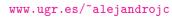
Gravitational wave solutions in metric-affine gravity

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Structure of this presentation

- Introduction to metric-affine geometry
- Metric-Affine gauge theory and dynamics
- 3 Gravitational waves in (quadratic) MAG
- Field equations for our the GW Ansatz
- **Solutions** of the field equations
- 6 Summary and conclusions

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[AJC, Obukhov 2021]

1. Introduction to metric-affine geometry

Geometric structures: metric

Geometric gravity (Einstein 1915) \rightsquigarrow The spacetime is modelled as a differentiable manifold \mathcal{M} .

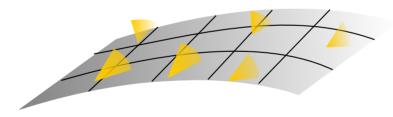
Geometric structures

- \square *Metric tensor:* $g_{\mu\nu}$
 - ⇒ Measuring (length, volume...)

$$s[\gamma](\sigma) = \int_0^\sigma \sqrt{|g_{\mu\nu}(\sigma')\dot{x}^{\mu}(\sigma')\dot{x}^{\nu}(\sigma')|} \,d\sigma'. \tag{1.1}$$

$$\operatorname{vol}(\mathcal{U}) = \int_{\mathcal{U}} \boldsymbol{\omega}_{\operatorname{vol}}, \qquad \boldsymbol{\omega}_{\operatorname{vol}} := \sqrt{|g|} \, \mathrm{d}x^{1} \wedge \dots \wedge \mathrm{d}x^{D} \qquad D := \dim(\mathcal{U}). \tag{1.2}$$

 \Rightarrow Module of a vector (not necessarily non-negative) \Rightarrow light cones \Rightarrow causality.



⇒ Notion of scale (conformal transformations...)

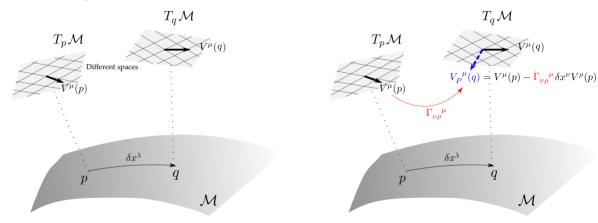
$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \,. \tag{1.3}$$

Geometric structures: connection

Geometric gravity (Einstein 1915) \leadsto The spacetime is modelled as a differentiable manifold \mathcal{M} .

Geometric structures

- \Box Connection: $\Gamma_{\mu\nu}^{\rho}$
 - \Rightarrow Notion of parallel in $\mathcal{M} \Rightarrow$ Covariant derivative ∇_{μ}



Geometric structures: connection

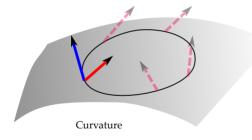
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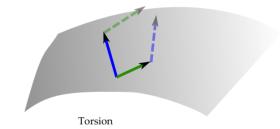
Geometric structures

- \Box Connection: $\Gamma_{\mu\nu}^{\ \rho}$
 - \Rightarrow Notion of parallel in $\mathcal{M} \Rightarrow$ Covariant derivative ∇_{μ}
 - Geometrical objects:

Curvature:
$$R_{\mu\nu\lambda}{}^{\rho} := \partial_{\mu}\Gamma_{\nu\lambda}{}^{\rho} - \partial_{\nu}\Gamma_{\mu\lambda}{}^{\rho} + \Gamma_{\mu\sigma}{}^{\rho}\Gamma_{\nu\lambda}{}^{\sigma} - \Gamma_{\nu\sigma}{}^{\rho}\Gamma_{\mu\lambda}{}^{\sigma}, \tag{1.4}$$

Torsion: $T_{\mu\nu}{}^{\rho} := \Gamma_{\mu\nu}{}^{\rho} - \Gamma_{\nu\mu}{}^{\rho}. \tag{1.5}$

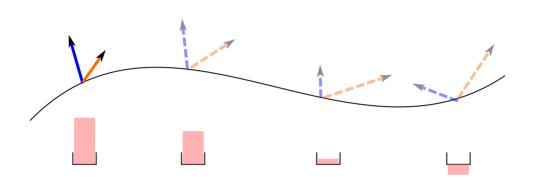




Geometric structures

Def.: In the presence of metric and connection we define the *nonmetricity tensor*:

$$Q_{\mu\nu\rho} := -\nabla_{\mu} g_{\nu\rho} \,. \tag{1.6}$$



Theorem. Given $g_{\mu\nu}$, there is only one connection that satisfies

$$T_{\mu\nu}^{\ \rho} = 0$$
 (torsionless condition), (1.7)

$$Q_{\mu\nu\rho} = 0$$
 (compatibility condition), (1.8)

the Levi-Civita connection:

$$\mathring{\Gamma}_{\mu\nu}{}^{\rho} = \frac{1}{2} g^{\rho\sigma} \left(\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu} \right). \tag{1.9}$$

Metric-affine theories

Instead of choosing $\mathring{\Gamma}$, they consider the metric and the (general) connection as independent fields.

Metric-Affine geometry in exterior notation

Metric-affine geometry arises naturally when formulating a gauge theory of the affine group $(Aff(4,\mathbb{R})=Tr_4 \rtimes GL(4,\mathbb{R})).$ [Hehl, McCrea, Mielke, Ne'eman 1995]

Three fundamental objects: coframe, metric and connection 1-form.

□ **Coframe**. Arbitrary basis of the cotangent space pointwise smooth:

$$\mathbf{e}_{a} = e^{\mu}{}_{a} \partial_{\mu}, \qquad \boxed{\boldsymbol{\vartheta}^{a} = e_{\mu}{}^{a} \mathrm{d}x^{\mu}} \qquad [\boldsymbol{\vartheta}^{a} (\mathbf{e}_{b}) = \delta^{a}_{b} \iff e_{\mu}{}^{a} e^{\mu}{}_{b} = \delta^{a}_{b}]. \qquad (1.10)$$

☐ **Metric**. Components of the metric in the arbitrary basis:

$$g_{ab} = e^{\mu}{}_{a} e^{\nu}{}_{b} g_{\mu\nu} \,. \tag{1.11}$$

For a *D*-dimensional manifold:

⇒ Canonical volume form

$$\boldsymbol{\omega}_{\text{vol}} := \frac{1}{D!} \mathcal{E}_{a_1 \dots a_D} \boldsymbol{\vartheta}^{a_1} \wedge \dots \wedge \boldsymbol{\vartheta}^{a_D} = \sqrt{|g|} dx^1 \wedge \dots \wedge dx^D, \qquad |g| \equiv |\det(g_{\mu\nu})|. \tag{1.12}$$

 \Rightarrow Hodge star of an arbitrary k-form $\alpha = \frac{1}{k!} \alpha_{a_1...a_k} \vartheta^{a_1} \wedge ... \wedge \vartheta^{a_k}$

$$\star : \Omega^{k}(\mathcal{M}) \longrightarrow \Omega^{D-k}(\mathcal{M})$$

$$\boldsymbol{\alpha} \longmapsto \star \boldsymbol{\alpha} := \frac{1}{(D-k)!k!} \alpha^{b_{1} \dots b_{k}} \mathcal{E}_{b_{1} \dots b_{k} c_{1} \dots c_{D-k}} \boldsymbol{\vartheta}^{c_{1}} \wedge \dots \wedge \boldsymbol{\vartheta}^{c_{D-k}}. \tag{1.13}$$

Metric-Affine geometry in exterior notation

□ Connection 1-form

$$\left| \boldsymbol{\omega}_{a}{}^{b} = \omega_{\mu a}{}^{b} \mathrm{d}x^{\mu} \right|. \tag{1.14}$$

where $\omega_{\mu a}{}^{b}$ are the components of the affine connection in the anholonomic basis:

$$\omega_{\mu a}{}^{b} = e^{\nu}{}_{a}e_{\lambda}{}^{b}\Gamma_{\mu\nu}{}^{\lambda} + e_{\sigma}{}^{b}\partial_{\mu}e^{\sigma}{}_{a}. \tag{1.15}$$

N.B. $\Gamma_{\mu\nu}^{\lambda}$ and $\omega_{\mu a}^{b}$ contain the same information (for a given frame/coframe).

⇒ Exterior covariant derivative (of algebra-valued forms)

$$\mathbf{D}\boldsymbol{\alpha}_{a...}^{b...} = \mathrm{d}\boldsymbol{\alpha}_{a...}^{b...} + \boldsymbol{\omega}_{c}^{b} \wedge \boldsymbol{\alpha}_{a...}^{c...} + ... - \boldsymbol{\omega}_{a}^{c} \wedge \boldsymbol{\alpha}_{c...}^{b...} - ... , \qquad (1.16)$$

Curvature, torsion and non-metricity forms:

$$\mathbf{R}_{a}^{b} := \mathrm{d}\boldsymbol{\omega}_{a}^{b} + \boldsymbol{\omega}_{c}^{b} \wedge \boldsymbol{\omega}_{a}^{c} \qquad \qquad = \frac{1}{2} R_{\mu\nu a}^{b} \mathrm{d}x^{\mu} \wedge \mathrm{d}x^{\nu} , \qquad (1.17)$$

$$T^{a} := \mathbf{D}\vartheta^{a} \qquad = \frac{1}{2}T_{\mu\nu}{}^{a}\mathrm{d}x^{\mu} \wedge \mathrm{d}x^{\nu} , \qquad (1.18)$$

$$\mathbf{Q}_{ab} \coloneqq -\mathbf{D}g_{ab} \qquad \qquad = Q_{\mu ab} \mathrm{d}x^{\mu} \,. \tag{1.19}$$

They can be decomposed according to irreps of $GL(4, \mathbb{R})$:

$$\boldsymbol{T}^{a} = \underbrace{\overset{(1)}{\boldsymbol{T}^{a}}}_{\text{tensor}} + \underbrace{\overset{(2)}{\boldsymbol{T}^{a}}}_{\text{trace}} + \underbrace{\overset{(3)}{\boldsymbol{T}^{a}}}_{\text{axial}}, \qquad \qquad \boldsymbol{Q}_{ab} = \underbrace{\overset{(1)}{\boldsymbol{Q}_{ab}}}_{\text{tot. symm}} + \underbrace{\overset{(2)}{\boldsymbol{Q}_{ab}}}_{\text{tens.}} + \underbrace{\overset{(3)}{\boldsymbol{Q}_{ab}}}_{\text{traces}} + \underbrace{\overset{(3)}{\boldsymbol{Q}_{ab}}}_{\text{traces}}$$

$$\boldsymbol{R}^{ab} = \boldsymbol{W}^{[ab]} + \boldsymbol{Z}^{(ab)} \quad \Rightarrow \left\{ \begin{array}{ll} \boldsymbol{W}^{ab} &= {}^{(1)}\boldsymbol{W}^{ab} + {}^{(2)}\boldsymbol{W}^{ab} + \underbrace{{}^{(3)}\boldsymbol{W}^{ab}}_{\text{tot. antis.}} + {}^{(4)}\boldsymbol{W}^{ab} + {}^{(5)}\boldsymbol{W}^{ab} + \underbrace{{}^{(6)}\boldsymbol{W}^{ab}}_{\text{Ric. scalar}} \\ \boldsymbol{Z}^{ab} &= {}^{(1)}\boldsymbol{Z}^{ab} + {}^{(2)}\boldsymbol{Z}^{ab} + {}^{(3)}\boldsymbol{Z}^{ab} + {}^{(4)}\boldsymbol{Z}^{ab} + \underbrace{{}^{(5)}\boldsymbol{Z}^{ab}}_{\text{trace}} \end{array} \right.$$

2. Metric-Affine gauge theory and dynamics

- \square The fundamental gravitational fields in metric-affine gravity are: g_{ab} , ϑ^a and $\omega_a{}^b$.
- □ Gauge symmetry \Rightarrow the most general metric-affine Lagrangian *D*-form is:

$$L = L(g_{ab}, \vartheta^a, Q_{ab}, T^a, R_a{}^b). \tag{2.1}$$

Equations of motion

$$\mathbf{D}M^{ab} - m^{ab} = 0 \qquad [\text{EoM } g_{ab}], \tag{2.2}$$

$$\mathbf{D}\boldsymbol{H}_a - \boldsymbol{E}_a = 0 \qquad [\text{EoM } \boldsymbol{\vartheta}^a], \tag{2.3}$$

$$\mathbf{D}\boldsymbol{H}^{a}{}_{b}-\boldsymbol{E}^{a}{}_{b}=0 \qquad [\text{EoM}\,\boldsymbol{\omega}_{a}{}^{b}], \qquad (2.4)$$

where

$$M^{ab} := -2\frac{\partial L}{\partial Q_{ab}}, \qquad H_a := -\frac{\partial L}{\partial T^a}, \qquad H^a{}_b := -\frac{\partial L}{\partial R_a{}^b},$$

$$(2.5)$$

$$m{m}^{ab} := 2rac{\partial m{L}}{\partial g_{ab}} \qquad m{E}_a := rac{\partial m{L}}{\partial m{artheta}^a} \,, \qquad m{E}^a{}_b := -m{artheta}^a \wedge m{H}_b - g_{bc}m{M}^{ac} \,.$$

□ Noether identity under $GL(4, \mathbb{R})_{local} \Rightarrow EoM$ of g_{ab} is redundant.

$$\mathbf{D}\boldsymbol{M}^{ab} - \boldsymbol{m}^{ab} = 0 \qquad [\text{EoM } g_{ab}],$$

$$\mathbf{D}\boldsymbol{H}_a - \boldsymbol{E}_a = 0 \qquad [\text{EoM } \boldsymbol{\vartheta}^a],$$

$$\mathbf{D}\boldsymbol{H}^a_b - \boldsymbol{E}^a_b = 0 \qquad [\text{EoM } \boldsymbol{\omega}_a^b].$$
(2.6)

In practice, one fixes the gauge (e.g. by taking $g_{ab} \equiv \eta_{ab}$).

□ Noether identity under Diff(\mathcal{M}) \Rightarrow E_a is determined by $H^a{}_b$, H_a and M^{ab} via:

$$\boldsymbol{E}_{a} = \boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{L} + (\boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{T}^{b}) \wedge \boldsymbol{H}_{b} + (\boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{R}_{b}^{c}) \wedge \boldsymbol{H}_{c}^{b} + \frac{1}{2} (\boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{Q}_{bc}) \boldsymbol{M}^{bc}.$$
(2.8)

Metric-Affine gauge theory and dynamics

Standard procedure to find solutions

Compute

$$M^{ab} := -2 \frac{\partial L}{\partial Q_{ab}}, \qquad H_a := -\frac{\partial L}{\partial T^a}, \qquad H^a{}_b := -\frac{\partial L}{\partial R_a{}^b}.$$
 (2.9)

② Determine

$$\boldsymbol{E}^{a}{}_{b} := -\boldsymbol{\vartheta}^{a} \wedge \boldsymbol{H}_{b} - g_{bc} \boldsymbol{M}^{ac} , \qquad (2.10)$$

$$\boldsymbol{E}_{a} = \boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{L} + (\boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{T}^{b}) \wedge \boldsymbol{H}_{b} + (\boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{R}_{b}^{c}) \wedge \boldsymbol{H}_{c}^{b} + \frac{1}{2} (\boldsymbol{e}_{a} \boldsymbol{\perp} \boldsymbol{Q}_{bc}) \boldsymbol{M}^{bc}.$$
(2.11)

- **3** Evaluate H_a , H^a_b , E^a_b and E_a in the chosen Ansatz (fix the gauge a take a constant anholonomic metric g_{ab} , e.g. Minkowski).
- Solve the differential equations

$$\mathbf{D}\boldsymbol{H}_a - \boldsymbol{E}_a = 0 \qquad [\text{EoM } \boldsymbol{\vartheta}^a] \,, \tag{2.12}$$

$$\mathbf{D}\boldsymbol{H}^{a}{}_{b} - \boldsymbol{E}^{a}{}_{b} = 0 \qquad [\text{EoM}\,\boldsymbol{\omega}_{a}{}^{b}]. \tag{2.13}$$

3. Gravitational waves in (quadratic) MAG

The Lagrangian

(Quadratic) MAG Lagrangian

The most general one containing linear and quadratic invariants of Q_{ab} , T^a and R_a .

$$L = \frac{1}{2\kappa c} \Big\{ a_0 \star (\boldsymbol{\vartheta}_a \wedge \boldsymbol{\vartheta}_b) \wedge \boldsymbol{R}^{ab} - \boldsymbol{T}^a \wedge \sum_{I=1}^3 a_I \star (^{(I)}\boldsymbol{T}_a) \qquad \sim R + TT$$

$$- \boldsymbol{Q}_{ab} \wedge \sum_{I=1}^4 b_I \star (^{(I)}\boldsymbol{Q}^{ab}) - 2b_5 (^{(3)}\boldsymbol{Q}_{ac} \wedge \boldsymbol{\vartheta}^a) \wedge \star (^{(4)}\boldsymbol{Q}^{bc} \wedge \boldsymbol{\vartheta}_b) \qquad \sim QQ$$

$$- 2\boldsymbol{\vartheta}^a \wedge \star \boldsymbol{T}^b \wedge \sum_{I=1}^3 c_I {}^{(I+1)}\boldsymbol{Q}_{ab} \Big\} \qquad \sim QT$$

$$- \frac{\ell_\rho^2}{2\kappa c} \boldsymbol{R}^{ab} \wedge \star \Big[\sum_{I=1}^6 w_I {}^{(I)}\boldsymbol{W}_{ab} + v_1 \boldsymbol{\vartheta}_a \wedge (\boldsymbol{e}_c \boldsymbol{\bot}^{(5)}\boldsymbol{W}^c{}_b) \Big] \qquad \sim RR$$

$$+ \sum_{I=1}^5 z_I {}^{(I)}\boldsymbol{Z}_{ab} + v_2 \boldsymbol{\vartheta}_c \wedge (\boldsymbol{e}_a \boldsymbol{\bot}^{(2)}\boldsymbol{Z}^c{}_b) + \sum_{I=2}^5 v_I \boldsymbol{\vartheta}_a \wedge (\boldsymbol{e}_c \boldsymbol{\bot}^{(I)}\boldsymbol{Z}^c{}_b) \Big]. \qquad \sim RR \qquad (3.1)$$

(we do not consider the cosmological constant term)

- \square κ and ℓ_{ρ} are the gravitational couplings.
- \blacksquare Term with a_0 is the metric-affine version of the Einstein term.
- ☐ This Lagrangian has in total a_I (3) $+b_I$ (5) $+c_I$ (3) $+w_I$ (6) $+z_I$ (5) $+v_I$ (5) = 27 parameters.

Ansatz for the geometry: basic fields

■ **Metric**: we fix the gauge $g_{ab} = diag(+1, -1, -1, -1)$ (Minkowski metric).

Coframe

$$\boldsymbol{\vartheta}^{\widehat{0}} = \frac{1}{2}(\boldsymbol{U}+1)\mathrm{d}\sigma + \frac{1}{2}\mathrm{d}\rho,$$

 $\vartheta^{\widehat{A}} = \mathrm{d}x^A. \qquad A = 2, 3.$

 $\vartheta^{\widehat{1}} = \frac{1}{2}(U-1)\mathrm{d}\sigma + \frac{1}{2}\mathrm{d}\rho,$

where $U = U(\sigma, x^A)$. This implies that $g_{\mu\nu}$ is of the Brinkmann type:

$$ds^2 = d\sigma d\rho + U d\sigma^2 - \underbrace{\delta_{AB} dx^A dx^B}_{AB}.$$

We introduce

$$oldsymbol{k} := \mathrm{d}\sigma = oldsymbol{artheta}^{\widehat{0}} - oldsymbol{artheta}^{\widehat{1}}$$
 (wave 1-form) o dual to $\partial_{
ho} = k^{\mu}\partial_{\mu}$.

$$\omega_a{}^b = -\mathbf{k} \left(k_a V^b + k^b W_a \right) + k_a k^b u_c \vartheta^c.$$

Connection

where
$$W$$
 V and u depend on σ and x^A and are transversal.

where W_a , V_a and u_a depend on σ and x^A and are transversal:

$$W^a = \delta^a_A W^A(\sigma, x^B), \quad V^a = \delta^a_A V^A(\sigma, x^B), \quad u_a = \delta^A_A u_A(\sigma, x^B), \quad A = 2, 3.$$

Unknowns: Wave's profile determined by 7 variables:
$$U$$
, W^A , V^A , and u_A .

transversal 2D space

(3.2)

(3.3)

(3.4)

(3.5)

(3.6)

(3.7)

(3.8)

Ansatz for the geometry: derived fields

Ansatz:

$$\{\boldsymbol{\vartheta}^{\widehat{0}},\boldsymbol{\vartheta}^{\widehat{1}},\boldsymbol{\vartheta}^{\widehat{A}}\} = \{\frac{1}{2}(U+1)\mathrm{d}\sigma + \frac{1}{2}\mathrm{d}\rho , \frac{1}{2}(U-1)\mathrm{d}\sigma + \frac{1}{2}\mathrm{d}\rho , \mathrm{d}x^{A}\},$$

$$g_{ab} = \mathrm{diag}(1,-1,-1,-1), \qquad \boldsymbol{\omega}_{a}{}^{b} = -\boldsymbol{k}\left(k_{a}V^{b} + k^{b}W_{a}\right) + k_{a}k^{b}u_{c}\boldsymbol{\vartheta}^{c},$$

Torsion

$$\mathbf{T}^{a} = -\mathbf{k} \wedge k^{a} \left[\frac{1}{2} \partial_{A} \mathbf{U} - \delta_{AB} \mathbf{W}^{B} + u_{A} \right] \boldsymbol{\vartheta}^{A} \qquad = {}^{(1)} \mathbf{T}^{a}$$

$$\left(\underbrace{{}^{(2)} \mathbf{T}^{a}}_{\text{trace}} = \underbrace{{}^{(3)} \mathbf{T}^{a}}_{\text{axial}} = 0 \right).$$
(3.9)

Nonmetricity

$$Q_{ab} = -2\mathbf{k} \ k_{(a}(\mathbf{W}_{b)} + V_{b)}) + 2k_{a}k_{b}u_{A}\boldsymbol{\vartheta}^{A} \qquad = {}^{(1)}Q_{ab} + {}^{(2)}Q_{ab}$$

$$\left(\underbrace{{}^{(3)}Q_{ab} = {}^{(4)}Q_{ab}}_{\text{traces}} = 0\right).$$

$$(3.10)$$

Curvature

$$\mathbf{R}_{a}{}^{b} = \mathbf{k} \wedge (k_{a}\underline{\mathbf{d}}V^{b} + k^{b}\underline{\mathbf{d}}W_{a}) + k_{a}k^{b}\mathbf{d}(u_{A}\boldsymbol{\vartheta}^{A}) \qquad (\underline{\mathbf{d}} := \boldsymbol{\vartheta}^{A}\boldsymbol{e}_{A} \cdot \underline{\mathbf{d}} = \mathbf{d}^{A}\partial_{A})$$
(3.11)
$$\begin{pmatrix} \mathbf{W}^{ab} = \mathbf{k} \wedge k^{[b}\underline{\mathbf{d}}(\mathbf{W}^{a]} - V^{a]} \end{pmatrix} = {}^{(1)}\mathbf{W}^{ab} + {}^{(2)}\mathbf{W}^{ab} + {}^{(4)}\mathbf{W}^{ab} \\ \begin{pmatrix} {}^{(3)}\mathbf{W}^{ab} = {}^{(5)}\mathbf{W}^{ab} = {}^{(6)}\mathbf{W}^{ab} = 0 \end{pmatrix}$$

$$\Leftrightarrow \left\{ \begin{array}{ll} \boldsymbol{W}^{ab} = \boldsymbol{k} \wedge k^{[b}\underline{\mathbf{d}}(\boldsymbol{W}^{a]} - \boldsymbol{V}^{a]}) & = & ^{(1)}\boldsymbol{W}^{ab} + ^{(2)}\boldsymbol{W}^{ab} + ^{(4)}\boldsymbol{W} \\ & \left(^{(3)}\boldsymbol{W}^{ab} = ^{(5)}\boldsymbol{W}^{ab} = ^{(6)}\boldsymbol{V} \right) \\ \boldsymbol{Z}^{ab} = \boldsymbol{k} \wedge k^{(b}\underline{\mathbf{d}}(\boldsymbol{W}^{a)} + \boldsymbol{V}^{a)}) + k^{a}k^{b}\mathbf{d}(\boldsymbol{u}_{A}\boldsymbol{\vartheta}^{A}) & = & ^{(1)}\boldsymbol{Z}^{ab} + ^{(2)}\boldsymbol{Z}^{ab} + ^{(4)}\boldsymbol{Z}^{ab} \\ & \left(^{(3)}\boldsymbol{Z}^{ab} = ^{(5)}\boldsymbol{Z}^{ab} = 0 \right) \end{array} \right.$$

(3.12)

4. Field equations for our the GW Ansatz

Field equations

$$[\text{EoM }\boldsymbol{\vartheta}^{a}] \qquad 0 = -\frac{a_{1}}{2}\Delta\boldsymbol{U} + \left[\frac{a_{0}}{2} + a_{1} - c_{1}\right]\partial_{A}\boldsymbol{W}^{A} - \left[\frac{a_{0}}{2} + c_{1}\right]\partial_{A}\boldsymbol{V}^{A} - (a_{1} - 2c_{1})\partial_{A}\underline{\boldsymbol{u}}^{A}, \tag{4.1}$$

$$[\text{EoM } \boldsymbol{\omega}_{[ab]}] \qquad 0 = \frac{a_0 + a_1}{2} \partial_A \boldsymbol{U} + \left[-\frac{a_0}{2} - a_1 + c_1 \right] \underline{\boldsymbol{W}}_A + \left[\frac{a_0}{2} + c_1 \right] \underline{\boldsymbol{V}}_A + (a_1 - 2c_1) \boldsymbol{u}_A$$

$$- \frac{\ell_\rho^2}{4} \left[2w_1 \Delta \underline{\boldsymbol{W}}_A - 2w_1 \Delta \underline{\boldsymbol{V}}_A + (v_4 + 2w_4) \partial_A \partial_B \boldsymbol{W}^B + (v_4 - 2w_4) \partial_A \partial_B \boldsymbol{V}^B \right]$$

$$- \frac{\ell_\rho^2}{4} \epsilon_{AB} \epsilon^{CD} \underline{\partial}^B \left[(v_2 - 2w_2) \partial_C \underline{\boldsymbol{W}}_D + (v_2 + 2w_2) \partial_C \underline{\boldsymbol{V}}_D - 2v_2 \partial_C \boldsymbol{u}_D \right].$$

$$(4.2)$$

$$[EoM \,\omega_{(ab)}] \qquad 0 = \frac{a_1 - 2c_1}{2} \partial_A U + \left[\frac{a_0}{2} - a_1 - \frac{4(2b_1 + b_2)}{3} + 3c_1 \right] \underline{W}_A$$

$$+ \left[\frac{a_0}{2} - \frac{4(2b_1 + b_2)}{3} + c_1 \right] \underline{V}_A + \left[a_0 + a_1 - 4c_1 - \frac{8(b_1 - b_2)}{3} \right] \underline{u}_A$$

$$- \frac{\ell_\rho^2}{4} \left[2z_1 \Delta \underline{W}_A + 2z_1 \Delta \underline{V}_A + (z_1 + z_4 + 3v_4) \partial_A \partial_B \underline{W}^B + (z_1 + z_4 + v_4) \partial_A \partial_B \underline{V}^B \right]$$

$$- \frac{\ell_\rho^2}{4} \epsilon_{AB} \epsilon^{CD} \underline{\partial}^B \left[(2v_2 - z_1 - z_2) \partial_C \underline{W}_D - (z_1 + z_2) \partial_C \underline{V}_D - 2(z_1 - z_2 + v_2) \partial_C \underline{u}_D \right]. \qquad (4.3)$$

$$0 = \frac{2c_1 - a_1}{2} \partial_A \underline{U} + \left[\frac{a_0}{2} + a_1 - \frac{4(b_1 - b_2)}{3} - 3c_1 \right] \underline{W}_A$$

$$+ \left[\frac{a_0}{2} - \frac{4(b_1 - b_2)}{3} - c_1 \right] \underline{V}_A + \left[4c_1 - a_1 - \frac{4(b_1 + 2b_2)}{3} \right] \underline{u}_A$$

$$+ \frac{\ell_\rho^2}{4} \epsilon_{AB} \epsilon^{CD} \underline{\partial}^B \left[(z_1 - z_2 + 2v_2) \partial_C \underline{W}_D + (z_1 - z_2) \partial_C \underline{V}_D + 2(z_1 + z_2 - v_2) \partial_C \underline{u}_D \right]. \qquad (4.4)$$

$$0 = \partial_\sigma \left[(z_4 - z_1 + 3v_4) \partial_A \underline{W}^A + (z_4 - z_1 + v_4) \partial_A \underline{V}^A - 4z_1 \partial_A \underline{u}^A \right]. \qquad (4.5)$$

Field equations

$$[\text{EoM } \vartheta^a] \qquad 0 = (\dots)\Delta U + (\dots)\partial_A W^A + (\dots)\partial_A V^A + (\dots)\partial_A \underline{u}^A, \tag{4.6}$$

$$[\text{EoM }\omega_{[ab]}] \qquad 0 = (\dots)\partial_A U + (\dots)\underline{W}_A + (\dots)\underline{V}_A + (\dots)u_A$$

$$-\frac{\ell_\rho^2}{4} \left[(\dots)\Delta\underline{W}_A + (\dots)\Delta\underline{V}_A + (\dots)\partial_A\partial_B W^B + (\dots)\partial_A\partial_B V^B \right]$$

$$-\frac{\ell_\rho^2}{4} \epsilon_{AB}\underline{\partial}^B \left\{ \epsilon^{CD} \left[(\dots)\partial_{[C}\underline{W}_{D]} + (\dots)\partial_{[C}\underline{V}_{D]} + (\dots)\partial_{[C}u_{D]} \right] \right\}. \tag{4.7}$$

[EoM
$$\omega_{(ab)}$$
] 0 = (same structure as the previous one), (4.8)

0 =(same structure as the previous one without the second line),

$$0 = \partial_{\sigma} \left[(...) \partial_{A} W^{A} + (...) \partial_{A} V^{A} + (...) \partial_{A} \underline{u}^{A} \right]. \tag{4.10}$$

where

$$\underline{\underline{W}}_A := \delta_{AB} \underline{W}^B, \qquad \underline{\underline{V}}_A := \delta_{AB} \underline{V}^B, \qquad \underline{\underline{u}}^A := \delta^{AB} \underline{u}_B, \qquad \underline{\partial}^A := \delta^{AB} \partial_B, \qquad \Delta := \delta^{AB} \partial_A \partial_B \quad (4.11)$$

and ϵ_{AB} , ϵ^{CD} correspond to the 2-dimensional Levi-Civita symbol (convention: $\epsilon_{23} := 1$, $\epsilon^{23} := 1$).

(4.9)

5. Solutions of the field equations

Exploring special cases: Riemannian case (T = 0, Q = 0)

Conditions: Nullity of torsion and nonmetricity is equivalent to

$$u_A = 0, \qquad W^A = -V^A = \frac{1}{2} \delta^{AB} \partial_B U.$$
 (5.1)

Non-trivial equations

$$a_0 \Delta \underline{U} = 0, \tag{5.2}$$

$$v_4 \,\partial_\sigma \Delta \underline{U} = 0, \tag{5.3}$$

$$v_4 \ell_\rho^2 \partial_A \Delta \underline{U} = 0, \tag{5.4}$$

$$(w_1 + w_4)\ell_\rho^2 \partial_A \Delta \frac{\mathbf{U}}{\mathbf{U}} = 0. \tag{5.5}$$

Remarks

- ☐ The solution of GR, $\Delta U = 0$ (and Levi-Civita), is a solution of all MAG models.
- For **L** without the RR sector ($w_I = z_I = v_I = 0$):

$$u_A = 0,$$
 $W^A = -V^A = \frac{1}{2}\delta^{AB}\partial_B U,$ $\frac{a_0}{2}\Delta U = 0$ (5.6)

is the general solution, except for very specific parameters:

[Obukhov, Vlachynsky, Esser, Hehl 1997]

$$-a_1 = \frac{a_2}{2} = 2a_3 = 2c_1 = -c_2 = -c_3 = a_0, \tag{5.7}$$

$$4b_1 = 2b_2 = -8b_3 = \frac{8b_4}{3} = 2b_5 = a_0. (5.8)$$

Exploring special cases: Teleparallel case (R = 0)

Conditions: Nullity of curvature is equivalent to

$$W^A = W^A(\sigma), \qquad V^A = V^A(\sigma), \qquad u_A = \frac{1}{2}\partial_A \mathcal{U} \quad (\mathcal{U} = \mathcal{U}(x^B)).$$
 (5.9)

Coframe equation

$$a_1 \Delta U + (a_1 - 2c_1) \Delta U = 0.$$
 (5.10)

Connection equation

→ New variables

$$\Theta_A = \frac{1}{2} \partial_A \mathbf{U} - \underline{\mathbf{W}}_A + u_A, \tag{5.11}$$

$$\Phi_A := \underline{W}_A + \underline{V}_A + u_A, \tag{5.12}$$

$$\Psi_A := \underline{W}_A + \underline{V}_A - 2u_A. \tag{5.13}$$

 \rightarrow Non-trivial equations

$$\begin{pmatrix} 0 & (a_0 - 4b_1) & 0 \\ 2(a_0 + a_1) & 0 & (a_0 + 2c_1) \\ 3(a_0 + 2c_1) & 0 & 2(a_0 + 2b_2) \end{pmatrix} \begin{pmatrix} \Theta_A \\ \Phi_A \\ \Psi_A \end{pmatrix} = 0.$$
 (5.14)

Remarks

☐ If the determinant vanishes we have non-trivial solutions.

Exploring special cases: Teleparallel case (R = 0) and Q = 0 (Standard teleparallelism)

Conditions: Nullity of curvature and nonmetricity is equivalent to

$$-V^{A} = W^{A} = W^{A}(\sigma), u_{A} = 0. (5.15)$$

Coframe equation

$$a_1 \Delta \underline{\boldsymbol{U}} = 0. \tag{5.16}$$

Connection equation

→ New variables become

$$\Theta_A = \frac{1}{2} \partial_A \mathbf{U} - \underline{\mathbf{W}}_A, \qquad \Phi_A = \Psi_A = 0. \tag{5.17}$$

 \rightarrow Non-trivial equations

$$(a_0 + a_1)\Theta_A = 0$$
 $(a_0 + 2c_1)\Theta_A = 0$ (5.18)

Remarks

☐ Non-trivial solutions for

$$a_0 + a_1 = 0, a_0 + 2c_1 = 0.$$
 (5.19)

so

$$a_0 \Delta \underline{U} = 0. \tag{5.20}$$

Same metric structure as in the GR solution (but $\Gamma \neq$ Levi-Civita).

Exploring special cases: Teleparallel case (R = 0) and T = 0 (Symmetric teleparallelism)

Conditions: Nullity of curvature and torsion is equivalent to

$$W^{A} = W^{A}(\sigma), \qquad V^{A} = V^{A}(\sigma), \qquad u_{A} = \frac{1}{2} \partial_{A} \mathcal{U}(x^{B}), \qquad \underbrace{\partial_{A}(U + \mathcal{U}) - 2\underline{W}_{A}}_{W^{A}(\sigma)} = 0. \tag{5.21}$$

Coframe equation

$$c_1 \Delta \underline{U} = 0. ag{5.22}$$

Connection equation

 \rightarrow New variables become

$$\Theta_A = 0, \qquad \Phi_A = \underline{W}_A + \underline{V}_A + \frac{1}{2}\partial_A \mathcal{U}, \qquad \Psi_A = \underline{W}_A + \underline{V}_A - \partial_A \mathcal{U}.$$
 (5.23)

 \rightarrow Non-trivial equations

$$(a_0 - 4b_1)\Phi_A = 0,$$
 $(a_0 + 2b_2)\Psi_A = 0,$ $(a_0 + 2c_1)\Psi_A = 0.$ (5.24)

Remarks

☐ There are non-trivial solutions for some values of the parameters.

☐ Step 1. Potential-copotential decomposition

$$W^{A} =: \frac{1}{2} \left(\delta^{AB} \partial_{B} W + \epsilon^{AB} \partial_{B} \overline{W} \right), \tag{5.25}$$

$$V^{A} =: \frac{1}{2} \left(\delta^{AB} \partial_{B} \mathcal{V} + \epsilon^{AB} \partial_{B} \overline{\mathcal{V}} \right), \tag{5.26}$$

$$u_A =: \frac{1}{2} \left(\partial_A \mathcal{U} + \epsilon_{AB} \delta^{BC} \partial_C \overline{\mathcal{U}} \right). \tag{5.27}$$

Useful, since

$$F^{A} = \frac{1}{2} \left(\delta^{AB} \partial_{B} \mathcal{F} + \epsilon^{AB} \partial_{B} \overline{\mathcal{F}} \right) \qquad \Rightarrow \qquad \boxed{\partial_{A} F^{A} = \frac{1}{2} \Delta \mathcal{F} \quad \text{and} \quad \epsilon_{AB} \delta^{BC} \partial_{C} F^{A} = \frac{1}{2} \Delta \overline{\mathcal{F}}} \ . \tag{5.28}$$

- □ **Step 2. Splitting of the equations**. By a similar decomposition, the equations can be split into even and odd parts. [Blagojević, Cvetković, Obukhov 2017]
- ☐ Step 3. Convenient change of variables

$$\overline{\mathcal{X}}_{1} = -\overline{\mathcal{W}} + \overline{\mathcal{U}},
\overline{\mathcal{X}}_{2} = \overline{\mathcal{W}} + \overline{\mathcal{V}} + \overline{\mathcal{U}},
\overline{\mathcal{X}}_{3} = \overline{\mathcal{W}} + \overline{\mathcal{V}} - 2\overline{\mathcal{U}}.$$

$$\overline{\mathcal{W}} = -\overline{\mathcal{X}}_{1} + \frac{1}{3}\overline{\mathcal{X}}_{2} - \frac{1}{3}\overline{\mathcal{X}}_{3},
\overline{\mathcal{V}} = \overline{\mathcal{X}}_{1} + \frac{1}{3}\overline{\mathcal{X}}_{2} + \frac{2}{3}\overline{\mathcal{X}}_{3},
\overline{\mathcal{U}} = \frac{1}{3}\overline{\mathcal{X}}_{2} - \frac{1}{3}\overline{\mathcal{X}}_{3}.$$

$$\frac{\mathcal{W}}{\overline{\mathcal{V}}} = -\mathcal{X}_1 + \frac{1}{3}\mathcal{X}_2 - \frac{1}{3}\mathcal{X}_3,
\Leftrightarrow \overline{\mathcal{V}} = \overline{\mathcal{X}}_1 + \frac{1}{3}\overline{\mathcal{X}}_2 + \frac{2}{3}\overline{\mathcal{X}}_3,
\overline{\mathcal{U}} = \frac{1}{2}\overline{\mathcal{X}}_2 - \frac{1}{2}\overline{\mathcal{X}}_3.$$
(5.30)

ODD SECTOR

 \square By combining the equations: $\overline{\mathcal{X}}_2$ is decoupled:

$$(a_0 - 4b_1) \overline{\mathcal{X}}_2 - \ell_\rho^2 z_1 \Delta \overline{\mathcal{X}}_2 = 0,$$
 (5.31)

whereas $\overline{\mathcal{X}}_1$ and $\overline{\mathcal{X}}_3$ verify

$$(a_{0} + 2c_{1})\overline{\mathcal{X}}_{1} + \frac{2}{3}(a_{0} + 2b_{2})\overline{\mathcal{X}}_{3} - \frac{\ell_{\rho}^{2}}{4} \left\{ -2\left[2w_{1} + 2w_{2} + v_{2}\right] \Delta \overline{\mathcal{X}}_{1} - \left[2w_{1} + 2w_{2} + v_{2} + \frac{1}{3}(z_{1} + 3z_{2})\right] \Delta \overline{\mathcal{X}}_{3} \right\} = 0, \quad (5.32)$$

$$(a_{0} + a_{1})\overline{\mathcal{X}}_{1} + \left(\frac{a_{0}}{2} + c_{1}\right)\overline{\mathcal{X}}_{3} - \frac{\ell_{\rho}^{2}}{4} \left[-4(w_{1} + w_{2}) \Delta \overline{\mathcal{X}}_{1} - (2w_{1} + 2w_{2} + v_{2}) \Delta \overline{\mathcal{X}}_{3} \right] = 0 \quad (5.33)$$

ODD SECTOR

$$\begin{pmatrix}
0 & a_0 - 4b_1 + 4z_1 \overline{Q}^2 & 0 \\
a_0 + 2c_1 - 2\overline{Q}^2 \Lambda_2 & 0 & \frac{2}{3}(a_0 + 2b_2) - \overline{Q}^2(\Lambda_2 + \Lambda_3) \\
a_0 + a_1 - \overline{Q}^2 \Lambda_1 & 0 & \frac{a_0}{2} + c_1 - \overline{Q}^2 \Lambda_2
\end{pmatrix}
\begin{pmatrix}
\overline{\mathcal{X}}_1^{(0)} \\
\overline{\mathcal{X}}_2^{(0)} \\
\overline{\mathcal{X}}_3^{(0)}
\end{pmatrix} = 0. (5.34)$$

Abbreviations:

$$\overline{Q}^2 := \frac{\ell_\rho^2}{4} \overline{q}_A \overline{q}_B \delta^{AB}, \qquad \Lambda_1 := 4(w_1 + w_2), \qquad \Lambda_2 := 2(w_1 + w_2) + v_2, \qquad \Lambda_3 := \frac{1}{3} (z_1 + 3z_2). \tag{5.35}$$

The three modes propagate if

$$a_0 - 4b_1 + 4z_1\overline{\mathcal{Q}}^2 = 0, \qquad \mathcal{A}\overline{\mathcal{Q}}^4 + \mathcal{B}\overline{\mathcal{Q}}^2 + \mathcal{C} = 0,$$
 (5.36)

where we denoted the combinations of the coupling constants

$$\mathcal{A} := 2\Lambda_2^2 + \Lambda_1(\Lambda_2 + \Lambda_3), \tag{5.37}$$

$$\mathcal{B} := -4\left(\frac{a_0}{2} + c_1\right)\Lambda_2 + (a_0 + a_1)(\Lambda_2 + \Lambda_3) - \frac{2}{3}(a_0 + 2b_2)\Lambda_1, \tag{5.38}$$

$$C := 2\left(\frac{a_0}{2} + c_1\right)^2 - \frac{2}{3}(a_0 + 2b_2)(a_0 + a_1). \tag{5.39}$$

□ The amplitudes $\overline{\mathcal{X}}_I^{(0)}(\sigma)$ are arbitrary functions of σ .

EVEN SECTOR

The equations are

$$(2c_{1} - a_{1}) \mathcal{X}_{1} + \frac{1}{3}(a_{0} - 4b_{1}) \mathcal{X}_{2} + \left[-\frac{a_{0}}{2} - c_{1} + \frac{2}{3}(a_{0} + 2b_{2}) \right] \mathcal{X}_{3} = 0, \quad (5.40)$$

$$(a_{0} + a_{1}) \mathcal{X}_{1} + \left(\frac{a_{0}}{2} + c_{1} \right) \mathcal{X}_{3} - \frac{\ell_{\rho}^{2}}{4} \left[2(w_{1} + w_{4}) \Delta \mathcal{X}_{0} + \frac{2}{3} v_{4} \Delta \mathcal{X}_{2} + \frac{1}{3} v_{4} \Delta \mathcal{X}_{3} \right] = 0, \quad (5.41)$$

$$(a_{1} - 2c_{1}) \mathcal{X}_{1} + \frac{2}{3}(a_{0} - 4b_{1}) \mathcal{X}_{2} + \left[\frac{a_{0}}{2} + c_{1} - \frac{2}{3}(a_{0} + 2b_{2}) \right] \mathcal{X}_{3}$$

$$- \frac{\ell_{\rho}^{2}}{4} \left[v_{4} \Delta \mathcal{X}_{0} + \frac{2}{3} (3z_{1} + z_{4} + 2v_{4}) \Delta \mathcal{X}_{2} + \frac{1}{3} (3z_{1} + z_{4} + 2v_{4}) \Delta \mathcal{X}_{3} \right] = 0, \quad (5.42)$$

$$\frac{a_{0}}{2} \Delta \mathcal{X}_{0} - a_{1} \Delta \mathcal{X}_{1} - c_{1} \Delta \mathcal{X}_{3} = 0, \quad (5.43)$$

$$\partial_{\sigma} \left\{ v_{4} \Delta \mathcal{X}_{0} + \frac{2}{3} (z_{4} - 3z_{1} + 2v_{4}) \Delta \mathcal{X}_{2} + \frac{1}{3} (z_{4} + 3z_{1} + 2v_{4}) \Delta \mathcal{X}_{3} \right\} = 0. \quad (5.44)$$

Special solution

$$\mathcal{X}_2 = \mathcal{X}_3 = 0 \tag{5.45}$$

If we assume $v_4 \neq 0$ and $a_1 \neq 2c_1$, the system becomes

$$\Delta U = 0, \qquad \mathcal{W} = -\mathcal{V} = U, \qquad \mathcal{U} = 0. \tag{5.46}$$

(massless graviton mode).

EVEN SECTOR

We take $\mathcal{X}_I = \mathcal{X}_I^{(0)}(\sigma) e^{iq_A x^A}$

☐ First four equations

$$\begin{pmatrix}
-\frac{a_0}{2} & a_1 & 0 & c_1 \\
0 & 2c_1 - a_1 & \frac{1}{3}(a_0 - 4b_1) & -\frac{a_0}{2} - c_1 + \frac{2}{3}(a_0 + 2b_2) \\
2(w_1 + w_4)\mathcal{Q}^2 & a_0 + a_1 & \frac{2}{3}v_4\mathcal{Q}^2 & \frac{a_0}{2} + c_1 + \frac{1}{3}v_4\mathcal{Q}^2 \\
\mathcal{Q}^2v_4 & 0 & a_0 - 4b_1 + \frac{2}{3}\mathcal{Q}^2\Lambda_0 & \frac{1}{3}\mathcal{Q}^2\Lambda_0
\end{pmatrix}
\begin{pmatrix}
\mathcal{X}_0^{(0)} \\
\mathcal{X}_1^{(0)} \\
\mathcal{X}_2^{(0)} \\
\mathcal{X}_3^{(0)}
\end{pmatrix} = 0.$$
(5.47)

Abbreviations:

$$Q^2 := \frac{\ell_\rho^2}{4} q_A q_B \delta^{AB}, \qquad \Lambda_0 := 3z_1 + z_4 + 2v_4.$$
 (5.48)

- \Rightarrow This is a 4 × 4 matrix but its determinant is a 2nd degree polynomial in Q^2 .
- \Rightarrow Solutions for $Q^2 \rightsquigarrow 2$ propagating massive modes.

[Blagojević, Cvetković, Obukhov 2017]

I Last equation. It constrains the σ dependence:

$$v_4 \,\partial_\sigma \mathcal{X}_0^{(0)} + \frac{2}{3} (\Lambda_0 - 6z_1) \,\partial_\sigma \mathcal{X}_2^{(0)} + \frac{1}{3} \Lambda_0 \,\partial_\sigma \mathcal{X}_3^{(0)} = 0. \tag{5.49}$$

Exploring special cases: Vassiliev pseudo-instantons

In [Vassiliev 2002]:

- \blacksquare $L \sim RR$ (v's are omitted).
- \square Def. of pseudo-instantons: $Q_{ab} = 0$ irreducible curvature solving vacuum EoM.

So:

Vanishing nonmetricity for our Ansatz means that

$$\begin{array}{cccc}
\mathcal{U} = \overline{\mathcal{U}} &= 0, \\
\underline{\mathcal{W}} &= -\mathcal{V}, & \Leftrightarrow \\
\overline{\mathcal{W}} &= -\overline{\mathcal{V}}.
\end{array}
\Leftrightarrow
\begin{cases}
\mathcal{X}_0 &= 2\mathcal{W}, \\
\mathcal{X}_1 &= \overline{\mathcal{U}} - \mathcal{W}, \\
\mathcal{X}_2 = \mathcal{X}_3 &= 0,
\end{cases}
\overline{\mathcal{X}}_1 &= -\overline{\mathcal{W}}, \\
\mathcal{X}_2 = \overline{\mathcal{X}}_3 &= 0.
\end{cases}$$
(5.50)

Eqs. for the purely quadratic model (only w_I , z_J , v_K are nonvanishing):

$$\Delta \mathcal{W} = 0, \quad \Delta \overline{\mathcal{W}} = 0 \tag{5.51}$$

This automatically imply

$$\mathbf{R}^{ab} = {}^{(1)}\mathbf{W}^{ab}, \tag{5.52}$$

which is irreducible.

6. Summary and conclusions

Summary and conclusions

Results

- Quadratic (even) metric-affine action in vacuum
- ☐ Ansatz with 7 independent functions
- Particular solutions explored: Riemannian, teleparallel...
- ☐ Method to find the general solutions → potential + co-potential decomp.
- ☐ Solutions for large families of MAG theories.

Limitations of this work / future work

- Solutions with matter
- Non-trivial cosmological constant
- Odd parity invariants:

$$\mathbf{R}^{ab} \wedge \mathbf{R}_{ab}, \qquad {}^{(I)}\mathbf{T}^a \wedge {}^{(J)}\mathbf{T}_a \qquad \dots$$

lacktriangledown Different Ansatz (Kundt metric, other non-trivial irreds for $oldsymbol{T}^a$ and $oldsymbol{Q}_{ab}$).

Thanks for your attention!

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