Symmetries and conservation laws

Exercise 1: Conservation of angular momentum

A spin $s = \frac{1}{2}$ particle decays into two particles with $s_1 = 0$ and $s_2 = \frac{3}{2}$.

i) Find the possible values of the orbital angular momentum of the two particles in the final state.

ii) Plot the angular distribution of the final state in the following two cases:

(a) The initial particle is polarized along the positive direction of the $\hat{z}$ axis, i.e., the initial spin is $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$. What happens if parity is conserved?

(b) The initial particle is not polarized, i.e., we have a statistical mixture with equal proportion of $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$ and $\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$.

Exercise 2: Parity violation in weak decays

A K^+ meson is a spin zero particle with the decay modes

$$K^+ \to \pi^+\pi^0 \ \ (21.0\%), \quad K^+ \to \pi^+\pi^+\pi^- \ \ (5.5\%).$$

Show that if the pion has $J_\pi^P = 0^-$, in these decays parity is not conserved.

Exercise 3: Isospin violation in weak decays

The $\Sigma^\pm$ baryons decay weakly through the non-leptonic channels

$$\Sigma^+ \to p + \pi^0, \quad \Sigma^+ \to n + \pi^+, \quad \Sigma^- \to n + \pi^-.$$

Suppose that the operator mediating these decays is the component of a tensor operator $T^{1/2}$ in isospin space and that the transition amplitude is $\mathcal{A}(i \to f) \propto \langle f|T_M^{1/2}|i\rangle$.

i) Find the relation among the amplitudes of these three processes.

ii) If the decay widths are proportional to the modulus squared of the corresponding amplitude, what will be the relation among the three widths?

The isospin $|t\ t_3\rangle$ of the different particles involved in these processes is

$$\begin{aligned}
&|p\rangle = \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle, && |n\rangle = \left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle, && \\
&|\pi^+\rangle = -\,|1\ 1\rangle, && |\pi^0\rangle = |1\ 0\rangle, && |\pi^-\rangle = |1\ -1\rangle \\
&|\Sigma^+\rangle = -\,|1\ 1\rangle, && |\Sigma^0\rangle = |1\ 0\rangle, && |\Sigma^-\rangle = |1\ -1\rangle.
\end{aligned}$$

Hint: Use the Wigner-Eckart theorem to write the matrix elements in terms of the reduced matrix elements $A_{2t} \equiv \langle t||T^{1/2}||1\rangle,\ t = 1/2,\ 3/2.$

Exercise 4: Time reversal and Kramers degeneracy

If the Hamiltonian H of a quantum system commutes with the time reversal operator Θ, the energy eigenstate $|n\rangle$ and $\Theta|n\rangle$ will have the same energy eigenvalue

i) Do $|n\rangle$ and $\Theta|n\rangle$ represent the same physical state?

ii) Show that for systems with half-integer spin the two states do not coincide, something that is known as the *Kramers degeneracy*.

iii) Show that the Kramers degeneracy disappears if we apply an external magnetic field $\vec{B}$.

Hint: $\vec{B}$, which does not change under a time reversal transformation, adds in the Hamiltonian a term proportional to $\vec{S}\cdot\vec{B}$.

Exercise 1 must be turned in [deadline: 27 November 2020 at 15:00]