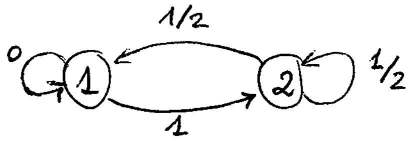
HOJA 2

4. $\{1\} = \{\text{ir al lago}\}$
$\{2\} = \{\text{ir al río}\}$

a) $P = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$



b) $\det(P-\lambda I) = \begin{vmatrix} -\lambda & 1 \\ 1/2 & 1/2-\lambda \end{vmatrix} = \lambda(\lambda - \frac{1}{2}) - \frac{1}{2} = \lambda^2 - \frac{\lambda}{2} - \frac{1}{2} \Rightarrow$

$\Rightarrow \lambda = \dfrac{1/2 \pm \sqrt{1/4 + 2}}{2} = \begin{cases} \lambda_1 = 1 \longrightarrow v_1 = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \\ \lambda_2 = -1/2 \longrightarrow v_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \end{cases}$

c) $P = BJB^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & -1/3 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$

$\lim_{n\to\infty} P^n = B \lim_{n\to\infty} \underbrace{J^n}_{= \begin{pmatrix} 1^n & 0 \\ 0 & (-\frac{1}{2})^n \end{pmatrix}} \cdot B^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & -1/3 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix}$

d) Buscamos $(x_1, x_2) = (x_1, x_2)P$ de tal manera $\lim_{n\to\infty} (x_1,x_2)P^n = (x_1,$

$x_1 = \dfrac{x_2}{2}$

$x_2 = x_1 + \dfrac{x_2}{2}$

$\Longrightarrow \pi = \left(\frac{x_2}{2}, x_2\right) \xrightarrow[\frac{x_2}{2} + x_2 = 1]{\text{normalizamos}} \left(1/3, 2/3\right)$ estado estacionari

3.

$$\begin{pmatrix} 1/4 & 0 & 2 \\ 3/4 & 1/2 & 0 \\ 0 & 1/2 & 1 \end{pmatrix}$$

Calculamos autovalores: $\begin{vmatrix} 1/4-\lambda & 0 & 2 \\ 3/4 & 1/2-\lambda & 0 \\ 0 & 1/2 & 1 \end{vmatrix} = \cdots =$ [illegible] $\to \lambda_1 = 1'5$, complejos

$$\begin{pmatrix} -1'33 & 0 & 2 \\ 3/4 & -1'05 & 0 \\ 0 & 1/2 & -0'55 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 1'35\,V_1 = 2V_3 \Rightarrow V_1 = 1'48 \\ 3/4\,V_1 = 1'05\,V_2 \Rightarrow V_1 = 1'4V \\ 1/2\,V_2 = 0'55\,V_3 \Rightarrow V_2 = 1'1V \end{cases}$$

$$\cancel{\Rightarrow \begin{cases} 1'48\,V_3 = 1'4\,V_2 \to V_2 = 1'06\,V_3 \\ 0'55\,V_3 = 1/2\,V_2 \to V_3 = \frac{0'8}{0'55}(1'06\,V_3) \end{cases}} =$$

$\vec{V} = (\cancel{[illegible]}, 1'48, 1'057, 1)$

normalizamos: $\vec{V} = (0'42, 0'3, 0'28)$

$$(x_1, x_2, x_3)\cdot\frac{1}{\lambda_1^n} = \frac{\lambda_1^n}{\lambda_1^n} a V_1 + \frac{\lambda_2^n}{\lambda_1^n} b V_2 + \frac{\lambda_3^n}{\lambda_1^n} c V_3$$

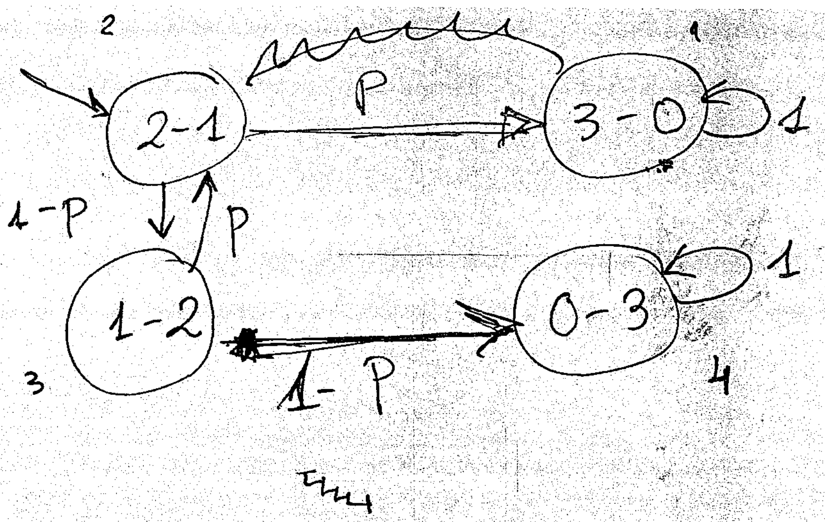
$$\vec{X}(n) = \underline{A}^k X(0)$$

$$\vec{X}(n) = (\lambda_1^k \cancel{\alpha_1} + \lambda_2^k V_2 + \lambda_3^k V_3)\cdot \vec{X}(0)$$

$$\vec{X}(n) = \lambda_1^k C_1 u_1$$

~~[illegible]~~ pensamientos

5.



$$M = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ p & 0 & 1-p & 0 \\ 0 & p & 0 & 1-p \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Podemos sumar

$\cancel{\det(P - I\lambda}$ $\quad \det(M - \lambda I) = (1-\lambda)^2 \begin{vmatrix} -\lambda & 1-p \\ p & -\lambda \end{vmatrix} = (\lambda-1)^2(\lambda^2 - p + p^2) =$

$\cancel{= (\lambda-1)^2}$ → 1 (doble) $\quad \boxed{\lambda_1 = 1}$

→ 2 ~~complejos~~

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 =$

1.

a) $A = \begin{pmatrix} 0 & 1'5 \\ 0'1 & 0'8 \end{pmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1'5 \\ 0'1 & 0'8-\lambda \end{vmatrix} = -0'8\lambda + \lambda^2 - 0'15 =$$

$$= \lambda^2 - 0'8\lambda - 0'15 \quad \begin{matrix} \nearrow 0'957 \\ \searrow -0'157 \end{matrix}$$

autovalor dominante $\lambda_1 = 0'957$

$$B = \begin{pmatrix} 0 & 1'2 \\ 0'5 & 0'7 \end{pmatrix}$$

$$\det(B - \lambda I) = \begin{vmatrix} -\lambda & 1'2 \\ 0'5 & 0'7-\lambda \end{vmatrix} = -0'7\lambda + \lambda^2 - 0'6 =$$

$$= \lambda^2 - 0'7\lambda - 0'6 \quad \begin{matrix} \nearrow 1'2 \\ \searrow -0'5 \end{matrix}$$

autovalor dominante $\lambda_1 = 1'2$

Buscamos estabilizar B:

$$\begin{vmatrix} -\lambda & 1'2 \\ 0'5 & \alpha-\lambda \end{vmatrix} = -\alpha\lambda + \lambda^2 - 0'6 = \lambda^2 - \alpha\lambda - 0'6 \Rightarrow$$

$$\Rightarrow \frac{\alpha \pm \sqrt{\alpha^2 - 4.(-0'6)}}{2} = \frac{\alpha \pm \sqrt{\alpha^2 + 2'4}}{2} = 1$$

$$\Rightarrow \alpha \pm \sqrt{\alpha^2 + 2'4} = 2 \Rightarrow \cancel{\alpha^2} + 2'4 = \cancel{\alpha^2} - 4\alpha + 4 \Rightarrow$$

$$\Rightarrow -4\alpha + 4 - 2'4 = 0 \Rightarrow -4\alpha = -1'6 \Rightarrow \alpha = \frac{1'6}{4} = 0'4$$

$$\begin{pmatrix} -1 & 1'2 \\ 0'5 & -0'6 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow \begin{cases} -u_1 + 1'2u_2 = 0 \\ 0'5u_1 - 0'6u_2 = 0 \end{cases} \longrightarrow u = \begin{pmatrix} 1'2 \\ 1 \end{pmatrix}$$

$$\Rightarrow 1'2\beta + 1\beta = 1 \Rightarrow 2'2\beta = 1 \Rightarrow \beta = \frac{1}{2'2} = 0'45$$

$$\Rightarrow u = \begin{pmatrix} 0'55 \\ 0'45 \end{pmatrix} \text{ (normalizado)} \Rightarrow \underline{55\%}.$$