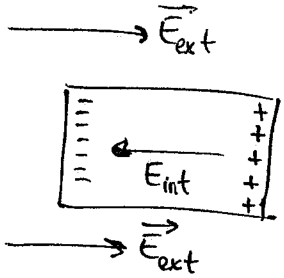
TEMA 3: CAMPO ELÉCTRICO EN MEDIOS MATERIALES

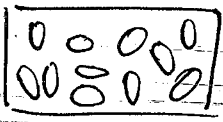
- → CONDUCTORES (metales) (cargas móviles)
- → AISLANTES (dieléctricos) (cargas no móviles (o muy poco)).

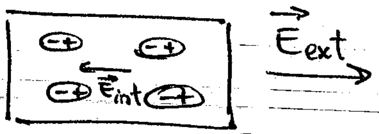


equilibrio: $\vec{E}_{ext} = -\vec{E}_{int}$

E_{TOTAL} interior conductor es cero

$$\left.\begin{array}{l}\vec{E} = -\vec{\nabla} V \\ \downarrow \qquad \downarrow \\ 0 \qquad \text{constante}\end{array}\right\} \begin{array}{l}\text{Si } \vec{E} = 0 \\ V = \text{cte.}\end{array}$$





$$K = \frac{E_{ext}}{E_{int}}$$ ↪ permitividad relativa

$E_{int} < E_{ext}$

Si es metal $\Rightarrow$ $K \to \infty$

Aire $\Rightarrow K \to 1$

K siempre ≥ 1

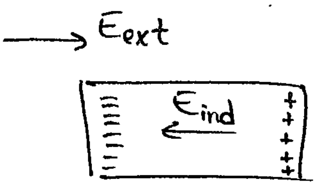
Energía almacenada en un conductor: $U = \frac{1}{2} \sum q_i V_i = \frac{1}{2} V \sum q_i =$

$$= \frac{1}{2} Q_T V$$

RECUERDO

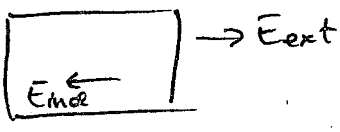
Materiales bajo $\vec{E}_{ext}$

conductores



$$\left.\begin{array}{l}\vec{E}_{ind} = -\vec{E}_{ext} \\ E_{int} = \vec{E}_{ext} + \vec{E}_{ind} = 0\end{array}\right\} \begin{array}{l}\text{en equilibrio} \\ E_{int} = 0 \\ V = \text{cte}\end{array}$$

Aislantes/dieléctrico

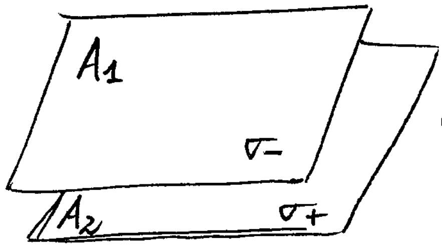


$E_{ind} << E_{ext}$

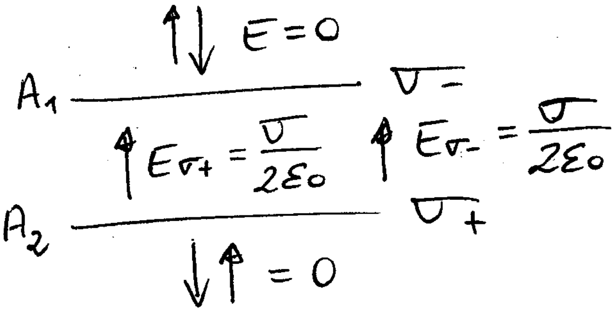
$E_{int} = E_{ext} - E_{ind} \neq 0$

$\varepsilon_r = K = \frac{E_{ext}}{E_{int}}$ $\quad E_{int} < E_{ext}$ $\quad K > 1$ $\quad$ metal $K \to \infty$

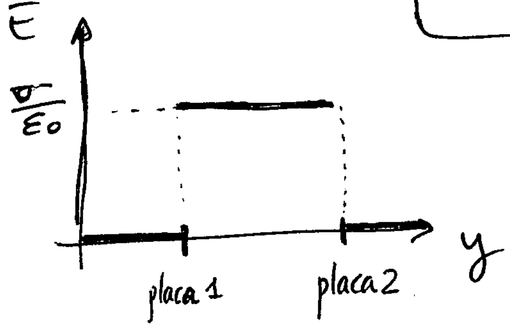
CONDENSADORES



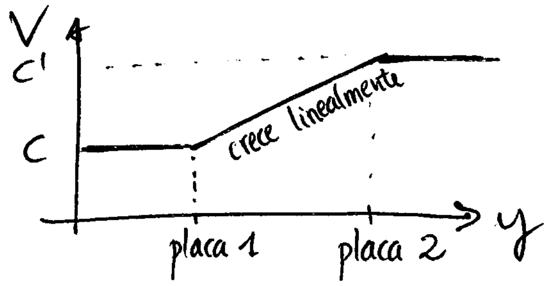
$A_1 = A_2$



$$\boxed{E_{int} = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}} \qquad \boxed{E_{ext} = 0}$$



$$\boxed{\sigma = \frac{Q}{A}}$$



$$\Delta V = \int \vec{E} \cdot d\vec{l} = \frac{\sigma}{\varepsilon_0} \int d\vec{l} = \frac{\sigma}{\varepsilon_0} \cdot d \Rightarrow \boxed{\Delta V = \frac{\sigma}{\varepsilon_0} \cdot d}$$

- Capacidad de un condensador (C):

$$\boxed{C = \frac{Q}{\Delta V}} = \frac{\sigma \cdot A}{\sigma/\varepsilon_0 \cdot d} = \frac{\varepsilon_0 A}{d} \Rightarrow \boxed{C = \frac{\varepsilon_0 \cdot A}{d}}$$

sólo depende de la geometría

$\rightarrow$ S.I: $F = \frac{C}{V}$

- Energía almacenada en un condensador

$$U = \int V dq = \int \frac{q}{C} dq = \frac{1}{C}\int q\, dq = \frac{1}{C} \cdot \frac{Q^2}{2} = \frac{1}{2} \cdot \frac{1}{C} \cdot Q^2 =$$

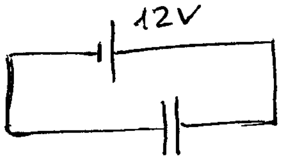
$$= \frac{1}{2} Q V = \frac{1}{2} C V^2 \Rightarrow$$

$C = \frac{Q}{\Delta V}$

$$\Rightarrow \boxed{U = \frac{1}{2} \cdot Q \cdot \underset{V}{\underset{\shortparallel}{\Delta V}} = \frac{1}{2} C \underset{V}{\underset{\shortparallel}{\Delta V}}^2}$$

Ejemplo

Condensador $\dashv\vdash$ $l = 14\,cm$ y $d = 2\,mm$ se conecta a una batería de 12 V.



a) Carga condensador

$$\Delta V_{pl} = \Delta V_{bat} = 12V$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \, l^2}{d} = \frac{\varepsilon_0 \, . \, 14 \, . \, 14 \, . \, 10^{-4}}{2 . 10^{-3}} = 8'6 . 10^{-11} F$$

$$Q = C . \Delta V = 8'6 . 10^{-11} F . \, 12V = 1'04 . 10^{-9} C$$

b) ¿U?

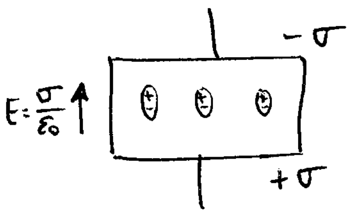
$$U = \frac{1}{2} V^2 C = \frac{1}{2} . 12V . \, 8'6 . 10^{-11} F = 6'2 . 10^{-9} J$$

c) Desconectamos las baterías y separamos las placas hasta $d_f = 3'5\,mm$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 . 14 . 14 . 10^{-4}}{3'5 . 10^{-3}} = 4'95 . 10^{-11} F$$

$$\Delta V = \frac{Q}{C} = \frac{1'04 . 10^{-9} C}{4'95 . 10^{-11} F} = 21'01 V$$

DIELÉCTRICO EN EL INTERIOR DE UN CONDENSADOR



$E = \frac{\sigma}{\varepsilon_0}$

$\Delta V_{pl} = E_0 \cdot d$ – antes

$E_{int} = \frac{E_{ext}}{K} \longrightarrow \Delta V_{pl} = \frac{E_0}{K} \cdot d$ con dieléctrico

antes:

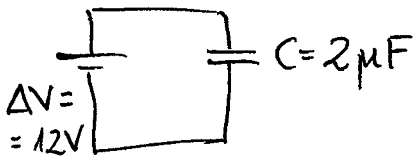
$\left[C_0 = \frac{Q}{V_0} \right] \Rightarrow$

con dieléctrico:

$$\boxed{C_d = \frac{Q}{V_0/K} = \frac{KQ}{V_0} = K\,C_0}$$

Ejemplo: Considerar $C = 2\mu F$ conectado a batería de $12V$

a) ¿cual es la carga en las placas?



$C = \frac{Q}{\Delta V_{placas}} \qquad \Delta V_{placas} = \Delta V_{bat}$

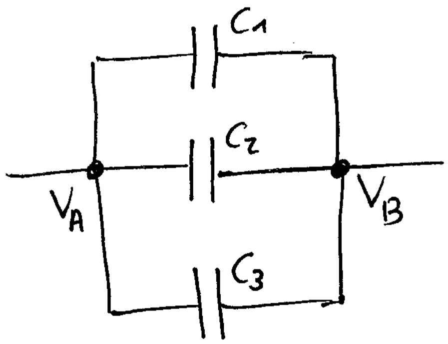
$Q = C \cdot \Delta V_{placas} = 2 \cdot 10^{-6}\,F \cdot 12V = 2'4 \cdot 10^{-5}\,C$

b) Si entre las placas ponemos un dieléctrico con $K = 2'5$ ¿cual es la Q_d?

$C_d = K\,C_0 = \frac{Q_d}{\Delta V_d} \qquad \Delta V_d = \Delta V_{antes}$

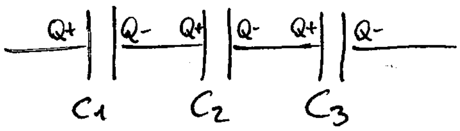
$Q_d = \Delta V \cdot C_d = K \cdot C_0 \cdot \Delta V = 2'5 \cdot 2 \cdot 10^{-6} \cdot 12 = 6 \cdot 10^{-5}\,C$

• Asociación de condensadores



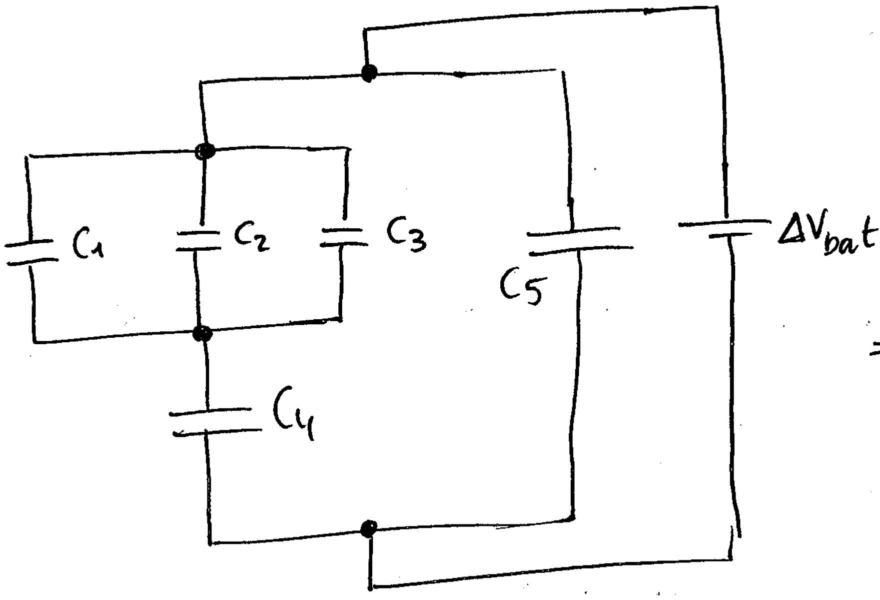
Todos tienen la misma ΔV

$$\left.\begin{aligned} Q_1 &= C_1 \Delta V \\ Q_2 &= C_2 \Delta V \\ Q_3 &= C_3 \Delta V \end{aligned}\right\} \overbrace{Q_1 + Q_2 + Q_3}^{Q_T} = \Delta V \underbrace{(C_1 + C_2 + C_3)}_{C_{eq.}}$$

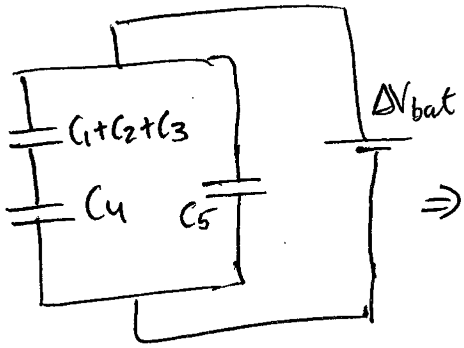


Todos tienen la misma carga

$$\Delta V_T = \Delta V_1 + \Delta V_2 + \Delta V_3 =$$
$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} =$$
$$= Q \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)}_{1/C_{eq.}}$$



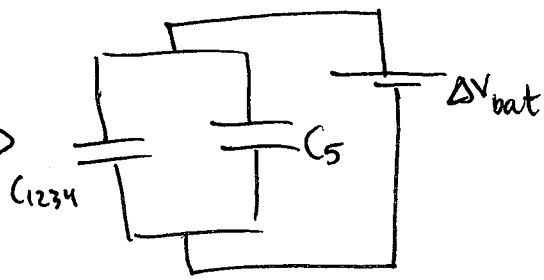
$\Rightarrow$



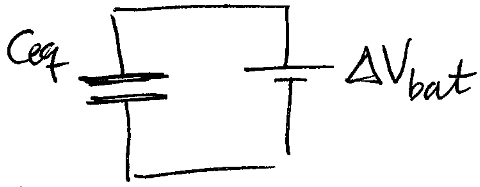
$\Rightarrow$

$$C_{1234} = \frac{C_{123}\, C_4}{C_{123} + C_4}$$

$\Rightarrow$

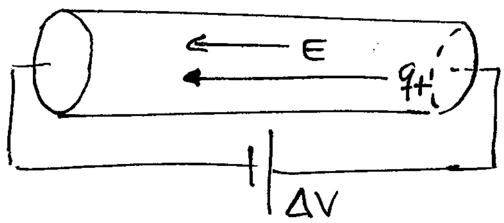


$\Rightarrow$

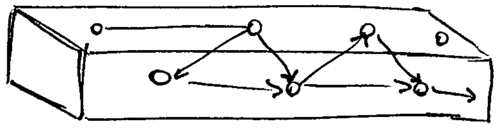


$$C_{eq} = C_{1234} + C_5$$

4 CORRIENTES ELÉCTRICAS ESTACIONARIAS (CORRIENTE CONTINU



movimiento de cargas en el interior de un conductor



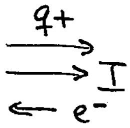
$V_{entre} \approx 10^6$ m/s

$V_d \approx 10^{-2}$ m/s

Intensidad de corriente

$$I = \frac{\Delta Q}{\Delta t} = \frac{C}{S}$$

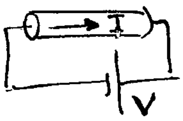
S.I: $A = \frac{C}{S}$

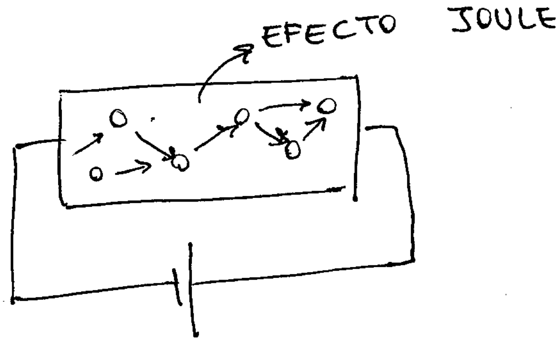


Resistencia eléctrica

$$R = \frac{V}{I}$$

S.I = $\Omega = \frac{V}{A}$





- **Intensidad de corriente**

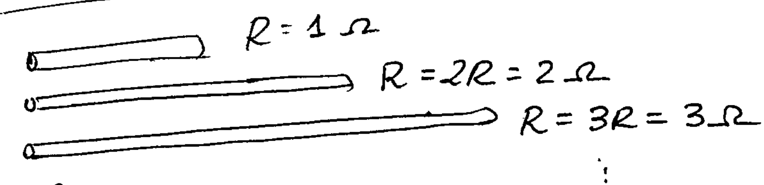
$$I = \frac{V}{R}$$

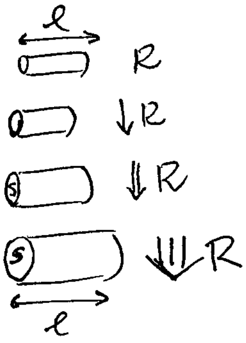
- **Resistencia eléctrica**

$$R = \frac{V}{I}$$

Resistencia (R) y resistividad (ρ)

$$R = \rho \frac{l}{S} \implies \rho = \frac{RS}{l}$$





Cobre:

$\rho = 1'7 \cdot 10^{-8}\ \Omega \cdot m$

Calcular la resistencia de un cable de Cu de 2mm de radio y 1 ~~cm~~ m ~~kg~~ longitud

$$R = \rho \frac{L}{S} = \rho \frac{L}{\pi r^2} = 1'7 \cdot 10^{-8} \frac{1m}{\pi (2 \cdot 10^{-3})^2} =$$

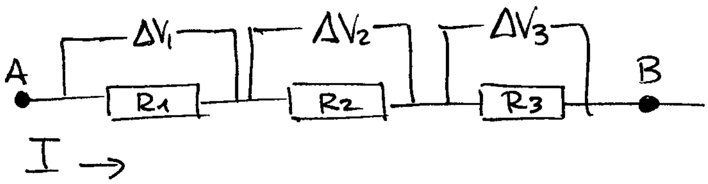
$$= 1'3 \cdot 10^{-3}\ \Omega$$

Calcular la R si reducimos el radio a 0'5 mm

$R = \dots = 1'2 \cdot 10^{-2}\ \Omega$

▶ Potencia disipada en una resistencia

$$\boxed{P = \frac{\Delta U}{\Delta t} = V \cdot \frac{\Delta Q}{\Delta t} = V \cdot I = \frac{V^2}{R} = I^2 \cdot R}$$

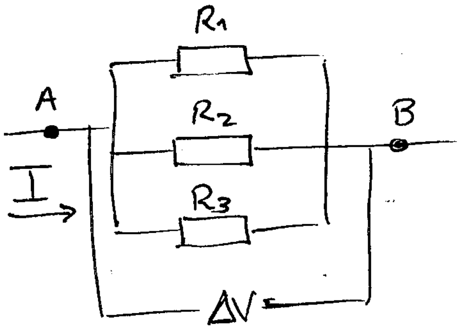


la I que circula es la misma

$$\Delta V_{ab} = \Delta V_1 + \Delta V_2 + \Delta V_3 =$$

$$= I_1 R_1 + I_2 R_2 + I_3 R_3 = I(R_1 + R_2 + R_3)$$

SERIE: $\boxed{R_{eq} = R_1 + R_2 + R_3}$



La I que circula es distinta por rama.

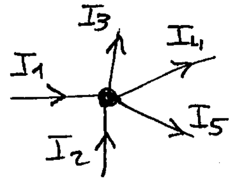
$$R_{eq}^{-1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

PARALELO: $\boxed{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}}$

RESOLUCIÓN DE CIRCUITOS: LEYES DE KIRCHOFF

→ [1] NODOS
→ [2] BUCLE CERRADO

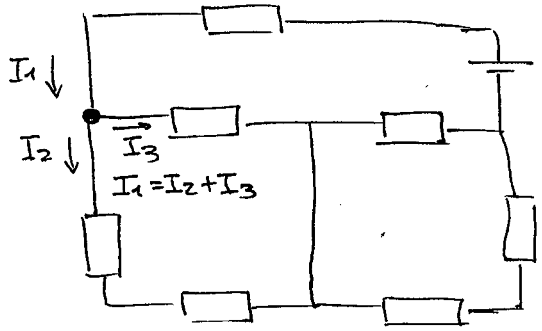
1. NODOS: "Toda corriente que entra en un nodo sale".



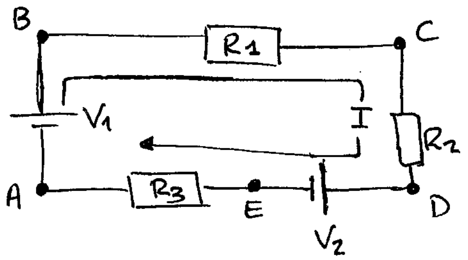
$$I_1 + I_2 = I_3 + I_4 + I_5$$

$$\boxed{\begin{array}{l} I_{ent} > 0 \\ I_{sal} < 0 \end{array}}$$

$$\sum I_{entr} = \sum I_{sal}$$

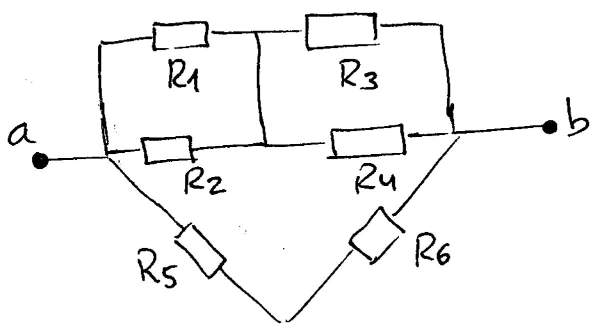


2. BUCLE CERRADO: "En un bucle (malla o circuito) la $\Delta V_T = 0$.

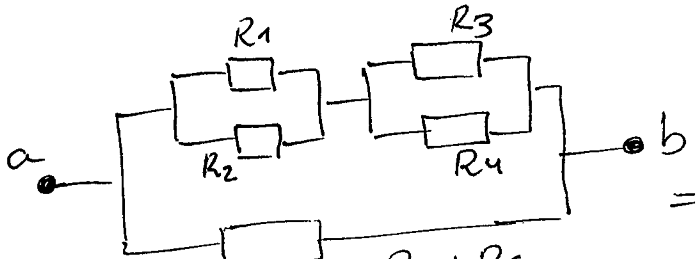


$$V_1 - IR_1 - IR_2 - V_2 - IR_3 = 0$$

Ejemplo simplificación



$\Rightarrow$



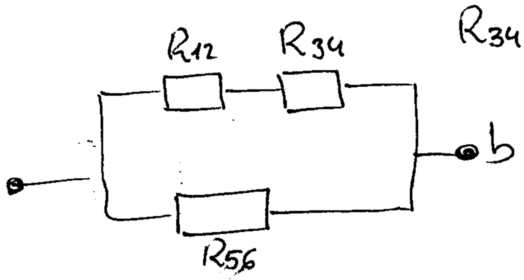
$$R_{56} = R_5 + R_6$$

=

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

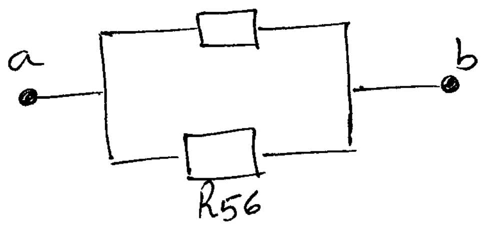
$$R_{34} = \frac{R_3 R_4}{R_3 + R_4}$$

$\Rightarrow$



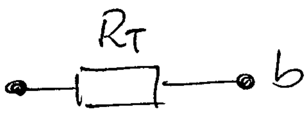
$\Rightarrow$

$$R_{1234} = R_{12} + R_{34}$$



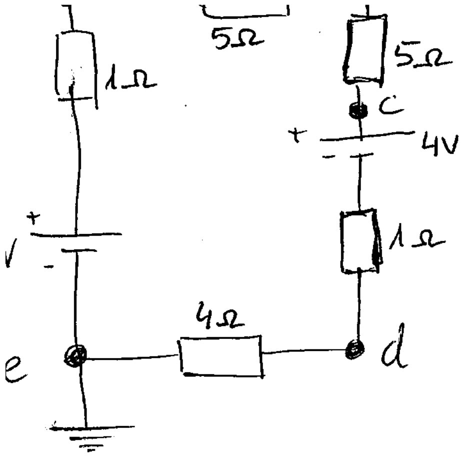
$\Rightarrow$

$\Rightarrow$



$$R_T = \frac{R_{1234} \cdot R_{56}}{R_{1234} + R_{56}} = \frac{(R_5 + R_6) \cdot \left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}\right)}{(R_5 + R_6) + \left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}\right)}$$

R



▶ Aplico la ley del bucle cerrado:

$$0 = 12 - 1I - 5I - 5I - 4 - 1I - 4I$$

$$0 = 8 - 16I \Rightarrow \boxed{I = 0'5A}$$

$\underline{\Delta V_{ab}} = I \cdot 5 = 0'5 \cdot 5 = 2'5V$

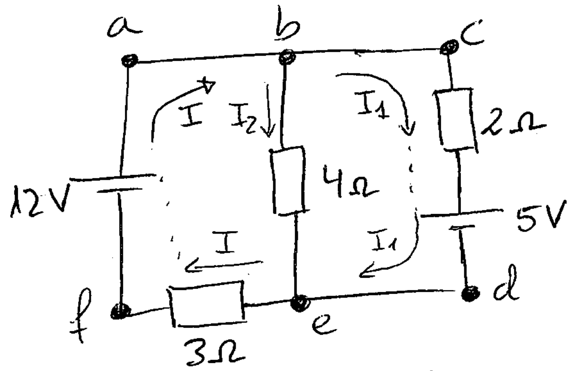
$\underline{\Delta V_{bc}} = I \cdot 5 = 0'5 \cdot 5 = 2'5V$

$\underline{\Delta V_{cd}} = 4 + 1 \cdot I = 4 + 0'5 = 4'5V$

$\underline{\Delta V_{de}} = 4 \cdot I = 4 \cdot 0'5 = 2V$

$\underline{\Delta V_{ea}} = 12 + I = 12 + 0'5 = 12'5V$

2 MALLAS



▶ Aplico la ley de nodos

$$I = I_1 + I_2$$

2 bucles cerrados

- Malla exterior: abcdefa

$$12 - 2I_1 - 5 - 3I = 0$$

- Malla interna: bcdeb la corriente va en sentido contrario

$$-2I_1 - 5 \boxed{+ 4I_2} = 0$$

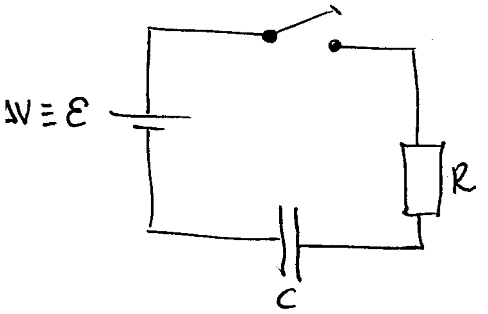
$$\begin{cases} I = I_1 + I_2 \\ 7 - 2I_1 - 3I = 0 \\ -2I_1 + 4I_2 - 5 = 0 \end{cases} \Rightarrow \begin{cases} I = 2A \\ I_1 = 0'5A \\ I_2 = 1'5A \end{cases}$$

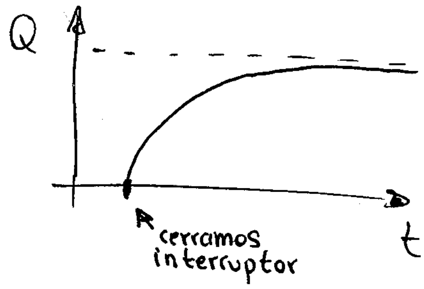
ΔV_{ad} (por arriba) $= I_1 \cdot 2 + 5 = \mathbf{6}V$ ✓

ΔV_{ad} (por abajo) $= 12 - 3 \cdot I_1 = 6V$

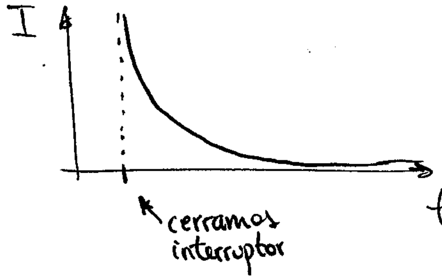
corriente en sentido contrario

CONDENSADORES EN CIRCUITOS DE CORRIENTE CONTINUA





El comportamiento de un condensador en un circuito cambia drásticamente dependiendo si está ya cargado o no.



$$\boxed{\varepsilon - IR - \Delta V_{condensador} = 0}$$ en el circuito anterior

$$C = \frac{Q}{\Delta V_{cond}} \Rightarrow \Delta V_{cond} = \frac{Q}{C}$$

$$\varepsilon - \underbrace{I}_{\frac{dQ}{dt}} R - \frac{Q}{C} = 0 \Rightarrow \varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0 \Rightarrow$$

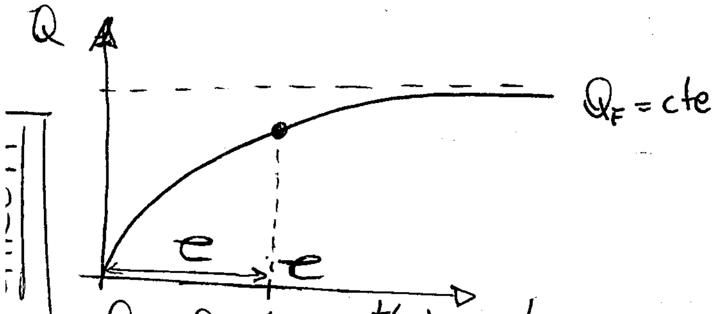
$$\Rightarrow \frac{dQ}{Q - C.\varepsilon} = \frac{1}{R.C}dt \Rightarrow \int_{Q_i=0}^{Q_F} \frac{dQ}{Q - C.\varepsilon} = \int -\frac{1}{RC}dt \Rightarrow$$

$$\Rightarrow \ln\frac{Q - C.\varepsilon}{-C.\varepsilon} = \frac{-t}{RC} \Rightarrow \frac{Q - C.\varepsilon}{C.\varepsilon} = e^{-t/RC} \Rightarrow$$

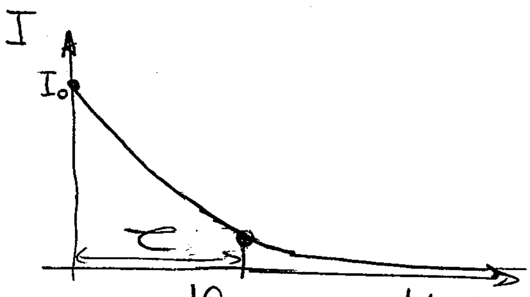
$$\Rightarrow \boxed{Q = C.\varepsilon\left(1 - e^{-t/RC}\right)}$$

τ tiempo característico $t = RC$

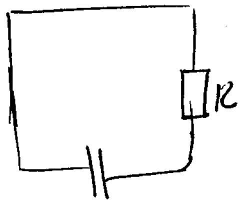
$$Q = Q_F\left(1 - e^{-1}\right) = 0'63\,Q_F$$



$$Q = \underbrace{Q_F}_{C.\varepsilon}\left(1 - e^{-t/RC}\right)$$



$$I = \frac{dQ}{dt} = \frac{\varepsilon}{R}e^{-t/RC}$$



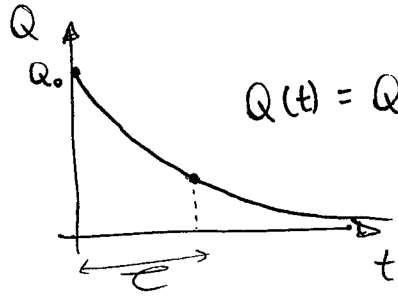
C inicialmente cargado

$$IR = \frac{Q}{C} \longrightarrow Q(t) = Q_0\, e^{-t/RC}$$

$$I = I_0\, e^{-t/RC}$$

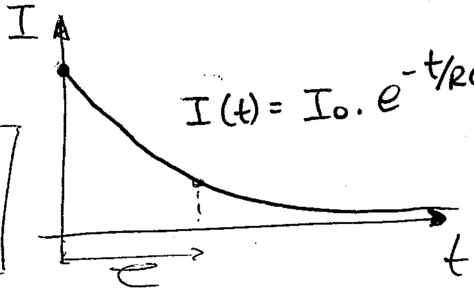
$$I_0 = \frac{Q_0}{RC}$$

DESCARGA



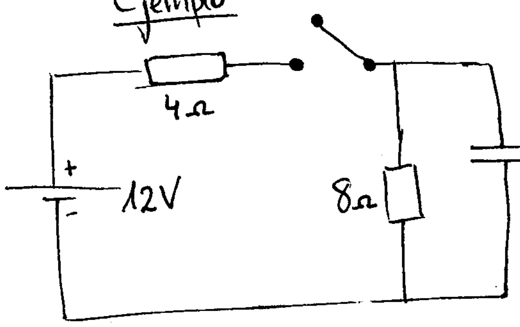
$$Q(t) = Q_0\, e^{-t/RC}$$

$\boxed{\tau = RC}$ → tiempo característico



$$I(t) = I_0 \cdot e^{-t/RC} = \frac{Q_0}{RC}\, e^{-t/RC}$$

Ejemplo



En $t=0$ cerramos el interruptor

Hallar la I en las resistencias

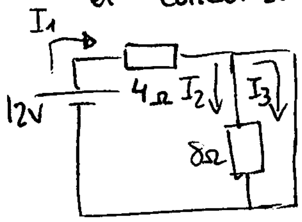
a) en $t=0$

b) Después de un largo tiempo

c) determinar la Q_c pasado un largo tiempo.

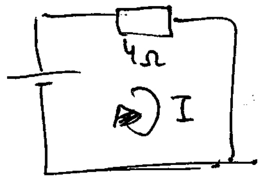
a) En $t=0$

el condensador $Q=0$ $\Delta V_c = 0$ es un cable



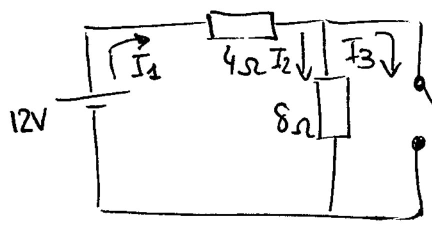
$I_3 = I_1$

$I_2 = 0$



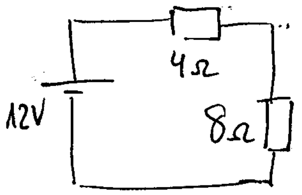
$12 - 4I_1 = 0 \Rightarrow I_1 = 3A$

b) En $t = \infty$



$I_3 = 0$

$I_1 = I_2$



Leyes de Kirchoff:

$12 - I_1 4 - I_1 8 = 0 \Rightarrow I_1 =$

Ley de Ohm:

$R_T = 12\,\Omega$ $\quad I = \frac{V}{R} = \frac{12}{12} = 1A$

c) Q_c en $t = \infty$

$C = \frac{Q}{\Delta V_c} \Rightarrow Q = C\,\Delta V_c$; $\Delta V_c = \Delta V_{8\Omega} = I_2 \cdot 8 = 8V \Rightarrow$

$\Rightarrow Q = 6 \cdot 10^{-6}\,F \cdot 8V = 48\,\mu C$