

# Bootstrapping General Relativity and beyond

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- 1 Introduction
- 2 Bootstrapping GR and other metric theories
- 3 Extension to the metric-affine framework
- 4 Summary

## 1. Introduction

*Bootstrapping*: consistent way of generating non-linear interactions for a given linear theory.

## Bootstrapping GR

- [Rosen 1940 (I)] [Rosen 1940 (II)]: GR = self-interacting field theory in flat space
- [Gupta 1954]: GR from PF +  $\infty$  non-linear terms
- Feynman (Lec.): consistency requires self-coupling to its own  $T_{\mu\nu}$
- Huggins PhD Thesis: ambiguities in the process.
- [Deser 1970]: Deser's argument in the 1st order formalism.
- ...

## Other lines of research

- [Wald 1986]: order-by-order preservation of the Bianchi id
- ...

## Remarkable

- [Padmanabhan 2008]: GR just from Fierz-Pauli +  $T_{\mu\nu}$  self-coupling? GR unique?  
→ clarified in [Butcher et al. 2009]

## Beyond GR

- Higher order der. theories bootstrap in the same way [Butcher et al. 2009] [Ortin 2017]
- [Deser 2017]: 1st order formalism does not work unless one imposes  $\Gamma = \mathring{\Gamma}$

Goal: extend [Butcher et al. 2009] approach beyond standard metric theories

Consider a (globally) Poincaré invariant theory,  $\mathcal{L} = \mathcal{L}(\Phi^A, \partial_\mu \Phi^A)$ .

**Canonical currents:** defined via Noether theorem

$$\text{translations} \rightsquigarrow T_{\text{can}}{}^\mu{}_\nu := \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi^A} \partial_\nu \Phi^A - \mathcal{L} \delta^\mu{}_\nu, \quad (1.1)$$

$$\text{Lorentz} \rightsquigarrow J_{\text{can}}{}^{\mu\nu\lambda} := \underbrace{T_{\text{can}}{}^{\mu[\lambda} x^{\nu]}}_{\text{orbital}} + \underbrace{\frac{1}{2} \sum_A \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi^A} (\Lambda_{\Phi^A}^{\nu\lambda}) \Phi^A}_{\text{spin } S_{\text{can}}{}^{\mu\nu\lambda}}. \quad (1.2)$$

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**Example 1.** Massless free scalar field

$$S[\Phi] = -\frac{1}{2} \int d^D x \, \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi, \quad (1.3)$$

$$\Rightarrow T_{\text{can}\mu\nu} = -\partial_\mu \Phi \partial_\nu \Phi + \frac{1}{2} \eta_{\mu\nu} \partial_\rho \Phi \partial^\rho \Phi, \quad S_{\text{can}}{}^{\mu\nu\lambda} = 0. \quad (1.4)$$

**Example 2.** Massive Dirac spinor

$$S[\Psi] = \int d^4 x \left[ \frac{i}{2} (\bar{\Psi} \gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma^\mu \Psi) - m \bar{\Psi} \Psi \right] \quad (1.5)$$

$$\Rightarrow T_{\text{can}\mu\nu} = \frac{i}{2} (\bar{\Psi} \gamma_\mu \partial_\nu \Psi - \partial_\nu \bar{\Psi} \gamma_\mu \Psi) - \mathcal{L} \eta_{\mu\nu}, \quad S_{\text{can}}{}^{\mu\nu\lambda} = \frac{i}{4} \bar{\Psi} \gamma^{[\mu} \gamma^\nu \gamma^{\lambda]} \Psi. \quad (1.6)$$

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## Hilbert prescription

- ① Minimal coupling:  $\eta^{\mu\nu} \rightarrow g^{\mu\nu}$  and  $\partial_\mu \rightarrow \nabla_\mu$  (torsionful but metric-compatible).



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- ③ I define:

$$T_{\text{H}\mu\nu} := \frac{-2}{\sqrt{-g}} \frac{\delta S_{\text{M}}[g, K, \Phi]}{\delta g^{\mu\nu}} \Bigg|_{g=\eta, K=0}, \quad (1.9)$$

$$S_{\text{H}}{}^{\mu\nu\lambda} \eta_{\lambda\rho} := \frac{1}{\sqrt{-g}} \frac{\delta S_{\text{M}}[g, K, \Phi]}{\delta K_{\mu\nu}{}^\rho} \Bigg|_{g=\eta, K=0}. \quad (1.10)$$

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$$\text{Lorentz} \rightsquigarrow J_{\text{can}}{}^{\mu\nu\lambda} := \underbrace{T_{\text{can}}{}^{\mu[\lambda} x^{\nu]} }_{\text{orbital}} + \underbrace{\frac{1}{2} \sum_A \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi^A} (\Lambda_{\Phi^A}^{\nu\lambda}) \Phi^A}_{\text{spin } S_{\text{can}}{}^{\mu\nu\lambda}}. \quad (1.12)$$

## Hilbert prescription in the vielbein formulation

- ① Minimal coupling:  $\eta^{\mu\nu} \rightarrow \eta^{ab} e^\mu{}_a e^\nu{}_b$  and  $\partial_\mu \rightarrow \nabla_\mu$  (torsionful but metric-compatible), as well as  $\gamma^\mu \rightarrow \gamma^a e^\mu{}_a$ .
- ② The resulting action is a functional  $S_M[e, K, \Phi]$  (where  $K_{\mu a}{}^b := \omega_{\mu a}{}^b - \dot{\omega}_{\mu a}{}^b$ ).
- ③ I define:

$$T_{V\mu\nu} := e_\nu{}^c \eta_{ca} \frac{1}{|e|} \frac{\delta S_M[e, K, \Phi]}{\delta e^\mu{}_a} \Big|_{e=\delta, K=0}, \quad (1.13)$$

$$S_V{}^{\mu\nu\lambda} := e^\nu{}_a e^\lambda{}_b \frac{1}{|e|} \frac{\delta S_M[e, K, \Phi]}{\delta K_{\mu ab}} \Big|_{e=\delta, K=0}. \quad (1.14)$$

□ Both approaches coincide.

- There is a connection between

superpotentials (canonical pres.)  $\leftrightarrow$  minimal couplings (Hilbert's pres.)

**Example.** Massless free scalar field

We can add to the canonical e-m tensor:

$$\Delta T_{\mu\nu} = \alpha (\partial_\mu \partial_\nu \Phi - \eta_{\mu\nu} \partial^2 \Phi) \equiv \partial^\rho \chi_{\rho\mu\nu}, \quad (1.15)$$

where the associated superpotential is  $\chi_{\rho\mu\nu} = 2\alpha \partial_{[\rho} \Phi \eta_{\mu]\nu}$ .

This corresponds in Hilbert's approach with the non-minimal coupling:

$$S_{\text{nm}}[g, \Phi] = -\frac{\alpha}{2} \int d^D x \sqrt{-g} \Phi \mathring{R}(g). \quad (1.16)$$

(We are free to add them between the 1st and the 2nd steps of the prescription).

- Irrelevant for the physical charges.
- But an appropriate choice is crucial for the bootstrapping to succeed.

For an action that is quadratic on  $\Phi^A$ , we get the EoM:

$$\mathcal{D}_{AB}\Phi^B = 0, \quad (1.17)$$

**Step 1:** We identify a current  $j^A$  which we want to add as a source:

$$\mathcal{D}_{AB}\Phi^B = \lambda j_A, \quad (1.18)$$

Consistent variational principle requires adding (at the action level)

$$\Delta S \sim \lambda \int d^D x j_A \Phi^A. \quad (1.19)$$

**Step 2:**  $j_A = j_A(\Phi^B)$ , so the new current is  $j_A + \lambda \Delta j_A \dots$

$$\mathcal{D}_{AB}\Phi^B = \lambda(j_A + \lambda \Delta j_A), \quad (1.20)$$

**Step 3...**

### Examples

- Finite process in Yang-Mills theories.
- Infinite process for the Fierz-Pauli action.

## 2. Bootstrapping GR and other metric theories

Generic action  $S[Q^I]$  ( $\{Q^I(x)\}$  are spacetime fields).

We evaluate  $Q^I = \bar{Q}^I + \lambda q^I$ ,

$$S[Q] = \sum_{n=0}^{\infty} \lambda^n S^{(n)}[\bar{Q}, q], \quad (2.1)$$

where the partial actions  $S^{(n)}$  are given by

$$S^{(n)}[\bar{Q}, q] = \frac{1}{n!} \frac{d^n}{d\lambda^n} S[\bar{Q} + \lambda q] \Big|_{\lambda=0}. \quad (2.2)$$

### Bootstrapping recursive formula

$$\frac{\delta S^{(n)}[\bar{Q}, q]}{\delta q^I} = \frac{\delta S^{(n-1)}[\bar{Q}, q]}{\delta \bar{Q}^I}. \quad (2.3)$$

### Reconstruction formula

$$S^{(n)}[\bar{Q}, q] = \frac{2}{n!} \left[ \int d^D x \, q^I(x) \frac{\delta}{\delta \bar{Q}^I(x)} \right]^{n-2} S^{(2)}[\bar{Q}, q]. \quad (2.4)$$

We consider the reconstruction of the equations of a diff-invariant metric theory  $S[g]$ .

\* We do not know  $S^{(2)}[\bar{g}, h]$  for any arbitrary background metric  $\bar{g}^{\mu\nu}$

## Bootstrapping (à la [Butcher et al. 2009](#))

1. Starting point:  $S^{(2)}[\eta, h]$ .
2. Promote the metric to a general one.
3. Add the appropriate non-minimal couplings.
4. Apply the *recursive formula* to generate the source of the next order.

$$\frac{\lambda^n}{\sqrt{-\bar{g}}} \frac{\delta S^{(n)}[\bar{g}, h]}{\delta h^{\mu\nu}} = \lambda t_{\mu\nu}^{(n-1)}. \quad (2.5)$$

with

$$t_{\mu\nu}^{(n)} := -\frac{\lambda^n}{\sqrt{-\bar{g}}} \frac{\delta S^{(n)}[\bar{g}, h]}{\delta \bar{g}^{\mu\nu}}, \quad (2.6)$$

5. Use the *reconstruction formula* to obtain the next order action.
6. Go to step 4 (or 3) and iterate.



The Einstein-Hilbert action is given by

$$S[g] = \frac{1}{2\kappa_{(D)}} \int d^D x \sqrt{-g} \mathring{R}(g). \quad (2.7)$$

We expand  $g^{\mu\nu} = \bar{g}^{\mu\nu} + \lambda h^{\mu\nu}$ ,

$$S[\bar{g} + \lambda h] = \frac{1}{4\kappa_{(D)}} \int d^D x \sqrt{-\bar{g}} \sum_{n=2}^{\infty} \left[ M_{(n)}^{\alpha_1 \alpha_2}{}_{\mu_1 \nu_1 \dots \mu_n \nu_n} \bar{\nabla}_{\alpha_1} h^{\mu_1 \nu_1} \bar{\nabla}_{\alpha_2} h^{\mu_2 \nu_2} h^{\mu_3 \nu_3} \dots h^{\mu_n \nu_n} \right. \\ \left. + H_{(n)\mu_1 \nu_1 \dots \mu_n \nu_n} h^{\mu_1 \nu_1} \dots h^{\mu_n \nu_n} \right], \quad (2.8)$$

Only the term  $n = 2$  is required to reconstruct the whole dynamics.

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We expand  $g^{\mu\nu} = \bar{g}^{\mu\nu} + \lambda h^{\mu\nu}$ ,

$$S[\bar{g} + \lambda h] = \frac{1}{4\kappa_{(\text{D})}} \int d^{\text{D}}x \sqrt{-\bar{g}} \left[ M_{(2)}^{\alpha\beta}{}_{\mu\nu\rho\lambda} \bar{\nabla}_{\alpha} h^{\mu\nu} \bar{\nabla}_{\beta} h^{\rho\lambda} + H_{(2)\mu\nu\rho\lambda} h^{\mu\nu} h^{\rho\lambda} \right] + \dots \quad (2.10)$$

where

$$\begin{aligned} M_{(2)}^{\alpha\beta}{}_{\mu\nu\rho\lambda} &= -\frac{1}{2} \left[ \bar{g}^{\alpha\beta} \bar{g}_{\mu(\rho} \bar{g}_{\lambda)\nu} - \bar{g}^{\alpha\beta} \bar{g}_{\mu\nu} \bar{g}_{\rho\lambda} - 2\delta^{\alpha}{}_{(\rho} \bar{g}_{\lambda)(\mu} \delta^{\beta}{}_{\nu)} + \delta^{\alpha}{}_{(\rho} \delta^{\beta}{}_{\lambda)} \bar{g}_{\mu\nu} + \delta^{\beta}{}_{(\mu} \delta^{\alpha}{}_{\nu)} \bar{g}_{\rho\lambda} \right], \\ H_{(2)\mu\nu\rho\lambda} &= \frac{1}{2} \bar{R} \left( \bar{g}_{\mu\rho} \bar{g}_{\lambda\nu} + \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}_{\rho\lambda} \right) - \bar{R}_{\mu\nu} \bar{g}_{\rho\lambda} \end{aligned} \quad (2.11)$$

Both,

- the Fierz-Pauli term  $M_{(2)} \bar{\nabla} h \bar{\nabla} h$
- **and the non-minimal coupling**  $H_{(2)} h h$

are needed to reconstruct Einstein eqs.!

[Butcher et al. 2009]

In [Butcher et al. 2009] only quadratic matter actions were considered.

## Generalization

$$S[g, \Phi] = S_g[g] + S_M[g, \Phi], \quad S_M[g, \Phi] = \sum_{p=2}^N \mathcal{A}_M^{(p)}[g, \Phi], \quad (2.12)$$

where  $\mathcal{A}_M^{(p)} \propto \Phi^p$  (including derivatives).

Everything works the same way, but now:

$$t_{\mu\nu}^{(n)} = t_{g\mu\nu}^{(n)} + t_{M\mu\nu}^{(n)} \quad t_{M\mu\nu}^{(n)} := -\frac{\lambda^n}{\sqrt{-\bar{g}}} \frac{\delta S_M^{(n)}[\bar{g}, h, \bar{\Phi}, \phi]}{\delta \bar{g}^{\mu\nu}}. \quad (2.13)$$

**Particular case: small matter perturbations** ( $\Phi = 0 + \lambda\phi$ )

$$t_{M\mu\nu}^{(n)}|_{\bar{\Phi}=0} = -\frac{\lambda^n}{\sqrt{-\bar{g}}} \frac{\delta}{\delta \bar{g}^{\mu\nu}} \left[ \sum_{p=0}^n \frac{1}{(n-p)!} \left( \int d^D x \, h^{\mu\nu} \frac{\delta}{\delta \bar{g}^{\mu\nu}(x)} \right)^{n-p} \mathcal{A}_M^{(p)}[\bar{g} + \lambda h, \phi] \right]_{\lambda=0} \quad (2.14)$$

Every order of  $\lambda$  mixes terms  $\mathcal{A}^{(p)}$  and  $\mathcal{A}^{(q)}$  with  $p + q = n$ .

Alternative (ghostfree) Lagrangian for the massless spin-2 field in Minkowski spacetime:

$$\mathcal{L}_{\text{WTDiff}} \sim \frac{1}{2} \partial_\sigma h_{\mu\nu} \partial^\sigma h^{\mu\nu} - \partial_\rho h^{\rho\mu} \partial_\sigma h^\sigma{}_\mu + \frac{2}{D} \partial_\sigma h^{\sigma\mu} \partial_\mu h - \frac{D+2}{2D^2} \partial_\mu h \partial^\mu h. \quad (2.15)$$

WTDiff symmetry:

$$\delta_\xi h_{\mu\nu} = 2\partial_{(\mu} \xi_{\nu)} \quad \text{with} \quad \partial_\mu \xi^\mu = 0 \quad (2.16)$$

$$\delta_\phi h_{\mu\nu} = \phi \eta_{\mu\nu}, \quad (2.17)$$

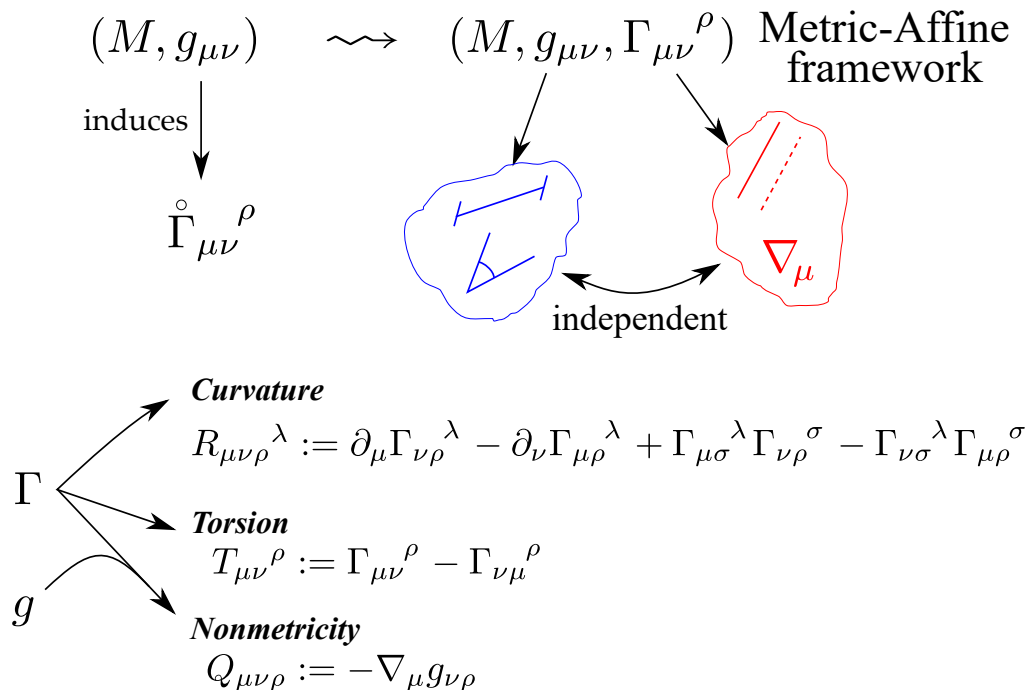
Coupling to the traceless energy-momentum tensor!

$$\frac{\delta S^{(n)}[\bar{g}, h]}{\delta h^{\mu\nu}} = -\sqrt{-\bar{g}} \left( t_{\mu\nu}^{(n-1)} - \frac{1}{D} \bar{g}_{\mu\nu} \bar{g}^{\alpha\beta} t_{\alpha\beta}^{(n-1)} \right), \quad (2.18)$$

$\Rightarrow$  we reconstruct the eqs. of Unimodular gravity

[Carballo et al. 2022]

### 3. Extension to the metric-affine framework



\* Everything works in the vielbein formulation.

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We add the contorsion  $K_{\mu ab} := \omega_{\mu ab} - \dot{\omega}_{\mu ab}$  (we assume  $\nabla g = 0$ ):

$$S[e, K, \Phi] = S_g[g(e)] + S_F[e, K, \Phi], \quad S_F[g, \Phi] = \sum_{p=2}^N \mathcal{A}_F^{(p)}[e, K, \Phi], \quad (3.1)$$

where  $\mathcal{A}_F^{(p)}$  is the sector  $\propto \Phi^p$  (including derivatives).

□ EoM of  $e$  reconstructed order by order by:

$$\mathfrak{t}_{\mu}^{(n) a} := - \frac{\tilde{\lambda}^n}{|\bar{e}|} \frac{\delta S^{(n)}[\bar{e}, \epsilon, \bar{K}, k, \bar{\Phi}, \phi]}{\delta \bar{e}^{\mu}_a} \quad (3.2)$$

□ EoM of  $K$  reconstructed order by order by:

$$\mathfrak{s}^{(n)\mu ab} = \mathfrak{s}_F^{(n)\mu ab} := \frac{\tilde{\lambda}^n}{|\bar{e}|} \frac{\delta S_F^{(n)}[\bar{e}, \epsilon, \bar{K}, k, \bar{\Phi}, \phi]}{\delta \bar{K}_{\mu ab}}. \quad (3.3)$$



$$S_{\text{Dirac}}[e, K, \Psi, \bar{\Psi}] = \int d^4x |e| \left[ \frac{i}{2} e^\mu{}_c \left( \bar{\Psi} \gamma^c \hat{\nabla}_\mu \Psi - \text{h.c.} \right) - m \bar{\Psi} \Psi \right] \quad (3.4)$$

$$= S_{\text{Dirac}}[e, 0, \Psi, \bar{\Psi}] + \int d^4x |e| \left[ -\frac{1}{4} K_{\mu ab} e^\mu{}_c (i \bar{\Psi} \gamma^{[a} \gamma^b \gamma^c] \Psi) \right], \quad (3.5)$$

- $K_{\mu ab}$  only enters linearly and without derivatives  
 $\Rightarrow$  the bootstrapping of  $K_{\mu ab}$  closes after 1st non-trivial iteration.

- For small matter perturbations  $\Psi = 0 + \tilde{\lambda} \psi$

$$\sum_{n=0}^{\infty} \mathfrak{s}^{(n)\mu ab} |_{\bar{\Psi}=0} = \mathfrak{s}^{(2)\mu ab} |_{\bar{\Psi}=0} = \frac{1}{|\bar{e}|} \frac{\delta S_{\text{Dirac}}[\bar{e}, \bar{K}, \psi, \bar{\psi}]}{\delta \bar{K}_{\mu ab}} = \frac{i}{4} \bar{e}^\mu{}_c \bar{\psi} \gamma^{[c} \gamma^a \gamma^b] \psi, \quad (3.6)$$

## 4. Summary

\* Matter fields are just spectators.

### **Bootstrapping process with metric** (à la [\[Butcher et al. 2009\]](#))

1. Starting point:  $S^{(2)}[\eta, h]$ .
2. Promote the metric to a general one.
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## New results / extensions

- ☐ Vielbein formulation.
- ☐ Arbitrary analytic matter.
- ☐ WTDiff: coupling to the traceless part of  $T_{\mu\nu}$  (see also [\[Carballo et al. 2022\]](#)).
- ☐ Torsionful case (a coupling to the partial spin density tensors appear order by order).
- ☐ Analogously, the nonmetricity couples to the corresponding currents (GL theories).

### □ Disappearing degrees of freedom in New General Relativity

$$S_{\text{NGR}}[e] = \int \frac{1}{2\kappa} (a_1 T_{\mu\nu\rho} T^{\mu\nu\rho} + a_2 T_{\mu\nu\rho} T^{\rho\nu\mu} + a_3 T^{\rho}_{\rho\mu} T^{\nu\mu}_{\nu}) \sqrt{|g|} \, d^4x. \quad (4.1)$$

GR equivalent for  $(a_1, a_2, a_3) = (-\frac{1}{4}, -\frac{1}{2}, 1)$ .

### □ Junction conditions in bi-scalar Poincaré

$$S_{\text{bSPG}}[g, S, T] = \int \left( \frac{M_P^2}{2} R + \beta R^2 + \alpha \mathcal{H}^2 + \frac{m_T^2}{2} T_\mu T^\mu + \frac{m_S^2}{2} S_\mu S^\mu \right) \sqrt{|g|} \, d^4x \quad (4.2)$$

### □ Spatially-isotropic/homogeneous metrics in GR

### □ pp-waves in MAG

### □ Hawking radiation with torsion

### □ ...

**Thanks for your attention**  
**and Merry Christmas! :)**