Homework 2 Report and code output

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February 17, 2021

In [224…
```
import numpy as np
import matplotlib.pyplot as plt
```

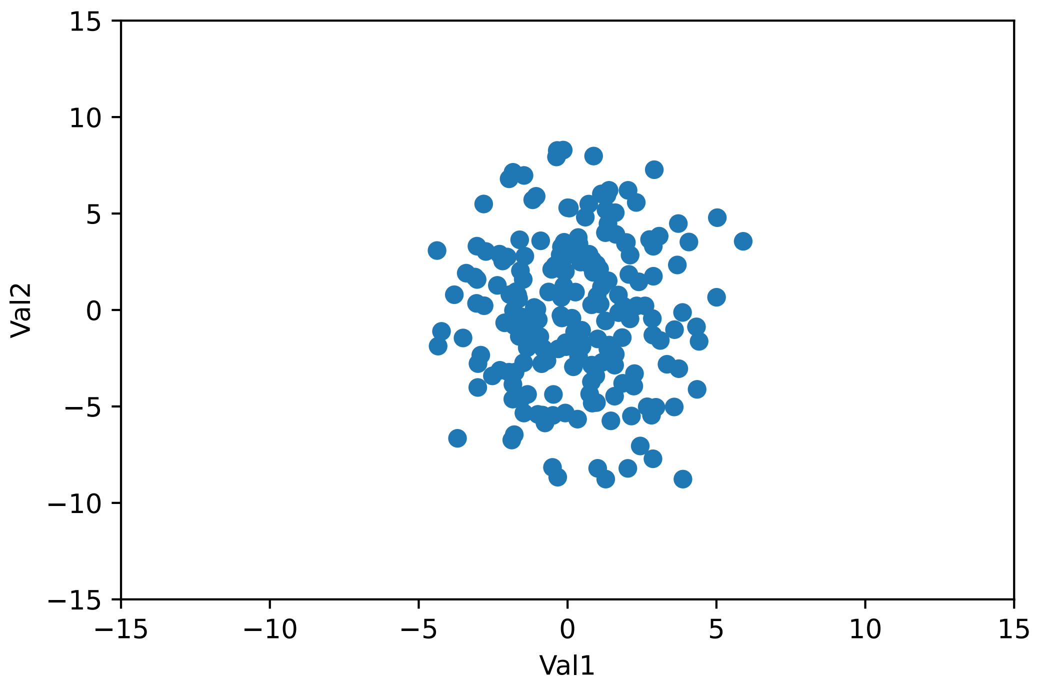
In [225…
```
data1 = np.loadtxt('./data/lindata1.csv',delimiter=',')
data2 = np.loadtxt('./data/lindata2.csv',delimiter=',')
data3 = np.loadtxt('./data/lindata3.csv',delimiter=',')
```

Data Visualization

Here we will visualize the data provided

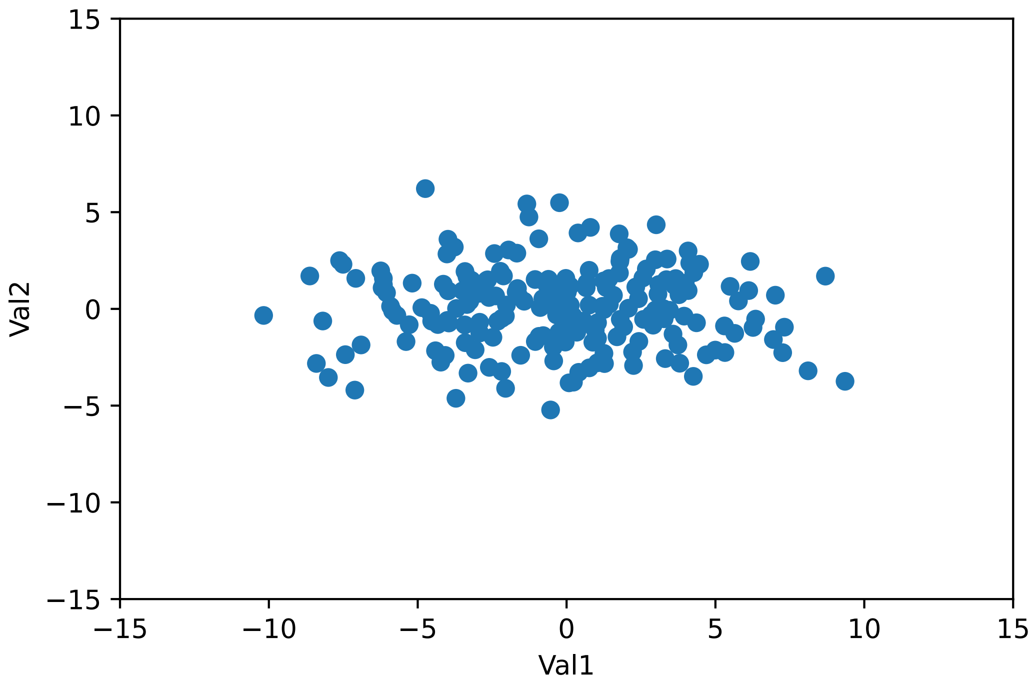
In [226…
```
plt.figure()
plt.scatter(data1[:,[0]],data1[:,[1]])
plt.xlabel("Val1")
plt.ylabel("Val2")
plt.xlim(-15,15)
plt.ylim(-15,15)
```

Out[226…
```
(-15.0, 15.0)
```



In [227…
```
plt.figure()
plt.scatter(data2[:,[0]],data2[:,[1]])
plt.xlabel("Val1")
plt.ylabel("Val2")
plt.xlim(-15,15)
plt.ylim(-15,15)
```

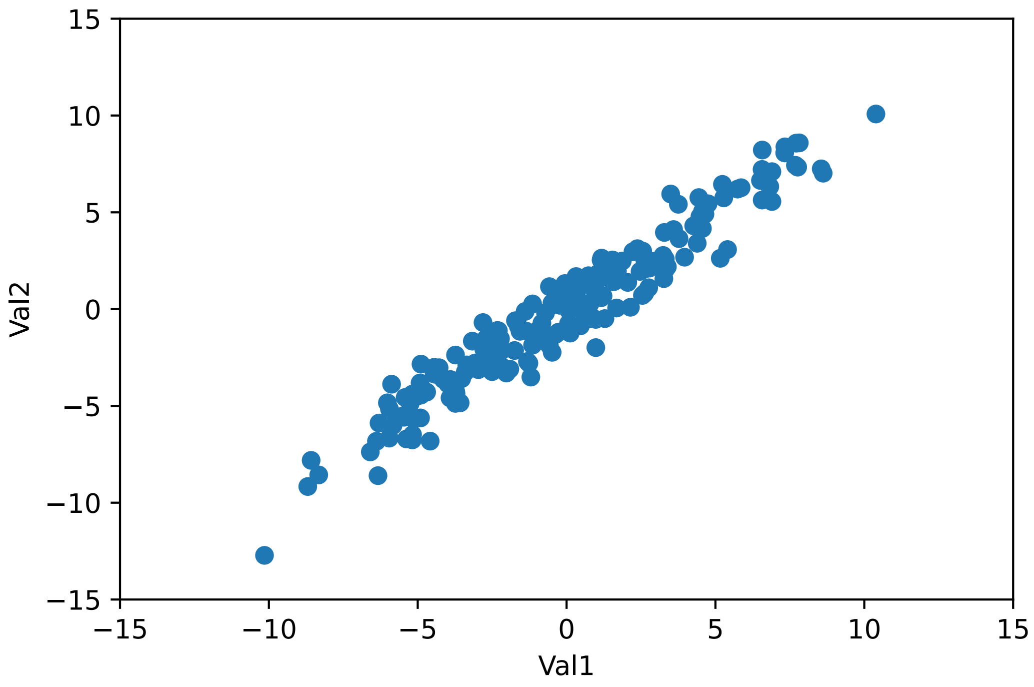
Out[227… (-15.0, 15.0)



In [228…

```
plt.figure()
plt.scatter(data3[:,[0]],data3[:,[1]])
plt.xlabel("Val1")
plt.ylabel("Val2")
plt.xlim(-15,15)
plt.ylim(-15,15)
```

Out[228… (-15.0, 15.0)



Plotting Eigenvalues 5.1

In this section, I have created a visualization principal components of the datasets in the files Lindata1.csv, Lindata2.csv, and Lindata3.csv. This part was particularly challenging to set up the algorythm correctly and getting the proper scale for the values.

To briefly describe the PCA function, I mean center the data and compute the covarice matrix using the formula $(X^T X)/(n-1)$. From that matrix I can obtain the eigen values and vectors which I later scale for my final principal component. I repeat this three times with each dataset.

The eigenvectors of a covariance matrix represent the directions (vector) in which the data varies the most while the eigenvalues represent the magnitude (or length of arrow) of this direction.

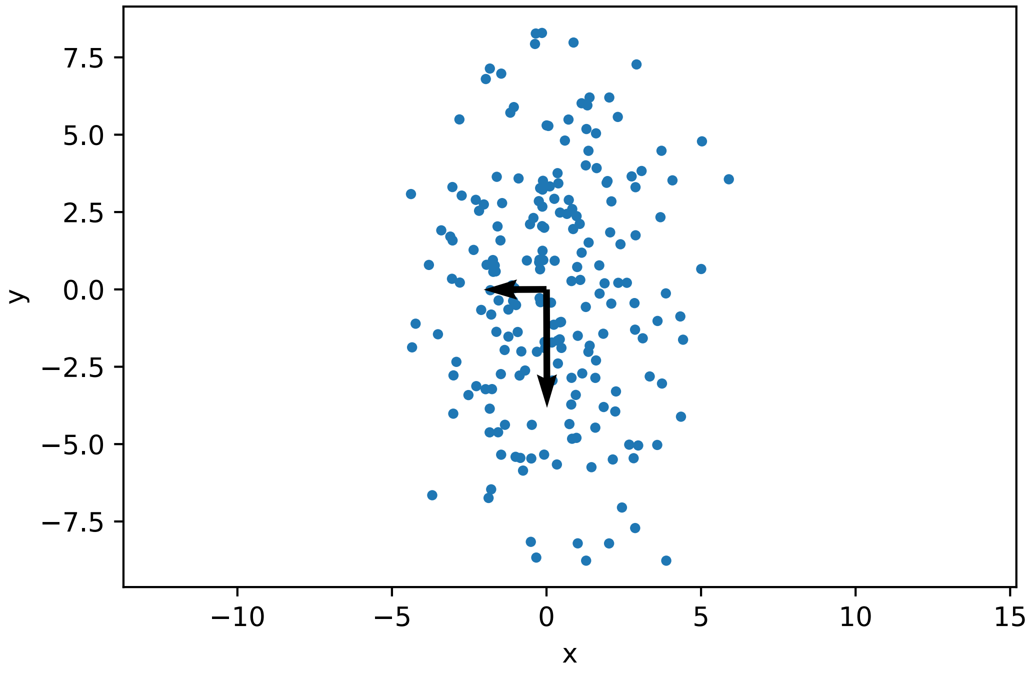
The vectors shown in the following graphs are the eigenvectors of the covariance matrix scaled by the square root of the corresponding eigenvalue, and shifted so their tails are at the mean.

In [229…

```
def pca(data):
    #mean centering
    mean = np.mean(data,axis=0)
    data = data - np.mean(data,axis=0)
    mean = np.mean(data,axis=0)
    #compute covarice matrix
    cov = np.dot(data.T, data)/(data.shape[0]-1)
    #eigen values
    eigen_vals, eigen_vecs = np.linalg.eig(cov)
    #get scaled pc
    pc = np.array([np.sqrt(eigen_vals[0])*eigen_vecs[:,0],np.sqrt(eigen_vals[1])*eigen_
    return mean, pc[0,:], pc[1,:]
```

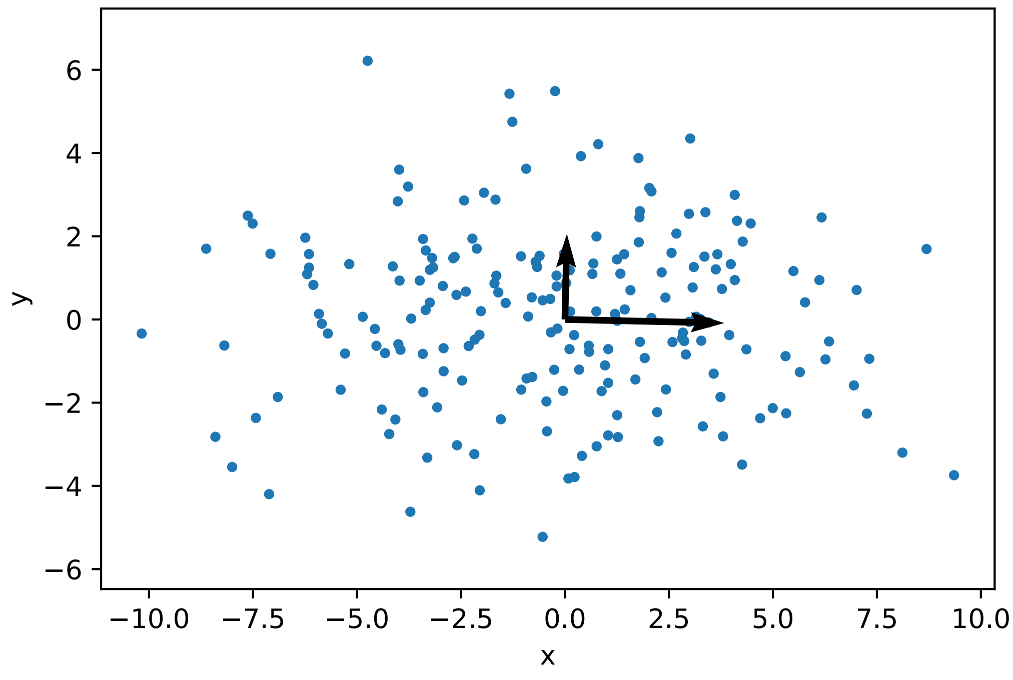
In [230…

```
mean, pc1, pc2 = pca(data1)
plt.figure()
plt.scatter(data1[:,[0]],data1[:,[1]],s=8)
plt.xlabel("x")
plt.ylabel("y")
plt.quiver(*mean,*pc1,scale=1,scale_units='xy')
plt.quiver(*mean,*pc2,scale=1,scale_units='xy')
plt.axis('equal')
plt.show()
```



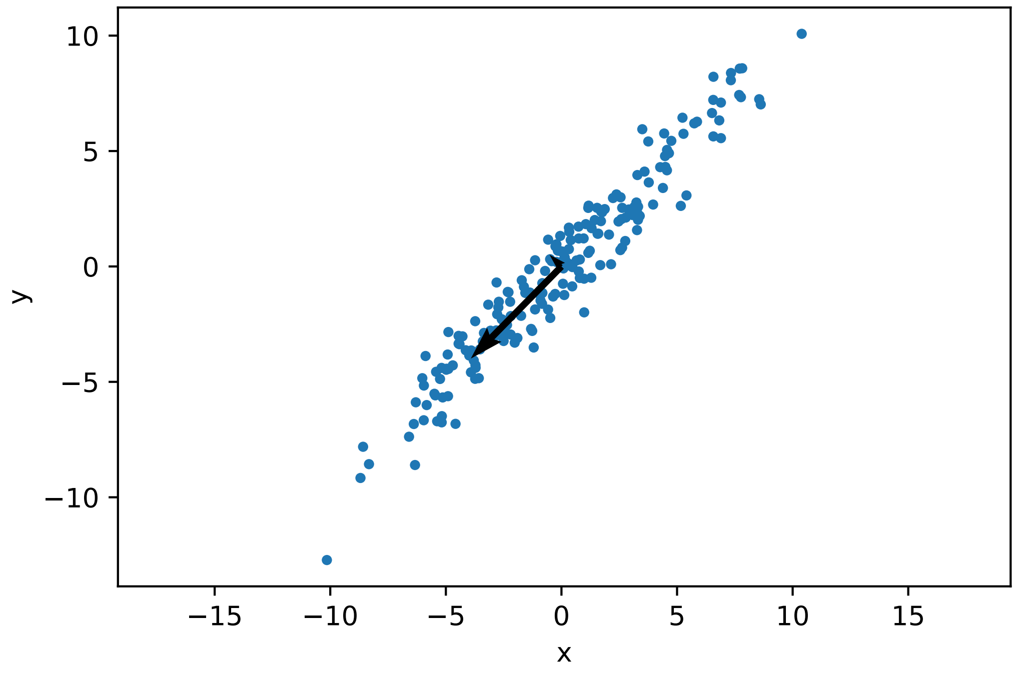
In [231…

```
mean, pc1, pc2 = pca(data2)
plt.figure()
plt.scatter(data2[:,[0]],data2[:,[1]],s=8)
plt.xlabel("x")
plt.ylabel("y")
plt.quiver(*mean,*pc1,scale=1,scale_units='xy')
plt.quiver(*mean,*pc2,scale=1,scale_units='xy')
plt.axis('equal')
plt.show()
```



In [232…

```
mean, pc1, pc2 = pca(data3)
plt.figure()
plt.scatter(data3[:,[0]],data3[:,[1]],s=8)
plt.xlabel("x")
plt.ylabel("y")
plt.quiver(*mean,*pc1,scale=1,scale_units='xy')
plt.quiver(*mean,*pc2,scale=1,scale_units='xy')
plt.axis('equal')
plt.show()
```



Least Squares fitting 6

In this section, will now fit polynomials to the data in the files Nonlindata1.csv, Nonlindata2.csv, and Nonlindata3.csv. These files are located in the data folder. I will use two different methods:

- Linear Least Squares
- Ridge Regression

Both of these models will be trained on the loaded csv data and later tested with the data generated by:

```
x_test = np.arange(-2,2,.01)
```

Introduction

In this report I will show the benefits of using regularization. Regularization can reduce overfitting by biaring a solution towards the value 0. I'll show the differences between the Linear Least Squares Model (without regularization) and Ridge Regression (with regularization). Although Ridge Regression is a great algorithm, it still has the limitation of struggling with a large polinomial order. Another downfall of the model is that we are traiding variance for bias which means that we need to make sure to use a proper lambda value.

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:

```
data1 = np.loadtxt('./data/nonlindata1.csv',delimiter=',')
data2 = np.loadtxt('./data/nonlindata2.csv',delimiter=',')
data3 = np.loadtxt('./data/nonlindata3.csv',delimiter=',')
```

Data visualization

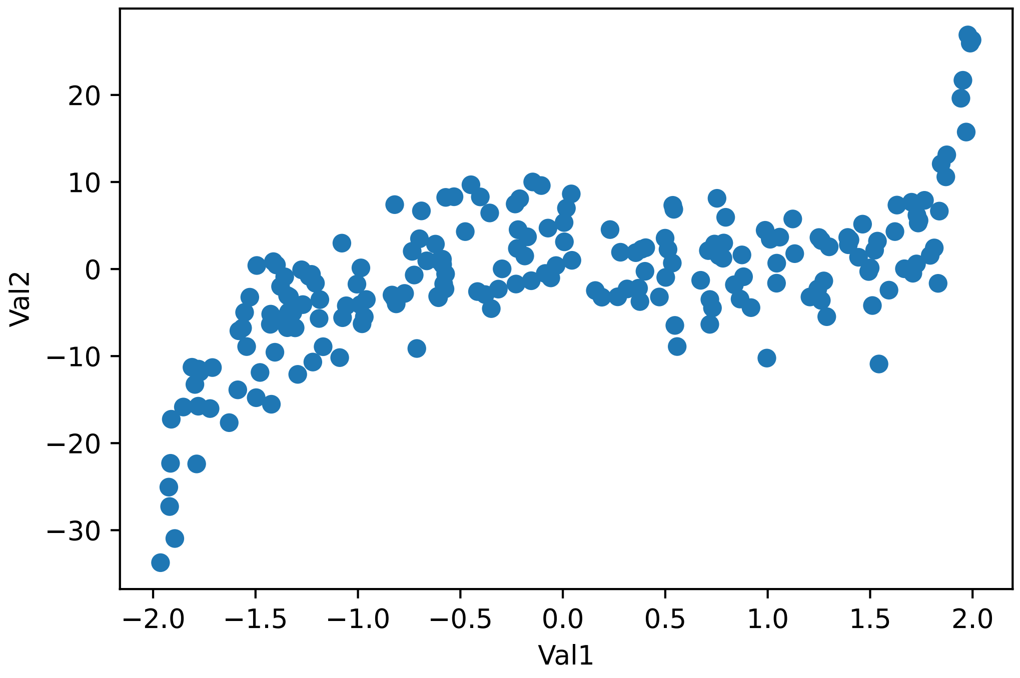
Data visualized: Nonlindata1.csv, Nonlindata2.csv, and Nonlindata3.csv

In [3]:

```
plt.figure()
plt.scatter(data1[:,[0]],data1[:,[1]])
plt.xlabel("Val1")
plt.ylabel("Val2")
```

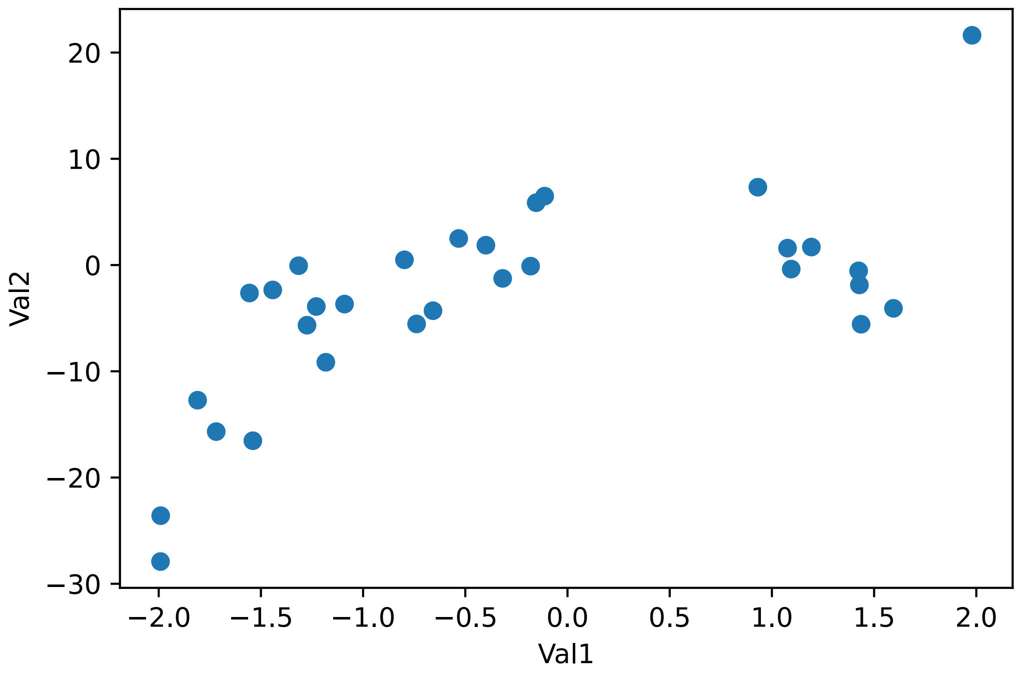
Out[3]:

```
Text(0, 0.5, 'Val2')
```



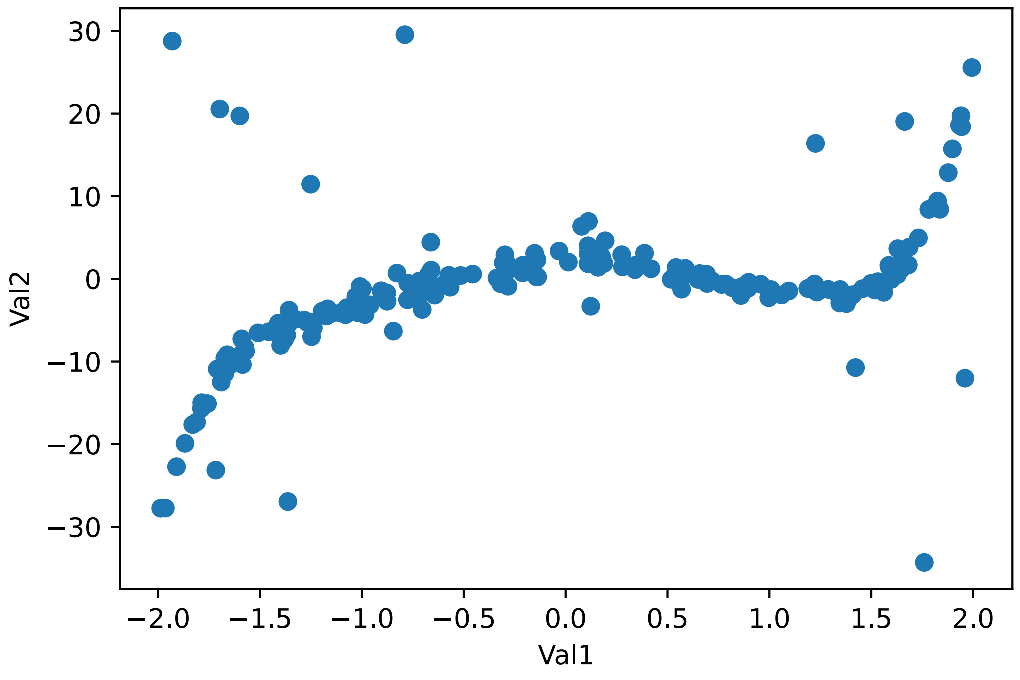
In [4]:
```python
plt.figure()
plt.scatter(data2[:,[0]],data2[:,[1]])
plt.xlabel("Val1")
plt.ylabel("Val2")
```

Out[4]:
```
Text(0, 0.5, 'Val2')
```



In [5]:
```python
plt.figure()
plt.scatter(data3[:,[0]],data3[:,[1]])
plt.xlabel("Val1")
plt.ylabel("Val2")
```

Out[5]:
```
Text(0, 0.5, 'Val2')
```



Provided helper function and test data

In [6]:

```
# provided helper function
def lift(x,n_poly=12):
    N = np.size(x, 0)
    X = np.zeros([N,n_poly+1])
    for p in range(n_poly+1):
        X[:,p]=x**p
    return X
```

In [7]:

```
x_test=np.arange(-2,2,.01)
```

Linear least squares

In this section I will present my Linear Least Square function and test it with data1, data2, and data3. The data is first fit with the loaded csv file and then tested with the x_test data.

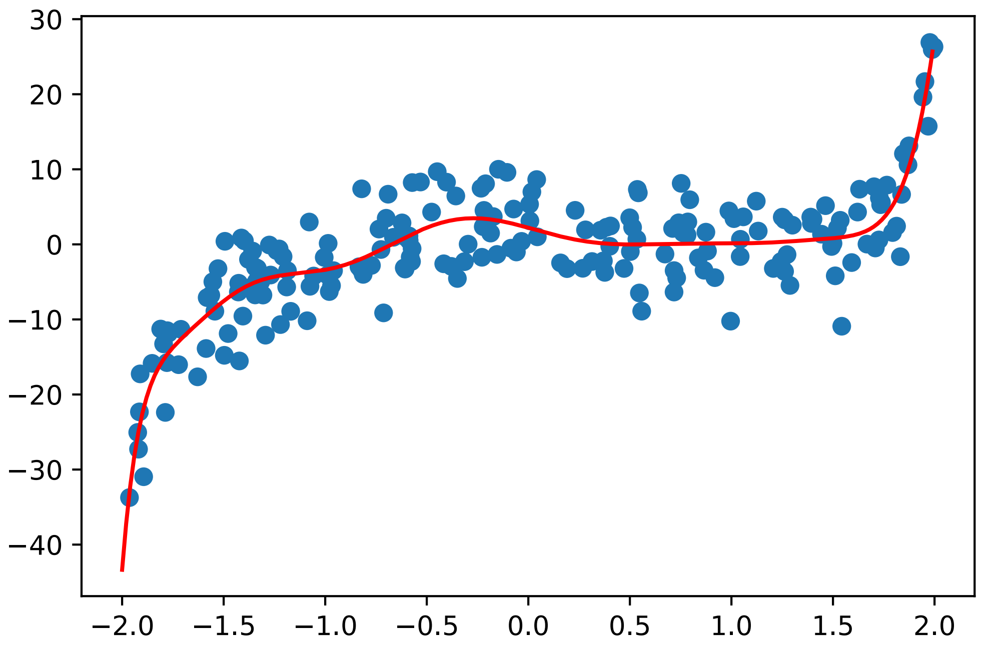
In [8]:

```
def leastSquaresFit(data):
    x = data[:,0]
    y = data[:,1]
    x = lift(x)
    return np.dot(np.linalg.inv(np.dot(x.T, x) ), np.dot(x.T, y))
```

Data 1

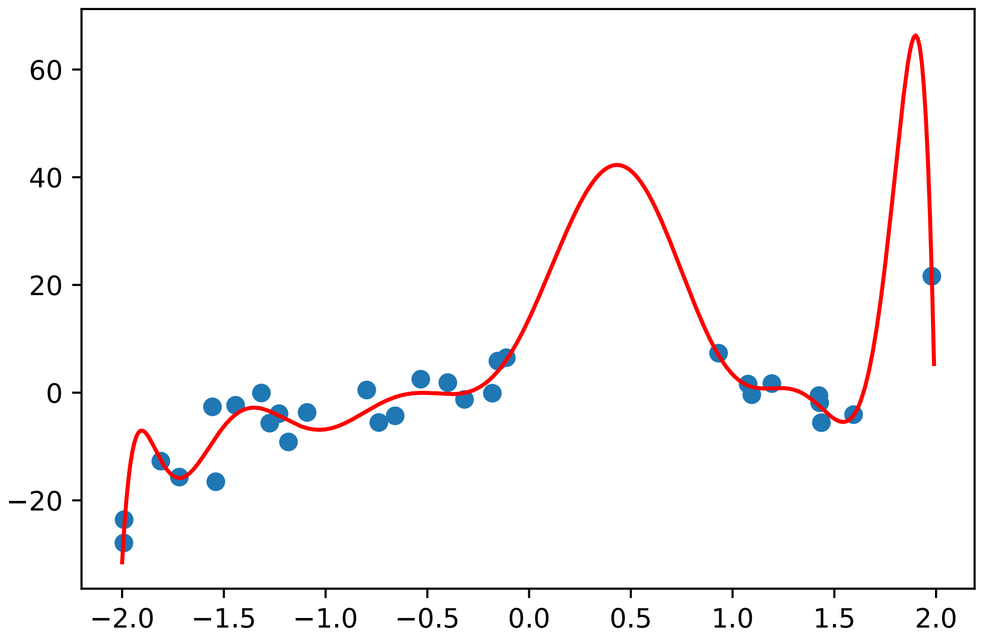
In [9]:

```
# fit on data 1 and test with x_test
output = np.dot(lift(x_test), leastSquaresFit(data1))
plt.scatter(data1[:,0], data1[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



Data 2

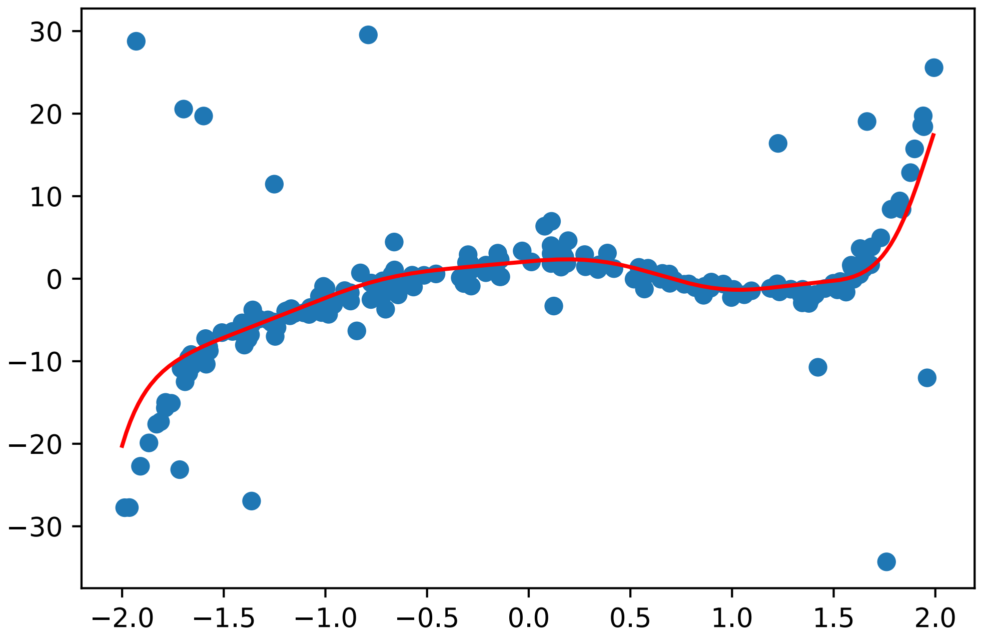
In [10]:
```
# fit on data 2 and test with x_test
output = np.dot(lift(x_test), leastSquaresFit(data2))
plt.scatter(data2[:,0], data2[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



Data 3

In [11]:

```
# fit on data 3 and test with x_test
output = np.dot(lift(x_test), leastSquaresFit(data3))
plt.scatter(data3[:,0], data3[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



Ridge Regression

In this section I will present my Ridge Regression function and test it with data1, data2, and data3. The data is first fit with the loaded csv file and then tested with the x_test data and with different values for lambda.

In [12]:

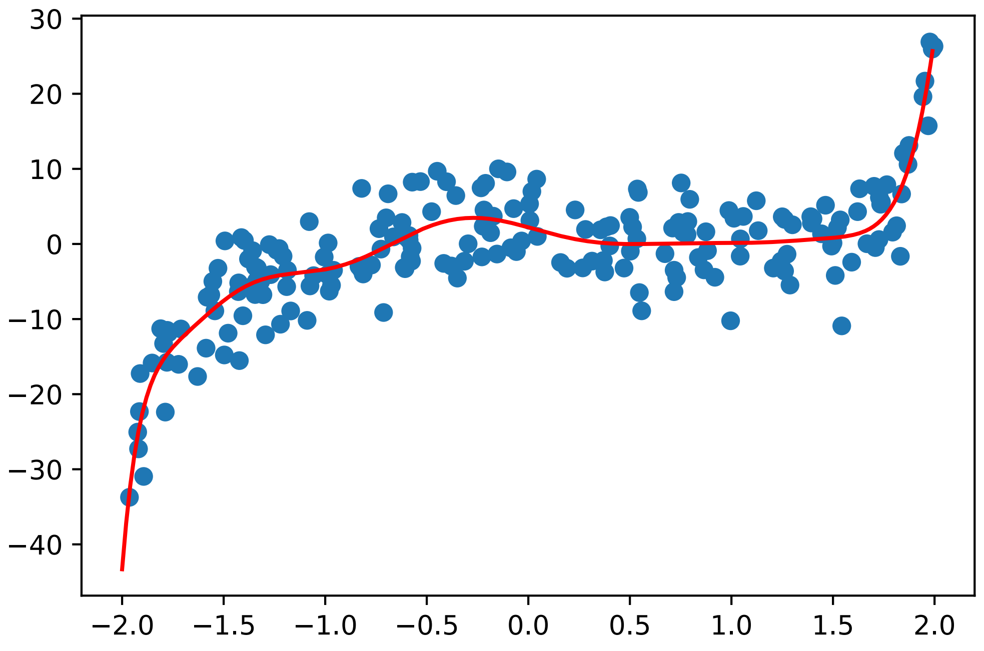
```
def ridgeFit(data,lamb):
    x = data[:,0]
    y = data[:,1]
    x = lift(x)
    al = lamb * np.eye(x.shape[1])
    al[0, 0] = 0
    return np.dot(np.linalg.inv(np.dot(x.T, x) + np.dot(al.T, al)), np.dot(x.T, y))
```

Data 1

Here I test different values of lambda after fitting ridge regression to data 1 and testing on x_test

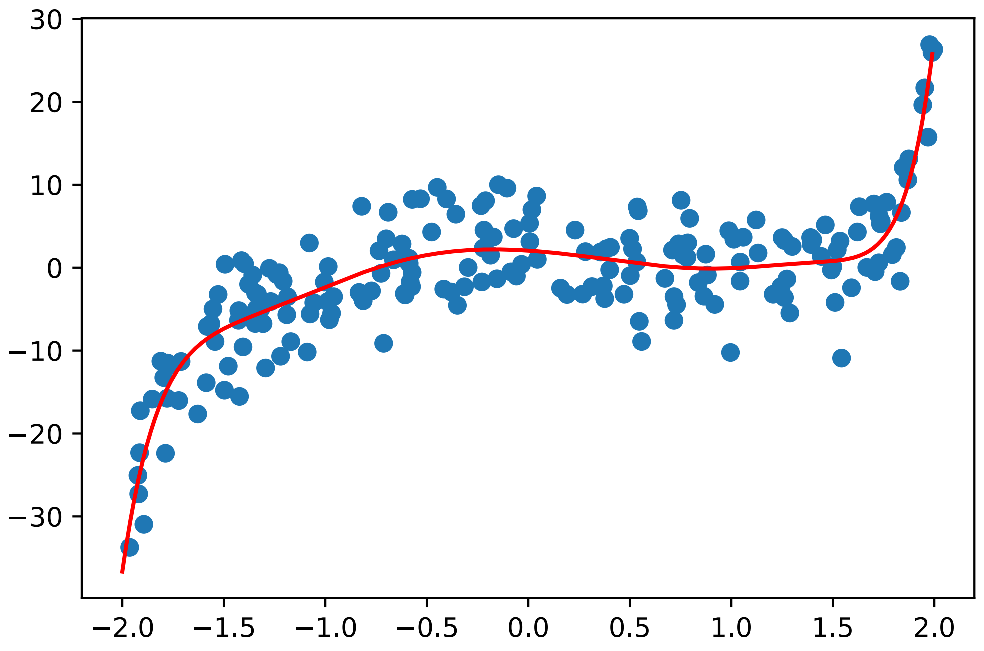
In [13]:

```
# fit on data 1 and test with x_test and lambda = 0.01
output = np.dot(lift(x_test), ridgeFit(data1,0.01))
plt.scatter(data1[:,0], data1[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



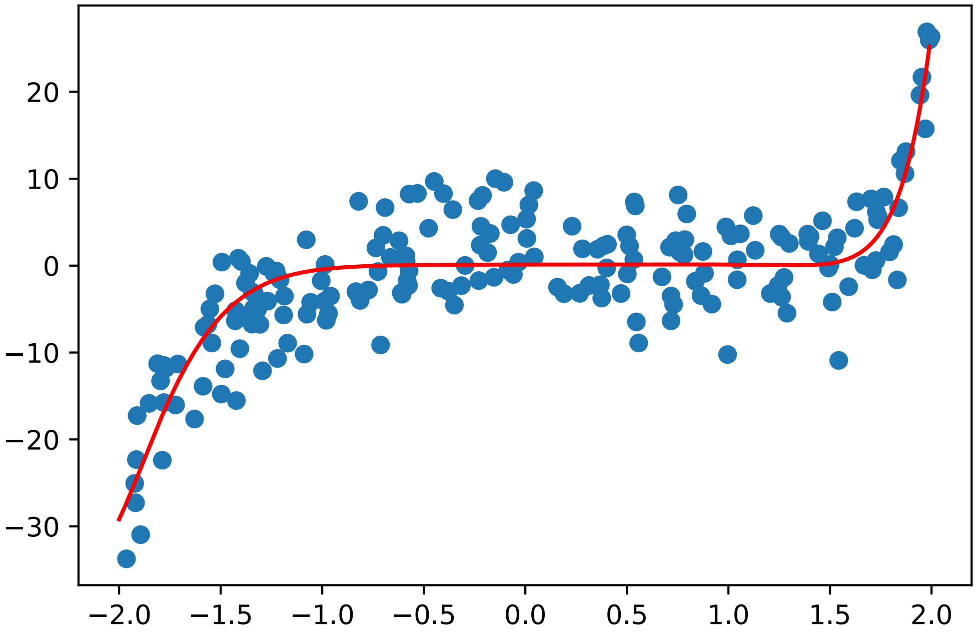
In [14]:

```
# fit on data 1 and test with x_test and lambda = 1
output = np.dot(lift(x_test), ridgeFit(data1,1))
plt.scatter(data1[:,0], data1[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



In [15]:

```
# fit on data 1 and test with x_test and lambda = 50
output = np.dot(lift(x_test), ridgeFit(data1,50))
plt.scatter(data1[:,0], data1[:,1])
plt.plot(x_test,output, "r")
plt.show()
```

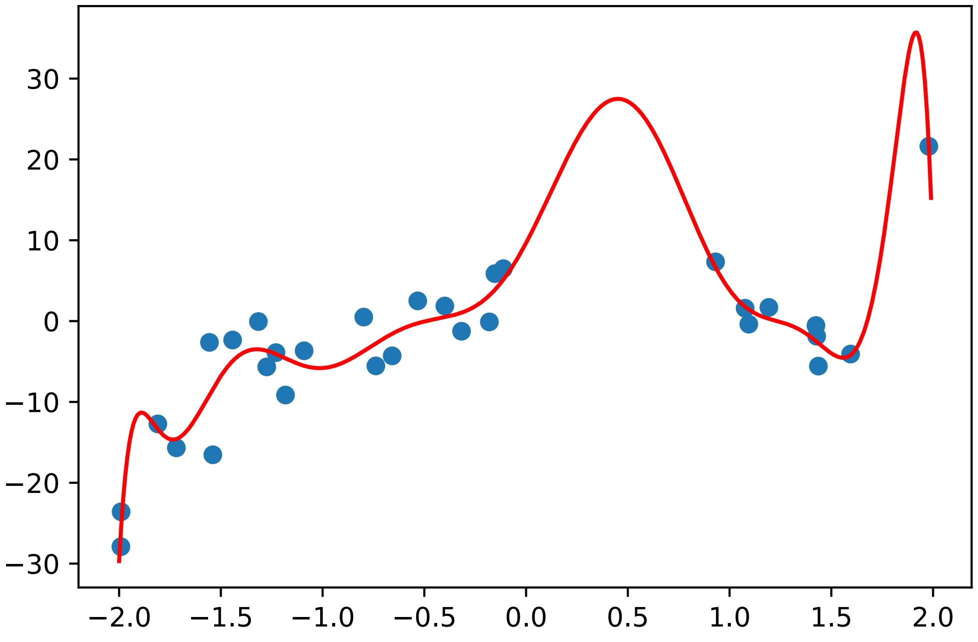


Data 2

Here I test different values of lambda after fitting ridge regression to data 2 and testing on x_test

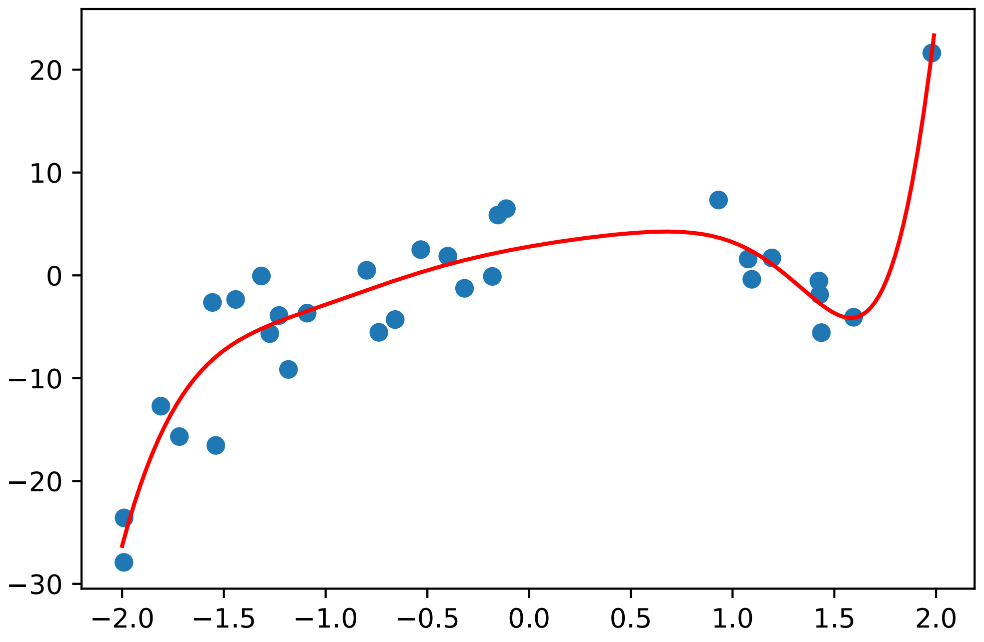
In [16]:

```
# fit on data 2 and test with x_test and lambda = 0.01
output = np.dot(lift(x_test), ridgeFit(data2,0.01))
plt.scatter(data2[:,0], data2[:,1])
plt.plot(x_test,output, "r")
plt.show()
```

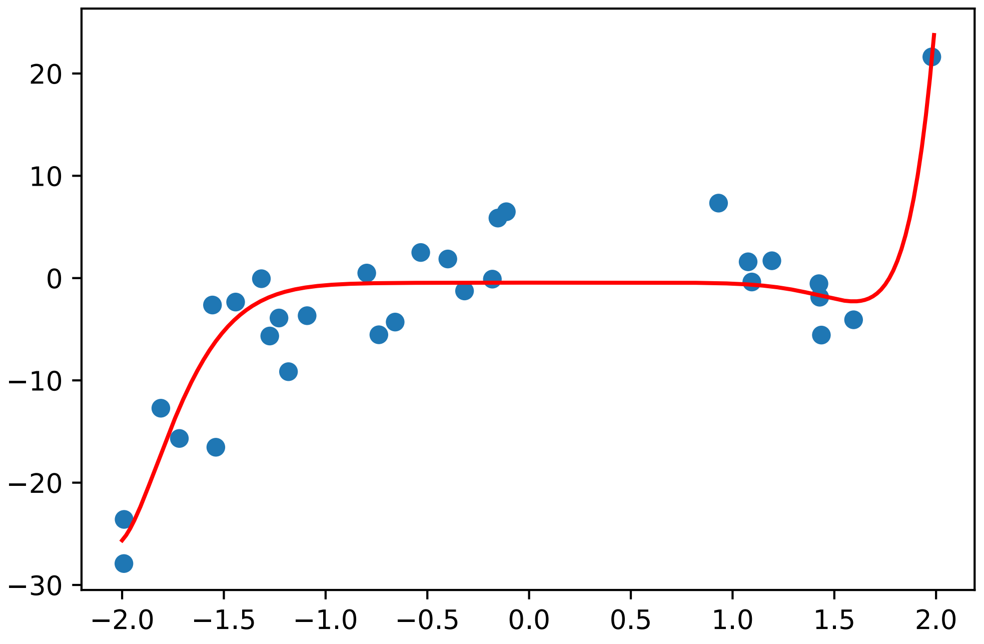


In [17]:

```
# fit on data 2 and test with x_test and lambda = 1
output = np.dot(lift(x_test), ridgeFit(data2,1))
plt.scatter(data2[:,0], data2[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



In [18]:
```
# fit on data 2 and test with x_test and lambda = 50
output = np.dot(lift(x_test), ridgeFit(data2,50))
plt.scatter(data2[:,0], data2[:,1])
plt.plot(x_test,output, "r")
plt.show()
```

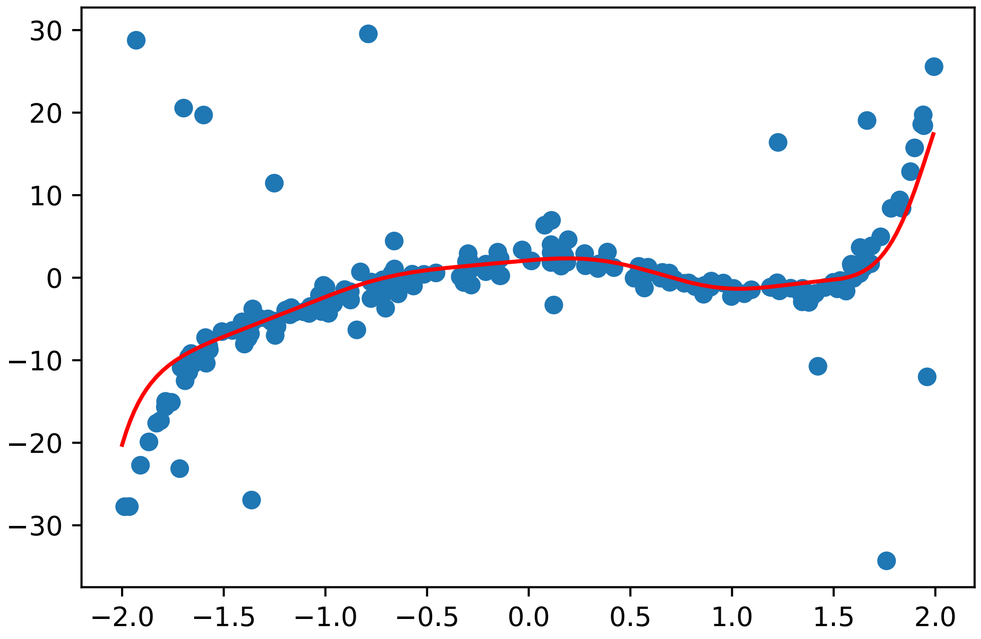


Data 3

Here I test different values of lambda after fitting ridge regression to data 3 and testing on x_test

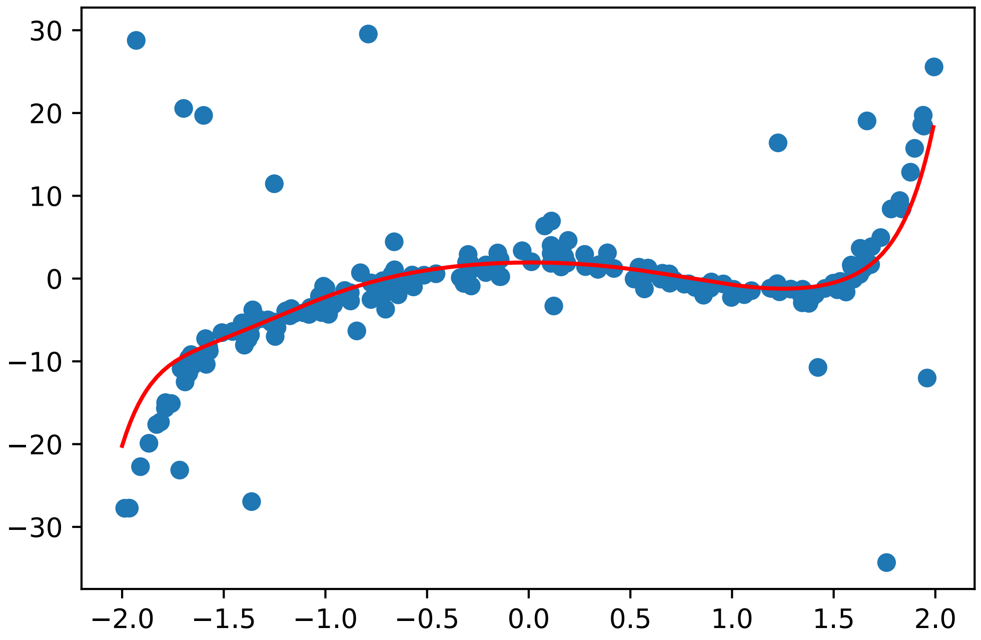
In [19]:
```
# fit on data 3 and test with x_test and lambda = 0.01
```

```
output = np.dot(lift(x_test), ridgeFit(data3,0.01))
plt.scatter(data3[:,0], data3[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



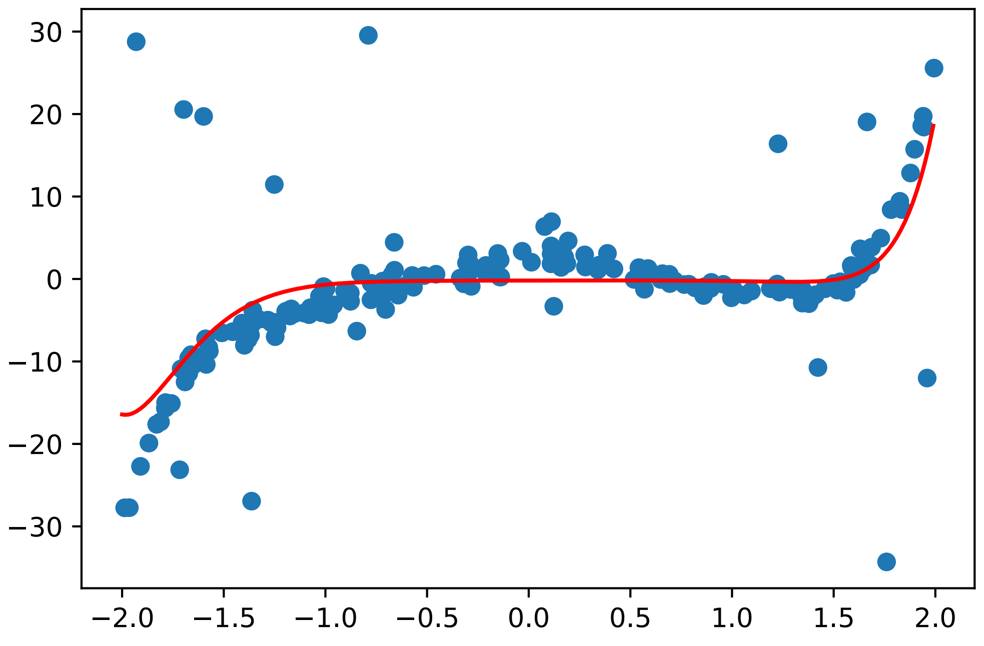
In [20]:

```
# fit on data 3 and test with x_test and lambda = 1
output = np.dot(lift(x_test), ridgeFit(data3,1))
plt.scatter(data3[:,0], data3[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



In [21]:

```
# fit on data 3 and test with x_test and lambda = 50
output = np.dot(lift(x_test), ridgeFit(data3,50))
plt.scatter(data3[:,0], data3[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



EXTRA CREDIT

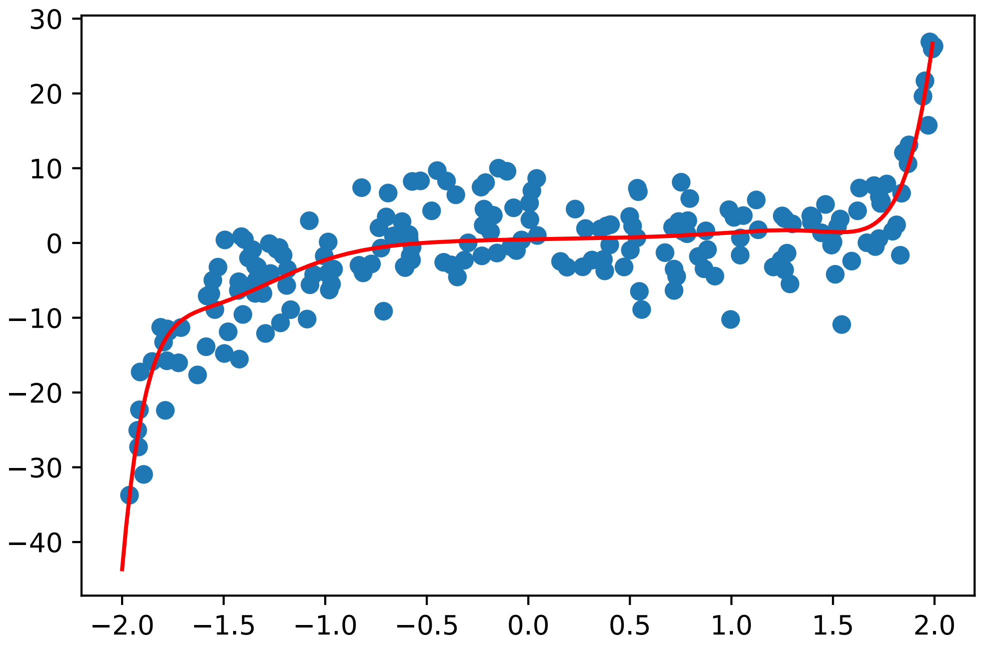
sklearn's Huber robust regressor model

Why and How it works: Huber is a linear regression model that is robust to outliers. The fact that this model is a linear regression model means that we can use it the same way as we have used the previously implemented models in this report. From the graphs we can see that it manages to fit the data1 dataset relatively well. This model has a parameter called sigma that can help us controll how influenced the model is going to be by outliers.

In [22]:
```
from sklearn.linear_model import HuberRegressor
```

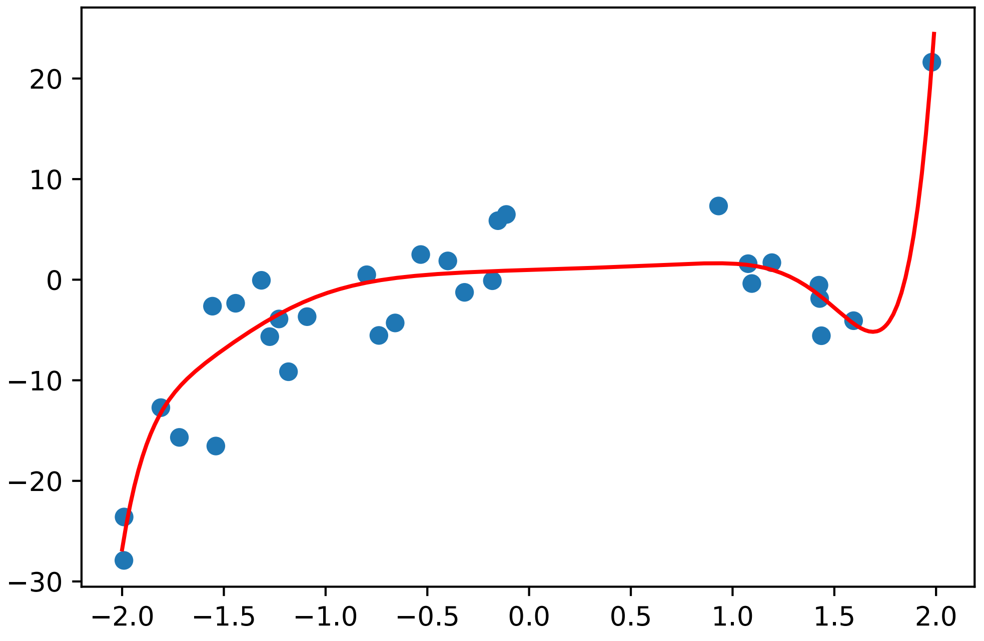
Data 1

In [23]:
```
x = data1[:,0]
y = data1[:,1]
x = lift(x)
huber = HuberRegressor().fit(x,y)
output = huber.predict(lift(x_test))
plt.scatter(data1[:,0], data1[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



Data 2

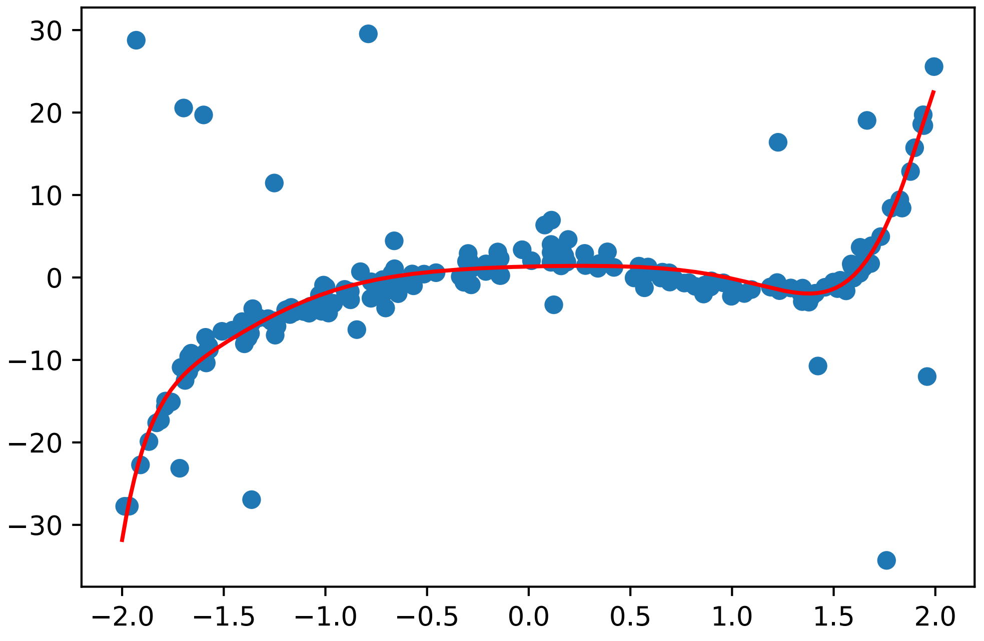
In [24]:
```
x = data2[:,0]
y = data2[:,1]
x = lift(x)
huber = HuberRegressor().fit(x,y)
output = huber.predict(lift(x_test))
plt.scatter(data2[:,0], data2[:,1])
plt.plot(x_test,output, "r")
plt.show()
```



Data 3

In [25]:

```
x = data3[:,0]
y = data3[:,1]
x = lift(x)
huber = HuberRegressor().fit(x,y)
output = huber.predict(lift(x_test))
plt.scatter(data3[:,0], data3[:,1])
plt.plot(x_test,output, "r")
plt.show()
```


30
20
10
0
−10
−20
−30
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0