \frac{31}{48}t^4 + \frac{73}{480}t^6$$

$$z(t) = 0.5 + t - \frac{1}{4}t^2 - t^3$$

Tabla resumen

	Taylor	
t	y(t)	z(t)
0	-0.5	0.5
0.1	-0.4875644	0.5965
0.2	-0.4510236	0.682
0.3	-0.3926204	0.7505
0.4	-0.3159104	0.796
0.5	-0.2254883	0.8125
0.6	-0.1266044	0.794
0.7	-0.0246721	0.7345
0.8	0.0753344	0.628
0.9	0.16959207	0.4685
1	0.25625	0.25

e) Partiendo de:

$$y' = f(t, y)$$

$$y' = sen(2t) - ty$$

$$y_{i+1} = y_i + hf(t_i, y_i)$$

$$z' = f(t, z)$$

$$z' = cos(2t) - tz$$

$$z_{i+1} = z_i + hf(t_i, z_i)$$

Para i = 0

$$y_1 = y_0 + hf(t_0, y_0)$$

$$y_1 = y_0 + h(sen(2t_0) - t_0 y_0)$$

$$y_1 = -0.5 + 0.1\left(sen\big(2(0)\big) - (0)(-0.5)\right) = -0.5$$

$$z_1 = z_0 + hf(t_0, z_0)$$

$$z_1 = z_0 + \ h(cos(2t_0) - t_0 z_0)$$

$$z_1 = 0.5 + 0.1\left(cos\big(2(0)\big) - (0)(0.5)\right) = 0.6$$

Para i = 1:

$$y_2 = y_1 + hf(t_0, y_0)$$

$$y_2 = y_1 + h(sen(2t_1) - t_1 y_1)$$

$$y_2 = -0.5 + 0.1\left(sen\big(2(0.1)\big) - (0.1)(-0.5)\right) = \ -0.475133067$$

$$z_2 = z_1 + hf(t_1, z_1)$$

$$z_1 = z_1 + h(cos(2t_1) - t_0 z_1)$$

$$z_1 = 0.5 + 0.1\left(cos\big(2(0.1)\big) - (0.1)(0.6)\right) = 0.692006658$$

Tabla resumen:

i	t	$y_1 = y_0 + h(sen(2t_0) - t_0 y_0)$	$z_1 = z_0 + \ h(cos(2t_0) - t_0 z_0)$
0	0	-0.5	0.5
1	0.1	-0.5	0.6
2	0.2	-0.475133067	0.692006658
3	0.3	-0.426688571	0.770272624
4	0.4	-0.357423667	0.829698007
5	0.5	-0.271391111	0.866180757
6	0.6	-0.173674457	0.87690195
7	0.7	-0.070050081	0.860523609
8	0.8	0.033398398	0.81728367
9	0.9	0.130683886	0.748981024
10	1	0.216307099	0.658852523

f) Para poder aproximarnos tendremos que realizar ciertos cálculos previos

$$y_{i+1} = y_i + \frac{h}{2}(k_1 + k_2)$$

$$z_{i+1} = z_i + \frac{h}{2}(k_1 + k_2)$$

$$k_{1,y} = f(t_0, y_0)$$

$$k_{1,z} = f(t_0, z_0)$$

$$k_{2,y} = f(t_o + h, y_0 + hK_1)$$

$$k_{2,z} = f(t_o + h, z_0 + hK_1)$$

$$para\ k = 1,2,3, \dots$$

Para i=0 tenemos

$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2)$$

$$k_{1,y} = f(t_0, y_0)$$

$$k_{2,y} = f(t_o + h, y_0 + hK_1)$$

$$z_1 = z_0 + \frac{h}{2}(k_1 + k_2)$$

$$k_{1,z} = f(t_0, z_0)$$

$$k_{2,z} = f(t_o + h, z_0 + hK_1)$$

Evaluando

$$k_{1,y} = f(0, -0.5) = sen\big(2(0)\big) - (0)(-0.5) = 0$$

$$k_{2,y} = f(0.1, -0.5) = sen\big(2(0.1)\big) - (0.1)(-0.5) = \ 0.2486693308$$

$$y_1 = -0.5 + \frac{0.1}{2}(0 + \ 0.2486693308) = \ -0.487566533$$

$$k_{1,z} = f(0,0.5) = cos\big(2(0)\big) - (0)(0.5) = 1$$

$$k_{2,z} = f(0.1,0.6) = cos\big(2(0.1)\big) - (0.1)(0.6) = \ 0.9200665778$$

$$y_1 = 0.5 + \frac{0.1}{2}(1 + \ 0.9200665778) = \ 0.596003329$$

Tabla resumen para y(t)

i	t	K1=f(ti,yi)	ti+h	yi+hk1	K2=F(ti+h,ti+hk1)	Y=yi+(h/2)(k1+k2)
0	0					-0.5
1	0.1	0	0.1	-0.5	0.248669331	-0.487566533
2	0.2	0.247425984	0.2	-0.462823935	0.481983129	-0.451096078
3	0.3	0.479637558	0.3	-0.403132322	0.68558217	-0.392835091
4	0.4	0.682493001	0.4	-0.324585791	0.847190407	-0.316350921
5	0.5	0.843896459	0.5	-0.231961275	0.957451622	-0.226283517
6	0.6	0.954612743	0.6	-0.130822243	1.010532432	-0.128026258
7	0.7	1.008854841	0.7	-0.027140774	1.004448272	-0.027361103
8	0.8	1.004602502	0.8	0.073099148	0.941094285	0.069923737
9	0.9	0.943634614	0.9	0.164287198	0.825989153	0.158404925
10	1	0.831283198	1	0.241533245	0.667764182	0.233357294

Tabla resumen para z(t)

i	t	K1=f(ti,zi)	ti+h	zi+hk1	K2=F(ti+h,zi+hk1)	z=zi+(h/2)(k1+k2)
0	0					0.5
1	0.1	1	0.1	0.6	0.920066578	0.596003329
2	0.2	0.920466245	0.2	0.688049953	0.783451003	0.681199191
3	0.3	0.784821156	0.3	0.759681307	0.597431223	0.75031181
4	0.4	0.600242072	0.4	0.810336017	0.372572302	0.798952529
5	0.5	0.377125698	0.5	0.836665099	0.121969757	0.823907302
6	0.6	0.128348655	0.6	0.836742167	-0.139687546	0.823340357
7	0.7	-0.13164646	0.7	0.810175711	-0.397155855	0.796900241
8	0.8	-0.387863026	0.8	0.758113939	-0.635690673	0.745722556
9	0.9	-0.625777567	0.9	0.6831448	-0.842032414	0.672332057
10	1	-0.832300946	1	0.589101963	-1.005248799	0.58045457

Conclusión

Con los resultados obtenidos a partir de los tres métodos, se observa que los métodos de Taylor y Runge-Kutta de segundo orden se tienen resultados más parecidos, mientras que con Euler los resultados difieren más, por lo que se puede decir que Taylor y Runge-Kutta de segundo orden son más precisos a comparación de Euler.

6. Resuelva el problema de valores iniciales definidos por la ecuación diferencial $y'' - y' - 2y = 0$, con las condiciones de frontera $y(0) = 0.1$; $y'(0) = 0.2$, utilice el método de Euler.

Desarrollo

Solución real

$$y(x) = 0.1e^{2x}$$

Realizando un cambio de variable

$$y' = z$$

$$y'' = z'$$

Sustituyendo en la ecuación nos queda

$$z' - z - 2y = 0$$

Nos queda el sistema

$$y' = z$$

$$z' = 2y + z$$

Las condiciones iniciales ahora son $y(x_0) = 0.1$ y $z(x_0) = 0.2$

Partiendo de

$$y' = f(z)$$

$$y' = z$$

$$y_{i+1} = y_i + hf(z_i)$$

$$z' = f(y, z)$$

$$z' = 2y + z$$

$$z_{i+1} = z_i + hf(y_i, z_i)$$

Para i = 0

$$y_1 = y_0 + hf(z_0)$$

$$y_1 = y_0 + h(z_0)$$

$$y_1 = 0.1 + 0.1(0.2) = 0.12$$

$$z_1 = z_0 + hf(y_0, z_0)$$

$$z_1 = z_0 + \ h(2y_0 + z_0)$$

$$z_1 = 0.2 + 0.1(2(0.1) + (0.2)) = 0.24$$

Para i = 1:

$$y_2 = y_1 + hf(z_1)$$

$$y_2 = y_1 + h(z_1)$$

$$y_2 = 0.12 + 0.1(0.24) = 0.144$$

$$z_2 = z_1 + hf(y_1, z_1)$$

$$z_1 = z_1 + h(2y_1 + z_1)$$

$$z_1 = 0.24 + 0.1(2(0.12) + 0.24) = 0.288$$

Soluciones reales

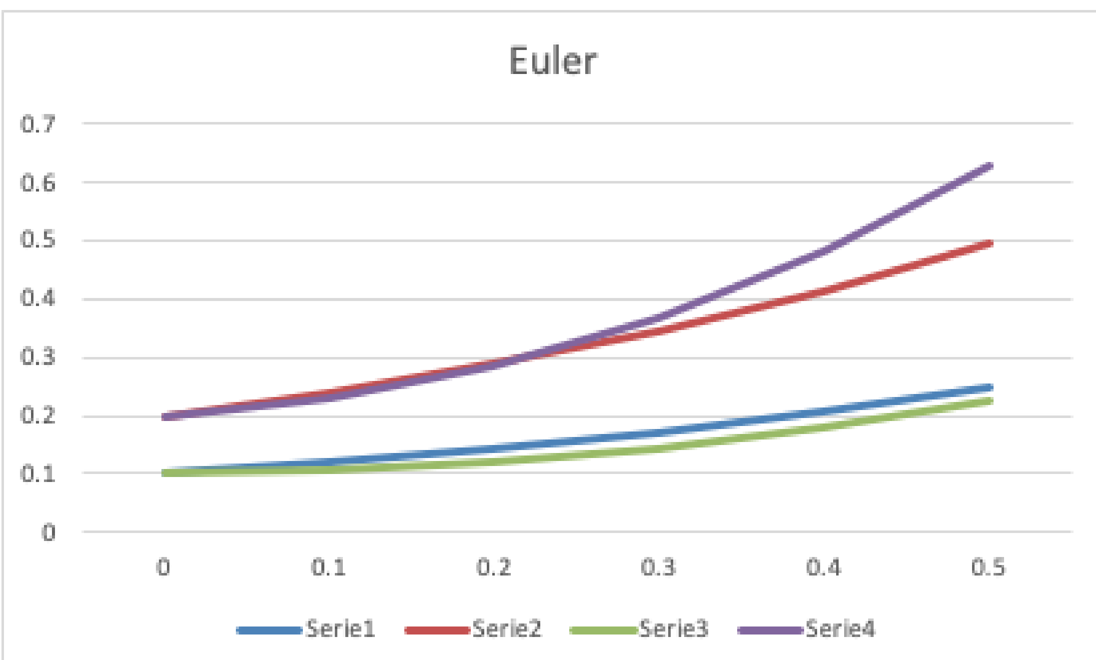
$$z(y) = -2y + 2.2e^{y} - 2$$

$$y(z) = 0.5\,(z^2 + 0.2)$$

Tabla resumen

i	x	$y_{i+1} = y_i + hf(z_i)$	$z_{i+1} = z_i + hf(y_i, z_i)$	$y(z) = 0.5\,(z^2 + 0.2)$	$z(y) = -2y + 2.2e^{y} - 2$
0	0	0.1	0.2	0.1	0.2
1	0.1	0.12	0.24	0.105	0.23137602
2	0.2	0.144	0.288	0.12	0.287086068
3	0.3	0.1728	0.3456	0.145	0.369689377
4	0.4	0.20736	0.41472	0.18	0.482014335
5	0.5	0.248832	0.497664	0.225	0.627186796

Gráfica



Resuelva la siguiente ecuación diferencial $x'' + 2x' + x = 2e^t$ sujeta a las siguientes condiciones iniciales $x(0) = 0$ y $x'(0) = 1$ en el intervalo $0 \le t \le 3.5$con h=0.5

Desarrollo

El polinomio de Taylor tiene la siguiente forma

$$x(t) = x(t_0) + x'(t_0)(t - t_0) + \frac{x''(t_0)(t - t_0)^2}{2!} + \frac{x'''(t_0)(t - t_0)^3}{3!} + \cdots$$

Valuando las diferentes derivadas en $t_0 = 0$

Por condiciones iniciales se tiene que $x(t_0) = 0$ y $x'(t_0) = 1$

$$x'' = 2e^t - 2x' - x\ ;\ valuando \to x''(t_0) = 2e^0 - 2 = 0$$

$$x''' = 2e^t - 2x'' - x';\ valuando \to x'''(t_0) = 2e^0 - 2(0) - 1 = 1$$

$$x^{iv} = 2e^t - 2x''' - x'';\ valuando \to x^{iv}(t_0) = 2e^0 - 2(1) - 0 = 0$$

$$x^{v} = 2e^t - 2x^{iv} - x''';\ valuando \to x^{v}(t_0) = 2e^0 - 2(0) - 1 = 1$$

Repitiendo el patrón que se tiene, llegamos a que el pilonomio tiene la siguiente forma

$$x(t) = t + \frac{t^3}{6} + \frac{t^5}{120} + \frac{t^7}{5040}$$

Solución real

$$x(t) = \frac{1}{2}(e^t - e^{-t})$$

Tabla resumen

i	X	$x(t) = t + \frac{t^3}{6} + \frac{t^5}{120} + \frac{t^7}{5040}$	$x(t) = \frac{1}{2}(e^t - e^{-t})$
0	0	0	0
1	0.5	0.5210953	0.521095305
2	1	1.175198413	1.175201194
3	1.5	2.129171317	2.129279455
4	2	3.625396825	3.626860408
5	2.5	6.03907025	6.050204481

Gráfica