Quantum Information Preliminaries

Resource:

Djordjevic, I. B. (2021). *Quantum information processing, quantum computing, and quantum error correction: An engineering approach* (2nd ed.). CRC Press.

Overview

Linear Superposition

Entanglement

Quantum Parallelism

Quantum Bit

- Photon Polarization
- State Vector Representation
- Phase

Quantum Gates

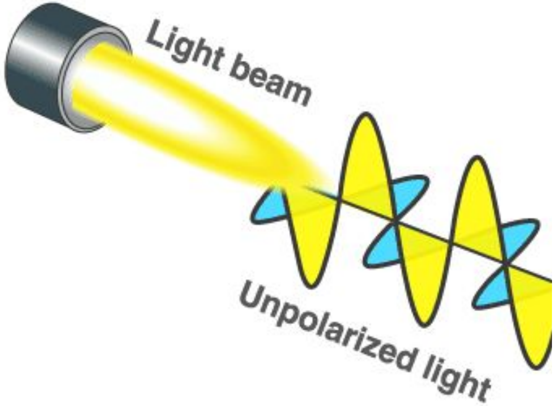
- Bit-Flip and Phase Flip

Quantum Error

Linear Superposition

- **Quantum Superposition**:
  Unlike a classical bit, which can be either 0 or 1, a qubit can be in a combination of both states at once.

- **Linear Combination of States**:
  Quantum states can be expressed as linear superpositions of the 0 and 1 states, reflecting a key feature of quantum systems.



Entanglement

- **Quantum Entanglement:** When two quantum objects become entangled, they form a single entity that cannot be described as a simple combination of independent quantum states. This phenomenon defies a description using tensor products of individual quantum states and implies that superluminal signal propagation would be required otherwise.

- **Exponential Information Growth**: Entangled states of N qubits contain exponentially more information compared to the linear growth observed with classical bits. This is due to the no-cloning theorem, which prevents copying arbitrary quantum states and thus rules out superluminal communication.

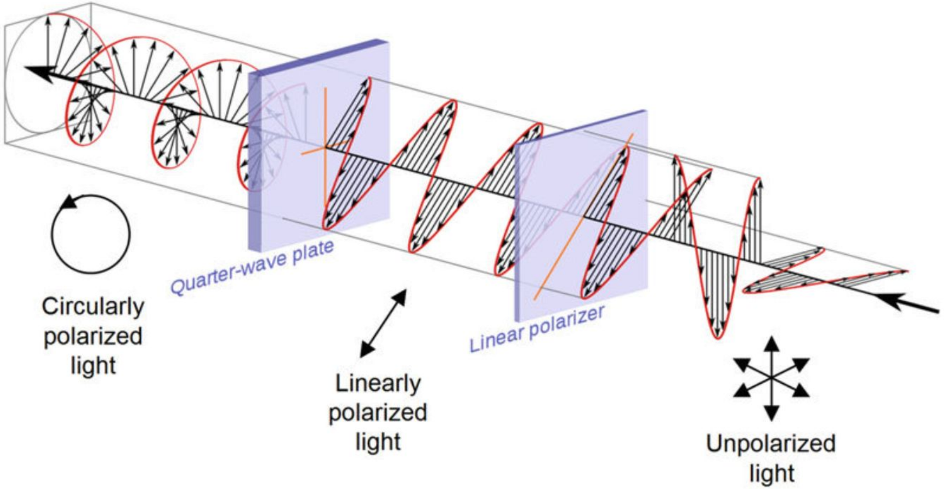
Quantum Parallelism

- **Quantum Parallelism**: Quantum computing can perform many operations simultaneously, unlike classical computing, which operates sequentially.

- **State Uncertainty**: Due to the no-cloning theorem, the internal state of a quantum computer cannot be known, unlike in classical computing where the internal state can be observed.

Quantum Bit

Photon Polarization



$$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix}, |L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

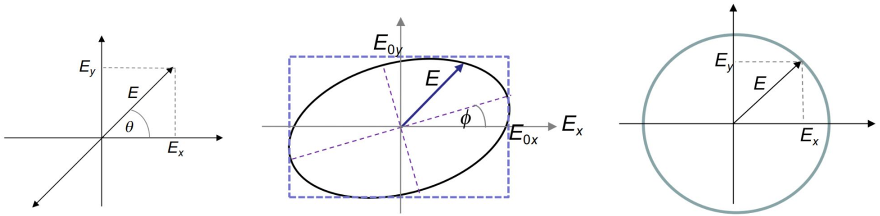
State Vector Representation

The electric/magnetic field of plane linearly polarized waves is described as follows [13]:

$$A(\boldsymbol{r}, t) = \boldsymbol{p} A_0 \exp[j(\omega t - \boldsymbol{k} \cdot \boldsymbol{r})], \quad A \in \{\boldsymbol{E}, \boldsymbol{H}\} \tag{1.1}$$

Combining x-polarization E_x of photon and y-polarization E_y of photon resultant wave obtained

$$\boldsymbol{E}(z,t) = \boldsymbol{E}_x(z,t) + \boldsymbol{E}_y(z,t) = \boldsymbol{e}_x E_{0x} \cos(\omega t - kz) + \boldsymbol{e}_y E_{0y}$$



(A) (B) (C)

FIGURE 1.1

Various forms of polarizations: (A) linear polarization, (B) elliptic polarization, and (C) circular polarization.

State Vectors and Poincare Sphere

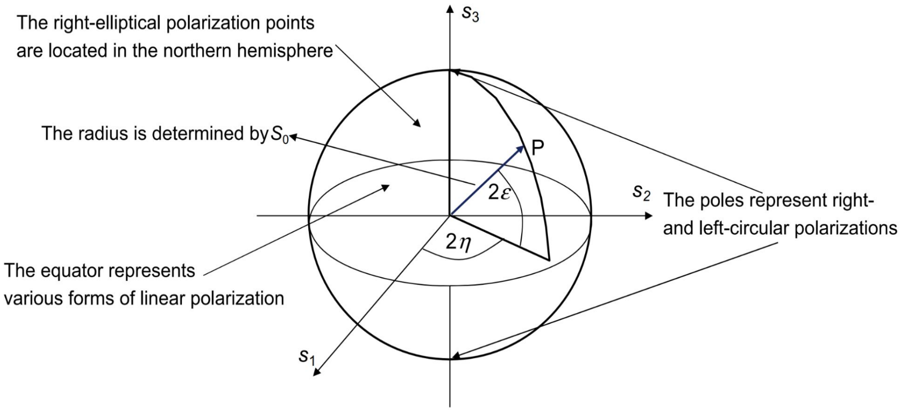
Jones vector representation of the polarization wave

$$\boldsymbol{E}(t) = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} = E \begin{bmatrix} \sqrt{1-\kappa} \\ \sqrt{\kappa} e^{j\delta} \end{bmatrix} e^{j(\omega t - kz)}$$

Stokes vector representation

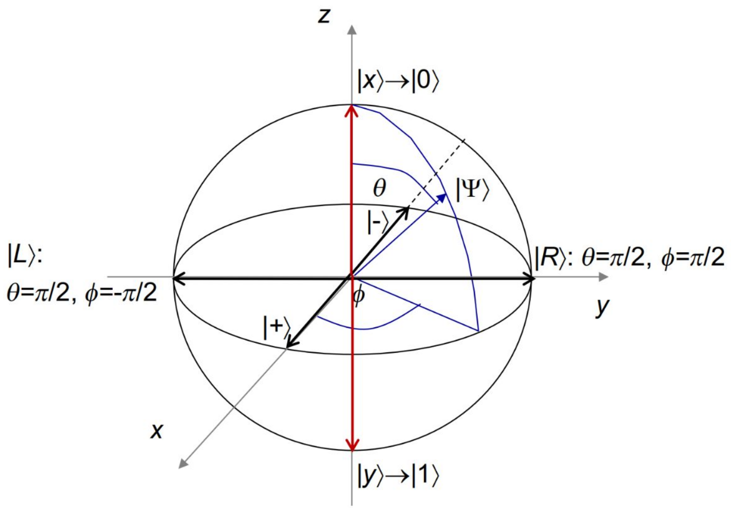
$$\boldsymbol{S}(t) = \begin{bmatrix} S_0(t) \\ S_1(t) \\ S_2(t) \\ S_3(t) \end{bmatrix}$$

Poincare Sphere with Stokes Representation



Phase

$$|\psi(\theta,\phi)\rangle = \cos(\theta/2)|0\rangle + e^{j\phi}\sin(\theta/2)|1\rangle \doteq \begin{pmatrix} \cos(\theta/2) \\ e^{j\phi}\sin(\theta/2) \end{pmatrix}$$



|0> (|x>-polarization)

|1> (|y>-polarization)

Relative vs Global Phase and its effect on Interference

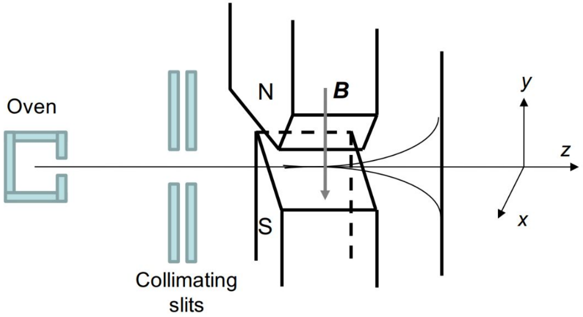
Quantum Gates

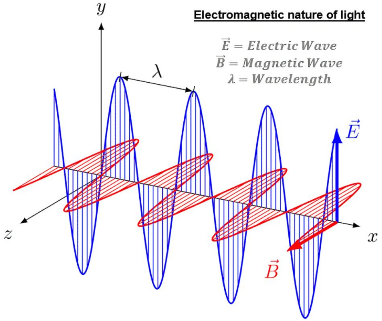
Quantum Gates

- **Errors from Imperfect Gates**: Imperfect control gates can introduce errors by applying incorrect operations in a quantum sequence.

- **Enhanced Error Correction**: Quantum Error Correction Codes (QECC) must address errors from both the quantum channel and imperfections in quantum gates during encoding and decoding.

Quantum Gates and the Electric/Magnetic Field





- superposition state can be represented in terms of these bases as follows

$$\psi_+ = e^{-j\phi/2}\cos(\theta/2) \quad \psi_- = e^{j\phi/2}\sin(\theta/2)$$

$$|\psi\rangle = e^{-j\phi/2}\cos(\theta/2)|+\rangle + e^{j\phi/2}\sin(\theta/2)|-\rangle$$

Bit-Flip and Phase-Flip

- X gates introduce bit flip
- Z gates introduce phase flip
- Y gates introduce simultaneously bit and phase flips

$$X(a|0\rangle + b|1\rangle) = a|1\rangle + b|0\rangle, \quad Y(a|0\rangle + b|1\rangle) = j(a|1\rangle - b|0\rangle), \quad Z(a|0\rangle + b|1\rangle) = a|0\rangle - b|1\rangle$$

Quantum Error

Quantum Error

- **Gate Imperfections**: Imperfect quantum gates can introduce additional errors in computations, so QECC must address errors from both the quantum channel and faulty gate operations.

- **Decoherence**: Interaction with the environment can blur superposition states and introduce errors, requiring quantum registers to be well-isolated to minimize these effects.

- **Error Correction Needs**: Quantum Error Correction Codes (QECC) are essential for protecting quantum information from errors caused by decoherence and during dynamic quantum computations.

Quantum Error

- **Imperfect Gates**: Quantum Error Correction Codes (QECC) face challenges due to imperfections in gates used for encoding, decoding, and syndrome extraction.

- **Syndrome Extraction**: The process of syndrome extraction involves entangling ancilla qubits with code blocks, which can introduce errors.

- **Accuracy Threshold**: Error recovery involves using controlled operations to correct the errors.

- **Accuracy Threshold**: Despite these issues, effective quantum error protection is possible if the error probability per gate is below a certain threshold, as stated by the accuracy threshold theorem.