$B = \{V_1, V_2, V_3\} \subset \mathbb{R}^3$

tal que $V_1^* = f$, $f(x,y,z) = x - y$

$V_1^* = e_1^* - e_2^*$

$$\begin{cases} V_1^*(V_1) = 1 \\ V_1^*(V_2) = 0 \\ V_1^*(V_3) = 0 \end{cases} \Rightarrow V_2, V_3 \in \operatorname{Ker} V_1^* = \operatorname{Ker} f = \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$V_1 \notin \operatorname{Ker} f = (1,0,0)$

$$\begin{aligned} V_1^*(V_1) &= 1 = f(V_1) \\ V_1^*(V_2) &= 0 = f(V_2) \\ V_1^*(V_3) &= 0 = f(V_3) \end{aligned} \xrightarrow{\text{base}} V_1^* = f$$

$\forall\, v = \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3$

$V_1^*(v) = \alpha_1 V_1^*(V_1) + \alpha_2 V_1^*(V_2) + \alpha_3 V_1^*(V_3) =$

$= \alpha_1 f(V_1) + \alpha_2 f(V_2) + \alpha_3 f(V_3) = f(v)$

Si A es cuadrada , $A \cdot \operatorname{adj}(A) = |A| \cdot I_{n\times n}$

Si $|A| \neq 0 \rightsquigarrow A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj}(A)$

Si $\exists A^{-1} \longrightarrow \underset{1}{\underset{\shortparallel}{\det(I)}} = \det(A \cdot A^{-1}) = \underset{0}{\underset{\nparallel}{\det(A)}} \cdot \det(A^{-1})$

$V = \langle V_1 = (1, -1, 2) \ , \ V_2 = (2, 1, -1) \rangle \subset \mathbb{R}^3$

Escribir V como la solución de un sistema de ecuaciones

Es lo mismo calcular (una base de) V°

$$\mathcal{B} = \{\varphi_1, \varphi_2, \dots, \varphi_k\}$$

Ecuaciones $\begin{cases} \varphi_1(x,y,z) = 0 \\ \vdots \\ \varphi_k(x,y,z) = 0 \end{cases}$

$$ax + by + cz = 0$$

$$\begin{cases} a \cdot 1 + b \cdot (-1) + c \cdot 2 = 0 \\ a \cdot 2 + b \cdot 1 + c \cdot (-1) = 0 \end{cases}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \end{pmatrix} \rightsquigarrow \text{solución} \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\Downarrow$$

$$\{-e_1^* + 5e_2^* + 3e_3^*\}$$

$$\boxed{\text{Ecuación: } -x + 5y + 3z = 0}$$

$$(\operatorname{Ker} f)^\circ = \operatorname{Im} f^*$$

<u>Ejemplo:</u>

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \end{pmatrix} \rightsquigarrow \left\langle \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\rangle = \operatorname{Ker} f$$

$(0 \quad 1 \quad -1) \rightsquigarrow$ sist. homogéneo

$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$ coords de base de $(\operatorname{Ker} f)^\circ$

$$\{ e_1^*, \ e_2^* + e_3^* \}$$

Vamos a calcular ahora $\operatorname{Im} f^*$

$f^* = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \to$ miras las columnas independientes

$$\text{Base} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$\{ e_1^*, \ -e_1^* + e_2^* + e_3^* \}$$

$f: M_{2\times 2}(\mathbb{R}) \longrightarrow \mathbb{R}^3$

$f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b, 0, d)$

$\{V_1^*, V_2^*, V_3^*\}$ base dual de $\{V_1 = (1,0,0)\ ,\ V_2 = (1,1,0)\ ,\ V_3 = (1,1,1)\}$

CALCULAR $f^*(V_3^*)$

$$\begin{array}{ccc} M_{2\times 2}(\mathbb{R})^* & \xleftarrow[f^*]{} & (\mathbb{R}^3)^* \\ f^*(V_3^*) & \longleftarrow & V_3^* \end{array}$$

$$[f^*(V_3^*)]\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = V_3^*\ f\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = V_3^*(3,0,4) = V_3^*(3V_1 - 4V_2 + 4V_3) = 4$$

$\Rightarrow$ LO MISMO CON $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Describir el $\operatorname{Ker} f^*$ y $(\operatorname{Im} f)^\circ$

$$\operatorname{Im} f = \langle f(e_1), f(e_2), f(e_3), f(e_4) \rangle = \langle (1,0,0)\ ,\ (0,0,1) \rangle$$

$$f(e_1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\quad f(e_2) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\quad f(e_3) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},\quad f(e_4) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$