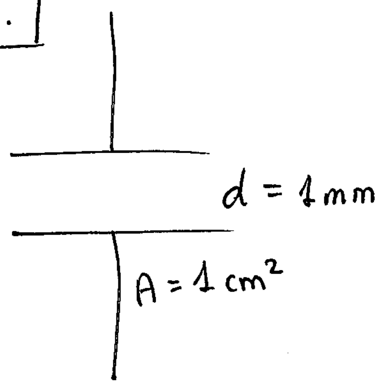
EJERCICIOS TEMA 3

5.



a) Aislado $Q = 2pC$ ¿ΔV? ΔU ($d_i = 1mm \rightarrow$ $\rightarrow d_F = 2mm$)

$$C = \frac{A\varepsilon_0}{d} \quad ; \quad \Delta V = \frac{Q}{C} = \frac{Qd}{A\varepsilon_0} =$$

$$= \frac{2.10^{-12}C \cdot 1.10^{-3}}{8'85.10^{-12} \cdot 1(10^{-4})} = 2'26\,V$$

$$U = \frac{1}{2}QV = \frac{1}{2}\cdot\frac{Q^2}{C} \quad ; \quad \Delta U = U_F - U_i = \frac{1}{2}\cdot\frac{Q_F^2}{C_F} - \frac{1}{2}\cdot\frac{Q_i^2}{C_i} =$$

$$= \frac{1}{2}Q^2\left(\frac{1}{C_F} - \frac{1}{C_i}\right) = \frac{1}{2}Q^2\left(\frac{d_F}{\varepsilon_0 A} - \frac{d_i}{\varepsilon_0 A}\right) =$$

$$= \frac{1}{2}\cdot\frac{Q^2}{\varepsilon_0 \cdot A}\left(d_F - d_i\right) = 2'26.10^{-12}\,J$$

b) Conectado a batería 3V ΔU ($d_i = 1mm \rightarrow d_F = 2mm$)

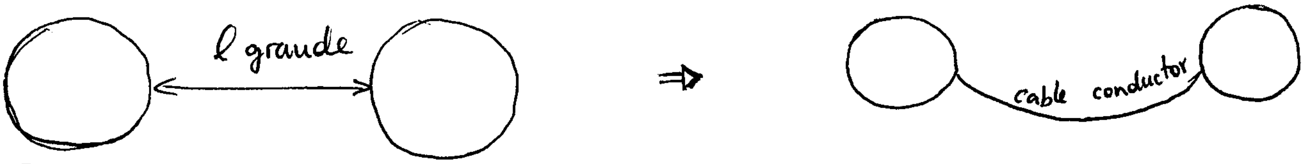
$$C = \frac{A\varepsilon_0}{d} \quad ; \quad \Delta V = \frac{Q}{C} \quad ; \quad Q = C \cdot \Delta V = \frac{A\cdot\varepsilon_0}{d}\cdot\Delta V_{bat} = 2'65\,pC$$

$$U = \frac{1}{2}QV = \frac{1}{2}\cdot\frac{Q^2}{C} \quad ; \quad \Delta U = U_F - U_i = \frac{1}{2}C_F V_F^2 - \frac{1}{2}C_i V_i^2 =$$

$$= \frac{1}{2}\cdot\frac{A\varepsilon_0 V^2}{d_i} - \frac{1}{2}\cdot\frac{A\cdot\varepsilon_0}{d_F}V^2 =$$

$$= \frac{1}{2}\varepsilon_0 \cdot A \cdot V^2\left(\frac{1}{d_i} - \frac{1}{d_F}\right) \approx 2'0\,pJ$$

3.



$R_1 = 5cm$ $\qquad$ $R_2 = 10cm$

$Q_1 = 3nC$ $\qquad$ $Q_2 = -9nC$

en equilibrio : a) V_1, V_2 ? $\qquad$ b) ΔU

a) $V_1 = V_2$ intercambio de carga $Q_{F1} \neq Q_{i1}$ y $Q_{F2} \neq Q_{i2}$

$$V_{1i} = K \frac{Q_{1i}}{R_1} \qquad V_{2i} = K \frac{Q_{2i}}{R_2}$$

Sabemos que después de alcanzar el equilibrio:

$$\begin{cases} V_{F1} = V_{F2} = K \dfrac{Q_{1F}}{R_1} = K \dfrac{Q_{2F}}{R_2} \\ Q_{1i} + Q_{2i} = Q_{1F} + Q_{2F} \end{cases}$$

Sist. de ecuaciones: Q_{1F}? Q_{2F}?

RESPUESTA: $Q_{1F} = -2nC$ $\qquad$ $Q_{2F} = -4nC$

$$V_{1F} = K \frac{Q_{1F}}{R_1} = 360V \qquad V_{2F} = K \frac{Q_{2F}}{R_2} = 360 V$$

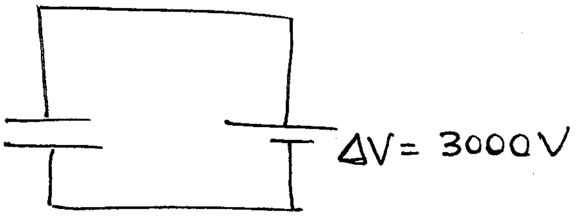
b) ¿ΔU?

$U = \frac{1}{2} QV$; $V = k \frac{Q}{R}$ esfera

$$\left.\begin{aligned} U_{inicial} &= \frac{1}{2} K \left(\frac{Q_{1i}^2}{R_1} + \frac{Q_{2i}^2}{R_2} \right) \\ U_F &= \frac{1}{2} K \left(\frac{Q_{1F}^2}{R_1} + \frac{Q_{2F}^2}{R_2} \right) \end{aligned}\right\} \Delta U = -3'33 J$$

8.

$d = 2cm$
$A = 100cm^2$



a) E

Con bateria E_f si $d_f = 5\,cm$

U_{antes} y $U_{después}$

$$E_{antes} = \frac{\Delta V}{d_{antes}} = \frac{3000\,V}{2.10^{-2}} = 1'5 . 10^5\ N/C \text{ ó } V/m$$

$$E_{después} = \frac{\Delta V}{d_{después}} = \frac{3000\,V}{5.10^{-2}} = 0'6 . 10^5\ \frac{N}{C} \text{ ó } V/m$$

$$\boxed{U = \frac{1}{2} Q V = \frac{1}{2} C V^2}$$

$$U_{antes} = \frac{1}{2} \cdot \frac{\varepsilon_0 A}{d} \cdot V^2 = \frac{1}{2} \cdot \frac{\varepsilon_0 \cdot 100.10^{-4}}{0'02} \cdot (3000)^2 = 1'9.10^{-5}\ J$$

$$U_{después} = \frac{1}{2} \cdot \frac{\varepsilon_0 . A}{d} \cdot V^2 = \frac{1}{2} \cdot \frac{\varepsilon_0 . 100.10^{-4}}{0'05} (3000)^2 = 7'9.10^{-6}\ J$$

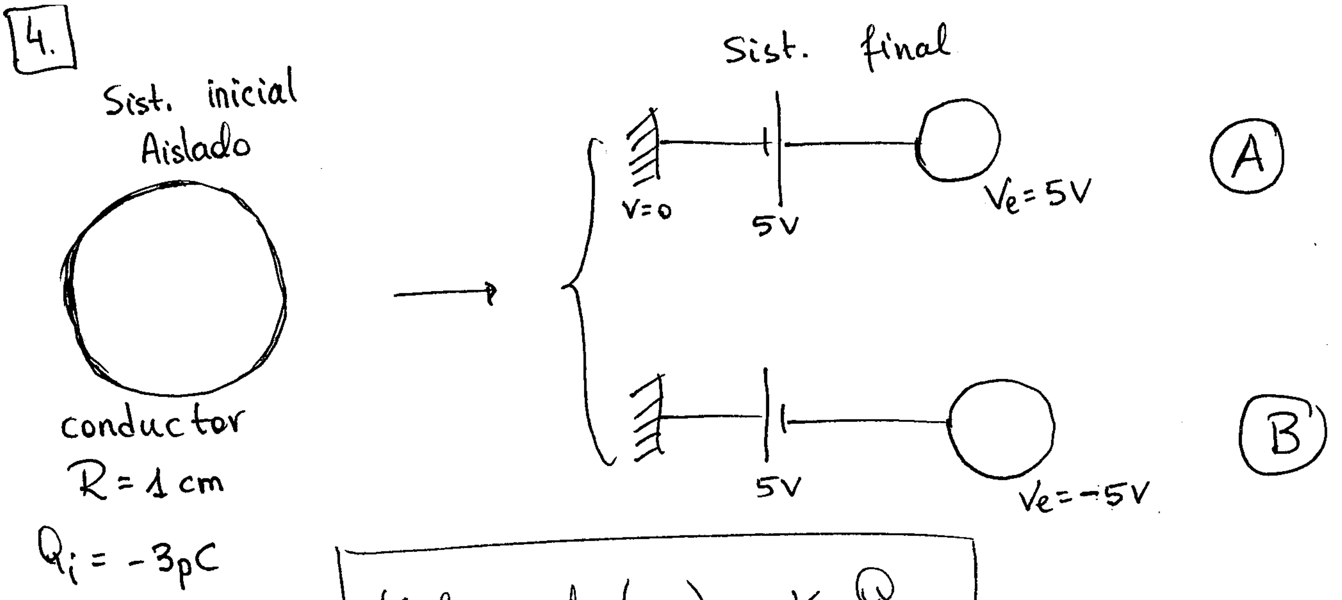
▶ Si se desconecta la batería:

$$Q_F = Q_i \qquad Q_i = CV_i = \frac{\varepsilon_0 A}{d} V_i = \frac{\varepsilon_0 . 100 . 10^{-4}}{2.10^{-2}} . 3000 = 1'32.10^{-8}\ C$$

U_i = apartado anterior

$$U_F = \frac{1}{2} Q_F V_F = \frac{1}{2} Q_F \cdot \frac{Q_F}{C_F} = \frac{1}{2} \cdot \frac{Q_F^2}{C_F} = \frac{1}{2} \cdot Q_i^2 \cdot \frac{d_F}{\varepsilon_0 . A} = 4'95 . 10^{-5}\ J$$

4.



$$\boxed{V(\text{esfera conductora}) = K\frac{Q}{R}}$$

(A)

SISTEMA INICIAL

$$U = \frac{1}{2}Q_iV_i = \frac{1}{2}Q_i K\frac{Q_i}{R} = 4'05\,pJ$$

SISTEMA FINAL

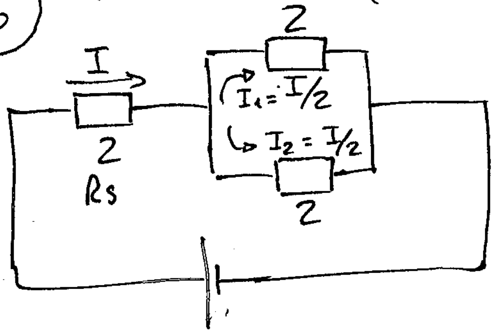
$$V = \frac{KQ_F}{R} \quad ; \quad Q_F = \frac{VR}{K} = 5'5pC$$

$$U_F = \frac{1}{2}QV = \frac{1}{2}\cdot 5'5\,pC \cdot 5 =$$

(B) Sistema inicial igual que en (A).

$$V = \frac{KQ_F}{R} \quad ; \quad Q_F = \frac{VR}{K} = -5'5\,pC$$

(4.6) $P_{max} = 10$ W (enunciado)



¿Potencia máxima de un circuito.

$$P = I \cdot V = I^2 \cdot R$$

Rs soportará Imax.

¿Imax? $Pot_{max} = 18 W$; $P = I^2 R \Rightarrow P_{max} = I_{max}^2 \cdot R \Rightarrow$

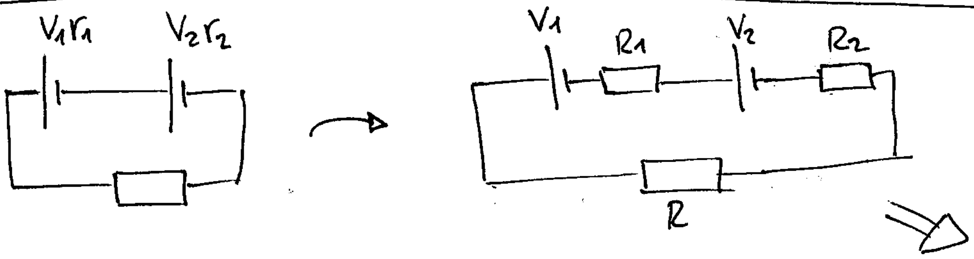
$$\Rightarrow I_{max} = \sqrt{\frac{P_{max}}{R}} = 3A$$

Si $I = 3A \Rightarrow I_1 = I_2 = \frac{I}{2} = 1'5 A$

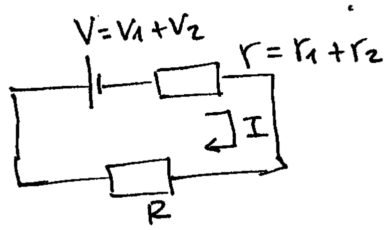
$$Pot\ Total = P_{Rs} + P_{R_{arriba}} + P_{R_{abajo}} = 18 + (1'5^2 \cdot 2) + (1'5^2 \cdot 2) =$$

$$= \boxed{27 W}$$

(4.1)



¿R para que Pot_R sea máxima?



$$P_R = I\,V = I^2 \cdot R$$

Quiero $P(R) \longrightarrow \frac{dP}{dR} = 0 \longrightarrow$ encontrar máximo

Para $I(R)$ Kirchoff en el circuito

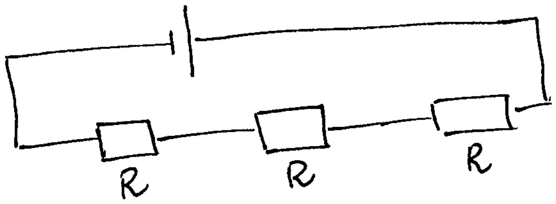
$$V - Ir - IR = 0 \Rightarrow I = \frac{V}{R+r}$$

$$P_R = \left(\frac{V}{R+r}\right)^2 \cdot R \quad ; \quad \frac{dP}{dR} = \left(\frac{V}{R+r}\right)^2 - R\frac{2V^2}{(R+r)^3} = \left(\frac{V}{R+r}\right)^2\left(1 - \frac{2R}{R+r}\right)$$

$$\frac{dP}{dR} = 0 \Rightarrow \left(\frac{V}{R+r}\right)^2\left(1 - \frac{2R}{R+r}\right) = 0 \Rightarrow \boxed{r = R}$$

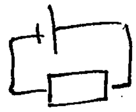
$$\boxed{R = r = r_1 + r_2}$$

4.5 Tres resistencias iguales consumen 10 W en serie. ¿Cuál es la Pot. consumida en paralelo?



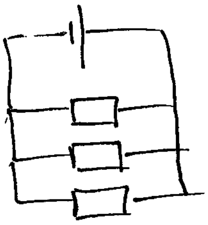
$$R_{eq_s} = R + R + R = 3R$$

$$P = I \cdot V = \frac{V^2}{R} \quad \text{en serie}$$



Req_s

$$P_s = \frac{V^2}{3R} \quad \text{serie}$$



$$P = \frac{V^2}{Req}$$

$$Req_p = \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{R}{3}$$

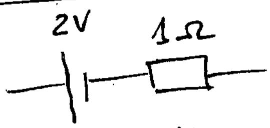
Req_p

$$P_p = \frac{V^2}{R/3} = \frac{3V^2}{R}$$

$$\Rightarrow \boxed{P_p = 9 P_s}$$

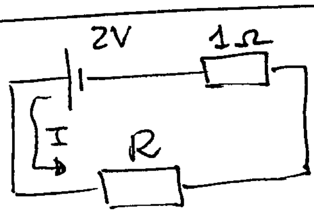
4.3

Ⓐ



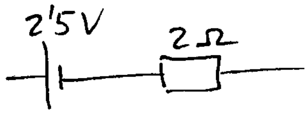
¿R?

Fem = ΔV

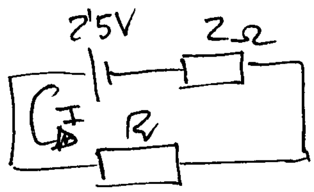


$$2 - IR - I \cdot 1 = 0$$

Ⓑ



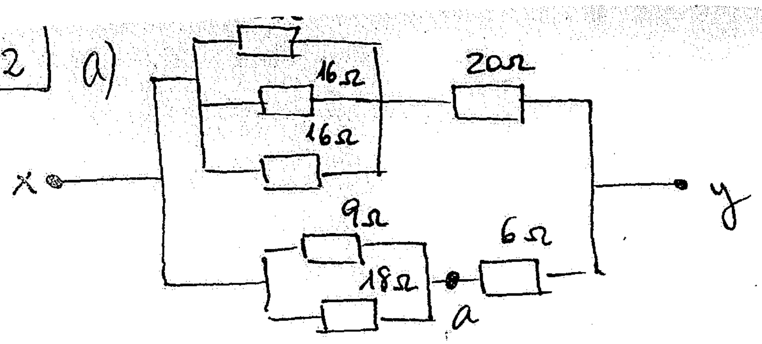
¿R?



$$2'5 - I \cdot R - 2I = 0$$

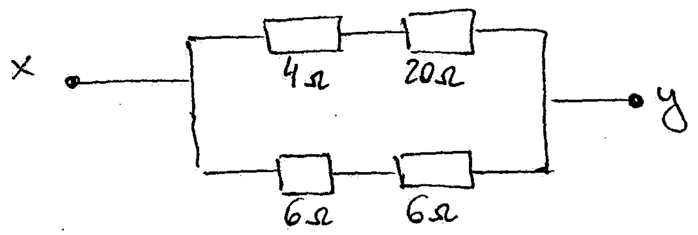
$$\begin{cases} 2 - IR - I = 0 \\ 2'5 - I \cdot R - 2I = 0 \end{cases} \Rightarrow \boxed{I = 0'5\,A} \wedge \boxed{R = 3\,\Omega}$$

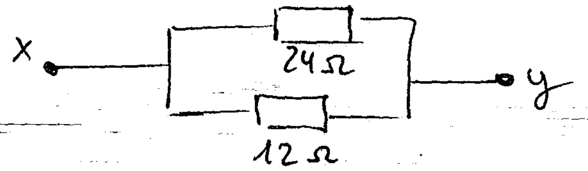
4.2 a)



$$\left(\frac{1}{8}+\frac{1}{16}+\frac{1}{16}\right)^{-1}=4$$

$$\left(\frac{1}{9}+\frac{1}{18}\right)^{-1}=6$$

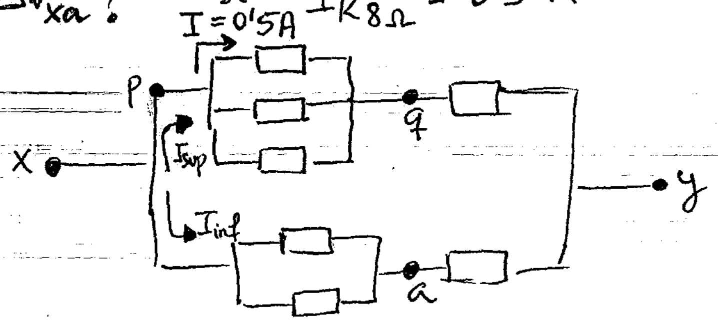




$$\left(\frac{1}{24}+\frac{1}{12}\right)^{-1}=8\,\Omega$$

$$\boxed{R_{eq_T}=8\,\Omega}$$

b) ΔV_{xa}? Si $I_{R8\Omega}=0'5\ A$



$V_{pq}=0'5\cdot 8=4V$

$I_{sup}=\frac{4V}{4\Omega}=1A$

$V_T=I_{sup}\cdot 24\Omega=24V$

$I_{inf}=\frac{V_T}{12\Omega}=2A$

$\Delta V_{xa}=I_{inf}\cdot 6\,\Omega=2A\cdot 6\,\Omega=12V$