

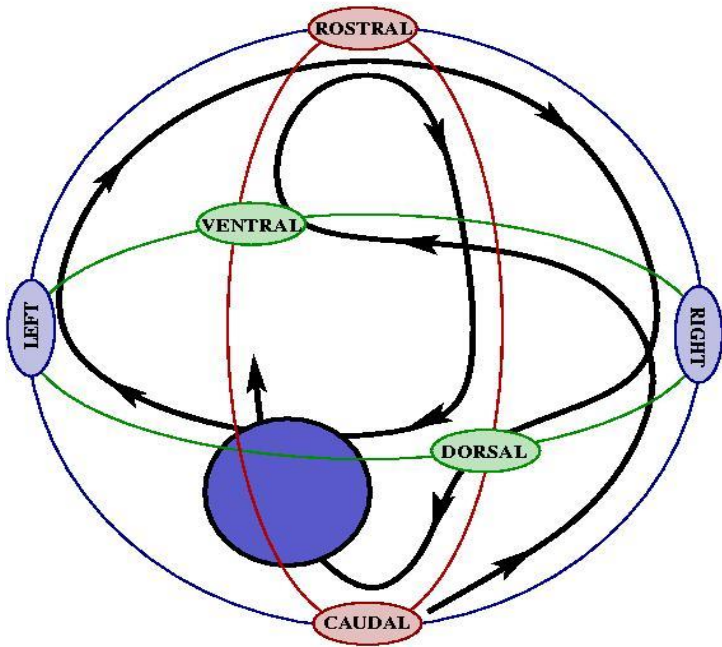
# Motivación a las redes competitivas

# Animal modelo: el molusco clione



# Comportamiento normal

La posición cabeza-arriba la controlan dos estatocistos



# Comportamiento de caza

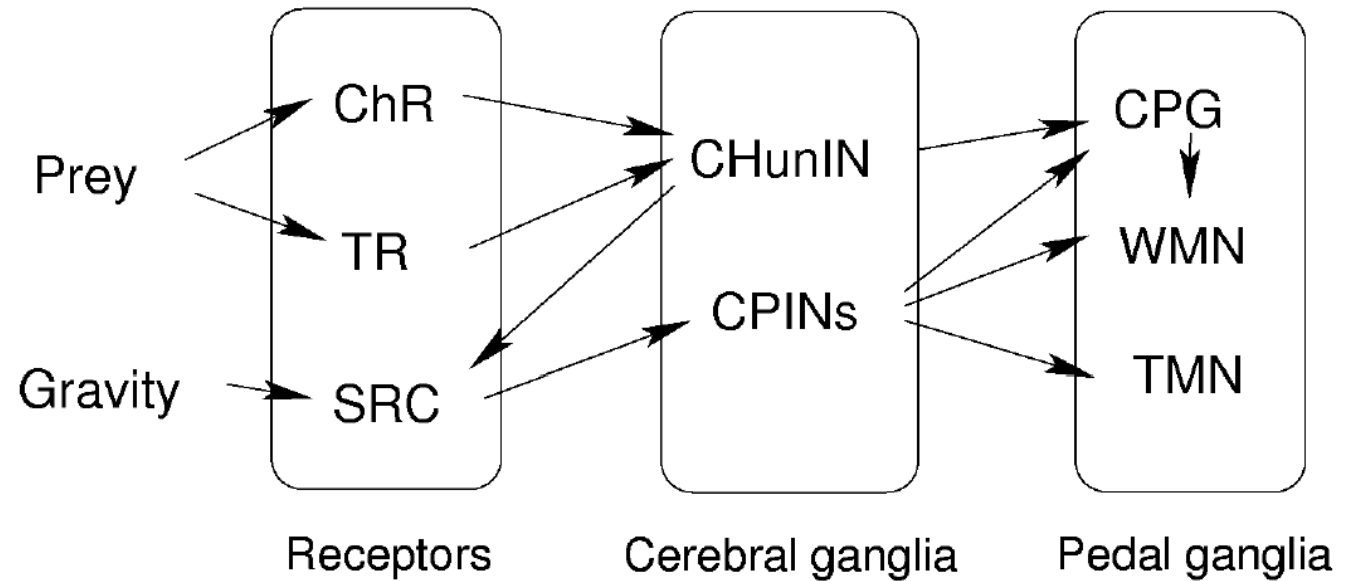
Presa: *Limacina helicina*



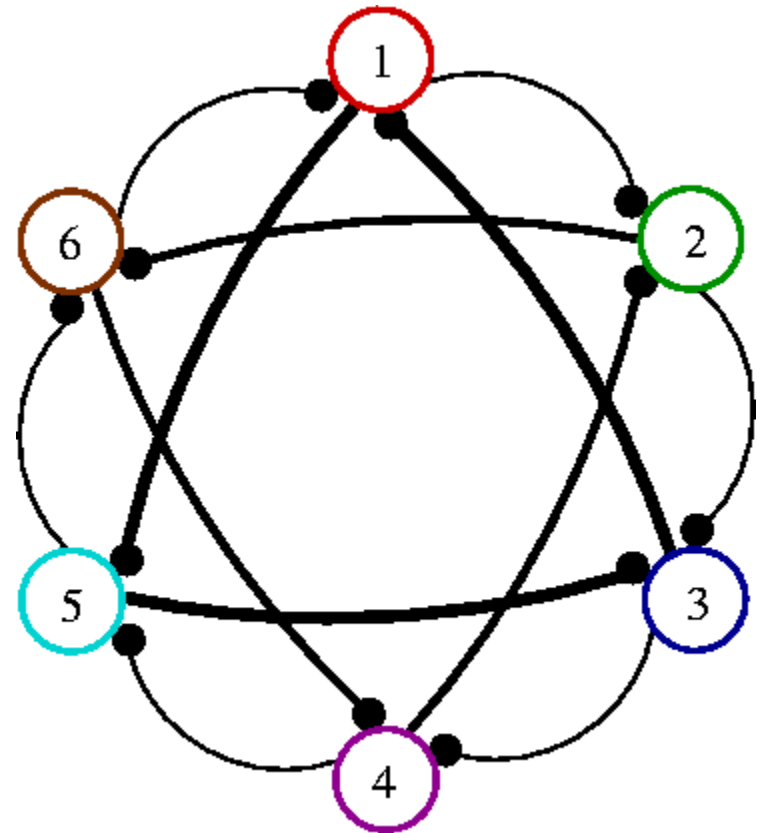
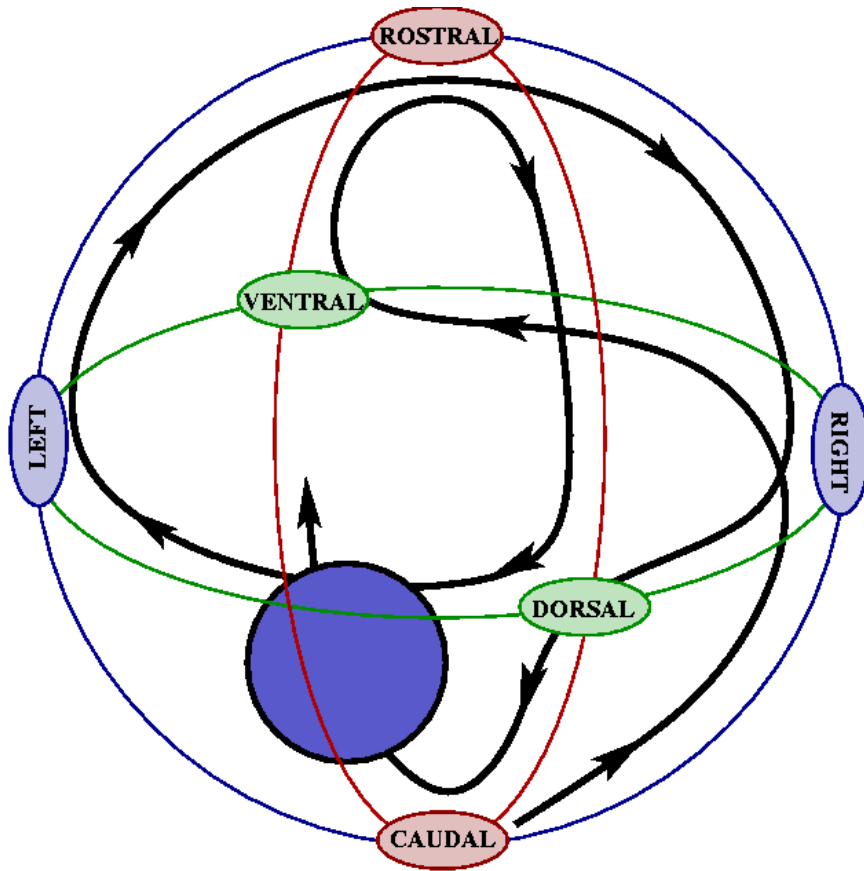
Depredador: *Clione limacina*



# El sistema nervioso del clione



# Modelo de la red de receptores del estatocisto



# Modelo de actividad promedio

$$\dot{a}_i = a_i(\sigma(\mathbf{H}, \mathbf{S}) - \sum_{j=1}^N \rho_{ij} a_j + H_i(t)) + S_i(t)$$

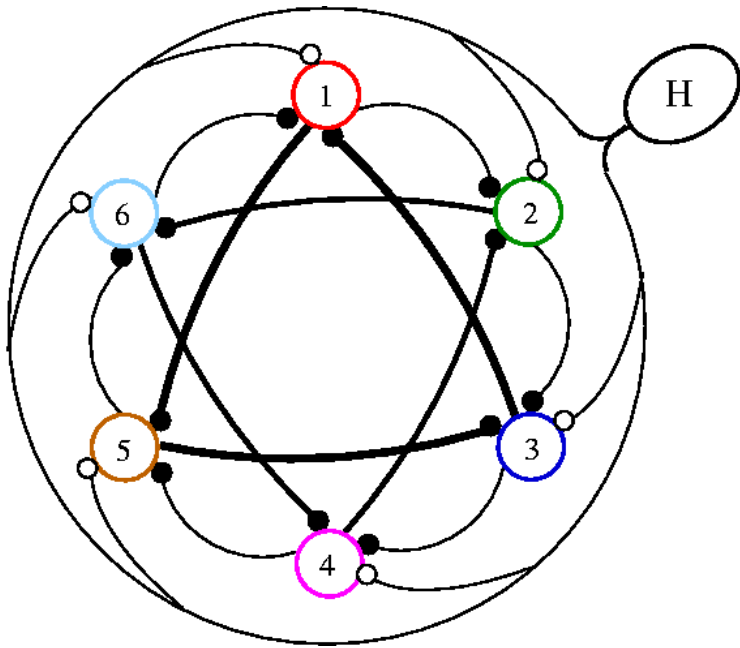
$a_i$  Es el promedio de disparos de la neurona  $i$

$\rho_{ij}$  Es la fuerza de la inhibición en  $i$  debida a  $j$

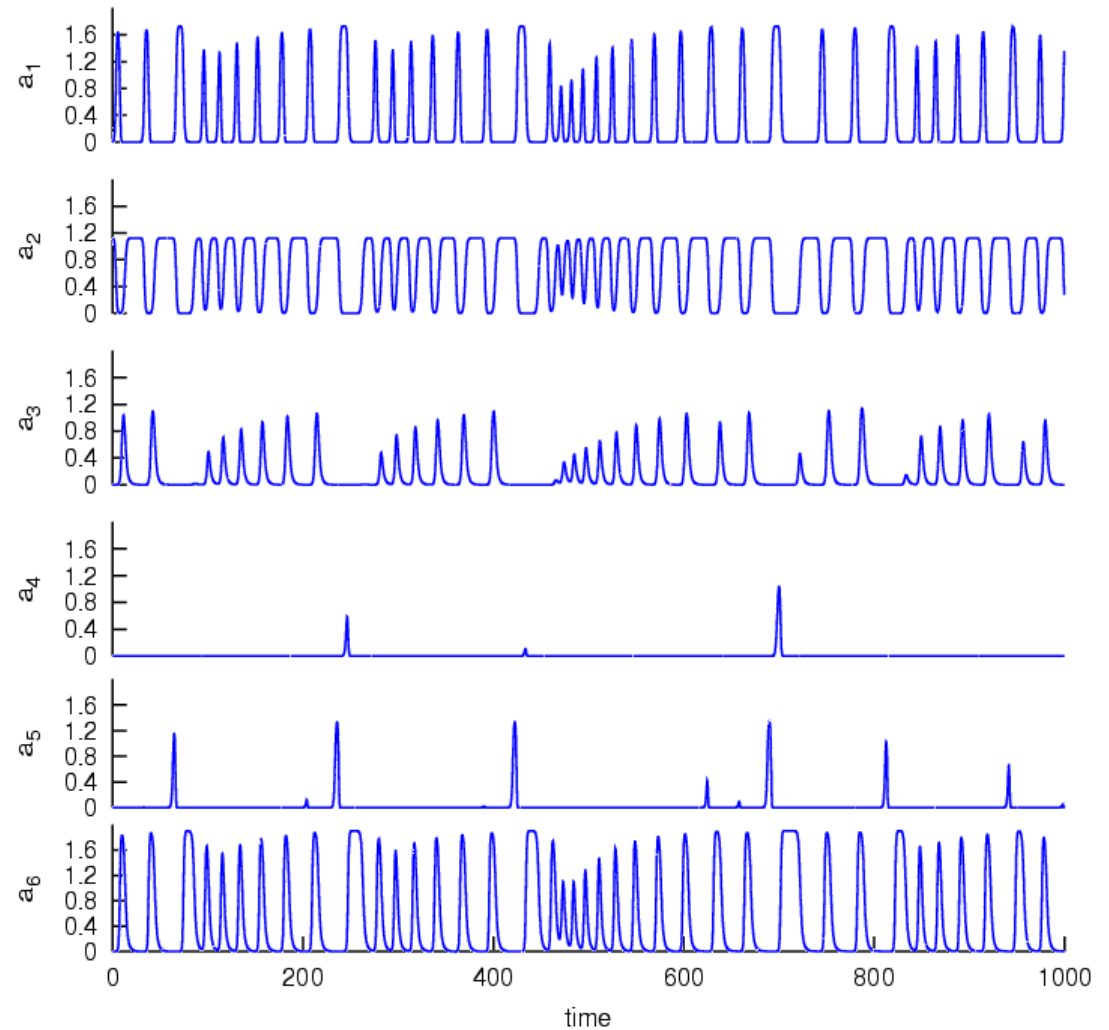
$\mathbf{H}$  Es la excitación debida a la interneurona cerebral de caza

$\mathbf{S}$  Es la excitación debida a la presión del estatolito

# Competición sin ganador entre los estatocistos

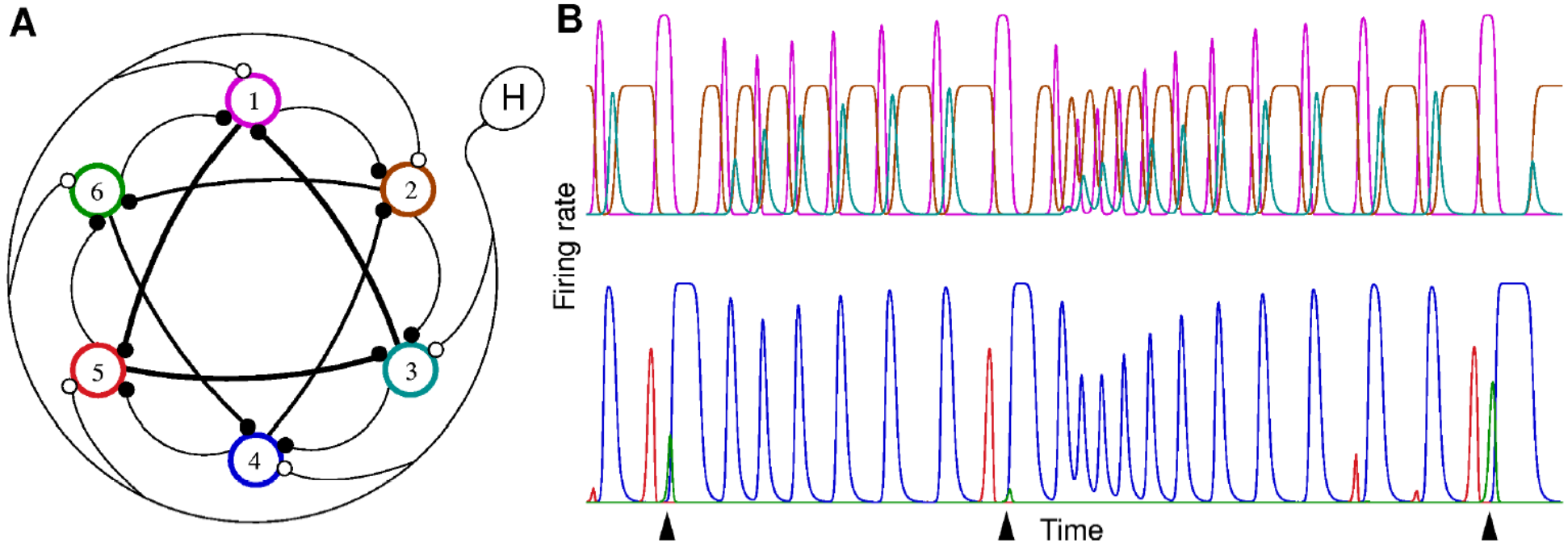


Neurona de caza (H) activa

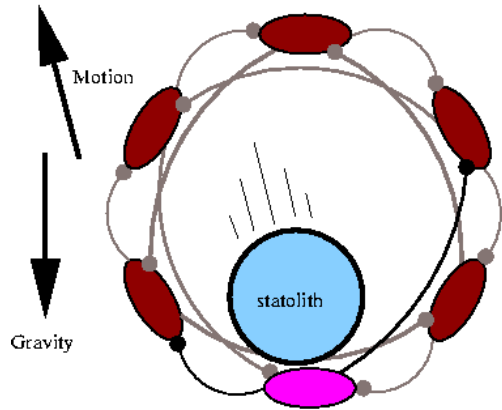




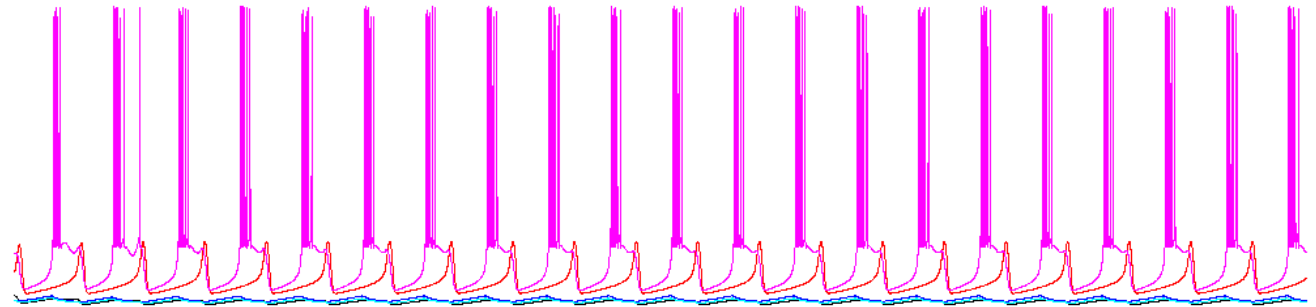
# Actividad irregular con bloqueo de la fase entre las neuronas activas



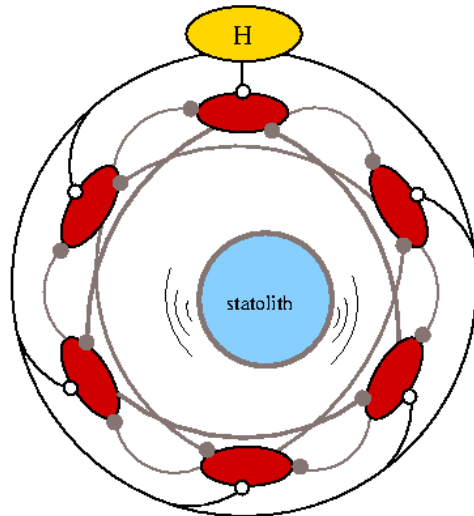
# Dinámica dual en un modelo H-H



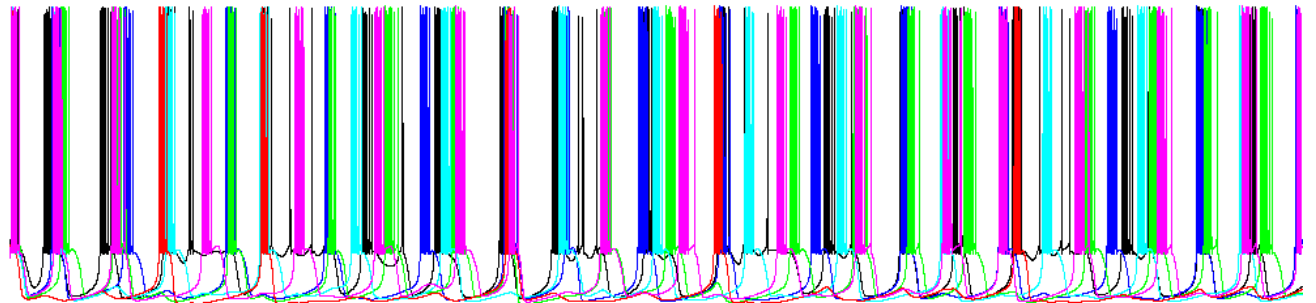
Winner-take-all in the SRC network



**A**



Winnerless competition in the SRC network



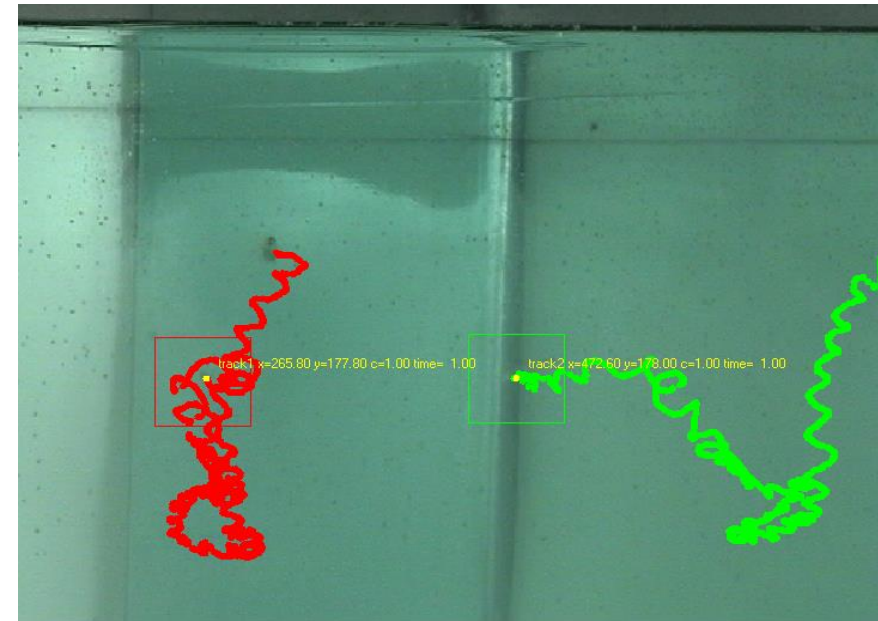
**B**

# Experimentos de laboratorio



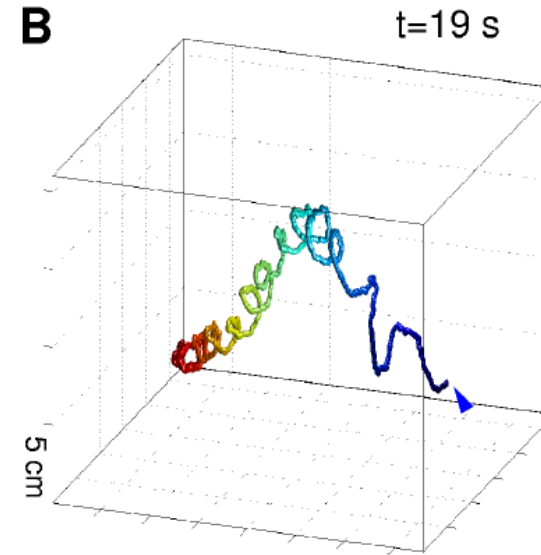
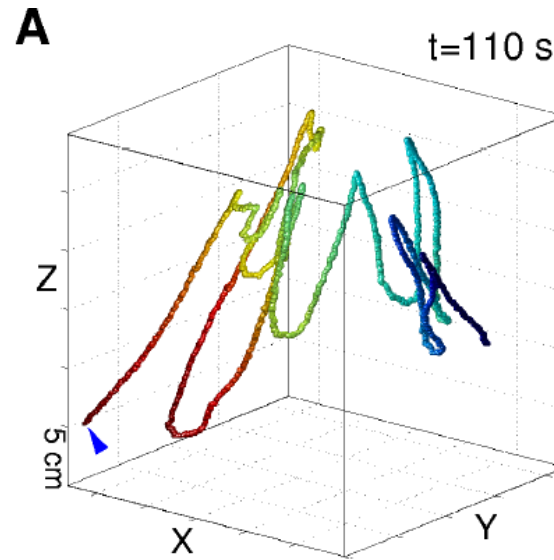
- Animales intactos durante el comportamiento normal y de caza
- Animales sin estatocistos
- Animales bajo el efecto de physostigmine que induce la caza

Las trayectorias 2-D registradas por la cámara se utilizan para reconstruir los movimientos en 3-D



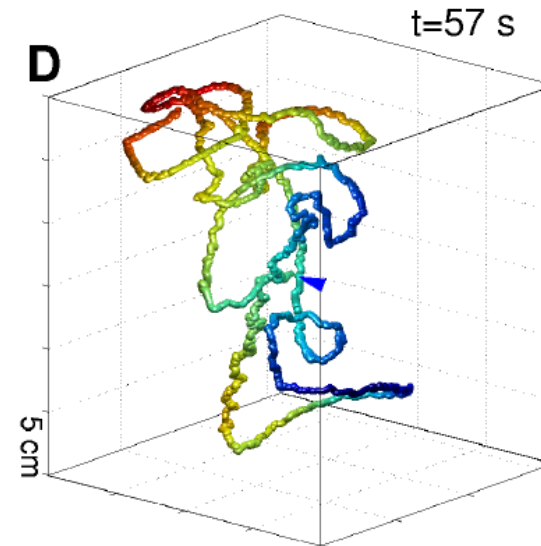
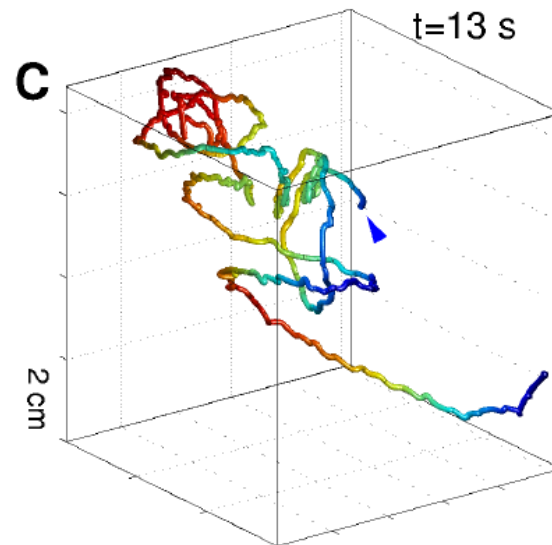
# 3-D trajectories

Comportamiento normal



Animal sin estatocistos

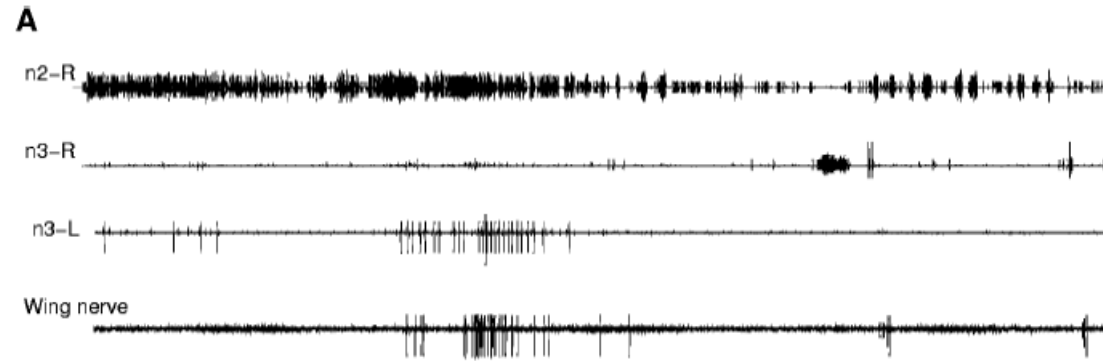
Caza



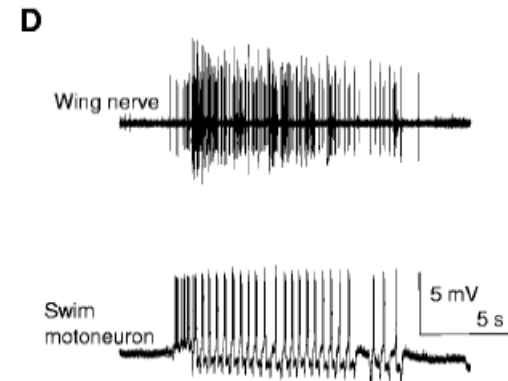
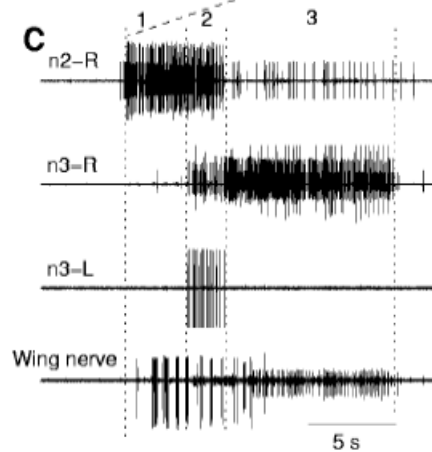
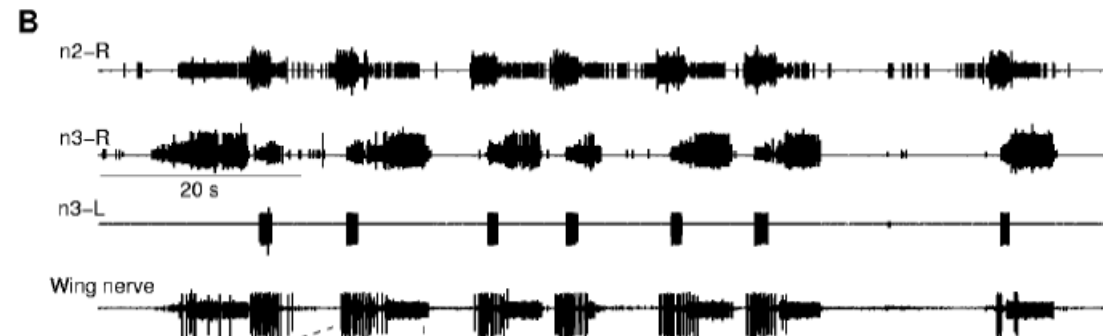
Caza provocada por la droga

# Electrofisiología

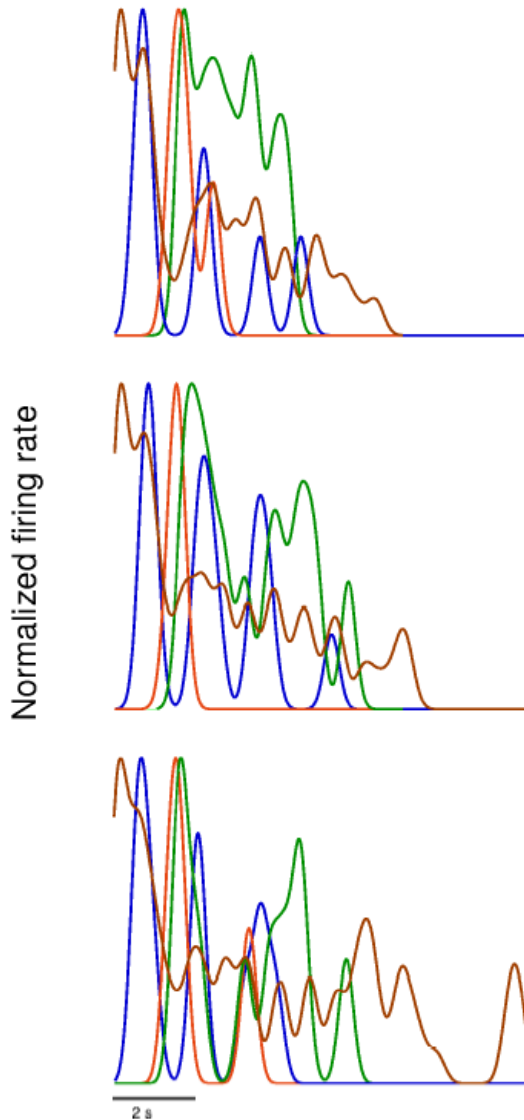
Control



Physostigmine



# Bloqueo de fase en las motoneuronas

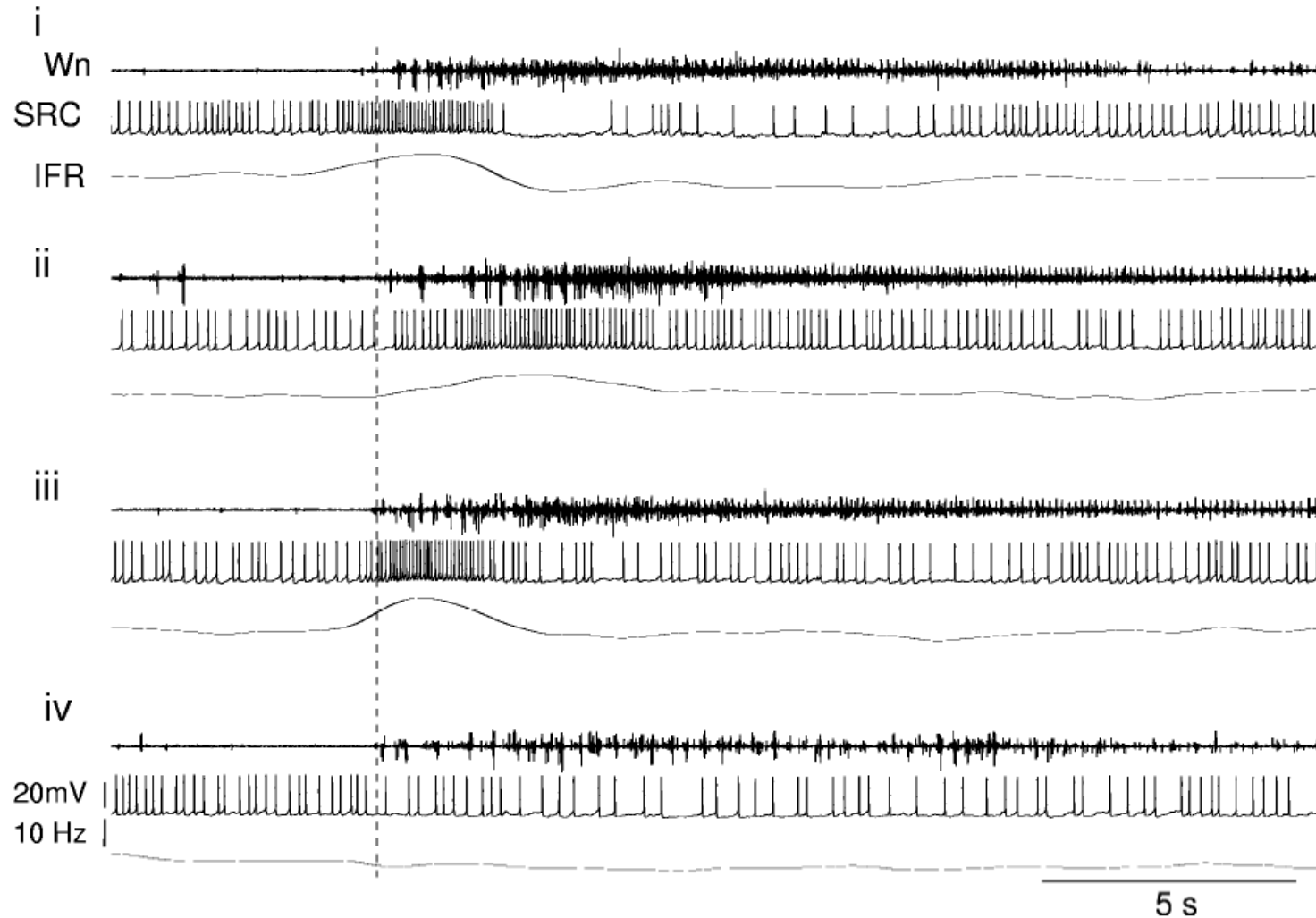


- Cuatro neuronas distintas de los nervios de la cola.
- Tres diferentes episodios de caza provocados por la droga.
- Consideramos el principio de la actividad de cada episodio el principio de la actividad en los nervios de las alas.

# Actividad correlacionada de los receptores y de las motoneuronas

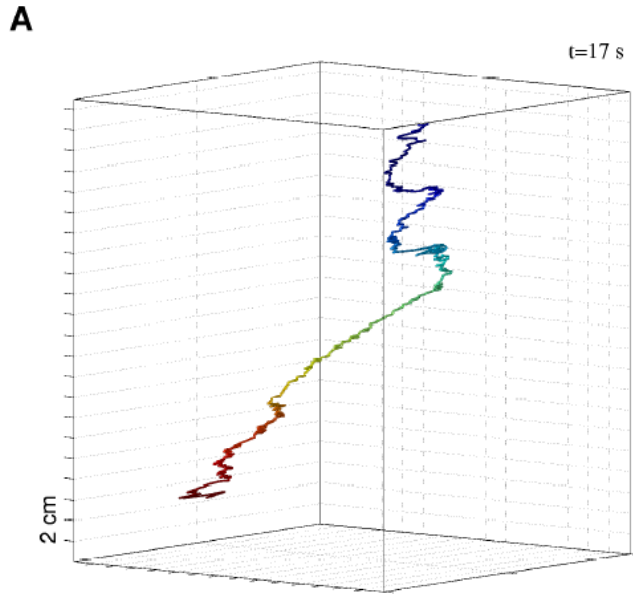


# Fase de la actividad de los receptores durante la caza

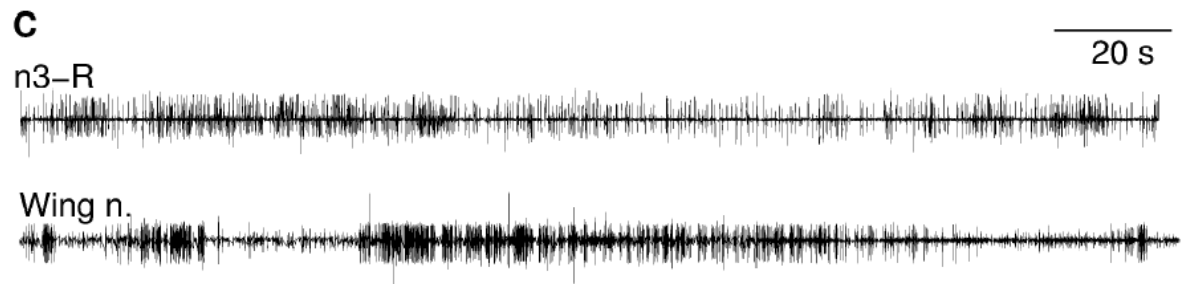
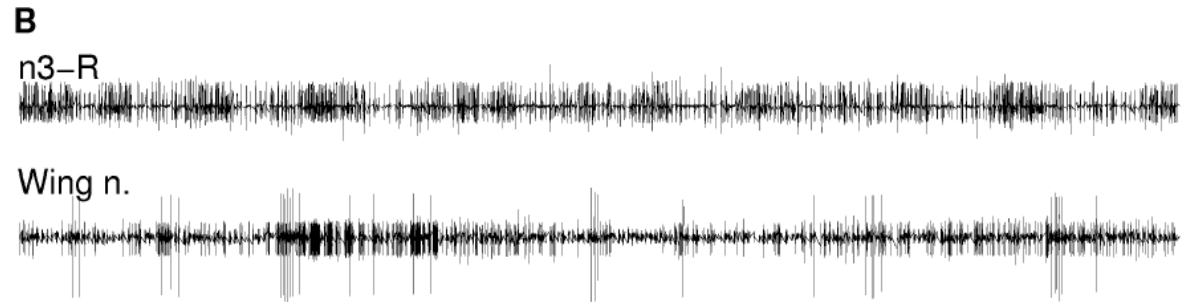




# El comportamiento de caza no ocurre sin los estatocistos



Sin estatocistos y con la droga



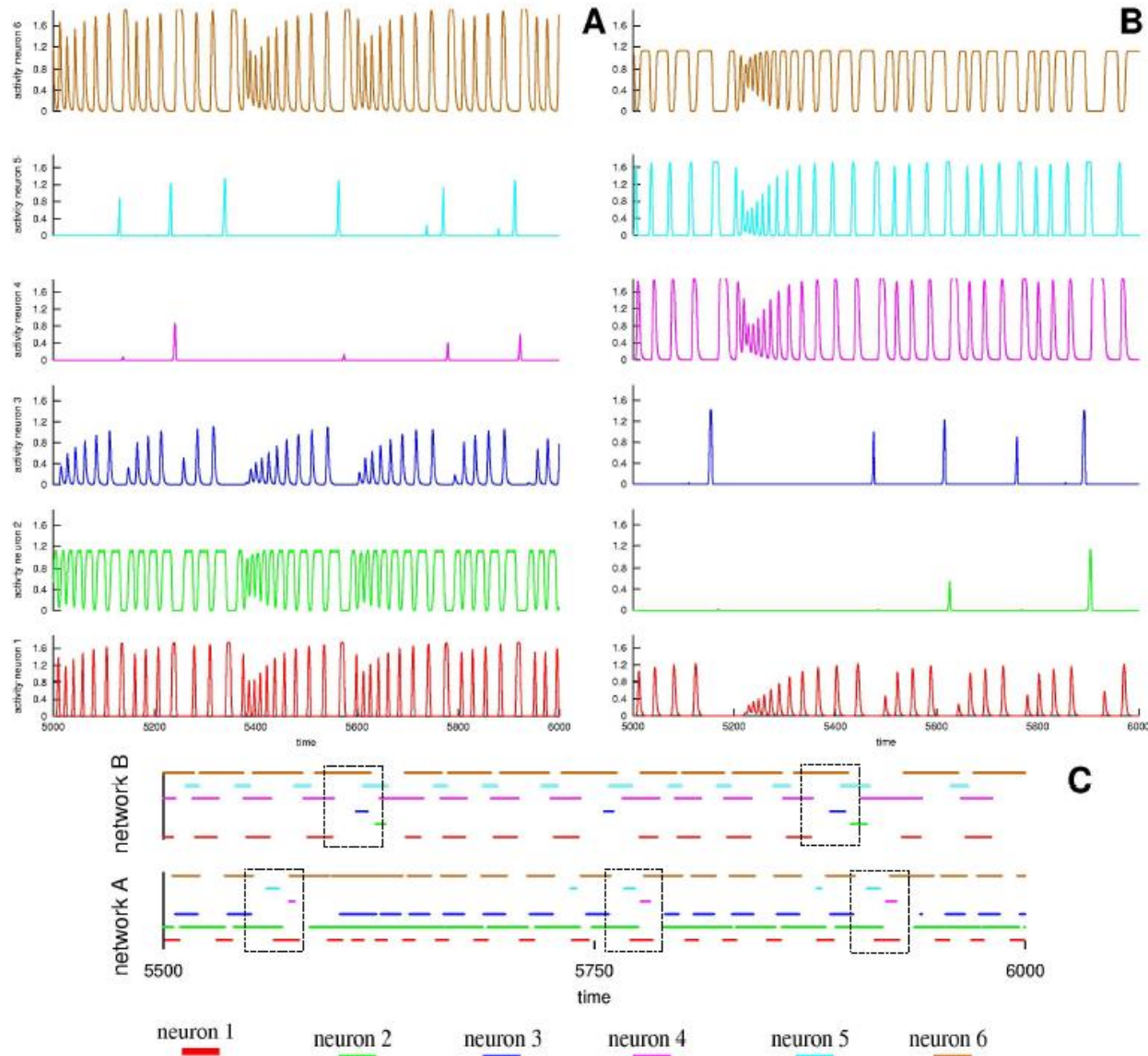
Sin estatocisto y con la droga en un animal operado dos días antes

# Conclusiones:

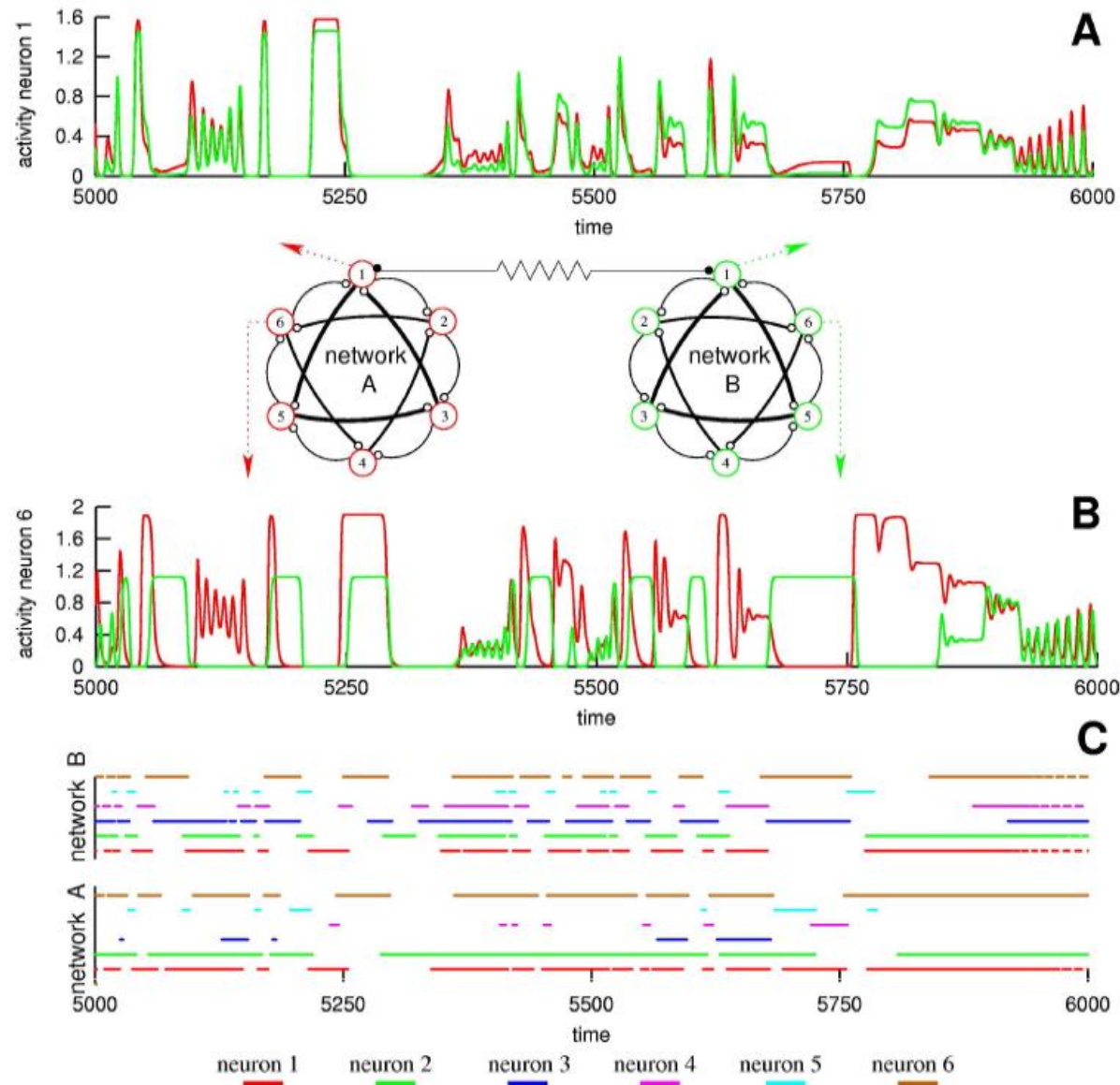


- La red inhibitoria de estatocistos es necesaria para producir el comportamiento de caza.
- La actividad de las neuronas sensoriales sigue una dinámica de winner-take-all y de competición sin ganador en el modo de caza en el modo de natación normal.
- Un receptor está activo durante diferentes fases de los episodios de caza. En algunos casos la actividad del receptor es anterior a la actividad de las ráfagas de las motoneuronas de la cola.
- El patrón de activación de los receptores está correlacionado con el patrón motor.
- La red de receptores tiene una función dual.

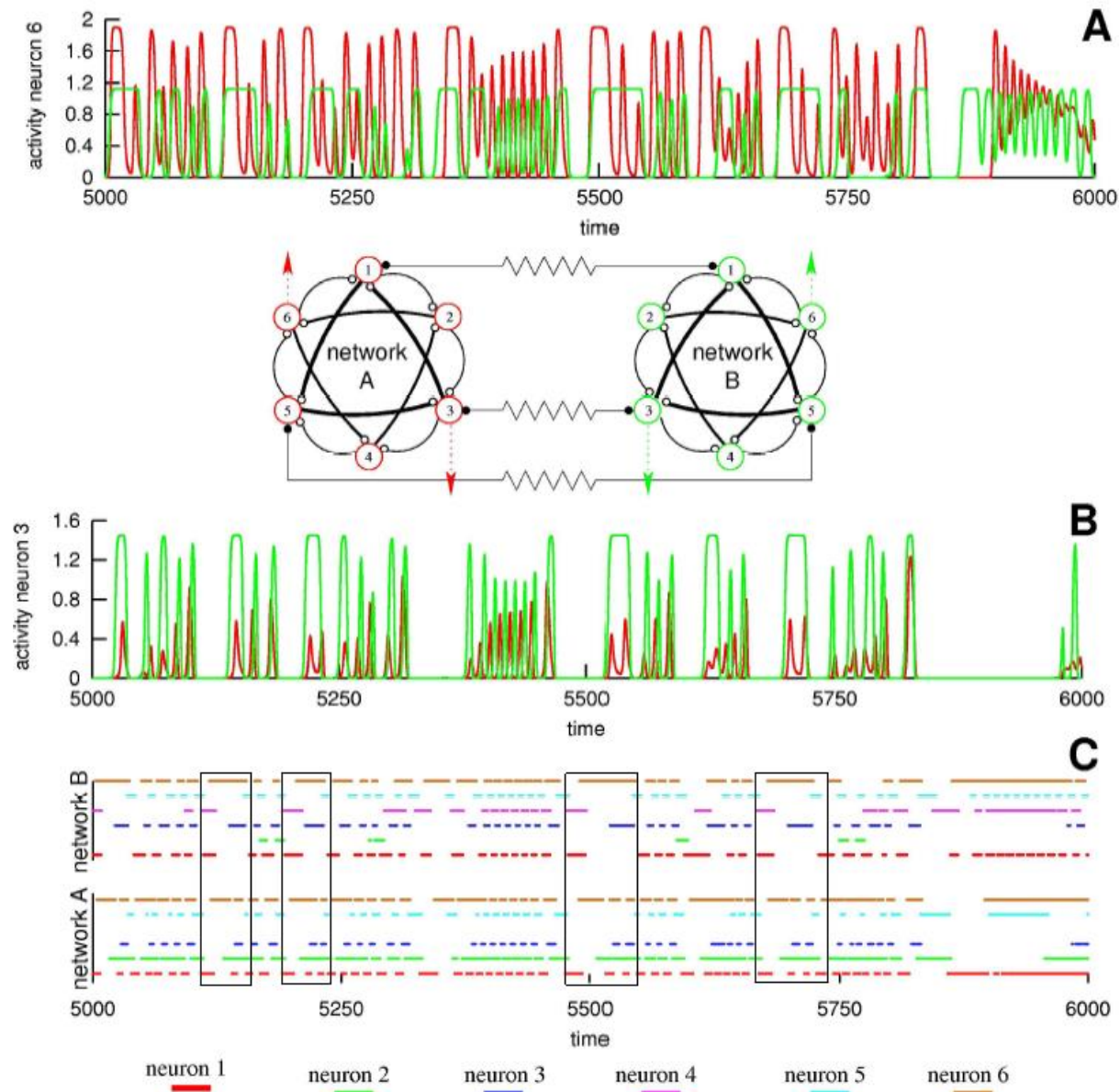
# Dos redes WLC independientes



# Dos redes WLC acopladas

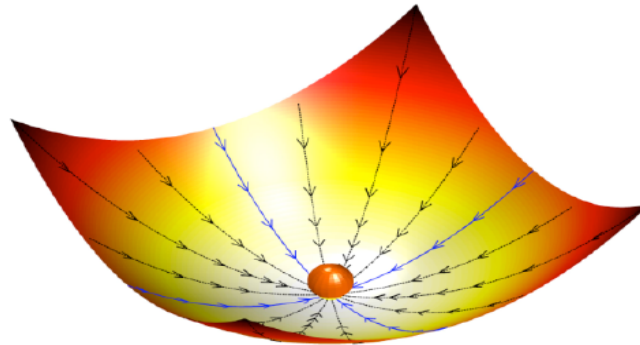


# Dos redes WLC acopladas II





# Visión tradicional: computación con atractores



Stable node (simple attractor) in the phase space of a strongly dissipative dynamical system. A dissipative system (or dissipative structure) is a thermodynamically open system, operating far from thermodynamic equilibrium in an environment with which it exchanges energy, matter and/or information. An elementary volume of the 'phase flow' of a dissipative system is compressed in the neighborhood of stable trajectories.

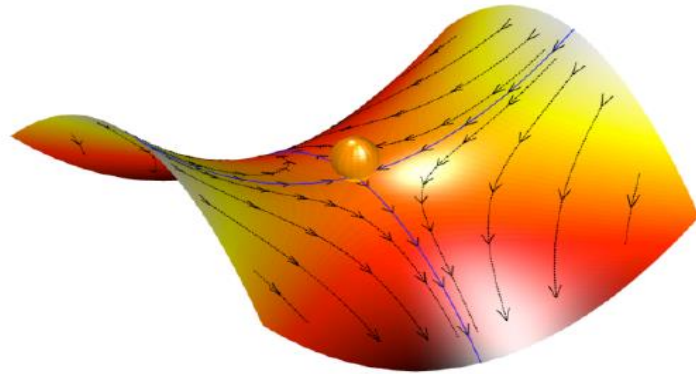
# **Visión complementaria**

## **Computación con Competición**

### **Sin Ganador:**

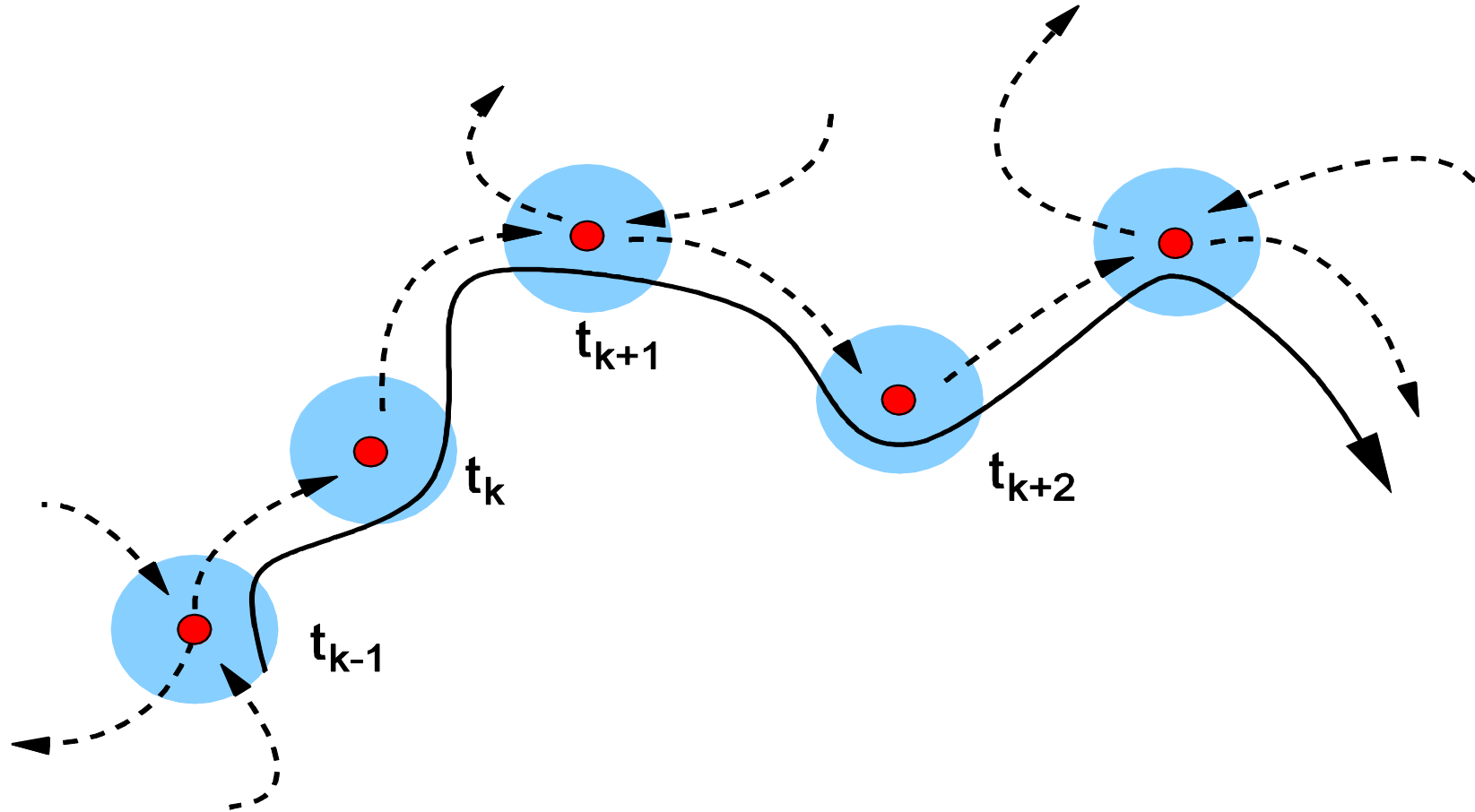
### **Silla con dos separatrices**

### **estables e inestables**



A separatrix is a manifold (surface or curve) that refers to the boundary separating two modes of behavior in the phase space of a dynamical system.

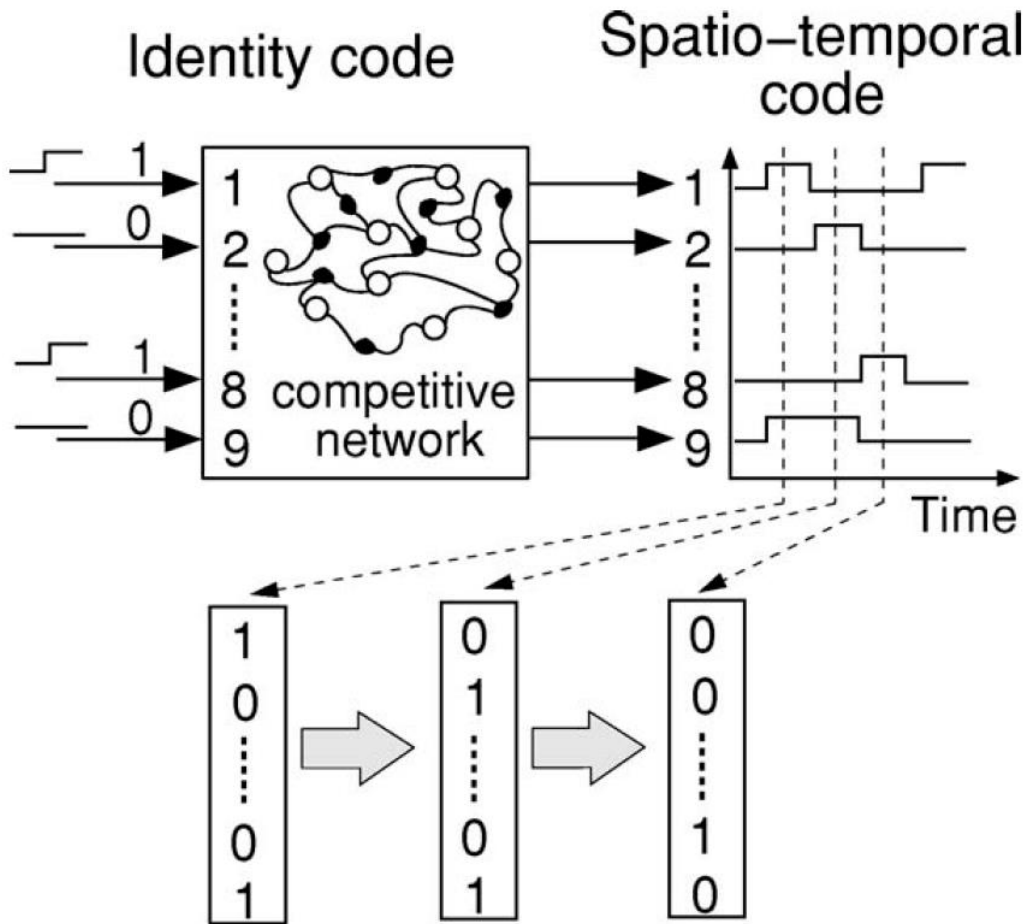
# Stable heteroclinic channel (SHC)



A set of dissipative saddles that are sequentially connected by unstable separatrices. The stability of a channel means that trajectories in the channel do not leave it until the end of the channel is reached.



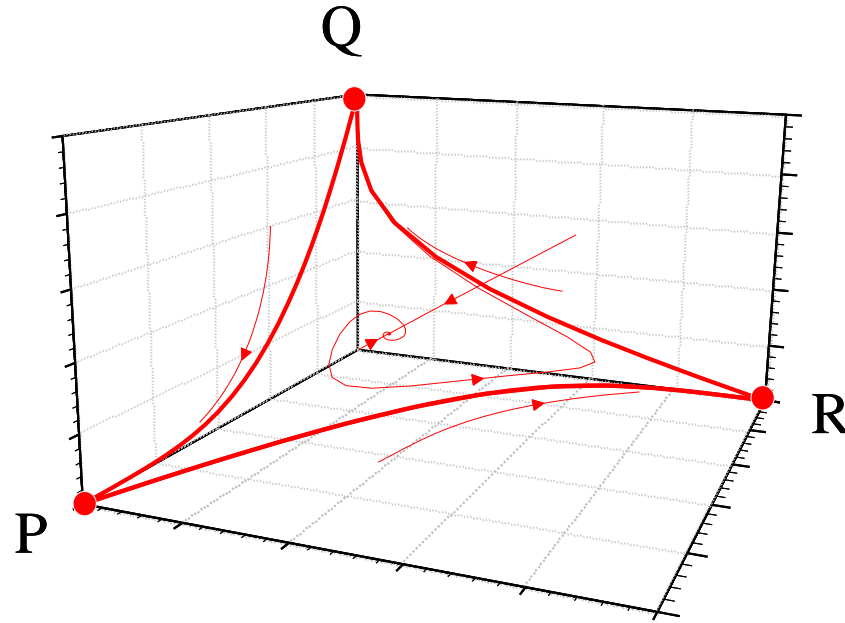
# Winnerless Competition (WLC)



WLC: the nonautonomous transient dynamics of neural systems receiving external stimuli and exhibiting sequential switching among temporal winners. The main feature of the WLC is the transformation of incoming inputs into spatiotemporal outputs based on the intrinsic switching dynamics of the neuronal ensemble.

Competition (through inhibitory connections) is a mechanism that maintains the highest level of variability and stability of neural dynamics, even if it is a transient behavior.

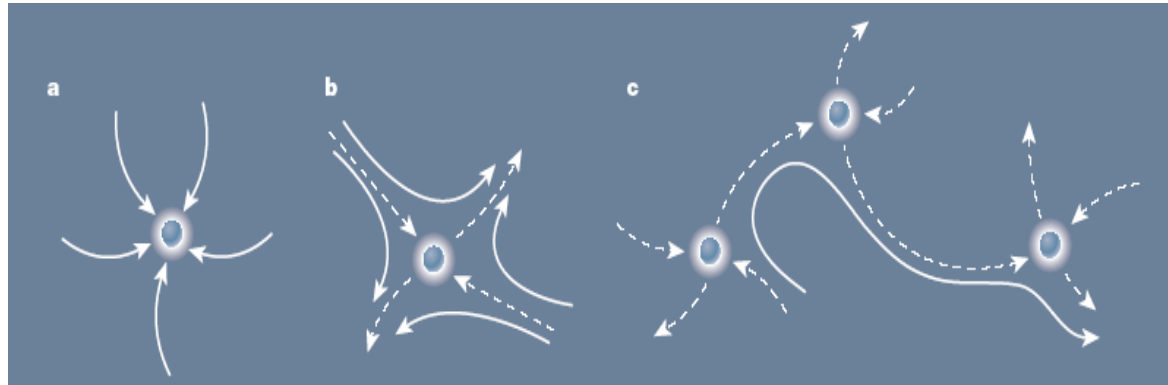
# Mathematical image of WLC



The mathematical image of Winnerless competitive dynamics is a number of saddle states whose vicinities are connected by their unstable manifolds to form a heteroclinic sequence. This is a natural dynamical image for many types of neural activity.

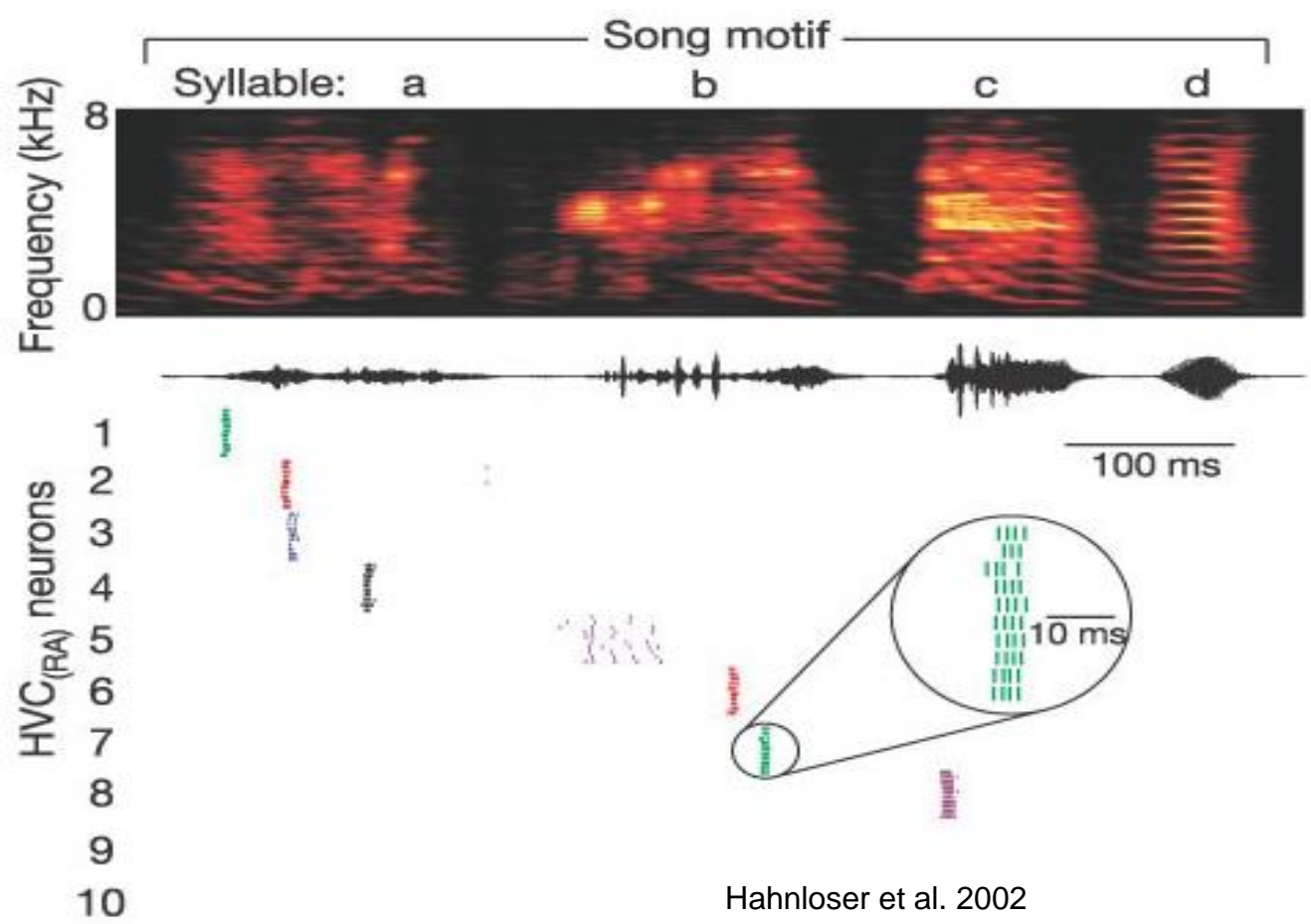
# WLC dynamics is good for:

- Generation of sequential activity
- Transient dynamics as opposed to (or together with attractors).
- Temporal coordination and synchronization
- Reproducibility and robustness



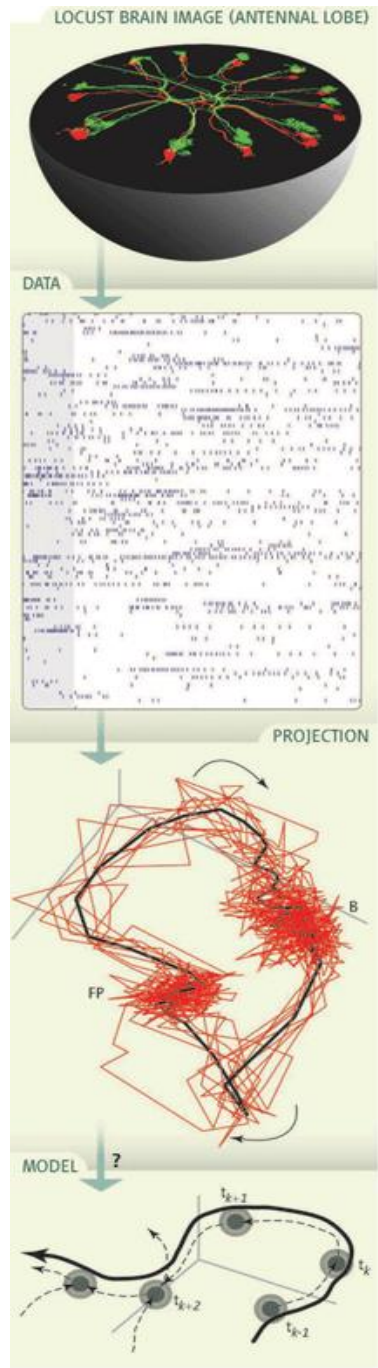
# Examples of neural systems that seem to display WLC

Vocal brain regions of songbirds

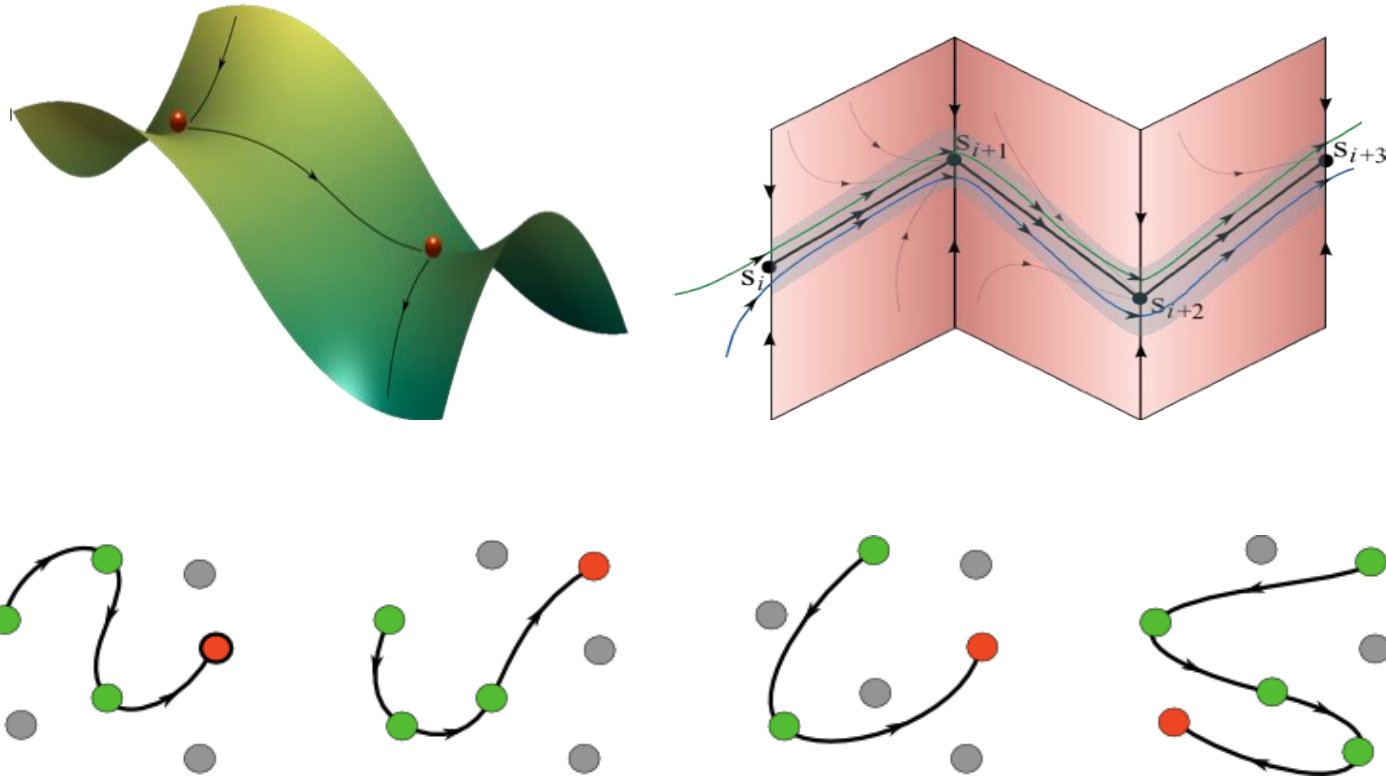


Hahnloser et al. 2002

Odor encoding of insects



# Robust transient sequential dynamics

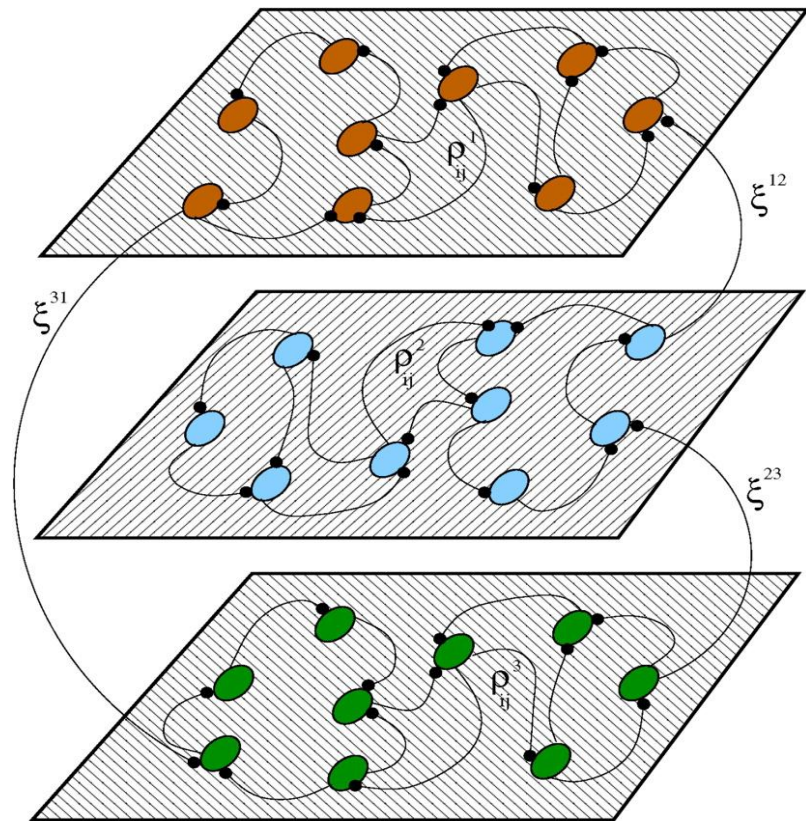


## IDEAS for the dynamical modeling of robust sequential dynamics:

- The existence in the phase space of metastable states that represent the activity of individual modes;
- Metastable states must be connected by separatrices to build the sequence.

# Generalized Lotka-Volterra models

$$\tau_i \frac{dx_i^m}{dt} = x_i^m \left( \sigma_i^m - \sum_{j=1}^N \rho_{ij}^m x_j^m - \sum_{k=1}^M \sum_{j=1}^N \xi_{ij}^{mk} x_j^k \right), \quad i, j = 1, \dots, N; \quad m, k = 1, \dots, M$$



$x_i^m \geq 0$  is the instantaneous amplitude of the  $(i^m)$ -mode

$\sigma_i \geq 0$  is the growth rate for the mode depending on external stimuli

$\rho_{ij}^m \geq 0$  and  $\xi_{ij}^{mk} \geq 0$  represent asymmetric interaction strengths between the modes

$m, k$  indicate different brain activity modalities

$i, j$  indicate different modes within the same modality



## Two fundamental principles to build a model of transient sequential dynamics

1. The transient dynamics generated has to be **robust against small perturbations but at the same time sensitive to external information signals** from the environment and internal signals that inform about the state of the system itself.
2. **Finite resources:** e.g., in terms of attention and working memory

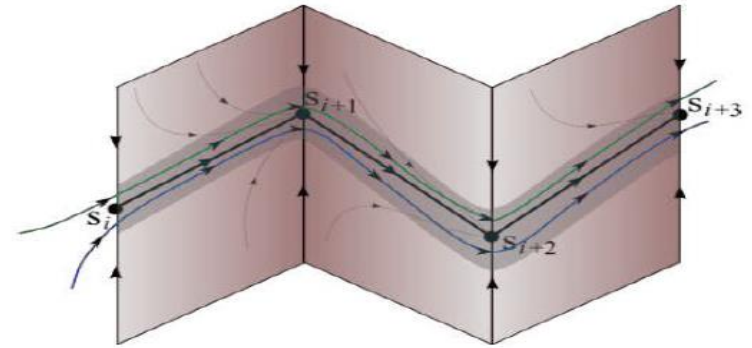
# Conditions for the stability of the heteroclinic channel

$$\lambda_1^{(i)} > 0 > \operatorname{Re} \lambda_2^{(i)} \geq \operatorname{Re} \lambda_3^{(i)} \geq \dots \geq \operatorname{Re} \lambda_d^{(i)}$$

Here the eigenvalues  $\lambda_j^{(i)}$  of the saddles are ordered and we introduce the saddle values as:

$$\nu_i = -\operatorname{Re} \lambda_2^{(i)} / \lambda_1^{(i)}$$

$$\text{If } \prod_i^g \nu_i > 1,$$



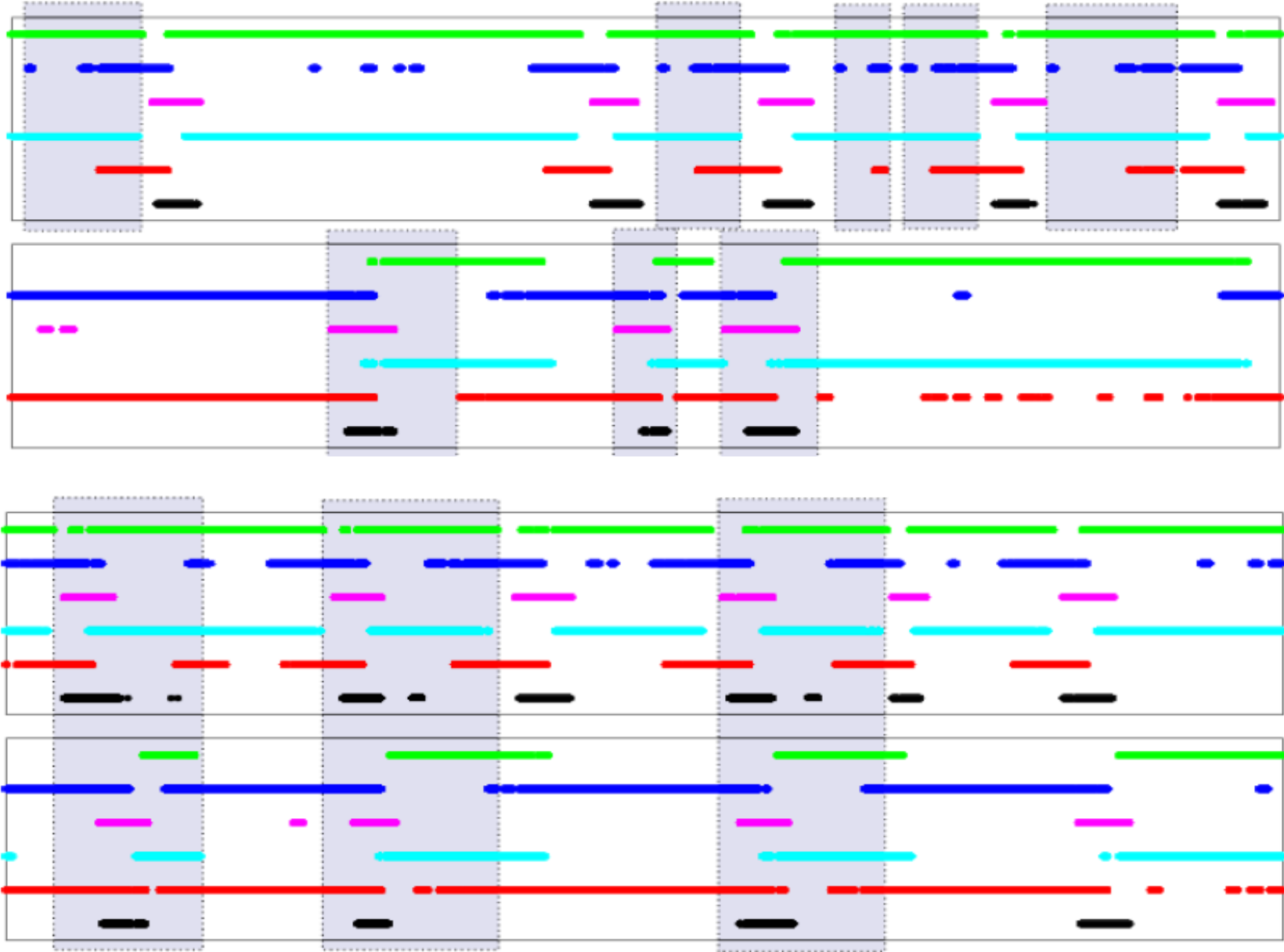
where  $g$  is the number of metastable states along the heteroclinic chain, the trajectories in the vicinity of a heteroclinic chain cannot scape from the channel, providing its robustness.

Rabinovich et al. 2012. Information flow dynamics in the brain. *Physics of Life Reviews* 9(1): 51-73.

M.I. Rabinovich et al. Transient Cognitive Dynamics, Metastability and Decision Making. *PLoS Computational Biology* 4(5): e1000072.



# Heteroclinic Binding



# Heteroclinic binding: Information Flow Capacity

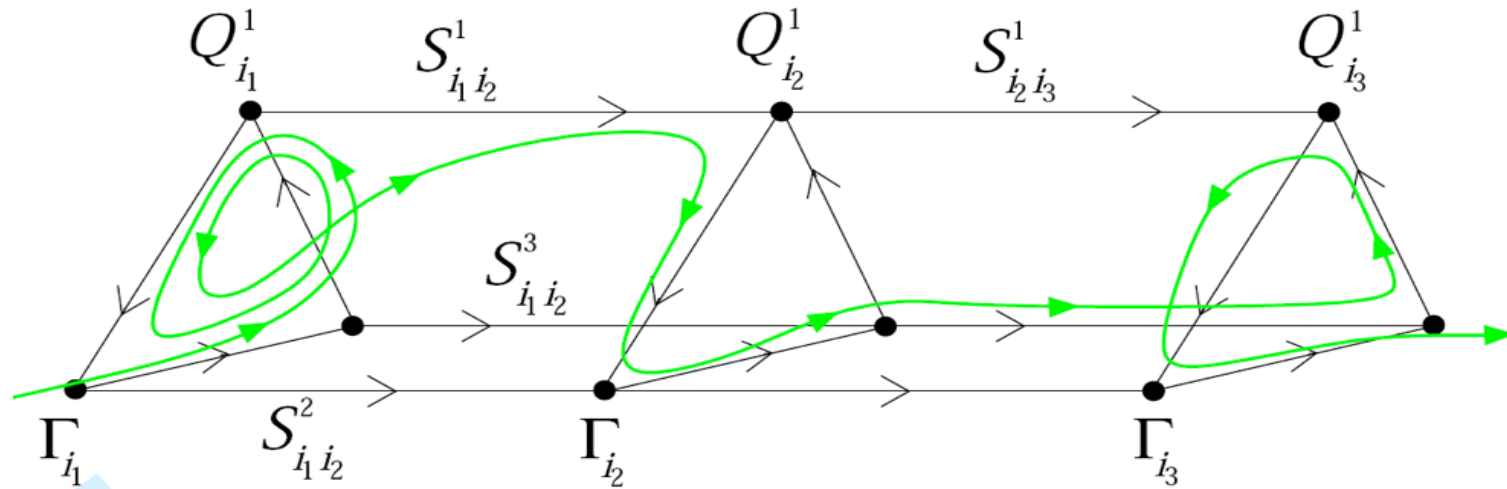
The Information Flow Capacity  $C_{IF}$  can quantitatively characterize the effectiveness of the heteroclinic binding:

$$C_{IF}(L) = \sum_l^L (\Delta C_{IF}(l)) \quad \text{where} \quad \Delta C_{IF}(l) = J_l + \sum_{j=1}^{J_l} \frac{\text{Re } \lambda_j^l}{|\lambda_{J_{l+1}}^l|}$$

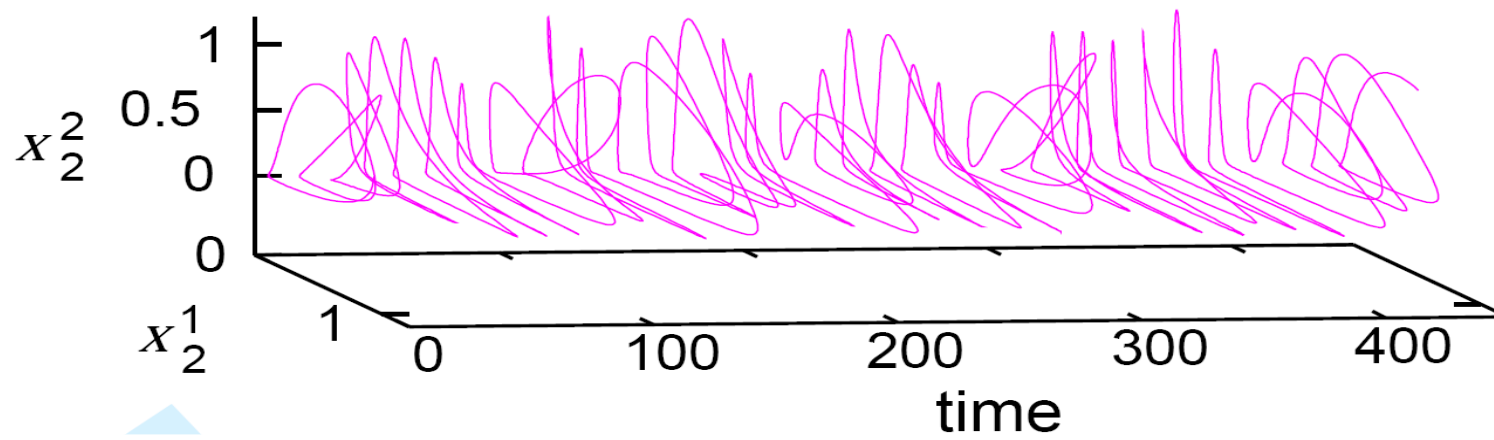
Here  $l$  is the index of the metastable state (saddle) along a channel,  $L$  is the number of saddles that the system passes until time  $t_L$ , and the integer  $J$  is determined by the following conditions:

$$\sum_{j=1}^J \text{Re } \lambda_j > 0, \quad \sum_{j=1}^{J+1} \text{Re } \lambda_j < 0$$

# Heteroclinic Binding



Each saddle along the binded heteroclinic channels has 2 unstable separatrices. This means all  $J_i=2$ . Thus, the estimation of the  $C_{IF}$  tells us that the flow capacity for a binding channel is at least two times larger than the  $C_{IF}$  of 3 independent channels



# Simple model of the interaction between cognition and emotion

$$\tau_{A_i} \frac{dA_i(t)}{dt} = A_i(t) \cdot \left[ \sigma_i(\mathbf{S}, \mathbf{B}, \mathbf{R}_A) - \sum_{j=1}^N \rho_{ij} A_j(t) \right] + A_i(t) \eta_A(t),$$

$$i = 1, \dots, N$$

$$\tau_{B_i} \frac{dB_i(t)}{dt} = B_i(t) \cdot \left[ \zeta_i(\mathbf{S}, \mathbf{B}, \mathbf{R}_B) - \sum_{j=1}^M \xi_{ij} B_j(t) \right] + B_i(t) \eta_B(t),$$

$$i = 1, \dots, M$$

$$\theta_A^i \frac{dR_A^i(t)}{dt} = R_A^i(t) \cdot \left[ \sum_{j=1}^N A_j(t) - \sum_{m=1}^{K_A} R_A^m - \phi_A \sum_{m=1}^{K_B} R_B^m + d_A(t) \right],$$

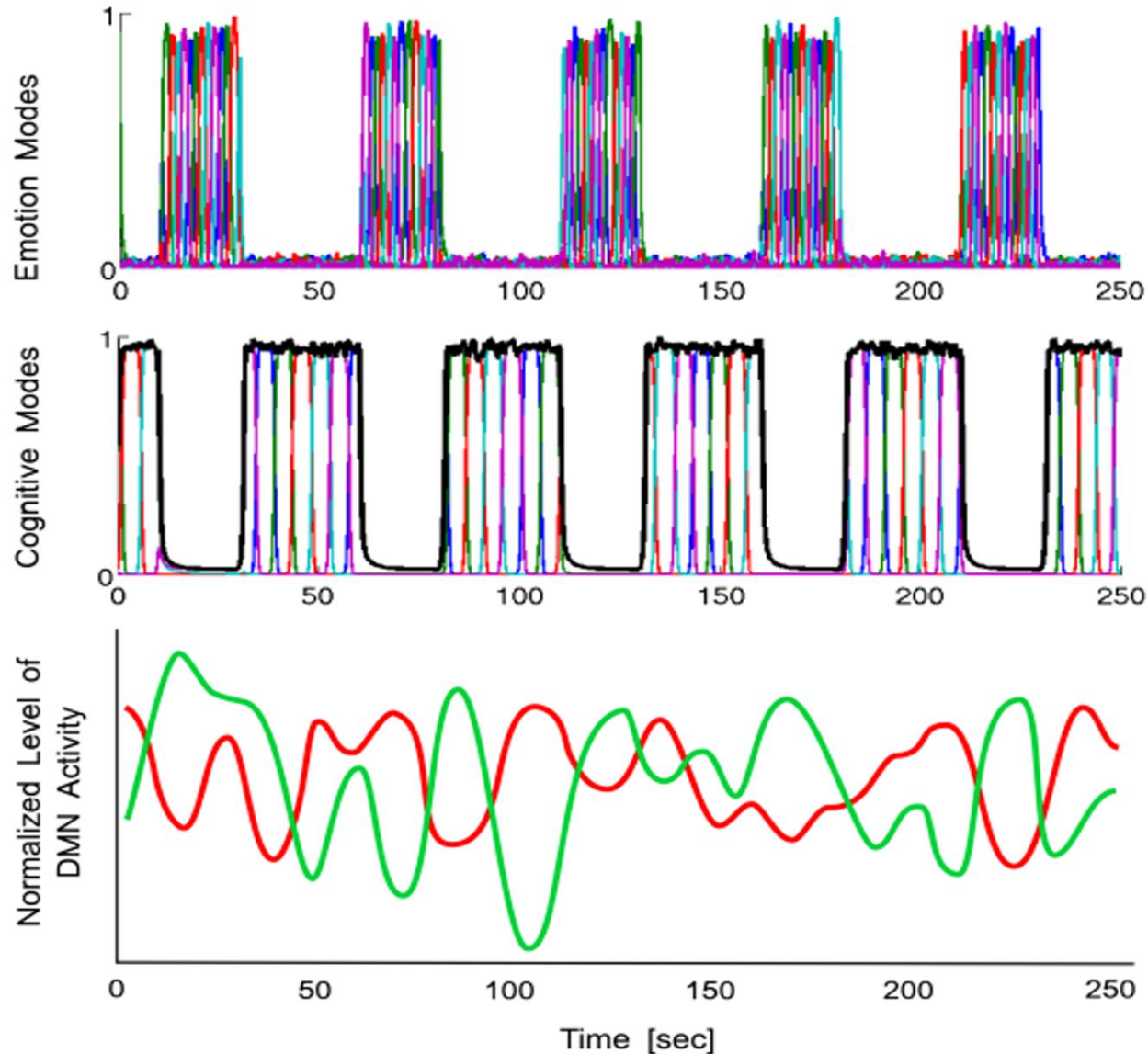
$$i = 1, \dots, K_A$$

$$\theta_B^i \frac{dR_B^i(t)}{dt} = R_B^i(t) \cdot \left[ \sum_{j=1}^M B_j(t) - \sum_{m=1}^{K_B} R_B^m - \phi_B \sum_{m=1}^{K_A} R_A^m + d_B(t) \right],$$

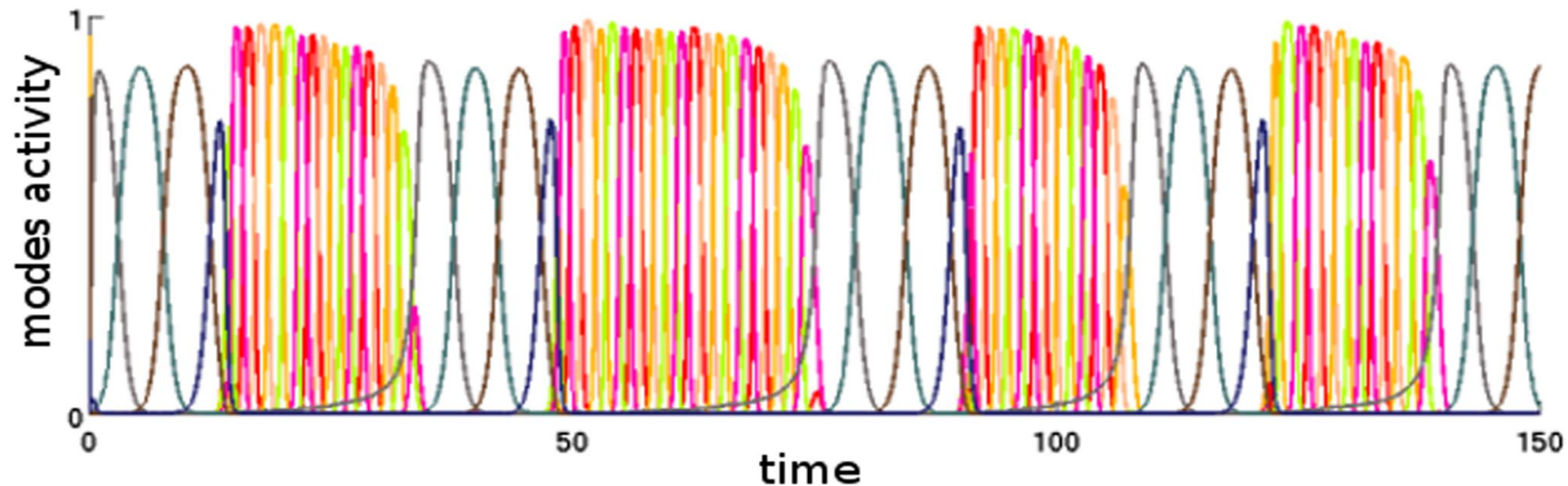
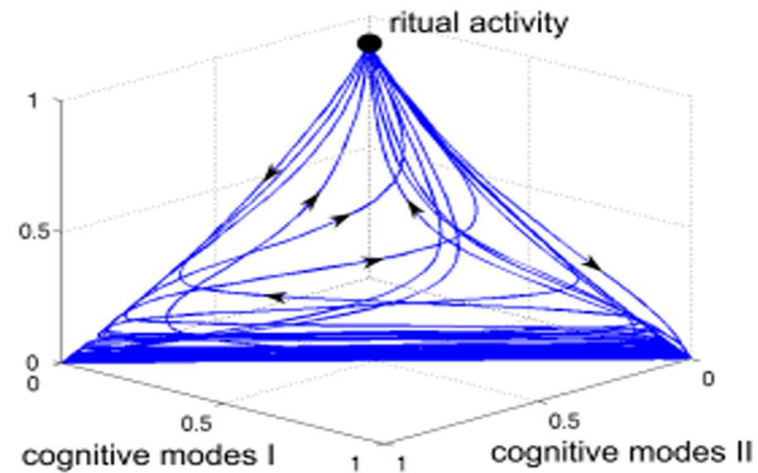
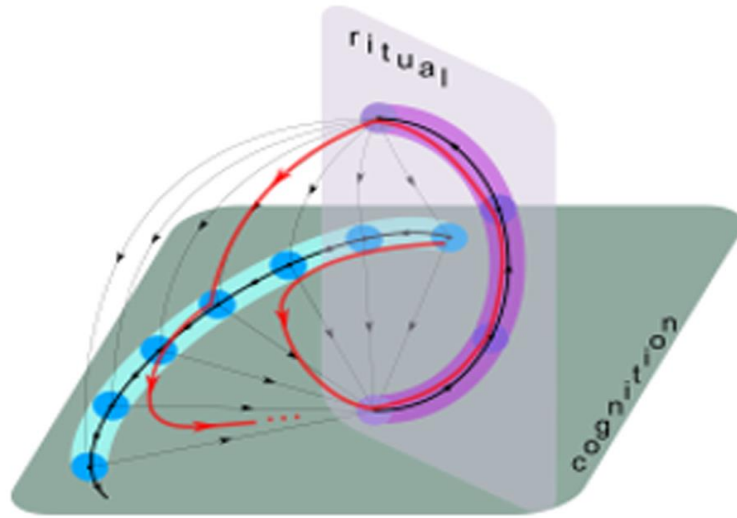
$$i = 1, \dots, K_B$$

Parameter	Role
$\tau_{A_i}$	Time constants for cognitive modes
$\tau_{B_i}$	Time constants for cognitive modes
$\theta_A^i, \theta_B^i$	Time constants for resource dynamics
$\rho, \xi$	Competition matrices – inducing metastable state sequence
$\sigma_i$	Increments to the cognition modes – locating the metastable states
$\zeta_i$	Increments to the emotion modes
$\eta_A, \eta_B, d_A, d_B$	Noise components
$\phi_A, \phi_B$	Regulate the resource modes competition

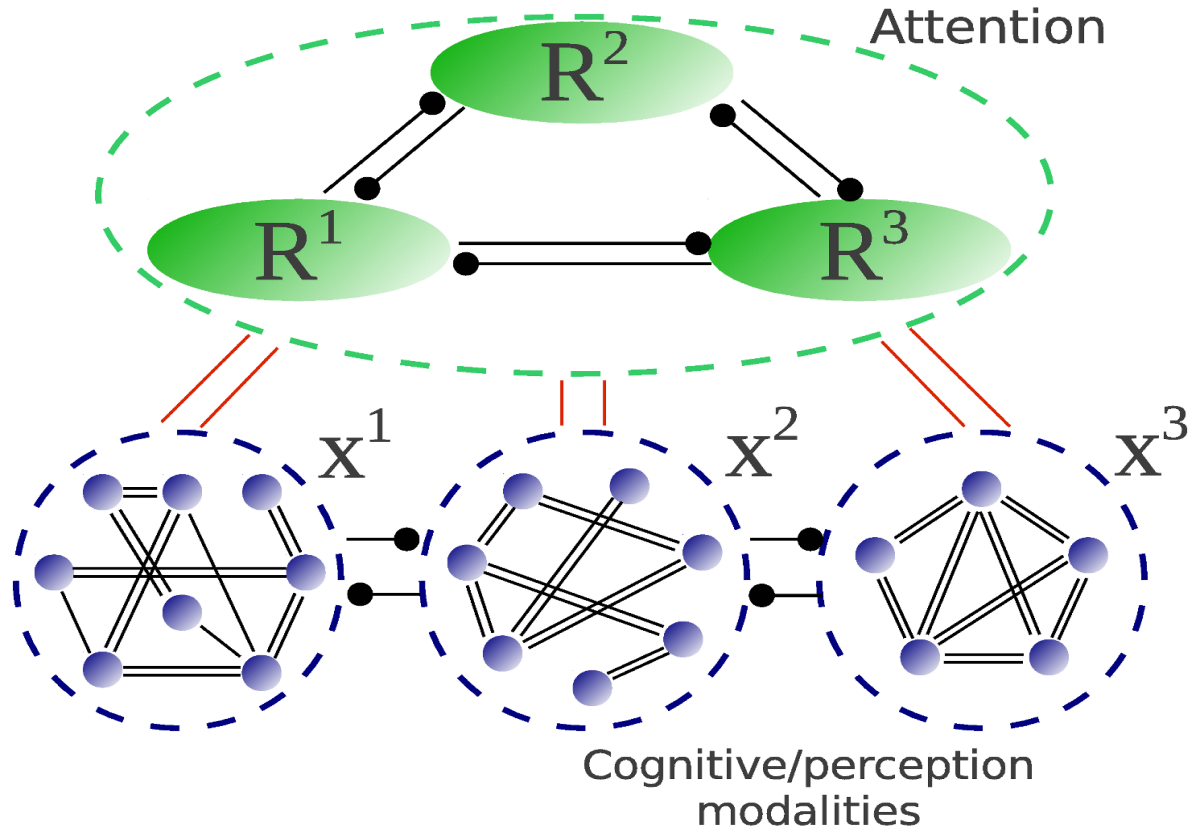
# Simple model of the interaction between cognition and emotion



# Dynamical representation of Obsessive Compulsive Disorder (OCD)



# Modeling attention dynamics

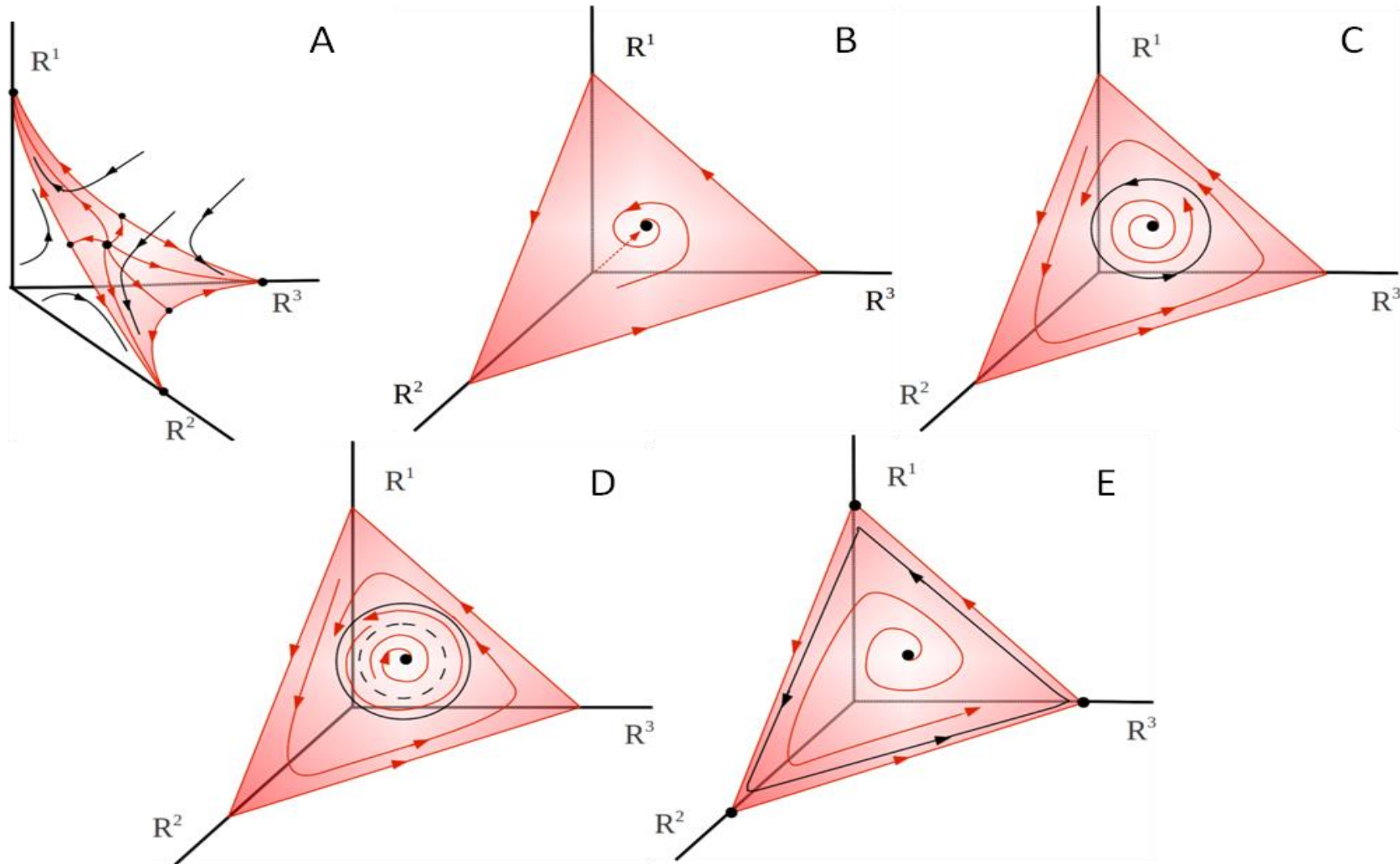


Architecture of the attention mode interaction in the case of three modality processing ( $X^1, X^2, X^3$ ).  $R^1, R^2, R^3$  represent attention resource modes corresponding to these modalities.

$$\theta_m \frac{dR^m}{dt} = R^m \cdot \left[ \gamma^m(X^m, S^m) - \sum_{k=1}^M \varsigma^{mk}(R^k, S^m) R^k \right]$$

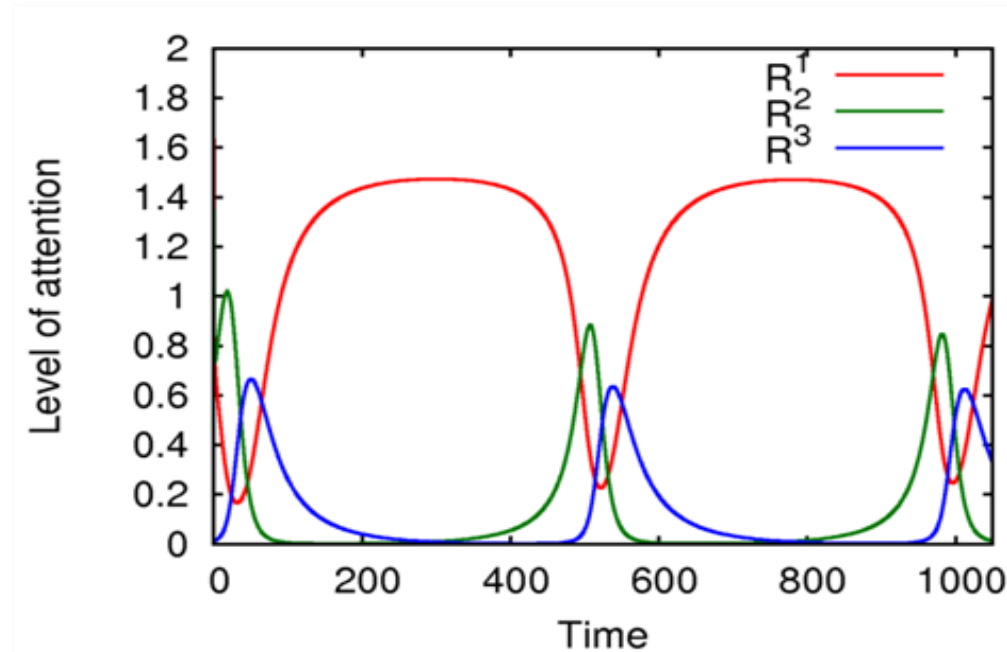


# Examples of different attention crossmodality dynamics

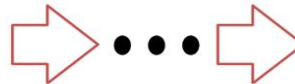
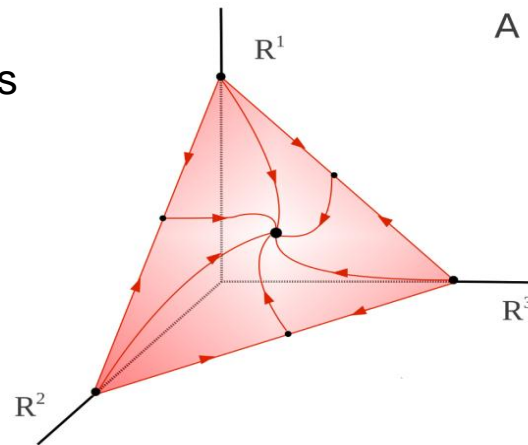




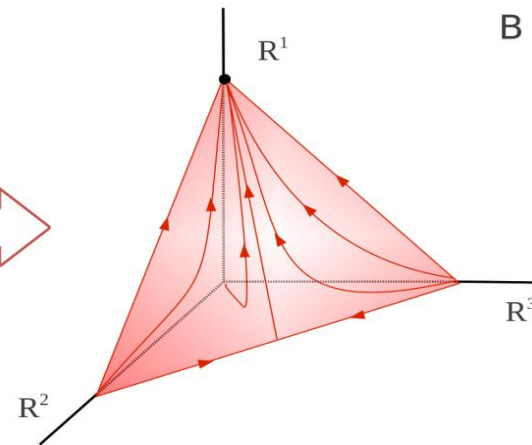
# Competition among attention modes



Coexisting tasks  
each of them  
requiring only a  
little attention



B Only one specific  
modality attracts all  
attention resources –  
'winner take all'  
attentional regime



# CONCLUSIONS:

- There are alternatives and complementary views to attractor dynamics in neural systems.
- Models proposed to describe neural sequential transient activity can be applied in a wide variety of problems.
- In this formalism, the informational flow is described and analyzed as a heteroclinic flow in the phase space.
- The discussed models provide essential mechanisms that can be used as control systems to drive sequential interactions in closed-loops.