Seminar

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Modified gravity theories
Metric-Affine framework and instabilities

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Structure of this presentation

1. Introduction to modified gravity

What to modify?

General Relativity (GR) is...

Geometrical point of view

- Einstein equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R}_{\text{geometry}} = \kappa \underbrace{\mathcal{T}_{\mu\nu}}_{\text{matter}} , \qquad \kappa := 8\pi G_{\mathrm{N}} , \tag{1.1}$$

  where
  $\mathcal{T}_{\mu\nu}$ is the energy-momentum tensor of the matter
  $R_{\mu\nu}$ and R are the Ricci tensor and Ricci scalar of the metric $g_{\mu\nu}$

- Lagrangian formulation: matter action S_{matt} plus the Einstein-Hilbert action

$$S_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{|g|}\, R(g, \partial g, \partial^2 g) \tag{1.2}$$

  $\Rightarrow$ 1 fundamental tensor: the metric $g_{\mu\nu}$

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Particle physics point of view

- The fundamental "thing" is the graviton, a massless spin-2 particle.
- This can be described by a symmetric field $h_{\mu\nu}$, via the (Massless) Fierz-Pauli Lagrangian

$$\mathcal{L}_{\mathrm{FP}} = \tfrac{1}{2}\partial_\mu h_{\nu\rho} \partial^\mu h^{\nu\rho} - \partial_\rho h^{\rho\mu} \partial_\sigma h^\sigma{}_\mu + \partial_\sigma h^\sigma{}_\mu \partial^\mu h^\lambda{}_\lambda - \tfrac{1}{2}\partial_\mu h^\sigma{}_\sigma \partial^\mu h^\lambda{}_\lambda , \tag{1.3}$$

  (this admits the Einstein Lagrangian as non-linear completion).

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But why?

Dark matter problem

- CMB, galaxy rotation curves, Bullet cluster, gravitational lenses...
- Density parameter:

$$\Omega_{\mathrm{DM}} = 0{,}265(7) \tag{1.4}$$

[Planck C. et al 2020]

Dark energy problem

- Expansion of the universe is accelerating (!!)
- Barotropic fluid behavior ($p = w\rho$) with

$$w = -1{,}03 \pm 0{,}03, \tag{1.5}$$

[Planck C. et al 2020]

Compatible with a cosmological constant ($w = -1$).

Hubble tension

Disagreement between:

- Value from Cosmic Microwave Background (CMB) and ΛCDM

$$\mathrm{H}_0 = (67{,}4 \pm 0{,}5)\ \mathrm{km/s/Mpc} \tag{1.6}$$

[Planck C. et al 2020]

- Value from late-time cosmological observations

$$\mathrm{H}_0 = (73{,}52 \pm 1{,}62)\ \mathrm{km/s/Mpc}. \tag{1.7}$$

[Riess et al 2018]

Singularities

- Black holes, cosmology...
- Solutions without singularities (?)

The cosmological constant problem

- Quantum corrections are many orders of magnitude greater than Λ

$$\Lambda = \Lambda_0 + \underbrace{(\text{quantum corrections})}_{\propto(\mathrm{M_P})^4 \sim 10^{76}\,\mathrm{GeV}^4} \,. \tag{1.8}$$

Naturalness problem (!) (is it really a problem?...)

Renormalizability and unitarity

- GR is non renormalizable. Fixed by adding quadratic gravitational terms:

$$R^2, \qquad R_{\mu\nu}R^{\mu\nu} \tag{1.9}$$

But... it violates unitarity (because a ghost propagates).

2. The landscape of modified gravity theories

Massive graviton

- Problems of the original formulations: vDVZ discontinuity, Boulwer-Deser ghost
- Solution: dRGT massive gravity (non-linear completion) [de Rham, Gabadadze, Tolley 2011]
- Very constrained by GW observations.

Violation of Lorentz symmetry

Examples:

- Einstein-Aether [Gasperini 1987]
- Hořava-Lifshitz gravity [Hořava 2009]

More that 4 dimensions

Examples:

- Randall-Sundrum model [Randall, Sundrum 1999]
- DGP model [Dvali, Gabadadze, Porrati 2000]

+ extra scalars

- Starting point: Brans-Dicke [Brans, Dicke 1961]

$$S_{\mathrm{BD}}[g_{\mu\nu},\phi] = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{|g|}\left(\phi R - \frac{\omega_{\mathrm{BD}}}{\phi}\partial_\mu\phi\partial^\mu\phi\right) + \text{matter}\,. \tag{2.1}$$

  $\rightsquigarrow$ point-dependent gravitational coupling
- More general $\rightsquigarrow$ Horndeski gravity ($\mathcal{L}(\phi,\partial\phi,\partial^2\phi)$ with 2nd order EoM) [Horndeski 1974]
- More general $\rightsquigarrow$ Beyond Horndeski gravity

+ extra vectors

- U(1) gauge vector $\to$ no cosmology
  $\Rightarrow$ Solution 1: non-abelian groups
  $\Rightarrow$ Solution 2: massive vector $\to$ Generalized Proca theory [Heisenberg 2014]
- More general: multi-Proca gravity...

+ extra metrics

- Example: bigravity $\{g_{\mu\nu}, f_{\mu\nu}\}$ [Hassan, Rosen 2012]
- More general: multi-gravity...

+ several fields of different kinds

Later $\to$ metric-affine framework (which contains different tensors of these and other types)

$f(R)$ gravity [De Felice, Tsujikawa 2010] [Sotiriou, Faraoni 2010]

$$S_{f(R)}[g_{\mu\nu}] = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{|g|}f(R) + \text{matter}. \tag{2.2}$$

Under an appropriate field redefinition:

$$\bar{S}_{f(R)}[g_{\mu\nu},\phi] = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{|g|}\Big(\phi R - V(\phi)\Big) + \text{matter}\,. \tag{2.3}$$

(Brans-Dicke with $\omega_{\text{BD}} = 0$ and a potential).

Unimodular gravity [Alvarez, Herrero-Valea 2013]

- GR under the constraint $\sqrt{|g|} = 1$.
- Cosmological constant is an integration constant and solves its naturalness problem.

Promote the metric to a Finsler metric [Lämmerzahl, Perlick 2018]

Intrinsic anisotropies in the microscopic behavior of the metric.

Promote the Christoffel symbols to a general connection

Let us see this in more detail...

3. A particular framework: Metric-Affine gravity

Geometric gravity (Einstein 1915) $\rightsquigarrow$ The spacetime is modelled as a *differentiable manifold* $\mathcal{M}$.

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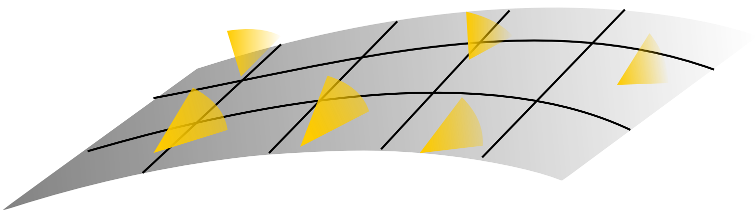
Geometric structures

- *Metric tensor:* $g_{\mu\nu}$
  - Measuring (length, volume...)

$$s[\gamma](\sigma) = \int_0^\sigma \sqrt{|g_{\mu\nu}(\sigma')\dot{x}^\mu(\sigma')\dot{x}^\nu(\sigma')|}\,\mathrm{d}\sigma'\,. \tag{3.1}$$

$$\mathrm{vol}(\mathcal{U}) = \int_{\mathcal{U}} \boldsymbol{\omega}_{\mathrm{vol}}\,, \qquad \boldsymbol{\omega}_{\mathrm{vol}} := \sqrt{|g|}\,\mathrm{d}x^1\wedge\ldots\wedge\mathrm{d}x^D \qquad D := \dim(\mathcal{U}). \tag{3.2}$$

  - Module of a vector (not necessarily non-negative) $\Rightarrow$ light cones $\Rightarrow$ causality.



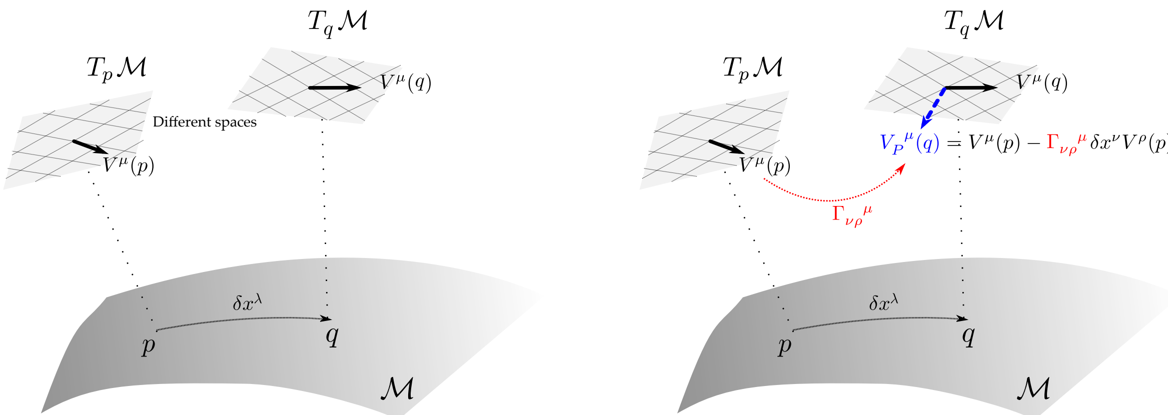
  - Notion of scale (conformal transformations...)

$$g_{\mu\nu} \to \mathrm{e}^{2\Omega} g_{\mu\nu}\,. \tag{3.3}$$

Geometric gravity (Einstein 1915) $\rightsquigarrow$ The spacetime is modelled as a *differentiable manifold* $\mathcal{M}$.

Geometric structures

- *Connection:* $\Gamma_{\mu\nu}{}^{\rho}$
  - Notion of parallel in $\mathcal{M}$ $\Rightarrow$ Covariant derivative ∇_μ



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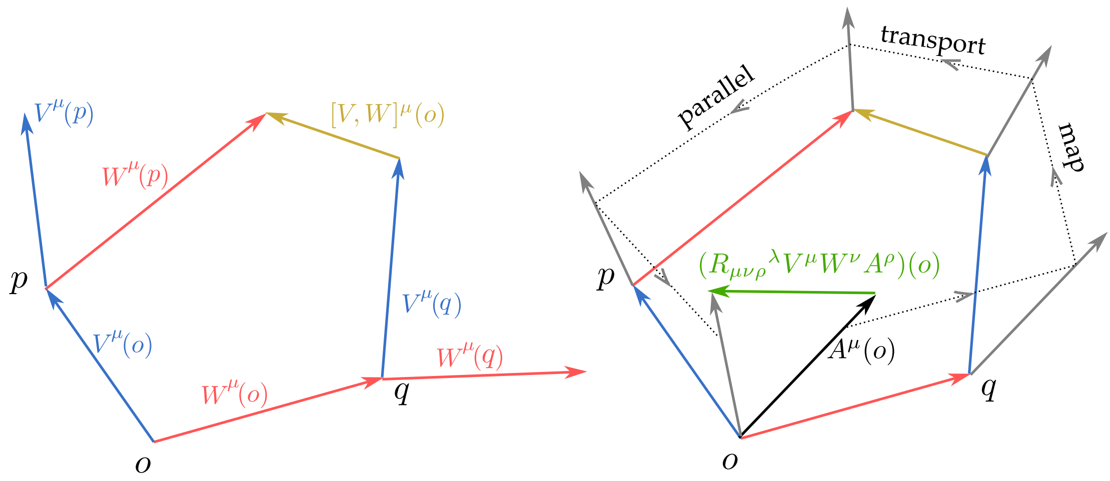
Geometric structures

- *Connection:* $\Gamma_{\mu\nu}{}^{\rho}$
  - Geometrical objects:

Curvature: $$R_{\mu\nu\lambda}{}^{\rho} := \partial_\mu \Gamma_{\nu\lambda}{}^{\rho} - \partial_\nu \Gamma_{\mu\lambda}{}^{\rho} + \Gamma_{\mu\sigma}{}^{\rho}\Gamma_{\nu\lambda}{}^{\sigma} - \Gamma_{\nu\sigma}{}^{\rho}\Gamma_{\mu\lambda}{}^{\sigma}\,, \tag{3.4}$$

Torsion: $$T_{\mu\nu}{}^{\rho} := \Gamma_{\mu\nu}{}^{\rho} - \Gamma_{\nu\mu}{}^{\rho}\,. \tag{3.5}$$



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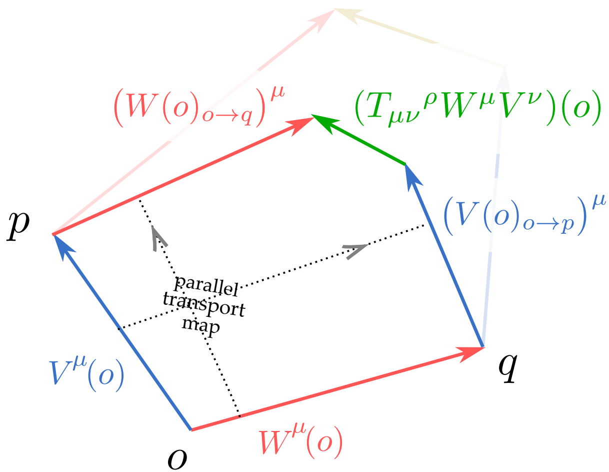
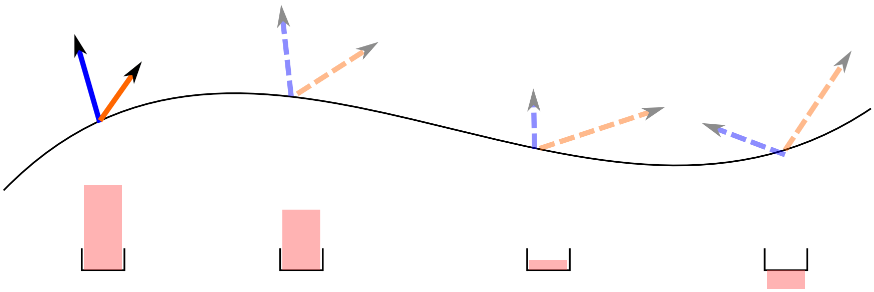


Def.: In the presence of metric and connection we define the *nonmetricity tensor*:

$$Q_{\mu\nu\rho} := -\nabla_\mu g_{\nu\rho} \,. \tag{3.6}$$



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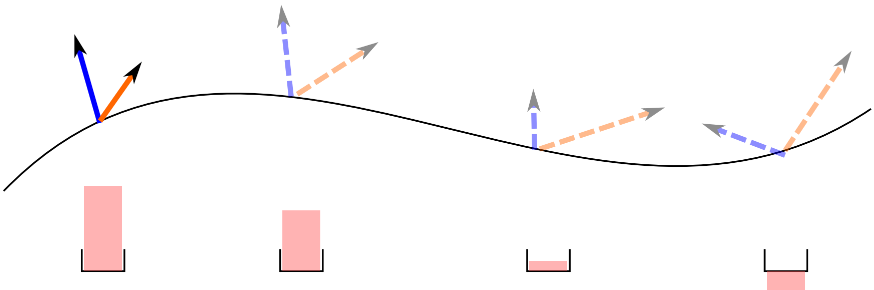

Theorem. Given $g_{\mu\nu}$, there is only one connection that satisfies

$$T_{\mu\nu}{}^{\rho} = 0 \quad \text{(torsionless condition)}, \tag{3.7}$$

$$Q_{\mu\nu\rho} = 0 \quad \text{(compatibility condition)}, \tag{3.8}$$

the *Levi-Civita connection:*

$$\mathring{\Gamma}_{\mu\nu}{}^{\rho} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right) . \tag{3.9}$$

Metric-affine theories
Instead of choosing $\mathring{\Gamma}$, they consider the metric and the (general) connection as independent fields.

Metric-Affine Gauge (MAG) gravity

- Gauge theory of Affine group (translations + linear transformations) gives two non-linear connections

$$e_\mu{}^a \quad \text{(translations)} \qquad\qquad \omega_{\mu a}{}^b \quad \text{(linear transformations)} \tag{3.10}$$

- (Resp.) Torsion and curvature are the field strengths.
- This description is equivalent to

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b g_{ab}\,, \qquad\qquad g_{ab} = \eta_{ab} \text{ (fixing the gauge)} \tag{3.11}$$

$$\Gamma_{\mu\nu}{}^\rho = e_\nu{}^a e^\rho{}_b \omega_{\mu a}{}^b + e^\rho{}_c \partial_\mu e_\nu{}^c\,, \tag{3.12}$$

Quadratic gravitational Lagrangian

$$S_{\text{qMAG}}[g_{\mu\nu}, \Gamma_{\mu\nu}{}^\rho] = \int \mathcal{L}_{\text{qMAG}} \sqrt{|g|}\mathrm{d}^D x, \tag{3.13}$$

where

$$\mathcal{L}_{\text{qMAG}} := \frac{1}{2\kappa}\Big(-2\kappa\Lambda + a_0 R \qquad \text{2 terms} \tag{3.14}$$

$$+ T_{\mu\nu}{}^\rho \mathcal{G}^{(\text{TT})\mu\nu}{}_\rho{}^{\alpha\beta}{}_\gamma T_{\alpha\beta}{}^\gamma \qquad \text{3 terms} \tag{3.15}$$

$$+ T_{\mu\nu}{}^\rho \mathcal{G}^{(\text{TQ})\mu\nu}{}_\rho{}^{\alpha\beta\gamma} Q_{\alpha\beta\gamma} \qquad \text{3 terms} \tag{3.16}$$

$$+ Q_{\mu\nu\rho} \mathcal{G}^{(\text{QQ})\mu\nu\rho\alpha\beta\gamma} Q_{\alpha\beta\gamma}\Big) \qquad \text{5 terms} \tag{3.17}$$

$$+ \frac{1}{2\varrho} R_{\mu\nu\rho}{}^\lambda \mathcal{G}^{(\text{RR})\mu\nu\rho}{}_\lambda{}^{\alpha\beta\gamma}{}_\delta R_{\alpha\beta\gamma}{}^\delta \qquad \text{16 terms} \tag{3.18}$$

- Complicated theory with many couplings and 10+64 a priori degrees of freedom !! (Allowing parity violation, there are even more terms in $D = 4$ prop. to $\epsilon_{\mu\nu\rho\lambda}$).

Teleparallel: constructed under the assumption of vanishing curvature, $R_{\mu\nu\rho}{}^{\lambda} = 0$.

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Trinity: equiv. theories (up to boundary terms)

- Teleparallel Equivalent of General Relativity $(R_{\mu\nu\rho}{}^{\lambda} = Q_{\mu\nu\rho} = 0)$

$$\mathcal{L}_{\text{TEGR}} := \frac{1}{2\kappa}\mathbb{T}, \qquad \mathbb{T} := \frac{1}{4}T_{\mu\nu\rho}T^{\mu\nu\rho} + \frac{1}{2}T_{\mu\nu\rho}T^{\mu\rho\nu} - T_{\mu\rho}{}^{\rho}T^{\mu\lambda}{}_{\lambda}, \tag{3.19}$$

- Symmetric Teleparallel Equivalent of General Relativity $(R_{\mu\nu\rho}{}^{\lambda} = T_{\mu\nu}{}^{\lambda} = 0)$

$$\mathcal{L}_{\text{STEGR}} := \frac{1}{2\kappa}\mathbb{Q}, \qquad \mathbb{Q} := \frac{1}{4}Q_{\mu\nu\rho}Q^{\mu\nu\rho} - \frac{1}{2}Q_{\mu\nu\rho}Q^{\nu\mu\rho} - \frac{1}{4}Q^{\rho}{}_{\rho\mu}Q_{\lambda}{}^{\lambda\mu} + \frac{1}{2}Q^{\rho}{}_{\rho\mu}Q^{\mu\lambda}{}_{\lambda}, \tag{3.20}$$

- Einstein-Hilbert Lagrangian $(T_{\mu\nu}{}^{\lambda} = Q_{\mu\nu\rho} = 0)$

$$\mathcal{L}_{\text{EH}} := \frac{1}{2\kappa}\mathring{R}. \tag{3.21}$$

Other teleparallel theories: $f(\mathbb{T})$, $f(\mathbb{Q})$, new GR, newer GR, General Teleparallel Equivalent,...

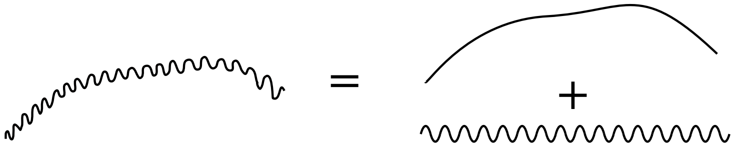
4. The dark side: instabilities

Background-dependent instabilities in field theory

Example: perturbation $\Phi = \Phi_0 + \phi$ around the background solution Φ_0,

$$\mathcal{L} = F_1(\Phi_0)\dot{\phi}^2 - F_2(\Phi_0)|\vec{\nabla}\phi|^2 - F_3(\Phi_0)\phi^2 \,. \tag{4.1}$$

Slow-varying background approx.



so

$$\mathcal{L} = \frac{1}{2a}\dot{\phi}^2 - \frac{1}{2}b|\vec{\nabla}\phi|^2 - \frac{1}{2}m^2\phi^2 \qquad \text{where} \quad b, m^2 \in \mathbb{R}, \quad a \in \mathbb{R}\setminus\{0\}\,. \tag{4.2}$$

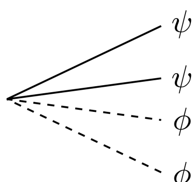
EoM in Fourier space

$$\boxed{\omega^2 = a(b|\vec{k}|^2 + m^2)}\,. \tag{4.3}$$

ω imaginary $\rightsquigarrow$ $\mathrm{e}^{\pm|\omega|t}$ allowed

- **Tachyons** (wrong sign of the mass term $m^2 < 0$)
- **Gradient instabilities** (wrong sign of the gradient term $b < 0$)
- **Ghosts** (wrong sign of the kinetic term $a < 0$)
  - Unbounded H.
  - Certain sectors become highly excited (no viol. of E conservation).
  - Quantum-m.: couplings (healthy)+(ghost) make the vacuum unstable.

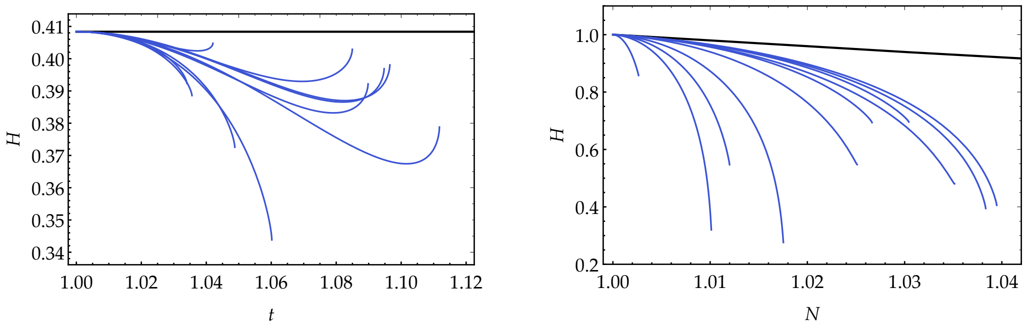
$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\psi\partial^\mu\psi - \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha\phi^2\psi^2 \qquad \rightsquigarrow$$



Strong coupling

- Discontinuity in the number of dynamical degrees of freedom at different orders.
- Singular surface in phase space.
- Perturbation expansion does not capture the full solution.

Particular case of (cosmological) Einsteinian Cubic gravity From [Beltrán, AJC 2021]
Numerical result around the solution dominated by Λ and by radiation, respectively.



Ostrogradski Theorem.
Let a Lagrangian involve n-th order finite time derivatives of variables. If $n \geq 2$ and the Lagrangian is non-degenerate with respect to the highest-order derivatives, the Hamiltonian of this system linearly depends on a canonical momentum.

$\rightsquigarrow$ Ghost instability

Theory Taken from [Joyce et al 2015]

$$\mathcal{L}_{\mathrm{A}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{2}(\Box\phi)^2, \qquad \Box := \eta^{\mu\nu}\partial_\mu\partial_\nu. \tag{4.4}$$

$\lambda \neq 0 \quad \Rightarrow \quad$ non-degenerate.

Explicit ghost
Consider

$$\mathcal{L}_{\mathrm{B}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \chi\Box\phi - \frac{1}{2\lambda}\chi^2\,. \tag{4.5}$$

On-shell $\left[\mathrm{EoM}(\chi)\ :\ \chi = \lambda\Box\phi\right]$ equivalent to $\mathcal{L}_{\mathrm{A}}$.

$$\mathcal{L}_{\mathrm{B}} \quad \xrightarrow[\phi\to\chi-\alpha]{\text{field redef.}} \quad \frac{1}{2}\partial_\mu\alpha\partial^\mu\alpha \overset{⚠}{-} \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2\lambda}\chi^2\,. \tag{4.6}$$

$\Rightarrow \quad \chi$ is the Ostrogradski ghost.

❒ Gauge symmetries usually introduce degeneracy and help to eliminate the ghosts.

Healthy theories

Theory	Field content
Lovelock (GR, GB,...)	Graviton
$f(\mathring{R})$, $f(\mathrm{GB})$	Graviton + scalar
$f(R)$	Graviton [+ non-dynamical scalar]
Horndeski gravity	Graviton + scalar
Generalized Proca	Graviton + (massive) vector
Ricci-Based Gravity	Graviton + scalar

Pathological theories

Theory	Some known pathologies
Massive gravity (original formulation)	Boulware-Deser ghost
Ricci-Based Gravity with $R_{[\mu\nu]}$	Ghost (projective mode)
Generic higher curvature gravity	Potential ghosts (massive spin-2, scalar)
$f(T)$	FLRW are strongly coupled
$f(Q)$	Max. Sym. are strongly coupled
qMAG with $Q_{\mu\nu\rho} = 0$ (Quadratic Poincaré gravity)	Ghosts and tachyons (and strong c.)

General quadratic MAG gravity $\rightarrow$ Same problems as in the Poincaré case (or even worse)

5. Previous projects

Metric-affine Lovelock is not a boundary term in critical dimension [Janssen, AJC 2019]

Example: Gauss-Bonnet in 4 dimensions

$$\sqrt{|g|}\delta^{\alpha}_{[\mu}\delta^{\beta}_{\nu}\delta^{\gamma}_{\rho}\delta^{\delta}_{\lambda]}R^{\mu\nu}{}_{\alpha\beta}R^{\rho\lambda}{}_{\gamma\delta} \qquad \neq \partial_{\mu}(...)^{\mu}\,. \tag{5.1}$$

Exact solutions in qMAG [AJC, Obukhov 2021]

We studied pp-wave solutions of S_{qMAG}

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + H(u,x,y,)\mathrm{d}u^2 - \mathrm{d}x^2 - \mathrm{d}y^2\,, \tag{5.2}$$

with non-trivial connection with both torsion and nonmetricity.

General quadratic teleparallel gravity [Beltrán et al 2020]

- Trinity as gauge fixed versions of a "general" equivalent.
- Check spectrum in flat space (+ gauge symmetries to avoid ghosts)

Strong coupling in Einsteinian Cubic Gravity and extensions [Beltrán, AJC 2021]

- Cosmological flat solutions are strongly coupled in (cosmological) ECG.
- At higher orders in the generalized quasi-topological theories $\rightarrow$ Ghosts (potentially) propagate in cosmologies.

Shown that 4DEGB is not viable [Arrechea, Delhom, AJC 2021]

Among others:

- Equations of motion not well-defined.
- The problem is absent at first order but not at higher orders in perturbation theory.

6. Ideas to remember

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- In GR there are several phenomenological and theoretical problems.
- One way of tackling them is by modifying GR (there are MANY ways to do it!).
- One particular framework in modified gravity is metric-affine gravity (metric+connection).
- The construction of modified gravity theories is plagued with instabilities (be aware of them!).

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Thanks for your attention!