DSolve[
$$\{y'''[x] - ((3/x)*(y''[x])) == (b*Log[x]), y''[2] == 3, y'[2] == 1/2, y[2] == 2\}, y[x], x$$
]

Outlie-

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{2886} \left( 9792 + 512 \, b - 2880 \, x - 720 \, b \, x + 80 \, b \, x^3 + 54 \, x^5 + 9 \, b \, x^5 - 1536 \, b \, Log[2] + 1440 \, b \, x \, Log[2] + 18 \, b \, x^5 \, Log[2] - 240 \, b \, x^3 \, Log[x]) \right\} \right\}$$

buttin-

(\*II\*)

DSolve[
$$\{y'''[t] - y''[t] == (b*(Exp[3*t]*Log[t])), y''[2] == 3, y'[2] == 1/2, y[2] == 2\}, y[t], t$$
]

Outlie-

$$\left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \right\} \left\{ \left\{ y[t] \rightarrow \frac{1}{18 \, 6^2} \right\} \left\{$$

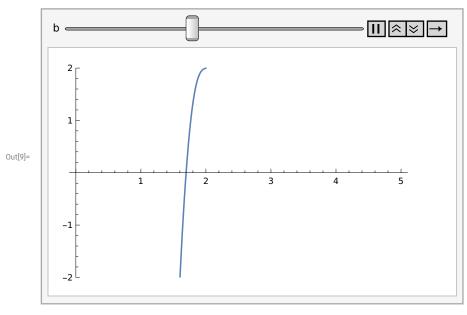
Out[8]= 
$$\left\{ \left\{ -\left( y[t] \rightarrow \frac{1}{18} \left( 72 + 2 \text{ b } e^6 + 54 \, e^{-2 + t} - 2 \text{ b } e^3 \, t - 45 \, t + 9 \text{ b } e^t \, \text{ExpIntegralEi[4]} - 8 \, \text{b ExpIntegralEi[6]} - 9 \, \text{b } e^t \, \text{ExpIntegralEi[2} \, t \right] + 2 \, \text{b} \left( 4 + 3 \, t \right) \, \text{ExpIntegralEi[3} \, t \right] - 4 \, \text{b } e^6 \, \text{Log[2]} - 9 \, \text{b } e^{4 + t} \, \text{Log[2]} + \text{b } e^6 \, t \, \text{Log[64]} + \text{b } e^{3 \, t} \, \text{Log[t]} \right) + \left( y[x] \rightarrow \frac{1}{2880} \right)$$

$$\left( 18 \left( 544 - 160 \, x + 3 \, x^5 \right) + \text{b} \left( 80 \, x^3 + 720 \, x \, (-1 + \text{Log[4]}) + 9 \, x^5 \, (1 + \text{Log[4]}) - 512 \, (-1 + \text{Log[8]}) \right) - 240 \, \text{b} \, x^3 \, \text{Log[x]} \right) \right) \right\}$$

(\*IV\*)

In[6]:= Manipulate[DSolve[
$$\{y''''[t] - y''[t] == ((b*Exp[3*t]*Log[t])), y''[2] == 3, y'[2] == 1/2, y[2] == 2\}, y[t], t], {b, 0.5, 5.5, 0.5}]$$

Plot[
$$\left\{ \left\{ \frac{1}{18} \left( 72 + 2 \, b \, e^6 + 54 \, e^{-2+t} - 2 \, b \, e^{3\,t} - 45 \, t + 9 \, b \, e^t \, \text{ExpIntegralEi[4]} - 8 \, b \, \text{ExpIntegralEi[6]} - 6 \, b \, t \, \text{ExpIntegralEi[6]} - 9 \, b \, e^t \, \text{ExpIntegralEi[2} \, t \right] + 2 \, b \, \left( 4 + 3 \, t \right) \, \text{ExpIntegralEi[3} \, t \right] - 4 \, b \, e^6 \, \text{Log[2]} - 9 \, b \, e^{4+t} \, \text{Log[2]} + b \, e^6 \, t \, \text{Log[64]} + b \, e^{3\,t} \, \text{Log[t]} \right) \right\} \right\}, \, \{t, \, 0, \, 5\}, \, \text{PlotRange} \rightarrow 2 \right], \, \{b, \, 0, \, 5\} \right]$$



(\*V\*)

$$\ln[12] = \left\{ \left\{ y \left[ t \right] \rightarrow \frac{1}{18 e^2} \right\} \right\}$$

 $\left(72\,\boldsymbol{e}^2 + 2\,b\,\boldsymbol{e}^8 + 54\,\boldsymbol{e}^t - 2\,b\,\boldsymbol{e}^{2+3\,t} - 45\,\boldsymbol{e}^2\,t + 9\,b\,\boldsymbol{e}^{2+t}\,\operatorname{ExpIntegralEi}[4] - 8\,b\,\boldsymbol{e}^2\,\operatorname{ExpIntegralEi}[6] - 6\,b\,\boldsymbol{e}^2\,t\,\operatorname{ExpIntegralEi}[6] - 9\,b\,\boldsymbol{e}^{2+t}\,\operatorname{ExpIntegralEi}[2\,t] + 8\,b\,\boldsymbol{e}^2\,\operatorname{ExpIntegralEi}[3\,t] + 6\,b\,\boldsymbol{e}^2\,t\,\operatorname{ExpIntegralEi}[3\,t] - 4\,b\,\boldsymbol{e}^8\,\operatorname{Log}[2] - 9\,b\,\boldsymbol{e}^{6+t}\,\operatorname{Log}[2] + 6\,b\,\boldsymbol{e}^8\,t\,\operatorname{Log}[2] + b\,\boldsymbol{e}^{2+3\,t}\,\operatorname{Log}[t]\right) \right\} \right\}$ 

Out[12]=

$$\left\{\left\{y[t] \rightarrow \frac{1}{18 e^2}\right\} \right\}$$

$$\left(72 e^2 + 2 b e^8 + 54 e^t - 2 b e^{2+3 t} - 45 e^2 t + 9 b e^{2+t} \text{ ExpIntegralEi}[4] - 8 b e^2 \text{ ExpIntegralEi}[6] - 6 b e^2 t \text{ ExpIntegralEi}[6] - 9 b e^{2+t} \text{ ExpIntegralEi}[2 t] + 8 b e^2 \text{ ExpIntegralEi}[3 t] + 6 b e^2 t \text{ ExpIntegralEi}[3 t] - 4 b e^8 \text{ Log}[2] - 9 b e^{6+t} \text{ Log}[2] + 6 b e^8 t \text{ Log}[2] + b e^{2+3 t} \text{ Log}[t]\right\}$$

$$\begin{aligned} &\text{In}[13] = & \text{f[t\_]} := \left\{ \left\{ \frac{1}{18 \, e^2} \right. \\ & \left( 72 \, e^2 + 2 \, b \, e^8 + 54 \, e^t - 2 \, b \, e^{2+3 \, t} - 45 \, e^2 \, t + 9 \, b \, e^{2+t} \, \text{ExpIntegralEi}[4] - 8 \, b \, e^2 \, \text{ExpIntegralEi}[6] - 6 \, b \, e^2 \, t \, \text{ExpIntegralEi}[6] - 9 \, b \, e^{2+t} \, \text{ExpIntegralEi}[2 \, t] + 8 \, b \, e^2 \, \text{ExpIntegralEi}[3 \, t] + \\ & 6 \, b \, e^2 \, t \, \text{ExpIntegralEi}[3 \, t] - 4 \, b \, e^8 \, \text{Log}[2] - 9 \, b \, e^{6+t} \, \text{Log}[2] + 6 \, b \, e^8 \, t \, \text{Log}[2] + b \, e^{2+3 \, t} \, \text{Log}[t]) \right\} \right\} \\ & \text{In}[19] = & \text{yp}[t\_] := b * (\text{Exp}[3 * t] * \text{Log}[t]) \\ & \text{m[t\_]} := \text{Integrate[yp[t], t]} \\ & \text{m[t]} \end{aligned}$$

$$& \text{out}[24] = \\ & \text{b} \left( -\frac{1}{3} \, \text{ExpIntegralEi}[3 \, t] + \frac{1}{3} \, e^{3 \, t} \, \text{Log}[t] \right) \\ & \text{(*Por lo que la función m[x] no es constante,} \\ & \text{y no es posible integrar sin recurrir a ortos métodos, tal vez.*)} \end{aligned}$$