**1.**

$[m] = M$

$[\rho] = M.L^{-3}$

$[V] = L^3$

$[S] = L^2$

con magnitudes elementales

Masa = M    Longitud = L

Matriz de dimensiones

$$\begin{array}{c} M \\ L \end{array}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 3 & 2 \end{pmatrix} = A$$

Rang(A) = 2 $\Rightarrow$ Existen 4-2=2 magnitudes adimensionales independientes $\Pi_1$ y $\Pi_2$.

Teorema Pi : $f(m, \rho, V, S) = 0 \implies F(\Pi_1, \Pi_2) = 0$

$$\Pi_1 = \frac{\rho V}{m} \qquad \Pi_2 = \frac{S}{V^{2/3}} \qquad V^{2/3} = \left(\frac{m}{\rho}\right)^{2/3}$$

$$\Pi_2 = \frac{S}{(m/\rho)^{2/3}}$$

$$S = V^{2/3} g\left(\frac{\rho V}{m}\right) \qquad S = \left(\frac{m}{\rho}\right)^{2/3} g\left(\frac{\rho V}{m}\right)$$

incompleto

$$\frac{\rho}{m} = \lambda \qquad \boxed{S = \lambda^{-2/3} g(\lambda V)}$$

temperatura

**2.** $u(\tau,t) = \frac{e}{c}(\kappa t)^{-3/2}\, g\left(\frac{\tau^2}{\kappa t}\right)$

$[u] = u$

$[t] = t$ (tiempo)

$[\tau] = x$ (longitud)

$[e] = e$ (energía)

$[c] = e \cdot u^{-1} \cdot x^{-3}$ (const. calorífica)

$[\kappa] = x^2 \cdot t^{-1}$ (difusión de calor)

$\Longrightarrow$ 2 magnitudes adimensionales

$$[\Pi_1] = \left[\frac{\tau^2}{\kappa t}\right] = \frac{x^2}{x^2 t^{-1} \cdot t} = 1$$

$$\Pi_2 = \frac{c u}{e(\kappa t)^{-3/2}} \longrightarrow [\Pi_2] = \frac{[c]\cdot[u]}{[e]\,[\kappa t]^{-3/2}} = \frac{e\, u^{-1} x^{-3}\, u}{e\, x^{-3}} = 1$$

$$g(\Pi_1, \Pi_2) = 0$$

$$\Pi_2 = g(\Pi_1) \longrightarrow u = \frac{e}{c}(\kappa t)^{-3/2}\, g\left(\frac{\tau^2}{\kappa t}\right)$$

$$N(0,\sigma) \sim \left(\exp\left(\frac{-x^2}{2\sigma}\right)\right) \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \qquad \tau = (x_1^2 + x_2^2 + x_3^2)^{1/2}$$

En 3 dimensiones tenemos: $\exp\left(\frac{-\tau^2}{2\sigma}\right) \cdot \frac{1}{(2\pi\sigma)^{3/2}}$

RECUERDO: $\frac{GM}{R^2} = g$

$\boxed{4.}$ $h''(t) = \frac{-gR^2}{(h(t)+R)^2}$ , $h(0) = 0$ , $h'(0) = v$

$F_G = \frac{-GMm}{(h(t)+R)^2}$

a) Magnitudes, magnitudes elementales y magnitudes adimensionale

$q_i$ : $h, t, g, R, v$

$[h] = L = \{\text{longitud}\}$

$[t] = T = \{\text{tiempo}\}$

$[g] = L.T^{-2}$ (aceleración)

$[v] = L.T^{-1}$ (velocidad)

$[R] = L$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & -2 & -1 & 0 \end{pmatrix}$$

rango 2

$\Rightarrow$ Tenemos $5-2=3$ magnitudes adimensionales:

$$\frac{h}{R}, \quad \frac{tV}{R}, \quad \frac{V}{\sqrt{gR}} = \frac{V^2}{gR}$$

b) Relación de la altura máxima a alcanzar el proyectil con respecto a $V, g, R$.

$0 = f(h, t, v, g, R)$

$0 = F\left(\frac{h}{R}, \frac{tV}{R}, \frac{V}{\sqrt{gR}}\right)$

$\frac{h}{R} = G\left(\underbrace{\frac{tV}{R}}_{\Pi_2}, \frac{V}{\sqrt{gR}}\right)$

$0 = F\left(\frac{h_{máx}}{R}, \frac{t_{máx}V}{R}, \frac{V}{\sqrt{gR}}\right)$

Tma. Función Implícita: $\frac{h_{máx}}{R} = G\left(\frac{t_{máx}V}{R}, \frac{V}{\sqrt{gR}}\right)$

esto también lo tengo para esto

Como alcanzamos la altura máxima : $h'(t_{max}) = 0 \Rightarrow$

$\Rightarrow \frac{\partial G}{\partial \Pi_2}\left(\frac{t_{máx}V}{R}, \frac{V}{\sqrt{gR}}\right) = 0$

Aplicamos otra vez Tma. Func. Implicita:

$$\frac{t_{máx} V}{R} = \varphi\left(\frac{V}{\sqrt{gR}}\right) \implies \frac{h_{máx}}{R} = G\left(\varphi\left(\frac{V}{\sqrt{gR}}\right), \frac{V}{\sqrt{gR}}\right) = \varphi^*\left(\frac{V}{\sqrt{gR}}\right)$$

$$\implies h_{máx} = R \cdot \varphi^*\left(\frac{V}{\sqrt{gR}}\right)$$

c) Identificar las escalas privilegiadas del problema y como se simplifica el problema en dichas escalas.

$$h''(t) = \frac{-gR^2}{(h(t)+R)^2} \qquad h(0) = 0, \quad h'(0) = V$$

Cambio variable: $\bar{t} = \frac{t}{t_c}$ $\qquad \bar{h}(\bar{t}) = \frac{h(t_c \bar{t})}{h_c}$

$$\mathring{\bar{h}}(\bar{t}) = \frac{\partial \bar{h}}{\partial \bar{t}}(\bar{t}) = \frac{\partial}{\partial \bar{t}}\left(\frac{h(t_c\bar{t})}{h_c}\right) = \frac{t_c}{h_c} h'(t_c\bar{t})$$

$$\ddot{\bar{h}}(\bar{t}) = \frac{\partial}{\partial \bar{t}}\left(\frac{t_c}{h_c} h'(t_c\bar{t})\right) = \frac{t_c^2}{h_c} h''(t_c\bar{t})$$

$\bar{h}(0) = 0$

$$\mathring{\bar{h}}(0) = \frac{t_c}{h_c} h'(0) = \frac{t_c}{h_c} \cdot V$$

$$\left.\begin{array}{l} h''(t) = \frac{h_c}{t_c^2} \ddot{\bar{h}}(\bar{t}) \\ h(t) = h_c \bar{h}(\bar{t}) \end{array}\right\} \implies \frac{h_c}{t_c^2} \ddot{\bar{h}}(\bar{t}) = \frac{-gR^2}{(h_c\bar{h}(\bar{t}) + R)^2}$$

cambio 1: $\frac{h}{R}$ $\quad h_c = R$

$\frac{t}{R/V}$ $\quad t_c = \frac{R}{V}$

cambio 2: $\frac{h}{R}$ $\quad h_c = R$

$\frac{t}{\sqrt{R/g}}$ $\quad t_c = \sqrt{R/g}$

cambio 3: $\frac{h}{V^2/g}$ $\quad h_c = \frac{V^2}{g}$

$\frac{t}{V/g}$ $\quad t_c = \frac{V}{g}$

EDO 1 : $\dfrac{R}{(R/v)^2}\ddot{\bar{h}} = \dfrac{-gR^2}{(R\bar{h}+R)^2}$ $\qquad \dot{\bar{h}}(0) = \dfrac{t_c}{h_c} v = 1$

$$\frac{R^3\ddot{\bar{h}}}{(R/v)^2 gR^2} = \frac{-1}{(1+\bar{h})^2}$$

$$\frac{v^2}{Rg}\ddot{\bar{h}} = \frac{-1}{(1+\bar{h})^2} \xrightarrow[\varepsilon^2=\frac{v^2}{Rg}]{R>>1} \varepsilon^2\ddot{\bar{h}} = \frac{-1}{(1+\bar{h})^2} \Longrightarrow 0 = \frac{-1}{(1+\bar{h})^2}$$

$\varepsilon$ muy pequeño

todo lo que hemos hecho no nos ha servido

EDO 2 : $\dot{\bar{h}}(0) = \dfrac{t_c}{h_c} v = \dfrac{\sqrt{R/g}}{R} v = \dfrac{v}{\sqrt{gR}} = \varepsilon$

$$\frac{R}{R/g}\ddot{\bar{h}} = \frac{-gR^2}{(R\bar{h}+R)^2} \Longrightarrow g\ddot{\bar{h}} = \frac{-g}{(\bar{h}+1)^2} \Longrightarrow \ddot{\bar{h}} = \frac{-1}{(1+\bar{h})^2}$$

$$\Longrightarrow \begin{cases} \ddot{\bar{h}} = \dfrac{-1}{(1+\bar{h})^2} \\ h(0) = 0 \\ h'(0) = \varepsilon \qquad \varepsilon = 0 \end{cases}$$

No nos sirve tampoco
No tiene sentido físico

EDO 3 : $\dot{\bar{h}}(0) = \dfrac{v/g}{v^2/g} \cdot v = 1$

$$\frac{v^2/g}{(v/g)^2}\ddot{\bar{h}}(t) = \frac{-gR^2}{(\frac{v^2}{g}\bar{h}+R)^2} \Longrightarrow \ddot{\bar{h}}(t) = \frac{-1}{(\frac{v^2}{gR}\bar{h}+1)^2} \Longrightarrow$$

$$\Longrightarrow \begin{cases} \bar{h}(0) = 0 \\ \dot{\bar{h}}(0) = 1 \\ \ddot{\bar{h}} = \dfrac{-1}{(\varepsilon^2\bar{h}+1)^2} \xrightarrow{\varepsilon\to 0} \ddot{\bar{h}} = -1 \end{cases}$$

Vamos a resolver la EDO : $\bar{h}(\bar{t}) = \dfrac{-\bar{t}^2}{2} + A\bar{t} + B$

$\bar{h}(0) = 0 \Longrightarrow B = 0$

$\dot{\bar{h}}(\bar{t}) = -\bar{t} + A$

En conclusión: $\bar{h}(\bar{t}) = \frac{-\bar{t}^2}{2} + \bar{t}$

Recordemos ahora que $\bar{h}(\bar{t}) = \frac{h(t_c\bar{t})}{h_c}$:

$$\frac{h(\overbrace{t_c\bar{t}}^{t})}{h_c} = \frac{-1}{2}\left(\frac{t}{t_c}\right)^2 + \left(\frac{t}{t_c}\right)$$

$$h(t) = -\frac{h_c}{2t_c^2}t^2 + \frac{h_c t}{t_c}$$

$$h(t) = -\frac{v^2/g}{2(v/g)^2}t^2 + \frac{v^2/g}{v/g}t = \frac{-gt^2}{2} + vt$$

En el apartado b obtuvimos: $\frac{h_{max}}{R} = \varphi^*\left(\frac{v}{\sqrt{gR}}\right)$

$$h'(t) = 0$$

$$-gt + v = 0 \longrightarrow t = \frac{v}{g}$$

$$h_{max} = -\frac{g}{2}\left(\frac{v}{g}\right)^2 + v\left(\frac{v}{g}\right) = \frac{v^2}{2g}$$

¿Esto coincide?

$\Downarrow$ vemos que sí

$$\frac{h_{max}}{R} = \frac{1}{2}\frac{v^2}{gR}$$

$$\Downarrow$$

$$h_{max} = \frac{v^2}{2g}$$