CS 162 Programming languages

Lecture 7: Operational Semantics I

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What does a program mean?

- We have learned how to specify syntax.
  - Example: let x = *lambda lambda* is not a valid λ^+ program
  - But we have not yet talked about what the meaning of a program is.
- First question: What is the meaning of a program in λ^+?
  - Answer: The value the program evaluates to
  - Example: let x=3 in x Value:3

How to specify meaning of programs

- Option 1: Don’t worry too much
- Developer of language has some informal concept of the intended meaning, implement a compiler/interpreter that does whatever the language designers believe to be reasonable.
- Then, declare the meaning to be whatever the compiler produces
- A terrible idea

How to specify meaning of programs

- Why is this such a bad idea?
- This approach promotes bugs/inconsistencies to expected behavior.
- Hides specification of language in many implementation details
- Makes it almost impossible to implement another compiler that accepts the same language
- Unfortunately, this is (still) a very common approach
- Languages designed this way: C, C++ (to some extent), Perl, PHP, JavaScript, ...

How to specify meaning of programs

- Option 2: Try to write out precisely the meaning of each language construct in documentation, then follow this description in implementation
- Example: Describe the meaning of !e in the λ^+ language:
- First attempt: “This evaluates to the head of e”
- What if e is not a list?
- Second attempt: “This evaluates to the head of e if e is a list, and to e otherwise”
- What if e is Nil?...

How to specify meaning of programs

- Written language is, by nature, ambiguous. It is very difficult to fully specify the meaning of all language constructs this way
- Easy to miss cases
- Results in long, complicated and difficult to understand specifications, but an improvement over no specification

Written specification in practice

- Let’s look at the ISO C++ standard: page 34:

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2 A declaration is a *definition* unless it declares a function without specifying the function’s body (8.4), it contains the `extern` specifier (7.1.1) or a *linkage-specification*[25] (7.5) and neither an *initializer* nor a *function-body*, it declares a static data member in a class definition (9.2, 9.4), it is a class name declaration (9.1), it is an *opaque-enum-declaration* (7.2), it is a *template-parameter* (14.1), it is a *parameter-declaration* (8.3.5) in a function declarator that is not the *declarator* of a *function-definition*, or it is a `typedef` declaration (7.1.3), an *alias-declaration* (7.1.3), a *using-declaration* (7.3.3), a *static_assert-declaration* (Clause 7), an *attribute-declaration* (Clause 7), an *empty-declaration* (Clause 7), a *using-directive* (7.3.4), an explicit instantiation declaration (14.7.2), or an explicit specialization (14.7.3) whose *declaration* is not a definition.

[*Example:* all but one of the following are definitions:

```
int a;                          // defines a
extern const int c = 1;         // defines c
int f(int x) { return x+a; }    // defines f and defines x
struct S { int a; int b; };     // defines S, S::a, and S::b
struct X {                      // defines X
  int x;                        // defines non-static data member x
  static int y;                 // declares static data member y
  X(): x(0) { }                 // defines a constructor of X
};
int X::y = 1;                   // defines X::y
enum { up, down };              // defines up and down
namespace N { int d; }          // defines N and N::d
namespace N1 = N;               // defines N1
X anX;                          // defines anX
```

whereas these are just declarations:

```
extern int a;                   // declares a
extern const int c;             // declares c
int f(int);                     // declares f
struct S;                       // declares S
typedef int Int;                // declares Int
extern X anotherX;              // declares anotherX
using N::d;                     // declares d
```

Machine model

- To study the operational semantics, we must understand what our machine model will require.
- First, need to know when our machine is “done” executing a program: the final expressions are values.
- Second, need environments that keep track of variables bound by let-expressions.

Values in λ^+

- We define a value in an inductive way:
  - Any integer i is a value.
  - Any lambda expression lambda x. e is a value.
  - Nil is a value.
  - If v_1, v_2 are values, then $v_1@v_2$ are values.
  - No other expression is a value.

Values in λ$^+$

- Those expressions are values:

$$10$$
$$\mathsf{lambda}\, x.\ 1 + 2$$
$$\mathsf{Nil}$$
$$10 \mathbin{@} \mathsf{lambda}\, y.\ y$$

- Those expressions are NOT values:

$$1 + 2$$
$$(\mathsf{lambda}\, x.\ 1 + 2)10$$
$$\mathsf{if}\ \mathsf{Nil}\ \mathsf{then}\ 10\ \mathsf{else}\ 20$$
$$(1 + 2) \mathbin{@} \mathsf{Nil}$$

Environments

- An environment E is a mapping from each variable to a value, i.e. it keeps track of "what variable has what value"
- $E = [x_1 \rightarrow v_1, \ldots, x_n \rightarrow v_n]$ indicates that the value of identifier x_i is v_i
- We use the notation $E(x) = v$ to denote that the result of looking up the identifier x in E is v
- Update environment: if originally $E = [x \rightarrow 99, y \rightarrow 4]$

  Then $E[x \rightarrow 0] = [x \rightarrow 0, y \rightarrow 4]$

Inference rules

$$\begin{array}{c} \text{Hypothesis 1} \\ \dots \\ \text{Hypothesis N} \\ \hline \vdash \text{Conclusion} \end{array}$$

- This means“given hypothesis1,…N, the conclusion is provable”

$$\begin{array}{c} \text{Miterm 1 grade} >= 70 \\ \dots \\ \text{Final grade} >= 140 \\ \hline \vdash \text{Final grade: A} \end{array}$$

Operational Semantics

- Operational semantics: define how program states are related to final values

- The ***big-step*** evaluation relation asserts that *we can prove for any expression of the form e that the meaning of this expression will evaluate to v under the environment E*

$$\mathbb{E} \vdash e \Downarrow v$$

Operational Semantics

$$\frac{}{\mathbb{E} \vdash i \Downarrow i} \text{ INT}$$

Any integer constant i will evaluate to itself

$$\frac{\mathbb{E} \vdash e_1 \Downarrow i_1 \qquad \mathbb{E} \vdash e_2 \Downarrow i_2}{\mathbb{E} \vdash e_1 + e_2 \Downarrow i_1 + i_2} \text{ ADD}$$

if e_1 and e_2 both evaluate to integers, then $e_1 + e_2$ evaluates to the sum of those integers

$$\frac{\text{ADD}\ \dfrac{\text{INT}\ \dfrac{}{\vdash 1 \Downarrow 1} \qquad \dfrac{}{\vdash 2 \Downarrow 2}\ \text{INT}}{\vdash 1 + 2 \Downarrow 3} \qquad \dfrac{}{\vdash 4 \Downarrow 4}\ \text{INT}}{\vdash (1+2) + 4 \Downarrow 7} \text{ ADD}$$

Operational Semantics

$$\frac{\mathbb{E} \vdash e_1 \Downarrow i_1 \qquad \mathbb{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \odot i_2}{\mathbb{E} \vdash e_1 \odot e_2 \Downarrow 1} \textsc{PredTrue}$$

$$\frac{\mathbb{E} \vdash e_1 \Downarrow i_1 \qquad \mathbb{E} \vdash e_2 \Downarrow i_2 \qquad \neg(i_1 \odot i_2)}{\mathbb{E} \vdash e_1 \odot e_2 \Downarrow 0} \textsc{PredFalse}$$

The predicate operators $\odot \in \{=, <, >, \leq, \geq\}$ evaluate to 0 if the predicate does not hold and to 1 otherwise

$$\frac{\mathbb{E} \vdash e_1 \Downarrow i \qquad i \neq 0 \qquad \mathbb{E} \vdash e_2 \Downarrow v}{\mathbb{E} \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \textsc{IfTrue}$$

$$\frac{\mathbb{E} \vdash e_1 \Downarrow 0 \qquad \mathbb{E} \vdash e_3 \Downarrow v}{\mathbb{E} \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v} \textsc{IfFalse}$$

Operational Semantics

$$\frac{}{\mathbb{E} \vdash \mathsf{lambda}\, x.\ e \Downarrow \mathsf{lambda}\, x.\ e}\ \textsc{Lambda}$$

Lambda abstractions just evaluate to themselves

$$\frac{\mathbb{E} \vdash e_1 \Downarrow \mathsf{lambda}\, x.\ e_1' \qquad \mathbb{E} \vdash [x \mapsto e_2]e_1' \Downarrow v}{\mathbb{E} \vdash (e_1\ e_2) \Downarrow v}\ \textsc{App}$$

To evaluate the application (e_1 e_2), we first evaluate the expression e_1. The operational semantics "get stuck" if e_1 is not a lambda abstraction. This notion of "getting stuck" in the operational semantics corresponds to a runtime error. Assuming the expression e_1 evaluates to a lambda expression, we evaluate the application expression by binding e_2 to x and then evaluating the expression $[x \rightarrow e_2]e'_1$ as in β-reduction in lambda calculus.

Operational Semantics

$$\frac{\mathbb{E}(x) = v}{\mathbb{E} \vdash x \Downarrow v} \textsc{Var}$$

If variable x is bound to some value v in the environment E, then x evaluates to its bound value.

$$\frac{\mathbb{E} \vdash e_1 \Downarrow v_1 \qquad \mathbb{E}[x \mapsto v_1] \vdash e_2 \Downarrow v_2}{\mathbb{E} \vdash \mathsf{let}\, x = e_1 \,\mathsf{in}\, e_2 \Downarrow v_2} \textsc{Let}$$

First evaluate the initial expression e_1 in environment E, which yields value v_1. Then we obtain a new environment E′ by binding identifier x to value v_1, i.e., $E' = E[x \leftarrow v_1]$. Next, we evaluate the body e_2 in this new environment E′, which yields value v_2 , which is also the result of evaluating the entire let expression.

TODOs by next lecture

- HW3 is out. Please start ASAP