The last one!

Jackknife and Nonparametric Bootstrap.

- Say we have iid sample $\underline{X} = (x_1, \dots, x_n)$ from an unknown prob. distribution F on some space $\mathcal{X}$.

$$\underline{x_i} \overset{iid}{\sim} \underline{F} \quad \text{for } i = 1, \dots, n$$

$x_i \sim Poi(\mu)$, $x_i \sim Bin(10, .5)$

- We can compute $\hat{\theta}$ applying some algorithm $t(\cdot)$ to $\underline{x}$. (i.e. $\hat{\theta} = t(\underline{x})$)

- Our objective is to assign a <u>standard error</u> to $\hat{\theta}$.

<u>Case 1</u>. Estimator is simple and we can derive its variance directly.

$$V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right) = \frac{1}{n^2} V\left(\sum x_i\right) = \frac{1}{n^2} \sum V(x_i) = \cancel{\frac{n\sigma^2}{n^2}} = \frac{\sigma^2}{n}.$$

from the variance of the X_i's.

$$se(\bar{x}) = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$$

<u>Case 2</u>. MLE. By Fisher's Fundamental theorem of MLE, $V(\hat{\theta}) = [I_\theta]^{-1}$

Case 3. Bayesian Estimator. Then you have

a posterior for θ ; $g(\theta \mid x)$

get variance of this distr.

Case 4. Everything else.

For this case...

xi is removed

Let $x_{(-i)} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$

and $\hat{\theta}_{(-i)} = t(x_{(-i)})$

Definition: The Jackknife estimate of standard error is:

$$\hat{se}_{Jack} = \left[\frac{n-1}{n} \cdot \sum_{i=1}^{n} \left(\hat{\theta}_{(-i)} - \hat{\theta}_{(\cdot)} \right)^2 \right]^{1/2}$$

where $\hat{\theta}_{(\cdot)} = \sum_{i=1}^{n} \hat{\theta}_{(-i)} / n$

$\underline{x} = [170, 165, 180, 185, 172]$; $\hat{\theta} = \bar{x} = 174.4$

$\underline{x}_{(-3)} = [170, 165, 185, 172]$; $\hat{\theta}_{(-3)} = \bar{x}_{(-3)} = 173$

$\hat{\theta}_{(-1)}$

$\hat{\theta}_{(-2)}$

$\hat{\theta}_{(-3)}$

$\hat{\theta}_{(-4)}$

$\hat{\theta}_{(-5)}$

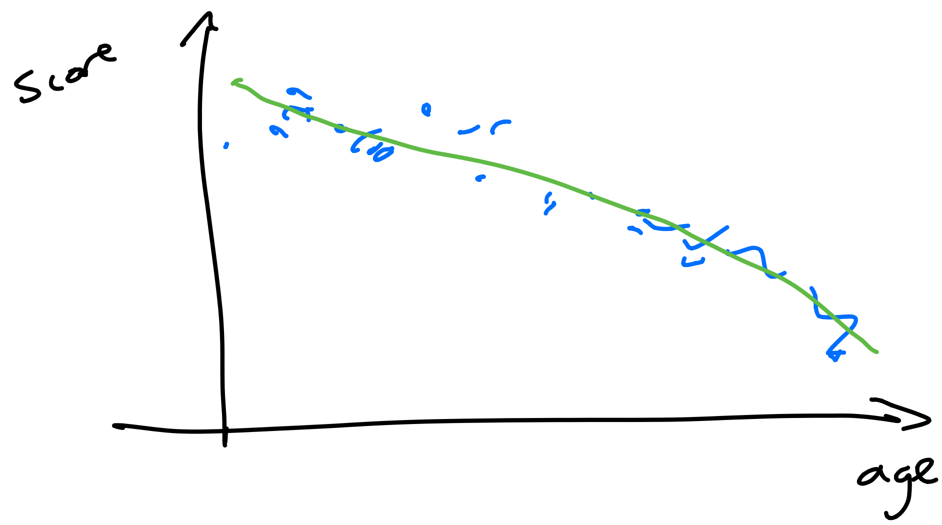
$$\hat{\theta}_{(\cdot)} = \frac{\hat{\theta}_{(-1)} + \hat{\theta}_{(-2)} + \hat{\theta}_{(-3)} + \hat{\theta}_{(-4)} + \hat{\theta}_{(-5)}}{5}$$

Example 1. Let $f(\cdot) = \bar{x}$. Then

$$\hat{\theta}_{(-i)} = \frac{n \cdot \overset{\sum x_i/n}{\bar{x}} - x_i}{(n-1)}$$

✓

Example 2. Using the lowess estimator in our <u>first ever</u> example (Chapter 1).



→ Jupyter to see Jackknife estimator.