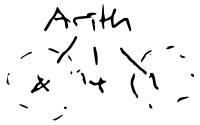
free vars

$x + 1$ 

$\{x\} \cup \emptyset = \{x\}$.

$\lambda x.\, x + y$ $\quad FV(x+y) \setminus \{x\} = \{y\}$

$\{x, y\}$

$\lambda x.\, \lambda y.\, x + y$ $\quad FV(\lambda y.\, x+y) \setminus \{x\} = \emptyset$

$\{x\}$

$(\lambda x.\, x + x)((\lambda y.\, x)\ 5)$ $\quad FV(\cdot) = \emptyset \cup \{x\} = \{x\}$.

$\emptyset$ $\qquad$ x $\qquad$ $\emptyset$

If $(\lambda x.\, x > y)\ 1$ then $2 + z$ else $x + z$

$\{y\} \quad \cup \quad \{z\} \quad \cup \quad \{x, z\} = \{x, y, z\}$

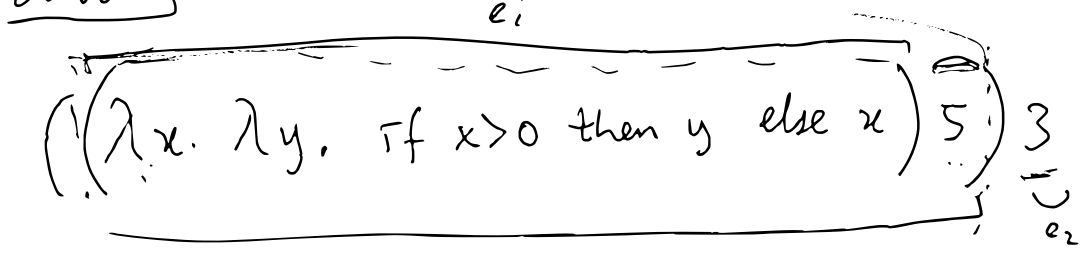
$FV(\text{let } x = 5 \text{ in } x + x) = \emptyset$

$\lambda y.\, x$

let $x = e_1$ in $e_2 \cong (\lambda x.\, e_2)\ e_1$

$FV(e_1) \cup (FV(e_2) \setminus \{x\}) = (FV(e_2) \setminus \{x\}) \cup FV(e_1)$

Subst



$$\frac{e_1 \Downarrow \lambda^{y} z.\, E \qquad e_2 \Downarrow v \qquad [z \to v]E \Downarrow v'}{e_1\ e_2 \Downarrow v'}\ \text{App}$$

$e_1 \Downarrow \lambda y.\text{if}\cdots$

$e_2 \Downarrow 3$

$e_1' \Downarrow \lambda x.\cdots$

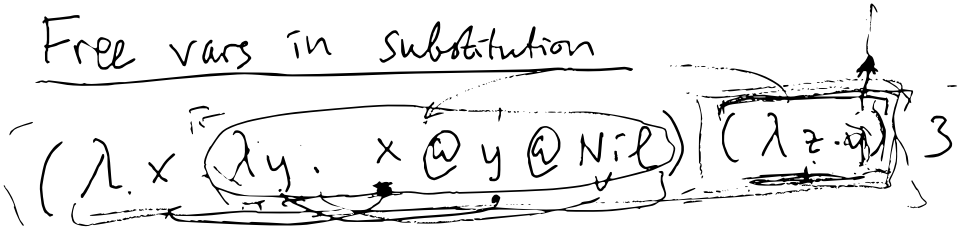
$e_2' \Downarrow 5$

$\Big\} \Rightarrow [x \to 5]\cdots \Downarrow$

$[x \to 5](\lambda y.\ \text{if } x>0 \text{ then } y \text{ else } x) = (\lambda y.\ \text{if } 5>0 \text{ then } y \text{ else } 5)$

$[y \to 3](\text{if } 5>0 \text{ then } y \text{ else } 5) = \text{if } 5>0 \text{ then } 3 \text{ else } 5 \Downarrow 3$

Free vars in Substitution



$\lambda y.3$

$f(x) = 3 + x$

$f(y) = 3 + y$

$(\lambda y.(\lambda z.y) @ y @ Nil)\ 3$

$\lambda z.3 @ 3 @ Nil$

} incorrect

what we want

$\lambda z.y @ 3 @ Nil$

$[x \to (\lambda z.y)]\ (\lambda y.\ x @ y @ Nil)$

FV = {y} ✓

$=_\alpha [x \to (\lambda z.y)]\ (\lambda y_0.\ x @ y_0 @ Nil)$

$= \lambda y_0\ (\lambda z.y) @ y_0 @ Nil.$

$(\lambda y_0 (\lambda z\ y) @ y_0 @ Nil)\ 3 \Downarrow (\lambda z\ y) @ 3 @ Nil$

$(\lambda y(\lambda y\ y+y))\ 5$

$(\lambda z.y)\ 5 \to y\ x$

$(\lambda x.\ \lambda y.\ x)\ (\lambda z\ y)\ 3\ 5$

$\to \lambda y.(\lambda z.y)\ 3\ 5$

$\to (\lambda z\ 3)\ 5 \to 3$

Subst 3 (factorial)

let $f = fix(\lambda f.\ \lambda x.\ \text{if } x<1 \text{ then } 0 \text{ else } x*(f\ (x-1)))$

in $f\ 3$ ⇓ E

$$\frac{(fix\ E) \Downarrow v \qquad [f \to v](f\ 3) \Downarrow v'}{\text{let } f = (fix\ E) \text{ in } f\ 3 \Downarrow v'}\ \text{LET}$$