HOJA 3

2. Suponer primero que X está tipificada
Luego destipificar.

$\mathbb{E}(X) = 0$ y $V(X) = 1$ (gratis: $\mathbb{E}(x^2) = 1$)

▸ $\text{asim}(X) = \mathbb{E}(x^3)$

▸ $\text{asim}(\bar{X}) =$

$\mathbb{E}(\bar{x}) = \mathbb{E}(X) = 0$

$V(\bar{x}) = \frac{1}{n}$

$= \frac{\mathbb{E}((\bar{x}-0)^3)}{(1/n)^{3/2}}$

Vamos a calcular $\mathbb{E}(\bar{x}^3)$:

$$\mathbb{E}(\bar{x}^3) = \frac{1}{n^3} \cdot \mathbb{E}((x_1 + \dots + x_n)^3) = \frac{1}{n^3} \mathbb{E}\begin{pmatrix} (x_1 + \dots + x_n) \cdot \\ \cdot (x_1 + \dots + x_n) \cdot \\ \cdot (x_1 + \dots + x_n) \end{pmatrix} =$$

$\uparrow$ n sumandos

estos sumandos pueden ser de la forma

$\to x_K^3 \to \mathbb{E} = \mathbb{E}(x$

$\to x_j^2 x_K \quad j \neq K \to \mathbb{E} = 0$

$\to x_j x_K x_i \quad i \neq j \neq K \neq$ $\to \mathbb{E} = 0$

$= \frac{n}{n^3} \mathbb{E}(x^3)$

$\Rightarrow \text{asim}(\bar{x}) = n^{3/2} \frac{\mathbb{E}(x^3)}{n^2} = \frac{\text{asim}(x)}{\sqrt{n}}$

demostrado para X tipificada.

En general, destipificamos:

$X \longrightarrow Y = \frac{X - \mathbb{E}(X)}{\sqrt{V(X)}}$ $\qquad \bar{Y} = \frac{\bar{X} - \mathbb{E}(X)}{\sqrt{V(X)}}$

$\uparrow$ aplicamos a $\bar{Y} \to$ deshacer cambios

5.1 $X \sim \operatorname{Exp}(\lambda)$ $\lambda > 0$ $X_1, \dots, X_n$ ciones independientes ...

$\Rightarrow \min(X_1, \dots, X_n) \sim \operatorname{Exp}(n\lambda)$ y calcular $n\mathbb{E}(m_n)$.

$X \sim \operatorname{Exp}(\lambda)$

$$f_X(t) = \begin{cases} 0 & t < 0 \\ \lambda e^{-\lambda t} & t \geq 0 \end{cases}$$

$$F_X(t) = 1 - e^{-\lambda t} \quad t \geq 0$$

Nos preguntamos cual es $F_{m_n}(t)$ $t \geq 0$:

$$F_{m_n}(t) = \mathbb{P}(\min(X_1, \dots, X_n) \leq t) = 1 - \mathbb{P}(\min(X_1, \dots, X_n) > t) =$$

$$= 1 - \mathbb{P}(X > t)^n = 1 - ((1 - \mathbb{P}(X < t))^n) =$$

ones e dependientes

$$= 1 - (e^{-\lambda t})^n = 1 - e^{-n\lambda t} \Rightarrow m_n \sim \operatorname{Exp}(n\lambda)$$

$$\mathbb{E}(m_n) = \frac{1}{\lambda n} \qquad \left(\text{porque } \mathbb{E}(X) = \frac{1}{\lambda} \text{ cuando } X \sim \operatorname{Exp}(\lambda)\right).$$

4. X

1	1/3
2	1/3
3	1/3

$n = 3$ $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ (el profesor le llamó a $Z_3 = U$).

U = estadístico de la muestra X_1, X_2, X_3 tras ordenar de menor a mayor nos quedamos con el segundo valor.

$U = 1$

1 1 1
1 1 ␣
1 ␣ 1
␣ 1 1

$$\mathbb{P}(U = 1) = 3 \cdot \frac{2}{27} + \frac{1}{27} = \frac{7}{27}$$

$U = 3$ $\quad \mathbb{P}(U = 3) = \frac{7}{27}$

$U = 2$ $\quad \mathbb{P}(U = 2) = 1 - (\text{el resto}) = 1 - 2 \cdot \frac{7}{27} = \frac{13}{27}$

$\boxed{6.}$ $X \sim \text{unif}([0,a])$ $\quad a>0$ $\qquad M_n = \max(X_1, \dots, X_n)$

Primero, calcular F_X:

$$f_X(x) = \begin{cases} 0 & x<0 \\ \frac{1}{a} & x \in [0,a] \\ 0 & x>a \end{cases} \qquad F_X(x) = \begin{cases} 0 & x<0 \\ \frac{x}{a} & x\in[0,a] \\ 1 & x>a \end{cases}$$

$$F_{M_n}(t) = \mathbb{P}(\max(X_1,\dots,X_n) \leq t) = \mathbb{P}(X \leq t)^n = \left(\frac{t}{a}\right)^n \quad 0 \leq t \leq a$$

$$f_{M_n}(t) = F'_{M_n}(t) = n\left(\frac{t}{a}\right)^{n-1} \cdot \frac{1}{a} \qquad 0 \leq t \leq a$$

$$\mathbb{E}(M_n) = \int_0^a x\, n\left(\frac{x}{a}\right)^{n-1} \cdot \frac{1}{a}\, dx = \frac{n}{n+1} \cdot a$$

$$\mathbb{E}(M_n^2) = \int_0^a x^2 n\left(\frac{x}{a}\right)^{n-1} \cdot \frac{1}{a}\, dx = \int_0^a n\left(\frac{x}{a}\right)^{n+1} dx = n\left[\frac{\left(\frac{x}{a}\right)^{n+2}}{n+2}\right]_0^a$$

$$\mathbb{V}(M_n) = \mathbb{E}(M_n^2) - \mathbb{E}(M_n)^2$$

b) $\varepsilon > 0$ $\quad \mathbb{P}(a - M_n > \varepsilon a)$

$$\mathbb{P}(a - M_n > \varepsilon a) = \mathbb{P}(M_n < a(1-\varepsilon)) = F_{M_n}(a(1-\varepsilon)) = \left(\frac{a(1-\varepsilon)}{a}\right)^n = (1-$$

1. (a) $X_1, \dots, X_{35}$ $\quad X$ es $N(1,2) \longrightarrow \bar{X} \approx N(1, \frac{2}{35})$, $\frac{2}{35} = V(\bar{X})$

$$¿\mathbb{P}(\bar{X} \leq 1'2,\ S \leq 1'2)?$$

$$\mathbb{P}(\bar{X} \leq 1'2,\ S \leq 1'2) \underset{\text{independientes Fisher-Cochran}}{=} \mathbb{P}(\bar{X} \leq 1'2) \cdot \boxed{\mathbb{P}(S \leq 1'2)} =$$

$\mathbb{P}(S^2 \leq 1'44)$ ← tipificamos

chi cuadrado n-1 grados

$$= \underbrace{\mathbb{P}\left(\frac{\bar{X}-1}{\sqrt{2/35}} \leq \frac{1'2-1}{\sqrt{2/35}}\right)}_{\Phi\left(\frac{1'2-1}{\sqrt{2/35}}\right)} \cdot$$

b) Probabilidad condicionada

independ (intersección de indep.)

$$\mathbb{P}(\bar{X} \leq 1'2 \mid S^2 \leq 1'2) \underset{\text{Bayes}}{=} \frac{\mathbb{P}(\bar{X} \leq 1'2) \cdot \cancel{\mathbb{P}(S^2 \leq 1'2)}}{\cancel{\mathbb{P}(S^2 \leq 1'2)}} = \mathbb{P}(\bar{X} \leq 1'2) = \Phi\left(\frac{1'2-1}{\sqrt{2/35}}\right)$$

calculado

2. X es $N(0, \sigma^2)$ $\quad X_1, \dots, X_{100}$ $\quad n=100$

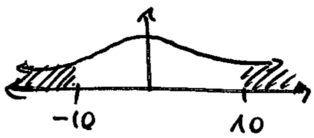
a) $¿\mathbb{P}(|\bar{x}| > \sigma,\ S^2 > 2\sigma^2)?$

b) $¿\mathbb{P}(\{|\bar{x}| \geq \sigma\} \cup \{S^2 > 2\sigma^2\})?$

a) $\mathbb{P}(|\bar{x}| \geq \sigma) \cdot \mathbb{P}(S^2 > 2\sigma^2)$

$$\mathbb{P}\left(\frac{99}{\sigma^2} S^2 > \frac{99}{\sigma^2} \cdot 2\sigma^2\right) = \mathbb{P}\left(\frac{99}{\sigma^2} S^2 > 198\right) = 1 - F_{\chi^2_{99}}(198)$$

$$\mathbb{P}\left(\frac{|\bar{x}|}{\sigma/10} \geq 10\right) = 2(1 - \Phi(10))$$



b) $$\mathbb{P}(\{|\bar{x}| \geq \sigma\} \cup \{S^2 > 2\sigma^2\}) = \frac{\mathbb{P}(|\bar{x}| \geq \sigma) + \mathbb{P}(S^2 > 2\sigma^2)}{\mathbb{P}(|\bar{x}| \geq \sigma) \cdot \mathbb{P}(S^2 > 2\sigma^2)} =$$

$$= \frac{2(1-\Phi(10)) + 1 - F_{\chi^2_{99}}(198)}{2(1-\Phi(10)) \cdot (1 - F_{\chi^2_{99}}(198))}$$

MAL! Este tendría que estar restando

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$\boxed{11.}$ $(X_1, \dots, X_n)$ $\qquad X \sim N(\mu, \sigma^2)$ $\qquad t \geq 0$

$$\mathbb{P}\left(n(\bar{x}-\mu)^2 + (n-1)S^2 \geq t\right) = \mathbb{P}\left(n\left(\frac{\bar{x}-\mu}{\sigma}\right)^2 + \frac{(n-1)}{\sigma^2}S^2 \geq \frac{t}{\sigma^2}\right) =$$

$$= \mathbb{P}\left(\underbrace{\cancel{n}\left(\frac{x-\mu}{\sigma/\cancel{\sqrt{n}}}\right)^2}_{\substack{\text{normal} \\ 0,1 \\ \text{al cuadrado} \\ \equiv \\ \chi^2_1}} + \underbrace{\frac{(n-1)}{\sigma^2}S^2}_{\chi^2_{n-1}} \geq \frac{t}{\sigma^2}\right) = \mathbb{P}\left(\chi^2_n \geq t/\sigma^2\right) =$$

$$= 1 - F_{\chi^2_n}(t/\sigma^2)$$

Tenemos: $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

$$\operatorname{asim}(\bar{X}) = \frac{\operatorname{asim}(X)}{\sqrt{n}}$$

a) Basta X_i tipificada

$$\operatorname{asim}(\bar{Z}) = \frac{\mathbb{E}((\bar{Z} - \mathbb{E}(\bar{Z}))^3)}{V(\bar{x})^{3/2}}$$

$$= \frac{\mathbb{E}((\bar{Z} - \mathbb{E}(\bar{Z}))^3)}{V}$$

$$\mathbb{E}\left(\left(\frac{\bar{Z} - \mathbb{E}(\bar{Z})}{\sqrt{V(\bar{Z})}} \right)^3 \right) = \mathbb{E}(\cancel{Z})$$

$\mathbb{E}(X)$

X tipificado