

# CS162 Discussion Session 3: $\lambda$ Calculus

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# Review

$$FV(x) = x$$

$$FV(\lambda x \rightarrow e) = \text{vars}(e) - x$$

$$FV(e_1 \ e_2) = FV(e_1) + FV(e_2)$$

$$FV(x \ y) = \{x, y\}$$

$$FV(\lambda y \rightarrow x \ y) = \{x\}$$

$$FV((\lambda x \rightarrow \lambda y \rightarrow y) \ x) = \{x\}$$

Figure 1: Free variables

$$\lambda x \rightarrow e =_{\alpha} \lambda y \rightarrow e[x := y]$$

- Rename a formal parameter and replace all its occurrences in the body
- $\lambda x \rightarrow e$   **$\alpha$ -equivalent** to  $\lambda y \rightarrow e[x := y]$

Figure 2:  $\alpha$  renaming

## Rules of $\alpha$ renaming: (from Wikipedia - Lambda Calculus)

- when  $\alpha$  renaming an abstraction, the **only** variable occurrences that are renamed are those that are **bound** to the same abstraction.
- $\alpha$  renaming is **not** possible if it would result in a variable getting captured by a different abstraction.

$\beta$ -Reduction:  $(\lambda x \rightarrow e_1) e_2 =_{\beta} e_1[x := e_2]$

where  $e_1[x := e_2]$  means “ $e_1$  with all **free occurrences** of  $x$  replaced with  $e_2$ ”  
In other words, If you see an *abstraction* applied to an *argument*, take the *body* of the abstraction and replace all free occurrences of the *formal* by that *argument*

Figure 3:  $\beta$  reduction

## Quiz

# Quiz - $\lambda$ Calculus

① Find all the free variables in the following expressions:

- $(\lambda x \rightarrow x y) (\lambda x \rightarrow y x)$
- $(\lambda x \rightarrow x z) (\lambda w \rightarrow w y z x)$

② True or False:

- $\lambda r \rightarrow r x = \alpha \lambda x \rightarrow x x$
- $\lambda x \rightarrow x x = \alpha \lambda x \rightarrow x y$
- $\lambda x \rightarrow x y = \alpha \lambda x \rightarrow x z$
- $\lambda x \rightarrow y z = \alpha \lambda y \rightarrow y z$
- $(\lambda x \rightarrow (\lambda x \rightarrow x) x) = \alpha (\lambda x \rightarrow (\lambda y \rightarrow y) x)$   
 $= \alpha (\lambda z \rightarrow (\lambda y \rightarrow y) z)$

③ Apply  $\beta$ -reduction (and  $\alpha$ -renaming if needed) to the following expressions as much as possible:

- $(\lambda y \rightarrow y) (\lambda z \rightarrow z x)$
- $(\lambda x \rightarrow (x y)) (\lambda z \rightarrow z)$
- $(\lambda y \rightarrow y (\lambda x \rightarrow x)) (\lambda x \rightarrow x)$



① Find all the free variables in the following expressions:

- $(\lambda x \rightarrow x \text{ y}) (\lambda x \rightarrow \text{y x})$
- $(\lambda x \rightarrow x \text{ z}) (\lambda w \rightarrow w \text{ y z x})$

② The  $\alpha$ -renaming of the following expressions is correct or not:

- $\lambda y \rightarrow y x = \alpha \lambda x \rightarrow x x$  ✗
- $\lambda x \rightarrow x x = \alpha \lambda x \rightarrow x y$  ✗
- $\lambda x \rightarrow x y = \alpha \lambda x \rightarrow x z$  ✗
- $\lambda x \rightarrow y z = \alpha \lambda y \rightarrow y z$  ✗
- $(\lambda x \rightarrow (\lambda x \rightarrow x) x) = \alpha (\lambda x \rightarrow (\lambda y \rightarrow y) x)$  ✓  
     $= \alpha (\lambda z \rightarrow (\lambda y \rightarrow y) z)$  ✓

③ Apply  $\beta$ -reduction (and  $\alpha$ -renaming if needed) to the following expressions as much as possible:

- $(\lambda y \rightarrow y) (\lambda z \rightarrow z x)$   $(\lambda z \rightarrow z x)$
- $(\lambda x \rightarrow (x y)) (\lambda z \rightarrow z)$   $y$
- $(\lambda y \rightarrow y (\lambda x \rightarrow x)) (\lambda x \rightarrow x)$  identity function

## Q & A