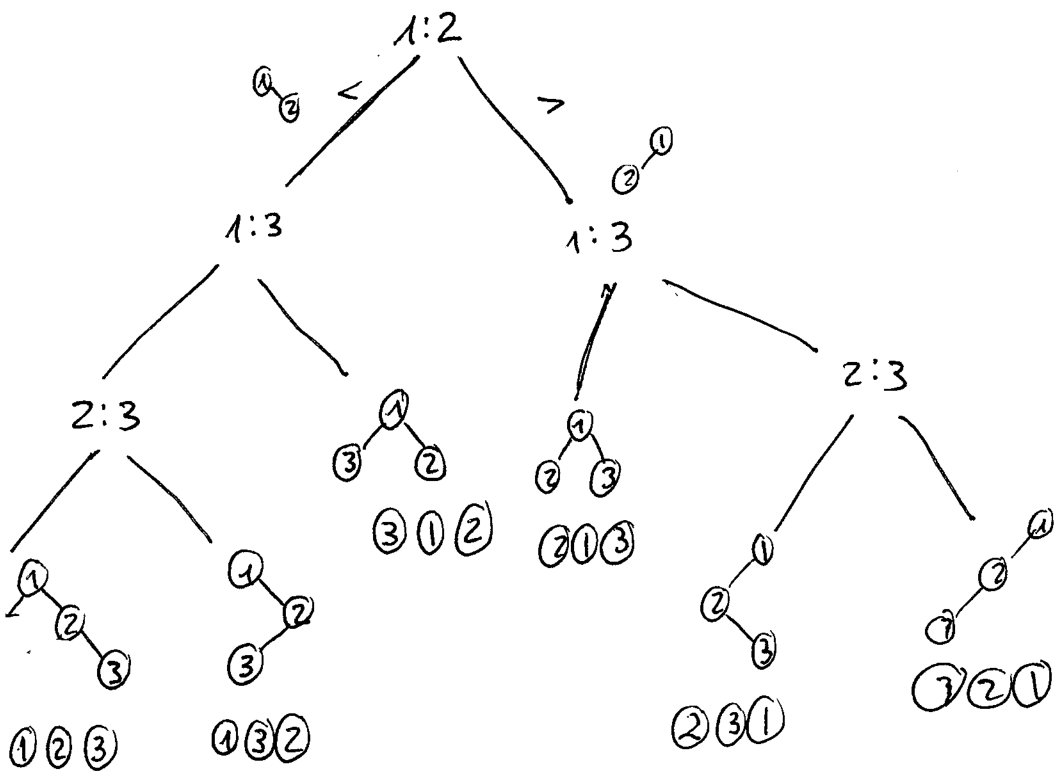
$\boxed{126}$ $G_N = \sum_{0}^{N} F_i$ F_i = i-ésimo nº de Fibonacci

$$G_N = \sum_{0}^{N} F_i \qquad G_{N'} + 1 = F_{N'+2} \quad \forall N' < N$$

N	G_N	
0	1	2 F_2
1	2	3
2	4	5
3	7	8
4	12	13

$\boxed{102}$ 1 2 3



124.

n	0	1	2	3	4	5
j	1	0	1	1	2	3
a	0	1	1	2	3	5
total	1	1	2	3	5	8

125. $n(N) = \#$ llamadas recursivas F_n

$$n(N) = 2 + n(N-1) + n(N-2) \qquad n(0) = n(1) = 0$$

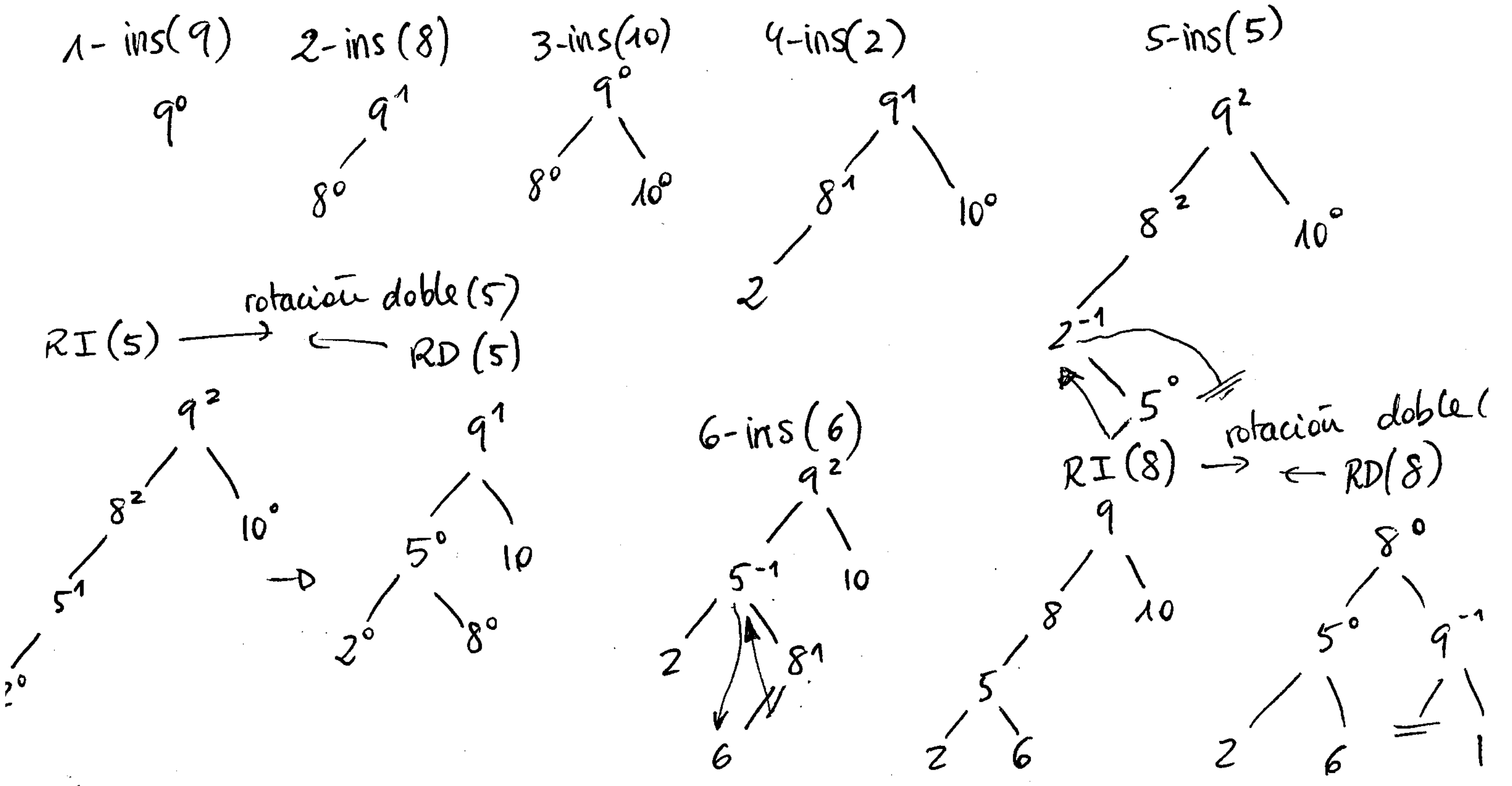
$$\Rightarrow \frac{n(N)}{2} = 1 + \frac{n(N-1)}{2} + \frac{n(N-2)}{2}$$

$F_n =$

pajarito

N	n_N	$n_{N/2}$	F
0	0	0	[illegible]
1	0	0	[illegible]
2	2	1	[illegible]
3	4	2	[illegible]
4	8	4	[illegible]
5	14	7	8

118 9 8 10 2 5 6



136 REHASHING

DA, SL

$\lambda = 0'5$

N $\quad$ N/2

T1 ———————— tam N $\quad \frac{N}{2}$

T2 ———————————— tam 2N

n_sondeos

① $\displaystyle\sum_{1}^{N/2} \underbrace{n_{ins}(D_i, T_i^1)}_{n_B^f(D_i, T_i^1) \approx A^f(i-1,N) = \phi\left(\frac{i-1}{N}\right)} \cong \sum_{i=1}^{N/2} \phi\left(\frac{i-1}{N}\right) = \sum_{i=0}^{\frac{N}{2}-1} \phi\left(\frac{i}{N}\right) \approx \int_0^{N/2} \phi\left(\frac{x}{N}\right) dx$

cambio de variable $x/N = u$

$$\int_0^{1/2} \phi(u)\, N\, du = N \int_0^{1/2} \phi(u)\, du$$

② $\displaystyle\sum_{1}^{N} n_{Ins}(D_i, T_i^2) \simeq \sum_{i=1}^{N} \phi\left(\frac{i-1}{2N}\right) = \sum_{i=0}^{N-1} \phi\left(\frac{i}{2N}\right) \approx \int_0^{N} \phi\left(\frac{x}{2N}\right) dx =$

c.v. $\frac{x}{2N} = u \quad = \displaystyle\int_0^{1/2} \phi(u)\, 2N = 2N \int_0^{1/2} \phi(u)\, du$

Coste total Rehashing $= 3N \displaystyle\int_0^{1/2} \phi(u)\, du$

$\boxed{137}$ $N=16$

$h(K) = K \% \underbrace{16}_{}$ → compuesto, mal hashing

$K^2 \% 16 \begin{cases} 0 \\ 1 \\ 4 \\ 9 \end{cases}$

$\boxed{138}$ M primo $\qquad h(K)+i^2$

$\lambda < \frac{1}{2} \qquad 1 \leq i < j < \frac{M}{2}$

$\lfloor \frac{M}{2} \rfloor < M/2 \qquad h(K)+i^2 \equiv h(K)+j^2 \pmod M$

$\Rightarrow \quad j^2 - i^2 \equiv 0 \quad (M) \quad \Rightarrow \quad M \mid j^2 - i^2 = (j-i)(j+i)$

$\Rightarrow$ ó $M \mid j-i$ ó $M \mid j+i$ pero $j-i < j < M$

y $j+i < j+j = 2j < 2\,\frac{M}{2} < M$ contradicción

$\boxed{135.}$ HASH DIERECCIONAMIENTO ABIERTO

$$A^e(N,m) = \frac{1}{\lambda}\int_0^\lambda \phi(u)\,du \leq \frac{1}{\lambda}\int_0^\lambda \phi(\lambda)\,du = \frac{\phi(\lambda)}{\lambda}\int_0^\lambda du$$

$$= \phi(\lambda) = A^f(N,m)$$

HASH ENLAZAMIENTO

$$A^e(N,m) = \frac{1}{\lambda}\int_0^\lambda (1+\phi(u))\,du = 1 + \frac{1}{\lambda}\int_0^\lambda \phi(u)\,du$$

$$\leq 1 + \phi(\lambda)$$

$\boxed{130.}$ HE 1000 4 colisiones

$$\left.\begin{array}{l} A^f(N,m) \leq 4 \\ A^e(f,m) \leq 4 \end{array}\right\} \quad *$$

* como no dice nada podemos suponer función hash uniforme.

$$A^f(1000,m) = \lambda = \frac{1000}{m} \leq 4 \Rightarrow \frac{1000}{4} \leq m \Rightarrow m \geq 250$$

$$A^e(1000,m) = 1 + \frac{\lambda}{2} = 1 + \frac{1000}{2m} \leq 4 \Rightarrow m \geq \frac{1000}{6} \approx 166$$

$\boxed{131.}$ SL

$$A^f(1000,m) = \frac{1}{2}\left(1 + \frac{1}{\left(1-\frac{1000}{m}\right)^2}\right) \leq 5 \rightarrow 1 + \frac{1}{(\ )^2} \leq 10 \rightarrow$$

$$\rightarrow \frac{1}{(\ )^2} \leq 9 \rightarrow m \geq 1500$$

$$A^e(1000,m) = \frac{1}{2}\left(1 + \frac{1}{1-\frac{1000}{m}}\right) \leq 5$$

128. ¿ Cuantos T_p hay ?

$$T_p \quad n_p$$

$$\begin{array}{ccc} & T_p & \\ \swarrow & & \searrow \\ {}_{p-1}\; T_{p-1} & & T_{p-2}\; n_{p-2} \end{array}$$

$\boxed{2 \times n_{p-1} \times n_{p-2}}$ → porque puede ser $T_{p-1}\ T_{p-2}$ ó $T_{p-2}\ T_{p-1}$

$$n_p = 2 \cdot n_{p-1} \cdot n_{p-2} \longleftrightarrow \log(n_p) = \log(2 \cdot n_{p-1} \cdot n_{p-2}) = 1 + \log(n_{p-1}) + \log(n_{p-2})$$

$$\Rightarrow \underbrace{n_p + 1}_{\mu_p} = \underbrace{1 + \log(n_{p-1})}_{\mu_{p-1}} + \underbrace{\log(n_{p-2}) + 1}_{\mu_{p-2}}$$

$$\mu_0 = 1 + \log n_0 = 1 = F$$
$$\mu_1 = 1 + \log n_1 = 2 = F$$

$$\mu_p = F_{p+1}$$

vaya numeraco

$$\delta_p = F_{p+1} - 1 \qquad n_p = 2^{\delta p} = \frac{1}{2} 2^{F_{p+1}} = \frac{1}{2} 2^{\phi^p} \quad \ldots$$

134. HASH DIRECCIONAMIENTO ABIERTO

$$A^f_{sx}(N,m) = 1 + \frac{1}{(1-\lambda)^2}$$

$$A^e_{sx}(N,m) = \frac{1}{\lambda}\int_0^{\lambda}\left(1 + \frac{1}{(1-u)^2}\right) du \qquad [\ldots]$$

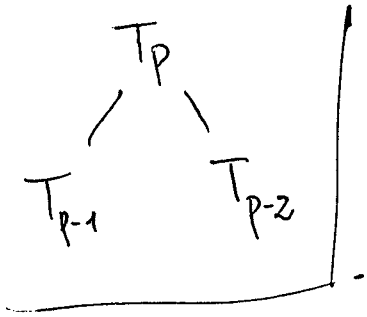
133. HASH POR ENCADENAMIENTO

$$A^f_{HE}(N,m) = \lambda^2 \qquad n^f_{T_i}(D_i, i-1, m) + 1$$

$$A^e_{HE}(N,m) = \frac{1}{N}\sum_{i=1}^{N} n^e_T(D_i, N, m) \simeq \frac{1}{N}\sum_{i=1}^{N}\left(1 + \left(\frac{i-1}{m}\right)^2\right) =$$

$$= 1 + \frac{1}{N}\sum_{0}^{N-1}\left(\frac{j}{m}\right)^2 = 1 + \frac{1}{N.m^2} \cdot \frac{1}{3}\left(N^3 + O(N^2)\right) = 1 + \frac{1}{3} \cdot \frac{N^2}{m^2} + O\left(\frac{N^2}{Nm^2}\right)$$

$$= 1 + \frac{1}{3}\lambda^2 + O\left(\lambda \cdot \frac{1}{m}\right) = 1 + \frac{1}{3}\lambda^2 + o_m(1)$$



(128)

$$n_p = 2 \cdot n_{p-1} \cdot n_{p-2} \qquad \begin{array}{l} n_0 = 1 \\ n_1 = 2 \end{array}$$

Aplicamos logaritmos

$$\log(n_p) = 1 + \log(n_{p-1}) + \log(n_{p-2})$$

$$L_p = 1 + l_p = F_{p+1} \qquad L_0 = 1$$

$$l_p = F_{p+1} - 1 \qquad L_1 = 2$$

$$n_p = 2^{l_p} = 2^{F_{p+1}} - 1$$

$$n_4 = 2^5 - 1 = 33$$

$$n_3 = 15$$

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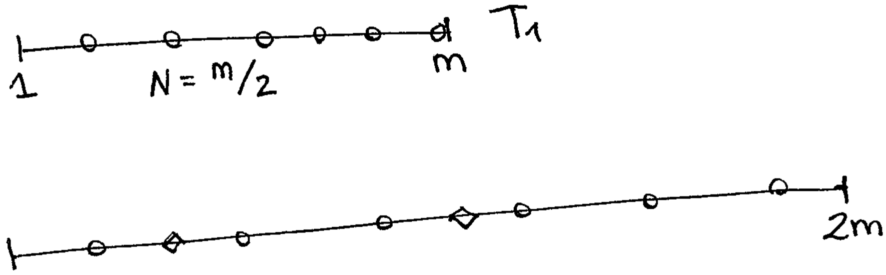
(136) $\lambda < \frac{1}{2}$



1. → hashing viejos
2. → rehashing viejos
3. → hashing nuevos

Sondeo Lineal

$$1.\ \sum_{j=1}^{m/2} n_I(j, T_1) = \sum_{j=1}^{m/2} n^f(j, T_1 j) \simeq \sum_{j=1}^{m/2} A^f_{SL}(j-1, m) \simeq \sum_{j=1}^{m/2} \phi\left(\frac{j-1}{m}\right) \simeq$$

$$\simeq \sum_{j=0}^{m/2} \phi\left(\frac{j}{m}\right) \simeq \int_0^{m/2} \phi\Big(\underbrace{\frac{u}{m}}_{x}\Big) \underbrace{du}_{m dx} = m \int_0^{1/2} \phi(x)\, dx$$

$$2.\ \sum_1^{m} n^f(j, T_2 j) \simeq \sum_1^{m} A^f_{SL}(j-1, 2m) \simeq \cdots = \int_0^{m} \phi\Big(\underbrace{\frac{u}{2m}}_{x}\Big) \underbrace{du}_{2m dx} =$$

$$= 2m \int_0^{1/2} \phi(x)\, dx$$

La suma: $3m \int_0^{1/2} \phi(x)\, dx$

$$n_N = 2 + n_{N-1} + n_{N-2}$$

$$\Downarrow$$

$$\underbrace{2 + n_N}_{r_n} = \underbrace{2 + n_{N-1}}_{r_{n-1}} + \underbrace{n_{N-2} + 2}_{r_{n-2}}$$

$$r(0) = 2 = r(1)$$

$$r_N = 2 + n_N$$

$$\frac{n_N + 2}{2} = F_N \Rightarrow n_N = 2F_n - 2$$

133.

$$A_E^f(N,m) = \lambda^2$$

$$\frac{1}{N}\sum_1^N n_e(D_i,T) = \frac{1}{N}\sum_1^N n_f(D_i,T_i) + 1 = \cdots =$$

$$= 1 + \sum_1^N \left(\frac{i-1}{m}\right)^2 = 1 + \frac{1}{N}\cdot\frac{1}{m^2}\sum_1^{N-1} i^2 =$$

$$= 1 + \frac{1}{Nm^2}\cdot\left\{\frac{N^3}{3} + O(N^2)\right\} =$$

$$= 1 + \frac{1}{3}\lambda^2 + O\left(\frac{N^2}{Nm^2}\right) =$$

$$= 1 + \frac{1}{3}\lambda^2 + O\left(\frac{\lambda}{m}\right)$$

ó $\updownarrow$ la integral

$$1 + \frac{1}{\lambda}\int_0^\lambda u^2\,du$$

134. Lo mismo sin sumar 1.

$$A^f_H(N,m) = \frac{1}{(1-\lambda)^2}$$ Hash direccionamiento abierto

$$A^e_H(N,m) = \frac{1}{\lambda}\int_0^\lambda \phi(u)\,du = \frac{1}{\lambda}\int_0^\lambda \frac{1}{(1-u)^2} = \frac{1}{\lambda}\left[\frac{1}{1-u}\right]_0^\lambda = \frac{1}{\lambda}\left[\frac{1}{1-\lambda}-1\right] =$$

$$= \frac{1-1+\lambda}{\lambda(1-\lambda)} = \frac{1}{1-\lambda}$$

$\Rightarrow$ esto si fueran 2 pts.

Como son 5 pts. :

$$\frac{1}{N}\sum_1^N \underbrace{n^e(D_i,T)}_{\parallel}$$

$$n^f(D_i,T_i) \simeq A^f(i-1,m) = \frac{1}{\left(1-\frac{i-1}{m}\right)^2}$$

<u>DISEÑO DE TABLAS</u>:

Tabla con direccionamiento abierto sondeos lineales $N=1000$

¿m?

$$A^f(1000,m) \leq 13 \qquad \frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$$

$$A^e(1000,m) \leq 13 \qquad \frac{1}{2}\left(1+\frac{1}{1-\lambda}\right)$$

$$\frac{1}{2}\left(1+\frac{1}{\left(1-\frac{N}{m}\right)^2}\right) = \frac{1}{2}\left(1+\frac{1}{\left(1-\frac{1000}{m}\right)^2}\right) \leq 13 \Rightarrow 1+\frac{1}{(\ \ )^2} \leq 26 \Rightarrow$$

$$\Rightarrow \frac{1}{(\ \ )^2} \leq 25 \Rightarrow \frac{1}{1-\frac{1000}{m}} \leq 5 \Rightarrow \frac{1}{5} \leq 1-\frac{1000}{m} \Rightarrow \frac{1000}{m} \leq \frac{4}{5}$$

$$\Rightarrow m \geq \frac{5000}{4} = 1250$$