HOJA 4

RECUERDO

Si C activo replicable, entonces su proceso de precios es:

$$C_0 = e^{-jR\Delta t}\,\mathbb{E}_P(C_j)$$

En el caso de una call europea: $\underset{\text{precio call}}{c} = e^{-R.M.\Delta t}\,\mathbb{E}_P(\underset{\text{flujo call}}{C})$, $T = M.\Delta t$

$$\mathbb{E}_P(C) = \underbrace{\sum_{j=0}^{M}}_{\substack{j+1 \\ \text{posibles} \\ \text{escenarios}}} \underbrace{C_j^{(M)}}_{\substack{\text{pago/valor} \\ \text{de cada} \\ \text{escenario}}} \cdot \underbrace{\binom{M}{j}\frac{1}{2^M}}_{\substack{\text{prob de} \\ \text{cada} \\ \text{escenario}}}$$

$$A(\sigma) = \frac{1}{\cosh(\sigma\sqrt{\Delta t})}$$

$$C_j^{(M)} = \left(S_j^{(M)} - K\right)^+$$

número de "subidas"

número "bajadas"

Jarrow-Rudd

$$S_j^{(M)} = S_0 \cdot u^j \cdot d^{M-j} \quad ; \quad u = e^{R\Delta t} A(\sigma)\, e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{R\Delta t} A(\sigma)\, e^{-\sigma\sqrt{\Delta t}}$$

$$\Rightarrow S_j^{(M)} = S_0\, e^{R\cdot M\cdot\Delta t} A(\sigma)^M \cdot e^{(2j-M)\sigma\sqrt{\Delta t}}$$

$$\Rightarrow C_j^{(M)} = \left(S_0 e^{R.M.\Delta t} A(\sigma)^M \cdot e^{(2j-M)\sigma\sqrt{\Delta t}} - K\right)^+$$

$$\Rightarrow \boxed{c = e^{-R.M.\Delta t}\sum_{j=0}^{M}\left(S_0 e^{RM\Delta t} A(\sigma)^M \cdot e^{(2j-M)\sigma\sqrt{\Delta t}} - K\right)^+ \binom{M}{j}\frac{1}{2^M}}$$

1.1

a) paga 1 si la cotización está por encima de S_0:

$$C_j = \begin{cases} 1 & \text{si } S_j^{(M)} > S_0 \\ 0 & \text{si } S_j^{(M)} \leq S_0 \end{cases}$$

$$S_j^{(M)} = S_0 e^{MR\Delta t} A(\sigma)^M \cdot e^{\sigma\sqrt{\Delta t}(2j-M)} > S_0 \iff$$

$$\iff e^{\sigma\sqrt{\Delta t}(2j-M)} > e^{-MR\Delta t} A(\sigma)^{-M} \iff$$

$$\iff 2j - M > \frac{-MR\Delta t - M\ln(A(\sigma))}{\sigma\sqrt{\Delta t}} \iff j > \frac{M}{2} + \frac{-M(R\Delta t + \ln(A(\sigma)))}{2\sigma\sqrt{\Delta t}}$$

$$\Rightarrow j^* := \left\lfloor \frac{M}{2} + \frac{-M(R\Delta t + \ln(A(\sigma)))}{2\sigma\sqrt{\Delta t}} \right\rfloor$$

$$\Rightarrow \boxed{C = e^{-R.M.\Delta t} \sum_{j=j^*}^{M} 1 \cdot \binom{M}{j} \frac{1}{2^M}}$$

b) Análogamente al apartado a)

$$S_0 e^{RM\Delta t} e^{(2j-M)\sigma\sqrt{\Delta t}} A(\sigma)^M > K \iff e^{(2j-M)\sigma\sqrt{\Delta t}} > \frac{K}{S_0 e^{RM\Delta t} A(\sigma)^M}$$

$$\iff (2j-M)\sigma\sqrt{\Delta t} > \ln(K) - \left[\ln(S_0) + RM\Delta t + M\ln(A(\sigma))\right] \iff$$

$$\iff j > \frac{M}{2} + \frac{\ln(K) - \ln(S_0) - RM\Delta t - M\ln(A(\sigma))}{2\sigma\sqrt{\Delta t}}$$

$$\Rightarrow \hat{j} := \left\lfloor \frac{M}{2} + \frac{\ln(K) - \ln(S_0) - RM\Delta t - M\ln(A(\sigma))}{2\sigma\sqrt{\Delta t}} \right\rfloor$$

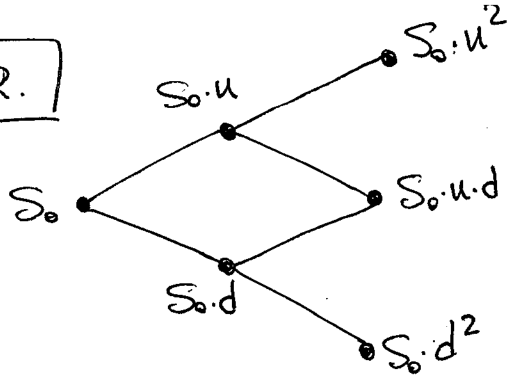
Por otro lado:

$$S_0 e^{RM\Delta t} e^{(2j-M)\sigma\sqrt{\Delta t}} A(\sigma)^M < 2K \iff j < \frac{M}{2} + \frac{\ln(2K) - \ln(S_0) - RM\Delta t - M\ln(A(\sigma))}{2\sigma\sqrt{\Delta t}}$$

$$\Rightarrow \tilde{j} := \left\lfloor \frac{M}{2} + \frac{\ln(2K) - \ln(S_0) - RM\Delta t - M\ln(A(\sigma))}{2\sigma\sqrt{\Delta t}} \right\rfloor$$

$$\Rightarrow \boxed{C = e^{-RM\Delta t} \sum_{j=\hat{j}}^{\tilde{j}} 1 \cdot \binom{M}{j} \frac{1}{2^M}}$$

$\boxed{2.}$



4 sendas posibles (equiprobables):

① $C_1 = \left(\frac{S_0 + S_0 \cdot u + S_0 \cdot u^2}{3} - S_0 \right)^+$

② $C_2 = \left(\frac{S_0 + S_0 \cdot u + S_0 \cdot u \cdot d}{3} - S_0 \right)^+$

③ $C_3 = \left(\frac{S_0 + S_0 . d + S_0 \cdot u \cdot d}{3} - S_0 \right)^+$

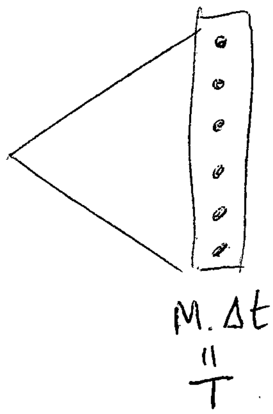
④ $C_4 = \left(\frac{S_0 + S_0 \cdot d + S_0 \cdot d^2}{3} - S_0 \right)^+$

$$\Rightarrow \boxed{c = e^{-2R\Delta t} \frac{1}{4} \left(C_1 + C_2 + C_3 + C_4 \right)}$$

$\boxed{3.}$

$\boxed{1.}$

$$A(\sigma) = \frac{1}{\cosh(\sigma\sqrt{\Delta t})}$$



Valores

$$S_M = S_0 \, e^{R.M.\Delta t} A(\sigma)^M \cdot e^{(2j-M)\sigma\sqrt{\Delta t}} \qquad j = 0, \ldots, M$$

con probs $\binom{M}{j} \frac{1}{2^M}$

Opción paga 1 si $S_M \geq S_0 = K$

paga 0 si no

$$\cancel{S_0}\, e^{R.M.\Delta t} A(\sigma)^M e^{(2j-M)\sigma\sqrt{\Delta t}} \geq \cancel{S_0} \cdot 1$$

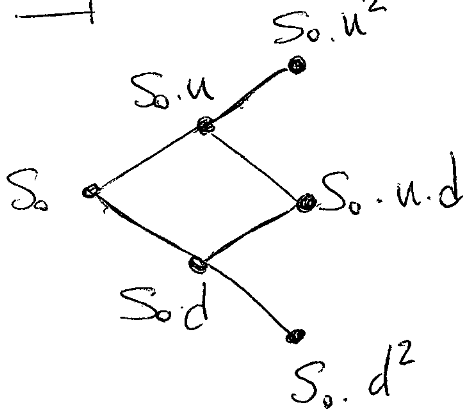
$$e^{(2j-M)\sigma\sqrt{\Delta t}} \geq e^{-R.M.\Delta t} A(\sigma)^{-M}$$

$$2j \geq \underbrace{\left\lfloor \frac{M}{2} - \frac{RM\Delta t + M\ln(A(\sigma))}{2\sigma\sqrt{\Delta t}} \right\rfloor}_{= j^*}$$

$$\begin{array}{l} \text{precio call} \\ \text{binaria} \end{array} = e^{-R.M.\Delta t} \sum_{j \geq j^*} 1 \cdot \binom{M}{j} \frac{1}{2^M} \qquad \text{entre } K \text{ y } \underset{\tilde{j}}{2K}$$

$$= \cdots = \sum_{j=\hat{j}}$$

$\boxed{2.}$



$$\left(\frac{S_0 + S_1 + S}{3} - S_0\right)^+$$

$$\text{precio} = e^{-R.2.\Delta t}\left[\frac{1}{4}\left(\frac{S_0 + S_0.u + S_0.u^2}{3} - S_0 \right. \right.$$

$$\left. + \frac{1}{4}\left(\cdots \right)^+ \cdots \right]$$

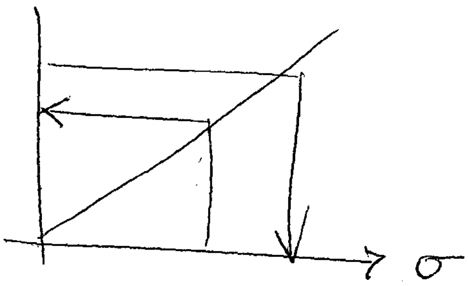
3. / 4. Lo mismo

$\downarrow$

¿Son estos precios consistentes con B-S?

$$\begin{bmatrix} 7 & \cdot & \cdot \\ 3'7 & \cdot & \cdot \\ 1'6 & \cdot & \cdot \end{bmatrix} \quad \text{precios}$$

$$\begin{array}{c} 45 \\ 50 \\ 55 \end{array} \begin{bmatrix} 41\% & \cdot & \cdot \\ 37\% & \cdot & \cdot \\ 65\% & \cdot & \cdot \end{bmatrix} \quad \text{volas implícitas}$$



$$\text{call}_{BS} = S\,\Phi(d_+) - Ke^{-rT}\,\Phi(d_-)$$

$$d_\pm = \frac{1}{\sigma\sqrt{T}} \ln\left(\frac{S}{Ke^{-rT}}\right) \pm \frac{1}{2}\sigma\sqrt{T}$$

<u>Previo</u>

$$\phi(d_\pm) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_\pm^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{1}{\sigma^2 T}\ln(\;)^2 + \frac{1}{4}\sigma^2 T \pm \ln\left(\frac{S}{Ke^{-rT}}\right)\right]}$$

$$\phi(d_+) = \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[\cdots + \cdots\right.}}_{\phi(d_-)}{}^{\left.- \ln\left[\frac{S}{Ke^{-rT}}\right]\right]}$$

$$\cdot\, \underbrace{e^{-\ln(S/Ke^{-rT})}}_{Ke^{-rT}/S}$$

$$\phi(d_+) = \frac{Ke^{-rT}}{S}\phi(d_-) \Longrightarrow \boxed{S\,\phi(d_+) = Ke^{-rT}\phi(d_-)}$$

$$\frac{\partial c}{\partial S} = \Phi(d_+) + S\,\Phi(d_+)\,\frac{\partial}{\partial S}$$