Chapter 6

The Turboprop cycle

6.1 Propellor efficiency

The turboprop cycle can be regarded as a very high bypass limit of a turbofan. Recall that the propulsive efficiency of a thruster with $P_e = P_0$ and $f \ll 1$ is

$$\eta_{pr} = \frac{2}{1 + U_e/U_0}. \tag{6.1}$$

This expression is relevant to a propeller also where U_e is replaced by U_∞ the velocity, shown schematically in Figure 6.1, that would occur far downstream of the propeller if there were no mixing of the propeller wake.

The achievement of high propulsive efficiency at a given thrust requires a large mass flow with a small velocity increment. The turboprop accomplishes this by using a low pressure turbine to produce shaft power to drive a propeller. Since the propeller disc is quite large the mass flow rate is large and a high propulsive efficiency can be achieved.

The power output of a turbine is proportional to the square of the blade speed and so turbines generally operate at high rotational speeds limited by compressibility effects at the blade tips. The propeller diameter is much larger than the turbine diameter and so to avoid compressibility losses over the outer portion of the propeller a gearbox is required to step the turbine rotational speed down to values that keep the propeller tip Mach numbers below one.

It should be pointed out that Figure 6.1 is purely a schematic and the relative size of the gearbox shown in this figure is quite unrealistic. The figure below shows a cross section of

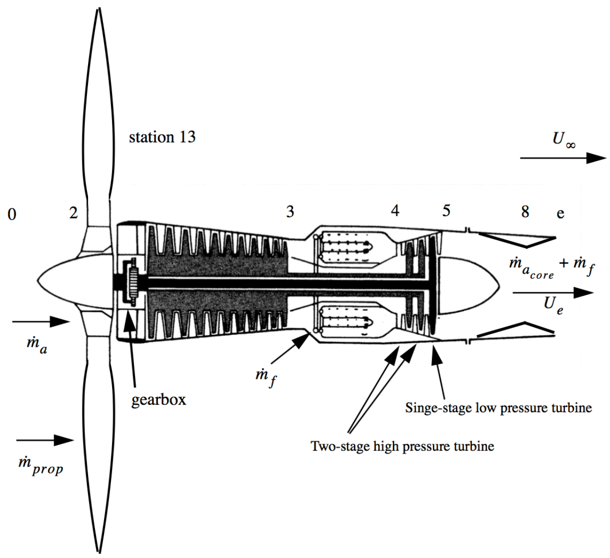

Figure 6.1: *Schematic of a turboprop engine.*

a current model of the Allison T56 turboprop engine. This is a widely used engine that has been in operation since 1954.

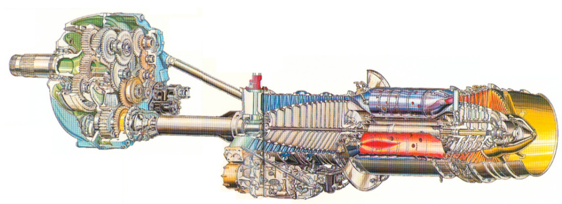

Figure 6.2: *Turboprop engine with its gearbox.*

As the figure shows, the gearbox is massive and contains a dizzyingly complex system of gears and bearings. In fact the main disadvantage of the turboprop is the weight and maintenance cost of the gearbox as well as the maintenance cost of the propeller. In terms of cycle performance, the power losses through the gearbox are also a significant factor.

In any case if an efficient gearbox and propeller system can be developed, a turboprop should be a more efficient cycle than a turbojet or a turbofan at low Mach number. Actual cycle analysis shows that at low to moderate subsonic flight Mach numbers fuel consumption is lower for the turboprop cycle. However in the transonic and supersonic flight regime propeller driven aircraft are prohibitively noisy and propeller efficiency falls off rapidly with

Mach number due to stagnation pressure losses at the blade tips.

Part of the thrust of a turboprop comes from the flow through the core engine. We can analyze the core flow using the same approach used in the turbojet. But a major portion of the thrust comes from the propeller and because the flow through the propeller is unducted there is no simple way to relate the thrust produced by the propeller to the usual flow variables that we can analyze using basic gas-dynamic tools. Such an analysis would require a means of determining the flow speed induced by the propeller infinitely far down stream of the engine. For this reason, the analysis of the turboprop begins with a definition of the propeller efficiency.

$$\eta_{prop} = \frac{T_{prop} U_0}{W_p} \tag{6.2}$$

where W_p is the power supplied to the propeller by the low pressure turbine. As long as the propeller efficiency is known then the propeller thrust is known in terms of the flow through the turbine. To understand the nature of the propeller efficiency it is useful to factor (6.2) as follows. The thrust produced by the propeller is

$$T_{prop} = \dot{m}_{prop} \left(U_\infty - U_0\right). \tag{6.3}$$

Using (6.3) the propeller efficiency can be factored as

$$\eta_{prop} = \left(\frac{\dot{m}_{prop}\left(U_\infty - U_0\right)U_0}{\frac{1}{2}\dot{m}_{prop}\left({U_\infty}^2 - {U_0}^2\right)}\right)\left(\frac{\frac{1}{2}\dot{m}_{prop}\left({U_\infty}^2 - {U_0}^2\right)}{W_p}\right). \tag{6.4}$$

The propeller efficiency factors into a product of a propulsive efficiency multiplying a term that compares the change in kinetic energy across the propeller to the shaft work.

$$\eta_{prop} = \left(\frac{2U_0}{U_\infty + U_0}\right)\left(\frac{\frac{1}{2}\dot{m}_{prop}\left({U_\infty}^2 - {U_0}^2\right)}{W_p}\right) \tag{6.5}$$

Let's look at this from a thermodynamic point of view. The stagnation enthalpy rise across the propeller produced by the shaft work is

$$W_p = \dot{m}_{prop} \left(h_{t13} - h_{t2}\right). \tag{6.6}$$

In the simplest model of propeller flow, the propeller is treated as an actuator disc, a uniform disc over which there is a pressure and temperature rise that is constant over the disc area. The flow velocity increases up to and through the disc while the flow velocity is the same just ahead and just behind the disc as shown in Figure 6.3. According to Froude's theorem the velocity change ahead of the propeller is the same as behind and so

$$U_2 = U_{13} = \frac{U_0 + U_\infty}{2}. \tag{6.7}$$

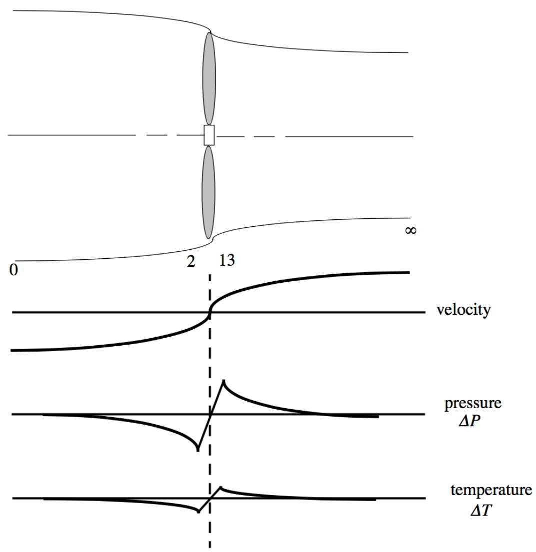

Figure 6.3: *Effect of propeller actuator disc on flow velocity, pressure and temperature.*

Since the velocity is the same before and after the propeller we can write

$$W_p = \dot{m}_{prop} C_p \left(T_{13} - T_2 \right). \tag{6.8}$$

The stagnation pressure and stagnation temperature across the propeller are related by a polytropic efficiency of compression which accounts for the entropy rise across the propeller.

$$\frac{P_{t13}}{P_{t2}} = \left(\frac{T_{t13}}{T_{t2}} \right)^{\frac{\gamma \eta_{pc}}{\gamma - 1}} \tag{6.9}$$

Equation (6.9) can be written as

$$\frac{P_{13}}{P_2}\left(\frac{1+\frac{\gamma-1}{2}{M_{13}}^2}{1+\frac{\gamma-1}{2}{M_2}^2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_{13}}{T_2}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}}\left(\frac{1+\frac{\gamma-1}{2}{M_{13}}^2}{1+\frac{\gamma-1}{2}{M_2}^2}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}}. \tag{6.10}$$

The Mach number change across the propeller is small due to the small temperature change and to a good approximation (6.10) relates the static temperatures and pressures

$$\frac{P_{13}}{P_2} = \left(\frac{T_{13}}{T_2}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}} \tag{6.11}$$

which we can write as

$$1+\left(\frac{P_{13}-P_2}{P_2}\right) = \left(1+\frac{T_{13}-T_2}{T_2}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}}. \tag{6.12}$$

For a propeller that is lightly loaded, (6.12) can be approximated as

$$\frac{P_{13}-P_2}{P_2} \cong \frac{\gamma\eta_{pc}}{\gamma-1}\left(\frac{T_{13}-T_2}{T_2}\right) \tag{6.13}$$

or

$$P_{13}-P_2 \cong \eta_{pc}\rho_2 C_p\left(T_{13}-T_2\right). \tag{6.14}$$

The propeller thrust is

$$T = \left(P_{13}-P_2\right)A \tag{6.15}$$

where A is the effective area of the actuator disc. Now combine (6.8), (6.14) and (6.15) to form the propeller efficiency.

$$\eta_{prop} = \frac{\eta_{pc}\rho_2 U_0 A C_p\left(T_{13}-T_2\right)}{\dot{m}_{prop}C_p\left(T_{13}-T_2\right)} = \frac{\eta_{pc}\rho_2 U_0 A}{\rho_2 U_2 A} \tag{6.16}$$

Using (6.7), the propeller efficiency becomes finally

$$\eta_{prop} = \left(\frac{2U_0}{U_0+U_\infty}\right)\eta_{pc} \tag{6.17}$$

which should be compared with (6.5). We can now interpret the energy factor in (6.5) as

$$\eta_{pc} \cong \frac{\frac{1}{2}\dot{m}_{prop}\left({U_\infty}^2 - {U_0}^2\right)}{W_p}. \tag{6.18}$$

The polytropic efficiency is also related to the entropy change across the propeller. From (6.13)

$$\frac{dP}{P} = \eta_{pc}\left(\frac{\gamma}{\gamma - 1}\right)\frac{dT}{T}. \tag{6.19}$$

According to the Gibbs equation, the entropy change across a differential part of the compression process is

$$\frac{ds}{C_p} = \frac{dT}{T} - \left(\frac{\gamma - 1}{\gamma}\right)\frac{dP}{P} = (1 - \eta_{pc})\frac{dT}{T} \tag{6.20}$$

and so the propeller efficiency can also be written as

$$\eta_{prop} = \left(\frac{2U_0}{U_0 + U_\infty}\right)\left(1 - \frac{T}{C_p}\frac{ds}{dT}\right). \tag{6.21}$$

These results tell us that the propeller efficiency is determined by two distinct mechanisms. The first is the propulsive efficiency that is directly related to the propeller loading (Thrust/Area). The higher the loading, the more power is lost to increasing the kinetic energy of the flow. A very highly loaded propeller is sensitive to blade stall and one of the advantages of putting a duct around the propeller, turning it into a fan, is that a higher thrust per unit area can be achieved.

The second mechanism is the entropy rise across the propeller due to viscous friction and stagnation pressure losses due to compressibility effects. A well designed propeller should achieve as low an entropy rise per unit temperature rise, ds/dT, as possible. Note that even if we could design a propeller that operated isentropically it would still have an efficiency that is less than one.

6.2 Work output coefficient

The thrust equation for the turboprop is

$$T_{total} = T_{core} + T_{prop}$$

$$or \tag{6.22}$$

$$T = \dot{m}_a \left(U_e - U_0\right) + \dot{m}_f U_e + \left(P_e - P_0\right) A_e + \dot{m}_{prop} \left(U_\infty - U_0\right).$$

Substitute the propeller efficiency

$$T_{total} = \dot{m}_a \left(U_e - U_0\right) + \dot{m}_f U_e + \left(P_e - P_0\right) A_e + \eta_{prop} \frac{W_p}{U_0}. \tag{6.23}$$

The core thrust has the usual form

$$\frac{T_{core}}{\dot{m}_a a_0} = M_0 \left(\left(1 + f\right) \frac{M_e}{M_0} \sqrt{\frac{T_e}{T_0}} - 1 \right) \tag{6.24}$$

where the nozzle is taken to be fully expanded $P_e = P_0$.

The presence of U_0 in the denominator of (6.23) indicates the inadequacy of the propeller efficiency for describing propeller thrust at low speeds. As a consequence the performance of a turboprop is usually characterized in terms of power output instead of thrust. Define the work output coefficient as

$$C_{total} = \frac{T_{total} U_0}{\dot{m}_a C_p T_0} = C_{core} + C_{prop} \tag{6.25}$$

where from (6.23)

$$C_{core} = \left(\gamma - 1\right) {M_0}^2 \left(\left(1 + f\right) \frac{M_e}{M_0} \sqrt{\frac{T_e}{T_0}} - 1 \right) \tag{6.26}$$

and

$$C_{prop} = \eta_{prop} \frac{W_p}{\dot{m}_a C_p T_0}. \tag{6.27}$$

The work output coefficient and the dimensionless thrust are directly proportional to one another

$$\frac{T}{P_0 A_0} = \frac{\gamma}{\gamma - 1} C_{total}. \tag{6.28}$$

The fuel efficiency of the turboprop is expressed in terms of the specific horsepower

$$SHP = \frac{pounds\ of\ fuel\ burned\ per\ hour}{output\ horsepower} = 3600 \frac{\dot{m}_f g}{T_{total} U_0}$$

$$or \tag{6.29}$$

$$SHP = \frac{2545}{C_p T_0} \left(\frac{f}{C_{total}} \right)$$

where the temperature is in degrees Rankine and the heat capacity is in BTU/lbm-hr.

6.3 Power balance

The turbine drives both the compressor and propeller. The power to the propeller is

$$W_p = \eta_g \left(\left(\dot{m}_a + \dot{m}_f \right) \eta_m \left(h_{t4} - h_{t5} \right) - \dot{m}_a \left(h_{t3} - h_{t2} \right) \right) \tag{6.30}$$

where η_g is the gearbox efficiency and η_m is the shaft mechanical efficiency. Substitute (6.30) into (6.27) and assume the gas is calorically perfect. The work output coefficient of the propeller is expressed in terms of cycle parameters as

$$C_{prop} = \eta_{prop} \eta_g \left((1 + f) \eta_m \tau_\lambda (1 - \tau_t) - \tau_r (\tau_c - 1) \right). \tag{6.31}$$

6.4 The ideal turboprop

The assumptions of the ideal turboprop are essentially the same as for the ideal turbojet, namely

$$\begin{aligned}
\pi_d &= 1 \\
\eta_{pc} &= 1 \\
\pi_b &= 1 \\
\eta_{pe} &= 1 \\
\pi_n &= 1
\end{aligned} \tag{6.32}$$

along with $P_e = P_0$. Notice that the propeller efficiency is not assumed to be one. The ideal turboprop cycle begins with a propeller efficiency below one as reflected in the proportionality of the propeller efficiency to a propulsive efficiency that is inherently less than one. There is a little bit of an inconsistency here in that the compressor is assumed to be isentropic, whereas a small part of the compression of the core air is accomplished by the propeller. This portion of the compression is assumed to be isentropic even though the rest of the propeller may behave non-isentropically. In any case there is no way to distinguish between the propulsive and frictional parts of the propeller efficiency in practice and so all of the entropy change across the propeller can be assigned to the portion of the mass flow that passes outside of the core engine.

The exit Mach number is generated in the same manner as for the ideal turbojet. The Mach number ratio is

$$\left(\frac{M_e}{M_0}\right)^2 = \left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right). \tag{6.33}$$

The temperature ratio is also generated in the same way.

$$\frac{T_e}{T_0} = \frac{\tau_\lambda}{\tau_r \tau_c} \tag{6.34}$$

The work output coefficient of the core is

$$C_{core} = 2\left(\tau_r - 1\right)\left(\left(1 + f\right)\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2}\left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right)^{1/2} - 1\right). \tag{6.35}$$

6.4.1 Optimization of the ideal turboprop cycle

The question now is: what fraction of the total thrust should be generated by the core in order to produce the maximum work output coefficient. The answer to this question is required in order to properly select the size of the turbine. Now determine an extremum in C_{total} with respect to τ_t.

$$\frac{\partial C_{total}}{\partial \tau_t} = \frac{\partial C_{core}}{\partial \tau_t} + \frac{\partial C_{prop}}{\partial \tau_t} = 0 \tag{6.36}$$

Substitute C_{core}, (6.35) and C_{prop}, (6.31) into (6.36) and carry out the differentiation

$$2\left(\tau_r - 1\right)\left(1+f\right)\left(\frac{\tau_\lambda}{\tau_r\tau_c}\right)^{1/2}\frac{1}{2}\left(\frac{\tau_r\tau_c\tau_t - 1}{\tau_r - 1}\right)^{-1/2}\left(\frac{\tau_r\tau_c}{\tau_r - 1}\right) - \eta_{prop}\eta_g\left(1+f\right)\eta_m\tau_\lambda = 0 \tag{6.37}$$

which simplifies to

$$\left(\tau_\lambda\tau_r\tau_c\right)^{1/2}\left(\frac{\tau_r\tau_c\tau_t - 1}{\tau_r - 1}\right)^{-1/2} = \eta_{prop}\eta_g\eta_m\tau_\lambda. \tag{6.38}$$

Notice that the propeller, gearbox and shaft efficiencies enter the analysis as one product. Let

$$\eta = \eta_{prop}\eta_g\eta_m. \tag{6.39}$$

Square (6.38) and solve for τ_t.

$$\tau_t\big|_{\;max\ thrust\ ideal\ turboprop} = \frac{1}{\tau_r\tau_c} + \frac{(\tau_r - 1)}{\eta^2\tau_\lambda} \tag{6.40}$$

This result essentially defines the size of the turbine needed to achieve maximum work output coefficient which is equivalent to maximum thrust. Lets see what core engine velocity ratio this corresponds to.

$$\left(\frac{U_e}{U_0}\right)^2\Bigg|_{max\,thrust\,ideal\,turboprop} = \frac{\tau_\lambda}{\tau_r\tau_c}\left(\frac{1}{\tau_r - 1}\right)\left(\tau_r\tau_c\,\tau_t\big|_{max\,thrust\,ideal\,turboprop} - 1\right) \tag{6.41}$$

Substitute (6.40) into (6.41). The result is the very simple relationship

$$\left.\frac{U_e}{U_0}\right|_{max\,thrust\,ideal\,turboprop} = \frac{1}{\eta}. \tag{6.42}$$

As the propeller-gearbox-shaft efficiency improves, the optimum turboprop cycle takes a larger and larger fraction of the thrust out of the propeller. This is accomplished with a larger turbine and a smaller core thrust.

The result (6.42) gives us some additional insight into the nature of the propeller efficiency. An ideal turboprop with a propeller that produced isentropic compression would have a core velocity that satisfies

$$\left.U_e\right|_{max\,thrust\,ideal\,turboprop} = \frac{U_0 + U_\infty}{2}. \tag{6.43}$$

The core exit speed would be the average of the upstream and far downstream velocities. Referring back to (6.7) we can see that in this limit the core thruster becomes an indistinguishable part of the propeller actuator disc.

6.4.2 Compression for maximum thrust of an ideal turboprop

Once the turbine has been sized according to the above, then the thrust due to the core engine is fixed by (6.42). It is then a matter of choosing the compressor that maximizes C_{prop}. Differentiate (6.31) with respect to τ_c. Neglect f.

$$\frac{\partial C_{prop}}{\partial \tau_c} = \eta_{prop}\eta_g\left(-\eta_m\tau_\lambda\left(\frac{\partial \tau_t}{\partial \tau_c}\right) - \tau_r\right) = \eta_{prop}\eta_g\left(-\eta_m\tau_\lambda\left(\frac{-1}{\tau_r{\tau_c}^2}\right) - \tau_r\right) \tag{6.44}$$

Maximum C_{prop} is achieved for

$$\tau_c = \frac{\sqrt{\eta_m\tau_\lambda}}{\tau_r} \tag{6.45}$$

which is essentially the same result we obtained for the turbojet (exactly the same if we had included the shaft efficiency in the turbojet analysis). At this point the required turbine temperature ratio can determined from (6.40).

6.5 Turbine sizing for the non-ideal turboprop

The optimization problem is still essentially the same; we need to select the turbine temperature ratio so as to maximize the total work output coefficient.

$$\frac{\partial C_{total}}{\partial \tau_t} = \frac{\partial C_{core}}{\partial \tau_t} + \frac{\partial C_{prop}}{\partial \tau_t} = 0 \tag{6.46}$$

Assume the core flow is fully expanded. The squared velocity ratio across the non-ideal core is

$$\left(\frac{U_e}{U_0}\right)^2 = \frac{1}{\tau_r - 1}\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)\left(\tau_r \tau_c \tau_t - \frac{{\tau_c}^{1-\eta_{pc}} {\tau_t}^{\left(1-\frac{1}{\eta_{pe}}\right)}}{\left(\pi_d \pi_b \pi_n\right)^{\frac{\gamma-1}{\gamma}}}\right) \tag{6.47}$$

and the core work output coefficient of the non-ideal turboprop is

$$C_{core} = 2\left(\tau_r - 1\right)\left(\frac{(1+f)}{\left(\tau_r - 1\right)^{1/2}}\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2}\left(\tau_r \tau_c \tau_t - \frac{{\tau_c}^{1-\eta_{pc}} {\tau_t}^{\left(1-\frac{1}{\eta_{pe}}\right)}}{\left(\pi_d \pi_b \pi_n\right)^{\frac{\gamma-1}{\gamma}}}\right)^{1/2} - 1\right). \tag{6.48}$$

For the non-ideal cycle the condition (6.46) becomes

$$\begin{gathered}\left(\tau_r - 1\right)^{1/2}\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2}\left(\tau_r \tau_c \tau_t - \frac{{\tau_c}^{1-\eta_{pc}} {\tau_t}^{\left(1-\frac{1}{\eta_{pe}}\right)}}{\left(\pi_d \pi_b \pi_n\right)^{\frac{\gamma-1}{\gamma}}}\right)^{-1/2} \times \\ \left(\tau_r \tau_c - \left(1 - \frac{1}{\eta_{pe}}\right)\frac{{\tau_c}^{1-\eta_{pc}} {\tau_t}^{\left(-\frac{1}{\eta_{pe}}\right)}}{\left(\pi_d \pi_b \pi_n\right)^{\frac{\gamma-1}{\gamma}}}\right) - \eta_{prop}\eta_g \eta_m \tau_\lambda = 0.\end{gathered} \tag{6.49}$$

Various flow parameters are specified in (6.49) and the turbine temperature ratio for maximum work output coefficient is determined implicitly. Figure 6.4 shows a typical calculation.

The optimum turbine temperature ratio increases with non-ideal effects indicating that a larger fraction of the total thrust is developed across the core engine of a non-ideal turboprop.

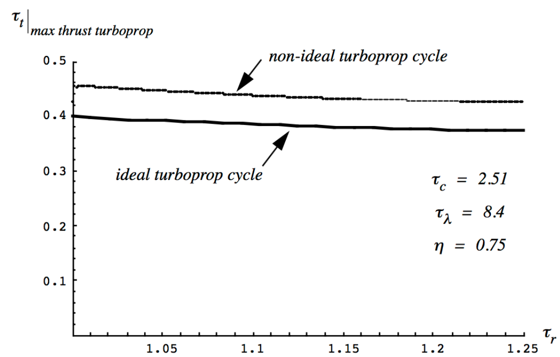

Figure 6.4: *Comparison of turbine selection for the ideal and non-ideal turboprop cycle. Parameters of the non-ideal cycle are* $\pi_d = 0.97$, $\eta_{pc} = 0.93$, $\pi_b = 0.96$, $\eta_{pe} = 0.95$, $\pi_n = 0.98$.

6.6 Problems

Problem 1 - Consider the propeller shown in Figure 6.5.

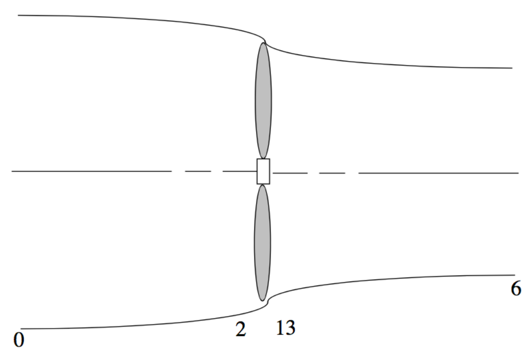

Figure 6.5: *Flow through an actuator disc.*

Show that for small Mach number, the velocity at the propeller is approximately

$$U_2 = U_{13} = \frac{U_0 + U_\infty}{2}. \tag{6.50}$$

In other words one-half the velocity change induced by the propeller occurs upstream of the propeller. This is known as Froude's theorem and is one of the cornerstones of propeller theory.

Problem 2 - Compare ducted versus unducted fans. Let the fan area be the same in both cases.

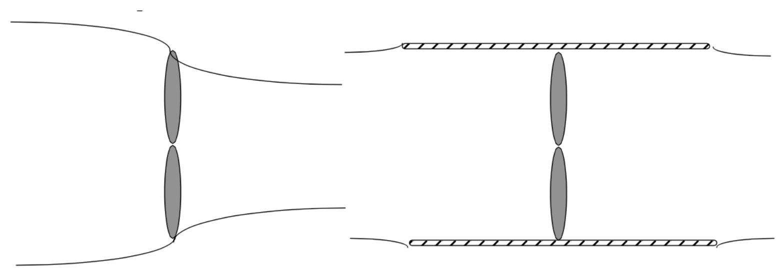

Figure 6.6: *Ducted and unducted fans.*

Show that, for the same power input, the ducted case produces more thrust. Note that the operating point of the ducted fan is chosen to produce a capture area equal to the area of the duct. Suppose the operating point were changed. How would your answer change?

Problem 3 - Derive equation (6.28).

Problem 4 - Use Matlab or Mathematica to develop a program that reproduces Figure 6.4.

Problem 5 - An ideal turboprop engine operates at a free stream Mach number, $M_0 = 0.7$. The propeller efficiency is, $\eta_{prop} = 0.8$ and the gearbox and shaft efficiencies are both 1.0. The turbine is chosen to maximize the total work output coefficient. The compressor is chosen according to, $\tau_c = \sqrt{\tau_\lambda}/\tau_r$ and $\tau_\lambda = 6$. Determine the dimensionless thrust, T/P_0A_0 where A_0 is the capture area corresponding to the air flow through the core engine. Assume $f \ll 1$. Is the exit nozzle choked?.

Problem 6 - A non-ideal turboprop engine operates at a free stream Mach number, $M_0 = 0.6$. The propeller efficiency is, $\eta_{prop} = 0.8$ and the gearbox and shaft efficiencies are both 1.0. The operating parameters of the engine are $\tau_\lambda = 7$, $\tau_c = 2.51$, $\pi_d = 0.97$, $\eta_{pc} = 0.93$, $\pi_b = 0.96$, $\eta_{pe} = 0.95$, $\pi_n = 0.98$. Determine the dimensionless thrust, T/P_0A_0 where A_0 is the capture area corresponding to the air flow through the core engine. Do not assume $f \ll 1$.

Problem 7 - A turboprop engine operates at a free stream Mach number, $M_0 = 0.6$. The propeller efficiency is $\eta_{prop} = 0.85$, the gearbox efficiency is $\eta_g = 0.95$, and the shaft efficiency is $\eta_m = 1$. All other components operate ideally and the exhaust is fully expanded $P_e = P_0$. The operating parameters of the engine are $\tau_\lambda = 7$ and $\tau_c = 2.51$. Assume the turbine is sized to maximize C_{total} and assume $f \ll 1$. Determine the total work output coefficient C_{total} and dimensionless thrust T/P_0A_0. The ambient temperature and pressure are $T_0 = 216\,K$ and $P_0 = 2 \times 10^4 N/m^2$.

Problem 8 - A propulsion engineer is asked by her supervisor to determine the thrust of a turboprop engine at cruise conditions. The engine is designed to cruise at $M_0 = 0.5$. At that Mach number the engine is known to be operating close to its maximum total work output coefficient C_{total}. She responds by asking the supervisor to provide some data on the operation of the engine at this condition. List the minimum information she would need in order to provide a rough estimate of the thrust of the engine. What assumptions would she need to make in order to produce this estimate?