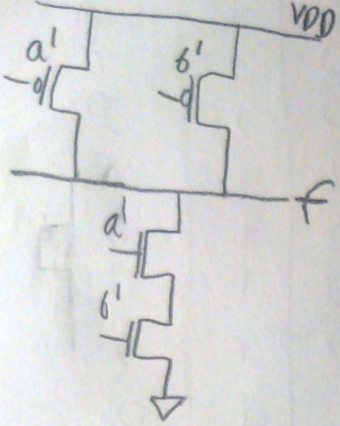
HW2 Problem 1 Design complementary gates for the following: Inverted inputs allowed!

a) $a+b$ $f = (a'b')'$

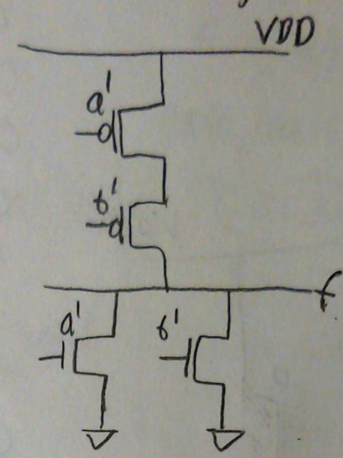
Truth Table

A	B	Out	$a'b'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0



b) ab $f = (a'+b')'$

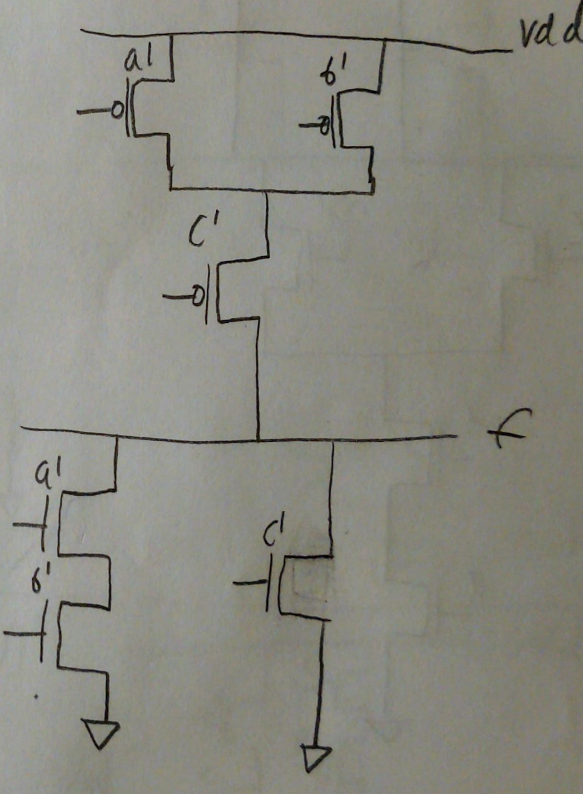
A	B	Out	$a'+b'$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0



c) $(a+b)c$ $f = ((a'b')+c')'$

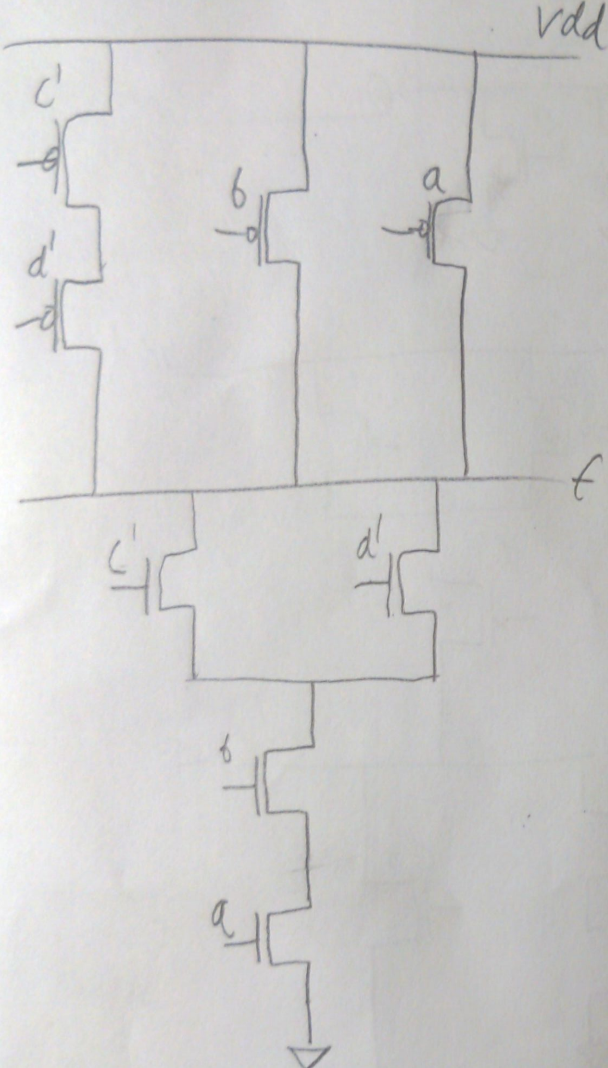
Truth Table

A	B	C	Out	$a'b'+c'$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0



d) $(ab)' \overset{out}{+} (cd)$ $\quad f = a' + b' + (cd)$ $\quad f = (a \cdot b \cdot (c' + d'))'$

A	B	C	D	$(AB)'$	CD	Out	$a \cdot b \cdot (c'+d')$
0	0	0	0	1	0	1	0
0	0	0	1	1	0	1	0
0	0	1	0	1	0	1	0
0	0	1	1	1	1	1	0
0	1	0	0	1	0	1	0
0	1	0	1	1	0	1	0
0	1	1	0	1	0	1	0
0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	0
1	0	0	1	1	0	1	0
1	0	1	0	1	0	1	0
1	0	1	1	1	1	1	0
1	1	0	0	0	0	0	1
1	1	0	1	0	0	0	1
1	1	1	0	0	0	0	1
1	1	1	1	0	1	1	0

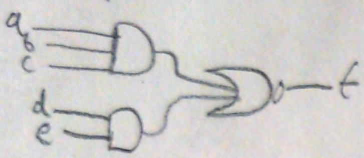


Question 2 Write the logic expressions and draw transistor level schematic

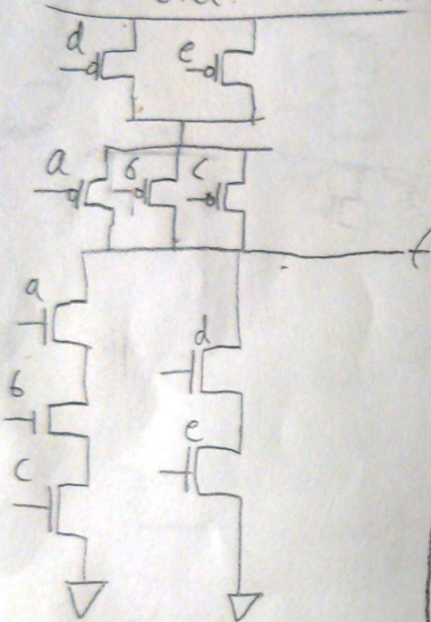
a) AOI32

$f = (abc + de)'$

gate logic:



Transistor level:

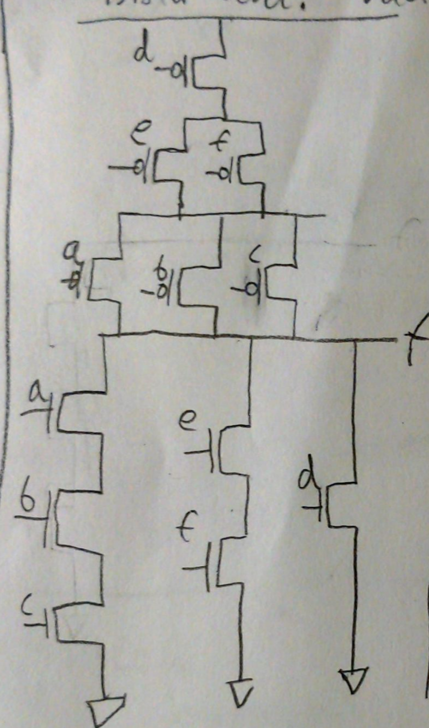


b) AOI 312

$f = (abc + d + ef)'$

gate logic:

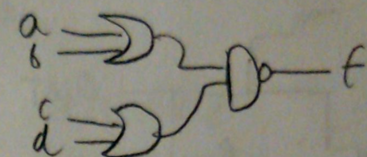
Transistor level:



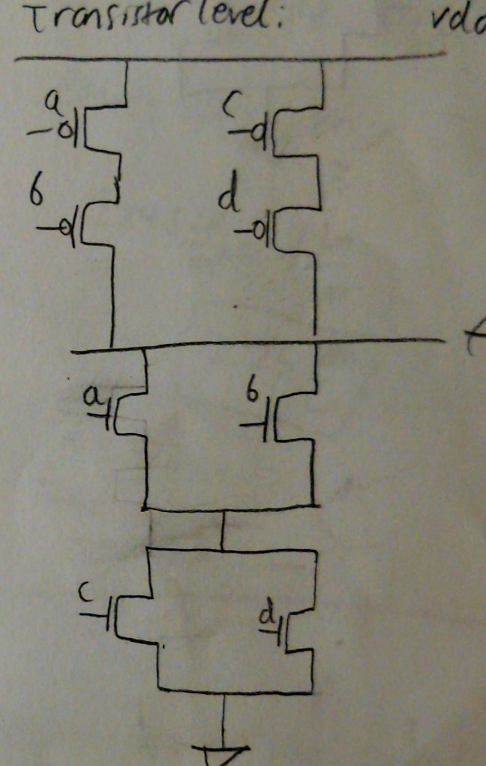
c) OAI22

$f = [(a+b)(c+d)]'$

gate logic

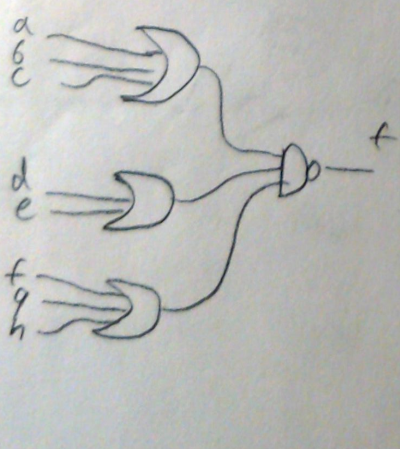


Transistor level:

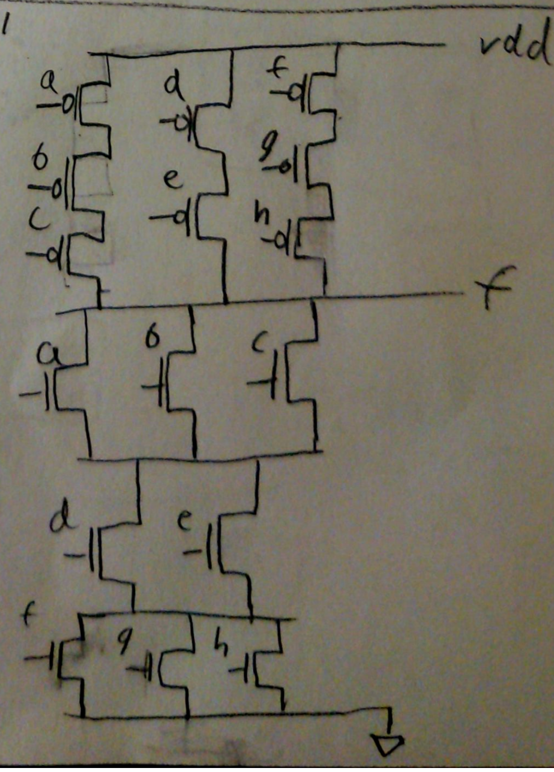


d) OAI323 $f = [(a+b+c)(d+e)(f+g+h)]'$

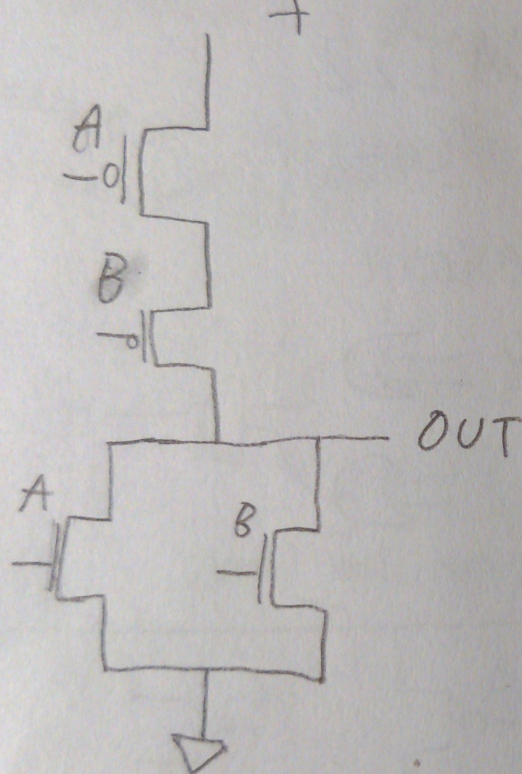
gate logic



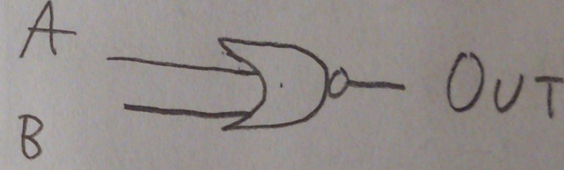
Transistor level:



Question 3: Draw the circuit level schematic. Write the logic



Nor gate



$OUT = \overline{(A+B)}$

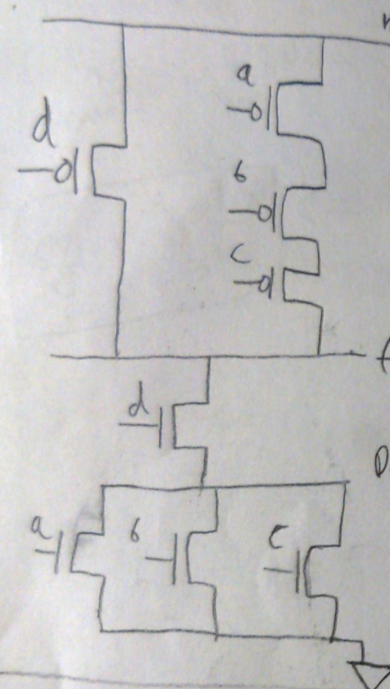
Problem 4: Size the transistors so the pullup and pulldown are approximately equal

NMOS $R_n = 6.47K\Omega$. PMOS $R_p = 29.6K\Omega$ CL $\underline{R_p = 4.5 \cdot R_n}$ from Lecture!

* assume all Pmos has the same W_p and L_p delay $= (R_{up} + R_L)C = (R_{down} + R_L)C$

* assume all Nmos has the same W_n and L_n $R_{up} = R_{down}$

a) $f = a'b'c' + d' = ((a+b+c)d)'$



UP
Worst Best
3 1

Down
2
(both)

worst case

$R_{up} = 3 \times \frac{L_p}{W_p} \times R_p$

$R_{down} = 2 \times \frac{L_n}{W_n} \times R_n$

$3 \times 4.5 \frac{L_p}{W_p} = 2 \frac{L_n}{W_n}$

$\Rightarrow \boxed{\frac{W_p}{L_p} = 6.75 \times \frac{W_n}{L_n}}$

Best case

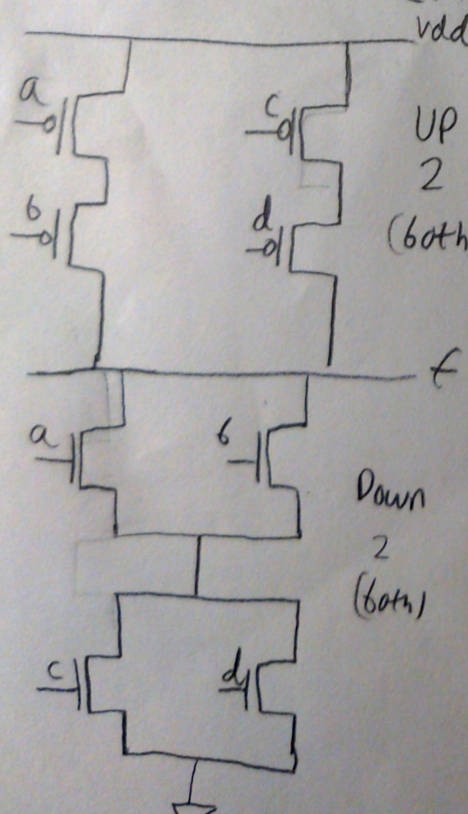
$R_{up} = 1 \times \frac{L_p}{W_p} \times R_p$

$R_{down} = 2 \times \frac{L_n}{W_n} \times R_n$

$1 \times 4.5 \frac{L_p}{W_p} = 2 \frac{L_n}{W_n}$

$\Rightarrow \boxed{2.25 \frac{W_n}{L_n}}$

b) $F = a'b' + c'd' = ((a+b) \cdot (c+d))'$



UP
2
(both)

Down
2
(both)

worst case

$R_{up} = 2 \times \frac{L_p}{W_p} \times R_p$

$R_{down} = 2 \times \frac{L_n}{W_n} \times R_n$

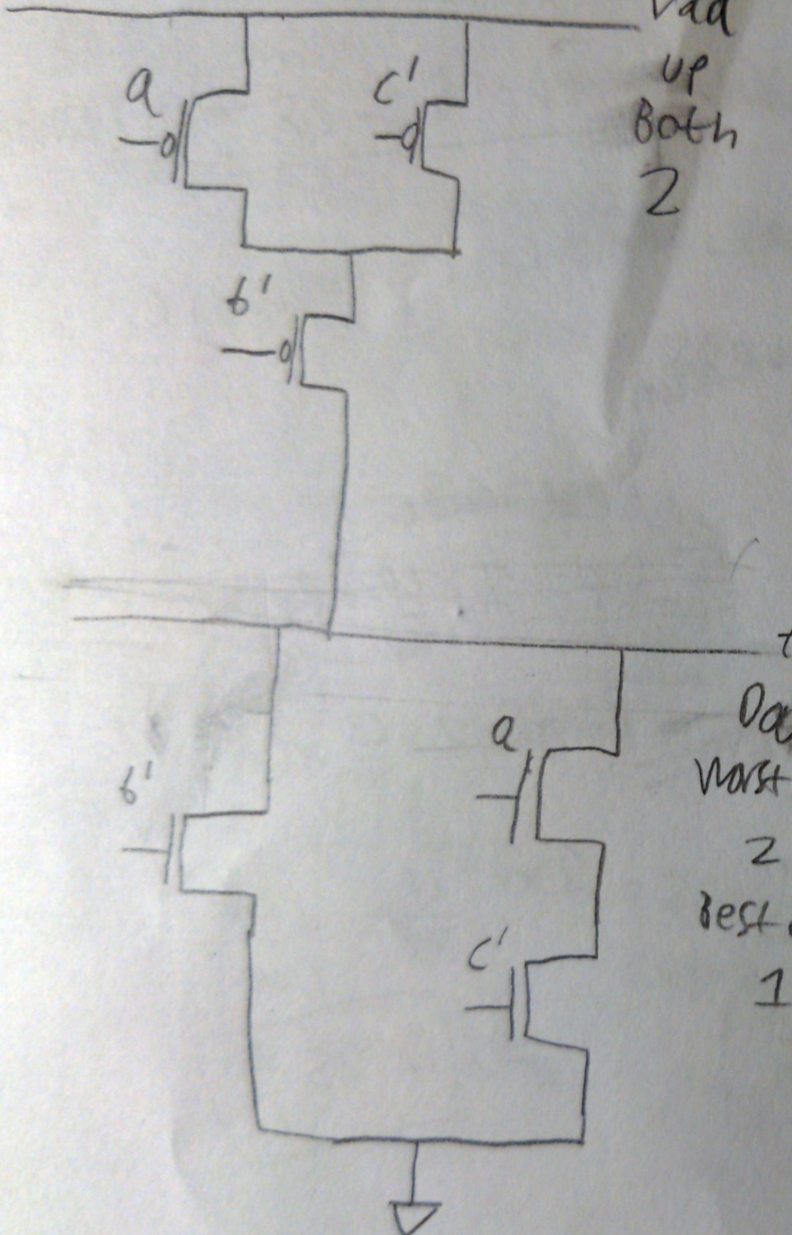
$2 \times 4.5 \frac{L_p}{W_p} = 2 \frac{L_n}{W_n}$

$\Rightarrow \boxed{\frac{W_p}{L_p} = 4.5 \frac{W_n}{L_n}}$

Best case is the same!

$\boxed{\frac{W_p}{L_p} = 4.5 \frac{W_n}{L_n}}$

c) $f = a'b + bc = b(a'+c) = (b' + ac')'$ assuming inverted inputs can be given directly without needing an inverter



up
Both
2

Down
worst case
2
Best case
1

worst case

$R_{up} = 2\frac{L_p}{W_p} \times R_p$

$R_{down} = 2 \times \frac{L_n}{W_n} \times R_n$

$2 \times 4.5 \frac{L_p}{W_p} = 2\frac{L_n}{W_n}$

$\Rightarrow \boxed{\frac{W_p}{L_p} = 4.5\frac{W_n}{L_n}}$

Best case:

$R_{up} = 2\frac{L_p}{W_p} \times R_p$

$R_{down} = 1 \times \frac{L_n}{W_n} \times R_n$

$2 \times 4.5 \frac{L_p}{W_p} = 1\frac{L_n}{W_n}$

$\Rightarrow \boxed{\frac{W_p}{L_p} = 9\frac{W_n}{L_n}}$

Question 5 Compute the value of Reff required to model an inverter that reaches 50% of its output value at 20ps with load C_L equal to $3C_1$, $4C_1$ and $5C_1$. What effect does load capacitance have on the effective resistance ($C_1 = 0.89 * 10^{-15}$)?

$R_n = 6.47K\Omega$ PMOS $= 29.6K\Omega$ NMOS $R_n = 6.47K\Omega$, $C_1 = 0.89fF$

Load $3C_1$ $C_L = 3 \times 0.89$

$$R_{eff} = \frac{t_s}{C_L \times \ln\left(\frac{V_{out}}{V_{dd}}\right)} = \frac{20 \times 10^{-12}}{0.89 \times 3 \times \ln\left(\frac{1}{2}\right) \times 10^{-15}} = 10.81 \times 10^3 \Omega \quad \boxed{10.81\ K\Omega}$$

As load capacitance increases the effective resistance decreases.

Load $4C_1$ $C_L = 4 \times 0.89$ $$= \frac{20 \times 10^{-12}}{0.89 \times 4 \times \ln\left(\frac{1}{2}\right) \times 10^{-15}} = 8.11 \times 10^3 \Omega \quad \boxed{8.11\ K\Omega}$$

Load $5C_1$ $C_L = 5 \times 0.89$ $$= \frac{20 \times 10^{-12}}{0.89 \times 5 \times \ln\left(\frac{1}{2}\right) \times 10^{-15}} = 6.48 \times 10^3 \Omega \quad \boxed{6.48\ K\Omega}$$

Question 6: Compute transition times for a four input NAND gate with 8/2 pulldown

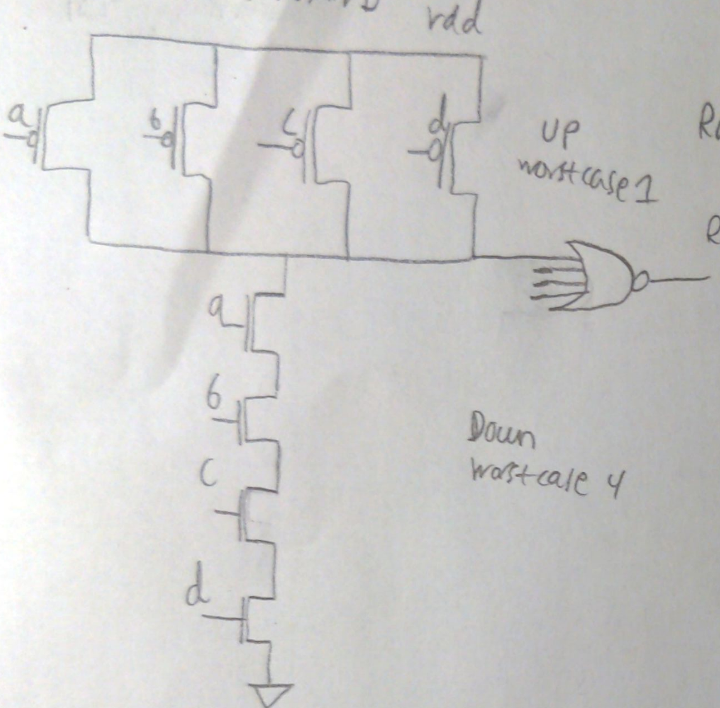
$R_n = 6.47k\Omega$ $\frac{W}{L} = \frac{8}{2}$ for n-type. $\frac{8}{2}$ pullup that drives NOR four input

$R_p = 29.6k\Omega$

fall time $t_f = 2.3 \times (R_{down} + R_L) C_L$

rising time $t_r = 2.3 \times (R_{up} + R_L) C_L$

Four input NAND



$R_{down} = 4 \times \frac{L_n}{W_n} \times R_n = 4 \times \frac{2}{8} \times 6.47k\Omega = 6.47k\Omega$

$R_{up} = 1 \times \frac{L_n}{W_n} \times R_p = 1 \times \frac{2}{8} \times 29.6k\Omega = 7.4k\Omega$

Gate capacitance

$C_L = \sum_{i=0}^{N} \frac{W_i}{L_i} C_L$

$C_L = \frac{8}{2} C_L + \frac{8}{2} C_L$

$= 4C_L + 4C_L = 8C_L = 8 \times 0.89fF$

$R_L = 0$

$= 7.12 \times 10^{-15} F$

$t_f = 2.3 \times 6.47 \times 10^3 \times 7.12 \times 10^{-15}$

$= 1.06 \times 10^{-10} = \boxed{106ps}$

$t_r = 2.3 \times 7.4 \times 10^3 \times 7.12 \times 10^{-15}$

$= 1.21 \times 10^{-10} = \boxed{121ps}$

Question 7 Compute rise time for a two-input NAND gate with 8/2 pulldown and 8/2 pullup that drives these wires (assume that the wire impedance is modeled as a single lump)

$\lambda = 90nm = 0.09um$

Poly resistance : $8 \frac{\Omega}{\square}$

Metal1 resistance: $0.08 \frac{\Omega}{\square}$

a) Poly wire with width 3λ, length 300λ

$R_L = \frac{L}{W} \times \frac{r_{poly}}{\square} = \frac{300\lambda}{3\lambda} \times 8 \frac{\Omega}{\square} = 800\,\Omega$

$R_{up} = \frac{L_P}{W_P} \times R_P = \frac{2}{8} \times 29.6K\Omega$

$= 7.4K\Omega$

$C_r = C_{plate} + C_{fringe}$

$A_C \times \underbrace{C_{plate-unit}}_{63 \times 10^{-18}} + P_C \times \underbrace{C_{finge-unit}}_{63\times10^{-18}}$ plug in $\lambda = 0.09um$

$= 3\lambda \times 300\lambda \times 63\times10^{-18} + (3\lambda + 300\lambda) \times 2 \times 63\times10^{-18}$

$= 4.59 \times 10^{-16} + 3.44 \times 10^{-15}$

$= \boxed{3.899 \times 10^{-15} F}$

$t_r = 2.3 \times (7.4\times10^3 + 0.8\times10^3) \times 3.899 \times 10^{-15} F$

$= 7.35 \times 10^{-11} s = \boxed{73.5 ps}$

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b) Metal 1 wire of width 4λ length 600λ

$R_1 = \frac{L}{W} \times \frac{r_{metal1}}{\square} = \frac{600\lambda}{4\lambda} \times 0.08 \frac{\Omega}{\square} = 12\,\Omega$

$C_r = C_{plate} + C_{fringe}$

$A_C \times \underbrace{C_{plate-unit}}_{36\times10^{-18}} + P_C \times \underbrace{C_{fring-unit}}_{54\times10^{-18}}$

$= 4\lambda \times 600\lambda \times 36\times10^{-18} + (4\lambda + 600\lambda) \times 2 \times 54\times10^{-18}$

$= 6.998 \times 10^{-16} + 5.87 \times 10^{-15}$

$= 6.5698 \times 10^{-15}$

$R_{up} = \frac{L_P}{W_P} \times R_P = \frac{2}{8} \times 29.6k\Omega$

$= 7.4k\Omega$

$t_r = 2.3 \times (7.4\times10^3 + 12) \times 6.5698 \times 10^{-15}$

$= 1.119 \times 10^{-10} = \boxed{111.9 ps}$

c) Metal2 wire of width with 4λ, length 1200λ

$R_L = \frac{L}{W} \times \frac{r_{metal2}}{\square} = \frac{1200\lambda}{4\lambda} \times 0.08\frac{\Omega}{\square} = 24\,\Omega$

$R_{UP} = \frac{L_P}{W_P} \times R_P = \frac{2}{8} \times 29.6\,k\Omega$

$= 7.4\,k\Omega$

$C_L = C_{plate} + C_{fringe}$

$A_C \times \frac{C_{plate\text{-}unit}}{36 \times 10^{-18}} + \frac{P_C \times f_{fringe\text{-}unit}}{51 \times 10^{-18}}$

Plugin λ

$\lambda = 0.09$

$= 4\lambda \times 1200\lambda \times 36 \times 10^{-18} + (4\lambda + 1200\lambda) \times 2 \times 51 \times 10^{-18}$

$= 1.4 \times 10^{-15} + 1.105 \times 10^{-14}$

$= 1.245 \times 10^{-14}$

$t_r = 2.3 \times (7.4 \times 10^3 + 24) \times 1.245 \times 10^{-14}$

$= 2.126 \times 10^{-10} = \boxed{212.6\,ps}$

Question 8 Metal 1 wire $R_{int} = 500\Omega$ $C_{int} = 200fF$

$R_0 = 6.47k\Omega$ $C_0 = 1.78fF$

a) If no additional buffer is inserted, then what is 50% delay?

$$\tau_{50\%} = K\left[0.7R_0\left(\frac{C_{int}}{K} + C_0\right) + \frac{R_{int}}{K}\left(0.4\frac{C_{int}}{K} + 0.7C_0\right)\right]$$

$$= 1\left[0.7 \times 6.47\times10^3 \times \left(\frac{200\times10^{-15}}{1} + 1.78\times10^{-15}\right) + \frac{500}{1}\left(0.4\times\frac{200\times10^{-15}}{1} + 0.7\times1.78\times10^{-15}\right)\right]$$

$$= 0.7\times6.47\times10^3 \times 2.0178\times10^{-13}$$

$$9.1386162\times10^{-10} + 4.0623\times10^{-11}$$

$$= 9.545\times10^{-10} = \boxed{954.5\ ps}$$

b) One buffer is inserted!

$$\tau_{50\%} = 2\left[0.7\times6.47\times10^3\times\left(\frac{200\times10^{-15}}{2} + 1.78\times10^{-15}\right) + \frac{500}{2}\left(0.4\times\frac{200\times10^{-15}}{2} + 0.7\times1.78\times10^{-15}\right)\right]$$

$$= 2\left[0.7\times6.47\times10^3 \times 1.0178\times10^{-13} + 1.03115\times10^{-11}\right]$$

$$2\left[4.6096162\times10^{-10} + 1.03115\times10^{-11}\right]$$

$$2\left[4.713\times10^{-10}\right]$$

$$= 9.425\times10^{-10} = \boxed{942.5\ ps}$$

c) $K=2 \quad h=2$ 3.235

$$T_{50\%} = K\left[0.7\frac{R_o}{h}\left(\frac{C_{int}}{K}+hC_o\right)+\frac{R_{int}}{K}\left(0.4\frac{C_{int}}{K}+0.7hC_o\right)\right]$$

$$2\left[0.7\times\frac{6.47\times10^3}{2}\left(\frac{200\times10^{-15}}{2}+2\times1.78\times10^{-15}\right)+\frac{500}{2}\left(0.4\times\frac{200\times10^{-15}}{2}+0.7\times2\times1.78\times10^{-15}\right)\right]$$

$$0.7\times\frac{6.47\times10^3}{2}\left(1.0356\times10^{-13}\right)+1.0623\times10^{-11}$$

$$2.3451162\times10^{-10}+1.0623\times10^{-11}$$

$$=2.4513\times10^{-10}=\boxed{245.1\,ps}$$

d) Optimal number of buffers, buffer size, and minimum 50% delay?

$$K=\sqrt{\frac{0.4R_{int}C_{int}}{0.7R_oC_o}}$$

$$K=\sqrt{\frac{0.4\times500\times200\times10^{-15}}{0.7\times6.47\times10^3\times1.78\times10^{-15}}}$$

$$K=\sqrt{4.967}$$

$$\boxed{=2.227}$$ Buffers

$$h=\sqrt{\frac{R_oC_{int}}{R_{int}C_o}}$$

$$h=\sqrt{\frac{6.47\times10^3\times200\times10^{-15}}{500\times1.78\times10^{-15}}}$$

$$\sqrt{1453.93}$$

$$\boxed{h=38.1}$$ Buffer size

Minimum 50% delay

$$T_{50\%}=2.5\sqrt{R_oC_oR_{int}C_{int}}$$

$$2.5\sqrt{6.47\times10^3\times1.78\times10^{-15}\times500\times200\times10^{-15}}$$

$$\sqrt{1.15166\times10^{-21}}$$

$$=2.5\cdot3.3936\times10^{-11}$$

$$=8.484\times10^{-11}=\boxed{84.8\,ps}$$