Ejercicios Laboratorio.pdf (parte1)

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1 EJERCICIOS Laboratorio.pdf - Alejandro Santorum

EJERCICIO 1 - Demuestra por inducción sobre $n \in \mathbb{N}$ las afirmaciones siguientes:

1. $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

```
In [1]: [(sum(k^2 for k in srange (1, n+1))) == ((n*(n+1)*((2*n)+1))/6)
         for n in srange(10, 20)]
        #incrementar el valor de n en el for para comprobar un mayor rango de números
```

```
Out[1]: [True, True, True, True, True, True, True, True, True, True]
```

2.$\frac{1}{1*2} + \frac{1}{2*3} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}, \forall n >= 1$

```
In [2]: [(sum(1/(k*(k+1)) for k in srange (1, n+1)))  == (n/(n+1)) for n in srange(1, 10)]
        #incrementar el valor de n en el for para comprobar un mayor rango de números
```

```
Out[2]: [True, True, True, True, True, True, True, True, True]
```

$3.1 * 1! + 2 * 2! + ... + n * n! = (n-1)! - 1$

```
In [3]: [(sum(k*factorial(k) for k in srange (1, n+1))) == (factorial(n+1) - 1)
         for n in srange (1, 10)]
        #incrementar el valor de n en el for para comprobar un mayor rango de números
```

```
Out[3]: [True, True, True, True, True, True, True, True, True]
```

4.$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + ... + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}$

```
In [4]: [(sum(k/(2^k) for k in srange (1, n+1))) == (2-((n+2)/(2^n)))
         for n in srange (1, 10)]
        #incrementar el valor de n en el for para comprobar un mayor rango de números
```

```
Out[4]: [True, True, True, True, True, True, True, True, True]
```

5.$(1+q)(1+q^2)(1+q^4)...(1+q^{2^n}) = \frac{1-q^{2^{n+1}}}{1-q}$

```
In [5]: [(mul(1+(q^(2^k)) for k in srange(0,n+1))) == ((1-(q^(2^(n+1))))/(1-q))
         for n in srange (1, 5) for q in srange (2, 5)]
        #incrementar el valor de n y el de q(NUNCA IGUAL A 1)
        #para comprobar un mayor rango de números
```

```
Out[5]: [True, True, True, True, True, True, True, True, True, True, True, True]
```

EJERCICIO 3 - Demostrar por inducción la fórmula para la suma de los n primeros cubos:

$$1^3 + 2^3 + ... + n^3 = \frac{(n+1)^2 n^2}{4}$$

```
In [6]: [(sum(k^3 for k in srange(1, n+1))) == ((((n+1)^2)*(n^2))/4) for n in srange (1,10)]
        #incrementar el valor de n en el for para comprobar un mayor rango de números
```

```
Out[6]: [True, True, True, True, True, True, True, True, True]
```

EJERCICIO 4 - Estudiar el límite de las siguientes sucesiones:

a) $\left\{\frac{n^2}{n+2}\right\}$

```
In [7]: var('n')
        l(n) = (n^2)/(n+2)
        l.limit(n = infinity)
```

```
Out[7]: n |--> +Infinity
```

b) $\left\{\frac{n^3}{n^3+2n+1}\right\}$

```
In [8]: var('n')
        l(n) = (n^3)/(n^3 + 2*n + 1)
        l.limit(n = infinity)
```

```
Out[8]: n |--> 1
```

c) $\left\{\frac{n}{n^2-n-4}\right\}$

```
In [9]: var('n')
        l(n)=(n)/(n^2 -n -4)
        l.limit(n = infinity)
```

```
Out[9]: n |--> 0
```

d) $\left\{\frac{\sqrt{2n^2-1}}{n+2}\right\}$

```
In [10]: var('n')
         l(n)=(sqrt(2*(n^2)-1))/(n+2)
         l.limit(n = infinity)
```

```
Out[10]: n |--> sqrt(2)
```

e) $\left\{\frac{\sqrt{n^3+2n}+n}{n^2+2}\right\}$

In [11]:
```
var('n')
l(n)=(sqrt(n^3+2*n)+n)/(n^2+2)
l.limit(n = infinity)
```

Out[11]:
```
n |--> 0
```

f) $\left\{\frac{\sqrt{n+1}+n^2}{\sqrt{n+2}}\right\}$

In [12]:
```
var('n')
l(n)=(sqrt(n+1)+n^2)/(sqrt(n+2))
l.limit(n = infinity)
```

Out[12]:
```
n |--> +Infinity
```

g) $\left\{\frac{(-1)^n n^2}{n^2+2}\right\}$

In [13]:
```
var('n')
l(n)=(((-1)^n)*n^2)/(n^2+2)
l.limit(n = infinity)
```

Out[13]:
```
n |--> ind
```

h) $\left\{\frac{n+(-1)^n}{n}\right\}$

In [14]:
```
var('n')
l(n)=(n+(-1)^n)/(n)
l.limit(n = infinity)
```

Out[14]:
```
n |--> 1
```

i) $\left\{\left(\frac{2}{3}\right)^n\right\}$

In [15]:
```
var('n')
l(n)=(2/3)^n
l.limit(n = infinity)
```

Out[15]:
```
n |--> 0
```

j) $\left\{\left(\frac{5}{3}^n\right)\right\}$

In [16]:
```
var('n')
l(n)=(5/3)^n
l.limit(n = infinity)
```

Out[16]:
```
n |--> +Infinity
```

k) $\left\{\frac{2^n}{4^n+1}\right\}$

```
In [17]: var('n')
         l(n) = (2^n)/(4^n+1)
         l.limit(n = infinity)
```

```
Out[17]: n |--> 0
```

l) $\left\{\frac{3^n+(-2)^n}{3^{n+1}+(-2)^{n+1}}\right\}$

```
In [18]: var('n')
         l(n) = (3^n+(-2)^n)/(3^(n+1)+(-2)^(n+1))
         l.limit(n = infinity)
```

```
Out[18]: n |--> 1/3
```

m) $\left\{\frac{n}{n+1} - \frac{n+1}{n}\right\}$

```
In [19]: var('n')
         l(n) = ((n)/(n + 1) - (n+1)/n)
         l.limit(n = infinity)
```

```
Out[19]: n |--> 0
```

n) $\left\{\sqrt{n+1} - \sqrt{n}\right\}$

```
In [20]: var('n')
         l(n) = (sqrt(n+1)-sqrt(n))
         l.limit(n = infinity)
```

```
Out[20]: n |--> 0
```

ñ) $\left\{\frac{1}{n^2} + \frac{2}{n^2} + ... + \frac{n}{n^2}\right\}$

EJERCICIO 5 - Calcular, si existen, los límites de las sucesiones que tienen como término general:

a) $a_n = (\frac{n^2+1}{n^2})^{2n^2-3}$

```
In [21]: var('n')
         l(n) = ((n^2+1)/(n^2)^(2*n^2-3))
         l.limit(n = infinity)
```

```
Out[21]: n |--> 0
```

b) $b_n = (\frac{n^2-1}{n^2})^{2n^2-3}$

```
In [22]: var('n')
         l(n) = ((n^2-1)/(n^2)^(2*n^2-3))
         l.limit(n = infinity)
```

```
Out[22]: n |--> 0
```

c) $c_n = a_n + \frac{1}{b_n}$

In [23]:
```
var('n')
f(n) = ((n^2+1)/(n^2)^(2*n^2-3))
g(n) = ((n^2-1)/(n^2)^(2*n^2-3))
l(n) = f(n)+(1/g(n))
l.limit(n = infinity)
```

Out[23]:
```
n |--> +Infinity
```