# TEMA 3 - VECTORES ALEATORIOS

**1.** Sea $X$ una variable aleatoria, $X$ = "suma de los puntos obtenidos en n tiradas de un dado".

$\mathbb{E}(X) = ?$

$X_i$ = "resultado obtenido en el i-ésimo lanzamiento"

$$\mathbb{E}(X_i) = \sum_{K=1}^{6} \frac{1}{6} \cdot K = 3'5$$

$$\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + \dots + X_n) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \dots + \mathbb{E}(X_n) = n \cdot 3'5$$

**3.** $X$ = "instante de llegada de A" $\qquad f(X,Y) = ?$

$Y$ = "instante de llegada de B"

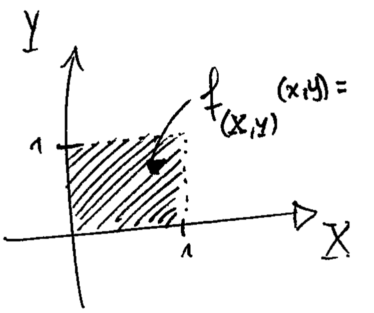
$f_{(X,Y)}(x,y) = f_X(x) \cdot f_Y(y)$ porque A y B llegan de forma independiente.

$$f_X(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$ es una densidad porque $\to$ 1. $f_X \geq 0$; 2. $\int_{\mathbb{R}} f_X(x)\,dx = 1$

$$f_Y(y) = \begin{cases} 1, & y \in [0,1] \\ 0, & y \notin [0,1] \end{cases}$$

Entonces:

$$f_{(X,Y)}(x,y) = f_X(x) f_Y(y) = \begin{cases} 1, & x \in [0,1],\ y \in [0,1] \\ 0, & x \notin [0,1],\ y \notin [0,1] \end{cases}$$




b) Si el primero que llega espera a el otro 15 minutos, ¿$\mathbb{P}$("A y B se encuentren")?

$$\mathbb{1}_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}$$

$$\mathbb{P}(\underbrace{\{|X-Y| \leq 1/4\}}_{E}) = \iint_E f_{(X,Y)}(x,y)\,dx\,dy = \iint_E 1 \cdot \mathbb{1}_{\underbrace{[0,1]\times[0,1]}_{A}}(x,y)\,dx\,dy =$$

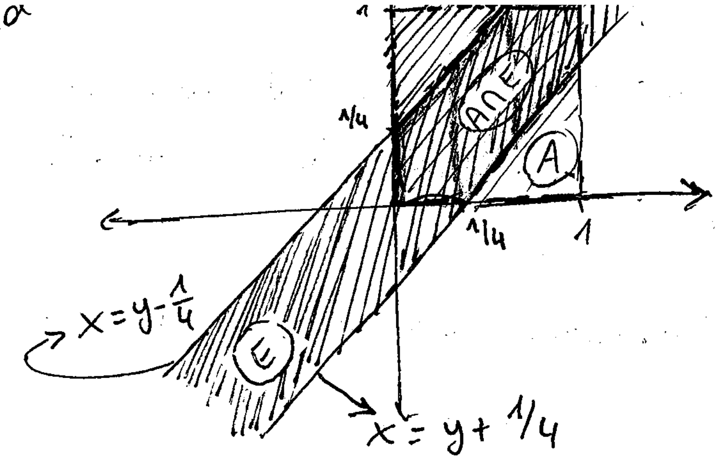
función densidad

$$= \int_0^{1/4} dx \int_0^{x+1/4} dy \; 1 \; +$$

$$+ \int_{1/4}^{3/4} dx \int_{x-1/4}^{x+1/4} dy \cdot 1 \; +$$

$$+ \int_{3/4}^{1} dx \int_{x-1/4}^{1} dy \cdot 1 \; =$$

$$= 1 - \left(\frac{3}{4}\right)^2 = 7/16$$




$$|x-y| \leq \frac{1}{4}$$

$$-\frac{1}{4} \leq x - y \leq \frac{1}{4}$$

$$y - \frac{1}{4} \leq x \leq y + \frac{1}{4}$$

$$\boxed{\mathbb{1}_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}}$$

c) $f_{(X,Y)}(x,y) = \begin{cases} 4xy, & (x,y) \in [0,1] \times [0,1] \\ 0, & \text{resto} \end{cases}$ $\quad f_X = ? \quad f_Y = ?$

$x \in \mathbb{R}$, $f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\, dy = \begin{cases} 0 & x \notin [0,1] \\ \underbrace{\int_0^1 4xy\, dy}_{= 2x} & x \in [0,1] \end{cases}$

$y \in \mathbb{R}$, $f_Y(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\, dx = \begin{cases} 0 & y \notin [0,1] \\ \underbrace{\int_0^1 4xy\, dx}_{= 2y} & y \in [0,1] \end{cases}$

$f_X(x) \cdot f_Y(y) = \begin{cases} 2x\,2y, & x \in [0,1],\, y \in [0,1] \\ 0, & \text{en el resto} \end{cases} \Rightarrow X$ e $Y$ son independientes.

**4.** $f_{(X,Y)}(x,y) = \begin{cases} K(x+xy) & , \quad x,y \in (0,1)\times(0,1) \\ 0 & , \quad \text{en el resto} \end{cases}$

a) ¿valor de $K$?

1.- $f \geq 0 \iff K \geq 0$

2.- $\iint_{\mathbb{R}^2} f_{(X,Y)}(x,y)\,dxdy = 1$

$$1 = K\int_0^1 dx \int_0^1 dy\,(x+xy) = K\left[\int_0^1 dx\int_0^1 dy\,x + \int_0^1 dx \int_0^1 dy\,xy\right] = \left[\frac{1}{2}+\frac{1}{4}\right]K$$

$$\Rightarrow K = 4/3$$

b) ¿ $f_X$, $f_Y$ ?

$$f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy = \begin{cases} 0 & , \quad x \notin [0,1] \\ \int_0^1 \frac{4}{3}(x+xy)\,dy & , \quad x \in [0,1] \end{cases}$$

$$\int_0^1 \frac{4}{3}(x+xy)\,dy = 4/3\,x\int_0^1 (1+y)\,dy = 2x$$

$$f_Y(y) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dx = \begin{cases} 0 & , \quad y \notin [0,1] \\ \int_0^1 \frac{4}{3}(x+xy)\,dx & , \quad y \in [0,1] \end{cases}$$

$$\int_0^1 \frac{4}{3}(x+xy)\,dx = \frac{4}{3}(1+y)\int_0^1 x\,dx = \frac{2}{3}(1+y)$$

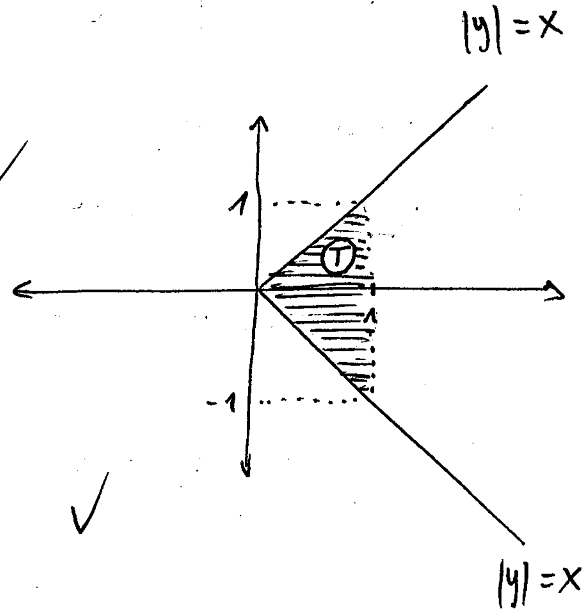
c) ¿ $X$ e $Y$ son independientes ?

Como $2x\left(\frac{2}{3}(1+y)\right) = \frac{4}{3}(x+xy)$ cuando $x,y \in [0,1]\times[0,1]$

y $0$ en el resto $\Rightarrow$ $X$, $Y$ son independientes

$\left(f_{(X,Y)}(x,y) = f_X(x)\cdot f_Y(y)\right)$

$$f_{(X,Y)}(x,y) = \begin{cases} \dots \\ 0 & \text{en el resto} \end{cases}$$




a) Comprobar que $f_{(X,Y)}$ es densidad

1.- $f_{(X,Y)}(x,y) \geq 0 \,, \quad \forall (x,y) \in \mathbb{R}^2$ ✓

2.- $\iint_{\mathbb{R}^2} f_{(X,Y)}(x,y)\,dxdy = 1$

$$\int_0^1 dx \int_{-x}^{x} dy \cdot 1 = \int_0^1 dx \; 2x = \left[x^2\right]_{x=0}^{x=1} = 1 \quad ✓$$

b) ¿$E(X)$, $E(Y)$?

$$E(X) = \int_{\mathbb{R}} x \, f_X(x)\,dx \quad \dots$$

$$\boxed{f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy}$$

pero además hay otra forma:

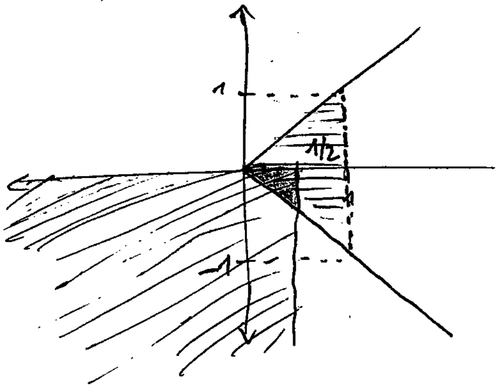
$$E(X) = \iint_{\mathbb{R}^2} x\, f_{(X,Y)}(x,y)\,dxdy = \int_{\mathbb{R}} dx \cdot x \cdot \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy$$

$$= \iint_{\mathbb{R}^2} x \cdot 1 \cdot \mathbb{1}_T(x,y)\,dxdy = \int_0^1 dx \int_{-x}^{x} x\,dy = \int_0^1 dx\; 2x^2 = \left[\frac{2x^3}{3}\right]_{x=0}^{x=1} = \frac{2}{3}$$

$$E(Y) = \iint_{\mathbb{R}^2} y \cdot 1 \cdot \mathbb{1}_T(x,y)\,dxdy = \int_0^1 dx \int_{-x}^{x} dy\, y = \int_0^1 dx \left[\frac{y^2}{2}\right]_{y=-x}^{y=x} = 0$$

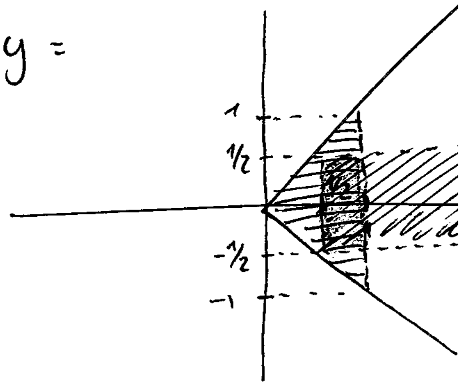
c) ¿ $\mathbb{P}(X < 1/2 \, , \, Y < 0) = ?$ ¿ $\mathbb{P}(X > 1/2 \, , \, -1/2 < Y < 1/2$

$$\mathbb{P}(X < 1/2 \, , \, Y < 0) = \iint_{\{X < 1/2 \, , \, Y < 0\}} f_{(X,Y)}(x,y)\, dx\, dy =$$




$$= \int_0^{1/2} dx \int_{-x}^{0} dy \cdot 1 = 1/8$$

$$\mathbb{P}(X > 1/2, \; -1/2 < Y < 1/2) = \iint_{\{X > 1/2, \, -1/2 < Y < 1/2\}} f_{(X,Y)}(x,y)\, dx\, dy =$$




$$= \int_{1/2}^{1} dx \int_{-1/2}^{1/2} dy = 1/2$$

6.

$$f_{(X,Y)}(x,y) = \begin{cases} K.ye^{-2x}e^{-y} & , x>0, y>0 \\ 0 & , \text{en el resto} \end{cases}$$

a) Hallar el K. ¿Son independientes X e Y?

b) ¿$\mathbb{E}(X)$?

a) $\int_0^\infty dx \int_0^\infty dy \left(Kye^{-2x}e^{-y}\right) = K\int_0^\infty e^{-2x}dx \int_0^\infty ye^{-y}dy =$

$= K\left(-\frac{1}{2}\left[e^{-2x}\right]_0^\infty\right)\int_0^\infty ye^{-y}dy = \frac{K}{2}\int_0^\infty ye^{-y}dy \Rightarrow$

$udv = uv - \int vdu$

$u = y \to du = 1d$

$dv = e^{-y} \to v = \int e^{-y} = -e^{-y}$

$= \frac{K}{2}\left[\underbrace{-ye^{-y}\Big]_0^\infty}_{=0} + \int_0^\infty e^{-y}dy\right] = \frac{K}{2}\left[-e^{-y}\right]_0^\infty = \frac{K}{2}.1 = 1 \Rightarrow \boxed{K=2}$

$$f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy = \begin{cases} 0 , & x\in(-\infty,0), y\in(-\infty,0) \\ \int_0^\infty 2ye^{-2x}e^{-y}dy & x\in(0,\infty), y\in(0,\infty) \end{cases}$$

$$\int_0^\infty 2ye^{-2x}e^{-y}dy = 2e^{-2x}\int_0^\infty e^{-y}.y\,dy = 2e^{-2x}$$

$$f_Y(y) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dx = \begin{cases} 0 , & x\in(-\infty,0), y\in(-\infty,0) \\ \int_0^\infty 2ye^{-2x}e^{-y}dx & x\in(0,\infty), y\in(0,\infty) \end{cases}$$

$$\int_0^\infty 2ye^{-2x}e^{-y}dx = 2ye^{-y}\int_0^\infty e^{-2x}dx = 2ye^{-y}\left(-\frac{1}{2}\left[e^{-2x}\right]_0^\infty\right) =$$

$$= 2ye^{-y}.\frac{1}{2} = ye^{-y}$$

Entonces: $f_X(x).f_Y(y) = \begin{cases} 0 & , x\in(-\infty,0), y\in(-\infty,0) \\ 2e^{-2x}.y.e^{-y} & x\in(-\infty,0), y\in(-\infty,0) \end{cases}$

$\Rightarrow$ Son independientes.

b) $\mathbb{E}(X) = \int_{\mathbb{R}} x \, f_X(x)\,dx = \int_0^{\infty} 2x \cdot e^{-2x}\,dx = 2\int_0^{\infty} x \cdot e^{-2x}\,dx \Rightarrow$

$u = x \rightarrow du = 1$

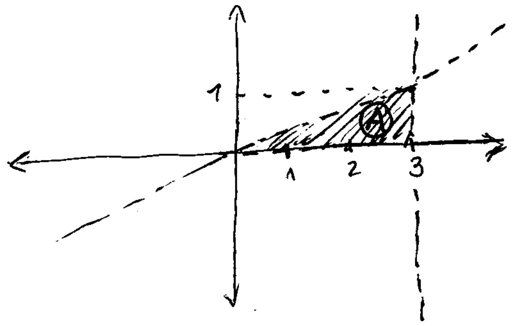
$dv = e^{-2x} \rightarrow \int e^{-2x} =$

$= -\frac{1}{2}e^{-2x}$

$$= \cancel{2}\Big( \underbrace{\left[-\tfrac{1}{\cancel{2}} x e^{-2x}\right]_0^{\infty}}_{0} + \tfrac{1}{\cancel{2}}\int_0^{\infty} e^{-2x}\,dx \Big) = \int_0^{\infty} e^{-2x}\,dx =$$

$$= \frac{-1}{2}\left[e^{-2x}\right]_0^{\infty} = \frac{-1}{2}\cdot(-1) = 1/2$$

$\boxed{11.}$ $(X,Y)$ el vector aleatorio con distribución uniforme en el recinto limitado por $y = x/3$, $x = 3$, $y = 0$. Calcular $f_{(X,Y)}$, $F_{(X,Y)}$, $f_X$, $f$

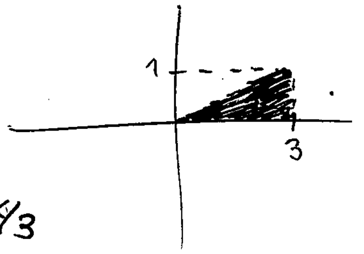


a) $\int_0^3 \int_0^{x/3} K\,dy\,dx = K\int_0^3 x/3\,dx = \frac{K}{3}\left[\frac{x^2}{2}\right]_0^3 =$

$= 1 \iff \frac{9}{6}K = 1 \iff K = \frac{6}{9} = \frac{2}{3}$

$$f_{(X,Y)}(x,y) = \begin{cases} 2/3 & \text{si } (x,y) \in A \\ 0 & \text{si } (x,y) \notin A \end{cases}$$

$F_{(X,Y)}(x,y)$ "



b) $\int_{-\infty}^{x}\int_{-\infty}^{y} f_{(X,Y)}(z,w)\,dz\,dw = \begin{cases} 0, & x<0 \text{ ó } y<0 \\ 1, & x>3 \text{ y } y>1 \\ \frac{x \cdot x/3}{2}\cdot\frac{2}{3} = \frac{x^2}{9} & x \in (0,3),\ y > x/3 \\ \text{faltan dos casos} & \end{cases}$

faltan dos casos → $y \in (0,1)$ $x > 3$

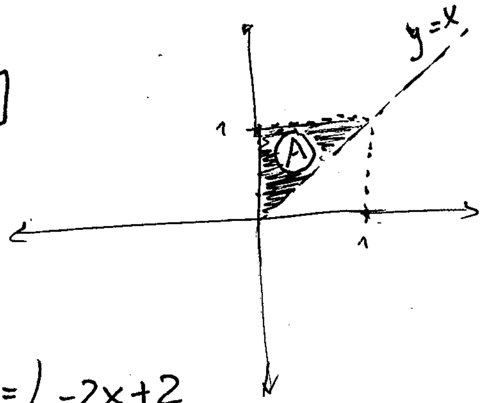
→ $x \in (0,3)$, $y \in (0, x/3)$

c) $f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy = \int_0^{x/3} 2/3\,dy = \frac{2}{3}\frac{x}{3} = \begin{cases} \frac{2x}{9} & \text{cuando } x \in [0,3] \\ 0 & \text{cuando } x \notin [0,3] \end{cases}$

$f_Y(y) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dx = \int_{3y}^{3} 2/3\,dx = \frac{2}{3}(3-3y) = \begin{cases} 2-2y & \text{cuando } y \in [0,1] \\ 0 & \text{cuando } y \notin [0,1] \end{cases}$

12\. La densidad de $(X, Y)$ es:

$$f_{(X,Y)}(x,y) = \begin{cases} 2 & x \in [0,y],\ y \in [0,1] \\ 0 & \text{en el resto} \end{cases}$$




Calcular $f_X,\ f_Y,\ f_{X|Y=y},\ f_{Y|X=x}$

$$f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy = \int_0^1 2\,dy = (-x+1)\cdot 2 = \begin{cases} -2x+2 & \text{cuando } x \in [0,1] \\ 0 & \text{cuando } x \notin [0,1] \end{cases}$$

$$f_Y(y) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dx = \int_0^y 2\,dx = \begin{cases} 2y & \text{cuando } y \in [0,1] \\ 0, & \text{resto} \end{cases}$$

$$f_{Y|X=x}(y) = \frac{f_{(X,Y)}(x,y)}{f_X(x)} = \begin{cases} 1/1-x, & 1 < y < x \\ 0, & \text{resto} \end{cases}$$

13\. Si $f_{X_1}(x_1) = \begin{cases} e^{-x_1}, & x_1 > 0 \\ 0, & x_1 \leq 0 \end{cases}$ y $f_{X_2|X_1=x_1}(x_2) = \begin{cases} \dfrac{x_1}{x_2^{x_1+1}}, & x_2 > 1 \\ 0, & x_2 \leq 1 \end{cases}$

Calcular $f_{(X_1,X_2)}$, $f_{X_2}$, $f_{X_1|X_2=x_2}$

15. Sea $(X,Y)$ un vector aleatorio con densidad uniforme en $(0,1)\times(0,1)$. a) Calcular $F_{X+Y}$? $\quad Z = X+Y$

b) Calcular $F_{(U,V)}$ donde $U = X+Y$, $V = X-Y$

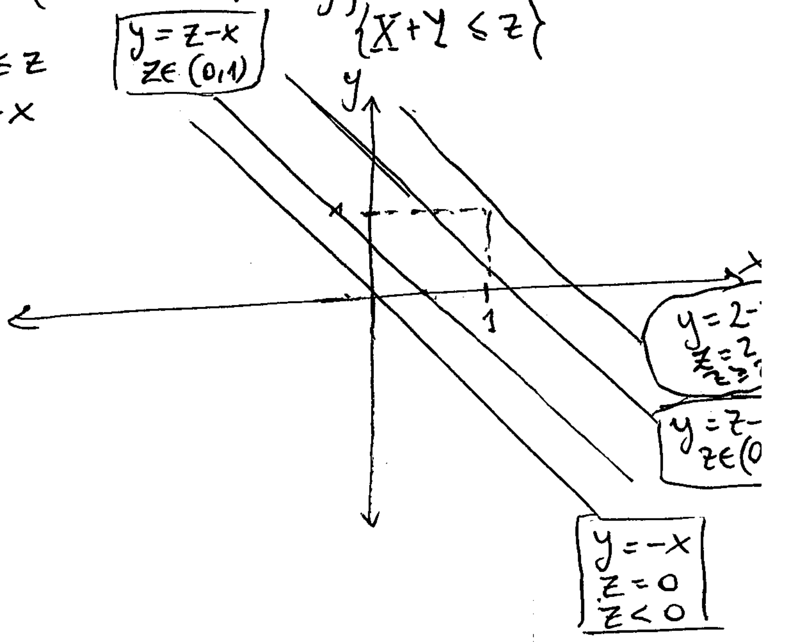
a) $$f_{(X,Y)}(x,y) = \begin{cases} 1 & (x,y)\in(0,1)\times(0,1) \\ 0 & \text{en el resto} \end{cases}$$

$$F_{X+Y}(z) = F_{Z}(z) = \mathbb{P}(Z\leq z) = \mathbb{P}(X+Y\leq z) = \iint_{\{X+Y\leq z\}} f_{(X,Y)}(x,y)\,dx\,dy$$

$x+y\leq z$

$y\leq z-x$

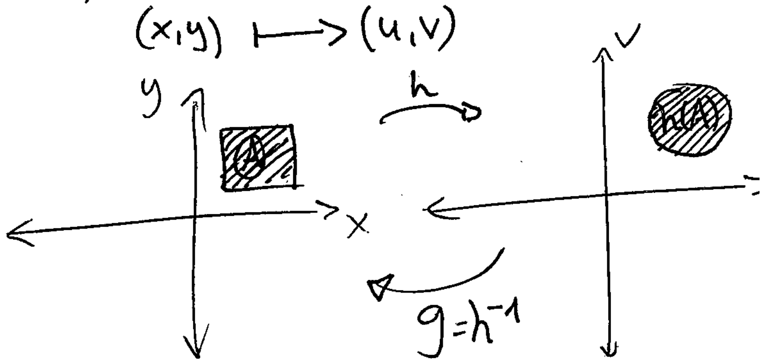
$$= \begin{cases} 0 & z\leq 0 \\ \int_0^z dx\int_0^{z-x} dy\ 1 = \frac{z^2}{2} & z\in(0,1) \\ 1-\frac{(2-z)^2}{2}, & z\in[1,2) \\ 1, & z\geq 2 \end{cases}$$




EN GENERAL

Si $\begin{cases} u = h_1(x,y) \\ v = h_2(x,y) \end{cases}$ $\qquad h = (h_1,h_2): \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$\qquad (x,y)\longmapsto(u,v)$




Si $h$ se puede invertir, $g := h^{-1}$

$$\begin{cases} x = g_1(u,v) \\ y = g_2(u,v) \end{cases}$$

Si $g$ es derivable y las derivadas son continuas.

$$|\det(J)| = \left|\det\begin{pmatrix} \frac{\partial g_1}{\partial u} & \frac{\partial g_1}{\partial v} \\ \frac{\partial g_2}{\partial u} & \frac{\partial g_2}{\partial v} \end{pmatrix}\right|$$

Entonces:

$$\mathbb{P}_{U,V}(h(A)) = \mathbb{P}_{X,Y}(A) = \iint_A f_{(X,Y)}(x,y)\,dx\,dy =$$

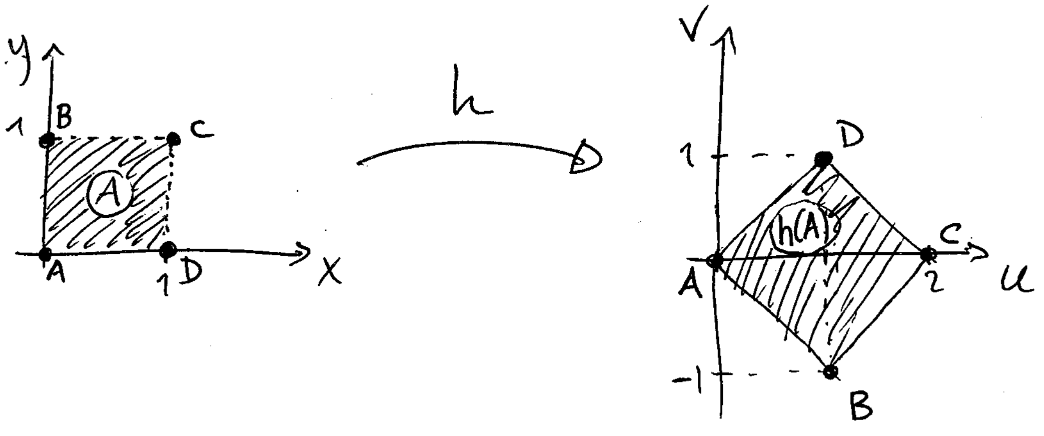
$$= \boxed{\iint_{h(A)} f_{(X,Y)}(g_1(u,v), g_2(u,v)) \cdot |J(u,v)|\,du\,dv}$$

b) $\begin{cases} u = x+y = h_1(x,y) \\ v = x-y = h_2(x,y) \end{cases}$ la inversa es $\begin{cases} x = \frac{u+v}{2} = g_1(u,v) \\ y = \frac{u-v}{2} = g_2(u,v) \end{cases}$

$$|\det(J(u,v))| = \left|\det\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}\right| = \left|-1/4 - 1/4\right| = 1/2$$

$$f_{(U,V)}(u,v) = f_{(X,Y)}\left(\frac{u+v}{2}, \frac{u-v}{2}\right) \cdot \frac{1}{2} = \begin{cases} 1/2, & \left(\frac{u+v}{2}, \frac{u-v}{2}\right) \in (0,1)^2 \\ 0, & \text{en el resto} \end{cases}$$

$$\left(\frac{u+v}{2}, \frac{u-v}{2}\right) = g(u,v) \in (0,1)^2 \iff (u,v) \in h((0,1)^2)$$




$$f_{(U,V)}(u,v) = f_{(X,Y)}\left(\frac{u+v}{2}, \frac{u-v}{2}\right) \cdot \frac{1}{2} = \begin{cases} 1/2, & u \in (0,2),\ v \in \left(|u-1|-1,\ 1-|u-1|\right) \\ 0, & \text{en el resto} \end{cases}$$

$\boxed{16.}$ $X$ e $Y$ son dos variables independientes y con la misma densidad.

$$f(x) = \begin{cases} e^{-x}, & x>0 \\ 0, & \text{en el resto} \end{cases}$$

Calcular $f_{(X,Y)}$ y $f_{(U,V)}$ donde $U = X+Y$, $V = \frac{X}{X+Y}$. ¿Son independientes $U$ y $V$?

a) Como $X$ e $Y$ son independientes

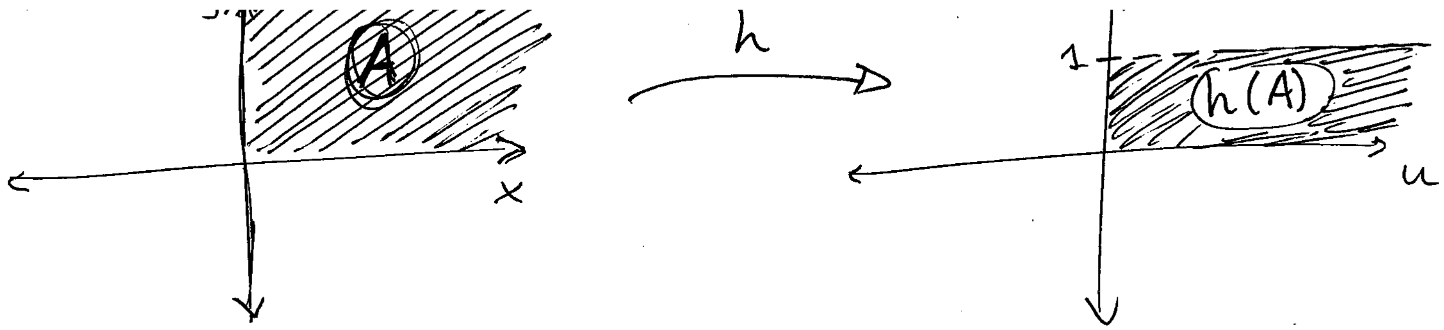
$$f_{(X,Y)}(x,y) = f_X(x)\, f_Y(y) = \begin{cases} e^{-x}\cdot e^{-y}, & x>0,\ y>0 \\ 0, & \text{en el resto} \end{cases}$$

$$\begin{cases} u = x+y = h_1(x,y) \\ v = \frac{x}{x+y} = h_2(x,y) \end{cases} \Longrightarrow \begin{cases} x = uv = g_1(u,v) \\ y = u(1-v) = g_2(u,v) \end{cases}$$

$$|\det(J)| = \left| \det\begin{pmatrix} v & u \\ 1-v & -u \end{pmatrix} \right| = |-uv - u(1-v)| = |u|$$

$$f_{(U,V)}(u,v) = f_{(X,Y)}(uv,\ u(1-v))\ |u| = \begin{cases} e^{-uv}\cdot e^{-u(1-v)}|u| & \text{cuando } (uv, u(1-v)) \in (0,\infty)^2 \\ 0 & \text{en el resto} \end{cases}$$

$$= \begin{cases} e^{-u}\cdot |u| & \text{cuando } (uv, u(1-v)) \in (0,\infty)^2 \\ 0 & \text{en el resto} \end{cases}$$




$$f_{(U,V)}(u,v) = f_{(X,Y)}(uv,\ u(1-u))\,|u| = \begin{cases} e^{-u}\cdot|u| & \text{cuando } \begin{matrix} u>0 \\ v\in(0,1) \end{matrix} \\ 0 & \text{en el resto} \end{cases}$$

b)
$$f_U(u) = \int_{\mathbb{R}} f_{(U,V)}(u,v)\,dv = \begin{cases} 0, & u<0 \\ \underbrace{\int_0^1 u e^{-u}\,dv}_{= \, u e^{-u}}, & u>0 \end{cases}$$

$$f_V(v) = \int_{\mathbb{R}} f_{(U,V)}(u,v)\,du = \begin{cases} 0 & v\notin(0,1) \\ \underbrace{\int_0^\infty u e^{-u}\,du}_{= \, 1} & v\in(0,1) \end{cases}$$

Como $f_U(u)\cdot f_V(v) = f_{(U,V)}(u,v) \Rightarrow U$ y $V$ son independiente

(14) H3

$(X,Y)$ vector aleatorio

no hay más casos posibles porque ya suman 1

$$\mathbb{P}(X=1,Y=0) = \mathbb{P}(X=0,Y=1) = \mathbb{P}(X=1,Y=1) = 1/3$$

¿$F_{(X,Y)} = ?$

$F_X, F_Y = ?$

| $X \backslash Y$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1/3 |
| 1 | 1/3 | 1/3 |

masa $P(x,y)$

$$F_{(X,Y)}(x,y) = \mathbb{P}(X \leq x, Y \leq y)$$

| $X \backslash Y$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1/3 |
| 1 | 1/3 | 1 |

distrib. $F_{(X,Y)}$

$\mathbb{P}(X=0) = 1/3$ $\qquad$ $\mathbb{P}(Y=0) = 1/3$

$\mathbb{P}(X=1) = 2/3$ $\qquad$ $\mathbb{P}(Y=1) = 2/3$

$$F_X(x) = \mathbb{P}(X \leq x) = \begin{cases} 1/3 & x = 0 \\ 1 & x = 1 \end{cases}$$

¿Varianza vectores aleatorios discretos?

¿Fórmulas finales de los modelos?

¿Ej 10 vectores aleatorios?

¿Ej 16 ~~comos~~ como sacar los nuevos intervalos de u y v.?

¿Aplicaciones teorema de los grandes números?

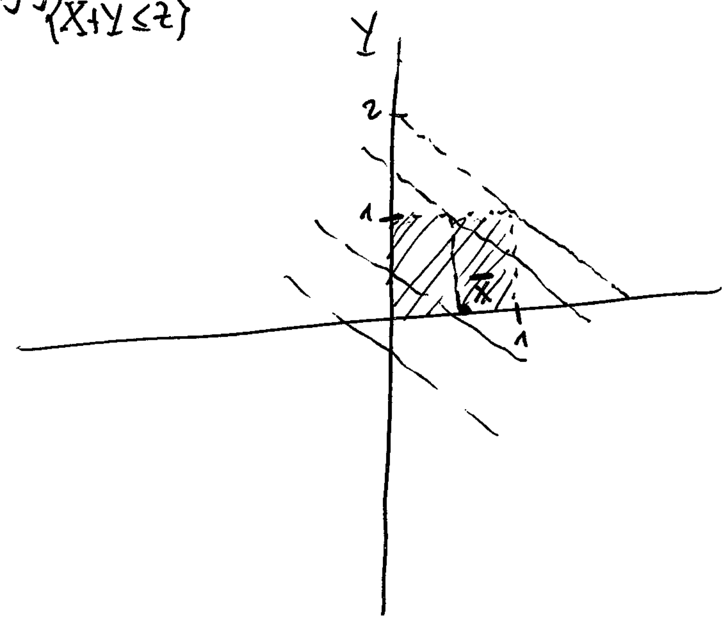
15.

$$f_{X,Y}(x,y) = \begin{cases} 1 & (x,y) \in [0,1]^2 \\ 0 & \text{resto} \end{cases}$$

a) ¿Distribución $X+Y$? ¿$F_{X+Y}$?

$Z = X+Y$

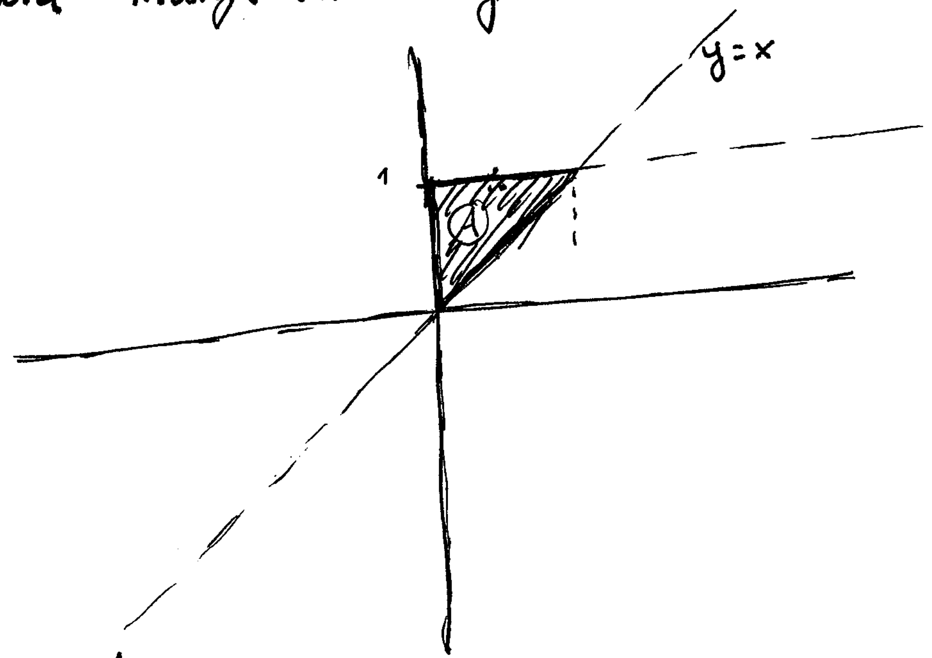
$$F_Z(z) = \mathbb{P}(Z \leq z) = \mathbb{P}(\{X+Y \leq z\}) = \iint_{\{X+Y \leq z\}} f_{(X,Y)}(x,y)\,dx\,dy = (*)$$




$$(*) = \begin{cases} 0 & \text{si } z < 0 \\ 1 & \text{si } z > 2 \\ \int_0^{\bar{x}} dx \int_0^1 dy \cdot 1 + \int_{\bar{x}}^1 dx \int_0^{y-\bar{x}} dy \cdot 1 & \end{cases}$$

$$f(x,y) = \begin{cases} 2 & \text{si } \underset{0=x}{0} < \underset{x=y}{x < y}, \ \underset{0=y}{0} < y < \underset{y=1}{1} \\ 0 & \text{en el resto} \end{cases}$$

¿Funciones de densidad marginales y condicionadas?




$$f_X(x) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dy = \int_x^1 2\,dy = 2(1-x) = 2-2x$$

$$f_Y(y) = \int_{\mathbb{R}} f_{(X,Y)}(x,y)\,dx = \int_0^y 2\,dx = \cancel{2-2y}\ 2y$$