ORDEN DEL ERROR DE TRUNCATURA DEL MÉTODO DE RUNGE

Suponemos $y_n = y(t_n)$. Recordemos que $y'(t) = f(t, y(t))$.

Veamos el error de $y(t_{n+1}) - y_{n+1}$:

$$y_{n+1} = y_n + h_n f\left(t_n + \frac{h_n}{2}, y_n + \frac{h_n}{2} f_n\right) =$$

$$= y(t_n) + h_n f\left(t_n + \frac{h_n}{2}, y(t_n) + \frac{h_n}{2} f_n\right) =$$

$$= y(t_n) + h_n f\left(t_n + \frac{h_n}{2}, y(t_n) + \frac{h_n}{2} y'_n\right) =$$

$$= y(t_n) + h_n f\Big(\underbrace{t_n + \frac{h_n}{2}}_{\hat{t} \in (t_n, t_{n+1})}, y\big(\underbrace{t_n + \frac{h_n}{2}}_{\hat{t} \in (t_n, t_{n+1})}\big)\Big) =$$

$$= y(t_n) + h_n y'\left(t_n + \frac{h_n}{2}\right) \qquad (*)$$

Usamos:

$$y(t_{n+1}) - y(t_n) = h_n f\left(t_n + \frac{h_n}{2}, y\left(t_n + \frac{h_n}{2}\right)\right) + \frac{1}{3} y'''(s) \left(\frac{h_n}{2}\right)^3$$

$s \in (t_n, t_{n+1})$

despejamos $y(t_n)$ y sustituimos en $(*)$

$$\Rightarrow (*) = y(t_{n+1}) - \cancel{h_n y'\left(t_n + \frac{h_n}{2}\right)} - \frac{1}{3} y'''(s) \left(\frac{h_n}{2}\right)^3 + \cancel{h_n y'\left(t_n + \frac{h_n}{2}\right)} =$$

$$= y(t_{n+1}) - \frac{1}{3} \cdot \left(\frac{h_n}{2}\right)^3 y'''(s)$$

$$\Rightarrow y_{n+1} - y(t_{n+1}) = -\frac{1}{3} \left(\frac{h_n}{2}\right)^3 y'''(s)$$

$$\Rightarrow \frac{y_{n+1} - y(t_{n+1})}{h_n} = \frac{-1}{3} \cdot \frac{h_n^2}{8} \left| y'''(s) \right| \quad s \in (t_n, t_{n+1})$$

LOCAL $\longrightarrow$ <u>ORDEN 2</u>

$$\max_{n=1,\dots,N} \tau_n = \max_{n=1,\dots,N} \frac{h_{n-1}^2}{-24} \left| y'''(s) \right| \leq \frac{-h_n^2}{24} \max_{s \in (t_n, t_{n+1})} y'''(s)$$

GLOBAL $\longrightarrow$ <u>ORDEN 2</u>