(*I.*)

(*Primer ejercicio*)

In[7]:=
```
A1 = {{3, 1, -2}, {-1, 2, 1}, {4, 1, -3}};
X[t_] = {x[t], y[t], z[t]};
S1 = X '[t] == A1.X[t];
```

In[10]:=
```
P1 = Det[A1 - λ * IdentityMatrix[3]];
Solve[P1 == 0, λ]
```

Out[11]=

$$\{\{\lambda \to -1\}, \{\lambda \to 1\}, \{\lambda \to 2\}\}$$

In[73]:=
```
Eigenvalues[A1];
```

In[78]:=
```
Eigenvectors[A1]
```

Out[78]=

{{1, 1, 1}, {7, -2, 13}, {1, 0, 1}}

In[22]:=
```
Simplify[DSolve[{S1, x[0] == -6, y[0] == 2, z[0] == -12}, {x, y, z}, t]]
```

Out[22]=

$$\left\{\left\{x \to \text{Function}\left[\{t\}, e^{-t}\left(-7 + e^{2t}\right)\right], y \to \text{Function}\left[\{t\}, 2\, e^{-t}\right], z \to \text{Function}\left[\{t\}, e^{-t}\left(-13 + e^{2t}\right)\right]\right\}\right\}$$

(*Segundo ejercicio*)

In[31]:=
```
LaplaceTransform[y[t] + y '[t] - Integrate[y[v] * Sin[t - v], {v, 0, t}] == -Sin[t], t, s]
```

Out[31]=

$$\text{LaplaceTransform}\left[-\int_0^t \text{Sin}[t - v] \times y[v]\, dv + y[t] + y'[t], t, s\right] == -\frac{1}{1 + s^2}$$

```
y[0] = 1;
Solve[-(LaplaceTransform[y[t], t, s] / s ^ 2) + s LaplaceTransform[y[t], t, s] - y[0]
== 1 / s ^ 2, LaplaceTransform[y[t], t, s]]
```

Out[27]=

$$\left\{\left\{\text{LaplaceTransform}[y[t], t, s] \to \frac{1}{-1 + s^3}\right\}\right\}$$

In[28]:=
```
InverseLaplaceTransform[1 / (s ^ 3 - 1), s, t]
```

Out[28]=

$$\frac{1}{3}\, e^{-t/2}\left(e^{3t/2} - \text{Cos}\left[\frac{\sqrt{3}\ t}{2}\right] - \sqrt{3}\ \text{Sin}\left[\frac{\sqrt{3}\ t}{2}\right]\right)$$

In[33]:=
```
LaplaceTransform[y '[t] - Integrate[(t - τ) * y[τ], {τ, 0, t}] == t, t, s]
```

Out[33]=

$$\text{LaplaceTransform}\left[-\int_0^t (t - \tau)\, y[\tau]\, d\tau + y'[t], t, s\right] == \frac{1}{s^2}$$

(*Tercer ejercicio*)

In[93]:=
```
InverseLaplaceTransform[(1 / (s ^ 2 + s + 4)) * (((1 - (2 * Exp[-2 * s])) / (s ^ 2)) + (2 / s)), s, t]
```

Out[93]=

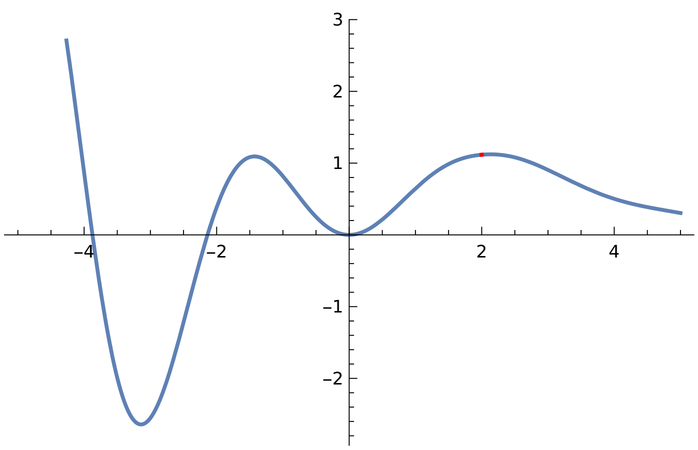
$$-\frac{1}{16}+\frac{t}{4}-2\,\text{HeavisideTheta}[-2+t]\left(-\frac{1}{16}+\frac{1}{4}(-2+t)+\frac{e^{\frac{2-t}{2}}\left(\sqrt{15}\,\text{Cos}\left[\frac{1}{2}\sqrt{15}\,(-2+t)\right]-7\,\text{Sin}\left[\frac{1}{2}\sqrt{15}\,(-2+t)\right]\right)}{16\sqrt{15}}\right)+\frac{e^{-t/2}\left(\sqrt{15}\,\text{Cos}\left[\frac{\sqrt{15}\,t}{2}\right]-7\,\text{Sin}\left[\frac{\sqrt{15}\,t}{2}\right]\right)}{16\sqrt{15}}+2\left(\frac{1}{4}-\frac{e^{-t/2}\left(\sqrt{15}\,\text{Cos}\left[\frac{\sqrt{15}\,t}{2}\right]+\text{Sin}\left[\frac{\sqrt{15}\,t}{2}\right]\right)}{4\sqrt{15}}\right)$$

In[95]:=

$$\text{Plot}\left[-\frac{1}{16}+\frac{t}{4}-2\,\text{HeavisideTheta}[-2+t]\left(-\frac{1}{16}+\frac{1}{4}(-2+t)+\frac{e^{\frac{2-t}{2}}\left(\sqrt{15}\,\text{Cos}\left[\frac{1}{2}\sqrt{15}\,(-2+t)\right]-7\,\text{Sin}\left[\frac{1}{2}\sqrt{15}\,(-2+t)\right]\right)}{16\sqrt{15}}\right)+\frac{e^{-t/2}\left(\sqrt{15}\,\text{Cos}\left[\frac{\sqrt{15}\,t}{2}\right]-7\,\text{Sin}\left[\frac{\sqrt{15}\,t}{2}\right]\right)}{16\sqrt{15}}+2\left(\frac{1}{4}-\frac{e^{-t/2}\left(\sqrt{15}\,\text{Cos}\left[\frac{\sqrt{15}\,t}{2}\right]+\text{Sin}\left[\frac{\sqrt{15}\,t}{2}\right]\right)}{4\sqrt{15}}\right),\right.$$

```
{t, -5, 5}, PlotStyle → Thick, ExclusionsStyle → Directive[Thick, Red]]
```

Out[95]=




```
(*II*)

(*Primer ejercicio*)
```

In[55]:=
```
A = {{3, 1, -2}, {-1, 2, 1}, {4, 1, -3}};
MatrixForm[A]
```

Out[56]//MatrixForm=

$$\begin{pmatrix} 3 & 1 & -2 \\ -1 & 2 & 1 \\ 4 & 1 & -3 \end{pmatrix}$$

```
In[61]:= M = λ * IdentityMatrix[3];

In[62]:= MatrixForm[M]
Out[62]//MatrixForm=
```

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

```
det = A - M;

In[65]:= MatrixForm[det]
Out[65]//MatrixForm=
```

$$\begin{pmatrix} 3-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 4 & 1 & -3-\lambda \end{pmatrix}$$

```
In[71]:= Solve[Simplify[((3 - λ) * (((2 - λ) * (-3 - λ)) - 1)) - ((3 + λ) - 4) - (2 * ((-1 - (4 (2 - λ)))))] == 0, λ]
Out[71]=
{{λ → -1}, {λ → 1}, {λ → 2}}

In[75]:= det1 = A - ((-1) * IdentityMatrix[3]);
MatrixForm[det1]
Out[76]//MatrixForm=
```

$$\begin{pmatrix} 4 & 1 & -2 \\ -1 & 3 & 1 \\ 4 & 1 & -2 \end{pmatrix}$$

```
RowReduce[det1] det1 = A - ((-1) * IdentityMatrix[3]);
Out[77]=
```

$$\left\{\left\{1, 0, -\frac{7}{13}\right\}, \left\{0, 1, \frac{2}{13}\right\}, \{0, 0, 0\}\right\}$$

```
In[71]:= Solve[Simplify[((3 - λ) * (((2 - λ) * (-3 - λ)) - 1)) - ((3 + λ) - 4) - (2 * ((-1 - (4 (2 - λ)))))] == 0, λ]
Out[71]=
{{λ → -1}, {λ → 1}, {λ → 2}}

In[82]:= det2 = A - ((1) * IdentityMatrix[3]);
MatrixForm[det2]
Out[83]//MatrixForm=
```

$$\begin{pmatrix} 2 & 1 & -2 \\ -1 & 1 & 1 \\ 4 & 1 & -4 \end{pmatrix}$$

```
In[84]:= RowReduce[det2]
Out[84]=
{{1, 0, -1}, {0, 1, 0}, {0, 0, 0}}
```

In[85]:=
```
det3 = A - ((2) * IdentityMatrix[3]);
MatrixForm[det3]
```

Out[86]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & -2 \\ -1 & 0 & 1 \\ 4 & 1 & -5 \end{pmatrix}$$

In[87]:=
```
RowReduce[det3]
```

Out[87]=
```
{{1, 0, -1}, {0, 1, -1}, {0, 0, 0}}
```

```
(*Tercer ejercicio*)
```

In[99]:=
```
y[0] = -1;
y'[0] = 0;
Solve[LaplaceTransform[
  y''[t] + (4 * y[t]) == 2 + (t * HeavisideTheta[-2 + t]) - 2 (-2 + t) HeavisideTheta[-2 + t],
  t, s], LaplaceTransform[y[t], t, s]]
```

Out[101]=
```
{}
```