Physics 7AW Homework 1 — Calculus Review & Vectors

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Physics 7AW - WAT 2020 edition

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Exercise 1. Given vectors $\boldsymbol{A} = 3\hat{\boldsymbol{i}} + 5\hat{\boldsymbol{j}} - 7\hat{\boldsymbol{k}}$ and $\boldsymbol{B} = 2\hat{\boldsymbol{i}} + 7\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}$ Find:
a) $\boldsymbol{B} + \mathbf{A}$
b) $\boldsymbol{A} - \boldsymbol{B}$
c) $|\boldsymbol{A}|$
d) $|\boldsymbol{B}|$
e) Angle θ *between* $\boldsymbol{A}$ *and* $\boldsymbol{B}$

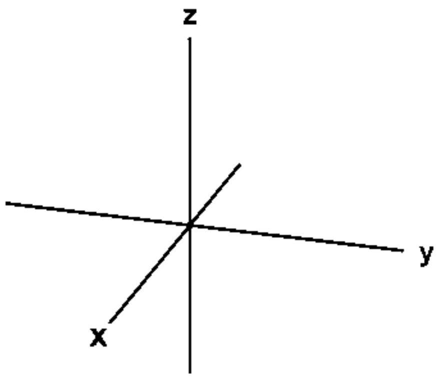
Exercise 2. Find a unit vector perpendicular to $\boldsymbol{A} = \hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}$ and
$\boldsymbol{B} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - 3\hat{\boldsymbol{k}}$
Answer $\hat{\boldsymbol{C}} = \pm\frac{1}{\sqrt{6}}(-2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}})$
Show your work!

Exercise 3. Compute $\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ 4 & 15 & 6 \\ 12 & 45 & 18 \end{vmatrix}$

Is there a quick way you could have solved this?

Exercise 4. Show that for an N-dimensional vector $\boldsymbol{A} = \mathrm{a}_0\hat{e_0} + a_1\hat{e_1} + ... + a_N\hat{e_N}$
$|\mathbf{A}| = \sqrt{\sum a_i^2}$

Exercise 5. Consider that standard $\mathbb{R}^3$ space.



Let $\hat{e_1}$ be a unit vector pointing in the x-axis, $\hat{e_2}$ unit vector in y-axis, and $\hat{e_3}$ unit vector in z-axis.
Find the following:
i) $\hat{e_1} \times \hat{e_2}$
ii) $\hat{e_1} \times \hat{e_3}$
iii) $\hat{e_2} \times \hat{e_1}$
iv) $\hat{e_3} \times \hat{e_1}$
v) $\hat{e_1} \times \hat{e_2} \times \hat{e_3}$

Exercise 6. Show that $B \times A = -A \times B$ in $\mathbb{R}^3$

Exercise 7. Solve the following:

i) $\frac{d}{dt}cos(\omega t + x\psi)$

ii) $\frac{d}{dx}sin(\omega t + x\psi)$

iii) $\frac{d}{dt}e^{i\theta t}cos(\omega t + \phi)$

iv) $\frac{d}{dt}(Acos(\omega t)e^{-i\Omega t} + B(t)ln(t + x))$

v) $\frac{d}{d\theta}cot(\theta)$

vi) $\int \frac{1}{at+b}dt$

Exercise 8. Show that $u(x, t) = e^{i(kx-\omega t)}$ satisfies the following equation:

$$\frac{d^2}{dt^2}u(x, t) = c^2\frac{d^2}{dx^2}u(x, t) \tag{1}$$

where $c = \sqrt{\frac{\omega}{k}}$. This is known as the *wave equation*