CS 162 Programming Languages

Lecture 4: OCaml Crash Course III

Yu Feng

Winter 2022

Outline for today

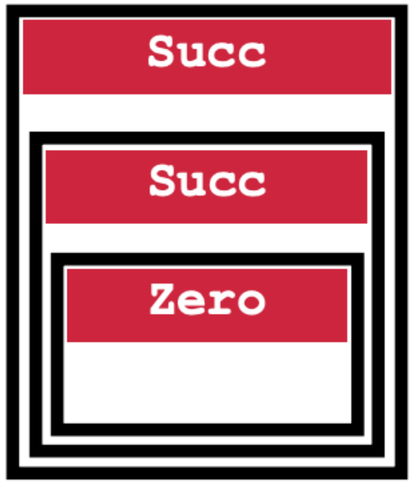
- Last lecture on OCaml
- Date types
- Higher-order functions

Recursive types

```
type nat = Zero | Succ of nat
```

What are values of nat ?
One nat contains another!

nat = recursive type



plus: nat*nat -> nat

Base pattern

Inductive pattern

```
type nat =
| Zero
| Succ of nat
```

Base pattern

Inductive pattern

```
let rec plus n m =
match m with
| Zero -> n
| Succ m’ -> Succ (plus n m’)
```

Base expression

Inductive expression

List datatype

```
type int_list =
  Nil
| Cons of int * int_list
```

Lists are a derived type: built using elegant core!

1. Each-of
2. One-of
3. Recursive

:: is just a syntactic sugar for "Cons"
[] is a syntactic sugar for "Nil"

List function: length

```
let rec len l =
    match l with
  | Nil -> 0
  | Cons(h,t) -> 1 + (len t)
```

Base pattern

Base expression

Inductive pattern

Inductive expression

List function: list_max

```
let rec list_max l =
    match l with
  | Nil -> 0
  | Cons(h,t) -> max h (list_max t)
```

Base pattern

Base expression

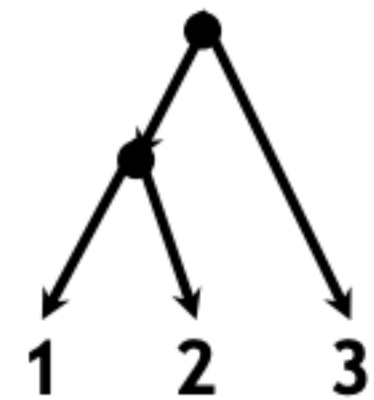
Inductive pattern

Inductive expression

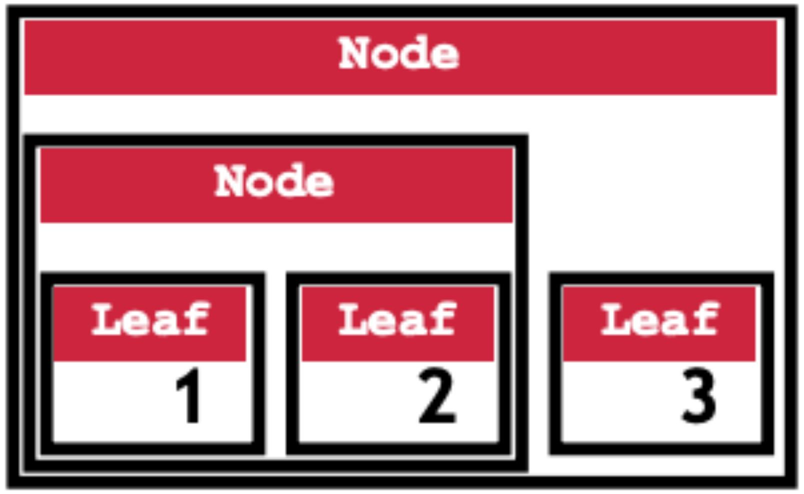
```
let max x y = if x > y then x else y;;
```

Representing Trees

```
type tree =
  Leaf of int
| Node of tree*tree
```



Node(Node(Leaf 1, Leaf 2), Leaf 3)



sum_leaf: tree -> int

```
type tree =
  Leaf of int
| Node of tree*tree
```

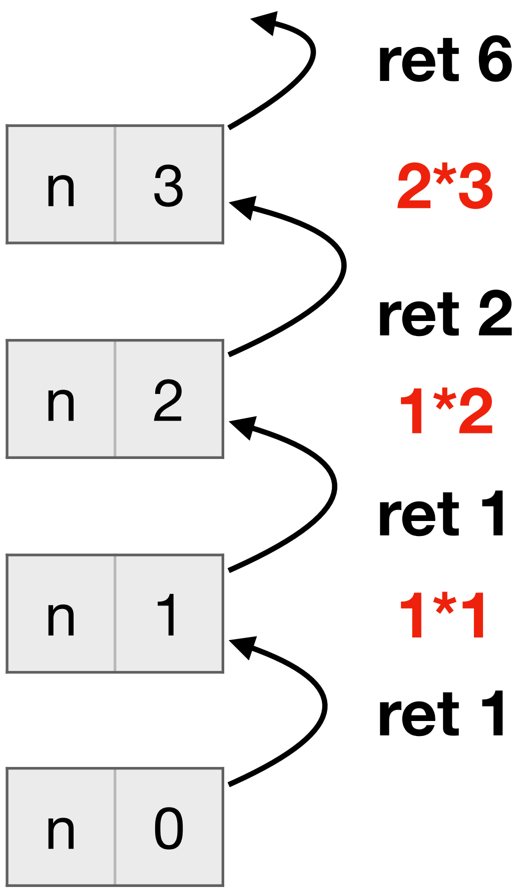
```
let rec sum_leaf t =
    match t with
    | Leaf n -> n
    | Node(t1,t2) -> (sum_leaf t1)
                    +(sum_leaf t2)
```

Factorial: int –> int

```
let rec fact n =
    if n<=0
    then 1
    else n * fact (n-1);;

fact 3;;
```

How does it execute?



Tail recursion

Tail recursion

- Recursion where all recursive calls are immediately followed by a return

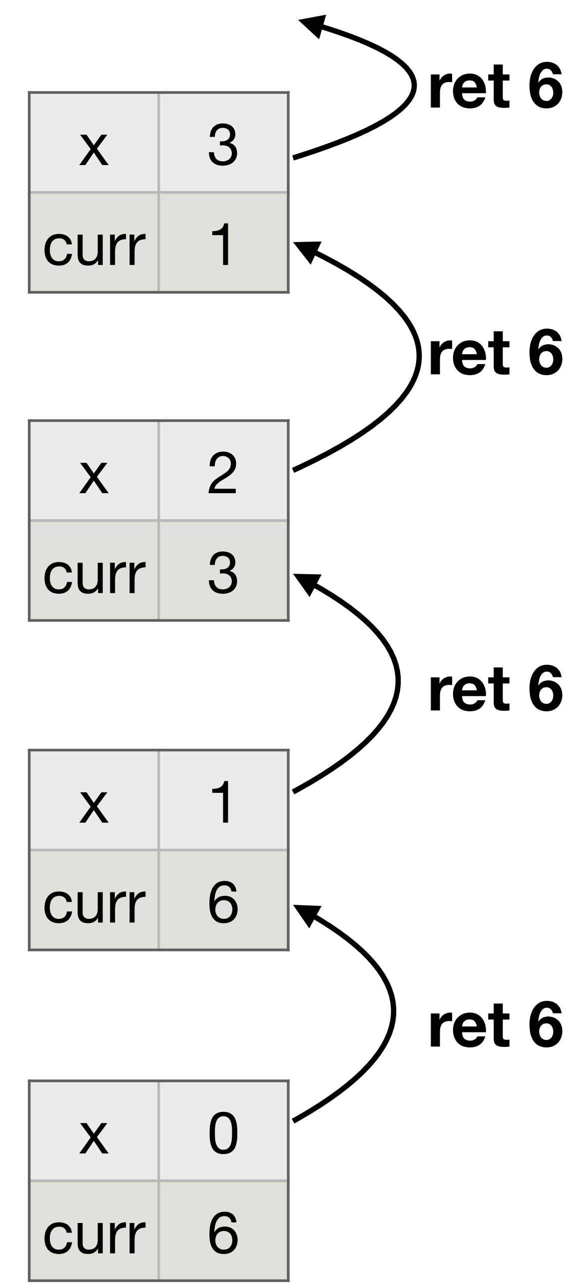
- In other words: not allowed to do anything between recursive call and return

Tail recursive Factorial

```
let fact x =
  let rec helper x curr =
     if x <= 0
     then curr
     else helper (x - 1) (x * curr)
  in
     helper x 1;;

fact 3;;
```

How does it execute?



Tail recursion

Tail recursion

- Recursion where all recursive calls are immediately followed by a return
- In other words: not allowed to do anything between recursive call and return

Why do we care about tail recursion?

- Tail recursion can be optimized into a simple loop

Compiler optimization

```
let fact x =
  let rec helper x curr =
     if x <= 0
     then curr
     else helper (x - 1) (x * curr)
  in
     helper x 1;;
```

```
fact(x) {
  curr := 1;
  while (1) {
  if (x <= 0)
  then { return curr }
  else { x := x - 1;
           curr := (x * curr) }}
}
```

Recursion

Loop

`max` function

```
let max x y = if x < y then y else x;;

(* return max element of list l *)
let list_max l =
   let rec l_max l =
      match l with
        [] -> 0
       | h::t -> max h (l_max t)
   in
      l_max l;;
```

A better `max` function

```
let max x y = if x < y then y else x;;

(* return max element of list l *)
let list_max2 l =
   let rec helper cur l =
      match l with
        [] -> cur
       | h::t -> helper (max cur h) t
   in
      helper 0 l;;
```

Tail recursion

`concat` function

```
(* concatenate all strings in a list *)
let concat l =
   let rec helper cur l =
      match l with
        [] -> cur
       | h::t -> helper (cur ^ h) t
   in
      helper "" l;;
```

What is the pattern?

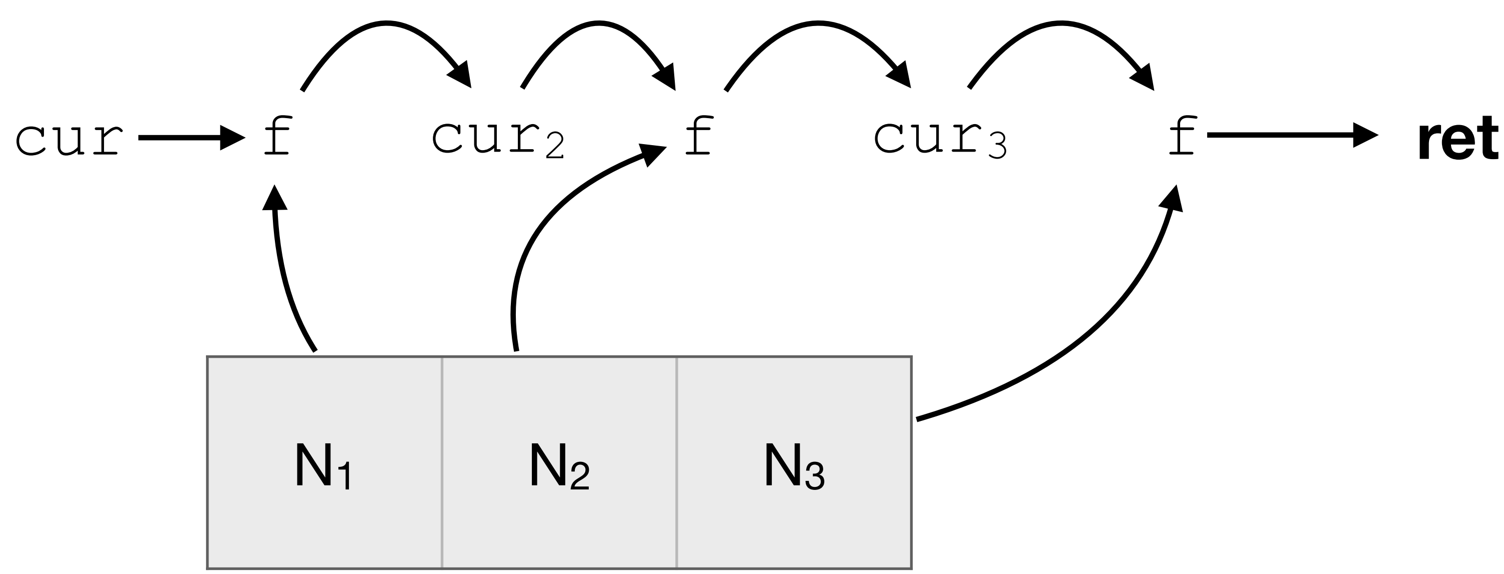
```
(* return max element of list l *)
let list_max2 l =
   let rec helper cur l =
      match l with
        [] -> cur
       | h::t -> helper (max cur h) t
   in
      helper 0 l;;
```

The two functions are sharing the same template!

```
(* concatenate all strings in a list *)
let concat l =
   let rec helper cur l =
      match l with
        [] -> cur
       | h::t -> helper (cur ^ h) t
   in
      helper "" l;;
```

fold

```
(* fold, the coolest function! *)
let rec fold f cur l =
   match l with
      [] -> cur
      | h::t -> fold f (f cur h) t;;
```



fold: examples

```
let list_max = fold max 0 l;;
```

```
let concat = fold (^) “" l;;
```

map

```
# (* return the list containing f(e)
     for each element e of l *)
let rec map f l =
    match l with
    [] -> []
    | h::t -> (f h)::(map f t);;
```

```
let incr x = x+1;;

let map_incr = map incr;;

map_incr [1;2;3];;
```

Composing functions

$$(f \circ g)(x) = f(g(x))$$

```
# (* return a function that given an argument x
applies f2 to x and then applies f1 to the result *)
let compose f1 f2 = fun x -> (f1 (f2 x));;

(* another way of writing it *)
let compose f1 f2 x = f1 (f2 x);;
```

Higher-order functions

```
let map_incr_2 = compose map_incr map_incr;;
map_incr_2 [1;2;3];;

let map_incr_3 = compose map_incr map_incr_2;;
map_incr_3 [1;2;3];;

let map_incr_3_pos = compose pos_filer map_incr_3;;
```

Instead of manipulating lists, we are manipulating the list manipulators!

Benefits of higher-order functions

Identify common computation patterns

- Iterate a function over a set, list, tree …
- Accumulate some value over a collection

Pull out (factor) “common” code:

- Computation Patterns
- Re-use in many different situations