# Quantum Error Code Correction Preliminaries

Resource:

Djordjevic, I. B. (2021). *Quantum information processing, quantum computing, and quantum error correction: An engineering approach* (2nd ed.). CRC Press.

https://www.cl.cam.ac.uk/teaching/1920/QuantComp/Quantum_Computing_Lecture_13.pdf

# Overview

Why is it difficult

What are the steps

Bit-Flip Channel

3-Qubit Flip Code [3,1]

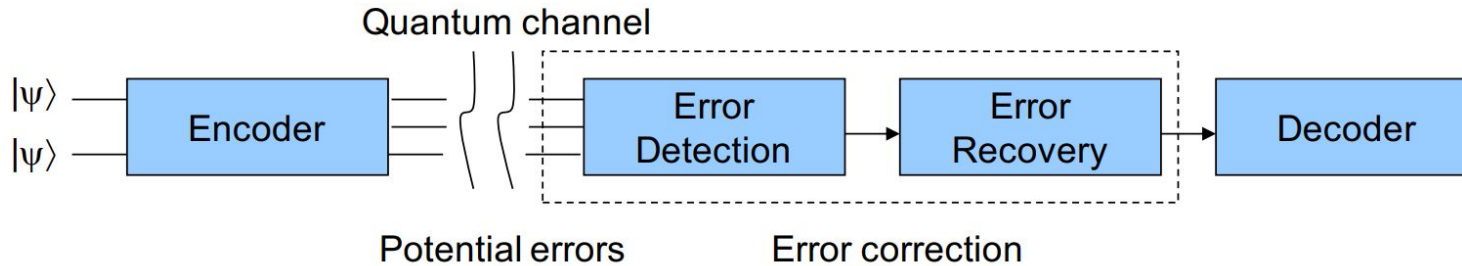   Encoding, Error Recognition, Error Reversing, Decoding

3-Qubit Phase-Flip Code

Shor 9-Qubit Code [9,1]

# Why is it difficult

- **No-Cloning Theorem**: It's impossible to create an exact copy of an arbitrary quantum state $|\psi\rangle=a|0\rangle+b|1\rangle|\psi\rangle = a|0\rangle + b|1\rangle|\psi\rangle=a|0\rangle+b|1\rangle$.

- **Continuous Quantum Errors**: Quantum errors are continuous, and a qubit can exist in any superposition of its basis states, complicating error correction.

- **Measurement Issues**: Measurement processes destroy quantum information, making it challenging to retrieve or correct information without altering the quantum state.

# What are the steps

- A quantum error correction consists of four major steps:
  - encoding
  - error detection
  - error recovery
  - decoding

# Bit-Flip Channel

- **Qubit Transmission Error**: A qubit $|\Psi\rangle = a|0\rangle + b|1\rangle$ can flip to $|\Psi'\rangle = b|0\rangle + a|1\rangle$ with probability $p$ during transmission.

Codewords:

$$|\overline{0}\rangle = |000\rangle \quad \text{and} \quad |\overline{1}\rangle = |111\rangle$$

# 3-Qubit Codes [3,1]

# Encoding

**Ancilla Qubit Operations**: The first ancilla qubit (second at the encoder input) is controlled by the information qubit (first at the encoder input) using CNOT gates:

- $\text{CNOT}_{12}$ on $a|000\rangle + b|100\rangle$ yields $a|000\rangle + b|110\rangle$

- This output then serves as input to $\text{CNOT}_{13}$, where the second ancilla qubit (third qubit) is controlled by the information qubit, resulting in $a|000\rangle + b|111\rangle$.

# Error Recognition, Error Reversing, Decoding

$\langle\psi_r|P_0|\psi_r\rangle = 0,\ \langle\psi_r|P_1|\psi_r\rangle = 1,\ \langle\psi_r|P_2|\psi_r\rangle = 0,\ \langle\psi_r|P_3|\psi_r\rangle = 0$, which can be represented as syndrome vector $S = [0\ 1\ 0\ 0]$, indicating that the error occurred in the first qubit

$$Z_1 Z_2 = (|00\rangle\langle11| + |11\rangle\langle11|) \otimes I - (|01\rangle\langle01| + |10\rangle\langle10|) \otimes I$$

$$Z_2 Z_3 = I \otimes (|00\rangle\langle11| + |11\rangle\langle11|) - I \otimes (|01\rangle\langle01| + |10\rangle\langle10|).$$

# Error Recognition, Error Reversing, Decoding

$\langle \psi_r | P_0 | \psi_r \rangle = 0$, $\langle \psi_r | P_1 | \psi_r \rangle = 1$, $\langle \psi_r | P_2 | \psi_r \rangle = 0$, $\langle \psi_r | P_3 | \psi_r \rangle = 0$, which can be represented as syndrome vector $S = [0\ 1\ 0\ 0]$, indicating that the error occurred in the first qubit

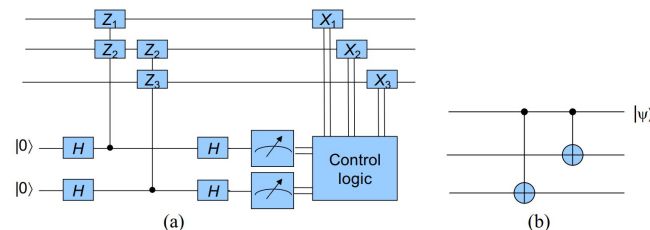| Bit-flip | $|\psi_1\rangle$ | $M_1$ | $M_2$ | Recovery | $|\psi_2\rangle$ |
|---|---|---|---|---|---|
| - | $\alpha|000\rangle + \beta|111\rangle$ | 0 | 0 | $I \otimes I \otimes I$ | $\alpha|000\rangle + \beta|111\rangle$ |
| 1 | $\alpha|100\rangle + \beta|011\rangle$ | 1 | 0 | $X \otimes I \otimes I$ | $\alpha|000\rangle + \beta|111\rangle$ |
| 2 | $\alpha|010\rangle + \beta|101\rangle$ | 1 | 1 | $I \otimes X \otimes I$ | $\alpha|000\rangle + \beta|111\rangle$ |
| 3 | $\alpha|001\rangle + \beta|110\rangle$ | 0 | 1 | $I \otimes I \otimes X$ | $\alpha|000\rangle + \beta|111\rangle$ |



**FIGURE 8.3**

(a) Three-qubit flip code error detection and error correction circuit. (b) Decoder circuit configuration.

# 3-Qubit Phase-Flip Code

**Qubit Phase Flip During Transmission**: A qubit $|\Psi\rangle = a|0\rangle + b|1\rangle$ can experience a phase flip to $a|0\rangle - b|1\rangle$ with probability $p$ during transmission through the quantum channel.

**Phase-Flip Error Protection**: To guard against phase-flip errors, we use a different computational basis, called the diagonal basis.

$$Z|+\rangle = \frac{Z|0\rangle + Z|1\rangle}{\sqrt{2}} = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \left|-\right\rangle, \; Z|-\rangle = \frac{Z|0\rangle - Z|1\rangle}{\sqrt{2}} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \left|+\right\rangle.$$

$$(8.6)$$

# Shor 9-Qubit Code [9,1]

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- **Error Correction**: The Shor 9-Qubit Code encodes one logical qubit into nine physical qubits, correcting both bit-flip and phase-flip errors.

- **Encoding/Decoding**: It uses entanglement to encode a logical qubit and measures qubits to correct errors, preserving the logical state.

- **Correction Mechanism**: The code employs CNOT and phase operations to detect and correct single-qubit errors, maintaining logical qubit integrity.