Assignment 1: Interplanetary Explorer Mission

2018-2019

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Group 2

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Contents

1 Introduction

In this assignment a feasibility study for an interplanetary mission will be presented. The mission consists of the preliminary mission analysis of the orbit transfers between the planets Neptune, Uranus and Jupiter. The mission starts from the departure planet Neptune and, through a gravity assist manoeuvre at Uranus, it will finally end with the arrival at Jupiter.
The mission analysis will be performed using the patched conics method: this implies a two body problem to be solved for each transfer without any perturbation. Ulterior approximations regarding the planets will be utilized, inasmuch as: they will be considered as having no atmospheres and no natural satellites or rings.

2 Design process

2.1 Initial choice for the time windows

Our goal is to design this kind of mission searching for the best possible way to do it in terms of ΔV and ΔT. This problem has 3 degrees of freedom: once we have defined when to start the mission, when to perform the flyby and when to arrive, we have fully defined the mission, both in terms of velocity vectors and times. It can be done by using the departure time and the time elapsed between the manoeuvre or, as we did, using the dates when they are performed.
First of all, a few considerations about our mission can be done, just knowing the orbit of the planets: Neptune and Uranus are the furthest planets of our solar system, they have periods of 164.8 and 84.02 years respectively. This fact determines an extremely high synodic period between the two planets and does not allow us to consider it all for our analysis. So we need to set a maximum time for the windows beyond which we cannot go, for all our degrees of freedom: we take vectors of 60 years of time length.

- Departure: 01/01/2020 to 01/01/2080
- Flyby manoeuvre: 01/01/2040 to 01/01/2100
- Arrival: 01/01/2060 to 01/01/2120

2.2 Mission constraints

Clearly, the great distances that our spacecraft has to cover, leads to long period transfer orbits. This, together with our intention of completing the mission within a lifetime period, forces us to choose a proper mission sequence that combines an affordable ΔV with a suitable ΔT.
To do so, we state that both the time of flights (between departure and flyby time and from flyby to arrival) in the genetic algorithm, have to be less than 35 years. This value is a very little fraction of the synodic period between our planets, so it could seem too short, but, as life of satellites and spacecrafts doesn't extend more than some decades, we are still stretching a bit technological limits (this constraint will be added later in the code). We have to keep into account that in the initial and final conditions the spacecraft is at the edge of the sphere of influence (SOI) of the respective planets. It means the body is far enough from the planet that it doesn't feel its gravitational attraction, but it's not

far enough to have a relative velocity (with respect to the planet) greater than zero. This means that the initial and final ΔV coincide with the excess velocities.

2.3 Mission assumptions

The mission will be studied using the patched conics method. It means we can consider 3 different legs, with respect to different focuses, that have to be linked with each other in terms of velocity and position to create the mission trajectory.
As we start and finish at the edges of the SOI of Neptune and Jupiter, we can just study the interplanetary leg around the Sun. It's important to remark that, in the method of patched conics, the SOI of a planet is considered as a point when we are orbiting around the Sun, while it has a finite radius when we are orbiting around the planet. We need to assume the planet is still during the flyby, so it's duration is small with respect to the total time.
We do not take into account the atmosphere of the planets nor any natural satellites or rings.
Constraints over the time have to be considered: first of all we cannot have a negative time of transfers; it is also important to notice that, as we do not escape from the solar system, transfer orbits can only be elliptical. In terms of code, it means that transfer times have to be grater than the parabolic time computed by Lambert's solver [1].
Is important to check that when we compute the hyperbola its radius of perigee is bigger than the one of the planet, otherwise we crush.
We also assume that the maximum velocity that spacecraft can provide us is equal to 40 km/s. We know it is a high value, but we only do it to have pork chop plots that are appreciable: we define the ΔV matrix a priori as equal to NaN and, only when all constraints are respected, the elements will become numbers; if we put a too low value of maximum ΔV the plot would be very poor. Of course, for the final choice we will have a much lower value to be given, so it is an assumption that does not create any kind of problem.
A preliminary analysis highlights the non-viability of a multi-revolution mission, due to high values of semi-major axis. Moreover, since the the planets have a counterclockwise motion, a clockwise motion trajectory would require a too high value of ΔV. (we computed ΔV for this maneuver that are 4 times higher than the best value).

2.4 Strategy

Preliminary considerations can be done considering an Hohmann [2] transfer: if we assume all the orbits circular, coplanar and with same orientation of the perigee, we can compute the ΔV needed to cover all the mission. It is around 1.5 km/s and 2.5 km/s depending on the choice of the radius of perigee of the flyby hyperbola. Of course, as we did very strong assumptions over the orbit shapes, we only take this value to have an idea of the order of magnitude of the target values.
Following the preliminary analysis we can expect that the interplanetary trajectory of interest will be a counterclockwise, with zero-revolution and an arc of ellipse.
what we have to decide is how to tackle the problem now: due to the dimension of temporal windows for our solutions, the nested-loop approach would request a number of points for the discretization of the problem too high to be performed on a personal computer in a reasonable amount of time. Following this limitation it has been decided to exploit

the two different approaches for two different aspect of the problem: the generation of pork-chop plots and the search for the optimum overall ΔV requested.

2.4.1 Nested-loop approach

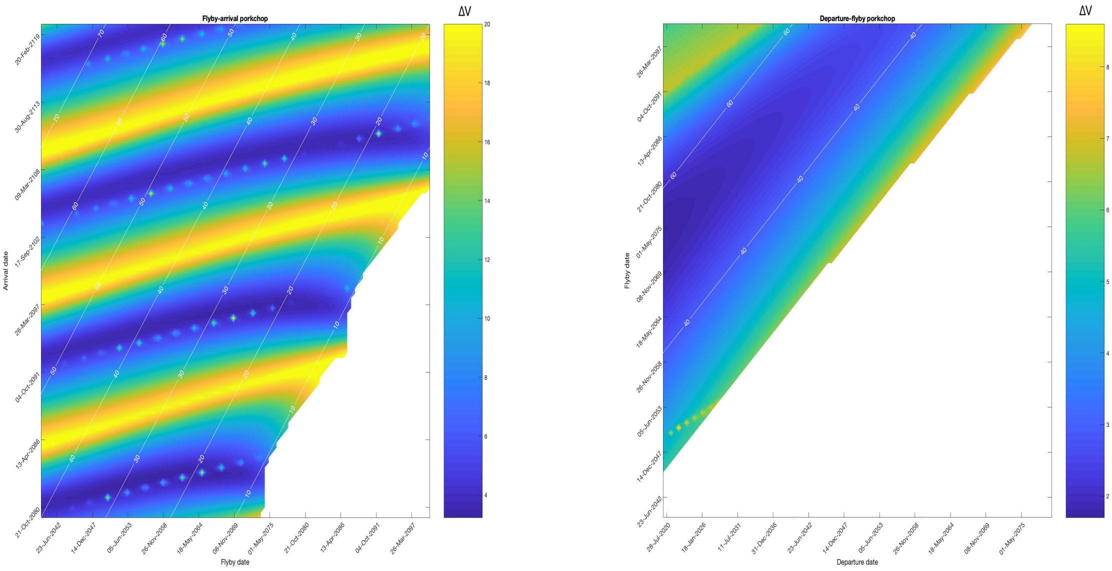
The easiest way to explore the possible solution, is to perform a triple loop over the 3 degrees of freedom: having the function "uplanet" [1], we know, at every instant of time, where all the planets of solar systems are; so we divide each time interval into an N value of discrete numbers; we take the first point in the vector of the departure dates and we evaluate the transfer arc between the linked position on the first orbit with all the possible points on the second one. In the mean time we do the same for the flyby vectors and arrival one, and we nested this two loop, in order to create a triple loop that scans all the combination between the 3 orbits over the time period under examination.
Despite this method results to be the easiest to implement, it is also very heavy from a computational point of view: if we divide the time vectors into 10 points, it will originate a 10^3 iteration to be computed. It means that, to find the absolute minimum we need a very high computational power that we do not have.

2.4.2 Optimization problem approach

To avoid the nested-loop problem we found useful a semi-analytic approach, that consists in the use of an optimization method as a genetic algorithm: it works without any kind of loop and its capable to find values of ΔV that represent local minimums of the analyzed function. To be sure that the value it gives us is not a local minimum, we perform a loop that launches it more than once.
Using this method and comparing it with the result we derived from the triple loop, the results obtained should be similar. If we run the algorithm number of times that tends to infinity, we have that it finds the absolute minimum.
It has to be noticed that in such a problem, with several decades of time of flight, also the time itself should be minimized. To perform this deeper analysis we have exploited a multi-objective optimization (Pareto optimization [3]): it provides the best combination of ΔV and time of flight. Both results (genetic algorithm and Pareto front) will be compared in order to see how far the minimum ΔV solution is from the curve of optimal solutions provided by the Pareto front.

2.4.3 Search of the solution

To look for the solution we created 3 pork chop plots: the first one, Figure 1 (b) showed in represents all the possible transfer from Neptune to Uranus (without keeping into account the ΔV of the flyby), the second one, Figure 1 (a), is like the previous one, but between Uranus and Jupiter; these two plots can give us the idea of where the final solution could be.



(a) flyby-arrival (b) departure-flyby

Figure 1: Pork-chop

The third one, showed in Figure 2, is from the departure time to the arrival one and it keeps into account the ΔV of flyby. In particular we created a 3 dimensional matrix (that has all the possible delta velocities), where we have a vector of possible ΔV of flyby, once we chose an initial and final date. To make the matrix two dimensional, we associated each choice of departure and arrival date, with the best value of the remaining flyby vector.

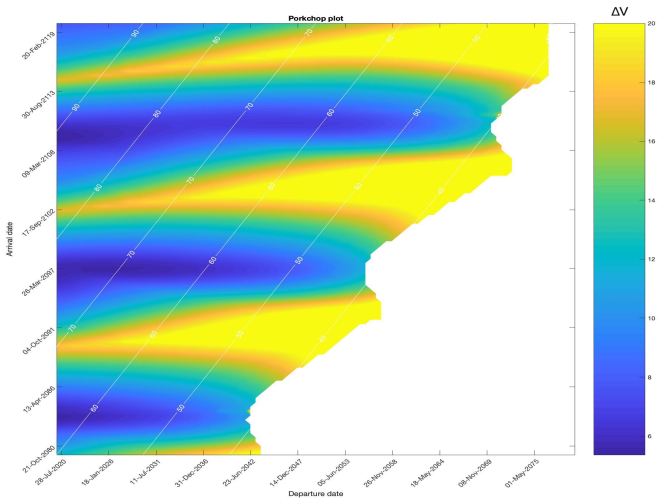


Figure 2: Pork-chop: departure-arrival

From these graphs we can notice that our starting and flyby planets are in an advantageous position at the beginning of our launch window. From Figure 1 (a), we see that there are some belts that have low values of ΔV. Considering both these comments, we will probably find easily the optimal launch time.

From the triple loop we have a $\Delta V = 5.3652 \ \ km/s$ and the departure date, as we expected, is in 2020.
After that, we used the optimization method to solve the constraint optimization problem, finding the result that satisfies all the constraints we imposed. The genetic algorithm found a transfer arc characterized by a $\Delta V = 7.3324 \ \ km/s$. We can explain the differences in the velocity terms reminding that in the loop we took a higher constraint on the transfer time, and a lower one in the optimization method. However we are still not sure that the genetic algorithm has managed to find the best value. It is possible to compare the last value with those taken from a multi-object optimization function, where we took into account also the time as a decision parameter. So far we have looked for the mission associated with the minimum ΔV. From now on we will look for a compromise solution, to find another mission which finishes sooner, even at the cost of rising a little the ΔV cost.

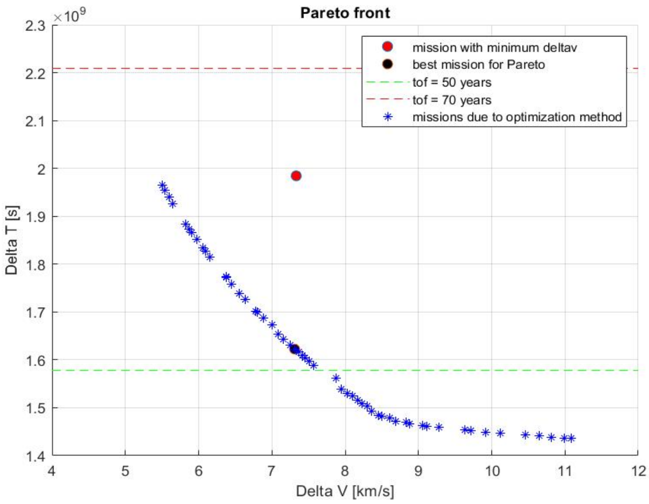


Figure 3: Pareto front

From Figure 3, we can notice that the value found by the genetic algorithm does not represent an absolute minimum, as the Pareto algorithm does. This curve shows a locus of points representing the missions, that minimize both ΔV and ΔT. The choice between these points is arbitrary, it's up to us to choose one. We decided to take the mission represented by the point on the graph that has the minimum difference, in terms of ΔV, from the value given by the genetic algorithm. The total minimum $\Delta V = 7.2419[km/s]$

3 Final solution

After we have found the final values of the two objective functions, we can determine the transfer arcs and the related dates.

3.1 Heliocentric Trajectory

In Figure 4 is shown the transfer arc related our final decision. The transfer time of the final decision mission is around 50 years.

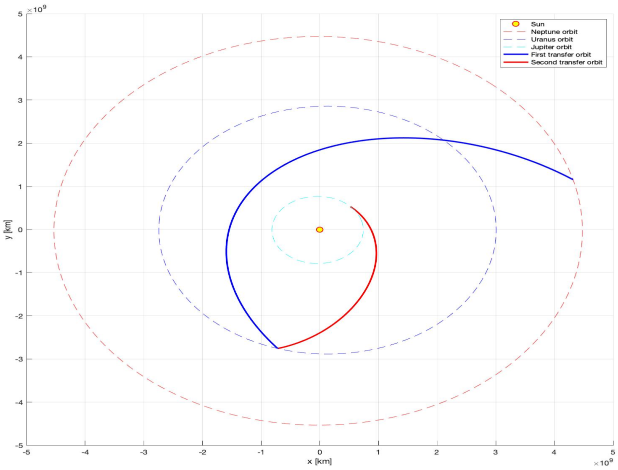


Figure 4: Transfer orbit

Date vectors

Manoeuvre	Year	Month	Day
departure	2032	01	01
flyby	2067	11	12
arrival	2083	06	01

3.2 Flyby: powered gravity assist manoeuvre

The hyperbolic trajectory will have the following distance from the surface of the planet and time to cover the hyperbola.

- $h = 8044.4km$
- ΔT=0.792 years

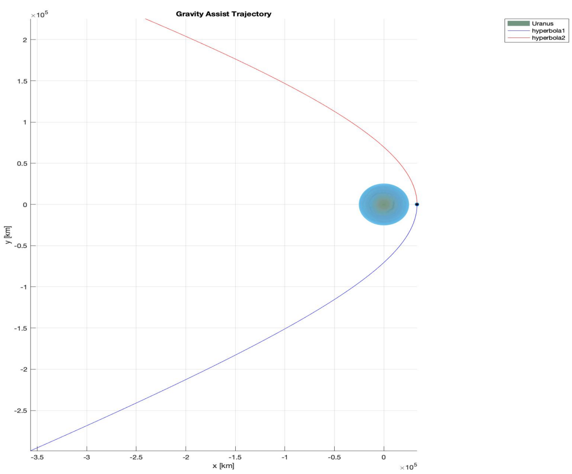


Figure 5: Hyperbola for the transfer orbit

3.3 Cost of the mission

The total cost of the mission is given by the sum (in absolute value) of the impulses the spacecraft is required to perform. A priori we cannot say if the flyby will be powered or not powered, so, in the luckiest case, we could have just to maneuver. The answer is given by Figure 5: it's shown that we have incoming arc different than the outgoing one, so we expect a powered flyby.

Delta Velocities

ΔV	$[km/s]$
departure	3.555
flyby	3.566
arrival	-0.12

The sign of the last ΔV means that we have to reduce the spacecraft velocity to follow the desired trajectory, in other words, to stay in Jupiter's orbit.

4 Conclusions

At the end of our analysis we can say that, without a very high computational power, it is better to look for our solution using optimization methods and using the nested-loop just to have an idea of the possible results. Considering a two objective optimization and without using the genetic algorithm (ga) too many times, we found a solution that is 0.1 km/s cheaper than the one found by ga and saving more than 10 years to finish the mission. Observing the sign of the ΔV of flyby, we notice that we are performing a leading-side flyby: the delta velocity vector has a negative component along the velocity vector of the planet. The last consideration is about the time needed to cover the flyby hyperbola: at the beginning we assumed that the SOI of Uranus is negligible with respect to the one of the Sun, and that the time to cover a flyby hyperbola is negligible. But, we probably should keep into account this time, as it's close to be 0.02* ΔT and Uranus has a SOI with radius equal to $5.1796 * 10^7 km$.

References

[1] Politecnico di Milano. *Orbital Mechanics course*. 2018.

[2] Howard D.Curtis. *Orbital Mechanics for Engineering Students*. 2015.

[3] MATLAB. *Mathwork documentation.*

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Contents

1 Introduction

In this assignment a planetary mission analysis will be presented. With the Earth observation as context of the mission, this assignment will discuss satellite ground track estimations, orbit analysis and perturbation propagation.
In this study, two different orbit perturbations will be considered, namely the J_2 effect and the solar radiation pressure perturbation. The evolution of the resulting orbit will be propagated in Cartesian coordinates and in Keplerian elements (obtained through Gauss planetary equations).

1.1 J_2 effect

The J2 perturbation effect is due to the fact that Earth is not a perfect sphere, but an oblate spheroid. This lack of symmetry affects the direction of the force of gravity with which an object is attracted to Earth.
Instead of being attracted to the center of the planet, there is a perturbation dependant on the latitude affecting this gravity pull. Thus, the parameter that measures this perturbation is J_2, and it changes for each planet.
The J_2 Earth value that we will use is $J_2 = 1.08263 \times 10^{-3}$ [1]

1.2 Solar Radiation Pressure

The solar radiation pressure is a phenomenon linked to the Sun and the photons it emits. Photons, despite their mass and size, have an important energy and momentum that can perturb the orbit of an Earth's satellite.
The total pressure that direct solar radiation can exert is $P_{SR} = 4.56 \times 10^{-6}[\frac{N}{m^2}]$[1]. In terms of acceleration (for Earth orbit, taken from [2]):

$$\mathbf{a}_{SRP} = -a_{SRP}\frac{\mathbf{r}_{sc-sun}}{||\mathbf{r}_{sc-sun}||} \tag{1}$$

$$a_{SRP} = p_{SR@1AU}\frac{AU^2}{||\mathbf{r}_{sc-sun}||^2}c_r\frac{A_{sun}}{m}\nu \tag{2}$$

where:

- c_r is the radiation pressure coefficient, dependent upon the optical properties of the satellite. It varies from 1 (black body that absorbs all the photons momentum) to 2 (all radiation is reflected, doubling the force). For this study $c_r = 1$.
- A_{sun}/m is the area to mass ratio relative to the Sun ($A_{sun}/m = 0.1[m^2/kg]$).
- ν is the binary shadow function ($0 < \nu < 1$).

Obviously, solar pressure only affects the satellite when it is in direct line of sight with the Sun ($\nu = 1$) The shadow function is taken from ([3]), and uses the hypothesis of sun rays parallel to the orbital plane. Thus, whenever Earth is in between the Sun and the satellite this phenomenon has no effect and should not be taken into account ($\nu = 0$).
Lastly, it is worth to note that it is not a particularly big perturbation when compared to others, for example atmospheric drag. However, whenever these other perturbations diminish (in the context of the previous example, at higher altitudes) and considering longer periods of time, solar radiation pressure acquires importance.

2 Mission analysis

In this section the mission will be properly explained and analyzed.
For the first part of the assignment the SRP perturbation will be ignored, so the ground tracks and the satellite orbit will be presented only for the J2 perturbation.
In order to study the effects of the perturbations, two cases are going to be compared, an orbit propagation including the J_2 effect and another one without any perturbation. The time evolution of the ground tracks will be presented for three time ranges, one orbit period, one day and ten days.
In addition will be proposed a solution for the repeating ground tracks, with the constraint of modifying the semi-major axis as little as possible.
After this preliminary analysis of the orbit and its characteristics, we will add to our model the additional effect of solar radiation pressure. Then we will study two different approaches to include the perturbation: at first a Cartesian coordinates approach (which integrate the state vectors through the two body problem) and then a Keplerian elements approach through the numerical integration of Gauss planetary equations.
After this step, the solar pressure perturbation effects, combined with the J2 effect, on the satellite orbit will be discussed through the analysis of the time evolution of the orbital elements, supported by a spectral analysis (frequency domain). At last, the results will be collated by comparing them with real satellite data.

2.1 Satellite orbit

The satellite orbit is defined by the following orbital elements:

Given orbit data

a [km]	39062	Semi-major axis
e [–]	0.3537	Eccentricity
i [rad]	1.2400	Inclination

As a degree of freedom we had to choose RAAN, true anomaly and argument of perigee:

Chosen orbital elements

Ω [deg]	270.00	RAAN
ω [deg]	45.000	Argument of perigee
θ [deg]	0.000	True anomaly

The main characteristic of this orbit is its semi-major axis. This value implies an orbital period of $T \approx 76832[s]$, which is not that distant from a geostationary orbit. In fact our satellite does circa 1.12 revolution around the Earth each day.
This allowed us in the last part of this assignment to compare our results with one of the many GEOs satellites that currently orbit around Earth.
With this basic knowledge we can easily imply that a real satellite in this regions is affected by SRP, J2 and lunar gravity perturbation. Also, from the given inclination we can expect a Westward rotation of the nodes.

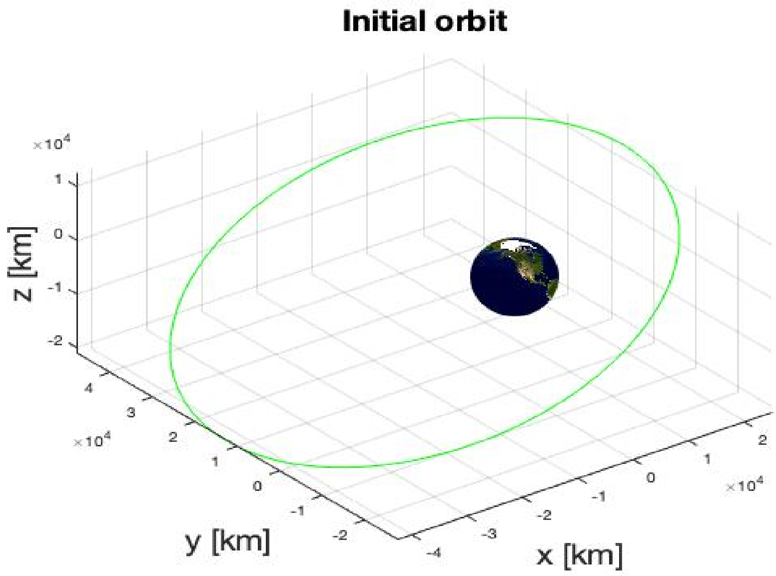


Figure 1: Initial satellite orbit around Earth

2.2 Ground track

In the following figures (2 3 4) are depicted the ground tracks of the satellite over the Earth for different periods of time.
The images offer also a comparison of the ground tracks with and without taking into account the J_2 effect. Even though they may look the same on the graph, they have different numerical values, but the difference between them for the orbit and the periods of our analysis is small enough to not be distinguishable.
In fact, calculating the secular effect of J_2 on the RAAN (nodal regression), the argument of perigee (perigee precession) and the true anomaly for this particular case yields:

$$\dot{\Omega}_{sec} = -\frac{3nR_E^2 J_2}{2p^2}\cos i = -4.135449452808462 \times 10^{-9}[\frac{rad}{s}] \tag{3}$$

$$\dot{\omega}_{sec} = \frac{3nR_E^2 J_2}{4p^2}(4 - 5\sin^2 i) = 6.955991849008000 \times 10^{-9}[\frac{rad}{s}] \tag{4}$$

$$\dot{M}_{sec} = -\frac{3nR_E^2 J_2\sqrt{1-e^2}}{4p^2}(3\sin^2 i - 2) = 2.603215850985405 \times 10^{-18}[\frac{rad}{s}] \tag{5}$$

It is important to notice that these are really small values, supporting the fact that is difficult to appreciate the difference in the figures for the orbit and the time intervals of interest.

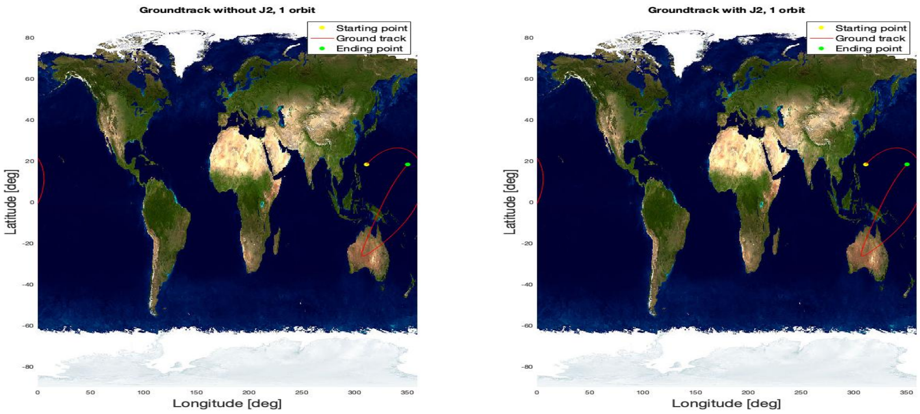


Figure 2: Satellite ground track for 1 orbit. The left one does not account the J2 .

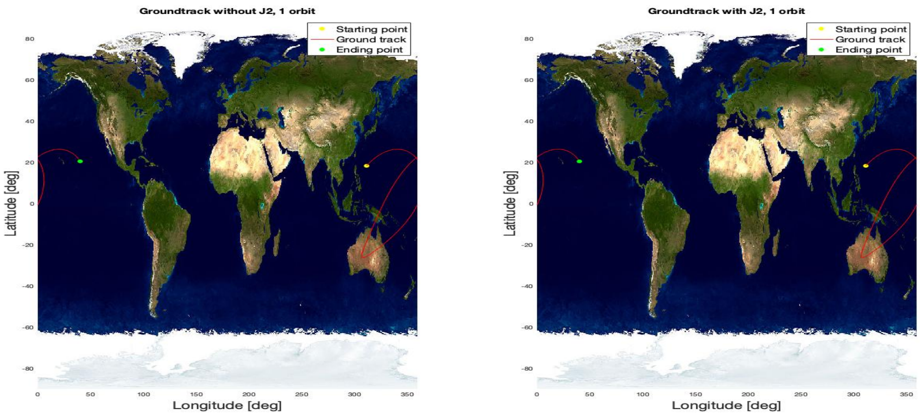


Figure 3: Satellite ground track for 1 day.The left one does not account the J2.

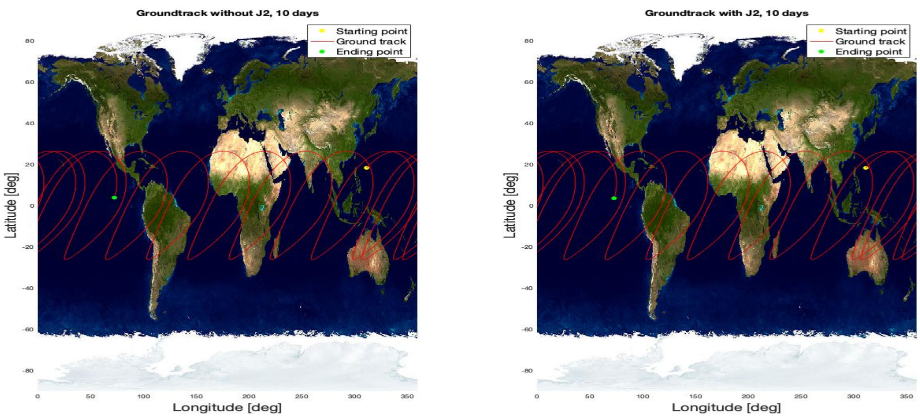


Figure 4: Satellite ground track for 10 days. The left one does not account the J2.

2.3 Repeating ground track

In order to improve the communications between the satellite and the ground stations a repeating ground track design will be presented.
To do so, it is necessary to modify the orbit period of the satellite, which intrinsically means changing its semi-major axis.
For the original orbit we can determine the number of revolution before the repetition, modifying a as little as possible as a constraint means that we should take the lowest integer of **n** (number of revolutions):

$$T = \frac{2\pi}{n\omega_E} \tag{6}$$

From this equation we can determine the new period and then the new semi-major axis, for which we can plot the ground tracks. Accounting the J2 effect adds a new level of difficulty to the problem, the regression of Ω can be seen as an acceleration (deceleration for retrograde orbits) of Earth's rotation. We insert equation 3 into equation 6 to get the non linear equation to solve for **a**:

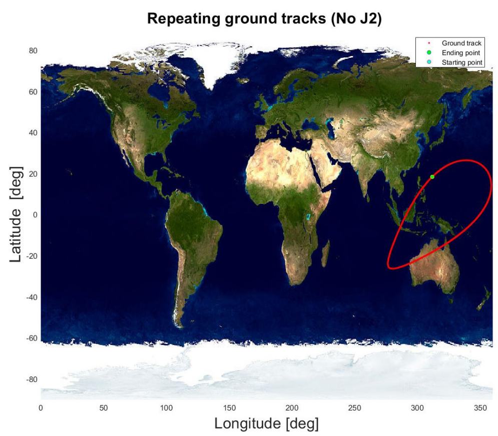
$$n = \frac{2\pi}{T(\omega_E - \dot{\Omega})} \tag{7}$$

$$\frac{2\pi}{(2\pi\sqrt{a^3/\mu})(\omega_E + \beta)\cos i} - n = 0 \tag{8}$$

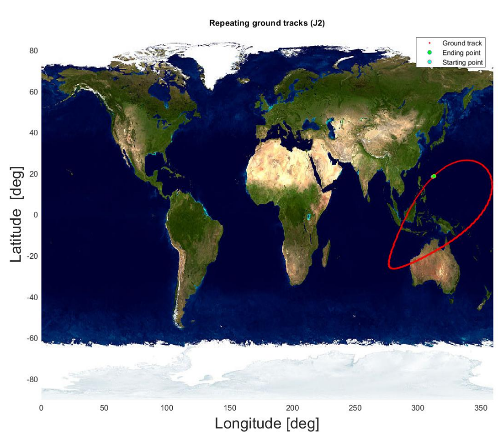
where

$$\beta = \frac{3}{2}\frac{\sqrt{\mu}J_2 R_e^2}{a^{7/2}(1-e^2)^2} \tag{9}$$

This equation was solved with a numerical algorithm (fsolve in MATLAB®), and the ground tracks repeated successfully.
As shown in the image, the repeating ground tracks are similar to some real satellites, because our orbit does not differ much from a GSO. As shown in the figure, the ground tracks do not repeat correctly, over longer periods, because we account only for $\dot{\Omega}$ and not for $\dot{\omega}$. Accounting for that effect, and for the geodetic latitude would give more accurate results.



(a) Repeating ground track for 10 days (no $J2$)



(b) Repeating ground track for 10 days ($J2$)

Figure 5: Repeating ground tracks

2.4 Orbit propagation with perturbations

2.4.1 Preliminary consideration

The spacecraft orbit propagation is computed taking into account only the effect of J_2 gravitational perturbation and the solar radiation pressure.
Due to the distance between the Sun and the Earth (ca. 1AU) the magnitude of the SRP acceleration will be much lower than the J_2 one. This will result in the J_2 effect overriding the secular variations due to the SRP perturbation as regards Ω, ω and θ.
In fact the SRP acceleration will have effect on each one of the Keplerian element of the orbit but, as said above, the secular effect will be of 10^{-4} order on a five year time window. So it will be expected a substantial variation in Ω, ω and θ values, along with a not very considerable variation for what concern a, e and i.
Knowing that the magnitude and the sign of the RAAN and argument of perigee depends on the value of inclination, it should be expected a variation in $\dot{\Omega}_{sec}$ and $\dot{\omega}_{sec}$, too.

2.4.2 Cartesian coordinates

For the first method of propagation, the effect of the perturbations are accounted for by including them in the two body problem differential equations as perturbing acceleration terms:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{a}_{J_2} + \mathbf{a}_{SRP} \tag{10}$$

Thus, solving equation 10 using the initial conditions of the satellite (r_0, v_0) will yield the orbit time evolution in terms of state vectors. We solved the equation system through MATLAB®'s solver **ode113**.

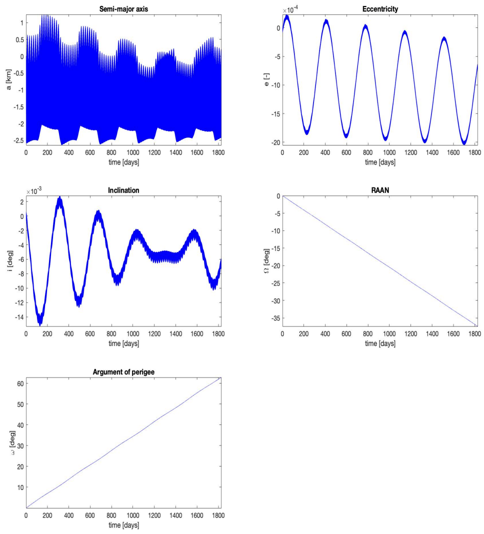
2.4.3 Keplerian elements (Gauss planetary equations)

In this case Gauss planetary equations(Battin [4] [pg.489]) will be used to calculate the effect of the perturbations of the Keplerian elements.
One of the benefits of using Gauss equations for the J_2 perturbations is that they show clearly the dependence of each elements from each component of the accelerations in the *tnh* reference frame.
In fact, regarding the Gauss equations is necessary to project the perturbing acceleration into the *tnh* reference frame, from the ECI.

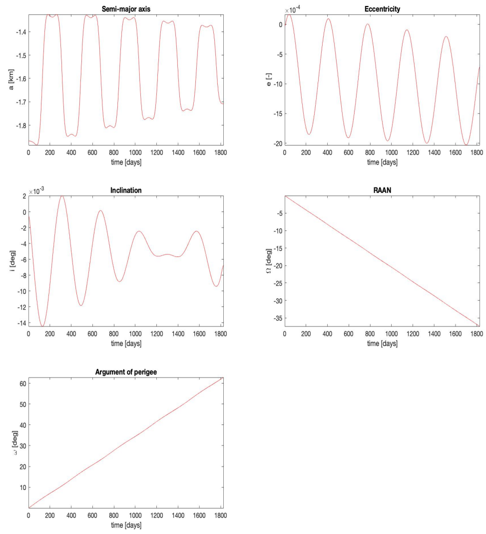
2.5 Keplerian elements evolution

As it was stated in the previous section, perturbations modify the orbits of satellites and so, the Keplerian elements change with time. Regarding the Keplerian elements, as it was briefly discussed previously, it is expected that the eccentricity increases due to the velocity changes at perigee and apogee. This is not always true, in fact as our results shows a decreasing eccentricity.
The effect of the SRP perturbation is to give a ΔV to the orbit, in particular at the perigee or the apogee. If the argument of perigee has particular values so that the velocity is directed parallel to the $\mathbf{a}_{SRP}$, the effect would be an acceleration of the satellite. The satellite spends more time in its apogee region, so the effect will be bigger in this region than in the perigee one. This, together with our value of ω, means that there will be a reduction of the eccentricity.

In our case the variation of eccentricity is of magnitude of 10^{-4} on a 5 year time span, so almost negligible. This is coherent with the compensation of the SRP effects of semi-major axis and eccentricity given by Earth revolution.
As Figure 6 shows, the considerations made above are supported.



(a) Keplerian elements evolution



(b) Keplerian elements evolution (smooth)

Figure 6: Evolution of Keplerian elements

2.6 Spectral analysis

Orbit perturbation effects can be divided in three different types: short period, long period and secular.
Short and long period variations will repeat the element variation pattern after a certain amount of time: few orbit revolutions and several days respectively. Due to their periodic behaviour these variations can be neglected in a long term analysis.
On the other hand secular effects consist in a constant variation of an element along time, so it results to be much more interesting from an orbit propagation analysis point of view. In order to obtain a clear view on the secular effect, it is necessary to filter the data from the short period oscillation.
To filter a set of data there are a lot of techniques, we decided to use two similar method. The first method uses an averaging of the values over a time windows that moves trough the data(movemean in MATLAB®).
We used the Fourier discrete transform to bring the data in the frequency domain, and we plotted in a 2-D stem plot, with logarithmic x axis in order to identify the cut-off frequency. We wanted to filter the short period(high frequency) variations in order to show the secular and the long period one, according to this we identified the time window to average the data.
The second method uses the function smoothn.m(Garcia [5], Curtis [1]), which is a function for robust smoothing of data, which worked very well autonomously finding the best smoothing parameter to eliminate short period variations. The results are shown below:

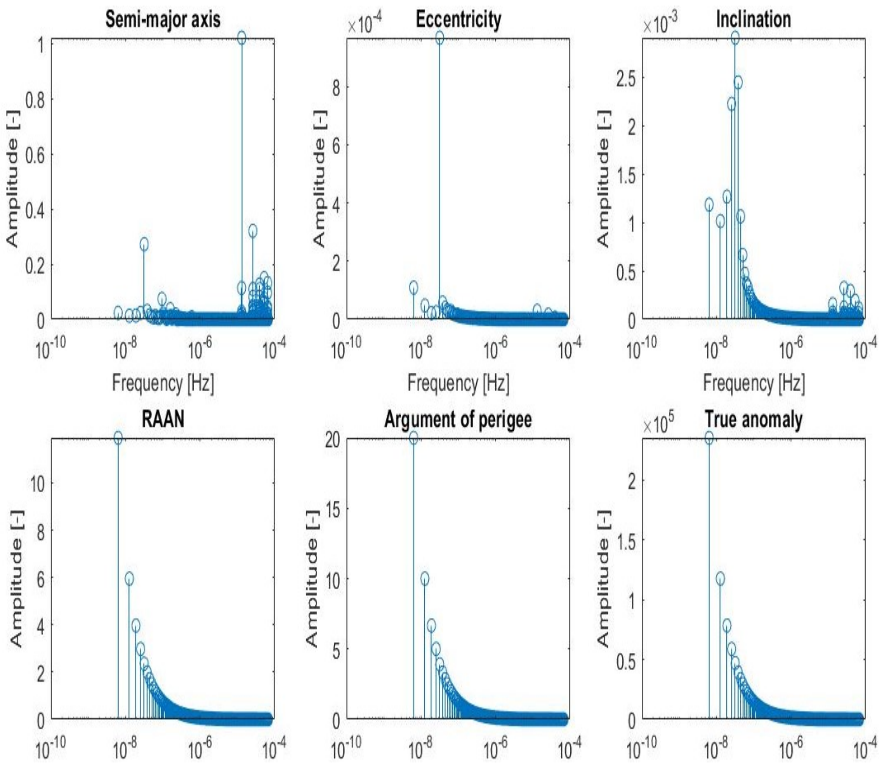


Figure 7: Fast Fourier Transform

2.7 Comparison with real data

The goal of this section is to compare the differences between the orbit propagation obtained by our model (J_2 and SRP perturbation effects) and the NASA JPL Horizon system, which provides orbital elements data for several spacecrafts. In order to obtain comparable data, a real satellite with an orbit similar to our case (and also with a significant period of retrievable information) has been chosen.
We choose the **GOES-6 satellite**, a geostationary weather monitoring satellite, with the following orbital elements for the date 01/01/2018: $a = 42180$ km, $e = 0.00031$, $i = 14.1°$, $\Omega = 348°$, $\omega = 280.5°$ We know the equations for $\dot{\Omega}_{sec}$ and $\dot{\omega}_{sec}$ (equations 3 and 4), and it results that they are both dependant from the inclination of the orbit. The difference in inclination between GOES-6 and our spacecraft orbits will be taken into account while making considerations. For this section, we plotted the real data obtained from Horizon, we propagated the orbit evolution with our propagator and we compared also with the filtered results. The figures below show the results.

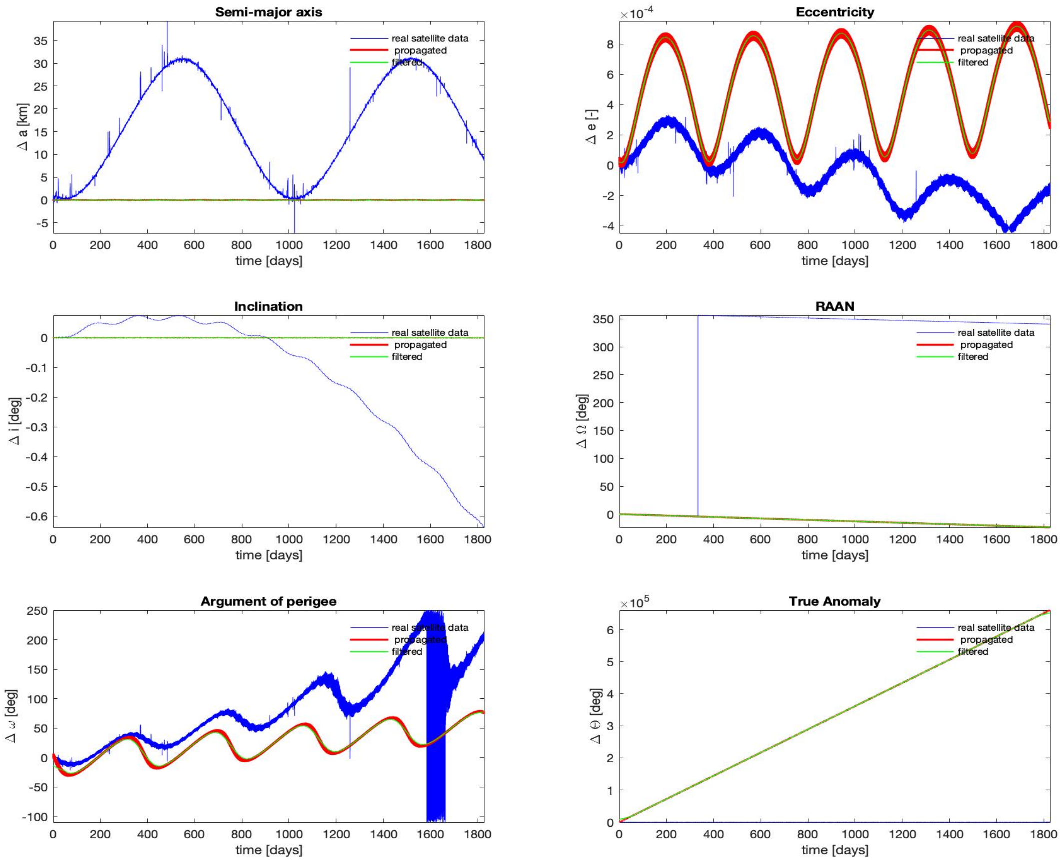


Figure 8: Real, propagated and filtered elements compared

Is it clear that there is a lot of difference for many of the orbital elements. This happens because we do not account for an accurate perturbation model, and we account only for two perturbations. For our orbital region, we should considered the lunar gravity, which would justify the inclination and eccentricity difference, and tesseral harmonics. The difference in the values of inclination will results in a faster variation of Ω and ω for the GOES-6 orbit.
The peak in the real data for the argument of perigee is probably due to orbital instability.

3 Conclusions

In this report we have analyzed the effects of different propagation methods and different perturbations models. In conclusion we can say that our model is inaccurate at a certain level because it does not account for some important effects occurring in our orbital region.

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