PARTE 1 DEL EXAMEN FINAL DE MOODLE

$\boxed{1.}$ c) ¿ $A^e_{BL}(N)$?

$$P(K == T[i]) = \frac{1}{C_N} \cdot i^{3/4}$$

$$A^e_{BL}(N) = \frac{S_N}{C_N} \quad ; \quad C_N = \sum_{i=1}^{N} i^{3/4}$$

$$A^e_{BL}(N) = \sum n \times p = \sum_{1}^{N} i \, \frac{i^{3/4}}{C_N} = \frac{1}{C_N} \cdot \underbrace{\sum_{1}^{N} i^{7/4}}_{S_N}$$

$$\int_0^N x^{7/4}\,dx \leq S_N \leq \int_1^{N+1} x^{7/4}\,dx \Rightarrow \left[\frac{4x^{11/4}}{11}\right]_0^N \leq S_N \leq \left[\frac{4x^{11/4}}{11}\right]_1^{N+1} \Rightarrow$$

$$\Rightarrow \frac{4N^{11/4}}{11} \leq S_N \leq \frac{4}{11}(N+1)^{11/4} - \frac{4}{11} \Rightarrow$$

$$\Rightarrow 1 \leq \frac{S_N}{\frac{4}{11}N^{11/4}} \leq \underbrace{\frac{\frac{4}{11}(N+1)^{11/4}}{\frac{4}{11}N^{11/4}}}_{\xrightarrow{n\to\infty} 1} - \underbrace{\frac{\frac{4}{11}\cdot 1}{\frac{4}{11}\cdot N^{11/4}}}_{\xrightarrow{n\to\infty} 0} \Rightarrow \frac{S_N}{\frac{4}{11}N^{11/4}} \xrightarrow{n\to\infty} 1$$

$$\Rightarrow \boxed{S_N \sim \frac{4}{11} N^{11/4}}$$

Análogamente, $\boxed{C_N \sim \frac{4}{7} N^{7/4}}$

$$A_{BL}(N) \sim \frac{S_N}{C_N} \sim \frac{\frac{4}{11}N^{11/4}}{\frac{4}{7}N^{7/4}} = \frac{7}{11} N$$

Demostración:

$$\frac{A_{BL}(N)}{\frac{7}{11}N} = \underbrace{\frac{S_N}{\frac{4}{11}N^{11/4}}}_{\to 1} \cdot \frac{1}{\underbrace{\frac{C_N}{\frac{4}{7}N^{7/4}}}_{\to 1}} \xrightarrow{n\to\infty} 1 \qquad \square$$

$\boxed{2.}$ A) $T_1 \sim f$ y $T_2 = o(f)$

a) $T_1 = f + O(T_2)$

¿$|T_1 - f| = O(T_2)$?

$f = N^2$; $T_1 = N^2 + N^{1+\varepsilon}$; $T_2 = N$ contraejemplo

b) $T_1 \cdot T_2 = o(f)$

$$\frac{T_1 T_2}{f} = \underset{?}{T_1} \cdot \underbrace{\frac{T_2}{f}}_{\to 0}$$

falso $\longrightarrow$ contraejemplo.

c) $T_1 T_2 = o(f^2)$

$$\frac{T_1 T_2}{f^2} = \underbrace{\frac{T_1}{f}}_{\to 1} \cdot \underbrace{\frac{T_2}{f}}_{\to 0} \longrightarrow 0$$

verdadero

e) $\frac{T_1}{T_2} = o(1)$

$$\underbrace{T_1/f}_{\to 1} \cdot \frac{1}{\underbrace{T_2/f}_{\to 0}} \longrightarrow \infty$$

falso

d) $\frac{T_2}{T_1} = o(1)$

$$\frac{T_2/T_1}{1} = \frac{T_2}{T_1} = \frac{T_2/f \;]\to 0}{T_1/f \;]\to 1} \longrightarrow 0$$

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$\boxed{2.}$

B) $\sigma = (\overbrace{2K+1 \_\_\_ 3K}^{B_1}, \overbrace{2K \_\_\_ K+1, K \_\_\_ 1}^{B_2})$

$$\boxed{n_A(\sigma) \geq inv(\sigma)}$$

$$inv(\sigma) = inv_{B_1 ext}(\sigma) + inv_{B_1 int}(\sigma) + inv_{B_2 ext}(\sigma) + inv_{B_2 int}(\sigma) =$$

$$= 2K^2 + 0 + 0 + \frac{2K(2K-1)}{2} = 2K^2 + 2K^2 - K =$$

$$= \frac{4N^2}{9} + O(N)$$

$\boxed{3.}$ $S_N = \sum_{n=1}^{N} \frac{\log\sqrt{n}}{n}$

$$\int_0^N \frac{\log\sqrt{x}}{x} dx \geq S_N \geq \int_1^{N+1} \frac{\log\sqrt{x}}{x} dx \Rightarrow \frac{1}{2}\int_0^N \frac{\log x}{x} dx \geq S_N \geq \frac{1}{2}\int_0^{N+1} \frac{\log x}{x} dx$$

$$\Rightarrow \frac{1}{2}\int_1^N \frac{\log x}{x} dx \geq S_N \geq \frac{1}{2}\int_2^{N+1} \frac{\log x}{x} dx \Rightarrow \frac{1}{2}\left[\frac{\log^2 x}{2}\right]_1^N \geq S_N \geq \frac{1}{4}\left[\log^2 x\right]_2^{N+1}$$

$$\Rightarrow \frac{1}{4}\log^2 N \geq S_N \geq \frac{1}{4}\left[\log^2(N+1) - \log^2 2\right] = \frac{\log^2(N+1)}{4} - \frac{\log^2 2}{4}$$

$$\Rightarrow 1 \geq \frac{S_N}{\frac{1}{4}\log^2 N} \geq \underbrace{\frac{\log^2(N+1)}{\cancel{4}} \cdot \frac{\cancel{4}}{\log^2 N}}_{\to 1} - \underbrace{\frac{\log^2 2}{\cancel{4}} \cdot \frac{\cancel{4}}{\log^2 N}}_{\to 0}$$

$$\boxed{S_N \sim \frac{\log^2 N}{4}}$$

$\boxed{4.}$ $T_1 \sim f \longrightarrow \lim_{n\to\infty} \frac{T_1}{f} = 1$ $\qquad T_2 = o(f) \longrightarrow \lim_{n\to\infty} \frac{T_2}{f} = 0$

a) $T_1 + T_2 \sim f$

$$\lim_{n\to\infty} \frac{T_1+T_2}{f} = \underbrace{\lim_{n\to\infty} \frac{T_1}{f}}_{\downarrow\, 1} + \underbrace{\lim_{n\to\infty} \frac{T_2}{f}}_{\downarrow\, 0} \longrightarrow 1$$ correcto

b) $T_1 + T_2 \sim T_1$

$$\lim_{n\to\infty} \frac{T_1+T_2}{T_1} = \underbrace{\lim_{n\to\infty} \frac{T_1}{T_1}}_{\downarrow\, 1} + \lim_{n\to\infty} \frac{T_2}{T_1} = \underbrace{1 + \underbrace{\lim_{n\to\infty} \frac{T_2/f \,]\to 0}{T_1/f \,]\to 1}}_{\downarrow\, 0}}_{\downarrow\, 1}$$ correcto

c) $T_1 - f = O(T_2)$

$T_1 = N^2 + N$

$f = N^2$ $\qquad$ contraejemplo

$T_2 = 1$

d) $T_1{}^2 - T_2{}^2 \sim T_1 \cdot f$

$$\lim_{n\to\infty} \frac{T_1{}^2 - T_2{}^2}{T_1 f} = \underbrace{\lim_{n\to\infty} \frac{\cancel{T_1} T_1}{\cancel{T_1} f}}_{\downarrow\, 1} - \underbrace{\lim_{n\to\infty} \frac{T_2 \cdot T_2}{T_1 \cdot f}}$$

$$\underbrace{\lim_{n\to\infty} \frac{T_2/f}{T_1/f}}_{\downarrow\, 0} \cdot \underbrace{\lim_{n\to\infty} \frac{T_2}{f}}_{\downarrow\, 0}$$

$\Rightarrow 1 - 0\ 0 = 1$ verdadero

$\boxed{5.}$

$$\sigma = (\underbrace{2K \quad 1 \quad 2K-1 \quad 2 \ldots K+2 \quad K-1 \quad K+1 \quad K}_{\substack{2K \text{ elem} \\ B_1}} \; \underbrace{3K \quad 3K-1 \ldots 2K+1}_{\substack{K \text{ elem} \\ B_2}})$$

B_1 int $= 2K-1 + 2K-3 + \cdots + 3 + 1 \longrightarrow$ Suma $2K$ primeros elementos impares. $\equiv S_{impar}$

$$S_{impar} = S_{2K} - S_{par} = \sum_{i=1}^{2K} 1 - 2\sum_{i=1}^{K} = \frac{\cancel{2}K(2K+1)}{\cancel{2}} - \frac{\cancel{2}K(K+1)}{\cancel{2}} =$$

$$= 2K^2 + K - K^2 - K = K^2$$

B_1 ext $= 0$

B_2 ext $= 0$

B_2 int $= \dfrac{K(K-1)}{2}$

$$n_A(\sigma) \geq inv(\sigma) = K^2 + \frac{K(K-1)}{2} = K^2 + \frac{K^2 - K}{2}$$

$N = 3K$

$$\Rightarrow \frac{3N^2}{18} - \frac{N}{8}$$

$\boxed{6.}$

```
print_bin (N):
    mientras N>0:
        print (N%2)
        N= N/2
            ↑ div. entera
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N	$n_{p-b}(N)$
5	3
4	3
7	3
8	4
9	4
5	3

$$\Rightarrow n_{p-b} = \lfloor \log_2 N \rfloor + 1$$