free vars

$x + 1$

$\lambda x . x + y$

$\lambda x . \lambda y . x + y$

$(\lambda x . x + x)((\lambda y . x)\ 5)$

If $(\lambda x . x > y)\ 1$ then $2 + z$ else $x + z$

let $x = 5$ in $x + x$

Subst

$(\lambda x.\ \lambda y.\ \text{if } x>0 \text{ then } y \text{ else } x)\ 5\ 3$

e_1 $\quad$ e_2

$e_1 \Downarrow \lambda y.\ E$ $\qquad$ $e_2 \Downarrow 3$

$[y \to 3]E \Downarrow v$

Free vars in substitution

$(\lambda x.\lambda y.\ x\ @\ y\ @\ Nil)\ (\lambda z.y)\ 3$

Subst 3 (factorial)

let $f = fix\,(\lambda f.\ \lambda x.\ \text{if } x<1 \text{ then } 0 \text{ else } x * (f\ (x-1)))$

in $f\ 3$ ⇓ E

$$\frac{(fix\ E) \Downarrow v \qquad [f \to v](f\,3) \Downarrow v'}{\text{let } f = (fix\ E) \text{ in } f\,3 \Downarrow v'}\ \text{LET}$$