HOJA 1

1. $C\left(1+\frac{R}{m}\right)^{d} = C(1+TAE) \Rightarrow \left(1+\frac{R}{m}\right)^{d} = 1+TAE$

tipo de interés nominal

$$m = \frac{\text{tipo interés}}{\text{tipo comp.}} \qquad d = \frac{\text{total periodo}}{\text{comp. en meses}}$$

total periodo $\equiv$ normalmente, 12 meses

a) 3% anual, comp. trimestral

$$m = \frac{12 \text{ meses}}{3 \text{ meses}} = 4 \qquad d = \frac{12}{3} = 4$$

$$\Rightarrow 1+TAE = \left(1+\frac{0'03}{4}\right)^{4} \Rightarrow \underline{TAE = 3'034\%}$$

b) 0'2% mensual, comp. mensual.

$$m = \frac{1 \text{ mes}}{1 \text{ mes}} = 1 \qquad d = \frac{12}{1} = 12$$

$$\Rightarrow 1+TAE = \left(1+\frac{0'2/100}{1}\right)^{12} \Rightarrow \underline{TAE = 2'427\%}$$

c) 1'3% semestral, comp. trimestral

$$m = \frac{6 \text{ meses}}{3 \text{ meses}} = 2 \qquad d = \frac{12}{3} = 4$$

$$\Rightarrow 1+TAE = \left(1+\frac{1'3/100}{2}\right)^{4} \Rightarrow \underline{TAE = 2'625\%}$$

d) 2'7% anual, comp. continua

$$1+TAE = e^{R} \Rightarrow \underline{TAE} = e^{2'7\%} - 1 = \underline{2'737\%}$$

e) interés 12% en dos años.

$$m = \frac{24 \text{ meses}}{24 \text{ meses}} = 1 \qquad d = \frac{12}{24} = \frac{1}{2}$$

$$\Rightarrow 1+TAE = \left(1+\frac{0'12}{1}\right)^{1/2} \Rightarrow \underline{TAE = 5'830\%}$$

2. TAE = 2'2% . ¿R?

a) R anual, comp. anual

$m = \frac{\tau}{\Delta T} = \frac{1 \text{ año}}{1 \text{ año}} = 1$. $d = \frac{12}{12} = 1$

$\Rightarrow 1 + TAE = \left(1 + \frac{R}{1}\right)^1 \Longrightarrow \underline{R = TAE = 2'20\%}$

b) R anual, comp. cuatrimestral

$m = \frac{\tau}{\Delta T} = \frac{12 \text{ meses}}{4 \text{ meses}} = 3 \quad d = \frac{12}{4} = 3$

$\Rightarrow 1 + TAE = \left(1 + \frac{R}{3}\right)^3 \Rightarrow 1 + \frac{2'2}{100} = \left(1 + \frac{R}{3}\right)^3 \Rightarrow \underline{R = 2'184\%}$

c) R semestral, comp. trimestral.

$m = \frac{6 \text{ meses}}{3 \text{ meses}} = 2 \qquad d = \frac{12}{3} = 4$

$\Rightarrow 1'022 = \left(1 + \frac{R}{2}\right)^4 \Longrightarrow \underline{R = 2\left[\sqrt[4]{1'022} - 1\right] = 1'091\%}$

d) R anual, comp. semestral

$m = \frac{12 \text{ meses}}{6 \text{ meses}} = 2 \qquad d = \frac{12}{6} = 2$

$\Rightarrow 1'022 = \left(1 + \frac{R}{2}\right)^2 \Longrightarrow \underline{R = 2\left[\sqrt{1'022} - 1\right] = 2'188\%}$

) R tipo continuo.

$1 + TAE = e^R \Longrightarrow R = \ln(1 + TAE) \Longrightarrow \underline{R = \ln(1'022) = 2'176}$

$\boxed{3.}$

0 —————— 6 años   R tipo nom. anual, comp. mens

$C = 1000$ €

$$\underbrace{\underset{\underset{1000}{\parallel}}{C}\left(1+\frac{R}{m}\right)^{d}}_{=1220} = \underbrace{1+TAE}_{1220}$$

$$m = \frac{12 \text{ meses}}{1 \text{ mes}} = 12$$

$d =$ ¿cuántos meses hay en 6 años?

$$d = 6.12 = 72$$

$$\Rightarrow \underbrace{1+TAE}_{=1220} = \left(1+\frac{R}{12}\right)^{72} \cdot \underset{\underset{1000}{\parallel}}{C} \Rightarrow \left(1+\frac{R}{12}\right)^{72} = \frac{1220}{1000} \Rightarrow$$

$$\Rightarrow \underline{R} = 12\left[\sqrt[72]{1'22} - 1\right] = \underline{3'319\,\%}$$

$\boxed{4.}$ opción a: duplicar capital en 15 años → $R = 100\,\%$
opción b: $R = 5\%$ anual, capitalización cuatrimestral

a) $m = \frac{\tau}{\Delta T} = \frac{15 \text{ años}}{15 \text{ años}} = 1$ $d = \frac{1 \text{ año}}{15 \text{ años}} = \frac{1}{15}$

$$1 + TAE = \left(1 + \frac{100\%}{1}\right)^{1/15} \Rightarrow \underline{TAE} = \sqrt[15]{2} - 1 = \underline{4'729\,\%}$$

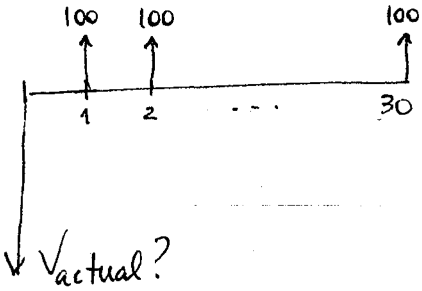
b) $m = \frac{12 \text{ meses}}{4 \text{ meses}} = 3$ $d = \frac{12 \text{ meses}}{4 \text{ meses}} = 3$

$$\left(1 + \frac{5/100}{3}\right)^{3} = 1 + TAE \Rightarrow \underline{TAE = 5'084\,\%}$$

$\Rightarrow$ La opción b es mejor, ya que $\underline{TAE_b > TAE_a}$.

5. $R = 2\%$ anual (efectivo)

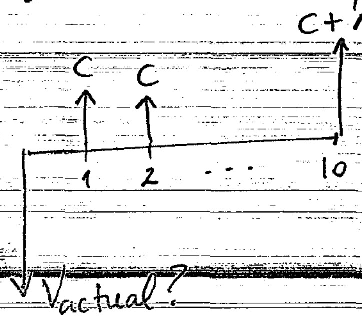
a) renta anual de 100 € durante 30 años.



$$\underline{V_{actual}} = \sum_{j=1}^{30} \frac{100}{(1+0'02)^j} = 100 \sum_{j=1}^{30} \frac{1}{(1+0'02)^j}$$

$$= 100\left[\frac{1}{R}\left(1 - \frac{1}{(1+R)^{30}}\right)\right] = \underline{2239'65}$$

b) bono nominal 100 €, con cupones anuales del $\underbrace{3\%}_{\to Q}$. Vencimiento, en 10 años, paga cupón y devuelve nominal.



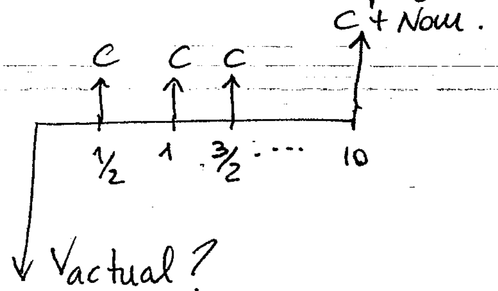
$$C = Q.\,Nom = 0'03 . 100 = 3$$

$$\underline{V_{actual}} = \sum_{j=1}^{10} \frac{3}{(1+R)^j} + \frac{Nom}{(1+R)^{10}} =$$

$$= 3\left[\frac{1}{R}\left(1 - \frac{1}{(1+R)^{10}}\right)\right] + \frac{100}{(1+R)^{10}} = \underline{108'98}$$

donde (recordemos) $R = 2\%$.

c) bono nominal 100 €, con cupones semestrales del $\underbrace{1\%}_{Q}$. Vencimiento en 10 años, paga cupón y devuelve nominal.



$$C = Q.\,Nom = 0'01 . 100 = 1$$

$$\underline{V_{actual}} = \sum_{j=1}^{20} \frac{1}{(1+R)^{j/2}} + \frac{Nom}{(1+R)^{10}} =$$

$$= \frac{\frac{1}{(1+R)^{1/2}} - \frac{1}{(1+R)^{10}} \cdot \frac{1}{(1+R)^{1/2}}}{1 - \frac{1}{(1+R)^{1/2}}} + \frac{100}{(1+R)^{10}} =$$

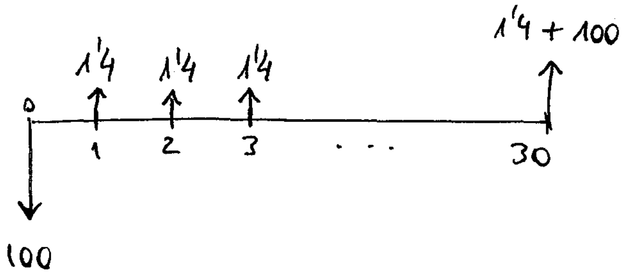
fórmula suma geométrica

$$\frac{P - UR}{1 - R}$$

$$= 18'059 + 82'0348 = \underline{100'09\ €}$$

6.

a)



$x := \text{TIR}$

$$100 = \sum_{j=1}^{30} \frac{1'4}{(1+x)^j} + \frac{100}{(1+x)^{30}} = 1'4 \cdot \frac{1}{x}\left(1 - \frac{1}{(1+x)^{30}}\right) + \frac{100}{(1+x)^{30}} \Longrightarrow$$

$$\Longrightarrow 100 = \frac{1'4}{x} - \frac{1'4}{x(1+x)^{30}} + \frac{100}{(1+x)^{30}} \Longrightarrow$$

$$\Longrightarrow 100\,x(1+x)^{30} = 1'4(1+x)^{30} - 1'4 + 100\,x \Longrightarrow$$

solver calculador

$$\Longrightarrow (100\,x - 1'4)(1+x)^{30} + 1'4 - 100\,x = 0 \Longrightarrow \underline{\underline{x = 1'4\,\%}}$$

b)

$$100 = \sum_{j=1}^{29} \frac{1'4}{(1+x)^j} + \frac{100}{(1+x)^{30}} = \frac{1'4}{x} - \frac{1'4}{x(1+x)^{29}} + \frac{100}{(1+x)^{30}} \Longrightarrow$$

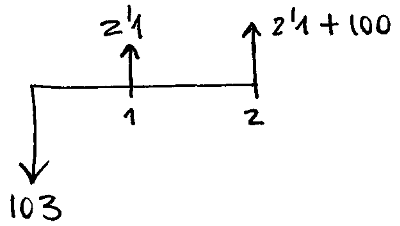
$$\Longrightarrow 100\,x(1+x)^{30} = 1'4(1+x)^{30} - 1'4(1+x) + 100\,x \Longrightarrow$$

solver calc.

$$\Longrightarrow (100\,x - 1'4)(1+x)^{30} + 1'4(1+x) - 100\,x = 0 \Longrightarrow$$

$$\Longrightarrow \underline{\underline{x = 1'362\,\%}}$$

c)



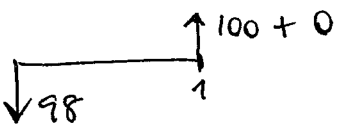
$$103 = \frac{2'1}{(1+x)} + \frac{2'1 + 100}{(1+x)^2} \Longrightarrow$$

$$\Longrightarrow 103(1+x)^2 = 2'1(1+x) + 2'1 + 100 \Longrightarrow$$

$$\Longrightarrow 103 + 206x + 103x^2 = 2'1 + 2'1x + 102'1 \Longrightarrow$$

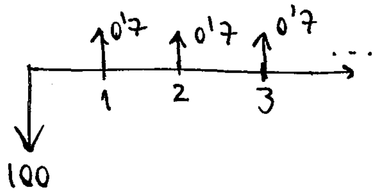
$$\Longrightarrow 103x^2 + 203'9x - 1'2 = 0 \Longrightarrow \underline{\underline{x = 0'5868\,\%}}$$

d)



$$98 = \frac{100}{1+x} \Longrightarrow \underline{\underline{x = 2'041\,\%}}$$

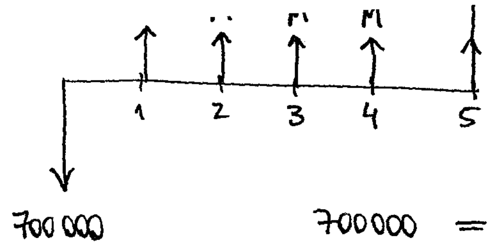
e)



$$100 = \sum_{j=1}^{\infty} \frac{0'7}{(1+x)^j} \Longrightarrow 100 = \frac{0'7}{x} \Longrightarrow$$

$$\Longrightarrow x = \frac{0'7}{100} = \underline{\underline{0'7\,\%}}$$

1.



$M = 210\,000$

$$700000 = \sum_{j=1}^{5} \frac{M}{(1+x)^j} + \frac{180000}{(1+x)^5} \Longrightarrow$$

$$\Longrightarrow 700\,000 = \frac{M}{x} - \frac{M}{x(1+x)^5} + \frac{180000}{(1+x)^5} \Longrightarrow$$

$$\Rightarrow 700\,000x(1+x)^5 = M(1+x)^5 - M + 180000\,x \Longrightarrow$$

$$\Rightarrow (700\,000x - M)(1+x)^5 + M - 180000\,x = 0 \Longrightarrow$$

solver calc.

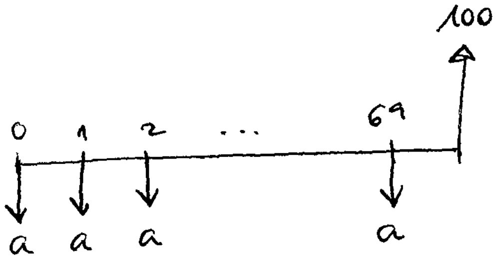
$$\Rightarrow (70x - 21)(1+x)^5 + 21 - 18x = 0 \Longrightarrow \underline{\underline{x = 20\%}}$$

2.

10.

$R = 2\%$

a)



Nos lo traemos todo a $T=0$:

$$\sum_{j=0}^{19}\frac{a}{(1+R)^j} = a + a\sum_{j=1}^{19}\frac{1}{(1+R)^j} = a + a\overbrace{\left(\frac{1}{R}\left(1-\frac{1}{(1+R)^{19}}\right)\right)}^{=15'6785} = \overbrace{\frac{100}{(1+R)^{20}}}^{=67'297}$$

$$\Longrightarrow 16'6785\, a = 67'2971 \Longrightarrow \boxed{a = 4'03\text{ €}}$$

b) Vamos a ponernos desde el punto de vista de la mutua:

$C_j :=$ recaudación mutua en tiempo j.

$$\left.\begin{aligned} C_0 &= a \cdot N \cdot \underbrace{\Pi_0}_{100\%} = aN \\ C_1 &= C_0(1+R) + aN\cdot \Pi_1 \\ &\ \vdots \\ C_{19} &= C_{18}(1+R) + a\cdot N\cdot \Pi_{19} \\ C_{20} &= C_{19}(1+R) \end{aligned}\right\} \quad \begin{aligned} &C_n = C_{n-1}(1+R) + aN\cdot\Pi_n, \\ &n = 1,\dots,19 \qquad C_0 = a\cdot N \end{aligned}$$

$$\Longrightarrow \boxed{C_{20} = N\cdot a\cdot \sum_{j=0}^{19}\Pi_j\,(1+R)^{20-j}}$$

Pero además, la mutua en $T=20$ paga $100\cdot N\cdot \Pi_{20}$, por lo que buscamos: $100\cdot N\cdot \Pi_{20} = N\cdot a \sum_{j=0}^{19}\Pi_j(1+R)^{20-j} \Longrightarrow$

$\Longrightarrow$ despejamos a. (me amarga)

↳ con a calculado, podríamos calcular el TIR.

c)