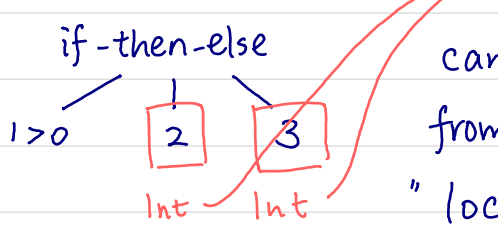
Type checking as type inference

$\Gamma \vdash$ if $1>0$ then 2 else 3 : Int



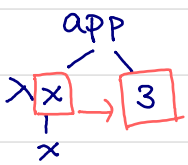
can be completely determined from types of sub-expressions.

"local" inference

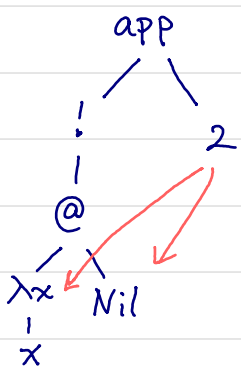
Question How to do inference if no annotation is provided?

Problem Local inference stops working.

$(\lambda x.x)\ 3$: Int



$(!((\lambda x.x)@Nil))\ 2$



Insights (1) Although we may not know exactly what an unknown type T is, we still have information that constrains what T can be.

(2) Constraints interact each other. If we consider all constraints at the same time, the constraint system may have a (unique) solution.

$x+y=0$, $z-x=2$ ⇒ $x=\cdots$, $y=\cdots$, $z=\cdots$

system of eqns ⇒ solution

Type Inference

= Constraint generation ⇒ global Constraint solving
(this section)

Constraint Generation:

Given an expr e, we can generate constraints mechanically, based on the shape of e.

$$\frac{\Gamma \vdash e : T \;\textcircled{2} \qquad \textcircled{1}\ X \text{ fresh} \quad \textcircled{3}\ T = \text{List}[X]}{\Gamma \vdash !e : X\ \textcircled{4}}\ \text{CT-Head}$$

Components:

(unknowns)

① A reservoir of type variables

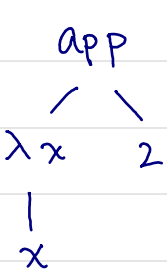
② Recursion

③ Generated constraints

④ Type of the current expression

Example 1

"$(\lambda x.\, x)\ 2$"



$$\frac{x : \gamma_3 \in \Gamma}{x : \gamma_3 \vdash x : \gamma_3}\ \text{CT-Var}$$

$$\frac{\dfrac{}{\emptyset \vdash \lambda x.\, x : \gamma_3 \to \gamma_3}\ \text{CT-}\lambda \qquad \dfrac{}{\emptyset \vdash 2 : \text{Int}}\ \text{CT-Int}}{\gamma_3 \to \gamma_3 = \gamma_1 \to \gamma_2 \qquad \gamma_1 = \text{Int}}\ \text{CT-App}$$

$$\emptyset \vdash (\lambda x.\, x)\ 2 : \gamma_2$$

(Global) Bag of Constraints

$\gamma_3 \to \gamma_3 = \gamma_1 \to \gamma_2$

$\text{Int} = \gamma_1$

$\gamma_3 \to \gamma_3 = \gamma_1 \to \gamma_2$, $\gamma_1 = \text{Int}$

$\Rightarrow$ $\gamma_3 = \gamma_1$, $\gamma_3 = \gamma_2$, $\gamma_1 = \text{Int}$

$\Rightarrow$ $\gamma_3 = \text{Int}$, $\gamma_3 = \gamma_2$, $\gamma_1 = \text{Int}$

$\Rightarrow$ $\gamma_1, \gamma_2, \gamma_3 = \text{Int}$

Solution

Example 2

" $\left(!((\lambda x.x)@\,\mathrm{Nil})\right)\ 5$ "