- SHOW THAT GAUSSIAN KERNEL IS PSD (POSITIVE SEMI-DEFINITIVE)

FOCUS ON 1D CASE:

$$k_s(x,y) = \exp\{-0.5/s\,(x-y)^2\}$$

WORK WITH A SCALED VERSION

$$\tilde{k}_s(x,y) = \frac{1}{\sqrt{2\pi s}} \cdot k_s(x,y) = \underline{\underline{N(y \mid x, s)}}$$

GAUSSIAN DENSITY WITH MEAN x AND VARIANCE s

WE USE THE GAUSSIAN PROPERTY

$$\int_{-\infty}^{\infty} N(z \mid x_i, s/2)\, N(z \mid x_j, s/2)\, dz =$$

$$= N(x_i \mid x_j, s) = \tilde{k}_s(x_i, x_j)$$

THE KERNEL $\tilde{k}_s(x,y)$ IS POSITIVE SEMI-DEFINITE IF

$$a^T K a \geq 0$$

$$\hookrightarrow \sum_{i,j} a_i a_j \tilde{k}_s(x_i, x_j) = \sum_{ij} a_i a_j N(x_i \mid x_j, s) =$$

$$= \sum_{ij} a_i a_j \int_{-\infty}^{\infty} N(z \mid x_i, s/2)\, N(z \mid x_j, s/2)\, dz =$$

$$= \int_{-\infty}^{\infty} \sum_{ij} a_i a_j N(z \mid x_i, s/2)\, N(z \mid x_j, s/2)\, dz =$$

$$= \int_{-\infty}^{\infty} \Big[\sum_{j} a_i N(z \mid x_i, s/2)\Big]^2 dz \geq 0$$