HOJA 10

1.

a) $6x^2 + 5y^2 + 7z^2 - 4xy - 4xz = 0$

$$\left(\begin{array}{ccc|c} 6 & -2 & -2 & 0 \\ -2 & 5 & 0 & 0 \\ -2 & 0 & 7 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & -c \end{pmatrix}$$

$\delta > 0 \rightsquigarrow$ CASO 1

$\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 = c \qquad \lambda_1, \lambda_2, \lambda_3 \neq 0$

$\Delta = -c\lambda_1\lambda_2\lambda_3 = 0 \Rightarrow \Delta = 0 \Rightarrow$ CASO 1.b

$\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 = 0$

Reescribiendo lo de arriba: $\pm \frac{x^2}{a^2} \pm \frac{y^2}{b^2} \pm \frac{z^2}{c^2} = 0$

¿Signatura? $\to$ cálculo del polinomio característico $\to$

$\to = -\lambda^3 + 18\lambda^2 - 99\lambda + 162$

$(-1, 18, -99, 162) \to$ 3 cambios de signo $\to$ 3 raíces positivas

$\to$ Signatura 3 $\Rightarrow$ $\boxed{\text{CASO 1.b.a}}$ Punto

d) $-2y^2 + xz - 4y + 6z + 5 = 0$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1/2 & 0 \\ 0 & -2 & 0 & -2 \\ 1/2 & 0 & 0 & 3 \\ \hline 0 & -2 & 3 & 5 \end{array}\right)$$

$\delta = -\frac{1}{2} \neq 0 \rightsquigarrow$ <u>CASO 1</u>

$$\lambda_1 x_1^2 + \lambda_2 y_1^2 + \lambda_3 z_1^2 = C$$

$\Delta = -C\lambda_1\lambda_2\lambda_3 = \frac{7}{2} > 0 \rightarrow$ <u>CASO 1.a</u> $\pm \frac{x_1^2}{a^2} \pm \frac{y_1^2}{b^2} \pm \frac{z_1^2}{c^2} = 1$

¿Signatura? $\rightarrow$ cálculo del polinomio característico $\rightarrow$

$\rightarrow \Rightarrow \det(A - \lambda I_3) = (2+\lambda)(-\lambda^2 + \frac{1}{4}) \rightarrow$ signatura $1 \rightarrow$

$\rightarrow$ Estamos en el <u>caso 1.a.b</u> porque $\Delta > 0$ y sign. $= 1$

<u>hiperboloide de 1 hoja</u>

<u>Ejemplo fuera de la hora</u> : $xy + 2xz + 2x + 1 = 0$

$$\left(\begin{array}{ccc|c} 0 & 1/2 & 1 & 1 \\ 1/2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 \end{array}\right)$$

$$S_2 = \begin{vmatrix} 0 & 1/2 \\ 1/2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -\frac{1}{4} - 1 = -\frac{5}{4} < 0$$

$\delta = 0$ $\begin{cases} \text{caso 2} : S_2 \neq 0 \\ \text{caso 3} : S_2 = 0 \end{cases}$ $\Rightarrow \delta = 0 \wedge S_2 \neq 0 \rightarrow$ <u>CASO 2</u>

$$\lambda_1 x^2 + \lambda_2 y^2 + Bz = C$$

$$\Delta = \det \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & B/2 \\ 0 & 0 & B/2 & -C \end{pmatrix} = \underbrace{\lambda_1\lambda_2}_{S_2}\left(\frac{-B^2}{4}\right)$$

; $\Delta = 0 \Rightarrow B = 0 \Rightarrow$ <u>CASO 2.b</u>

como $\Delta = 0 \wedge S_2 < 0 \Rightarrow$ <u>CASO 2.b.b.</u> ; falta ver si $C = 0$ ó $C \neq 0$

$S_3 = -S_2 . C$; $S_3 = \frac{-5}{4} \Rightarrow C \neq 0$ <u>cilindro hiperbólico</u>

Ejemplo 2 $\quad xy + 2xz + 2x + 2y + 2z + 1 = 0$

$$\bar{A} = \left(\begin{array}{ccc|c} 0 & 1/2 & 1 & 1 \\ 1/2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 \end{array}\right), \quad A = \begin{pmatrix} 0 & 1/2 & 1 \\ 1/2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$\delta = \det A = 0$

$$S_2 = \begin{vmatrix} 0 & 1/2 \\ 1/2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = \frac{-5}{4} < 0$$

$\underset{\lambda_1\lambda_2}{\overset{\parallel}{S_2}} \neq 0 \Rightarrow$ CASO 2

$$\lambda_1 x^2 + \lambda_2 y^2 + bz = C$$

$$\bar{A}' = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & b/2 \\ 0 & 0 & b/2 & -c \end{pmatrix}$$

$\Delta = |\bar{A}| = \frac{1}{4} \neq 0 \Rightarrow b \neq 0$

$\Rightarrow$ CASO 2.1

$$\Delta = |\bar{A}| = |\bar{A}'| = \underset{S_2}{\underset{\parallel}{\lambda_1\lambda_2}}\left(\frac{-b^2}{4}\right)$$

$$\lambda_1' x_1^2 + \lambda_2' y_1^2 = z_1$$

$$\pm \frac{x_1^2}{a^2} \pm \frac{y_1}{b^2} = z_1$$

Como $S_2 < 0 \Rightarrow$ Paraboloide hiperbólico

Ejemplo posicionamiento $\quad \underbrace{5x^2 + 3y^2 + 3z^2}_{\text{parte principal}} - 2xy - 2xz + 2yz - 10x + 6y - 2z - 10 = 0$

$R = \{O; e_1, e_2, e_3\}$ en $\mathbb{A}^3_{\mathbb{R}}$ o.n.

$$A = \begin{pmatrix} 5 & -1 & -1 \\ -1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

producto tres autovalores $\Rightarrow$ caso 1

$$|A - \lambda I| = -\lambda^3 + 11\lambda^2 - 36\lambda + 36 = -(\lambda - 2)(\lambda - 3)(\lambda - 6)$$ signatura 3

$\mathrm{Ker}(A - 2I) = \langle (0,1,-1) \rangle \Rightarrow u_1 = \frac{1}{\sqrt{2}}(0,1,-1)$

$\mathrm{Ker}(A - 3I) = \langle (1,1,1) \rangle \Rightarrow u_2 = \frac{1}{\sqrt{3}}(1,1,1)$

$\mathrm{Ker}(A - 6I) = \langle (2,-1,-1) \rangle \Rightarrow u_3 = \frac{1}{\sqrt{6}}(2,-1,-1)$

$\boxed{R' = \{O; u_1, u_2, u_3\}}$

Respecto a R': $2(x')^2 + 3(y')^2 + 6(z')^2 + 4\sqrt{2}\,x' - 2\sqrt{3}\,y' - 4\sqrt{6}\,z' - 10 = 0$

$\rightarrow$ traslación completando cuadrados: $2(\underbrace{x' + \sqrt{2}}_{x''})^2 - 4 + 3(\underbrace{y' - \frac{1}{\sqrt{3}}}_{y''})^2 - 1 + 6(\underbrace{z' - \frac{2}{\sqrt{6}}}_{z''})^2 - 4 - 10 = 0$

$R'' = \{P; u_1, u_2, u_3\} \rightarrow 2(x'')^2 + 3(y'')^2 + 6(z'')^2 = 19$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ -\frac{1}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ -\frac{1}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix} + \begin{pmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\boxed{2.}$ $x^2 - 2y^2 + \alpha z^2 - 2xz + 2yz + 2x + 1 = 0$ $\alpha \in \mathbb{R}$

¿para qué valores de α es un paraboloide?

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & -2 & 1 & 0 \\ -1 & 1 & \alpha & 0 \\ \hline 1 & 0 & 0 & 1 \end{array}\right)$$

$\delta = \det(A) = -2\alpha - \left(-2 + 1\right) = -2\alpha + 1$

δ tiene que ser igual a cero

$\delta = 0 \Leftrightarrow -2\alpha + 1 = 0 \Leftrightarrow \alpha = \frac{1}{2}$

$$S_2 = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 1/2 \end{vmatrix} + \begin{vmatrix} -2 & 1 \\ 1 & 1/2 \end{vmatrix} = -2 + \frac{1}{2} - 1 - 1 - 1 = -5 + \frac{1}{2} = \frac{-9}{2} < 0$$

$$\Delta = -\begin{vmatrix} 0 & -2 & 1 \\ -1 & 1 & \alpha \\ 1 & 0 & 0 \end{vmatrix} = -\left[-2\alpha - (1)\right] = -\left[-2 \cdot \frac{1}{2} - 1\right] = -(-2) = 2$$