EJERCICIOS BAYES

1.

$P(V) = 85\% = 0'85$

$P(A) = 15\% = 0'15$

1) A priori, 15%, porque $P(A) = 0'15$

2) $H = \{azul, verde\}$

Testigo dice azul $T = azul$

$$P(H=azul \mid T=azul) = \frac{P(T=azul \mid H=azul) \cdot P(H=azul)}{P(T=azul)} = (*)$$

$$P(T=azul) = P(T=azul \mid H=azul) \cdot P(H=azul) + P(T=azul \mid H=verde) \cdot P(H=ver$$

$$= 0'8 \cdot 0'15 + 0'2 \cdot 0'85 = 0'29$$

$$\Rightarrow (*) = \frac{0'8 \cdot 0'15}{0'29} = 0'41$$

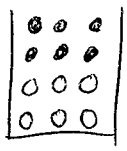
3) $P(T=azul \mid H=verde) = 0'2 < P(T=azul \mid H=azul) = 0'8$

$\Rightarrow H = azul$ (MV)

$$P(H=verde \mid T=azul) = \frac{P(T=azul \mid H=verde) \cdot P(H=verde)}{0'29} = \frac{0'2 \cdot 0'85}{0'29} = 0'5$$

$\Rightarrow$ (MAP) dice que $H = verde$

2.



1) Hay 5 blancas y 6 negras

$$\mathbb{P}(\text{'sacar blanca'}) = \frac{5}{5+6} = \frac{5}{11} = 0'\overline{45}$$

2)

$$\mathbb{P}(P=\text{'blanca'}) = \mathbb{P}(S=\text{'blanca'} \mid P=\text{'blanca'}) \cdot \mathbb{P}(P=\text{'blanca'}) +$$

$$+ \mathbb{P}(S=\text{'blanca'} \mid P=\text{'negra'}) \cdot \mathbb{P}(P=\text{'negra'}) =$$

$$= \frac{5}{11} \cdot \frac{6}{12} + \frac{6}{11} \cdot \frac{6}{12} = \frac{1}{2} = 0'5$$

$$\mathbb{P}(P=\text{'blanca'} \mid S=\text{'blanca'}) = \frac{\mathbb{P}(S=\text{'blanca'} \mid P=\text{'blanca'}) \cdot \mathbb{P}^{''}(\text{'blanca'})}{\mathbb{P}(S=\text{'blanca'})}$$

$$\mathbb{P}(S=\text{'blanca'}) = \mathbb{P}(S=\text{'blanca'} \mid P=\text{'blanca'}) \cdot \mathbb{P}(P=\text{'blanca'}) +$$

$$+ \mathbb{P}(S=\text{'blanca'} \mid P=\text{'negra'}) \cdot \mathbb{P}(P=\text{'negra'}) =$$

$$= \frac{5}{11} \cdot \frac{1}{2} + \frac{6}{11} \cdot \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \circledast = \frac{5/11 \cdot 1/2}{1/2} = \frac{5}{11} = 0'\overline{54}$$

3. $P(\text{lluvia}) = 0'2$

$P(\neg\text{lluvia}) = 0'8$

$P(\text{paraguas} \mid \text{llueve}) = 0'7$

$P(\text{paraguas} \mid \neg\text{llueve}) = 0'1$

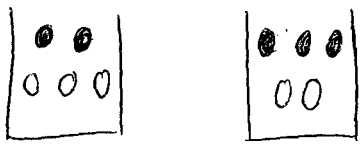
$$P(\text{llueve} \mid \text{paraguas}) = \frac{P(\text{paraguas} \mid \text{llueve}) \cdot P(\text{llueve})}{P(\text{paraguas})} = \circledast$$

$$P(\text{paraguas}) = P(\text{paraguas} \mid \text{llueve}) \cdot P(\text{llueve}) + P(\text{paraguas} \mid \neg\text{llueve}) \cdot P(\neg\text{llueve}) =$$

$$= 0'7 \cdot 0'2 + 0'1 \cdot 0'8 = 0'22$$

$$\Rightarrow \circledast = \frac{0'7 \cdot 0'2}{0'22} = \frac{7}{11} = 0'6\widehat{3}$$

4.



A B

$P(A) = 0'5 \Rightarrow P(B) = 0'5$

$P(N \mid A) = \frac{2}{5} = 0'4$

$P(B \mid A) = \frac{3}{5} = 0'6$

$P(N \mid B) = \frac{3}{5} = 0'6$

$P(B \mid B) = \frac{2}{5} = 0'4$

1) $P(N) = P(N \mid A) \cdot P(A) + P(N \mid B) \cdot P(B) = 0'4 \cdot 0'5 + 0'6 \cdot 0'5 = 0'5$

) $P(A \mid N) = \frac{P(N \mid A) \cdot P(A)}{P(N)} = \frac{0'4 \cdot 0'5}{0'5} = 0'4$

$P(B \mid N) = 0'6$

} MAP dice B

) MAP hecho en el 2)

MV: $P(N \mid A) = 0'4$, $P(N \mid B) = 0'6 \Rightarrow$ MV dice B

5. $\mathbb{P}(N|A) = \frac{2}{5} = 0'4$ $\quad \mathbb{P}(N|B) = 0'6$ $\quad \mathbb{P}(A) = 0'75$

$\mathbb{P}(B|A) = 0'6$ $\quad \mathbb{P}(B|B) = 0'4$ $\quad \mathbb{P}(B) = 0'25$

1) $\mathbb{P}(N) = \mathbb{P}(N|A) \cdot \mathbb{P}(A) + \mathbb{P}(N|B) \cdot \mathbb{P}(B) =$

$= 0'4 \cdot 0'75 + 0'6 \cdot 0'25 = 9/20$

2) $\mathbb{P}(A|N) = \dfrac{\mathbb{P}(N|A) \cdot \mathbb{P}(A)}{\mathbb{P}(N)} = \dfrac{0'4 \cdot 0'75}{9/20} = \dfrac{2}{3}$ $\Rightarrow$ MAP dice A

$\mathbb{P}(B|N) = 1 - \frac{2}{3} = \frac{1}{3}$

3) MAP hecho en 2)

MV: $\mathbb{P}(N|A) = 0'4$, $\mathbb{P}(N|B) = 0'6 \Longrightarrow$ MV dice B